A COMPLETE SOLUTION TO THE FORWARD-BIAS PUZZLE*

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A complete solution to the forward-bias puzzle should provide an econometric solution and an economic explanation for that solution. A complete solution should also explain the closely related failure of uncovered interest parity. In addition it should explain some related anomalies. One such anomaly is that variances for changes in exchange rates are over 100 times larger than variances for interest rate differentials and forward premiums. My econometric solution is that the relevant test equations omit two variables that covered interest parity implies should be included. For my data, the missing variables explain the failure of uncovered interest parity and the forward-bias puzzle. The missing variables also explain why the variance for changes in exchange rates is over 100 times larger than the variance for both interest rate differentials and forward premiums. My economic explanation is that, in general, forward rates do not equal expected future spot rates.

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Disciplines: Finance; Macroeconomics; International Economics.

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The forward-bias puzzle is probably the most important of several puzzles in international finance and open economy macroeconomics because it suggests that there are serious informational inefficiencies in foreign exchange markets. But after 30 years of research we still do not have a generally accepted solution.

As Frankel and Poonawala (2010) point out, most proposed solutions to the puzzle fall into one of two general categories. The first, and probably most common category, maintains the assumption of rational expectations and attributes the bias to a risk premium. The second attributes the bias to expectation errors.

My solution does not fall into either category. All I assume is that covered interest parity (CIP) holds. Covered interest parity provides both an econometric solution and an economic explanation. The econometric solution is that the standard test equation omits two variables that CIP implies should be included. The economic explanation is that in general forward rates do not equal expected future spot rates. As far as I am aware, the failure of forward rates to equal expected future spot rates has never before been suggested as the explanation for the forward-bias puzzle.

My solution for the forward-bias puzzle also explains the bias associated with uncovered interest parity and the fact that variances for changes in exchange rates are over 100 times larger than variances of both forward premiums and interest rate differentials.

1. The Forward-Bias Puzzle

The forward-bias puzzle is based on two assumptions: (1) Forward exchange rates equal expected future spot rates. (2) Expectations are rational. As shown later, it appears that, in general, forward rates do not equal expected future spot rates.

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1 For a discussion of the other puzzles see Obstfeld and Rogoff (2000).
Relying on rational expectations, tests of the assumption that forward rates equal expected future spot rates use actual future spot rates as proxies for expected future spot rates. Early tests regressed actual future spot rates against current forward rates. Such tests usually produced regression coefficients that were close to one. For a review of that early literature see Levich (1979).

Later the recognition of the possible effects of unit roots changed the standard test equation. To achieve stationarity, current spot rates were subtracted from both sides of the original test equation. That alteration produces equation (1), the new standard test equation.

Let \( s_t \) be the logarithm of the current spot price for foreign exchange. Let \( f_t \) be the logarithm of the current forward exchange rate. Finally let \( s_{t+1} \) be the logarithm of the future spot exchange rate.

\[
\Delta s_{t+1} = s_{t+1} - s_t = \beta_0 + \beta_1 (f_t - s_t)
\]

In equation (1) \( s_{t+1} \) acts as a proxy for the exchange rate that is expected in \( t+1 \) based on the information set available in \( t \) or \( E(s_{t+1}/\Phi_t) \).

A large literature shows that estimates of \( \beta_1 \) are usually closer to zero than to one and are often negative. For a discussion of these results and a review of the literature see Sarno (2005). Some of the more recent articles include Nikolaou and Sarno (2006), Sarno, Valente and Leon (2006), Sercu and Vinaimont (2006), Kearns (2007), Chakraborty and Haynes (2008), Chakraborty and Evans (2008), Wang and Wang (2009), Frankel and Poonawala (2010), Hochradl and Wagner (2010) and Pippenger (forthcoming). For examples of such estimates using the data described later, see Table 1.

Negative estimates of \( \beta_1 \) seem to imply an informational inefficiency. Exchange rates fall when the forward premium seems to predict that they will rise. That apparent predictive error is
the forward-bias puzzle. Covered interest parity, the no-arbitrage condition associated with the theory of covered interest rate arbitrage, provides an econometric explanation for the downward bias in the forward-bias puzzle and the similar bias in uncovered interest parity. The economic source of both downward biases appears to be that forward rates do not in general equal expected future spot rates.

2. Covered Interest Parity

Covered interest parity is the equilibrium condition, or what is often called the no-arbitrage condition, associated with the theory of covered interest arbitrage. At least between euro currencies, covered interest parity appears to hold on a daily basis. Effective arbitrage appears to quickly eliminate any potential profit from covered interest rate arbitrage. As Akram, Rime and Sarno (2008, 237) points out, “It seems generally accepted that financial markets do not offer risk-free arbitrage opportunities, at least when allowance is made for transaction costs.”

Equation (2) describes covered interest parity.

\[(f_t - s_t) - (i_t - i^*_t) = \pm e_t\]  

In equation (2) \(i\) is the domestic interest rate, \(i^*\) is the foreign interest rate and \(e\) captures the errors within the thresholds created by transaction costs, apparently mostly bid-ask spreads.\(^2\)

The interest rates should be risk free and their maturities must match the maturity of the forward exchange rate. With effective arbitrage, CIP holds whether or not expectations are rational and whether or not there is a risk premium.

In the Conclusions to their article, Akram, Rime and Sarno (2008) explain in more detail how covered interest rate arbitrage works.

\(^2\) See Balke and Wohar (1998) for evidence of the thresholds created by transaction costs.
This paper provides evidence that short-lived arbitrage opportunities arise in the major FX and capital markets in the form of violations of the CIP condition. The size of CIP arbitrage opportunities can be economically significant for the three exchange rates examined and across different maturities of the instruments involved in arbitrage. The duration of arbitrage opportunities is, on average, high enough to allow agents to exploit deviations from the CIP condition. However, duration is low enough to suggest that markets exploit arbitrage opportunities rapidly. These results, coupled with the unpredictability of the arbitrage opportunities, imply that a typical researcher in international macro-finance can safely assume arbitrage-free prices in the FX markets when working with daily or lower frequency data.

Since one “can safely assume arbitrage-free prices in FX markets when working with daily or lower frequency data” and my data are daily or weekly, I assume that covered interest parity holds for every t in my data set.

Since covered interest parity holds for all time periods, at time t I know that it will hold at time t+1. As a result, at time t I know that at time t+1 equation 3 will hold:

\[(f_{t+1} - s_{t+1}) - (i_{t+1} - i^*_{t+1}) = \pm e_{t+1}\]  

(3)

It is important to note that equation (3) does not involve expectations in the same way as equation (1). The f, s, i and i* in equation (3) are all the actual f, s, i and i* for t+1. None of these variables appear in equation (3) because they are proxies for expectations taken at time t. When one looks at equation (3) from the perspective of time t, all one assumes at t is that arbitrage will enforce covered interest parity at t+1. As a result, equation (3) does not depend on the information set \(\Phi_t\) in the same way as equation (1).

Arbitrage in financial markets is so effective that covered interest parity is sometimes used as though it were an identity. But covered interest parity is clearly not an identity.\(^3\) Indeed the quote from Akram, Rime and Sarno (2008) above proves that covered interest parity is not an

\(^3\) For the *New Palgrave Dictionary of Economics, Second Edition, Vol. 8* (2008, 451) CIP is clearly a theory. "Since it was clear that forward rates also reflected perceptions about future spot rates, it was a short step to the assumption of UIP, which builds on the theory of CIP by essentially postulating that market forces drive the forward exchange rate into equality with the expected future spot rate.” Italics added.
identity because there are short periods of time when, even after accounting for the transaction costs, covered interest parity does not hold. It is those short intervals that provide the occasional profits necessary to support effective arbitrage.

