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Integral Solutions for Transient Fluid Flow through Deformable Media

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Abstract

This paper presents an integral method for analyzing transient fluid flow through a deformable porous medium. Approximate analytical solutions have been obtained for one-dimensional linear and radial flow by the integral technique, in which the density of the fluid, and the porosity and permeability of the formation are treated as arbitrary functions of pressure. The integral solutions have been checked by comparison with exact solutions for the linear case, and with numerical simulation results for general non-linear problems; excellent agreement has been obtained.

In the study of transient flow of fluids through porous media, permeability of the formation has often been treated as a constant in order to avoid solving a non-linear problem. The present work shows that the assumption of a pressure-independent permeability may introduce significant errors for flow in deformable media. Application of the integral solutions to slightly compressible fluid flow in a horizontal fracture network is illustrated. The calculations show that neglect of changes in fracture permeability leads to large errors when injection pressure is high.
1. Introduction

The behavior of fluids in porous media is affected by coupled processes between fluid movement and rock deformation. Fluid flow through porous media is determined by the flow potential according to Darcy's law. When fluid flow occurs in porous media, the flow potential will change, and so will the pressure. The effective stress acting on the rock changes accordingly, and affects the deformation of the solids. The deformation of solids in turn changes the fluid flow channels and results in changes in the potential field.

In most theoretical studies of transient fluid flow in porous media, it has been assumed that the influence of the rock deformation on permeability is negligible, i.e., only fluid density and rock porosity are treated as functions of pressure. This assumption may be reasonable for slightly compressible fluid flow in purely porous media, such as sandstone, since the pore compressibility of sandstone is often small. However, even for flow in fractured media, the same assumption of constant permeability is often made. Neglect of effects of rock deformation on fluid mobility in fractures may introduce large errors, because the flow in fractures is very sensitive to changes in apertures, which will affect both permeability and porosity of the fracture system.

Raghavan et al. (1972) developed a numerical method for transient pressure response in well flow tests, which included effects of changes in rock and fluid properties with pressure. They defined a pseudopressure to represent the fluid and rock properties in the flow equation. Their model was studied analytically by Samaniego et al. (1977) after applying a linear approximation to the non-linear problem, and used for drawdown, buildup, injection, and falloff testing analysis. More recently, Pedrosa (1986) and Kikani et al. (1990) applied the perturbation analysis technique to look at pressure transient response in stress-sensitive formations, in which permeability was treated as a special kind of exponential pressure functions.

The effects of confining or overburden pressure on the permeability of porous media were studied experimentally by a number of authors (McLatchie, Hemstock and Young, 1957; Gary, Fatt and Bergamini, 1963; Gobran, Brigham and Ramey, 1987; Morrow and
Zhang, 1986; Fatt, 1958). The major conclusions of these studies are that the rock properties are dependent only on the effective stress, and that the relationship between rock properties and effective stress is history dependent. If external stress is kept constant, the absolute permeability of a porous medium can be expressed as a function of the difference between the confining pressure and the pore pressure. Some theoretical models have been proposed to relate rock permeability and confining pressure (Jennings, Carroll and Raile, 1981; Seeburger and Nur, 1984; Walsh and Brace, 1984).

The influence of fluid injection in a fractured porous medium was investigated numerically by Noorishad, Witherspoon and Maini (1972) using a finite element code. More recently, Noorishad and Tsang presented a numerical model (ROCMAS, 1987) for two-dimensional coupled flow and stress analysis in deformable, saturated, fractured rock media.

To the best of our knowledge, there are no analytical solutions available for the general coupled process of fluid flow and rock deformation without using linearization approximations, in particular for fluid flow in fractured reservoirs. This paper presents an analytical method for analyzing the non-linear coupled rock deformation and fluid flow problem. Approximate analytical solutions for one-dimensional linear and radial flow are obtained by an integral method, which is widely used in the study of steady and unsteady heat conduction problems (Özisik, 1980). The integral method as applied to heat transfer problems utilizes a parametric representation of the temperature profile by means of low-order polynomials, which is based on physical concepts such as a time-dependent penetration distance. An approximate solution to the heat transfer problem is then obtained from simple principles of continuity of temperature and heat flux, and energy conservation. This solution satisfies the governing partial differential equations only in an average, integral sense. However, the accuracy of integral solutions in heat transfer problems is generally acceptable for engineering applications. When applied to fluid flow problems in porous media, the integral method consists of assuming a pressure profile in the pressure-disturbance zone and determining the coefficients of the profile by making use of the integral mass balance equation.
The integral solutions obtained in this paper are very general, with fluid density and formation porosity and permeability arbitrary functions of pressure under isothermal conditions. The integral solutions are checked by comparison with the solutions for special linear cases when the exact solutions are available. Further, a numerical simulator is used to solve the general non-linear problem. It is found that the accuracy of the integral solutions for both linear and radial flow is surprisingly good when compared with the exact solution and with the numerical results for fluid flow through a semi-infinite system. We have found that the shape of the pressure profile for radial flow in a deformable medium can be quite different from temperature profiles typically recommended for heat conduction in radial flow systems (Lardner and Pohle, 1961). Analytical expressions for pressure profiles are proposed which result in very accurate results for transient fluid flow in a radial system.

The integral solutions are also applied to discuss the effects of fluid pressure upon fracture permeability during slightly-compressible fluid flow through a horizontal fracture system. The analytical results show that changes in fracture permeability due to changes in pressure can have a dominant influence on the flow field for high pressure injection. Neglect of effects of pressure on fracture permeability may introduce large errors in the flow behavior prediction.

The approximate integral solutions for transient fluid flow through a deformable permeable medium derived in this paper will find their applications in the following fields: i) to obtain physical insight into the phenomenon of coupled fluid flow and rock deformation; ii) to design and analyze well tests to determine formation and fluid properties; iii) to verify numerical simulators which include pressure-dependent fluid and formation properties.
2. Mathematical Formulation

To formulate the flow model, the basic assumptions used for fluid flow in porous media are as follows:

1) isothermal, isotropic and homogeneous formation;
2) single phase horizontal flow without gravity effects;
3) Darcy's law applies; and
4) physical properties of fluid and rock are purely elastic, depending only on stress (no hysteresis).

