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Phenomenologically Viable Models from Superstrings? *

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ABSTRACT

The phenomenology of a class of models based on superstring theories is discussed. If the supersymmetry breaking mechanism generates masses for scalars which are much larger than those for gauginos, then all models with only the weak and Planck scales are ruled out, provided a discrete symmetry eliminates tree level flavor changing neutral currents. Some restrictions on models with an intermediate scale are also given.

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1. INTRODUCTION

Recent developments[1-3] in string theory[4] have suggested that superstring models might provide a framework for the unification of elementary particle interactions. In particular, the field theoretic limit of these models has a large enough gauge symmetry to encompass the known interactions—strong and electroweak. There is still much to understand about how this limit is realized and how one goes from 10 to 4 dimensions[5,6]. It seems, however, of particular interest to study how these models survive the requirement of a successful phenomenology at low energy (under 10 TeV).

This requirement enables one to discriminate between the different models in a quite efficient way for the following reason. As shown by Witten[7], if the gauge symmetry includes the standard \( SU(3)_C \times SU(2)_L \times U(1)_Y \) symmetry, it must also include some extra \( U(1) \). We will show in this paper that, in many cases, mixing between the gauge boson of the extra \( U(1) \) and the neutral vector boson of the standard model endangers the successes of the standard model. Which \( U(1) \) remains at low energy, and how it is eventually broken, are questions closely connected to the symmetries of the theory at the scale of compactification, and hence to the properties of the underlying manifold. This is why a detailed study of the low energy constraints on these models is relevant in order to help in answering the more fundamental questions that remain open at the other end of the energy range (compactification scale). Let us emphasize, however, that although this paper was motivated by the recent upsurge of interest in superstring theories, many of its conclusions are applicable to other theories which result in low energy models with enlarged gauge groups and \( N = 1 \) supersymmetry.

To be precise, we consider here the phenomenology of a class of models based on \( N = 1 \) supergravity which are expected to occur as the low energy limit of the \( E_8 \times E_8 \) superstring theory[3]. When the 10-dimensional supergravity theory (the field theoretic limit of the superstring theory) is compactified on \( M_4 \times K \), where \( M_4 \) is the 4-dimensional Minkowski spacetime and \( K \) is a 6-dimensional Kähler
manifold (of radius $R$) with $SU(3)$ holonomy, the resulting 4-dimensional theory has an $N = 1$ unbroken supersymmetry and a $G \times Q$ gauge symmetry[7]. Here $G$ is $E_6$ or one of its subgroups and represents the gauge group of the observed fields. $Q$ is $E_6$ or one of its subgroups and constitutes a hidden sector[8] which may enable the remaining $N = 1$ supersymmetry to be broken. A catalogue of the possible groups $G$ was given in Ref. [9], where some of them were eliminated by looking at phenomenological considerations.

The models fall into two classes; those with and without an intermediate scale $M_I$ ($M_W \ll M_I \ll 1/R$) at which the group $G$ is broken down to a group $G'$ which contains $SU(3)_c \times SU(2)_L \times U(1)_Y$*. In this paper, we shall examine these models in more detail and show that under a set of assumptions discussed below, none of the models without an intermediate scale are candidates for a theory of low energy physics.

Particles with masses less than $1/R$ fall into representations of $G$ which can be classified into $N_I$ complete 27-dimensional representations of $E_6$, plus pieces of $27 + \overline{27}$ representations[7] (see below). We fix the scale $1/R$ by requiring that the gauge couplings have a common value at this scale. The value determined by this method is slightly less than the Planck mass ($M_P$), and is consistent with that expected from string dynamics[11]. Our conclusions are not sensitive to the value used for $1/R$.

We shall require that all the gauge and Yukawa couplings remain perturbative on all scales between $M_W$ and $1/R$. If this is not the case, we can make no predictions whatsoever. Models with four or more generations and no intermediate scale are not compatible with this requirement, if $G$ is of the form $SU(3)_c \times F$. In order to see this, consider the evolution of the $SU(3)$ coupling constant. Each generation of $27's$ contains three quarks ($u$, $d$ and a new flavor called $g$) and we have

$$\frac{1}{\alpha_s(\mu)} = \frac{1}{\alpha_s(M_W)} + \frac{1}{2\pi}(-9 + 3d) \ln (M_W/\mu)$$

(1.1)

where we have included the gluinos and the scalar quarks and $d$ is the number of generations. It can be seen from this that $\alpha_s$ becomes non-perturbative at order $10^8$ GeV if we assume that $\alpha_s(M_W) = 0.12$ and that there are four generations. Therefore, four generation models require the presence of an intermediate scale at which some of the species must decouple or the embedding of $SU(3)_c$ into some larger group at low energy. One can easily see that the same argument does not apply to 3 generation models.

As shown in Ref. [9], the possibility of having an intermediate scale is related to the presence of components of $27$ in the particle spectrum. It is easy to determine the number $N_I$ of complete 27 representations, since an index theorem gives the net number of chiral massless fields. The question of which pieces of $27 + \overline{27}$ survive (hence the name "survivor" that we will use for the members of the $27$) is a more intricate question. Actually, in the general case, there is no restriction on which components survive and on how many of them there are. In the most common case[7] (technically speaking, when the Kahler manifold has a Hodge number $b_{1,1} = 1$), the quantum numbers of the survivors are directly determined by the symmetries that fix the gauge group $G$.

Before we can analyze the models in detail, we have to specify the mechanisms for supersymmetry and electroweak symmetry breaking. Unfortunately, the issue of supersymmetry breaking has not yet received a satisfactory answer. Let us summarize the situation.

Soon after these models were introduced, it was realized that the hidden sector (subgroup $Q$ of $E_6$) could break supersymmetry by the formation of gaugino condensates[12,13]. The problem is in the transfer of that breaking to the observed sector. At tree level, in the observable sector, gauginos and scalars are massless and there are no terms in the low energy potential proportional to the

* Crudely speaking, models without an intermediate scale will have massive gauge bosons which will be observable at the SSC[10].
The masslessness of the observable gauginos is related to the absence of a cosmological constant \( \Lambda \) at tree level. Normally, a gaugino condensate in the hidden sector will give a huge contribution to \( \Lambda \). It turns out that this is exactly compensated for by the vacuum expectation value (vev) of a 3-form\(^{12}\) (to be complete, by \( F = dB + \omega_3 - \omega_3 \)) and the same compensation occurs for observable gaugino masses\(^{14}\). Since we expect a zero cosmological constant at all orders, it is difficult to guarantee that gaugino condensation in the hidden sector will give radiative masses to the observed gauginos.

In the case of scalars, it has been shown recently that they stay massless at the one-loop level\(^{11}\). The reason for this is not clear; it is rather improbable that this is a mere accident. No source of mass has been demonstrated to exist either for gauginos or for scalars. If the scalars and gauginos are massless, they will remain so when their masses evolve from the compactification scale to low energy through the renormalization group equations. Therefore, the gauge group \( G \) will remain unbroken (no scalar mass can become negative). We note that, as soon as \( G \) is broken, some mass terms will arise for the gauginos\(^{15}\), which will then generate scalar masses through the renormalization group equations. But, this mechanism cannot be used \textit{ab initio}.