Effective arbitrage and CIP are the keys to solving the forward-bias puzzle.

3. The Econometric Solution

A complete solution to the forward-bias puzzle should provide both an econometric solution and an economic explanation for that solution. This section provides the econometric solution. Section 7 provides the economic explanation.

The econometric solution starts with equation (4). Equation (4) is a direct implication of equation (3) and covered interest parity.

\[ s_{t+1} = f_{t+1} - (i_{t+1} - i_t^*) \pm e_{t+1} \]  

As in equation (3), the error term in equation (4) represents only the neutral range created by transaction costs. Equation (4) does not assume that the forward rate is the expected future spot rate. None of the variables in equation (4) represent proxies for their expected values in \( t \) conditional on the information set in \( t \). Equation (4) only assumes that covered interest parity holds for daily and lower frequency data.

Subtracting \( f_t \) from both sides of equation (4) produces equation (5).

\[ s_{t+1} - f_t = (f_{t+1} - f_t) - (i_{t+1} - i_t^*) \pm e_{t+1} \]  

Ignoring the transaction costs, \( s_{t+1} - f_t \), is the actual return from not covering a foreign investment. The domestic return \( i_t \) is the same with or without cover. The actual return from foreign assets without cover is \( i_t^* + s_{t+1} - s_t \). The certain return from foreign assets with cover is \( i_t^* + f_t - s_t \). So \( s_{t+1} - f_t \) is the actual or realized return on a foreign investment without cover minus the return with cover.
Ignoring the transaction costs, equation (5) says that the actual return from not covering equals the actual change in the forward rate \( (f_{t+1} - f_t) \) minus the actual future interest rate differential \( (i_{t+1} - i_{t+1}^*) \). As Section 7 shows, equation (5) suggests an economic explanation for the forward-bias puzzle and the similar bias associated with uncovered interest parity. That explanation is that forward rates in general do not equal expected future spot rates.

Like equation (5), the econometric solution for the forward-bias puzzle follows directly from equation (4). Subtracting \( s_t \) from both sides of equation (4) and then adding \( f_t \) to and subtracting \( f_t \) from the right-hand side of equation (4) produces equation (6).

\[
\Delta s_{t+1} = (f_t - s_t) + (f_{t+1} - f_t) - (i_{t+1} - i_{t+1}^*) \pm e_{t+1} \\
\text{(6)}
\]

As a restatement of covered interest parity, equation (6) describes the relationship between the actual future change in the exchange rate and the current forward premium implied by covered interest parity. One way to view equation (6) is that it says that the actual change in the future spot rate \( \Delta s_{t+1} \) equals the current forward premium \( f_t - s_t \) plus the actual return from not covering a foreign investment.

Unlike equation (1), equation (6) does not assume that \( s_{t+1} \) is a proxy for the expected future spot rate. As a result equation (6) does not contain any additional error terms that would be included by using an observed value in \( t+1 \) as a proxy for its expected value in \( t \).

Equation (6) contains two terms, \( (f_{t+1} - f_t) \) and \( (i_{t+1} - i_{t+1}^*) \), that do not appear in equation (1). Those two omitted variables appear to be the econometric source of the forward-bias puzzle.

The next section shows that, at least for my data, the bias due to omitting \( (f_{t+1} - f_t) \) and \( (i_{t+1} - i_{t+1}^*) \) explains my negative estimates of \( \beta_1 \). With those variables included, coefficients for the forward premium are closer to one than to zero. When forward premiums are positive, exchange rates tend to rise. When forward premiums are negative, exchange rates tend to fall.
4. The Evidence

My solution to the forward-bias puzzle assumes that, after accounting for the transaction costs, covered interest parity holds. At least for daily and lower frequency data, the evidence indicates that arbitrage effectively enforces that no arbitrage condition. That assumption implies an unusual econometric model. Transaction costs, mostly bid-ask spreads, produce upper and lower thresholds. Within the thresholds the relevant variables can move independently of each other. Outside the thresholds the no arbitrage condition implies that there are no errors because any error would imply arbitrage profits. Outside the thresholds any move in one variable must be offset exactly by movement in the remaining variables.

The neutral range within the thresholds affects my results in at least two ways: (1) It produces positively autocorrelated errors. Within the neutral range errors can persist indefinitely because the relevant variables are free to move independently of each other. As a result the errors are likely to be highly autocorrelated and could even be martingales. I attribute my low Durbin-Watson statistics to that autocorrelation. Note that the Durbin-Watson statistics improve when the omitted variables are included. (2) The neutral range also biases estimated coefficients toward zero because within the neutral range right-hand side variables can change without any corresponding change in the left-hand side variable. Within the thresholds the relevant coefficients are effectively zero while outside the thresholds they are plus or minus one.

4.1. The Data

The data cover two intervals between the United States and Canada and two intervals between the United States and the United Kingdom. For U.S.-Canada, the weekly interest rates are for 13 week Treasury bills. Those interest rates are from various issues of the *Federal Reserve Bulletin* starting with the issue of October 1964. Spot and forward exchange rates are
for noon and were supplied by the Bank of Canada. As the *Bulletin* makes clear, the Treasury bill rates are only approximations of the rates needed for arbitrage. The weekly data for U.S.-Canada run from January 1961 to June 1973. The first interval for Canada in Table 1 covers the era of pegged exchange rates that started *de facto* in December 1960 and ended in May 1970. The second interval covers a period of flexible exchange rates from June 1970 to June 1973.

For the U.S.-U.K., the daily data are from Balke and Wohar (1998). Their daily interest rates are one month euro rates. See Balke and Wohar (1998) for more details. Their daily data start in January 1974 and end in September 1993. To account for any possible effects of the switch to flexible rates in the early 1970s, the interval is divided into roughly two equal parts. The first begins in January 1974 and runs through early November 1983. The second begins the next day and ends in early September 1993.

For the Canadian data, where the interest rates are for 91 days, the future spot and future forward exchange rates are t plus 13 weeks. For the UK data, where the interest rates are for 30 days, the future spot and future forward exchange rates are t plus 22 observations.

The data are not ideal. Interest rates, future spot exchange rates and forward rates are not always matched exactly. Particularly for the US-Canadian data, the timing of the observations is not ideal. Future research should correct those shortcomings. However it seems unlikely that correcting any shortcomings in the data will change the basic message. The downward bias in the forward-bias puzzle is the result of omitting two important variables.

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4 For a detailed description of the interest rates, see the issue of October 1964.
5 The data start in January 1959 when rates were flexible. I start in January 1961 because the rates were pegged *de facto* in December of 1960. The data end in August 1973, but 13 weeks are lost due to the difference between spot and forward exchange rates.
6 For both U.S.-Canada and U.S.-U.K., missing observations are replaced with the previous observation. If two observations in a row are missing, the first is replaced with the previous observation and the second with the following observation.
7 The data in Balke and Wohar (1998) are bid and ask. Like them, I use the geometric mean of the bid and ask.
The next subsection discusses the theoretical effect of omitting variables. Subsection 4.3 reports the actual bias due to omitting $f_{t+1} - f_t$ and $i_{t+1} - i_t^*$. 