The governing equation derived by combining the mass conservation law, Darcy’s law and equations of state of the fluid and rock, is

\[ \nabla \cdot [ \rho \mathbf{u} ] = -\frac{\partial}{\partial t} [ \phi P ] \]  

where the volumetric flux \( \mathbf{u} \) is described by Darcy' law as

\[ \mathbf{u} = -\frac{k}{\mu} \nabla P \]  

and where \( k, \rho, \) and \( \mu \) are formation permeability, fluid density, and viscosity (constant), respectively; \( P \) is fluid phase pressure.

It has been shown (Terzaghi, 1943; Fatt, 1958; Gobran et al, 1987; Narasimhan, 1985) that hydrologic properties are functions only of the effective stress, defined as

\[ \sigma' = \sigma - \alpha P \]  

where \( \alpha \) is a parameter which depends on the mechanical properties of the rock and the geometry of the rock grains. The parametr is determined experimentally, and is usually found to be near 1. For a particular reservoir, the external stress \( \sigma \) is essentially a constant, depending on the overburden weight of the formation. Therefore, the effective stress \( \sigma' \) is a function of the pore-pressure only. We assume the following constitutive relations for fluid and rock:

\[ \rho = \rho(P) \]
\[ \phi = \phi(P) \]  
and
\[ k = k(P) \]
which are often called equations of state for fluid and rock.

Isothermal compressibilities of fluid and rock pore space are defined as follows:
for fluid
\[ C_r = \frac{1}{\rho} \left[ \frac{\partial \rho}{\partial P} \right]_T = -\frac{1}{V} \left[ \frac{\partial V}{\partial P} \right]_T \]  
and for rock pores
\[ C_r = \frac{1}{\phi} \left[ \frac{\partial \phi}{\partial P} \right]_T \]
where \( V \) is volume of fluid, and \( T \) is reservoir temperature. The compressibilities \( C_r \) and \( C_r \) may or may not be constants.

Introducing Equations (2), and (4)-(6) into (1), we have the flow equation
\[ \nabla \cdot \left[ \rho(P) k(P) \nabla P \right] = \frac{\partial}{\partial t} \left[ \phi(P) \rho(P) \right] \]  
By using Equations (7) and (8), another form of the flow equation can be obtained:
\[ \nabla \cdot \left[ \rho(P) \frac{k(P)}{\mu} \nabla P \right] = C_t \rho \phi \frac{\partial P}{\partial t} \]  
where
\[ C_t = C_f + C_r \]
is the total compressibility. Again, \( C_t \) is not necessarily a constant. Equations (9) and (10) are general non-linear equations and will be solved directly using the integral method with appropriate boundary and initial conditions in the following two sections.
3. Integral Solution for One-Dimensional Linear Flow

The integral method, which has been widely used in the heat transfer literature (Özisik, 1980), is applied here to obtain an analytical solution for the non-linear coupled fluid flow and rock deformation problem. The flow system of interest is a semi-infinite linear reservoir with a constant cross-sectional area \( A \), initially fully saturated with a single phase fluid. The same fluid is injected (or produced) at a given constant mass rate \( q_m \).

Then the problem to be solved from Equation (9) is as follows:

\[
\frac{\partial}{\partial x} \left[ \frac{\rho(P)k(P)}{\mu} \frac{\partial P}{\partial x} \right] = \frac{\partial}{\partial t} [\phi(P)\rho(P)]
\]

The initial condition is

\[
P(x, t=0) = P_i \quad \text{(constant)}
\]

The boundary conditions are as follows:

\[
-A \left[ \frac{k(P)\rho(P)}{\mu} \frac{\partial P}{\partial x} \right]_{x=0} = q_m
\]

\[
\lim_{x \to \infty} P(x, t) = P_i \quad \text{(constant)}
\]

The integral solution for the pressure profile in the pressure penetration zone is given by (see Appendix A),

\[
P(x, t) = P_i + \frac{\delta(t)}{3} \left[ \frac{q_m\mu}{A\rho(P_0)k(P_0)} \right] \left[ 1 - \frac{x}{\delta(t)} \right]^3
\]

where the pressure penetration distance, \( \delta(t) \), and the injection pressure, \( P_0 = P_0(t) \), are treated as unknowns, to be determined by the two following equations,

\[
\int_0^{\delta(t)} A\rho(P)\phi(P)dx = A\rho P_i \delta(t) + q_m t
\]

and

\[
P_0 = P_i + \frac{\delta(t)}{3} \left[ \frac{q_m\mu}{A\rho(P_0)k(P_0)} \right]
\]
Simultaneous solution of Equations (17) and (18) will give the two unknowns $P_0(t)$ and $\delta(t)$, and substituting $P_0(t)$ and $\delta(t)$ into Equation (16) yields the final solution of pressure distribution for the problem. It should be mentioned that the permeability function appears only at the inlet, as $k = k(P_0)$ in the integral solution (16).

Equation (17) is simply a mass balance equation for the fluid in the pressure penetration region of the system, namely

\[
\text{mass in disturbed zone} = \text{initial mass} + \text{mass injected} \quad (19).
\]

The "slightly compressible" fluid flow can be treated as a special case of the above solution. For small and constant compressibility, we can solve for the pressure penetration distance $\delta(t)$ (see Appendix A) as,

\[
\delta(t) = \left[\frac{12\rho(P_0)k(P_0)t}{\rho\phi C\mu}\right]^{1/2} \quad (20)
\]

Introducing Equation (20) into (18), we will have only one equation for one unknown $P_0(t)$. Solving $P_0(t)$ from the resulting equation for time $t$ and substituting it back into Equation (20), the penetration distance $\delta(t)$ is obtained. Then using the $P_0(t)$ and $\delta(t)$ in Equation (16), a final solution for the pressure profile will be obtained for the slightly compressible system.