Since we need nonzero scalar masses to break \( G \) and since the absence of gaugino masses may have a deeper origin (\( \Lambda = 0 \)), we will assume that at the compactification scale \( 1/R \), the scalar particles have a common mass \( m \), the gauginos have a zero or negligible mass and that there are no A or B terms in the potential. The renormalization group evolution of the scalar masses down to scales of order \( M_W \) will provide the mechanism for the breaking of the gauge symmetry. The presence of large Yukawa couplings (for example, the top quark Yukawa coupling) will drive one or more of the scalar mass-squared values negative, resulting in a symmetry breakdown\(^{16}\).

The rest of this paper is organized as follows. In section 2 we will discuss the minimal model based on the group \( SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_Y \) and will demonstrate how the model is constrained and, in its simplest form eliminated, by current data. Section 3 will discuss the remaining models in the case where they do not have an intermediate scale. Section 4 is devoted to comments on the models which can have such a scale and finally section 5 presents our conclusions.
2. THE SU(3)\(_e\) X SU(2)\(_L\) X U(1) X U(1) MODEL

When the additional six dimensions of the E\(_6\) x E\(_8\) superstring theory are compactified, the effective low energy theory has an N=1 supersymmetry\[3\]. If the manifold is simply connected, the low energy group is E\(_7\) x E\(_8\). The E\(_8\) is irrelevant for low energy physics although it may play a role in the breaking of N=1 supersymmetry by acting as a hidden sector. If the manifold is multiply connected, then the low energy group will be smaller than E\(_7\). As discussed in Ref. \[7\], this group will be the maximal subgroup of E\(_7\) which commutes with the discrete subgroup formed by the expectation values of Wilson lines which enclose holes in the manifold. This low energy group, if it is to contain SU(3) \(_e\) \times SU(2)\(_L\) \times U(1)\(_y\) \times U(1)\(_y'\), must contain SU(3) \(_e\) \times SU(2)\(_L\) \times U(1)\(_y\) \times U(1)\(_y'\). We shall first consider this simplest case.

The quarks and leptons of the standard model must fall into N\(_f\) 27 dimensional representations of E\(_6\). In the case where there is no intermediate scale, perturbative unification requires that N\(_f\) be less than four (see introduction). The 27 supermultiplet decomposes under SU(3) \(_e\) \times SU(2)\(_L\) \times U(1)\(_y\) \times U(1)\(_y'\), as follows

\[
\Phi[27] = (3, 2, 1/3, -2/3)_Q + (3, 1, -2/3, 4/3)_e + (3, 1, -4/3, -2/3)_{e'} + (\bar{3}, 1, 2/3, 1/3)_{e''} + (\bar{3}, 1, 2/3, 1/3)_{e'''} + (1, 2, 1/3, -2/3)_{H} + (1, 2, -1, 1/3)_{H'} + (1, 1, 2, -2/3)_{e'} + (1, 1, 0, -5/3)_{N} + (1, 1, 0, -5/3)_{N'}
\]

(2.1)

where \( g \) is a new charge -1/3 quark, and \( N \) and \( N' \) are new singlets. \( H \) and \( H' \) have the quantum numbers of the usual Higgs doublets of a supersymmetric model. The derivation of the U(1) quantum numbers is given in Appendix A.

The renormalization group scaling of the coupling constants in this model is easy to determine. At scale 1/R the SU(3) \(_e\) (\( g_2 \)) and SU(2)\(_L\) (\( g_2 \)) couplings are equal. The two U(1) couplings are also equal to each other and are given by

\[
\sqrt{5} g_1 = \sqrt{5} g_1' = g_2 = g_3.
\]

(2.2)

As long as we include complete representations of 27's or \( \overline{27} \)'s, the evolution of the U(1) couplings are the same and consequently they will be equal at low energies. In addition to the complete 27's of Eq. (2.1), some components of 27 and \( \overline{27} \) representations can survive to low energies. The prescription for determining the quantum numbers of these survivors has been given by Witten\[7\]. In the present case, one of two survivors from the \( \overline{27} \) are possible: either a charged field with the quantum numbers of \( e_R \) (i.e. \( (1, 1, -2, 2/3) \) or an SU(2)\(_L\) doublet \( (1, 2, -1, -4/3) \). If survivors are present, then the beta functions for the two U(1) coupling constants will not be the same, so that these couplings will not be equal at low energy. If there is one survivor, \( g_1' \) is approximately 3% smaller than \( g_1 \), a change which does not affect our conclusions.

Denoting the 27 by \( \Phi \) and the \( \overline{27} \) by \( \overline{\Phi} \), the superpotential will be an SU(3) \(_e\) \times SU(2)\(_L\) \times U(1)\(_y\) \times U(1)\(_y'\) truncation of the E\(_6\) invariant form

\[
g(\Phi, \overline{\Phi}) = a \phi^3 + b \phi'^3 + \frac{\lambda_N R}{M_P} \phi^2 \phi'^2.
\]

(2.3)

The survivors couple to the matter in the 27 representations only through the non-renormalizable term

\[
\delta g(\Phi, \overline{\Phi}) = \frac{\lambda_N R}{M_P} \phi^2 \overline{\phi}^2.
\]

(2.4)

In this model the survivors are not singlets with respect to SU(3)\(_e\) \times SU(2)\(_L\) \times U(1)\(_y\), hence it is not possible to use them to break the extra U(1)\(_y'\) at an intermediate scale (see Section 4). The model is therefore restricted to have three generations. The contribution of the survivors to the breaking of weak interactions is suppressed by a factor of order \( \sqrt{M_W/M_P} \) and will be neglected. The extra U(1)\(_y'\) can however be broken either by the vacuum expectation value (vev) of \( N \) or \( N' \). We will assume that only \( N \) receives a vev since, in this minimal model, \( N \) and \( N' \) have the same quantum numbers.
The most general $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_Y'$ invariant form for the superpotential is

$$g(\Phi) = \lambda_{u_{ik}} H_u Q_i u_k^c + \lambda_{d_{ik}} H'_i Q_j d_k^c + \lambda_{e_{ik}} H'_i L_j e_k^c + \lambda_{u_{ik}} H''_i N_k + \lambda_{u_{ik}} H'_i Q_j g_k + \lambda_{d_{ik}} Q_i g_j e_k^c + \lambda_{e_{ik}} Q_i g_j L_k + \lambda_{u_{ik}} L_j Q_i g_k + \lambda_{d_{ik}} L_j Q_i g_k + \lambda_{e_{ik}} L_j Q_i g_k$$

where $i,j,k$ are generation indices which run from 1 to 3. Wherever $N$ appears it is possible to add a similar term involving $N'$. Lepton number conservation forces us to take $\lambda_{1-6} = 0$ and constrains $\lambda_6$ (see below). If we require that baryon number be conserved then either $\lambda_6 = \lambda_Q = \lambda_1 = 0$ or $\lambda_6 = \lambda_Q' = \lambda_1 = \lambda_5 = 0$. Any other baryon number assignment results in a stable $g$-quark, which is cosmologically disastrous [17].