4.2 The Bias Due to Omitted Variables: Theory

Suppose the true model is

$$y = XB + u$$

where $X$ is a data matrix of $n \times k$. Then

$$E(\hat{B}/X) = (X'X)^{-1}X'y.$$ 

With the usual least squares assumptions

$$E(\hat{B}/X) = B.$$ 

But suppose the test equation mistakenly uses the data matrix $\tilde{X}$ where $\tilde{X}$ differs from $X$ merely in the exclusion of $k-r$ relevant variables $X_{r+1}, X_{r+2}, \ldots X_k$. The least squares estimator for the test equation is

$$E(\hat{b}/\tilde{X}) = (\tilde{X}'\tilde{X})^{-1}\tilde{X}'y.$$ 

Substituting $XB + u$ for $y$ produces

$$E(\hat{b}/\tilde{X}) = (\tilde{X}'\tilde{X})^{-1}\tilde{X}XB + (\tilde{X}'\tilde{X})^{-1}\tilde{X}'u$$

so that

$$E(\hat{b}/\tilde{X}) = PB$$

where

$$P = (\tilde{X}'\tilde{X})^{-1}\tilde{X}'X.$$ 

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8 Strictly speaking, those assumptions do not hold here because of the thresholds created by the transaction costs.

9 For a clear discussion of omitted variables see Wooldridge (2009).
Regressing each of the relevant variables $X_1$, $X_2$, ......,$X_k$ in turn on the set $\hat{X}$ and arranging the estimated coefficients as column vectors produces the matrix $P$ above where

$$
P = \begin{bmatrix}
1 & 0 & \ldots & 0 & p_{1,r+1} & \ldots & p_{1,k} \\
0 & 1 & \ldots & 0 & p_{2,r+1} & \ldots & p_{2,k} \\
& & \ddots & & \ddots & & \ddots \\
& & & & \ddots & & \ddots \\
0 & 0 & \ldots & 1 & p_{r,r+1} & \ldots & p_{r,k}
\end{bmatrix}
$$

So the bias in any estimated coefficient $\tilde{b}_i$ is

$$
E(\tilde{b}_i) - B_i = p_{i,r+1}B_{r+1} + \ldots + p_{i,k}B_k
$$

and

$$
E(\tilde{b}_i) = B_i + p_{i,r+1}B_{r+1} + \ldots + p_{i,k}B_k.
$$

According to covered interest parity, equation (7) corresponds to $y = XB + u$.

$$
\Delta s_{t+1} = \lambda_0 + \lambda_1(f_t - s_t) + \lambda_2 \Delta f_{t+1} + \lambda_3(i_{t+1} - i_t^*)
$$

Equation (1) corresponds to the test equation with the omitted variables.

$$
\Delta s_{t+1} = s_{t+1} - s_t = \beta_0 + \beta_1(f_t - s_t)
$$

If covered interest parity is the source of the downward bias in the forward-bias puzzle then the bias $\hat{\beta}_1 - \hat{\lambda}_1$ should equal $\hat{\phi}_1 \hat{\lambda}_2 + \hat{\omega}_1 \hat{\lambda}_3$ and $\hat{\beta}_1$ should equal $\hat{\lambda}_1 + \hat{\phi}_1 \hat{\lambda}_2 + \hat{\omega}_1 \hat{\lambda}_3$ where $\hat{\phi}_1$ and $\hat{\omega}_1$ are obtained from equations (8) and (9).

$$
\Delta f_{t+1} = \varphi_0 + \varphi_1(f_t - s_t)
$$

$$
(i_{t+1} - i_t^*) = \omega_0 + \omega_1(f_t - s_t)
$$

4.3 The Bias Due to Omitted Variables: Evidence

Table 1 reports the estimates of equation (1) using OLS and the data described above.\(^{10}\)

Most of the estimates of $\beta_1$ are negative. The average $\hat{\beta}_1$ in Table 1 is -1.154. The average $\hat{R}^2$ is only 0.015.

\(^{10}\) Regressions in all tables use RATS with “Robusterrors”.  

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Table 1  
Estimates of Equation 1  
\[ \Delta s_{t+1} = \beta_0 + \beta_1(f_t - s_t) \]

<table>
<thead>
<tr>
<th></th>
<th>( \hat{\beta}_0 )</th>
<th>( \hat{\beta}_1 )</th>
<th>( \hat{R}^2/DW )</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.-Canada</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 Jan 1961 to</td>
<td>0.251</td>
<td>-0.425</td>
<td>0.003</td>
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<tr>
<td>31 Dec 1969</td>
<td>(0.046)</td>
<td>(0.175)</td>
<td>0.100</td>
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<tr>
<td>U.S.-Canada</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 Jun 1970 to</td>
<td>-0.238</td>
<td>0.268</td>
<td>-0.003</td>
</tr>
<tr>
<td>29 Jun 1973</td>
<td>(0.093)</td>
<td>(0.372)</td>
<td>0.151</td>
</tr>
<tr>
<td>U.S.-UK</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 Jan 1974 to</td>
<td>0.657</td>
<td>-1.425</td>
<td>0.033</td>
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<tr>
<td>1 Nov 1983</td>
<td>(0.069)</td>
<td>(0.166)</td>
<td>0.087</td>
</tr>
<tr>
<td>U.S.-UK</td>
<td></td>
<td></td>
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<tr>
<td>2 Nov 1983 to</td>
<td>0.930</td>
<td>-3.034</td>
<td>0.025</td>
</tr>
<tr>
<td>30 Sep 1993</td>
<td>(0.129)</td>
<td>(0.406)</td>
<td>0.075</td>
</tr>
<tr>
<td>Averages</td>
<td>0.332</td>
<td>-1.154</td>
<td>0.015</td>
</tr>
<tr>
<td></td>
<td>(0.084)</td>
<td>(0.280)</td>
<td>0.103</td>
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</table>

Standard errors in parentheses.

Table 2  
Estimates of Equation 7  
\[ \Delta s_{t+1} = \lambda_0 + \lambda_1(f_t - s_t) + \lambda_2 \Delta f_{t+1} + \lambda_3 (i_{t+1} - i^*_{t+1}) \]

<table>
<thead>
<tr>
<th></th>
<th>( \hat{\lambda}_0 )</th>
<th>( \hat{\lambda}_1 )</th>
<th>( \hat{\lambda}_2 )</th>
<th>( \hat{\lambda}_3 )</th>
<th>( \hat{R}^2/DW )</th>
</tr>
</thead>
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<tr>
<td>5 Jan 1961 to</td>
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<td>0.605</td>
<td>1.002</td>
<td>-0.611</td>
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<td>31 Dec 1969</td>
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<td>(0.037)</td>
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<td>(0.043)</td>
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<td>U.S.-Canada</td>
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<tr>
<td>5 Jun 1970 to</td>
<td>-0.138</td>
<td>0.789</td>
<td>0.984</td>
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<td>0.984</td>
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<tr>
<td>29 Jun 1973</td>
<td>(0.021)</td>
<td>(0.049)</td>
<td>(0.013)</td>
<td>(0.074)</td>
<td>0.266</td>
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<td>U.S.-UK</td>
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</tr>
<tr>
<td>2 Jan 1974 to</td>
<td>0.007</td>
<td>0.990</td>
<td>1.000</td>
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<tr>
<td>1 Nov 1983</td>
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<td>(0.006)</td>
<td>(0.000)</td>
<td>(0.008)</td>
<td>1.468</td>
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<td>U.S.-UK</td>
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<tr>
<td>2 Nov. 1983 to</td>
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<td>0.992</td>
<td>0.999</td>
<td>-1.002</td>
<td>0.999</td>
</tr>
<tr>
<td>30 Sep 1993</td>
<td>(0.001)</td>
<td>(0.011)</td>
<td>(0.000)</td>
<td>(0.011)</td>
<td>1.776</td>
</tr>
<tr>
<td>Averages</td>
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<td>0.845</td>
<td>0.996</td>
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<td>0.993</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.026)</td>
<td>(0.004)</td>
<td>(0.034)</td>
<td>0.842</td>
</tr>
</tbody>
</table>