4. Integral Solution for One-Dimensional Radial Flow

The problem considered is fluid injection into a fully penetrating well in an infinite horizontal reservoir of constant thickness, and the formation is initially saturated with the same fluid. The governing equation (9) can be expressed in a radial coordinate system as

\[
\frac{1}{r} \frac{\partial}{\partial r} \left[ \frac{\rho(P)k(P)}{\mu} r \frac{\partial P}{\partial r} \right] = \frac{\partial}{\partial t} \left[ \rho(P)\phi(P) \right] \quad (21)
\]

The initial condition is
At the inner boundary at the well bore, $r = r_w$, the fluid is injected at a given mass injection rate $q_m$, i.e.

$$\frac{2\pi r_w h}{\mu} \left[ \rho(P)k(P) \frac{\partial P}{\partial r} \right]_{r=r_w} = q_m$$

(23)

Using three different functional forms for $P(r, t)$, the integral solutions for radial flow under a given mass injection rate $q_m$ are obtained in three alternative forms which are derived in Appendix B.

(i) The temperature profile recommended for radial heat conduction (Lardner and Pohle, 1961; Özisik, 1980) is

$$T(r, t) = \left[ P_n(r) \right] \ln(r)$$

(24)

where $P_n(r)$ is a nth-degree polynomial in $r$. Using a pressure trial function of the form (24), we have

$$P(r, t) = P_i - \frac{q_m \mu}{2 \pi h} \frac{1}{\rho(P_0)k(P_0)} \left[ 1 + \eta - r_D \right] \frac{\ln \left( \frac{r_D}{1+\eta} \right)}{\eta \ln(1+\eta)}$$

(25a)

for a first-degree polynomial $P_1(r)$, where $\eta = \delta(t)/r_w$, and $r_D = r/r_w$, and

$$P(r, t) = P_i - \frac{q_m \mu}{2 \pi h} \frac{1}{\rho(P_0)k(P_0)\eta} \left[ 1 + \eta - r_D \right]^2 \frac{\ln \left( \frac{r_D}{1+\eta} \right)}{\eta + 2 \ln(1+\eta)}$$

(25b)

for a second-degree polynomial $P_2(r)$.

(ii) From the Theis solution for radial flow with constant permeability, we know that the pressure at a given injection time is distributed as a logarithm in $(t/r^2)$. Thus for $t_D > 100$, the line-source Theis solution is simplified to:

$$P(r, t) - P_i = \frac{q_{inj}}{4\pi kh} \left[ \ln \frac{kt}{\phi \mu C_r^2} + .80907 \right]$$

(26)
which is very accurate except near the pressure penetration front. This suggests to look for a pressure profile in the deformable medium case in the form

\[ P(r, t) - P_i = \text{constant} \times \ln(P_n(r)) \]  

(27)

Using this profile, we find (Appendix B)

\[ P(r, t) = P_i - \frac{q_m \mu}{2 \pi \eta} \frac{1}{\rho(P_0)k(P_0)} \left[ 1 + \frac{1}{2 \eta} \right] \ln \left[ \left( \frac{2 \eta}{1+\eta} \right) - \left( \frac{r_D}{1+\eta} \right)^2 \right] \]  

(28)

(iii) In general, the pressure profile may deviate from a logarithmic distribution when dealing with a fluid flow problem in a deformable porous medium. It is instructive to try a pressure profile of the form:

\[ P(r, t) - P_i = [P_n(r)] \ln[Q_n(r)] \]  

(29)

where \( Q_n(r) \) is also a polynomial in \( r \). Then, we obtain the following integral solution (Appendix B),

\[ P(r, t) = P_i - \frac{q_m \mu}{2 \pi \eta} \frac{1}{\rho(P_0)k(P_0)} \left[ 1 + \eta - r_D \right] \ln \left[ \left( \frac{2 \eta}{1+\eta} \right) - \left( \frac{r_D}{1+\eta} \right)^2 \right] \]  

(30)

These expressions apply for \( r \leq r_w + \delta(t) \), while \( P(r, t) = P_i \) for \( r > r_w + \delta(t) \). The unknowns, \( P_0 = P(r_w, t) \), the wellbore pressure, and \( \delta(t) \), the pressure penetration distance, are determined by using any one of the Equations (25), (28), or (30) for \( r = r_w \), together with the following mass balance equation in the pressure disturbance region:

\[ \int_{r_w}^{r_w + \delta(t)} 2 \pi r \rho(P) \phi(P) \, dr = q_m t + \pi r_w ^2 \phi \left[ (r_w + \delta(t))^2 - r_w^2 \right] \]  

(31)

The applicability and accuracy of the three solutions, given by Equations (25), (28), and (30), will be discussed in the next section.
For slightly compressible fluid flow, we obtain the following explicit expressions of the integral mass balance equation for the different pressure profiles of Equations (25a), (28), or (30):

\[
q_m + \frac{r_w^2 \rho \phi_i C_{ml} q_m}{\rho(P_0) k(P_0) [\eta + \ln(1+\eta)]} \left[ \frac{5}{36} (1+\eta)^3 + \frac{1}{4} \eta^2 + \frac{5}{36} (\frac{1}{2} - \eta + \frac{1}{6}) \ln(1+\eta) \right] = 0 \quad (32)
\]

for the pressure profile Equation (25a),

\[
q_m + \frac{r_w^2 \rho \phi_i C_{ml} q_m}{\rho(P_0) k(P_0)} \left[ \frac{1+2\eta}{2\eta} \right] \left[ \frac{3}{2} (1+\eta)^2 + \frac{1}{2} + 4(1+\eta) \ln(1+\eta) \right] - \frac{1}{2} [1 - 4(1+\eta)^2] \ln \left[ \frac{1+2\eta}{(1+\eta)^2} \right] = 0 \quad (33)
\]

for the pressure profile Equation (28), and

\[
q_m + \frac{r_w^2 \rho \phi_i C_{ml} q_m}{\rho(P_0) k(P_0)} \left[ \frac{2\eta^2}{1+2\eta} - \ln \left[ \frac{1+2\eta}{(1+\eta)^2} \right] \right] \left[ \frac{7}{18} (1+\eta)^3 - \frac{1}{3} (1+\eta)^2 + \frac{1}{6} (1+\eta) - \frac{2}{9} \right] - \frac{2}{3} (1+\eta)^3 \ln(1+\eta) + \left[ \frac{2}{3} (1+\eta)^3 - \frac{1}{2} (1+\eta) + \frac{1}{3} \right] \ln \left[ \frac{1+2\eta}{(1+\eta)^2} \right] = 0 \quad (34)
\]

for the pressure profile Equation (30). Solving any pair of Equations (32)-(34) and (25a), (28), or (30) with \( r = r_w \) simultaneously for \( \delta(t) \) and \( P_0(t) \) and then substituting into (25a), (28), or (30) gives the final solution for the corresponding compressible system.

