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Particle Structure Function And Subbarrier Fusion In Hot Nuclei

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Abstract

The study of particle evaporation spectra can provide information about shape polarization phenomena induced by the nascent particle on the residual nucleus, and about optical modulations felt by the particle as it is preformed inside the nucleus. These aspects can be studied as a function temperature. Preliminary experimental evidence about these features has been obtained.

1 Introduction

The amalgamation of a particle with a nucleus in fusion reactions, and the segregation of the same particle in evaporation, pose the question of the existence of complex particles in the nuclear medium. Even for nucleons the interaction with the nuclear medium makes their existence as independent particles rather fleeting. In general, one wonders to what extent the total nuclear wave function is factorizable into the product of the wave function of the particle in question and that of the residual nucleus.

This problem is analogous to the problem of the interaction of a solute molecule with the molecules of the solvent in solutions. The properties of a solute molecule, or even of an individual solvent molecule, are dramatically renormalized by the interaction with the other molecules. Geometric conformation, bond lengths, and, in general, all other spectroscopic features, are at the very least changed with respect to the corresponding quantities in
vacuo. In the most extreme cases, the existence of the molecule itself may be compromised due to dissociation, or to other chemical reactions.

In the nuclear case, one can consider the nuclear medium as the solvent, and the various kinds of (complex) particles as the solute. Individual neutrons and protons, at least in ground state nuclei, behave more or less as non-interacting particles in the shell model potential. However, single particle excited states couple rather quickly into the many body degrees of freedom. The width of the corresponding strength function gives a direct estimate of the "lifetime" of the nucleon in the medium.

Qualitatively one can expect that tightly bound particles such as alphas may have more than a fleeting existence in the nuclear medium, certainly much longer than that of a weakly bound particle such as the deuteron. Cluster models have described complex particles within the nuclear medium. This problem has also been addressed, in the case of nuclear reactions, by the optical model theory. The interaction of the projectile with the nucleus is described in terms of a complex one body potential, where the imaginary part describes the removal of flux from the elastic channel, or the "absorption" of the particle into the collective unconscious of the nuclear many body system.

While the optical potential may adequately describe elastic scattering, it tells us little about the actual behavior of the particle inside the nucleus.

Although the problem has been studied theoretically for ground state nuclei, very little experimental information does in fact exist. No information is at hand concerning hot nuclei. This is true even for nucleons. We know nothing on how the shell/optical model potential evolves with temperature. Just as little we know about the existence and properties of clusters in hot nuclei.

One may wonder whether this information is at all accessible experimentally. Should it be so, it would provide us with an important and totally novel chapter of nuclear physics.

We believe that the questions outlined above, and possibly others, can be addressed by looking at compound nucleus evaporation of nucleons and complex particles in a fresh way.
2 Particle Evaporation From Compound Nuclei

As a particle is "segregated" from a compound nucleus state and prepares to exit, it senses its environment. This environment could be a mean field, like a shell/optical potential, or a local polarization field of some sort. This should result in states which acquire a width through the coupling with the continuum and the remaining many body degrees of freedom. A strength function should arise which modulates the spectrum of the emitted particle. This is illustrated qualitatively in Fig. 1. The states inside and above the well are the states of the particle in the nucleus, which, in the case of protons, tend to the shell model states well below the barrier, and to the optical model resonances in the continuum above the barrier.

Figure 1: Schematic drawing of the states of a particle in a potential well.

This modulation should be observable in very high statistics evaporation spectra. Since the particle to be emitted is in a hot nucleus, whose excitation energy is under experimental control, the strength function obtained from the modulation of the spectrum refers to that specific excitation energy or temperature. Thus the possibility exists of studying these fields not only for a variety of particles, but also for different temperatures.

Another way of looking at this problem is to consider the standard ex-
pression for the evaporation spectrum. Detailed balance between compound nucleus states $a$ and final states $b$ requires that

$$\rho_a \Gamma_{a \rightarrow b} = \rho_b \Gamma_{b \rightarrow a}$$  \hspace{1cm} (1)$$

where $\Gamma_{a \rightarrow b}$, $\Gamma_{b \rightarrow a}$ are the direct and inverse decay widths, and $\rho_a$, $\rho_b$ are the corresponding phase space volumes. The inverse width is formally expressed in terms of the "inverse" cross section

$$\Gamma_{a \rightarrow b} = \hbar \frac{\sigma_{\text{inv}} v}{V}$$  \hspace{1cm} (2)$$

where $v$ is the velocity of the particle, and $V$ is a normalization volume.

The combination of Eq. 1 and Eq. 2 gives the decay width differential in the particle kinetic energy:

$$\Gamma(\varepsilon) \propto \sigma_{\text{inv}}(\varepsilon) \varepsilon \rho(E - B - \varepsilon)$$  \hspace{1cm} (3)$$

where $B$ is the particle binding energy. A first order expansion of the log of the level density in the kinetic energy of the particle gives the transparent form:

$$\Gamma(\varepsilon) \propto \sigma_{\text{inv}}(\varepsilon) \varepsilon e^{-\varepsilon/T}$$  \hspace{1cm} (4)$$

Thus the spectrum is the product of a structure factor, namely the "inverse" cross section, and of a phase space factor. Removal of the latter should leave the former with its potentially interesting modulations.