Standard errors in parentheses.
Table 2 reports the results of estimating equation (7). All $\hat{\lambda}_j$ for $j$ greater than zero have the correct sign and all are significant at well beyond the 1 percent level. The smallest $\hat{R}^2$ in Table 2 is 0.984, which is many times larger than the largest $\hat{R}^2$ in Table 1.

For my data, equation (1) explains almost none of the variance in $\Delta s_{t+1}$. Equation (7) explains almost all of the variance in $\Delta s_{t+1}$. Since equation (7) is a restatement of covered interest parity, the much larger $\hat{R}^2$'s in Table 2 are additional evidence of the efficacy of arbitrage in international financial markets and are consistent with the literature on covered interest parity.

For the U.S.-Canada data, both $\hat{\lambda}_1$ are closer to one than to zero. For the data supplied by Balke and Wohar, which are much better than the data from the Bulletin, both $\hat{\lambda}_1$ are less than two standard errors from one. When the forward premium is part of a correctly specified equation that includes $\Delta f_{t+1}$ and $i_{t+1} - i^*_{t+1}$ there is no forward-bias puzzle in the sense that exchange rates fall when the forward premium predicts that they will rise. The results in Table 2 imply that exchange rates tend to rise when the forward premiums is positive and tend to fall when the forward premium is negative.

Whether $\lambda_1$ equals one remains an issue. The relatively low $\hat{\lambda}_1$ for U.S.-Canada can be explained at least in part by the relatively poor data for U.S.-Canada. The much smaller deviation from one for the U.S.-U.K. is due at least partly to the downward bias created by the transaction costs. Determining whether or not the coefficient for the forward premium is exactly one after accounting for the effects of the transaction costs requires additional research beyond the objectives of this paper.
Table 3
The Bias Due to Omitted Variables

<table>
<thead>
<tr>
<th></th>
<th>$\hat{\beta}_1$</th>
<th>$\hat{\phi}_1$</th>
<th>$\hat{\lambda}_2\hat{\phi}_1$</th>
<th>$\hat{\omega}_1$</th>
<th>$\hat{\lambda}_3\hat{\omega}_1$</th>
<th>$\hat{\lambda}_1$</th>
<th>$\hat{\lambda}_{2}\hat{\phi}_1+\hat{\lambda}_3\hat{\omega}_1$</th>
<th>$\hat{\lambda}<em>1+\hat{\lambda}</em>{2}\hat{\phi}_1+\hat{\lambda}_3\hat{\omega}_1$</th>
</tr>
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<tbody>
<tr>
<td>U.S.-Canada</td>
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<td></td>
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</tr>
<tr>
<td>1-5-1961 to 12-31-1969</td>
<td>-0.425</td>
<td>-0.845</td>
<td>-0.847</td>
<td>0.300</td>
<td>-0.183</td>
<td>0.605</td>
<td>-1.030</td>
<td>-0.425</td>
</tr>
<tr>
<td></td>
<td>(0.175)</td>
<td>(0.171)</td>
<td>(0.036)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S.-Canada</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6-5-1970 to 6-29-1973</td>
<td>0.268</td>
<td>-0.399</td>
<td>-0.393</td>
<td>0.118</td>
<td>-0.128</td>
<td>0.789</td>
<td>-0.521</td>
<td>0.268</td>
</tr>
<tr>
<td></td>
<td>(0.372)</td>
<td>(0.337)</td>
<td>(0.365)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S.-UK</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-2-1974 to 11-1-1983</td>
<td>-1.425</td>
<td>-1.587</td>
<td>-1.587</td>
<td>0.820</td>
<td>-0.827</td>
<td>0.990</td>
<td>-2.414</td>
<td>-1.424</td>
</tr>
<tr>
<td></td>
<td>(0.166)</td>
<td>(0.168)</td>
<td>(0.111)</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>U.S.-UK</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11-2-1983 to 9-30-1993</td>
<td>-3.034</td>
<td>-3.164</td>
<td>-3.161</td>
<td>0.863</td>
<td>-0.865</td>
<td>0.992</td>
<td>-4.026</td>
<td>-3.034</td>
</tr>
<tr>
<td></td>
<td>(0.406)</td>
<td>(0.395)</td>
<td>(0.029)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Standard errors in parentheses.

All of the relevant parameters needed to calculate the bias in equation (1) implied by covered interest parity are reported in Table 3. To save space, only $\phi_1$ and $\omega_1$, and their standard errors, are reported. Table 3 shows that the two omitted variables $\phi_{t+1}$ and $i_{t+1} - i^*_t$ explain all of the downward bias between Tables 1 and 2. The bias due to the omitted variables is $\lambda_2\phi_1 + \lambda_3\omega_1$. That bias is always negative and $\beta_1$ in Table 1 always equals $\lambda_1 + \lambda_2\phi_1 + \lambda_3\omega_1$ in Table 3.  

Tables 1, 2 and 3 include only two pairs of countries with only two intervals each. Before the forward-bias puzzle can be declared finally solved, the results in these three tables should be improved upon and confirmed across various countries and intervals. However, since these are

---

11 The value of $\lambda_1 + \lambda_2\phi_1 + \lambda_3\omega_1$ for the first U.S.-U.K. interval is -1.424 rather than -1.425. But that is the result of rounding errors in Table 3. The computer program reports a value for $\lambda_1 + \lambda_2\phi_1 + \lambda_3\omega_1$ of -1.42475, which rounds off to -1.425.
the only countries and intervals that I have analyzed, my results should hold up over space and time.

5. A Closely Related Anomaly

Wang and Wang (2009) recently discovered an anomaly that is closely related to the forward-bias puzzle. According to Wang and Wang (2009, 186), “… the variance of spot rate changes is in the range of 100-200 times the variance of the forward premium.” My data produce similar results. This large difference in variances, which standard explanations of the forward-bias puzzle do not explain, suggests the omission of at least one important variable.

Covered interest parity provides that variable. It is $\Delta f_{t+1} - (i_{t+1} - i_{t+1}^*)$. As shown in Table 4, the variance of $\Delta s_{t+1}$ is much larger than the variance of $f_t - s_t$ primarily because the variance of $\Delta f_{t+1}$ is about as large as the variance of $\Delta s_{t+1}$.

