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Publication Date
1963-06-01
University of California

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UNIVERSITY OF CALIFORNIA
Lawrence Radiation Laboratory
Berkeley, California
Contract No. W-7405-eng-48

AN INSTABILITY IN THREE PHASE MAGNETIC AMPLIFIER OPERATION

Leslie Terence Jackson
(M. S. Thesis)

June, 1963

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AN INSTABILITY IN
THREE-PHASE MAGNETIC AMPLIFIER OPERATION

Degree: Master of Science           Leslie T. Jackson

Major Subject: Electrical Engineering

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ABSTRACT

The three-phase magnetic amplifier with parallel control windings
is incrementally represented by the difference equations in Laplace
Transform notation,

\[
\frac{E_0(s)}{NE_c(s)} = \frac{1}{1 + GN^2 R_c - e^{-3Ts}} \left[ \frac{3a}{n} e^{-Ts} + e^{-2Ts} + e^{-3Ts} + (1 - \frac{3a}{n}) e^{-4Ts} \right]
\]

for greater than half-output, and

\[
\frac{E_0(s)}{NE_c(s)} = \frac{1}{1 + GN^2 R_c - e^{-3Ts}} \left[ e^{-2Ts} + e^{-3Ts} + e^{-4Ts} \right]
\]

for less than half-output, when the speed of response is eight half-cycles
or greater. These transfer functions are experimentally verified after
an unstable region is eliminated by degeneration.

The time constant expression from the difference equation is

\[
\tau = \frac{1}{\ln (1 + GN^2 R_c)} \text{ half-cycles.}
\]

This expression is verified by experimental time constant measurements
on a test magnetic amplifier. The difference equations are modified if
diode unblocking occurs or the resetting cores share voltage rather than
the core coming out of saturation absorbing all the voltage. For a control
resistance high enough that the exponential portion of the transient re-
sponse is eliminated, the delayed sixth-cycle steps which occur in response
to an input step are predicted by transfer functions incorporating the
diode unblocking and voltage sharing modes of operation.

The unstable region occurs below half-maximum output and is caused
by the positive feedback from the output through the gating core to the
resetting core becoming greater than unity. This regeneration is related
to voltage spikes coupled into the resetting core from the saturated gate
core when a core in another phase saturates. Sufficient degeneration to
eliminate the instability can be achieved by adding small chokes in each
parallel control branch, or slightly unbalancing the output by adding a
choke in one branch only. High control resistance or an inductive load
also eliminates the regeneration.
I. INTRODUCTION

The object of this thesis is to develop a dynamic representation of the three-phase bridge magnetic amplifier with the control windings of each phase in parallel. A further object is to describe the instabilities inherent in the circuit and the means of eliminating them.

The three-phase bridge magnetic amplifier is used extensively where the power requirements are in excess of one kilowatt. In these high power applications the larger number of components used with three-phase power in comparison to those used with single phase power is more than compensated for by:

1. Greater utilization of magnetic components and rectifiers.
2. Output ripple components greatly reduced,
3. Even distribution of the load over all three phases of the supply system.\(^2\)

The most commonly used control circuit for the three-phase magnetic amplifier has all six control windings connected in series. Severe instabilities can occur throughout the range of operation. The instability causes output voltage to change ten to twenty percent of the maximum voltage without any change in the applied control voltage. Two or three of these instabilities might occur over the range of operation, giving a multiple step appearance to the transfer curve of the output voltage versus control voltage.

The cause of these instabilities can be traced to dissimilarities in the characteristics of the magnetic cores. The different excitation current requirements of the cores creates a situation where a given
average control current satisfies the equilibrium requirements for two
different output levels. When one of these current levels is reached,
the instability will result.

A solution which has proven successful on large supplies is to
connect in parallel the three sets of two control windings per phase.
A resistor is also connected in each of these three branches. This
connection allows the sets of reactors to share dynamically the average
control voltage. This connection has been used extensively at the
Lawrence Radiation Laboratory in Berkeley where forty-two of this type
of large magnetic amplifier are now being used to power large magnets.

For many applications the three-phase magnetic amplifier must
perform in high performance closed-loop systems. In the operation of
large magnets used in experimental physics the current must be regulated
to better than ± 0.1% in the face of substantial perturbations in the
line voltage and magnet resistance. It is therefore most desirable to
know the dynamic characteristics of the magnetic amplifier in order to
determine the stability and response to perturbations of the system.
The need for accurate analytical representation is even greater when
the system is to follow a dynamic function where the absolute error is
critical. Because of the need for accurate representation and a clearer
understanding of the modes of operation of the three-phase magnetic
amplifier with parallel-connected control windings, this study was
undertaken.
DEFINITION OF SYMBOLS

e_s, E_s = instantaneous and half-cycle average gate supply voltage

e_o, E_o = instantaneous and half-cycle average output voltage

e_c, E_c = instantaneous and half-cycle average control voltage

e_A, e_B, E_A, E_B = instantaneous and half-cycle average induced voltages in reactors A and B of any chosen phase

\( G \) = core conductance (mhos)

\( G(s) \) = transfer function in the Laplace notation of \( \frac{E_o(s)}{E_c(s)} \)

\( i_c \) = instantaneous current in control winding of reactor

\( I_{m0} \) = magnetizing current corresponding to zero reset

\( K_{vo} \) = normalized voltage gain

\( L_c \) = control circuit inductance in each parallel branch

\( N_c, N_g, N \) = control turns, gate turns and turns ratio \( \frac{N_g}{N_c} \)

\( n \) = one-sixth cycle designation \( (n = 1, 2, \ldots, 6) \)

\( R_c \) = control circuit resistance in each parallel branch

\( \text{sgn} e \) = instantaneous polarity of voltage \( e \)

\( T \) = sixth cycle period of supply voltage \( (T = \frac{1}{6f}) \)

\( \tau \) = time constant (seconds)

\( \alpha \) = saturation angle

\( \Delta \Phi_s \) = amount of flux change during gating and resetting periods
II. THEORETICAL RESPONSE

The dynamic relationship between the control voltage and the output voltage of any magnetic amplifier always involves a time delay. Changes made in the control voltage during a reset period do not start to appear as changes in the output voltage until after the reactor gates during a following period. Usually the total excursion will extend over many periods before the output reaches the new steady state operating point required by the changed control voltage. This time delay is caused by the feedback from the output through the gating core to the resetting core. To gain faster speed of response the control circuit resistance is increased. Higher control resistance requires greater control voltage which decreased the affect of the feedback and gives faster response but lower gain.

The purpose of this chapter is to develop the relationship between the control and output voltages of a three-phase bridge magnetic amplifier with parallel control windings as shown in Figure 1. In developing the equations for this circuit the waveshapes shown in Figure 2 are very useful in determining the states of the various circuit elements at a given time. The three-phase bridge magnetic amplifier with all the control windings in series has been analyzed by Flairty and Storm. The circuit operation with parallel control windings is more readily analyzed than that of the series connection. With the control windings in series all the gating core voltages are effectively summed and then divided between the resetting cores.
Figure 1. Three-Phase Magnetic Amplifier
Experimental Set-up

N_c = 100 turns
N_g = 300 turns

Variable Voltage Source

Hewlett-Packard Current Probe 428B
oscilloscope trigger

R_c = 20Ω
L_3

Weston Lab Standard Voltmeter

10Ω

e_c

3A

2A

1N2074

100KL

1 ufd

e_s1

2B

3A

3B

25Ω

25 watts

vert.