We will require that there be no tree level flavor changing neutral currents involving observed quarks. This model has three pairs of Higgs doublets and hence potential trouble. These currents can be avoided by the introduction of discrete symmetries [17] to ensure that only one pair of doublets couples to quarks [18]. We shall denote the Higgs pair which gets a vev by $(H_u, H'_u)$ and the other Higgs doublets by $(H_a, H'_a)$ where $a = 1,2$. (Superstring models generally do have discrete symmetries [7]; the precise symmetry in a given case depends on the structure of the six dimensional manifold). An example of such a symmetry is $(H_u, H'_u, H_a, H'_a) \rightarrow (H_u, H'_u, -H_a, -H'_a)$. There are two choices for the behavior of $N$ under this symmetry. If $N \rightarrow N$, then the term $H_i H'_i N_k$ is diagonal, i.e. $\lambda_{H_i N_k}$ is proportional to $\delta_{ij}$. If $N \rightarrow -N$, then it is purely off-diagonal, i.e. it is zero if $i = j$. We will consider the former case and return to the latter at the end of this section. Once $N$ has obtained a vev, terms of the type $m_d^2 H_i H'_i$ can be generated by radiative corrections. The symmetry will guarantee that these terms will also be diagonal.

The potential now has a global symmetry of the form

$$H \rightarrow e^{i \alpha} H, \quad H' \rightarrow e^{-i \alpha} H'$$

$$H_a \rightarrow e^{i \beta} H_a, \quad H'_a \rightarrow e^{-i \beta} H'_a$$

$$Q_i \rightarrow e^{i \beta} Q_i, \quad u_i^c \rightarrow e^{-i (\alpha + \beta)} u_i^c$$

This symmetry will be broken by the vevs of $H$ and $H'$ and will result in a phenomenologically unacceptable axion if more than one pair of Higgs doublets gets a nonzero vev.

Since the $g$ quark acquires mass from the vev of $N$, the mass matrix of the charge -1/3 quarks will contain both the vev's of $H'$ and $N$. This will cause tree level flavor changing neutral currents involving observed quarks unless there is no mixing between the $d$-quarks and the $g$-quarks. Our assumption that a symmetry prohibits such tree level flavor changing neutral currents then eliminates the $QH'g^c$ and $gd''N$ terms from the superpotential. The $gd''N$ terms will not give tree level flavor changing neutral currents in the observable quark sector if $N$ does not get a vev. On the other hand, these terms will give a potentially dangerous contribution to the $K_L - K_S$ mass difference via a box diagram with $g$ and $N$ in the loop unless they are small. But, since the latter constraint is weak, we will allow the $gd''N$ terms if $N$ has no vev.

We will not include the $u^c d' g$ or $QQg$ terms. We can show that the analysis is unaffected by their presence. (If they are large, they will generate a negative $g$-quark mass-squared, breaking color. This constraint is sufficient to prove that the inequality $m^2_H < m^2_{H'}$, see below, is still valid.) One of these terms must be present at some level, however, to allow the $g$ quark to decay.

Without loss of generality, we can consider only one $N$ and $N'$ field (the additional ones will not affect our argument). Since $H$ and $N$ have nonzero vevs, lepton number conservation eliminates the $HLN$ terms. The term $\lambda_{u_{ij}} H_i H'_j N'$ is allowed if $N'$ has no vev since $N'$ can then have lepton number. But it will
generate a neutrino mass when $H$ gets a vev. We can estimate an upper bound on $\lambda_\nu$ as follows. The top quark's Yukawa coupling $\lambda_T$ cannot be too large at low energy or else it will become non-perturbative as it is run up to the scale $1/R$. This restricts $\lambda_T$ to be less than 1 at the scale $M_W$. Using the current limit on the top quark mass we can conclude that $(H) \equiv h > 40$ GeV, and hence that $\lambda_\nu$ is constrained by $\lambda_\nu \approx m_\nu/h$. The largest $\lambda_\nu$ is that for the tau neutrino which must satisfy $\lambda_\nu < 10^{-3}$. A similar argument applies to the term $\lambda_{HLN}$.

Finally, the terms $H L N'$ have no such constraint since neither $H_L$ nor $N'$ gets a vev. The presence of these terms is immaterial since they do not affect the conclusion that $m_H^2 < m_{H'}^2$ (see below).

We are now left with the following superpotential

$$g(\Phi) = \lambda_u H Q_i u_i^c + \lambda_d H' Q_i d_i^c + \lambda_e H' L_i e_i^c + \lambda_H H H' N + \lambda_{H_L} H_L H' N + \lambda_{H''} H'' N + \lambda_{H'''} H'' N$$

(2.7)

We have taken the $\lambda_\phi$ terms to be diagonal. (Our conclusions are not affected by this assumption).

As stated in the introduction, we assume that the mechanism which is responsible for the breaking of supersymmetry does not generate a gaugino mass or a term in the scalar potential proportional to the superpotential itself (an A term) but merely generates soft masses for all the scalars which are equal at scale $1/R$. The gaugino masses can be generated by radiative corrections from the scalar masses via the graphs shown in figure 1. These masses are soft on scales larger than $M_W$ and hence do not contribute to the renormalization group evolution of the scalar masses from $1/R$ to $M_W$. We neglect the radiatively generated A term since it is produced by two-loop diagrams and is consequently small.

A physically reasonable vacuum state must have nonzero vev's for $H$, $H'$ and $N$ only; the relevant scalar potential then has the following form

$$V(H, H', N) = \lambda_H^2 |N|^2 (|H|^2 + |H'|^2) + |H|^2 |H'|^2 + m_H^2 |H|^2$$

$$+ m_{H'}^2 |H'|^2 + m_N^2 |N|^2 + \frac{2}{5} (g_1^2 + g_2^2) (|H|^2 - |H'|^2)^3$$

$$+ \frac{g_1^2}{12} (4|H|^2 + |H'|^2 - 5|N|^2)^2.$$  

(2.8)

If a field is an $SU(2)$ doublet, only the component with no electric charge has been written. It is necessary for the vev's of $H$, $H'$ and $N$ all to be nonzero in order that the group be broken down to $SU(3)_c \times U(1)_{em}$ and that there be masses for all the quarks and leptons. Then all the $2 \times 2$ sub-determinants of the matrix of second derivatives of the potential must be positive. This implies that

$$\lambda_H^2 < \frac{5g_1^2}{36} [1 + (1 + \frac{9}{g_1^2} (g_1^2 + g_2^2)]^{1/2} = .13.$$  

(2.9)