<table>
<thead>
<tr>
<th></th>
<th>$\Delta s_{t+1}$</th>
<th>$f_t - s_t$</th>
<th>$\Delta f_{t+1}$</th>
<th>$i_{t+1} - i_{t+1}^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.-Canada</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 Jan 1961 to</td>
<td>0.983</td>
<td>0.023</td>
<td>1.009</td>
<td>0.019</td>
</tr>
<tr>
<td>31 Dec 1969</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US-Canada</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 Jun 1970 to</td>
<td>2.201</td>
<td>0.050</td>
<td>2.119</td>
<td>0.026</td>
</tr>
<tr>
<td>29 Jun 1973</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US-UK</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 Jan 1974 to</td>
<td>7.562</td>
<td>0.037</td>
<td>7.545</td>
<td>0.116</td>
</tr>
<tr>
<td>1 Nov 1983</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US-UK</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 Nov. 1983 to</td>
<td>15.315</td>
<td>0.042</td>
<td>15.400</td>
<td>0.039</td>
</tr>
<tr>
<td>30 Sep 1993</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Averages</td>
<td>6.515</td>
<td>0.038</td>
<td>6.518</td>
<td>0.050</td>
</tr>
</tbody>
</table>

As shown by equation (6), ignoring the transaction costs, covered interest parity implies that the variance of $\Delta s_{t+1}$ depends on the variances of $f_t - s_t$, $\Delta f_{t+1}$ and $i_{t+1} - i_{t+1}^*$ and the relevant co-variances.
\[ \Delta s_{t+1} = (f_t - s_t) + \Delta f_{t+1} - (i_{t+1} - i_{t+1}^*) \pm e_{t+1} \tag{6} \]

In Table 4 the variance for \( f_t - s_t \) and \( i_{t+1} - i_{t+1}^* \) are both very small as compared to the variance for \( \Delta s_{t+1} \). But the variance for \( \Delta f_{t+1} \) is essentially the same as the variance for \( \Delta s_{t+1} \). To save space and because they do not appear to be important, Table 4 does not report any co-variances.

Using the same data as the previous tables, Table 4 reports the relevant variances. In Table 4 the average variance for \( \Delta s_{t+1} \) is 6.515. But the average variances for \( f_t - s_t \) and \( i_{t+1} - i_{t+1}^* \) are respectively just 0.038 and 0.050. These estimates for \( \Delta s_{t+1} \) and \( f_t - s_t \) are consistent with those reported by Wang and Wang (2009). However the average variance for \( \Delta f_{t+1} \) is 6.518, which is essentially the same as the average variance for \( \Delta s_{t+1} \). As suggested by covered interest parity, the variance for \( \Delta s_{t+1} \) is large relative to the variance for \( f_t - s_t \) primarily because the variance for \( \Delta f_{t+1} \) is approximately the same as the variance for \( \Delta s_{t+1} \).

Note that here 'because' means only a statistical relationship, not causation. I will take up the issue of causation in future research.

Section 4 shows how covered interest parity can solve the forward-bias puzzle. This section shows how covered interest parity can solve the variance anomaly. The next section shows how covered interest parity solves similar problems with uncovered interest parity (UIP).

6. CIP and UIP

Chinn and Meredith (2004, 409) summarize the UIP literature succinctly as follows: "Few propositions are more widely accepted in international economics than that uncovered interest parity (UIP) is at best useless - or at worst perverse - as a predictor of future exchange rate movements." They also point out (p. 413) that the failure of UIP "must reflect two phenomena: (i) deviations from risk neutrality and/or rational expectations, and (ii) economic channels that generate a correlation between the interest differential and these deviations." They do not seem to consider the possibility that UIP might fail because the crucial assumption that forward exchange rates equal expected future spot rates might fail.

6.1 No Cover

Ignoring any transaction costs, equation (10) describes the *expected* return from investing abroad without cover.\(^{12}\)

\[
[E(s_{t+1}/\Phi_t) - s_t] - (i_t - i_t^*) = E(r/\Phi_t)
\]  

(10)

The expected return from investing abroad without cover denoted \(E(r/\Phi_t)\) equals the uncertain foreign return, which is \(i_t^* + E(s_{t+1}/\Phi_t) - s_t\), minus the certain domestic return, which is \(i_t\). The conventional interpretation of equation (10) is that, when investors require a risk premium to invest without cover, effective competition should produce an expected return for the marginal investor just large enough to compensate that investor for their perceived risk.

Note that, when covered interest parity holds, \(E(r/\Phi_t)\) also equals \(E(s_{t+1}/\Phi_t) - f_t\) because \(f_t\) equals \(s_t + (i_t - i_t^*)\). When \(E(r/\Phi_t)\) is zero, the risky expected return without cover equals the certain return with cover. Otherwise the risky expected return exceeds the certain return.

Typical discussions of UIP such as Aslan and Korap (2010) and Chinn and Meredith (2004) usually express equations like (10) in terms of a risk premium rather than an expected rate

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\(^{12}\) Investing without cover in this context is sometimes called 'uncovered interest arbitrage'. But such usage is an oxymoron because investing without cover involves risks that by definition arbitrage excludes.
of return. I do so in terms of an expected return primarily because as shown later covered
interest parity provides some interesting insights into the expected return.

6.2 Uncovered Interest Parity

Uncovered interest parity typically builds on equation (10) by assuming that investors are
risk neutral. The usual assumption is that, ignoring transaction costs, risk neutrality and effective
competition will drive the expected return for the marginal investor to zero.

\[ [E(s_{t+1}/\Phi_t) - s_t] - (i_t - i_t^*) = 0 \]  

(11)

One widely used approach to uncovered interest parity is through covered interest parity.\(^{13}\)

Using the conventional assumption behind equation (1) that forward exchange rates \( f_t \) equal
expected future spot rates \( E(s_{t+1}/\Phi_t) \) and ignoring transaction costs, covered interest parity
implies uncovered interest parity as expressed in equation (11) because covered interest parity
implies that \( f_t - s_t \) equals \( i_t - i_t^* \).

Of course this reasoning also works in the other direction. If CIP holds, then UIP
implicitly implies that forward exchange rates equal expected future spot rates and that \( E(r/\Phi_t) \) is
zero. As is shown later, the common assumption that current forward rates equal expected future
spot rates, and that therefore \( E(r/\Phi_t) \) is zero, is highly suspect.

Equation (11) is usually made operational by assuming rational expectations. With
rational expectations the actual future spot rate \( s_{t+1} \) equals the expected future spot rate \( E(s_{t+1}/\Phi_t) \)
plus a white noise error term \( u_t \) that is uncorrelated with the information set \( \Phi_t \). That information
set includes the current interest rates \( i_t \) and \( i_t^* \), and the current forward and spot rates \( f_t \) and \( s_t \).

Ignoring the transaction costs and assuming rational expectations produces the standard
test equation for uncovered interest parity.

\[ s_{t+1} - s_t = \alpha_0 + \alpha_1(i_t - i_t^*) + u_t \]  

(12)

\(^{13}\) See for example Chinn and Meredith (2004) and footnote 3.
The forward-bias puzzle and the failure of UIP as expressed by equation (12) are two sides of the same coin. When covered interest parity holds, ignoring the transaction costs, estimating equation (12) is essentially the same as estimating equation (1). Like estimates for $\beta_t$ in the forward-bias literature, estimates for $\alpha_t$ in the UIP literature are typically negative. For example, the average estimate of $\alpha_t$ from Table 1 in Chinn and Meredith (2004), which uses three maturities of 12 months or less and six countries versus the U.S., is -0.99.