Tektronics 502 Oscilloscope
Fig. 2 Supply and output voltage waveshapes of three-phase bridge magnetic amplifier at two different outputs (α = 45° and 90°)
This summation results in complicated core-resetting voltages whose characteristics logically divide the output range into four regions for purposes of analysis. There are two regions of operation when the output voltage is below one-half maximum and two regions of operation when it is above one-half maximum. With the parallel connection of the three sets of two control windings per phase only the two cores of a given phase act upon each other through the control circuit. The control source must be low impedance compared to the series resistors in each parallel branch for this isolation of the phases in the control circuit. With the parallel connection there are two modes of operation which occur above and below one-half maximum output.

The actual waveshapes of the output voltage of the model three-phase magnetic amplifier constructed for this study shown in figure 3 are in good agreement with those of figure 2. The high degree of uniformity maintained at all operating levels is due to the fact that factory matched grain-oriented fifty percent nickel-iron alloy cores are used in the circuit (see Appendix 1 for the parameters). The B-H loops shown in figure 4 and 5 for control resistors of 10 ohms and 100 ohms show that the increased control voltage required with greater control resistance to maintain the average exciting current produces a larger percentage of the reset flux.

Core voltages waveshapes and the corresponding B-H loops are shown in figures 6 through 11 for output voltage levels of 10, 7, and 5 volts. In addition the diode current waveshapes are shown in figures 6 and 8 and the diode voltage in figure 10. These curves in conjunction with those of figure 2 are useful in following core 1A through its cycle of
Fig. 3 Voltage wave shapes for $E_0 = 3, 6, 11, 18$ volts - 5 volts/cm

Fig. 4 Reactor B-H loop with $R_c = 100$ ohms 50 mv/cm vert. 5 mv/cm horiz.

Fig. 5 Reactor B-H loop with $R_c = 10$ ohms 50 mv/cm vert. 5 mv/cm horiz.

ZN-3799
operation. Figures 6 and 7 are for an output voltage just above half-maximum ($a < 60^\circ$) and are representative of the characteristics for $0^\circ < a < 60^\circ$. The negative polarity half of the voltage waveshape is the gating half-cycle, and the corresponding saturation current through the core is shown in the upper part of the figure. The saturation current is continuous as core 3B saturates before $e_{s_1} - e_{s_2}$ goes to zero (see figure 2).

When the output voltage is less than half-maximum the spike of voltage generated by core 2B saturating moves into the gating period of core 1A as shown in figure 8. The B-H loop of figure 9 also shows the occurrence of the voltage spikes that couple into core 1A from other saturating cores. These spikes occur when core 1A is at the beginning of gating, the beginning of resetting, and during the period of low reset occurring at the end of the resetting period. This last period of reset occurs when core 1B has saturated and core 1A is acted on only by the control voltage as expressed in equation 2.8 with $e_{1B} = 0$. The small amount of resetting done with $e_{1B} = 0$ moves the core closer to its static excitation level on the B-H loop which is most clearly observed with higher control resistance, as in figure 4. The effect of the higher control voltage necessary with larger $R_c$ on the excitation level is observed in comparing figures 4 and 5 for control resistances of 100 ohms and 10 ohms respectively.

The diode voltage shown in figure 10 exhibits a blocking voltage during the entire reset half cycle. In the analysis which follows, this continuity of blocking voltage at low outputs determines the mode of
operation assumed in this region. The presence of blocking voltage when there is no pulse of output voltage is due to the magnetizing current voltage drop across the load and the approximately 0.7 volt drop required across the silicon diode before current flows, both opposing the unblocking voltage of the resetting core. The derivation of the expressions for the operation of the three-phase magnetic amplifier will follow the approach used by Professor H. C. Bourne in describing single-phase operation and unpublished notes on three-phase magnetic amplifiers. The expression for the behavior of the core in the active region will also be that used by Professor Bourne. The core function is approximated by

\[ i_m = I_{m0} \text{sgn} \varepsilon_{\text{core}} + G \varepsilon_{\text{core}} \]  \hspace{1cm} (2.1)

in which all quantities are referred to the output or gate circuit. This expression states that the total exciting current at any instant is equal to the static exciting current plus the exciting current required for a given rate of change of flux. Square-loop core materials do not normally require an inductance term.

The description of the operation of the three-phase magnetic amplifier is divided into two modes because as the output goes through half-maximum output the saturation angle moves from one-sixth cycle period into the next period. The change of the period during which saturation occurs requires two sets of equations to describe the output voltage. Designating the sixth-cycle periods as \((n+1)\)--\((n+7)\) as in figure 2 the following expressions may be written for outputs greater and less than
Fig. 6. Top: diode current.  
Bottom: reactor voltage, $E_0 = 10$ volts.  
5 volts/cm.

Fig. 7. B-H loop for $E_0 = 10$ volts.  
5 mv/cm horiz., 50 mv/cm vert.

Fig. 8. Top: diode current.  
Bottom: reactor voltage, $E_0 = 7$ volts.

Fig. 9. B-H loop for $E_0 = 7$ volts.  
ZN-3800
Fig. 10  Top: diode voltage  
bottom: reactor voltage  
$E_0 = 5$ volts

Fig. 11  B-H loop for $E_0 = 5$ volts
half maximum. For \( a < 60^\circ \)

\[ E_0(n+6) = E_s - E_{1A}(n+6). \]  

For \( a > 60^\circ \)

\[ E_0(n+7) = \frac{E_s}{2} - E_{1A}(n+7). \]  

These expressions and most that follow are in terms of the sixth-cycle averages of the variables. The output voltages during the \((n+6)\) and \((n+7)\) periods are chosen to permit expression of these voltages in terms of the control voltages and output voltages during the preceding six periods. As the amount of flux change during resetting and gating must be equal, the net voltage applied to a core over a cycle must be zero. Therefore, the expression for core 1A during a cycle of operation is for \( a < 60^\circ \)

\[ E_{1A}(n+6) + E_{1A}(n+5) + E_{1A}(n+4) + E_{1A}(n+3) + E_{1A}(n+2) + E_{1A}(n+1) = 0. \]  

Substitution into equation 2.2 gives

\[ E_0(n+6) = E_s + E_{1A}(n+5) + E_{1A}(n+4) + E_{1A}(n+3) + E_{1A}(n+2) + E_{1A}(n+1). \]  

For \( a > 60^\circ \) the core voltage over a cycle is

\[ E_{1A}(n+7) + E_{1A}(n+6) + E_{1A}(n+5) + E_{1A}(n+4) + E_{1A}(n+3) + E_{1A}(n+2) = 0. \]  

Substitution into equation 2.3 yields

\[ E_0(n+7) = \frac{E_s}{2} + E_{1A}(n+6) + E_{1A}(n+5) + E_{1A}(n+4) + E_{1A}(n+3) + E_{1A}(n+2). \]  

The core 1A voltages for the \((n+1)\)--\((n+5)\) sixth-cycles for \( a < 60^\circ \) and the \((n+2)\)--\((n+6)\) sixth-cycles for \( a > 60^\circ \) can be found as functions
of the output and control voltages during each period.