Usually evaporation codes calculate the inverse cross section, or the corresponding transmission coefficients, from an optical model. There are two difficulties with this. The first is related to the fact that the residual nucleus to which the inverse cross section refers is an excited nucleus, while the optical potential is known for ground state nuclei. This is not a problem here, since we intend to extract the "inverse" cross section directly from the data, and infer the temperature dependence of the optical potential. The second is associated with shape polarization effects in the exit channel. These are due, for instance, to the Coulomb interaction, and are very different from those in the entrance channel. Let us consider this interesting problem in more detail.
3 Shape Polarization and Evaporation Spectra

In order to exit from a nucleus, a charged particle must overcome the Coulomb barrier. If the nucleus deforms in the direction of emission, the Coulomb barrier decreases. Therefore, it pays for the nucleus to invest some energy in deformation in order to lower the barrier. More quantitatively, let us consider a configuration formed by the emitted particle just in contact with the residual nucleus. Now we deform the residual nucleus always keeping the light particle in contact, and we plot the total energy as a function of deformation. The total energy has a minimum at some finite prolate deformation. This is the location of the saddle point, as shown in Fig. 2. The unbound mode, or reaction coordinate, is the distance between centroids. A particle crossing over the saddle point with zero kinetic energy acquires a kinetic energy at infinity smaller than the Coulomb barrier associated with a spherical configuration. This is not subbarrier emission, of course, in the sense that it is not associated with quantum barrier penetration.

Thermal fluctuations along this deformation coordinate $Z$ lead to large fluctuations in the Coulomb interaction energy, as shown in Fig. 2. While the total potential energy $V_T$ has a minimum at some prolate deformation, the fragment-fragment Coulomb interaction $V_{Coul}$ is a monotonically decreasing function of deformation coordinate.

Therefore we can expand the total potential energy and the Coulomb interaction energy about the saddle point along the deformation coordinate $Z$ ($Z = 0$ at the saddle):

$$V_T = V_T^0 + KZ^2, \quad V_{Coul} = V_{Coul}^0 - cZ.$$  \hfill (5)

Now, if we allow the shape to fluctuate involving an energy of the order of the temperature $T$, we obtain a corresponding fluctuation of the Coulomb energy:

$$\Delta V_{Coul} = 2\sqrt{\frac{c^2}{K}T} = 2\sqrt{pT}.$$  \hfill (6)

We call the parameter $p$ the amplification parameter. More quantitative considerations lead to an expression for the kinetic energy of the particle

$$P(x) \propto e^{-x/T} erfc \left( \frac{p - 2x}{2\sqrt{pT}} \right).$$  \hfill (7)
Saddle Point and Normal Modes

i) decay mode:

ii) non-amplifying mode:

iii) amplifying mode:

Figure 2: Top: Normal modes at the saddle point. Bottom: Total potential energy $V_T$ and Coulomb energy $V_{Coul}$ as a function of the deformation coordinate $Z$. 

$\Delta V_{Coul} = 2\sqrt{pT}$
where \( x = \varepsilon - V_{\text{Coul}}^0 \).

This formula is easily generalized to include: 1) a larger number of amplifying and non-amplifying degrees of freedom at the saddle; 2) their quantization; 3) true quantal barrier penetration along the reaction coordinate.

We see immediately that, by using a formula like that in Eq. 7 to fit experimental spectra, we can obtain information on the shape polarization associated with particle emission and on its excitation energy.

Furthermore, the fact that expressions like Eq. 7 are non polynomial helps by not introducing unwanted polynomial modulations in the extraction of the particle structure functions.

### 4 Experimental Particle Structure Functions

We are now going to explore high statistics evaporation spectra from the reaction \(^3\text{He} + ^{\text{nat}}\text{Ag}\) from 55 to 110 MeV bombarding energy in order to see whether physical modulations similar to those expected from optical potentials are present.

The plan is to fit the spectrum shown in the lower panel of each sextant in Fig. 3 with a smooth function, and to search the residuals for modulations. Standard expressions for the evaporation spectra do not provide adequate fits since they do not incorporate shape polarization, etc.

On the other hand, expressions like Eq. 7 have demonstrated the necessary flexibility. The fits to the experimental spectra are of extremely high quality.

The percentage residuals of the fits are also shown in Fig. 3, in the upper panel of each sextant. They are of the order of 1% throughout the energy range, which shows the goodness of the fitting functions. These residuals show a statistically significant modulation with an amplitude of about 1.5% which is approximately repeated in both amplitude and phase at all bombarding energies.