As one would expect, the econometric solution used earlier to solve the forward-bias puzzle also explains why estimates of $\alpha_t$ are usually negative. Covered interest parity implies that the same two variables are missing from equation (12) as from equation (1). Equation (13) adds those two missing variables to equation (12)

$$\Delta s_{t+1} = \gamma_0 + \gamma_1(i_t - i_t^*) + \gamma_2\Delta f_{t+1} + \gamma_3(i_{t+1} - i_{t+1}^*) + \epsilon_{t+1}$$

(13)

Table 5 shows the results of estimating equation (12) with the data used in previous tables.

<table>
<thead>
<tr>
<th></th>
<th>$\hat{\alpha}_0$</th>
<th>$\hat{\alpha}_1$</th>
<th>$R^2$/DW</th>
</tr>
</thead>
<tbody>
<tr>
<td>US-Canada 5 Jan 1961 to 31 Dec 1969</td>
<td>0.260</td>
<td>-0.399</td>
<td>0.001</td>
</tr>
<tr>
<td>(0.038)</td>
<td>(0.268)</td>
<td>0.100</td>
<td></td>
</tr>
<tr>
<td>US-Canada 5 Jun 1970 to 29 Jun 1973</td>
<td>-0.433</td>
<td>-0.913</td>
<td>0.013</td>
</tr>
<tr>
<td>(0.123)</td>
<td>(0.502)</td>
<td>0.157</td>
<td></td>
</tr>
<tr>
<td>US-UK 2 Jan 1974 to 1 Nov 1983</td>
<td>0.673</td>
<td>-1.479</td>
<td>0.033</td>
</tr>
<tr>
<td>(0.071)</td>
<td>(0.170)</td>
<td>0.088</td>
<td></td>
</tr>
<tr>
<td>US-UK 2 Nov 1983 to 30 Sep 1993</td>
<td>0.958</td>
<td>-3.125</td>
<td>0.024</td>
</tr>
<tr>
<td>(0.118)</td>
<td>(0.381)</td>
<td>0.071</td>
<td></td>
</tr>
<tr>
<td>Averages</td>
<td>0.364</td>
<td>-1.479</td>
<td>0.018</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.104</td>
</tr>
</tbody>
</table>

Standard errors in parentheses.
All of the $\hat{\alpha}_t$ are negative and the average $\hat{\alpha}_t$ is -1.479. All the $\bar{R}^2$ in Table 5 are small.

The average $\bar{R}^2$ is only 0.018. Like forward premiums, interest rate differentials explain almost none of the variance in $\Delta s_{t+1}$.

Table 6 shows the results of estimating equation (13). All of the $\hat{\gamma}_t$ are positive and significant at well beyond the 1 percent level. As in Table 2 for the forward-bias puzzle, in Table 6 there is no bias. When interest differentials predict that exchange rates will move in one direction they do not tend to move in the other direction.

Table 6
Estimates of Equation 13

<table>
<thead>
<tr>
<th></th>
<th>$\hat{\gamma}_0$</th>
<th>$\hat{\gamma}_1$</th>
<th>$\hat{\gamma}_2$</th>
<th>$\hat{\gamma}_3$</th>
<th>$\bar{R}^2$/DW</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.-Canada</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 Jan 1961 to</td>
<td>0.008</td>
<td>0.497</td>
<td>0.994</td>
<td>-0.591</td>
<td>0.983</td>
</tr>
<tr>
<td>31 Dec 1969</td>
<td>(0.007)</td>
<td>(0.055)</td>
<td>(0.004)</td>
<td>(0.061)</td>
<td>0.206</td>
</tr>
<tr>
<td>US-Canada</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 Jun 1970 to</td>
<td>0.118</td>
<td>0.854</td>
<td>1.063</td>
<td>-0.227</td>
<td>0.950</td>
</tr>
<tr>
<td>29 Jun 1973</td>
<td>(0.030)</td>
<td>(0.138)</td>
<td>(0.015)</td>
<td>(0.061)</td>
<td>0.134</td>
</tr>
<tr>
<td>US-UK</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 Jan 1974 to</td>
<td>-0.000</td>
<td>0.977</td>
<td>1.000</td>
<td>-0.975</td>
<td>0.999</td>
</tr>
<tr>
<td>1 Nov 1983</td>
<td>(0.001)</td>
<td>(0.010)</td>
<td>(0.001)</td>
<td>(0.010)</td>
<td>1.450</td>
</tr>
<tr>
<td>US-UK</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 Nov. 1983 to</td>
<td>0.000</td>
<td>0.966</td>
<td>0.999</td>
<td>-0.967</td>
<td>0.999</td>
</tr>
<tr>
<td>30 Sep 1993</td>
<td>(0.002)</td>
<td>(0.033)</td>
<td>(0.001)</td>
<td>(0.032)</td>
<td>1.733</td>
</tr>
<tr>
<td>Averages</td>
<td>0.032</td>
<td>0.824</td>
<td>1.014</td>
<td>-0.690</td>
<td>0.9823</td>
</tr>
</tbody>
</table>

Standard errors in parentheses.

Just as Table 3 shows the effects of the bias produced by omitting $\Delta f_{t+1}$ and $i_{t+1} - i_{t+1}^*$ from equation (1), Table 7 shows the effects of the bias produced by omitting $\Delta f_{t+1}$ and $i_{t+1} - i_{t+1}^*$ from
equation (12). The interpretation of Table 7 is the same as Table 3 except that in Table 7 $\hat{\phi_1}$ and $\hat{\omega_1}$ are obtained from equations (14) and (15) rather than from equations (8) and (9).

\[
\Delta f_{t+1} = \phi_0 + \phi_1(i_t - i_t^*) \tag{14}
\]

\[
(i_{t+1} - i_{t+1}^*) = \omega_0 + \omega_1(i_t - i_t^*) \tag{15}
\]

<table>
<thead>
<tr>
<th></th>
<th>$\hat{\beta_1}$</th>
<th>$\hat{\phi_1}$</th>
<th>$\hat{\gamma_2\phi_1}$</th>
<th>$\hat{\omega_1}$</th>
<th>$\hat{\gamma_3\omega_1}$</th>
<th>$\hat{\gamma_1}$</th>
<th>$\hat{\gamma_2\phi_1} + \hat{\gamma_3\omega_1}$</th>
<th>$\hat{\gamma_1} + \hat{\gamma_2\phi_1} + \hat{\gamma_3\omega_1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>US-Canada 1-5-1961 to 12-31-1969</td>
<td>-0.399</td>
<td>-0.606</td>
<td>-0.602</td>
<td>0.498</td>
<td>-0.295</td>
<td>0.497</td>
<td>-0.897</td>
<td>-0.399</td>
</tr>
<tr>
<td></td>
<td>(0.268)</td>
<td>(0.268)</td>
<td>(0.034)</td>
<td>(0.055)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US-Canada 6-5-1970 to 6-29-1973</td>
<td>-0.913</td>
<td>-1.377</td>
<td>-1.464</td>
<td>1.339</td>
<td>-0.304</td>
<td>0.854</td>
<td>-1.767</td>
<td>-0.913</td>
</tr>
<tr>
<td></td>
<td>(0.502)</td>
<td>(0.427)</td>
<td>(0.286)</td>
<td>(0.138)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US-UK 1-2-1974 to 11-1-1983</td>
<td>-1.479</td>
<td>-1.635</td>
<td>-1.635</td>
<td>0.842</td>
<td>-0.821</td>
<td>0.977</td>
<td>-2.456</td>
<td>-1.479</td>
</tr>
<tr>
<td></td>
<td>(0.170)</td>
<td>(0.171)</td>
<td>(0.011)</td>
<td>(0.010)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.381)</td>
<td>(0.382)</td>
<td>(0.009)</td>
<td>(0.033)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>-1.479</td>
<td>-1.724</td>
<td>-1.723</td>
<td>0.902</td>
<td>-0.580</td>
<td>0.824</td>
<td>-2.303</td>
<td>-1.479</td>
</tr>
</tbody>
</table>

Standard errors in parentheses.