By describing a complete cycle of core 1A of phase 1, all the possible states of all the cores are described since the other phases are displaced in time from phase 1. Having chosen core 1A to follow through a complete cycle of operation, the \((n+6)\) and \((n+7)\) periods are designated as the output periods because the periods which precede and control these periods are shown in figure 2. A requirement of this type of piecewise-linear analysis is that the starting point is again reached after one complete cycle. In all the periods to be described the control voltage is related to the core voltages by writing the control circuit equation, which in the case of the parallel control windings studied here only involves the control voltage and the core voltages of one phase.

The control circuit loop equation is

\[
N_0 c = N_1 R_c + e_{1A} + e_{1B}
\]

referred to the gate winding. During the sixth cycles other than those of equations 2.2 and 2.3 core 1A is resetting. The control current must satisfy the requirements of equation 2.1 because only the exciting current for the resetting core 1A flows as long as diode 1A is blocked. The diode is blocked for the test magnetic amplifier with \(R_c = 20 \text{ ohms}\) as is shown in figure 10 and previously described. The core voltage is negative during resetting and therefore equation 2.1 becomes

\[
\frac{i_c}{N} = -I_{mo} + Ge_{1A}
\]
Substituting into equation 2.6 and averaging over a sixth-cycle gives
\[ NE_c(n+m) = -I_{m0} N^2 R_c (n+m) + (1 + GN^2 R_c) E_{1A}(n+m) + E_{1B}(n+m) \]  \[ 2.10 \]
where \( m \) is the sixth cycle under consideration.

Equation 2.10 will now be used to solve for \( E_{1A} \) during all the resetting periods. The relationships will first be developed for \( \alpha < 60^\circ \).

**Period (n+1)**

\[ E_{1A}(n+1) = 0 \]  \[ 2.11 \]
as the core is saturated during this period.

**Period (n+2)**

When core 2A saturates during the \( (n+2) \) period, diode 1A blocks and core 1A comes out of saturation. Two possible modes of operation can then occur for the remainder of the \( (n+2) \) period. Core 1A and core 1B (which has been resetting) can share the control voltage. The second possibility is that core 1A will absorb all the control voltage because it has just come out of saturation and therefore requires less exciting current. Assuming the second alternative and substituting \( E_{1B}(n+2) = 0 \) into equation 2.10, the expression for \( E_{1A} \) is

\[ E_{1A}(n+2) = \frac{1 - \frac{3a}{n}}{1 + GN^2 R_c} (NE_c + I_{m0} N^2 R_c)(n+2). \]  \[ 2.12 \]

Equation 2.12 shows that \( E_{1A}(n+2) \) goes to zero as \( \alpha \rightarrow 60^\circ \).

**Period (n+3)**

The output voltage during the \( (n+3) \) period is described as

\[ E_0(n+3) = E_s - E_{1B}(n+3). \]  \[ 2.13 \]

Solving for \( E_{1B}(n+3) \) and substituting in equation 2.10 gives
\[ E_{1A}(n+3) = \frac{1}{1 + G N^2 R_c} \left[ (NE_c + I_{mo} N^2 R_c)(n+3) + E_o(n+3) - E_s \right]. \quad 2.14 \]

**Period (n+4)**

Since core 1B is saturated during this period, \( E_{1B} = 0 \), and equation 2.10 gives

\[ E_{1A}(n+4) = \frac{1}{1 + G N^2 R_c} (NE_c + I_{mo} N^2 R_c)(n+4). \quad 2.15 \]

**Period (n+5)**

The voltage absorbed by core 1A during the (n+5) period is the complement of that absorbed in the (n+2) period as here it is core 1B that comes out of saturation and absorbs all the control voltage for the remainder of the period. Equation 2.10 yields

\[ E_{1A}(n+5) = \frac{3a}{n} \left( NE_c + I_{mo} N^2 R_c \right)(n+5). \quad 2.16 \]

Substitution of these five voltage expressions into equation 2.5 yields

\[ E_o(n+6) = \frac{1}{1 + G N^2 R_c} \left[ (1 - \frac{3a}{n})(NE_c + I_{mo} N^2 R_c)(n+2) + \right. \]

\[ (NE_c + I_{mo} N^2 R_c)(n+4) + \left( \frac{3a}{n} \right)(NE_c + I_{mo} N^2 R_c)(n+5) + \]

\[ E_o(n+3) + G N^2 R_c E_s \] \quad 2.17

which is the difference equation representation of the output voltage for greater than half-maximum output.

The same procedure will now be followed to determine the output voltage for a >60°.
Period (n+2)

\[ E_{1A}(n+2) = 0 \]

as the core is saturated all during this period.

Period (n+3)

The saturation angle of core 1A has moved into the (n+4) period, and therefore

\[ E_{1B}(n+3) = \frac{E_s}{2} \]

Substitution in equation 2.10 and solution for \( E_{1A} \) gives

\[ E_{1A}(n+3) = \frac{1}{1 + G N^2 R_c} \left[ (N E_c + I_{mo} N^2 R_c)(n+3) - \frac{E_s}{2} \right] \].

Period (n+4)

\[ E_{1B} = \frac{E_s}{2} - E_o(n+4) \]

Substitution into equation 2.10 and solution for \( E_{1A} \) gives

\[ E_{1A}(n+4) = \frac{1}{1 + G N^2 R_c} \left[ (N E_c + I_{mo} N^2 R_c)(n+4) + E_o(n+4) - \frac{E_s}{2} \right] \].

Period (n+5)

Core 1B is saturated during the (n+5) period and core 1A is being reset by the control voltage during this time only as long as diode 1A is blocked. During the period after core 1B has saturated and conducted load current, but before it saturates again when core 3A fires, the only voltage blocking diode 1A is the voltage across the load resistor generated by the exciting current for core 3A. Of course, the diode itself requires approximately 0.7 volt before appreciable forward current will flow. In the magnetic amplifier shown in figure 1 the
exciting current through the 25 ohm load resistor is 0.7 volt which is sufficient to block diode 1A since only 0.4 volt appears across resetting core 1A in the opposite polarity (see figure 10 top). The equation for $E_{1A}$ from equation 2.10 is then

$$E_{1A}(n+5) = \frac{1}{1 + G N^2 R_c} \left( N E_c + I_{mo} N^2 R_c \right)(n+5). \quad 2.23$$

If the diode is unblocked during part of the cycle, gate-limited resetting results where the resetting core is clamped to the load voltage after the diode unblocks. When gate-limited reset occurs equation 2.23 is modified by introducing a coefficient before the existing term which goes to zero when the diode unblocks, and by adding a second term representing the gate-limited reset which appears when the diode unblocks. When the control resistance is much greater than that required to cause unblocking, the gate-limited reset term is negligible and the expression becomes

$$E_{1A}(n+5) = \frac{1}{1 + G N^2 R_c} \left[ 1 - \frac{2}{\pi} \left( \frac{\pi}{2} \right) \right] \left( N E_c + I_{mo} N^2 R_c \right)(n+5) \quad 2.24$$

Period $(n+6)$

$$E_{1A}(n+6) = 0 \quad 2.25$$
as the core is saturated all during the period.