Now consider the renormalization group equations [19] for $m_H$, $m_{H'}$, $m_N$, $m_\nu$, and for the left and right handed top squarks $m_Q$, and $m_\nu$. These equations can be written in the following form

$$8\pi^2 \frac{dM_1^2}{dt} = 3\lambda_H^2 M_1^2 + 4\lambda_H^2 M_2^2 + 3 \sum_i \lambda_{H_i}^2 M_i^2,$$

$$8\pi^2 \frac{dM_2^2}{dt} = 2\lambda_H^2 M_1^2 + 2\lambda_H^2 M_2^2 + 3 \sum_{i,j} \lambda_{H_{ij}}^2 M_{ij}^2,$$

$$8\pi^2 \frac{dM_3^3}{dt} = \lambda_{H'}^2 M_1^2 + 6\lambda_{H'}^2 M_2^2,$$

(2.10)

where

$$M_1^2 = m_{H_1}^2 + m_\nu^2 + m_N^2,$$

$$M_2^2 = m_{H'}^2 + m_{H''}^2 + m_N^2,$$

$$M_3^2 = m_{H'''}^2 + m_{H''}^2 + m_N^2.$$  

(2.11)

where $t = \log(\mu R)$ and $\mu$ is the renormalization scale. We neglect all the quark

* We take $g_1^2 = .126$, $g_2^2 = .45$ in all numerical estimates.
and lepton Yukawa couplings except for those involving the $g$ and $t$ quarks, since all the others will be much smaller ($\lambda_T \equiv \lambda_{u_3}$).

We can solve these equations subject to the boundary conditions

$$M_l^2(0) \equiv 3m_2^2, \quad M_l^2(t_0) \equiv m_t^2$$

(2.12)

where $t_0 = \log(M_W R)$. We obtain the following result:

$$I \equiv \int_0^{t_0} dt \lambda_T^2 M_3^2$$

(2.13)

$$= \frac{1}{69} (36m^2 + 5m_1^2 - 14m_2^2 - 3m_3^2).$$

It can be shown by numerical solution of the renormalization group equations that this quantity is positive for all choices of the parameters at $t = 0$, provided that the Yukawa couplings remain perturbative at all scales. Hence

$$\int_0^{t_0} dt \lambda_T^2 (m_H^2 + m_Q^2 + m_H^2) > 0.$$  

(2.14)

Now, consider the evolution of $m_H^2$ and $m_{H'}^2$:

$$8\pi^2 \frac{dm_H^2}{dt} = 3\lambda_T^2 (m_H^2 + m_Q^2 + m_H^2) + \lambda_T^2 (m_H^2 + m_{H'}^2 + m_{H'}^2)$$

$$8\pi^2 \frac{dm_{H'}^2}{dt} = \lambda_T^2 (m_H^2 + m_{H'}^2 + m_{H'}^2).$$  

(2.15)

Using Eq. (2.14), it follows that

$$m_H^2(M_W) < m_{H'}(M_W).$$  

(2.16)

This bound can now be combined with the equations which result from the first derivatives of the potential (Eq. (2.8)) to obtain the bound

$$n^2 < \frac{12}{5g_1^2} \left[ \frac{1}{2} (g_1^2 + g_2^2) - \lambda_H^2 [h^2 - h'^2] + \frac{1}{5} (4h^2 + h'^2) \right].$$  

(2.17)

We use the notation where the vev of a field is represented by the appropriate lower case letter.

We will first discuss the case where $r = h/h' < 1$. Using the bound on $\lambda_H$ from Eq. (2.9), we have

$$n^2 < \frac{1}{5} (4h^2 + h'^2).$$  

(2.18)

This implies that $n < 125$ GeV for any value of $r < 1$.

Is it possible to have a viable theory with such a small value of $n$? The model has two massive neutral gauge bosons whose masses are given by

$$M_{1,2}^2 = \frac{M_{1,2}^2}{2} + \frac{g_1^2 \xi}{36} \pm \frac{g_1^2 \xi}{36} [(1 - \frac{18M_{1,2}^2}{g_1^2 \xi})^2 + \frac{36g_1^2}{\xi^2} (h^2 - 4h'^2)^{1/2}]$$  

(2.19)

where $M_1$ ($M_2$) refers to the $+(-)$ sign. Here $M_{Z_a}$ is the mass of the $Z$ in the Weinberg-Salam model

$$M_{Z_a}^2 \equiv \frac{1}{2} (g_1^2 + g_2^2) (h^2 + h'^2)$$  

(2.20)

and

$$\xi \equiv 25n^2 + 16h^2 + h'^2$$  

(2.21)

These two eigenstates are mixtures of the standard $Z_0$ and the $U(1)'$ gauge boson (called $Z_a$). In the limit of very large $n$, the lightest of these two states becomes the standard $Z$ boson. For smaller values of $n$, the two states both become accessible experimentally. Unless $n$ is very large or $h' = 2h$, when there is no mixing, there will be no eigenstate with the standard $Z_0$ mass.
Therefore we can obtain a constraint on the model by requiring that there be one \( Z \) with mass close to the experimental value and another which could have escaped detection. The method that we adopt is to search the \( r \) range from 0 to 1. For a given value of \( r \), \( h' \) is given by

\[
h' = \frac{\sqrt{2}M_W}{9\sqrt{1 + r^2}}.
\] (2.22)

We then evaluate the upper bound on \( n \) from Eq. (2.18) and search the range of \( n \) for a solution in which there is a \( Z \) with mass \( 93 \pm 4 \text{ GeV} \); this restricts us to a value of \( r \) close to \( \frac{1}{3} \), and determines the other \( Z \) mass to be less than 65 GeV.

We then evaluate the production rate of the other \( Z \) at the \( S p \bar{p}S \) collider using the following couplings

\[
\mathcal{L}_{\text{INT}} = -\frac{g_2}{2\cos\theta_W}[\overline{R}R \gamma^\mu R + \overline{L}L \gamma^\mu L]Z_1\mu
+ \overline{R}R \gamma^\mu R + \overline{L}L \gamma^\mu L]Z_2\mu,
\] (2.23)

where \( Z_1 \) and \( Z_2 \) are the physical \( Z \) bosons with masses given by Eq. (2.19), \( R, L = (1 \pm \gamma_5)\psi/2 \)

\[
R_{2'} = -2Qx_w \sin\theta + \sqrt{2}Y'\cos\theta \]
\[
L_{2'} = (\tau_3 - 2Qx_w)\sin\theta + \sqrt{2}Y'\cos\theta \]
\[
R = -2Qx_w \cos\theta - \sqrt{2}Y'\sin\theta \]
\[
L = (\tau_3 - 2Qx_w)\cos\theta - \sqrt{2}Y'\sin\theta. \] (2.24)

\( \tau_3 \) and \( Q \) are the weak isospin and charge of a fermion(\( \psi \)) and the \( U(1)_Y \) quantum numbers are given in Appendix A. \( \theta \) parameterizes the mixing between the gauge boson eigenstates and is given by

\[
\sin^2\theta = \frac{M_2^2 - M_1^2}{M_2^2 - M_0^2}.
\] (2.25)

We require that there be less than 10 such new \( Z \)'s decaying to electron-positron pairs produced in \( p\bar{p} \) collisions at \( \sqrt{s} = 630 \text{ GeV} \) with an integrated luminosity of 200 nb\(^{-1} \). No values of \( r < 1 \) and \( n \) consistent with Eq. (2.18) can satisfy this constraint and so the model is unacceptable.