5. Discussion on Accuracy of Integral Solutions

The solutions from the integral method are approximate, and their accuracy needs to be confirmed by comparison with an exact solution or with numerical results. In this section, the integral solutions obtained in Sections 3 and 4 are checked by comparison with
exact solutions and numerical calculations for the one-dimensional linear and radial flow of slightly compressible fluid through a horizontal formation. The numerical code used is a modified version of MULKOM-GWF (Pruess and Wu, 1988). This is a fully implicit integral finite difference code for three-phase flow of gas, water and foam, which belongs to the "MULKOM" family developed by Pruess (1988).

The accuracy of integral solutions depends on the choice of pressure profiles, and on the nature of permeability dependence upon pressure as well, among other variables. We consider that pressure changes cause changes in porosity, which in turn affects permeability. For the permeability-porosity relationship, we use two alternative empirical models. One is a resistivity and pore shape model (Brace, 1977), which relates permeability to electrical resistivity by

\[ k = (m^2/k_0)F^{-2}\phi^{-1} \]  

(35)

where \( m \) is the hydraulic radius, the volume of the interconnected pores divided by their surface areas; \( k_0 \) a shape factor, \( 2 \leq k_0 \leq 3 \); \( F \) is the formation factor, the ratio of the resistivity of fluid-saturated rock to the resistivity of fluid alone, and

\[ F = \phi^{-\beta} \]  

(36)

where \( \beta \) is a constant close to 2. Thus, the permeability-porosity relationship is

\[ k = (m^2/k_0)\phi^{2\beta-1} \]  

(37)

We also consider a pore-geometry model often used in soil mechanics (Narasimhan, 1985),

\[ k = k_i \exp \left( \frac{2.303(\phi/(1-\phi) - \phi/(1-\phi_i))}{c_k} \right) \]  

(38)

where \( c_k \) is the slope of the void ratio \([ \phi/(1-\phi) ]\) plotted against \( \log (k) \). Figure 1 shows the relationships between the normalized permeability and the normalized porosity from these two models.
5.1 Check on Linear Flow Integral Solution

a) Comparison with Exact Solution

The exact solution of linear flow of a slightly compressible fluid in a semi-infinite system with constant permeability and constant injection rate at inlet \(x=0\), is given by (Carslaw and Jaeger, 1959):

\[
P(x, t) = P_i + \frac{q_{\text{inj}} t}{k A} \left[ \frac{1}{\sqrt{\pi \xi}} e^{-\xi^2} - \text{erfc}(\xi) \right]
\]  

(39)

where \(q_{\text{inj}}\) is the volumetric injection rate, and \(\xi\) is defined as

\[
\xi = \frac{x}{2 \sqrt{\frac{k t}{\phi_i C_w \mu}}}
\]  

(40)

The parameters as shown in Table 1 are used to evaluate both the exact solution (39) and the integral solution (16)-(18). A comparison of injection pressures calculated from the integral and the exact solutions is shown in Figure 2. The agreement between the two solutions is excellent for the entire transient period.

b) Comparison with Numerical Solution

The above example is simple, because we are dealing with a linear governing equation and the exact solutions exist. But for the case of a pressure-dependent permeability the governing equations are non-linear, and we no longer have exact solutions. Therefore, a numerical method is used to examine the integral solutions found in this work. The numerical code MULKOM-GWF has been modified by implementing the permeability functions (37) and (38) to take into account the effects of pressure on formation permeability. The parameters of fluids and rock used for this numerical simulation are
given in Table 1 and on the figures.

A comparison between the injection pressures at the inlet computed from the integral solution and the numerical model are shown in Figures 3 and 4, for the permeability functions of Equations (37) and (38), respectively. It is obvious that the integral solution matches the numerical results very well for the entire injection period, while the constant-permeability calculations lead to large errors as injection pressure increases. The integral solutions are always expected to introduce some error; however, comparisons in Figures 3 and 4 show that the integral solution for one-dimensional linear flow is excellent for engineering applications.

5.2 Check on Radial Flow Integral Solution

a) Comparison with Exact Solution

For an infinite-acting radial system with a constant permeability, the exact (Theis) solution for slightly compressible fluid flow is (Earlougher, 1977):

\[ P(r, t) = P_i + \frac{q_{inj} \mu}{4 \pi k \nu h} \left[ -\text{Ei} \left( -\frac{1}{4 t_D} \right) \right] \]  

where \( q_{inj} \) is the volumetric injection rate, a constant, and \( t_D \) is the dimensionless time,

\[ t_D = \frac{k \nu t}{\phi \mu C_r \nu} \]  

If we make use of Equation (24) to represent the pressure profile for radial fluid flow in porous media with a constant permeability, the results are shown in Figure 5, in which first-degree and second-degree polynomials in \( r \) are used, as given by Equation (25). The parameters of computation for comparing with the exact solution are given in Table 1. Figure 5 indicates that for a constant permeability case, the integral solution overestimates the pressure buildup due to injection by about 5-10%.
An example of the integral solution using Equation (27) as a pressure profile, with $P_n(r)$ being a second-degree polynomial, is given by Equation (28). A comparison of the Theis solution and the integral solution, Equation (28), is presented in Figure 6. Essentially, no differences can be observed in the wellbore injection pressure calculations from the two solutions.

Figure 7 shows the results from Equation (30), which also gives a very good approximation to the linear problem, when compared with the exact solution. However, about 1-2% errors are introduced by this integral solution.