Substitution of these five expressions for $E_{1A}$ into equation 2.7 gives

$$E_o(n+7) = \frac{1}{1 + G N^2 R_c} \left[ (N E_c + I_{mo} N^2 R_c)(n+3) + (N E_c + I_{mo} N^2 R_c)(n+4) 
+ (N E_c + I_{mo} N^2 R_c)(n+5) + E_o(n+4) - E_s \right] \quad 2.26$$
which is the representation of the output voltage for less than half-maximum output.

**Transformed output equation and transfer function**

To transform equations 2.17 and 2.26 into useful analytical expressions, the nature and limitations of the sixth-cycle averages used in the previous analysis must be understood. To use sixth-cycle averages to describe the output voltage the actual waveshape cannot be of importance. This is generally true in pulse-width modulated systems where the carrier is a much greater frequency than the signal, and only the average output is important. Since the change of flux in a core is a function of volt-seconds, the control voltage can assume any shape as long as abnormal behavior does not result and the sixth-cycle averages of the different inputs remain the same. Normally, however, the input is a continuous function of much slower frequency than the carrier and can be approximated by a train of pulses which have a constant magnitude during any sixth-cycle period. If the input and output are represented by a train of pulses, then an extended Laplace Transform technique, the jump function transform, is applicable. With this technique the Laplace transformed output during the n period is now $E^o_0(s)$ and that during the (n+1) period is $E^o_0(s)e^{-Ts}$.

The incremental portions of equations 2.17 and 2.26 may now be transformed as

$$E^o_0(s) = \frac{1}{1 + GN^2R_c} \left[ NE^c(s) \left( \frac{2a}{n} e^{-Ts} + e^{-2Ts} + e^{-3Ts} + (1 - \frac{2a}{n}) e^{-4Ts} \right) + E^o_0(s)e^{-3Ts} \right]$$

2.27
for a $< 60^\circ$, and
\[
E_o(s) = \frac{\frac{1}{1 + GN^2R_c}}{\left[NE_c(s)(e^{-2Ts} + e^{-3Ts} + e^{-4Ts}) + E_o(s)e^{-3Ts}\right] 2.28}
\]

for a $< 60^\circ$. Factoring equations 2.27 and 2.28 provides the following transfer functions:

\[
\frac{E_o(s)}{NE_o(s)} = G(s) = \frac{1}{1 + GN^2R_c e^{-3Ts}} \left[\frac{2\pi}{\pi} e^{-Ts} + e^{-2Ts} + e^{-3Ts} + (1 - \frac{3a}{\pi}) e^{-4Ts}\right] 2.29
\]

for a $< 60^\circ$, and

\[
\frac{E_o(s)}{NE_o(s)} = G(s) = \frac{1}{1 + GN^2R_c e^{-3Ts}} \left[e^{-2Ts} + e^{-3Ts} + e^{-4Ts}\right] 2.30
\]

for a $> 60^\circ$. The steady-state voltage gain may be obtained by letting $s = 0$ to yield

\[
K_vo = \frac{E_o}{NE_o} \bigg|_{s=0} = \frac{3}{GN^2R_c} 2.31
\]

for both expressions.

The poles of the transfer functions are the same and may be determined from the denominator of equation 2.29 as follows:

\[
l + GN^2R_c - e^{-3Ts} = 0
\]

\[
ln e^{-3Ts} = ln (1 + GN^2R_c)
\]

\[
3Ts \pm j2\pi k = -ln (1 + GN^2R_c) \quad k = 0,1,2,3---
\]

\[
s = -\frac{1}{3T} ln (1 + GN^2R_c) + j \frac{2\pi}{T} k.
\]
The dominant pole on the negative real axis corresponds to a time constant of
\[ \tau = \frac{3T}{\ln (1 + GN^2 R_c)} \text{ seconds.} \quad 2.32 \]

There are an infinite number of complex poles associated with this pole with the same negative real parts and spaced at phase intervals of $2\pi$ on the imaginary axis.

If $GN^2 R_c \ll 1$, a useful approximation for the time constant is
\[ \tau = \frac{3T}{GN^2 R_c} \text{ seconds} \quad 2.33 \]

or
\[ \tau = \frac{2}{GN^2 R_c} = K_{vo} \text{ sixth-cycles.} \quad 2.34 \]

Equation 2.34 is also obtained by using the expansion for the time delay
\[ e^{-mTs} = 1 - mTs \]

in equation 2.29.

Summary

If the time response is long enough to permit the use of the approximation
\[ \tau = \frac{2}{GN^2 R_c} = K_{vo} \text{ sixth-cycles} \quad 2.35 \]

then the time delay terms in the numerators of equations 2.29 and 2.30 may be neglected. The approximate and exact expressions for the time constant are very nearly equal at eight half-cycles, so for speeds faster than this the exact expression should be used. In the limit as $R_c$ approaches infinity the time constant goes to zero, and the delay
functions predict ramp responses in three successive steps, delayed by one or two sixth-cycles.

The assumption made for the (n+2) and (n+5) period that a core coming out of saturation will require less excitation than a core already resetting is difficult to observe experimentally. The alternative of assuming that the cores share the control voltage equally leads to the following transfer function for $\alpha < 60^\circ$.

$$
\frac{E(s)}{NE_c(s)} = \frac{e^{-2Ts} + e^{-3Ts} + e^{-Ts} \frac{2a}{\pi} + \frac{1 + G^2 R_c}{2 + G^2 R_c} \left[ 1 - \frac{3a}{\pi} (e^{-Ts} + e^{-4Ts}) \right]}{1 + G^2 R_c - e^{-3Ts}}.
$$

For $\alpha = 60^\circ$ this expression gives the same steady-state transfer function as equation 2.31

$$
\frac{E(s)}{NE_c(s)} \bigg|_{s=0} = K_v = \frac{3}{G^2 R_c}.
$$

However, when $\alpha = 0$ and $G^2 R_c \gg 1$, the gain is increased by the added term in the numerator to become

$$
K_v = \frac{4}{G^2 R_c}.
$$

Many magnetic amplifiers exhibit a higher gain region just before the maximum output is reached for perhaps this reason.

A second assumption which was justified for the parameters of the magnetic amplifier of figure 2 with $R_c = 20$ ohms is that the diode of the resetting core will remain blocked throughout the (n+5) period for $\alpha > 60^\circ$. When $R_c = 100$ ohms, however, the diode unblocks, and gate-limited reset occurs until core 3A saturates. The diode is unblocked
Since the available resetting voltage is 2.0 volts referred to the gate winding, five times that for \( R_c = 20 \) ohms. The transfer function for unblocked operation, which follows from the resetting voltage equation 2.24, is

\[
\frac{E_o(s)}{NE_c(s)} = \frac{1}{1 + GN^2R_c} e^{-3Ts} \left[ 1 - \frac{2}{n} \left( 1 - \frac{2}{3} \right) e^{-2Ts} + e^{-3Ts} + e^{-4Ts} \right].
\]

For \( n = 60^\circ \) the steady-state gain is that found previously, but for \( n = 120^\circ \) it is

\[
K_{vo} = \frac{2}{GN^2R_c}.
\]

The lower gain produces a "tailing-off" of the transfer curve as minimum output is approached, which often is observed experimentally.
III. AN INSTABILITY

The second purpose of this study was to discover any interesting operating characteristics inherent in the three-phase magnetic amplifier with parallel control windings. To further this end six factory matched cores were used in the model magnetic amplifier, which proved to be very closely matched in operation (see figure 3). Nevertheless, an instability was still present in the region just below one-half maximum output. This unstable region of the transfer curve is shown in figure 12, along with the transfer curves for the stabilized circuits described in this chapter.