The case \( r > 1 \) is easier to deal with since now there must be mixing between the standard model \( Z \) and the \( Z_a \). The bound on \( n \) becomes

\[
n^2 < h^2 \left( \frac{1}{5} - \frac{6}{5g_1^2}(g_1^2 + g_2^2) \right) + h^2 \left( \frac{4}{5} + \frac{6}{5g_1^2}(g_1^2 + g_2^2) \right) < (440 \text{ GeV})^2.
\] (2.26)

The shift in the \( Z_0 \) mass is given for large \( n \) by

\[
|\delta M_2^2| = \frac{M_2^2 - M_0^2}{M_0^2} = \frac{(h^2 - 4h^2)^2}{25n^2(h^2 + h'^2)}
\] (2.27)

and we therefore have

\[
|\delta M_2^2| > \frac{1}{25} \frac{(1 - 4r^2)^2}{(6.2r^2 - 5.2)} > .08
\] (2.28)

which corresponds to a shift in the standard \( Z \) mass of more than 4 GeV. Consequently we conclude that the case \( r > 1 \) is ruled out.

The case where \( \lambda_{H_{i\alpha}} \) is off-diagonal is easily disposed of. The relevant parts of the superpotential are as follows

\[
g(\Phi) = \lambda_T H_1 Q t^c + \lambda_{H_1 H_2} H_2^c + \lambda_{H_2 H_1} H_1^c + \lambda_\theta g^c g N.
\] (2.29)

Here we have assumed that there are only two \( H, H' \) pairs, the argument is unaffected if there are more. \( H_1 \) must have a vev in order to give mass to the top quark, but either of \( H'_1 \) or \( H'_2 \) could give mass to the bottom quark. If it is \( H'_1 \), then the model has an axion and is unacceptable. If it is \( H'_2 \), then \( H_2 \) and \( H'_1 \) must have no vev in order to exclude an axion. We are now back to the case discussed in detail above: one simply relabels \( H_1 \rightarrow H \) and \( H'_1 \rightarrow H' \).
We have shown that the model based on $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_Y'$ is not phenomenologically viable in its simplest form. Which of our assumptions must be modified to make a viable model? The model cannot have an intermediate scale where it is broken to $SU(3)_c \times SU(2)_L \times U(1)_Y$. As indicated above, the survivor fields have the wrong quantum numbers to achieve the breakdown, and the vev of $N$ is restricted to be small since it would generate a large $D$ term and hence a large negative mass squared for some squarks and sleptons.

We assumed that all of the scalar masses have a common value at the scale $1/R$. Although the Yukawa couplings participate in renormalization group scaling below $1/R$, this is not necessarily the case for scalar masses. They evolve from the scale $M_C$ at which the hidden sector breaks supersymmetry[12,13] (condensation scale). This scale is not expected to be far below $1/R$[11]. Our results are not sensitive to this assumption.

A crucial assumption is that $m_{H_1}^2 = m_{H'}^2$ at the scale $M_C$. The equality of the other scalar masses was only used to derive Eq. (2.14). However, since $m_{H_2}^2$ and $m_{H'}^2$ are both positive for all scales (or else color is broken) and since $m_{H_2}^2$ is positive at large scales, it would be difficult to find a model which did not satisfy Eq. (2.14). We also found numerically that Eq. (2.14) is valid if, at $M_C$, $m_{H_1}^2 = m_{H'}^2 = m_{N}^2 = 0$ and the other scalar masses are equal. Thus, the only critical assumption which cannot be relaxed is that $m_{H_1}^2 = m_{H'}^2$ at the scale $M_C$. We know of no models in the literature in which this assumption is not made.

The assumption that $\lambda_T > \lambda_B$ seems natural since the top quark is much heavier than the bottom quark. If $\lambda_B > \lambda_T$, then we cannot prove that $m_{H_1}^2(M_W) < m_{H'}^2(M_W)$ which was crucial to our argument. In this case it may be possible to make an acceptable model, but only, we can show, at the expense of a very large ratio of vevs, $h/h' > 25$.

If we relax the constraint on flavor changing neutral currents, then it may be possible to construct a viable model. The simplest option couples the $N$ field to off-diagonal $H H'$ pairs and to top and bottom quarks. For example, a superpotential of the form

$$g(\Phi) = \lambda_T H_1 Q t^c + \lambda_T H_2 Q t^c + N (\lambda_{H_1} H_1 H'_1 + \lambda_{H_2} H_2 H'_2 + \lambda_{H_1} H_1 H'_2 + \lambda_{H_2} H_2 H'_1) + \lambda_{Q} g N + \lambda_{Q} Q H' g'_1 + \lambda_{Q} N h'$$

(2.30)
can be used. The constraint $m_{H_1}^2 < m_{H'}^2$ can then be evaded, since the $\lambda_{H_i}$ coupling can be used to drive $m_{H'}^2$. Although aesthetically unappealing, such flavor-changing neutral currents may not be phenomenologically disastrous[21].
3. OTHER MODELS WITH NO INTERMEDIATE SCALE

In this section, we consider the other cases where the gauge symmetry is described by a rank five or six subgroup $G$ of $E_6$. We will discuss here the case with no intermediate scale; i.e. the group $G$ is unbroken down to scales of order 1 TeV. As discussed in the introduction, the requirement of perturbative unification forces there to be three generations. We shall make the same assumptions concerning the pattern of supersymmetry breaking as in section 2. That is, we assume that all the scalars have a common mass at the unification scale, that the gaugino masses are radiative (and therefore do not enter the renormalization group equations for the scalar masses) and that there are no A or B terms at scale $1/R$. These cases are different from the one discussed in the previous section in that we can now have survivor fields which are singlets with respect to $SU(3)_c \times SU(2)_L \times U(1)_Y$ and which can therefore be used to break the symmetries of the model. Such survivor fields are listed in Table 1.

(a) $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)' \times U(1)''$

The model based on this group has been considered by Mangano[14]. One of the $U(1)'s$ can be broken by a vev for the $N$ field, the other must be broken by a vev for $N'$. The relevant survivor fields are either an $N$ or an $N'$ (see Table 1). As before, these survivors can only couple to the matter 27's in the superpotential via terms of the form

$$
\delta g(\Phi) = \frac{\lambda_H}{M_p} N^2 N^2 \text{ or } \frac{\lambda_H}{M_p} N'^2 N'^2.
$$

The contribution of these terms is supressed by powers of $M_W R$ if there is no intermediate scale and is therefore negligible.

The quantum numbers of the fields under the various $U(1)'s$ are not completely determined. One of the $U(1)'s$ will be taken to be $U(1)_Y$, but there is freedom in determining the quantum numbers $Y'$ and $Y''$ which can be parameterized in terms of a mixing angle $\theta_3$. This is discussed in Appendix A, where all the quantum numbers are given (see Table (A.1)). The angle could be chosen so that one of the $U(1)'s$ is the same as that discussed in the previous section. Alternatively, it could be chosen so that $N$ is neutral under one of them; this will be used in the next section to discuss the case where the vev of $N$ generates an intermediate scale.