The total mass of fluid in the pressure penetration zone is always overestimated in the integral method because of the assumption of a finite pressure-penetration distance beyond which no fluid can migrate. The amount of fluid across the front is negligible for one-dimensional linear flow due to very small pressure gradients there compared with flux near the inlet. However, for radial flow, the increasing cross-sectional area, which is proportional to the radial distance $r$, may make the fluid mass across the front no longer negligible even with the very small pressure gradients. We have examined this effect by assuming that beyond the pressure penetration front, pressure and pressure gradient are described by the Theis solution. The total fluid mass across the pressure penetration front was calculated by integrating from $r = r_w + \delta(t)$ to $r = \infty$ using the Theis solution. It was found that a very small amount (less than .01%) of the injected fluid crosses the front, which can be neglected. Therefore, the assumptions Equations (B-1) and (B-2) used to define the pressure penetration front are reasonable to derive the radial flow integral solutions.

b) Comparison with Numerical Solution

Brace's permeability model (35) is used to examine the integral solution for the radial flow case. The input parameters are also given in Table 1. The calculations of the numerical, Theis and three integral solutions, are shown in Figure 8. The integral solution, Equation (25a) with a pressure profile like (24), gives the best approximation to the
problem, while both Theis and the other two integral solutions result in larger errors. It is very interesting to note that the integral solution (25) is poorest of the three forms of the integral solution for the constant permeability calculations. Obviously, the pressure profile for flow in a deformable medium deviates from the logarithmic distribution due to changes in permeability, and a pressure profile such as Equation (24), represents the physics best. A comparison of the different solutions for an order of magnitude smaller compressibility, $C_r = 5 \times 10^{-10} \text{ Pa}^{-1}$, is given in Figure 9. In this case the most accurate results are from the integral solution (30), with a pressure profile described by Equation (29). The integral solution (28) is here is also better than Equation (25a).

From many additional comparisons (not reported here) of the radial integral solution with both the Theis solution and numerical simulations, we have found that the pressure profile of Equation (24) should be used in the integral solution in order to include effects of significant changes in permeability. The integral solution (28) gives better accuracy if the medium is close to rigid, while for intermediate deformity of the medium, in general, Equation (30) gives the most accurate representation.

6. Flow through a Deformable Horizontal Fracture

Fluid flow through fractured media is of fundamental importance in many problems relating to energy recovery from the subsurface reservoirs and to nuclear waste disposal in geologic media. A number of physical models for fractures have been proposed to study transport phenomena in fractured media, and considerable progress has been made in understanding flow behavior of fluids through fractures since the 1950's. The simplest model is a network of parallel horizontal fractures, with constant spacings and initial apertures (see Figure 10). This fracture model and the integral solutions obtained in this paper are used to examine the effects of coupled stress and fluid flow through a horizontal fracture system. The parameters of fluid and rock for this study are shown in Table 2. The formation is assumed to be subject to vertical uniaxial stress. Then, the aperture $b$ is
given by

\[ b = b_i + \frac{2D}{E} (P - P_i) \]  

(43)

where \( b_i \) is the initial aperture, \( D \) is the half spacing between fractures, and \( E \) is Young's modulus of the intact rock. Fracture permeability is described by the cubic law (Wither­spoon et al., 1980) as,

\[ k_f = \frac{b_i^2}{12} = \frac{b_i^2}{12} \left[ 1 + \frac{2D}{Eb_i} (P - P_i) \right]^2 \]  

(44)

The effective permeability in the continuum sense, as used in Darcy's law, is then,

\[ k_{eff} = k_f \frac{b}{2D} = \frac{1}{3} \phi^3 \]  

(45)

where \( \phi \) is the porosity of the fracture system, given by

\[ \phi = \frac{b}{2D} = \frac{b_i}{2D} \left[ 1 + \frac{2D}{Eb_i} (P - P_i) \right] \]  

(46)

Note that the cubic dependence of permeability on porosity in (45) is identical to Brace’s model (37) for \( \beta = 2 \). Since the system is uniform and symmetric, only one basic section needs to be considered, as shown in Figure 10b.

A comparison of the injection pressures in a linear horizontal fracture system, for a constant and a pressure-dependent permeability, is given in Figure 11, for a constant mass injection rate condition at the inlet \( x=0 \). Here, the integral solution (25a) is used. Figure 11 shows that as the injection pressure gets higher the injection pressure would be overestimated if fracture permeability were taken as a constant.

Figure 12 shows the differences in injection pressures from constant and pressure-dependent fracture permeability solutions at the wellbore for a radial flow problem with a large mass injection rate. The constant-permeability solution always overestimates the pressure response at the wellbore as injection pressure reaches a very high value.

Since the parameters used in these calculations are reasonable for actual fluid flow through fractures, we conclude that neglect of permeability dependence on pressure will
lead to large errors to fluid flow through fractured media for a high injection pressure operation.

7. Conclusions

1. The integral method as commonly used for heat conduction analysis has been applied to study fluid flow through deformable porous media. The approximate analytical solutions for one-dimensional linear and radial flow in a semi-infinite system at a specified injection rate are obtained by the integral technique. The solutions are valid for a general non-linear governing flow equation with arbitrary constitutive correlations of permeability, porosity and fluid density as functions of pore pressure.

2. More suitable pressure profiles are proposed for integral solutions of radial flow through a deformable porous medium. Two published permeability models are used to examine the accuracy of the integral solutions by comparison with the exact solution and the numerical simulations for fluid flow through deformable porous media. Excellent agreement has been obtained for both linear and radial flow solutions. The integral solutions obtained from this paper are confirmed to give accurate results for engineering applications.

3. The effects of pressure on permeability are discussed by integral solutions for one-dimensional single-phase, slightly compressible linear and radial flow through a horizontal fracture. The results show that for flow in fractured media, ignoring pressure dependence of permeability may lead to large errors in flow behavior prediction under high-pressure operations.

4. The analytical solutions provided in this paper for the coupled fluid flow and rock deformation problem will find their applications in the following three fields: i) to obtain physical insight into flow phenomena in a deformable porous medium; ii) to determine some fluid and formation properties by well test analysis methods; iii) to verify
numerical codes that include effects of pressure-dependent fluid and rock properties.