The same instability as a function of time is shown in figures 13 and 14 for decreasing and increasing output voltage. The vertical position of the voltage on the oscilloscope was not changed in the two pictures, so the width of the hysteresis effect between decreasing and increasing voltage is observed. The pictures were obtained by making very small changes in the control voltage as the horizontal oscilloscope trace moved to the right. Eventually as the critical unstable region was reached the small change kept increasing, and during the next horizontal trace, the picture was taken. This technique was possible only because the positive exponential takes a few seconds to get into the rapidly changing region. When a new operating level is reached which satisfies the equilibrium requirements of the control voltage, the regeneration dies away rapidly.

The instability is able to occur if the output voltage can be in
\[ a = R_0 = 20 \]  
\[ B = R_0 = 20 \text{ V}, \quad L_0 = 3 \text{ mH} \]  
\[ b = R_0 = 20 \text{ V}, \quad L_0 = 1.5 \text{ mH}, \quad \text{as one control branch only} \]
Fig. 13  Output voltage during instability from 9 to 7 volts
0.5 volt/cm vert.
0.5 sec/cm horiz.

Fig. 14  Output voltage during instability from 8 to 10 volts
0.5 volt/cm vert.
0.5 sec/cm horiz.

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equilibrium with the same control voltage at two different outputs.
The spike of voltage which couples into the slowly resetting core at
the bottom of its B-H loop (seen in all the B-H loop figures) is a central
part of the instability due to the nonlinear response of the core to
equal but opposite applied volt-seconds. Although this spike is present
over the whole range of operation, the instability only occurs in a
small region below one-half maximum output. The state of the various
elements in this region is shown in figure 15 in a simplified circuit
diagram.

Figure 15. Simplified Circuit Showing Critical Elements Just Before
Core 3A Saturates.

At the time of this schematic the phase voltages of phase 1 and 2
are approximately equal. The ability of the core to saturate is repre-
seated by the switch in parallel with the winding of the core. Core 1B
has already been in saturation once at the moment shown here when it had
a return path through core 2A. Now core 3A is ready to saturate at
which time core 1B will again be driven far into saturation. When core
3A saturates, the current through the load increases suddenly, and there
is enough leakage inductance in core 1B to generate a spike in the
control circuit. Core 1A has been resetting from the time when core
1B saturated the previous sixth-cycle, and is easily reset by the
volt-seconds applied by the spike. The equal and opposite volt-second
area applied when core 1B first comes out of saturation does not cause
equivalent amount of gating because most of the volt-seconds are
absorbed by the control resistor.

The instability only occurs in the region below half output due to
the changing characteristics of the spike in association with the regen-
erative feedback that occurs. It is difficult to determine exactly the
sequence of events which occur during the period of instability or how
stability is regained. The regeneration starts when the saturation
angle reaches 60°, which is where the mode of normal operation changes
and also where the derivative of the core voltage reverses. The magneti-
ic amplifier then goes into the regenerative region where a small change
during the reset period causes a change in the output such that during
the next reset period the change is greater. The regeneration goes
on for a second or more (figure 13) until a new operating point is
reached where the magnetic amplifier is again stable with the same value
of steady-state control current.

The characteristics of the lower portion of the resetting B-H loop
where the spike occurs are shown in figures 16 and 17 at the beginning
and end of the instability respectively. The percentage of reset caused
by the spike compared to the d.c. control voltage is seen to increase
substantially from the upper to the lower voltage end of the instability.
The re-entrant loop at the beginning of the spike reset in figure 17 is
Fig. 16  Expanded view of B-H loop @ $E_0 = 9.5$ volts, 2 mv/cm vert. $\frac{1}{2}$ mv/cm horiz.

Fig. 17  Expanded view of B-H loop @ $E_0 = 6.2$ volts, 2 mv/cm vert. $\frac{1}{2}$ mv/cm horiz.

Fig. 18  Expanded view of B-H loop @ $E_0 = 9.5$ volts with 7 mh in each control circuit branch

Fig. 19  Expanded view of B-H loop @ $E_0 = 6.2$ volts with 7 mh in each control circuit branch
due to the voltage reversal occurring in the gating core as the current decreases to zero from saturation before being driven out the second time. As mentioned previously, the core would have to be driven to the gating side of the B-H loop before any appreciable change of flux could occur in the gating direction. The core is already resetting on the static B-H loop from which it is easily excited towards the dynamic loop as it is reset by the voltage spike.

Placing an inductor in each of the parallel control circuit branches is a way of reducing the feedback. The choke alters the spike voltage which is applied to the resetting core. As can be seen, in figures 18 and 19, the amount of flux reset contributed by the spike with the chokes inserted remains constant throughout the entire region of interest. The gating volt-seconds seen previously in figure 17 never appears in this mode of operation. The overall gain characteristic now appears as in figure 12, curve A.

The value of inductance required for this type of regeneration cancellation is related to the magnitude of the nonlinear impedance of the resetting reactor. Neglecting the resistive and capacitive components of the impedance, the slope of the B-H loop at the time of the spike can be used to find a dynamic equivalent inductance. The value is found to be 9 millihenries (see Appendix B for the calculation). An inductance of 4.5 millihenries is required in each branch of the control circuit to eliminate the regeneration. To determine the effect on the time constant of the magnetic amplifier, the inductance added to the circuit must be compared to the core parameter \( \frac{3T}{GN^2} \) which is the
equivalent inductance of the approximate time constant expression.

For the cores used in this circuit \( \frac{2T}{GN^2} \approx 1 \) henry. An added inductor of 9 millihenries is therefore approximately 100 times smaller than \( \frac{3T}{GN^2} \) and will not effect the speed of response of the magnetic amplifier.

A second approach to this problem eliminates the instability by altering the firing angles of the phases with respect to each other so that the spike cannot regenerate. This unbalancing is accomplished by introducing an inductor of 1.5 millihenries in only one of the three parallel control branches. With chokes in all three branches having one choke differ in value from the others by at least 1.5 millihenries will give the same results. The characteristics are shown in figure 12, curve a. The output voltage waveshape with this type of control is shown in figure 22 for two values of output. The phase voltage remaining highest is that which has the choke in its control winding. One of the disadvantages of this approach is the higher amount of 120 cps ripple introduced into the output.

Increasing the control resistance \( R_c \) will decrease the instability by absorbing a larger percentage of the spike voltage. The price paid to eliminate the unstable region in this fashion is lower gain and greater required control power. The effect on the B-H loop is shown in figures 20 and 21 for an increase in \( R_c \) from 10 to 100 ohms.

The instability can be visualized from the difference equations of 2.27 and 2.28 as an added coefficient of the output voltage feedback
Fig. 20  Expanded view of B-H loop
R = 20 ohms, top: 6.5=V₀, bottom:
16.5=V₀, 20 mv/cm vert., 5 mv/cm horiz.