In Table 7 the two omitted variables $\Delta f_{t+1}$ and $i_{t+1} - i_{t+1}^*$ explain all of the downward bias between Tables 5 and 6. The bias due to the omitted variables is $\hat{\gamma_2\phi_1} + \hat{\gamma_3\omega_1}$. That bias is always negative in Table 7 and $\hat{\alpha_1}$ in Table 5 always equals $\hat{\gamma_1} + \hat{\gamma_2\phi_1} + \hat{\gamma_3\omega_1}$ in Table 7.

The downward bias in the forward-bias puzzle and the downward bias in uncovered interest parity both appear to be the result of omitting the same two variables.
As with $\lambda$, whether or not $\gamma$ is different from one is unclear. The small $\hat{\gamma}$ for U.S.-Canada can be explained at least in part by the relatively poor data for U.S.-Canada. The much smaller deviation from one for the U.S.-U.K. is due at least partly to the downward bias due to the thresholds created by the transaction costs. Determining whether or not the coefficient for the interest rate differential is exactly one after accounting for the effects of the transaction costs will require additional research beyond the objectives of this paper.

As Table 4 shows, the anomaly that Wang and Wang (2009) points out regarding the relative variances of $\Delta s_{t+1}$ and $f_t - s_t$ also applies to $\Delta s_{t+1}$ and $i_t - i_t^*$. In Table 4 the variance for $\Delta s_{t+1}$ is over 100 times greater than the variance for $i_t - i_t^*$. As before, the variance for $\Delta f_{t+1}$ 'explains' the relatively large variance in $\Delta s_{t+1}$.

So far I have concentrated on an econometric solution for the downward bias in both the forward-bias puzzle and in uncovered interest parity. I have not offered an economic explanation for my econometric solution. The next section uses covered interest parity to provide an economic explanation for the forward-bias puzzle and the failure of uncovered interest parity.

7. Forward Rates and Expected Future Spot Rates

The forward-bias puzzle assumes that forward rates equal expected future spot rates. When combined with that assumption, CIP implies UIP. There is a large body of evidence supporting covered interest parity. There also is a comparable body of evidence supporting the forward-bias puzzle and rejecting uncovered interest parity. The obvious way to resolve this apparent conflict in empirical results is to drop the assumption that forward rates equal expected future spot rates. That solution is particularly attractive because covered interest parity suggests that forward rates often do not equal expected future spot rates.
Using covered interest parity and rational expectations, one can derive the conditions under which the forward rate equals the expected future spot rate and $E(r/\Phi_t)$ is zero. Although covered interest parity cannot determine the expected future spot rate it does impose important restrictions on $E(s_{t+1}/\Phi_t)$. Ignoring transaction costs and assuming that expectations are rational and that covered interest parity will hold in the future implies equation (16).

$$E(s_{t+1}/\Phi_t) = E(f_{t+1}/\Phi_t) - E(i_{t+1} - i^*_t/\Phi_t)$$

(16)

When covered interest parity holds and expectations are rational, expected future spot rates must equal expected future forward rates minus expected future interest rate differentials.

Subtracting $f_t$ from both sides of equation (16) describes the expected return from not covering $E(r/\Phi_t)$.

$$E(s_{t+1}/\Phi_t) - f_t = E(r/\Phi_t) = [E(f_{t+1}/\Phi_t) - f_t] - E(i_{t+1} - i^*_t/\Phi_t)$$

(17)

For forward rates to equal expected future spot rates and for $E(r/\Phi_t)$ to be zero so that UIP can hold, expected changes in forward rates must equal expected future interest rate differentials. That condition seems unlikely to hold in general.

Note that the conventional assumption that forward rates equal expected future spot rates eliminates any incentive to invest abroad without cover. When $E(s_{t+1}/\Phi_t)$ equals $f_t$ and $E(r/\Phi_t)$ is zero, only a risk lover would invest abroad without cover because the certain return with cover equals the risky return without cover. As a result, when CIP holds I find it difficult to see how a conventional risk premium can cause deviations from UIP. From the perspective of equation (17), none zero $E(r/\Phi_t)$ cause both the deviations from UIP and any resulting risk premiums.

Also note that $[E(f_{t+1}/\Phi_t) - f_t]$ and $E(i_{t+1} - i^*_t/\Phi_t)$ are the expectation equivalents of $(f_{t+1} - f_t)$ and $(i_{t+1} - i^*_t)$ respectively. These are the terms that covered interest parity implies are missing.
from equations (1) and (12). They appear to explain the downward bias in both uncovered interest parity and the forward-bias puzzle.

Why might the common and apparently reasonable assumption that forward rates equal expected future spot rates not hold in general? A full exploration of that issue is beyond the scope of this paper. But I can provide a starting point. Assume an initial equilibrium with no change in nominal or relative prices where domestic and foreign real interest rates are equal. In that equilibrium, with rational expectations \( E(f_{t+1}/\Phi_t) - f_t \) minus \( E(i_{t+1} - i_{t+1}^*/\Phi_t) \) is zero because both \( E(f_{t+1}/\Phi_t) - f_t \) and \( E(i_{t+1} - i_{t+1}^*/\Phi_t) \) are zero. In that steady state forward rates equal expected future spot rates. Both covered and uncovered interest parity hold.

Now consider an unanticipated increase in the stock of money in the home country at \( t \). In the new equilibrium all nominal domestic prices are higher by some percentage of the increase in the stock of money including spot and forward exchange rates. All relative prices are the same in the new equilibrium as in the old equilibrium.\(^{14}\) For simplicity of exposition, I also assume that the new equilibrium is achieved by \( t+2 \). From the old to the new equilibrium there is only pure inflation with no real effects in the sense that relative prices are the same in both equilibria.

In \( t+2 \) spot rates, forward rates and expected future spot rates are again all the same. They all are just higher than they were in \( t \). In addition \( [E(f_{t+3}/\Phi_{t+2}) - f_{t+2}] - E(i_{t+3} - i_{t+3}^*/\Phi_{t+2}) \) is zero in \( t+2 \) because both \( E(f_{t+3}/\Phi_{t+2}) - f_{t+2} \) and \( E(i_{t+3} - i_{t+3}^*/\Phi_{t+2}) \) are zero. So in the new equilibrium as in the old equilibrium forward rates equal expected future spot rates. Covered and uncovered interest parity hold in the new equilibrium as they did in the initial equilibrium.

But during the transition forward rates may not equal expected future spot rates. In that case there is an expected return from not covering even without a risk premium.