The contributions of the D terms to the scalar potential can be written in the form

$$
V_D = \frac{1}{8} (g_1^2 + g_2^2) \left[ \sum_i (|H_i|^2 - |H_i'|^2) \right]^2
$$

$$
+ \frac{g_1^2}{72} [\cos \theta_3 \left( \sum_i (|H_i|^2 + |H_i'|^2) - 5|N|^2 - 5|N'|^2 + 5|N'|^2 + 5|N|^2 \right)]
$$

$$
- \sqrt{15} \sin \theta_3 \left( \sum_i (|H_i|^2 - |N|^2 + |N|^2 + |N|^2 - |N'|^2) \right)^2
$$

$$
+ \frac{g_2^2}{72} [\sin \theta_3 \left( \sum_i (|H_i|^2 + |H_i'|^2) - 5|N|^2 - 5|N'|^2 + 5|N|^2 + 5|N|^2 \right)]
$$

$$
+ \sqrt{15} \sin \theta_3 \left( \sum_i (|H_i|^2 - |N|^2 + |N|^2 + |N|^2 - |N'|^2) \right)^2,
$$

where only the components with zero electric charge are written. We have given the potential assuming that both $N$ and $N'$ appear. This is not the case but the three options - no survivors, $\bar{N}$ or $\bar{N}'$ - can be obtained by deleting the appropriate terms. The coupling constants $g_1, g', g''$ are associated with $U(1)_Y, U(1)_Y'$, and $U(1)_Y''$, respectively. If there are no survivors, they are equal at all scales. This is not true if there are survivors. As in the previous model however, the differences between these couplings are at most 3% and so we shall neglect them and set $g_1 = g' = g''$. The $\theta_3$ dependence then cancels from the potential.

If we make the same assumptions as in section 2, we are left with the following form for the superpotential(cf. Eq. (2.7))

$$
g(\Phi) = \lambda_u H Q u^c_i + \lambda_d H' Q d^c_i + \lambda_e H' L e^c_i
$$

$$
+ \lambda_{n1} H H' N_1 + \lambda_{n2} H_a H_b N_2 + \lambda_{n3} g_i g_i N
$$

$$
+ \lambda_{n4} H_a L_i N'^i
$$

18
As in Section 2, \( i = 1, 2, 3 \) is a generation index. The vevs of \( H, H' \), at least one of the pairs \((N, N)\) and at least one of the pairs \((N', N')\) must be nonzero. The Higgs doublets which do not receive vevs are labelled by \( a, b = 1, 2 \). The scalar potential involving \( H, H', N, N' \) and \( N' \) is then,

\[
V = m_H^2 |H|^2 + m_{H'}^2 |H'|^2 + m_N^2 |N|^2 + m_{N'}^2 |N'|^2 + m_{N''}^2 |N''|^2 \\
+ \frac{1}{2} |(H|^2 + |H'|^2)| \\
+ \frac{1}{8} (g_1^2 + g_2^2) |(H|^2 - |H'|^2)|^2 \\
+ \frac{g_1^2}{2} [4 |H|^2 + |H'|^2 - 5 |N|^2 - 5 |N'|^2 + 5 |N''|^2] \\
+ \frac{g_1 h_1}{2} [2 h_2 + 2 (h_2 - 2 h_1)] + \frac{g_2 h_2}{2} [h_2 - h_1].
\]

(3.5)

Again, the cases of interest are obtained by deleting the relevant terms from this expression.

We begin by discussing the cases in which there is no survivor or the survivor field is an \( N \). An identical analysis to that of Section 2 gives the restriction \( m_H < m_{H'} \). From the potential of Eq. (3.5) we then find

\[
n^2 < \frac{6}{5g_1^2} (h^2 - h'^2) \left( \frac{5}{6} g_1^2 + \frac{1}{2} g_2^2 - \lambda_H^2 \right).
\]

(3.6)

Requiring that there be a stationary point of the potential for nonzero \( h \) and \( h' \) implies that,

\[
\lambda_H^2 < \frac{5}{6} g_1^2 + \frac{1}{2} g_2^2 = .34
\]

(3.7)

Hence, Eq. (3.6) implies that \( h^2 > h'^2 \) and \( n' < 320 \) GeV.

This bound is quite stringent and can be used to analyze the mass matrix for the neutral gauge bosons which is given by

\[
M^2 = \begin{pmatrix}
\frac{g_1^2}{18} (16 h^2 + h'^2 + 25 n^2) & \frac{g_1 \sqrt{h_1^2 + h_2^2}}{6} (h^2 - 4 h'^2) \\
\frac{g_1 \sqrt{h_1^2 + h_2^2}}{6} (h' + n^2) & \frac{5g_1^2}{6} (h'^2 + n^2) & -\frac{g_1 \sqrt{h_1^2 + h_2^2}}{2} \sqrt{5 h^2} \\
\frac{g_1 \sqrt{h_1^2 + h_2^2}}{6} (h^2 - 4 h'^2) & -\frac{g_1 \sqrt{h_1^2 + h_2^2}}{2} \sqrt{5 h'^2} & M_{N''}^2
\end{pmatrix}
\]

(3.8)

where

\[
\vec{n}^2 = n^2 + n^2 + \bar{n}^2 + \bar{n}'^2
\]

\[
\bar{n}^2 = n^2 - n^2 - \bar{n}^2 + \bar{n}'^2.
\]

An analysis of the \( Z \) mass matrix for arbitrary values of \( n \) and \( \bar{n} \) with \( n'' = 0 \) and \( n' < 320 \) GeV yields no solution with an eigenvalue within 4 GeV of the observed \( Z \) mass and with no other massive neutral gauge bosons lighter than 110 GeV. The model with no survivors or an \( N \) survivor is therefore ruled out.

We will now consider the case where the survivor is an \( N' \). If both \( n' \) and \( \bar{n}' \) are nonzero, then minimizing the potential of Eq. (3.5) gives,

\[
\frac{1}{n'} \frac{\partial V}{\partial N'} + \frac{1}{\bar{n}'} \frac{\partial V}{\partial \bar{N'}} = 0
\]

(3.9a)

or

\[
m_{N'}^2 + m_{N''}^2 = 0.
\]

(3.9b)

Hence, \( \bar{n}' = 0 \) is eliminated by the analysis of the model with no survivors. We are left with the case where the only nonzero vevs are \( h, h', n' \) and \( n \).

Minimization of the potential, Eq. (3.5), gives the relationship,

\[
0 = \frac{2}{\bar{n}'} \frac{\partial V}{\partial \bar{n}'} + \frac{1}{n'} \frac{\partial V}{\partial n'}
\]

(3.10)

\[
= m_{N'}^2 + 4 m_{N''}^2 + \lambda^2 (h^2 + h'^2) + \frac{5g_1^2}{6} (2h^2 - 2h'^2 + 5\bar{n}'^2).
\]

The requirement that there be a gauge boson within 4 GeV of the observed \( Z \) and no other gauge boson lighter than 110 GeV in the mass matrix of Eq. (3.8)
gives

$$\frac{n^2}{h^2 + h^2} > 3.8.$$  \hspace{1cm} (3.11)

Sustituting Eq. (3.11) into Eq. (3.10) gives,

$$m_N^2 < -4m_N^2 = -4m^2.$$  \hspace{1cm} (3.12)

We now show that the presence of a fixed point in the renormalization group equations for the scalar masses forbids reaching the region of parameter space defined by Eq. (3.12).