Fig. 21  Expanded view of B-H loop
R = 100 ohms, top: 6.5=V₀, bottom:
16.5=V₀, 20 mv/cm vert., 5 mv/cm horiz.

Fig. 22  Output voltage unbalanced by 1.5 mh in
one control circuit branch. Top: E₀ = 10 volts,
bottom: E₀ = 4 volts, 5 volts/cm
term \((1 + GN^2R_c)^{-1}\). In the region of instability the coefficient of
the output voltage feedback term must be greater than unity as regeneration does occur. The coefficient of the output voltage feedback term
may be determined from the positive exponential curves of figures 13
and 14. Plotting these curves on the semilog paper as a function of
time determines the power of the exponential, which in turn determines
the coefficient of the voltage feedback term (see Appendix C). In this
case the coefficient is 1.017, whereas the normal coefficient
\((1 + GN^2R_c)^{-1} = 0.87\), for \(R_c = 20\) ohms. The very small amount of
positive feedback accounts for the many cycles required to move through
the unstable region.
IV EXPERIMENTAL TIME CONSTANTS

The two time constant expressions developed in Chapter II must now be compared to actual time responses to determine the accuracy and range of the representation. The time constant expression, equation 2.23 is

\[ \tau = \frac{1}{\ln (1 + GN^2 R_c)} \text{ half-cycles,} \]

and the approximation when \( GN^2 R_c \gg 1 \), equation 2.33, is

\[ \tau = \frac{1}{GN^2 R_c} = \frac{K_v}{3} \text{ half-cycles.} \]

The approximation can be used for time constants greater than eight half-cycles. For responses where the exact expression must be used the half-cycle delay contributed by the three sixth-cycle steps should also be included.

To calculate the time constants the gain of the magnetic amplifier for the different control resistances must be known. Three gain curves for \( R_c = 5, 20, \) and 100 ohms are plotted in figure 22A. The greater excitation required as the control resistance is increased is not predicted by the core function of equation 2.1. This linear representation of the core can only approximate a very complicated nonlinear phenomena which is sensitive to the changing wave shapes in the control circuit and the change in coupling between the gate and control circuits caused by the increased control resistance. The translation of transfer curve to higher currents as the control resistance is increased occurs in a
\( A \) - Curve No. 1, \( R_C = 5 \) Ohms
\( \alpha \) - Curve No. 2, \( R_C = 20 \) Ohms
\( \beta \) - Curve No. 3, \( R_C = 100 \) Ohms

Figure 224: Output Voltage versus Total Control Current For A Parallel Control Winding. Three-Phase Magnetic Amplifier For \( R_C = 5, 20, 100 \) Ohms
similar manner in single-phase magnetic amplifiers.

It is interesting to note the increased gain at high output exhibited by the magnetic amplifier when $R_c = 100$ ohms. In Chapter II it was noted in the summary that if the core coming out of saturation shared the control voltage with the already resetting core rather than taking it all, an additional term appeared in the output expression for $\alpha = 0^\circ$. This additional term would cause the gain to increase near maximum output in the same way it does for $R_c = 100$ ohms.

The other possible mode of operation introduced in Chapter II occurs when the diode of the resetting core unblocks at low output. In figure 10 the diode was shown to remain blocked for $R_c = 20$ ohms. With high control resistance the diode would be more likely to unblock because the resetting voltage applied by the control voltage after the gating core has saturated is greater. This resetting voltage is in a direction to unblock the diode. If the diode unblocks, the gain falls off below half-maximum output. As can be seen from figure 22A, the curve below six volts for $R_c = 100$ ohms falls off more rapidly than the other two curves. The unblocking of the diode for $R_c = 100$ ohms can be observed with an oscilloscope.

The time constants for the exponential portion of the response are calculated by equations 2.23 or 2.33 depending on the speed of the response (see Appendix D) and are tabulated in table 1. The actual time constant measurements of figures 23 through 26 are also in table 1.
Fig. 23  Transient response from $E_0 = 15 \pm 12.5$ volts. $R = 20$ ohms
0.5 volts/cm vert., 20 msec/cm horiz.

Fig. 24  Transient response from $E_0 = 6 \pm 3.5$ volts. $R = 20$ ohms
0.5 volts/cm vert., 20 msec/cm horiz.

Fig. 25  Transient response from $E_0 = 15 \pm 13$ volts. $R = 5$ ohms
0.5 volts/cm vert., 50 msec/cm horiz.

Fig. 26  Transient response from $E_0 = 6 \pm 3.5$ volts. $R = 5$ ohms
0.5 volts/cm vert., 50 msec/cm horiz.
<table>
<thead>
<tr>
<th>$R_c$ (ohms)</th>
<th>$E_o$ (volts)</th>
<th>Calculated</th>
<th></th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>msec</td>
<td>half cycles</td>
<td>msec</td>
</tr>
<tr>
<td>5</td>
<td>15-12</td>
<td>358</td>
<td>44.7</td>
<td>355-315</td>
</tr>
<tr>
<td>20</td>
<td>15-12</td>
<td>89</td>
<td>11</td>
<td>100-110</td>
</tr>
<tr>
<td>100</td>
<td>15-12</td>
<td>24.5</td>
<td>3</td>
<td>40-36</td>
</tr>
</tbody>
</table>

Table 1. Theoretical and actual time constants for $R_c = 5$, 20 and 100 ohms.

The data in table 1 is plotted in figure 27 on semilog graph paper to show the divergence of the actual and calculated responses as the control resistance is increased.

Figure 27. Theoretical and actual time constants as functions of $R_c$. 

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The core conductance, $G$, may be calculated from the transfer curves of figure 22A to be \( \frac{1}{1920} \) mhos for $R_c = 20$ ohms, and \( \frac{1}{2200} \) mhos for $R_c = 100$ ohms (see Appendix D). By exciting one of the cores from the magnetic amplifier independently with a variable frequency sine-wave generator, another $G$ may be obtained. The variable frequency generator is used to determine the approximate d.c. exciting current by extending the curve of exciting current as a function of frequency (see Appendix D). The $G$ for 60 cps operation can then be calculated from equation 2.1 as follows:

$$ G = \frac{I_{md} - I_{mo}}{e_{core}} = \frac{(9.2 - 5.0) \text{ma}}{8.2 \text{ v}} = \frac{1}{1950} \text{ mhos}. $$

By this technique the approximate characteristics of the three-phase magnetic amplifier may be predicted. The variance between the $G$ obtained with the sine-wave generator and that found in the circuit while operating is a result of the complicated voltage waveshape actually impressed on a core.

When the control resistance is large enough, the output response approaches the response that occurs with a control current source. The exponential portion of the response disappears, and only delayed steps are predicted by equations 2.36 and 2.38. These steps are the "ramp" response discussed by Ellert.\(^5\) For large control resistance equation 2.36 and 2.38 accurately predict the transient behavior rather than equations 2.29 and 2.30 which were developed for low control resistance. As noted earlier in this chapter, the higher gain
at the upper end of the transfer curve for $R_c = 100$ ohms compared to the curve for $R_c = 20$ ohms in figure 22A indicated the change in gain described by equation 2.36 for steady state conditions. Equation 2.38 must be used as unblocking occurs for control resistances greater than approximately 50 ohms.