In order to extract information on the modulations observed at the various excitation energies, we have devised an analytical procedure based upon orthogonal polynomials. We write down the experimental spectrum as a linear combination of orthogonal polynomials

\[
F(\varepsilon) = \sum a_n S(\varepsilon) P_n(\varepsilon),
\]  

(8)
where $S(\varepsilon)$ is a suitably chosen weight function that generates the polynomials $P_n(\varepsilon)$. The orthogonality condition is

$$\int_a^b S^2(\varepsilon)P_n(\varepsilon)P_m(\varepsilon)d\varepsilon = \begin{cases} 0, & n \neq m, \\ 1, & n = m. \end{cases}$$  \hspace{1cm} (9)$$

The choice of $S(\varepsilon)$ is dictated by the desire of concentrating the bulk of the spectral shape into the single coefficient $a_0$. The modulations then appear in the higher order coefficients, hopefully in only one or two. This goal can be be achieved by choosing for $S(\varepsilon)$ the form given by Eq. 7 with parameters obtained from the least square fit. This guarantees that $a_0$ will take up the bulk of the spectrum. The amplitudes $a_n$ can be obtained from the dot product of the experimental spectrum with the $n^{th}$ polynomial

$$a_n = \int_a^b F(\varepsilon)S(\varepsilon)P_n(\varepsilon)d\varepsilon,$$  \hspace{1cm} (10)$$

and the corresponding strength $s_n$ can be defined as

$$s_n = a_n^2/\int_a^b F^2(\varepsilon)d\varepsilon.$$  \hspace{1cm} (11)$$

By definition we have $\sum s_n = 1$.

The result of this analysis is shown in Figs. 3 and 4. We see that indeed $a_0$ takes up the bulk of the spectrum, and that only a couple of coefficients $a_n$ suffice to exhaust the modulation.

The first task we are called upon, is to rule out: a) instrumental effects, like modulations introduced by detectors, amplification chain of electronics, ADC, etc.; b) fitting problems associated with the rigidity or modulations of the fitting functions.

This study is very preliminary. The data were not taken with all of the above in mind. Therefore, we must exercise the greatest caution before accepting for physics what may be a spurious effect. However, we can say the following. Regarding a), the effect is observed in several independent detector-electronic chains. Regarding b), the function used is non polynomial, thus it does not introduce the modulations we see. At close inspection it is clear that it is the data that wrap themselves around the fitting function, and not vice-versa.

It is clear that all of these measurements must be repeated with equipment designed explicitly to avoid the introduction of modulations. For the
Figure 3: Lower panels: The experimentally measured $\alpha$ spectra (●) and The linear combination of the orthogonal functions (Eq. 8) (solid lines). Upper panels: The dots are the percentage difference between the experimental data and the fits with Eq. 7. The error bars represent the statistical errors of the experimental data. The solid lines are the percentage difference of the combination of the orthogonal functions and the fits.
Figure 4: The strength $s_n$ of the $n^{th}$ order as defined in Eq. 11 plotted against the order $n$.

moment, we can hope that the effect will stand up, and can try speculating on its possible meaning.

5 Significance of the shape polarization parameters

The exceedingly good quality of the fit to the spectra by means of Eq. 7 suggests that it may be possible to extract information regarding shape polarization both as a function of particle energy and of excitation energy. It is
tempting to make a connection between this physics and that of subbarrier fusion. This connection is best seen by considering Eqs. 3 & 4. By eliminating the phase space part of the spectrum one is left with the inverse cross section as a function of particle energy. As we said before, this cross section refers to a hot target. By studying this cross section at low particle energy we are effectively exploring the “subbarrier” region. Thus the apparent oxymoron: “subbarrier fusion in hot nuclei.” This aspect of the problem is quite tantalizing. In principle one could attempt the same analysis as done in subbarrier fusion. The product $\varepsilon \sigma_{\text{inv}}(\varepsilon)$ can be extracted from the spectrum as shown by Eq. 4 and the barrier distribution can be obtained

$$B(\varepsilon) = \frac{\partial^2 \varepsilon \sigma_{\text{inv}}}{\partial \varepsilon^2}.$$  \hfill (12)

However, no effort has been made in this direction as yet.

6 Possible interpretations of the observed modulations

The “wavelength” of the modulations and their small amplitude suggest that they could be modulations associated with the optical potential felt by the $\alpha$-particle inside the nucleus. Although a fit with an optical model calculation may be premature, the features are consistent with a ground state-like optical potential with a moderate to large imaginary part. Interesting is the persistence and stability of the modulation with increasing energy. This can be seen from the weak evolution of the extracted coefficient $a_n$ with energy. These results are also consistent with the above interpretation. The temperature involved in the experiment is small and changes little over the excitation energy range investigated. In the temperature range covered by the experiment we expect that the geometries associated with the decay are not greatly altered yet (nuclei do not expand so easily).

Similarly, there are good reasons to expect that the strength of the potential has a weak temperature dependence. While we hope that a finer inspection might reveal temperature effects, the gross stability of the modulation with temperature is comforting.
7 Conclusions

Two distinct, though connected, aspects can be discussed in the physics of evaporation spectra:
1) a gross aspect relating to the polarization of the nucleus in the process of emitting the particle. This should become more and more relevant as the atomic number of the particle out of the compound nucleus increases.
2) a fine aspect dealing with the lifetime of the particle inside the nucleus. This should be most important for evaporated nucleons and light particles. In particular one may learn about the dependence of the imaginary part of the optical potential upon particle binding energy.
In both cases the possibility of performing the studies as a function of excitation energy or temperature is of paramount interest.

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