The renormalization group equations which the masses satisfy are,

$$\frac{dm_H^2}{dt} = \frac{\lambda_H^2}{8\pi^2}(m_H^2 + m_{H_1}^2 + m_N^2)$$

$$\frac{dm_{H_1}^2}{dt} = \frac{\lambda_{H_1}^2}{8\pi^2}(m_{H_1}^2 + m_N^2)$$

$$\frac{dm_L^2}{dt} = 2\frac{\lambda_L^2}{8\pi^2}(m_L^2 + m_{L_i}^2 + m_{H_1}^2)$$

$$\frac{dm_{Q_i}^2}{dt} = \frac{\lambda_{Q_i}^2}{8\pi^2}(m_{Q_i}^2 + m_{H_1}^2 + m_N^2)$$

$$\frac{dm_{L_i}^2}{dt} = \frac{\lambda_{L_i}^2}{8\pi^2}(m_{L_i}^2 + m_{H_1}^2 + m_N^2)$$

$$\frac{dm_{Q_i}^2}{dt} = \frac{\lambda_{Q_i}^2}{8\pi^2}(m_{Q_i}^2 + m_{H_1}^2 + m_N^2)$$

$$\frac{dm_{Q_i}^2}{dt} = \frac{\lambda_{Q_i}^2}{8\pi^2}(m_{Q_i}^2 + m_{H_1}^2 + m_N^2)$$

$$\frac{dm_{L_i}^2}{dt} = \frac{\lambda_{L_i}^2}{8\pi^2}(m_{L_i}^2 + m_{H_1}^2 + m_N^2)$$

$$\frac{dm_{Q_i}^2}{dt} = \frac{\lambda_{Q_i}^2}{8\pi^2}(m_{Q_i}^2 + m_{H_1}^2 + m_N^2)$$

$$\frac{dm_{L_i}^2}{dt} = \frac{\lambda_{L_i}^2}{8\pi^2}(m_{L_i}^2 + m_{H_1}^2 + m_N^2)$$

where we have neglected the Yukawa couplings of all fermions except the top quark. From Eq. (3.13) it is straightforward to obtain the following mass relations,

$$m_{H_1}^2 - m_H^2 + 3m_{Q_i}^2 = 3m^2$$

$$2m_{Q_i}^2 - m_{L_i}^2 = m^2$$

$$m_{Q_i}^2 = m_{L_i}^2$$

$$m_{H_1}^2 = m_{H_1}^2$$

$$\sum_{i}(m_{H_1}^2 - m_{H_1}^2) + \sum_i m_{L_i}^2 = 3m^2$$

$$3 \sum_i m_{Q_i}^2 + 2 \sum_i m_{H_1}^2 + 2m_{H_1}^2 - m_N^2 = (3N_g + 2N_h + 1)m^2$$

where \(m\) is the common mass of the scalars at \(1/R\), \(N_g\) is the number of \(g\) quarks and \(N_{H_1}\) is the number of pairs of \((H_1, H_1')\) Higgs fields. There is an obvious fixed point given by

$$2m_{Q_i}^2 + m_N^2 = 0$$

$$m_{H_1}^2 + m_{H_1}^2 + m_N^2 = 0$$

$$m_{H_1}^2 + m_{H_1}^2 + m_N^2 = 0$$

$$m_{Q_i}^2 + m_{Q_i}^2 + m_{Q_i}^2 = 0.$$  \hspace{1cm} (3.15)
Using Eqs. (3.14) and (3.15), we find that at the fixed point (assuming for simplicity that \( \lambda_{\text{ext}} \propto \delta_{\text{ext}} \))

\[
m^2_N \bigg|_{\text{fixed point}} = \frac{3N_v + 2N_H (4N_{N^c} + 1)}{3N_v + 2N_H (4N_{N^c} + 1) + \frac{14}{3} (2m^2)}.
\]

(3.16)

Hence,

\[
m^2_N > -2m^2
\]

(3.17)

which is in direct contradiction with Eq. (3.12). *

Our conclusion is that without an intermediate scale, it is impossible to generate the correct \( Z_0 \) mass in this model without having the extra massive neutral gauge bosons lighter than 110 GeV.

(b) \( SU(3)_c \times SU(2)_L \times U(1)_L \times SU(2)_R \times U(1)_R' \)

Among the members of a 27 representation of \( E_6 \), the set of fields which transform as doublets under \( SU(2)_R \) is

\[
\begin{pmatrix}
d^c \\ u^c \\ H^c \\ H^0 \\ \ell^c \\ N^c 
\end{pmatrix}.
\]

(3.18)

The \( U(1) \) quantum numbers can be determined by considering the normalizations of the gauge coupling constants \( g_{1L} \) and \( g_{1R'} \) of \( U(1)_L \) and \( U(1)_R' \). They can be chosen so that they are equal to the conventional \( U(1)_Y \) coupling at the unification scale. In that case, the quantum numbers are given in terms of the quantum numbers of the previous model (with \( \theta_3 = 0 \) ) by

\[
\begin{align*}
Y_L &= \frac{1}{\sqrt{5}} Y - \frac{2}{\sqrt{5}} Y' \\
Y_R &= \frac{1}{\sqrt{5}} Y + \frac{1}{2\sqrt{5}} Y' + \frac{\sqrt{3}}{2} Y'' \\
T_{3R} &= \frac{3}{10} Y + \frac{3}{20} Y' - \frac{1}{4} \sqrt{5} Y''.
\end{align*}
\]

(3.19)

Then \( Y = 2T_{3R} + \frac{1}{\sqrt{5}} (Y_L + Y_R) \) and the values of \( (Y_L, Y_R) \) are \((-\frac{\sqrt{5}}{5}, \frac{\sqrt{5}}{5})\) for \( H_1 \) and \( H_1' \), \((\frac{2\sqrt{5}}{5}, -\frac{\sqrt{5}}{5})\) for \( N \) and \((\frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{5})\) for \( N' \). The beta functions of \( g_{1L} \) and \( g_{1R'} \) are equal at the one-loop level to that of the \( U(1)_Y \) coupling \( g_1 \) if we include only full 27 representations and we neglect the contribution of the survivors.