Acknowledgement

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Nomenclature

\begin{itemize}
\item $A$: cross-sectional area \quad (m^2)
\item $b$: aperture of fracture \quad (m)
\item $b_i$: initial aperture of fracture \quad (m)
\item $C_f$: fluid compressibility \quad (Pa^{-1})
\item $c_k$: slope of $\phi/(1-\phi)$ versus log($k$) straight line
\item $C_r$: rock compressibility \quad (Pa^{-1})
\item $C_t$: total compressibility \quad (Pa^{-1})
\item $D$: half spacing between fractures \quad (m)
\item $E$: Young's modulus
\item $F$: formation factor of Brace's permeability model
\item $h$: thickness of formation \quad (m)
\item $k$: absolute permeability \quad (m^2)
\item $k_{eff}$: effective permeability, in Equation (45) \quad (m^2)
\item $k_f$: fracture permeability \quad (m^2)
\item $k_i$: coefficient of pore-geometry model \quad (m^2)
\end{itemize}
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k_0)</td>
<td>shape factor of Brace's permeability model</td>
<td></td>
</tr>
<tr>
<td>(m)</td>
<td>hydraulic radius</td>
<td>(m)</td>
</tr>
<tr>
<td>(P)</td>
<td>pressure</td>
<td>(Pa)</td>
</tr>
<tr>
<td>(P_0)</td>
<td>pressure at inlet, (x=0, r=r_w)</td>
<td>(Pa)</td>
</tr>
<tr>
<td>(P_i)</td>
<td>initial formation pressure</td>
<td>(Pa)</td>
</tr>
<tr>
<td>(P_{inj})</td>
<td>injection pressure</td>
<td>(Pa)</td>
</tr>
<tr>
<td>(P_n(r))</td>
<td>nth-degree polynomial in (r)</td>
<td></td>
</tr>
<tr>
<td>(P_T)</td>
<td>pressure function from Theis solution</td>
<td>(Pa)</td>
</tr>
<tr>
<td>(P^*)</td>
<td>pressure difference, in Equation (C-3)</td>
<td></td>
</tr>
<tr>
<td>(q_{inj})</td>
<td>mass injection rate</td>
<td>(kg/s)</td>
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<tr>
<td>(Q_n(r))</td>
<td>polynomial in (r)</td>
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<tr>
<td>(q_{inj})</td>
<td>volumetric injection rate</td>
<td>(m³/s)</td>
</tr>
<tr>
<td>(x)</td>
<td>distance to inlet</td>
<td>(m)</td>
</tr>
<tr>
<td>(r)</td>
<td>radial distance</td>
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</tr>
<tr>
<td>(r_w)</td>
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</tr>
<tr>
<td>(t)</td>
<td>time</td>
<td>(s)</td>
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<td>(T)</td>
<td>reservoir temperature</td>
<td>(°C)</td>
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<td>(t_D)</td>
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<tr>
<td>(\bar{u})</td>
<td>Darcy's velocity</td>
<td>(m/s)</td>
</tr>
<tr>
<td>(V)</td>
<td>volume of fluid</td>
<td>(m³)</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>constant, Equation (3)</td>
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<tr>
<td>(\beta)</td>
<td>exponential of Brace's permeability model</td>
<td></td>
</tr>
<tr>
<td>(\delta(t))</td>
<td>pressure penetration depth</td>
<td>(m)</td>
</tr>
<tr>
<td>(\theta(t))</td>
<td>defined in Equation (A-9)</td>
<td></td>
</tr>
<tr>
<td>(\mu)</td>
<td>fluid viscosity</td>
<td>(Pa·s)</td>
</tr>
<tr>
<td>(\xi)</td>
<td>defined in Equation (40)</td>
<td></td>
</tr>
<tr>
<td>(\rho)</td>
<td>fluid density</td>
<td>(kg/m³)</td>
</tr>
<tr>
<td>(\rho_i)</td>
<td>initial fluid density</td>
<td>(kg/m³)</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>stress</td>
<td>(Pa)</td>
</tr>
<tr>
<td>(\sigma')</td>
<td>effective stress</td>
<td>(Pa)</td>
</tr>
</tbody>
</table>
Φ defined in Equation (B-17)
φ porosity
φ_i initial formation porosity

References


Appendix A. Derivation of Integral Solution for Linear Flow

We define a pressure penetration (disturbance) distance \( \delta(t) \), in analogy to the thermal layer thickness in a heat conduction problem (Özisik, 1980), such that,

\[
\delta(t) = 0 \quad \text{at} \quad t = 0
\]  

(A-1)

and

\[
P(x \geq \delta(t), t) = P_i
\]

(A-2)

Similarly to the definition of a thermal layer, we postulate

\[
\frac{\partial P}{\partial x} \bigg|_{x=\delta(t)} = 0
\]

(A-3)

Ahead of the penetration front, the system is assumed undisturbed by injection (or production) and it remains at initial equilibrium. Another constraint on \( \delta(t) \) can be derived by assuming that Equation (10) is satisfied at the front,

\[
\frac{\partial}{\partial x} \left[ \frac{k(P)\rho(P)}{\mu} \right] \frac{\partial P}{\partial x} + \frac{k(P)\rho(P)}{\mu} \frac{\partial^2 P}{\partial x^2} = C_f \phi(P) \rho(P) \frac{\partial P}{\partial t}
\]

(A-4)

Substituting Equation (A-2)-(A-3) into (A-4), and assuming that \( \partial P/\partial t \) is continuous across the front, we will have

\[
\left. \frac{\partial^2 P}{\partial x^2} \right|_{x=\delta(t)} = 0
\]

(A-5)

Assume that the pressure profile in the pressure penetration zone be given by

\[
P(x, t) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \quad [0 \leq x \leq \delta(t)]
\]

(A-6)

The coefficients \( a_0, a_1, a_2, \) and \( a_3 \) depend on time and can be determined from Equations (14), (A-2), (A-3), and (A-5) to yield Equation (16).

The only unknowns in Equation (16) are \( \delta(t) \) and \( P_0(t) \), to be determined as follows.
Write

\[ \theta(t) = \int_{0}^{\delta(t)} A \rho(P) \phi(P) \, dx \]  

(A-7)

which is the total fluid mass from \( x = 0 \) to \( x = \delta(t) \) within the pressure disturbance zone.

Multiplying Equation (12) on both sides by \( A \) and integrating it with respect to \( x \) from \( x = 0 \) to \( x = \delta(t) \) yields

\[
\left[ A \frac{\rho(P)k(P)}{\mu} \frac{\partial P}{\partial x} \right]_{x=\delta(t)} - \left[ A \frac{\rho(P)k(P)}{\mu} \frac{\partial P}{\partial x} \right]_{x=0} = \int_{0}^{\delta(t)} A \frac{\partial}{\partial t} [\rho(P)\phi(P)] \, dx = \frac{d}{dt} \left[ \theta - A \delta(t) \rho_i \phi_i \right] 
\]

(A-8)

where

\( \rho_i = \rho(P_i) \) (constant)

and

\( \phi_i = \phi(P_i) \) (constant)

which are initial fluid density and formation porosity, respectively.