\(^{14}\) The assumption of constant relative prices assures the existence of a unique real interest rate.
For there to be no expected return between $t$ and $t+1$, $E(s_{t+1}/\Phi_t) - f_i$ must be zero. For that to be true $[E(f_{t+1}/\Phi_t) - f_i]$ minus $E(i_{t+1} - i_{t+1}^*/\Phi_t)$ must be zero. While it is obvious that expected returns will be zero in both the initial and new equilibria, it is not obvious that they must be zero during the transition. Suppose a liquidity effect produces a zero or negative $E(i_{t+1} - i_{t+1}^*/\Phi_t)$ while an expected rise in commodity prices produces a positive $[E(f_{t+1}/\Phi_t) - f_i]$. Under those conditions the expected return in $t$ would be positive.

Now assume a similar initial condition, but this time the shock is real. Domestic real interest rates rise while foreign real interest rates do not. With no change in the stock of money, in the new 'equilibrium' nominal commodity prices presumably are constant and the expected rate of inflation is zero in both countries.15 But the expected interest differential in the new equilibrium is positive because the domestic real rate is higher than the foreign real rate. Under these conditions, the only way that $E(s_{t+1}/\Phi_t)$ can equal $f_i$ in the new equilibrium is for the expected depreciation of forward and spot rates to equal the difference in real interest rates.16 But in the absence of any changes in the stock of money, and therefore presumably any changes in nominal commodity prices, continual expected depreciation does not seem possible. It would appear that forward rates could not equal expected future spot rates in the new 'equilibrium'.

Both the forward-bias puzzle and uncovered interest parity assume that forward rates equal expected future spot rates. When covered interest parity holds, that assumption implies that $E(r/\Phi_t)$ is zero. But that assumption is highly suspect. It appears that forward rates can deviate from expected future spot rates under a variety of reasonable conditions. Those deviations can

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15 Equilibrium appears as 'equilibrium' because real interest rate differentials create internal inconsistencies that are difficult to reconcile with a true equilibrium.

16 Note that CIP implies that $E(s_{t+1}/\Phi_t) - s_i = [E(f_{t+1}/\Phi_t) - f_i] - [E(i_{t+1} - i_{t+1}^*/\Phi_t) - (i_{t+1} - i_{t+1}^*)]$. Since the interest rate differential does not change in the new equilibrium, in the new equilibrium $E(s_{t+1}/\Phi_t) - s_i$ equals $E(f_{t+1}/\Phi_t) - f_i$. 

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explain the bias in both uncovered interest parity and the forward-bias puzzle without resorting to either a risk premium or the rejection of rational expectations.

As Tables (3) and (7) show, the econometric source of the downward bias is the correlation between either the forward premium or the interest differential on the one hand and the future change in the forward exchange rate and future interest rate differential on the other hand. These correlations can be explained without resorting to either a risk premium or the rejection of rational expectations.

As equation (5) shows, ignoring transaction costs, the actual return from not covering a foreign investment is \((f_{t+1} - f_t) - i_{t+1}^* - i_{t+1}^*\). With rational expectations, \((f_{t+1} - f_t)\) equals \([E(f_{t+1}/\Phi_t) - f_t]\) plus an error denoted \(\mu_t\). The actual future interest rate differential \(i_{t+1} - i_{t+1}^*\) equals \(E(i_{t+1} - i_{t+1}^*/\Phi_t)\) plus an error denoted \(\nu_t\). \([E(f_{t+1}/\Phi_t) - f_t] - E(i_{t+1} - i_{t+1}^*/\Phi_t)\) or \(E(r/\Phi_t)\) is the expected return from not covering and \(\mu_t - \nu_t\) is the unexpected return from not covering.

Rational expectations imply that both \(\mu_t\) and \(\nu_t\) are uncorrelated with the observed current forward premium in Table 3 and the observed current interest rate differential in Table 7. With rational expectations, neither error can be the source of the significant estimates for \(\hat{\phi}_1\) and \(\hat{\omega}_1\) in Tables (3) and (7).\(^{17}\)

However rational expectations do not preclude \([E(f_{t+1}/\Phi_t) - f_t]\) and \(E(i_{t+1} - i_{t+1}^*/\Phi_t)\) from being correlated with the current forward premium and the current interest rate differential because both \(f_t - s_t\) and \(i_t - i_t^*\) are part of the information set \(\Phi_t\) on which the expectations \([E(f_{t+1}) - f_t]\) and \(E(i_{t+1} - i_{t+1}^*)\) are based. As a result, current forward premiums and current interest rate differentials can be correlated with expected returns from not covering even when expectations are rational and there is no risk premium.

\(^{17}\) Neither \(\hat{\phi}_1\) nor \(\hat{\omega}_1\) is statistically significant for the second Canadian interval in Table 3. But both \(\hat{\phi}_1\) and \(\hat{\omega}_1\) have the correct sign.
I believe that the econometric results in Tables (3) and (7) are at least partly due to the failure of the assumption that forward exchange rates equal expected future spot rates. Whether or not the failure of rational expectations and/or the existence of a risk premium also contribute to those results remains an open issue. But neither the failure of rational expectations nor the existence of a risk premium is required to explain the forward-bias puzzle or the failure of uncovered interest parity.

There are two related anomalies that I make no attempt to explain here. Frankel and Poonawala (2010) show that the forward-bias is smaller for emerging-market currencies than for advanced country currencies. Given the similarity between the forward-bias puzzle and uncovered interest parity, one would expect something similar to hold for uncovered interest parity.

Chinn and Meredith (2004) show that uncovered interest parity works better for assets with a maturity of over 12 months than for assets with a maturity of 12 months or less. Given the similarity between uncovered interest parity and the forward-bias puzzle, one would expect something similar to hold for the forward-bias puzzle.

If the failure of forward rates to equal expected future spot rates is the source of both the forward-bias puzzle and the failure of uncovered interest parity, then that inequality should help explain the anomalies discovered by Frankel and Poonawala (2010) and Chinn and Meredith (2004). I leave it to others to determine whether or not that is the case.

8. Summary and Conclusions

A complete solution to the forward-bias puzzle should provide an econometric solution and an economic explanation for that econometric solution. It should also do the same for the failure of the closely related theory of uncovered interest parity.
In addition, a complete solution to the forward-bias puzzle should explain three related anomalies: (1) Why is the variance for changes in exchange rates over 100 times greater than the variance for either the forward premium or the interest rate differential? (2) Why are forward markets in emerging currencies less biased than in major currencies? (3) Why does uncovered interest parity work better for maturities over 12 months than for maturities of 12 months and less?

I provide an econometric solution for the forward-bias puzzle and for why uncovered interest parity fails. I also provide an economic explanation for that solution. My econometric solution is that in both cases the standard test equation omits two variables that covered interest parity implies should be included. My economic explanation for that econometric solution is that, contrary to the conventional assumption, covered interest parity implies that in general forward exchange rates do not equal expected future spot rates.

My solution to the forward-bias puzzle and the failure of uncovered interest parity provides a simple explanation for why the variance for changes in exchange rates is over 100 times larger than the variance for either forward premiums or interest rate differentials.

For my solution and explanation to be fully accepted future research needs to do at least three things: (1) Confirm my econometric solution for the forward-bias puzzle and the failure of uncovered interest parity using better data and econometric techniques that more fully account for the effects of transaction costs. (2) If possible, confirm that forward rates do not in general equal expected future spot rates. (3) Show that my econometric solution helps explain why the forward-bias is smaller for emerging currencies than for major currencies and why uncovered interest parity works better for long maturities than for short maturities.
References