To measure responses of approximately a half-cycle duration a Cyclic Integrator which takes an average of the output each sixth-cycle is convenient to use. To use this basic type of Cyclic Integrator, time must be allowed for the capacitor to reset, so the output response was taken between 6.5 and 2.5 volts where the output voltage goes to zero each sixth-cycle. The output responses recorded by the Cyclic Integrator for increasing and decreasing control voltage steps are shown in figures 28 and 29 for $R_c = 1000$ ohms. The shorting transistor around the integrating capacitor takes approximately 0.6 msec to discharge the capacitor and therefore the resetting period appears to form the second part of a half-sine wave with the integrated signal.

As the output is less than half-maximum, equation 2.38 is used to determine the transient response. The delayed steps are predicted to occur in the $(n+2)$, $(n+3)$ and $(n+4)$ sixth-cycles for a change in the $n$ sixth-cycle. As $u$ approaches $120^\circ$ the response during the $(n+2)$ sixth-cycle will not occur. The mercury relay which shorts out a small series resistor in the control circuit to initiate the step also triggers the oscilloscope, and therefore the first sixth-cycle in figure 28 is n. No output change occurs in the $(n+1)$ sixth-cycle, and none is discernable in the $(n+2)$ sixth-cycle. In the $(n+3)$ and
Fig. 28  Transient response from $E_0 = 2.5\cdot6.5$ volts through cyclic integrator. $R_c = 1000$ ohms.
150 v/cm vert., 2 msec/cm horiz.

Fig. 29  Transient response from $E_0 = 6.5\cdot2.5$ volts through cyclic integrator. $R_c = 1000$ ohms.
.05 v/cm, 2 msec/cm horiz.

Fig. 30  Transient response from $E_0 = 6.5\cdot2.5$ volts. $R = 1000$ ohms
4°v/cm vert., 2 msec/cm horiz.

Fig. 31  Transient response from $E = 16.5\cdot8.5$ volts. $R = 1000$ ohms
4°v/cm vert., 2 msec/cm horiz.

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(n+4) sixth-cycle changes occur as predicted for large $a$ with the new steady-state output level reached in the (n+4) sixth-cycle. The output response for a step occurring at some point other than at the beginning of a sixth-cycle must be determined by superposition of the predicted response for the average of the fractional sixth-cycle step plus the predicted response to the remainder of the step-change average beginning the next sixth-cycle.

The output voltage step responses for $R_c = 1000$ ohms without cyclic integration are shown in figures 30 and 31 for low and high output levels respectively. The response at low output in figure 30 duplicates figure 29, except that changes in the average voltage level must now be detected as changes in the firing angle. The response at high output of figure 31 exhibits changes in the sixth-cycle average voltages during the $(n+1), (n+2), (n+3)$ and $(n+4)$ sixth-cycles, where $n$ is the first complete sixth-cycle. This response is predicted by equation 2.36 when $a$ is small. These delayed step responses at both low and high outputs provide, along with the long time constant data, verification of the basic difference equation representation.
V. SUMMARY

The difference equations in Laplace transform notation,

\[
\frac{E_0(s)}{NE_c(s)} = \frac{1}{1 + GnR_c - e^{-3Ts}} \left[ \frac{2\pi}{n} e^{-Ts} + e^{-2Ts} + e^{-3Ts} + (1 - \frac{2\pi}{n}) e^{-4Ts} \right]
\]

for \( \theta < 60^\circ \), and

\[
\frac{E_0(s)}{NE_c(s)} = \frac{1}{1 + GnR_c - e^{-3Ts}} \left[ e^{-2Ts} + e^{-3Ts} + e^{-4Ts} \right]
\]

for \( \theta > 60^\circ \), accurately represent the incremental operation of the three-phase magnetic amplifier with parallel control windings when the speed of response is eight half-cycles or greater. These equations are derived by expressing the average output voltage over a sixth-cycle as the difference between the supply voltage and the gating core voltage averaged over that sixth-cycle. The gating core voltage during this sixth-cycle in turn may be described in terms of the control and output voltage during the preceding five sixth-cycles by utilizing the fact that the net volt-second area applied to a core over a cycle of resetting and gating is zero. By factoring and combining terms the expressions for \( E_0(s) \) as a function of \( NE_c(s) \) are obtained. The difference in the delays associated with the sixth-cycle average control voltages in equations 2.27 and 2.28 arises from expressing the variable in terms of sixth-cycle averages. For a change in the control voltage in a given
sixth-cycle, the first change in the load voltage occurs in the next sixth-cycle if \( a < 60^\circ \) and in the next sixth-cycle plus one if \( a > 60^\circ \). Therefore at \( a = 60^\circ \) the expressions are not identical because the output is resolved only to the nearest sixth-cycle.

The denominator of equation 2.29 may be equated to zero to determine the expression for the dominate pole on the real axis. This dominate pole corresponds to a time constant for the exponential portion of the response of

\[
\tau = \frac{1}{\ln (1 + GN^2 R_c)} \text{ half-cycles.} \tag{2.32}
\]

When the time constant is greater than eight half-cycles, the approximation

\[
\tau = \frac{1}{GN^2 R_c} \text{ half-cycles} \tag{2.33}
\]

may be used. In the latter case the half-cycle delay inherent in the response can usually be neglected. Experimental time constants are found to be in agreement with those calculated using equations 2.32 and 2.33.

In deriving the difference equation representation of the three-phase magnetic amplifier with parallel control windings, choices must be made at two points during the cycle of operation regarding the characteristics. The first choice occurs when the core chosen to follow through a cycle of operation comes out of saturation and is ready to reset. At this point the core can either share the resetting control voltage with the other core or take it all. If the core and
circuit characteristics are such that the cores share the voltage, then the difference equation 2.27 is modified by additional terms so that when \( a = 0 \) the gain is increased.

The second time in the cycle of operation where the circuit characteristics determine the mode of operation is when \( a > 60^0 \) and the diode of the resetting core may unblock and cause gate-limited reset. The unblocking will occur if the resetting voltage applied to the core referred to the gate winding is great enough to overcome the blocking voltage applied by the magnetizing current flowing through the load resistor. In circuits using germanium rather than silicon diodes the unblocking will occur more readily due to the lower-diode forward voltage. Unblocking of the diode modifies equation 2.28 and results in a lower steady-state gain at minimum output.

As the control resistance is increased to give step responses of fewer half-cycles, the diode unblocking and voltage sharing modes of operation mentioned previously both occur. The transfer curve shows an increase in the gain at high output and a tailing-off in gain as minimum output is approached.

For a control resistance high enough that the exponential portion of the transient response is eliminated, the delayed sixth-cycle steps which occur in response to an input step are predicted by transfer functions incorporating the diode unblocking and voltage sharing modes of operation.

The experimental three-phase magnetic amplifier used in this study with parallel control windings and factory matched square-loop cores exhibits an unstable region just below half-maximum output. This
regenerative region is caused by complicated nonlinear characteristics which can be approximated by letting the coefficient of the output voltage feedback term in equations 2.27 and 2.28 assume the proper values over the range of operation. For the instability to occur the coefficient must be greater than unity during the regenerative period. The numerical value of this coefficient was calculated from the positive exponential of the instability to be 1.017. The coefficient over the remainder of the output range is 0.87.