Introducing the boundary conditions (14) and (A-3) into (A-8), we have,

\[
q_m = \frac{d}{dt} [\theta - A \rho_i \phi_i \delta(t)] 
\]

(A-9)

Integrating Equation (A-9) with respect to \( t \) from \( t = 0 \) to \( t = t \), we obtain,

\[
\int_{0}^{t} q_m \, dt = \theta - A \rho_i \phi_i \delta(t) + C_x 
\]

(A-10)

Since at \( t = 0 \) the term on the left hand side is zero, and both \( \theta \) and \( \delta(t) \) are zero by their definition, the integration constant in (A-10) is shown to be

\[ C_x = 0 \]  

(A-11)

Then, we have the integral Equation (17).
For "slightly compressible" rock and fluid, compressibilities $C_f$ and $C_r$ are constants and very small, such that

$$\rho(P) = \rho_0 e^{C_f(P-P_i)} = \rho_0[1 + C_f(P - P_i)] \quad (A-12)$$

and

$$\phi(P) = \phi_0 e^{C_r(P-P_i)} = \phi_0[1 + C_r(P - P_i)] \quad (A-13)$$

then,

$$\rho(P)\phi(P) = \rho_0\phi_0[1 + C_f(P - P_i)][1 + C_r(P - P_i)]$$

$$= \rho_0\phi_0[1 + C_t(P - P_i)] \quad (A-14)$$

where $C_t = C_f + C_r$ is total compressibility. Now, we have

$$\theta = \int_0^\delta(t) \int_a^b \rho'(P)\phi(P)dx = \int_0^\delta(t) [1 + C_t(P - P_i)]dx$$

$$= A\rho_0\phi_0 \delta(t) + \frac{\rho_0\phi_0 C_t \mu}{12} \frac{q_m \delta^2(t)}{\rho(P_0)k(P_0)} \quad (A-15)$$

Substituting Equation (A-15) into (17) gives

$$A\rho_0\phi_0 \delta(t) + \frac{\rho_0\phi_0 C_t \mu q_m \delta^2(t)}{12 \rho(P_0)k(P)}$$

$$= A\rho_0\phi_0 \delta(t) + q_m \delta(t) \quad (A-16)$$

Then, the only physically meaningful root for $\delta(t)$ in Equation (A-16) can be obtained as in Equation (20).
Appendix B. Derivation of Solution for Radial Flow

The pressure penetration distance is defined as \( r = r_w + \delta(t) \), beyond which,

\[
P(r \geq r_w + \delta(t), t) = P_i
\]  \hspace{1cm} (B-1)

and the assumption of continuous derivatives of pressure on the front gives,

\[
\left. \frac{\partial P}{\partial r} \right|_{r=r_w+\delta(t)} = 0
\]  \hspace{1cm} (B-2)

Define

\[
P^*(r, t) = P(r, t) - P_i
\]  \hspace{1cm} (B-3)

Then, Equation (21) becomes

\[
\frac{1}{r} \frac{\partial}{\partial r} \left[ \frac{P(P)k(P)}{\mu} r \frac{\partial P^*}{\partial r} \right] = \frac{\partial}{\partial t} [\rho(P)\phi(P)]
\]  \hspace{1cm} (B-4)

The initial, boundary and constraint conditions now become,

\[
P^*(r, t=0) = 0
\]  \hspace{1cm} (B-5)

\[
P^*(r \geq r_w + \delta(t), t) = 0
\]  \hspace{1cm} (B-6)

\[
\left. \frac{\partial P^*}{\partial r} \right|_{r=r_w+\delta(t)} = 0
\]  \hspace{1cm} (B-7)

and a known mass injection rate is specified at the wellbore,

\[
\frac{2\pi r_w h}{\mu} \rho(P_0)k(P_0) \left. \frac{\partial P^*}{\partial r} \right|_{r=r_w} = -q_m
\]  \hspace{1cm} (B-8)

Three different pressure profiles are used in this study to describe the pressure distribution within the pressure disturbance zone. The first pressure profile, in analogy to Equation (24) used for heat conduction problems (Lardner and Pohle, 1961; Özisik, 1980), is given by
\[ P^*(r, t) = (b_0 + b_1 r) \ln(b_2 r) \quad (r_w \leq r \leq r_w + \delta(t)) \]  
(B-9a)

with \( P_1(r) \) being a first-degree polynomial, and

\[ P^*(r, t) = (b_0 + b_1 r + b_2 r^2) \ln \left[ \frac{r}{r_w + \delta(t)} \right] \quad (r_w \leq r \leq r_w + \delta(t)) \]  
(B-9b)

with \( P_2(r) \) being a second-degree polynomial. These result in Equation (25) after making use of the constraint conditions (B-6)-(B-8).

Equation (27) is used as the second profile,

\[ P^*(r, t) = C \ln(A r + B r^2) \quad (r_w \leq r \leq r_w + \delta(t)) \]  
(B-10)

Then, by satisfying the constraint conditions (B-6)-(B-8), we can obtain Equation (28).

The third pressure profile is,

\[ P^*(r, t) = c_1 (r_w + \delta(t) - r) \ln(c_2 r + c_3 r^2) \quad (r_w \leq r \leq r_w + \delta(t)) \]  
(B-11)

as a special case of Equation (29). It can be shown that the conditions (B-6)-(B-8) are satisfied for the choice of \( c_1 - c_3 \) as given by Equation (30).