The spike of voltage appearing across the resetting core when it is being reset only from the control voltage is closely related to the instability. This spike occurs when the already saturated gate core is redriven into saturation when the core of another phase fires. The resetting core accepts the volt-seconds of reset generated by the spike easily, whereas the reverse voltage generated by the gated core when first returning from saturation does not gate the resetting core. There is no gating as the core is excited for resetting, and the opposite polarity of voltage must first move the core to the gating side of the B-H loop.

Chokes introduced into the parallel branches of the circuit or an inductive load reduce the coupling of this spike into the resetting core and eliminate the unstable region. High control resistors which reduce the percentage of reset caused by the spike are also effective in eliminating the regeneration.
ACKNOWLEDGMENTS

The work described in this report was done in collaboration with Professor Henry C. Bourne, Jr., whose advice and encouragement are gratefully acknowledged.

This work was performed under the auspices of the U. S. Atomic Energy Commission.
APPENDIX A

Core Characteristics:

Core 3T4178D2 Deltamax 2 mil

$A_c = 0.685 \text{ cm}^2$

$l_c = 17.87 \text{ cm}$

$A_A w = 2.471$

$h_{dc} = 0.08 \text{ oersteds}$

$h_{60\omega} = 0.23 \text{ oersteds}$

$h_{400\omega} = 0.38 \text{ oersteds}$

for $N = 300$ turns

$E_{core-rms_{60\omega}} = 4.44 \text{ BANf} = (4.44)(1.5 \times 10^{-4})(0.685)(300)(60) = 8.2 \text{ V}$

$E_{line to line-rms} = 2E_{core} = 16.4 \text{ V}$

$E_{dc} = \frac{\sqrt{2}}{\pi} E_{line to line-rms} = 22 \text{ V}$

let

$I_{dc} = 1 \text{ amp}$ \hspace{1cm} $R_L = 22 \Omega - 22 \text{ Watts}$

$I_{md} = \frac{h_{60\omega}}{0.4 \pi} \frac{l_c}{N} = \frac{0.23 (17.8)}{0.4 \pi \times 300} = 10.6 \text{ mA}$

To allow $400 \text{ cps}$ usage and variations in the number of control turns

five winding of AWG #24 wire were wound with 25, 50, 100, 200, and 300 turns.
APPENDIX B

Equivalent Inductance of Core when Absorbing Spike Voltage

\[ e = L \frac{di}{dt} \]

and therefore

\[ \int_0^{t_1} e \, dt = \int_0^i \, i \, i \, \text{d}i = RCE_0 \]

where RC is the time constant of integrating network and \( E_0 \) the voltage across the capacitor. Solving for the inductance \( L \) gives:

\[ L = \frac{RCE_0}{i} \]

\[ L_{\text{gate}} = \frac{(100 \, \text{K})(1 \, \text{ufd})(2 \, \text{mv})}{(2.5 \, \text{mv})(1 \, \Omega)} = 80 \, \text{mH} \]

\[ L_{\text{control}} = \left( \frac{1}{N} \right)^2 L_{\text{gate}} = \left( \frac{100}{300} \right)^2 80 \, \text{mH} \approx 9 \, \text{mH} \]
APPENDIX C

Calculation of the Output Voltage Feedback Coefficient of Equation 2.17
During Regeneration.

The equation

\[ E_o(s) = \frac{N E(s)}{1 + \frac{G N}{R} e^{-Ts} + e^{-2Ts} + e^{-3Ts}} + x E_o(s) e^{-3Ts} \]

represents the output during regeneration where \( x \) is the unknown coefficient of the voltage feedback term. This coefficient \( x \) can be found from the time constant of the positive exponentials of figures 13 and 14. This time constant can be found as the inverse of the slope of the linear portion of the natural log of figure 13 plotted as a function of time.

\[
\text{slope} = \frac{1}{2.0 - 1.35} \geq 2.0
\]

let \( f(t) = e^{-t} \) in the region of linear slope, then

\[
\frac{1}{\tau} = -2.0
\]

\[ \tau = 0.5 \]
APPENDIX C (cont.)

Solving equation C.1 for $E_o(s)$ as a function of $NE_c(s)$ and approximately $e^{-3Ts} \approx 1 - 3Ts$ gives

$$
\frac{E_o(s)}{NE_c(s)} = \frac{(1 + GN^2R_c)^{-1}(e^{-Ts} + e^{-2Ts} + e^{-3Ts})}{1 - x(1 - 3Ts)}.
$$

Solving for the time constant gives

$$
\tau = \frac{3Tx}{1 - x},
$$

which in turn yields

$$
x = \frac{\tau}{3T + \tau}.
$$

To avoid taking small differences $(1-x)$ is found which in turn gives new expression for $x$.

$$
1-x = 1 - \frac{\tau}{3T + \tau}
$$

$$
1-x = \frac{3T}{3T + \tau}
$$

$$
x = 1 - \frac{1}{1 + \frac{\tau}{3T}} = 1 - \frac{1}{1 - (0.5)(120)} = 1 + \frac{1}{59} = 1.017.
$$

The coefficient of the voltage feedback term is 1.017. The coefficient being greater than 1 creates positive feedback which causes the instability. The coefficient is 0.9 over most of the rest of the range of operation.
Gain and Time Constant Calculation.

From figure 2 for 15-12 volts output, \( R_c = 20 \, \Omega \)

\[ K_{vo} = \frac{E_o}{N E_c} = \frac{(18-9)(\text{volts})}{\frac{300}{100} (50-36) \text{ milliamps} \frac{20 \, \Omega}{3}} = 32 \]

\[ \tau = K_{vo} T = \frac{32}{(6)(60)} = 89 \text{ msec.} \]

From figure 22A for \( R_c = 100 \, \Omega \)

\[ K_{vo} = \frac{20 \, \text{volts}}{\left(\frac{300}{100}\right)(79-52)(10^{-3})(\frac{100}{3})} = 7.4 \]

\[ GN^2 R_c = \frac{3}{7.4} = 0.406 \]

\[ \tau = \frac{3T}{\ln(1 + GN^2 R_c)} = \frac{3T}{\ln(1 + 0.406)} = \frac{1}{(120)(0.34)} = 24.5 \text{ msec.} \]

G Calculation from Gain

\[ K_{vo} = \frac{3}{GN^2 R_c} \quad G = \frac{3}{K_{vo} N^2 R_c} \]

for \( R_c = 20 \, \Omega \)

\[ G = \frac{3}{(32)(3)^2(20)} = \frac{1}{1920} \]

for \( R_c = 100 \, \Omega \)

\[ G = \frac{3}{(7.4)(3)^2(100)} = \frac{1}{2200} \]
G Calculation from Variable Frequency Generator

Test setup for B-H loop:

![Diagram of test setup]

Curve of magnetizing current vs. core voltage.

- \( i_m = I_{m0} \text{sgn} e_{core} + G e_{core} \)
- \( G = \frac{I_{md} - I_{m0}}{e_{core}} = \frac{(9.2 - 5.0) \text{ma}}{8.2 \text{ volts}} = \frac{1}{1950} \text{ mhos} \)
Cyclic Integrator for Magnetic Amplifier

When there is output voltage the diode conducts, turning off transistor TR1 and charging capacitor C1. As the output voltage goes to zero, diode CR1 stops conducting and TR1 turns on, discharging the capacitor C1.
REFERENCES


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