Perform the integration:

\[ \int_{r_w}^{r_w + \delta(t)} \pi \rho r h dr \]
on both sides of Equation (21), yielding

\[ \frac{2\pi h}{\mu} \left[ \rho \phi \left( \frac{\partial P}{\partial r} \right) \right]_{r_w}^{r_w + \delta(t)} = 2\pi h \int_{r_w}^{r_w + \delta(t)} r \frac{\partial}{\partial t} \left[ \rho \phi(P) \right] dr \]  
(B-12)

Using boundary condition (B-8) and changing the order of integration and derivative on the right side in Equation (B-12), we have

\[ q_m = \frac{d}{dt} \left[ \Phi - \pi h (r_w + \delta(t))^2 \rho \phi \right] \]  
(B-13)

where

\[ \Phi = \int_{r_w}^{r_w + \delta(t)} 2\pi \rho h (P) \phi(P) dr \]  
(B-14)
Integrating Equation (B-13) with respect to $t$ from $t = 0$ to $t = t$ gives

$$q_m t = \Phi - \pi h [r_w + \delta(t)]^2 \rho_i \phi_i + C_r$$

(B-15)

Since at $t = 0$, $\delta(t) = 0$, and $\Phi = 0$, we have

$$C_r = \pi hr_w^2 \rho_i \phi_i$$

(B-16)

Introducing (B-16) into (B-15) gives the integral equation (31).

In the special case of slightly compressible fluid and rock, whose constitutive equations are given by Equation (A-13)-(A-15), the explicit expressions of the mass balance equations are obtained by evaluating the integral in Equation (31) for the choice of pressure profiles, as given by Equations (32)-(34).
Table 1
Parameters for Checking Integral Solution

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>Initial pressure</td>
<td>$P_i=10^7$ Pa</td>
</tr>
<tr>
<td>Initial Porosity</td>
<td>$\phi_i=.20$</td>
</tr>
<tr>
<td>Initial Fluid Density</td>
<td>$\rho_i=975.9$ kg/m$^3$</td>
</tr>
<tr>
<td>Cross-Sectional Area</td>
<td>$A=1.0$ m$^2$</td>
</tr>
<tr>
<td>Formation Thickness</td>
<td>$h=1.0$ m</td>
</tr>
<tr>
<td>Fluid Viscosity</td>
<td>$\mu_f=3.5132\times10^{-3}$ Pa.s</td>
</tr>
<tr>
<td>Fluid Compressibility</td>
<td>$C_f=4.556\times10^{-10}$ Pa$^{-1}$</td>
</tr>
<tr>
<td>Rock Compressibility</td>
<td>$C_r=2, 5\times10^{-9}, 5\times10^{-10}$ Pa$^{-1}$</td>
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<tr>
<td>Mass Injection Rate</td>
<td>$q_m=0.01, 1.0, 10$ kg/s</td>
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<tr>
<td>Initial Permeability</td>
<td>$k=9.869\times10^{-13}$ m$^2$</td>
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<tr>
<td>Wellbore Radius</td>
<td>$r_w=.1$ m</td>
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<tr>
<td>Hydraulic Radius</td>
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<tr>
<td>Shape Factor</td>
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<td>Coefficient, Equation (38)</td>
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Table 2
Parameters for Flow through a Horizontal Fracture

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<td>Initial pressure</td>
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</tr>
<tr>
<td>Initial Aperture</td>
<td>$b_i=1\times10^{-3}$ m</td>
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<tr>
<td>Initial Fluid Density</td>
<td>$\rho_i=975.9$ kg/m$^3$</td>
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<tr>
<td>Half Spacing</td>
<td>$D=0.25$ m</td>
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<tr>
<td>Fluid Viscosity</td>
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<tr>
<td>Fluid Compressibility</td>
<td>$C_f=4.556\times10^{-10}$ Pa$^{-1}$</td>
</tr>
<tr>
<td>Young’s Modulus</td>
<td>$E=5\times10^{11}$ Pa</td>
</tr>
<tr>
<td>Mass Injection Rate</td>
<td>$q_m=1.0$ kg/s</td>
</tr>
<tr>
<td>Initial Fracture Permeability</td>
<td>$k_{eff}=1.66667\times10^{-10}$ m$^2$</td>
</tr>
<tr>
<td>Wellbore Radius</td>
<td>$r_w=1$ m</td>
</tr>
</tbody>
</table>
Figure 1. Permeability Functions for Checking Integral Solutions.
Comparison of Injection Pressures Calculated from Integral and Exact Solutions for Linear Flow in a Constant Permeability Medium.
Figure 3. Comparison of Injection Pressures Calculated from Integral, Exact (Constant Permeability) and Numerical Solutions for Linear Flow in a Deformable Medium with Permeability Function (37).
Comparison of Injection Pressures Calculated from Integral, Exact (Constant Permeability) and Numerical Solutions for Linear Flow in a Deformable Medium with Permeability Function (38).
Figure 5. Comparison of Injection Pressures Calculated from Integral and Exact Solutions (Constant Permeability) for Radial Flow in a Constant Permeability Medium with Pressure Profiles recommended in Heat Transfer.
Figure 6. Comparison of Injection Pressures Calculated from Integral and Exact Solutions (Constant Permeability) for Radial Flow in a Constant Permeability Medium with Pressure Profile (27) in This Work.

\[ k = \text{constant} \]

- Exact Solution, Eq. (41)
- Integral Solution, Eq. (28)

\[ q_m = (1 \text{ kg/s}) \]
\[ C_r = 5 \times 10^{-9} \text{ (Pa}^{-1}) \]
Comparison of Injection Pressures Calculated from Integral and Exact Solutions (Constant Permeability) for Radial Flow in a Constant Permeability Medium with Pressure Profile (29) in This Work.
Figure 8. Comparison of Injection Pressures Calculated from Integral and Numerical Solutions for Radial Flow in a Strongly Deformable Medium with Permeability Function (35).
Figure 9. Comparison of Injection Pressures Calculated from Integral and Numerical Solutions for Radial Flow in a Weakly Deformable Medium with Permeability Function (35).
(a) Basic Model - Uniform horizontal fracture

(b) Basic Section

Figure 10. Schematic of a Horizontal Fracture System.
Figure 11. Comparison of Injection Pressures in a Linear Fracture System with and without Including Effects of Permeability Changes.
Figure 12. Comparison of Injection Pressures in a Radial Fracture System with and without Including Effects of Permeability Changes.

- \( k_f = \text{Constant} \)
- \( k_f = k(P) \)

\[
D = 0.1 \text{ (m)} \\
b_i = 10^{-4} \text{ (m)}
\]