Therefore, at all scales

\[
g_{1L} \simeq g_{1R'} \simeq g_1.
\]

(3.20)

The same is true for the \( SU(2)_L \) and \( SU(2)_R \) couplings, since there are equal numbers of \( SU(2)_L \) and \( SU(2)_R \) doublets (neglecting survivors)

\[
g_{2L} = g_{2R}.
\]

(3.21)

There are two options for the survivor fields from the \( \overline{27} \). There can be an \( \overline{N} \) and a field \( \overline{\Phi} \) transforming as a \((2, 2)\) under \( SU(2)_L \times SU(2)_R \),

\[
\overline{\Phi} \equiv \begin{pmatrix} H^i \ 
\end{pmatrix}.
\]

(3.22)

or alternatively, the \( SU(2)_R \) doublet

\[
\overline{N} \equiv \begin{pmatrix} N' 
\end{pmatrix}.
\]

(3.23)

Because of the presence of an \( SU(2)_R \) symmetry, low energy phenomenology provides us with a rather strict bound on vevs of some of the fields. Including

* Since the parameter space is multidimensional, one might worry that \( m^2_N \) could approach its fixed point from a region in which Eq. (3.17) is invalid. We have demonstrated that this does not occur in the simple case where \( N \) couples to two \( g \)‘s only, and this is actually enough to obtain a bound which contradicts Eq. (3.12).
all the fields that could possibly contribute, we obtain the following form for the charged gauge boson mass matrix,

\[ M^2 = \frac{1}{2} \begin{pmatrix} g_{2L}^2 (h^2 + h'^2 + h'^2 + h'') & -2g_{2L}g_{2R}(hh' + hh'') \\ -2g_{2L}g_{2R}(hh' + hh'') & g_{2R}^2 (k^2 + h'^2 + h'^2 + n'^2 + n^2) \end{pmatrix} \]  

(3.24)

There is a phenomenological bound[22] on the mass of a right-handed W which is

\[ M_{W_R}^2 \geq (400 \text{ GeV})^2. \]  

(3.25)

This implies that \( n'^2 + n^2 > > h^2, \tilde{h}^2, h'^2, \tilde{h}'^2 \) and hence

\[ M_{W_R}^2 \approx \frac{1}{2} g_{2R}^2 (n'^2 + n^2). \]  

(3.26)

The scalar potential for this model has the following contribution from D terms (once again we give all possible contributions although they may not all appear simultaneously)

\[ V_D = \frac{g_{2L}^2}{8} (|H|^2 - |H|^2 - |H'|^2 + |H'|^2)^2 \]
\[ + \frac{g_{2R}^2}{8} (|H|^2 - |H|^2 - |H'|^2 + |H'|^2 - |N'|^2 + |N|^2)^2 \]
\[ + \frac{5g_{2L}^2}{72} (-|H|^2 + |H|^2 - |H'|^2 + |H'|^2 + 2|N|^2 - 2|N'|^2 + 2|N|^2 + 2|N'|^2 - 2|N|^2)^2 \]
\[ + \frac{5g_{2R}^2}{72} (|H|^2 - |H|^2 - |H'|^2 - |H'|^2 - 2|N|^2 + 2|N|^2 + |N'|^2 - |N|^2)^2 \].  

(3.27)

The terms allowed in the superpotential are identical to the \( U(1)^3 \) case (sub-section (a)) and therefore if we make the same assumptions as in Section 2, we are left with the superpotential of Eq. (3.4). The difference is that the \( SU(2)_R \) symmetry imposes \( \lambda_u = \lambda_d \). Moreover, in the case where \( \bar{N} \) and \( \bar{\Phi} \) survive from the \( \bar{\Phi}^3 \), there is an additional term coming from the \( \bar{\Phi}^3 \) term of Eq. (2.3):

\[ g(\Phi) = \bar{\lambda}_H \bar{H} \bar{H'} \bar{N}. \]  

(3.28)

To recapitulate, the scalar potential involving \( H, H', H', \bar{H}, \bar{H}', N, \bar{N}, N', \bar{N}' \) is

\[ V = m_H^2 |H|^2 + m_{H'}^2 |H'|^2 + m_{\bar{N}}^2 |N'|^2 + m_{\bar{N}}^2 |N|^2 + m_{\bar{N}}^2 |\bar{N}'|^2 \]
\[ + m_{\bar{N}}^2 |\bar{H}'|^2 + m_{\bar{H}}^2 |\bar{H}|^2 + \lambda^2_H (|H|^2 + |H'|^2 + |\bar{H}'|^2) \]
\[ + \lambda^2_R (|\bar{N}'|^2 + |N|^2 + |\bar{H}'|^2 + |\bar{H}|^2) + V_D. \]  

(3.29)

Requiring that the potential has a minimum for nonzero values of \( h' \) and \( h \) gives

\[ \lambda_H^2 < \frac{g_{2L}^2 + g_{2R}^2}{2}. \]  

(3.30)

Since \( (H, H') \) transforms as a \( (2, 2) \) under \( SU(2)_L \times SU(2)_R \), their Yukawa couplings are identical and hence \( m^2_H = m^2_{H'} \). Therefore,

\[ \frac{1}{2} g_{2R}^2 (n'^2 + n^2) = (h^2 - \tilde{h}^2) (\lambda_H^2 - \frac{g_{2L}^2 + g_{2R}^2}{2}) + (h'^2 - \tilde{h}'^2) (\frac{g_{2L}^2 + g_{2R}^2}{2}) \]  

(3.31)

which in turn gives, using Eq. (3.30),

\[ \frac{1}{2} g_{2R}^2 (n'^2 + n^2) < (h^2 + \tilde{h}^2 + h'^2 + \tilde{h}'^2) (\frac{g_{2L}^2 + g_{2R}^2}{2}). \]  

(3.32)

If \( \bar{N}' \) does not exist or if \( \bar{N}' = 0 \), then this is simply the constraint

\[ M_{W_R}^2 < 2M_{W_L}^2. \]  

(3.33)

which is in contradiction with the bound from Eq. (3.25).
We conclude that $N'$ exists and has a nonzero vev. In other words, we are in the case where the doublet $\bar{M}$ survives, and $n' \neq 0$. Then, using the same argument as used in the preceding subsection (Eq. (3.9) remains valid), we conclude that $n' = 0$. We return to the potential (Eq. (3.29)), delete $H, H', N$, set $n_1 = 0$ and obtain

$$
\frac{2}{n} \frac{\partial V}{\partial n} + \frac{1}{2n} \frac{\partial V}{\partial n'} = 0
$$

$$
= m^2_N + 4m^2_{N'} + g^2_R(h^2 - h'^2) + \lambda_R^2(h^2 + h'^2) + (g^2_R + \frac{5}{2}g^2)\bar{n'}^2.
$$

(3.34)

Since, from (3.25) and (3.26), $\bar{n'}^2 > 4(h^2 + h'^2)$, we obtain

$$
m^2_N < -4m^2_{N'} = -4m^2
$$

(3.35)

where $m$ is the common scalar mass at the unification scale. This is beyond the fixed point value for $m^2_N$ and therefore cannot be reached (see Eq. (3.17)).

To conclude, there is no way to reconcile this model with the phenomenological bound on the $W_R$ mass.

(c) $SU(3)_c \times SU(2)_L \times U(1)_Y \times SU(2)_N \times U(1)_{Y'}$

In this model, the group $SU(2)_N$ is the subgroup of $SU(2)_R$ which commutes with electric charge. Its representation content includes the following doublets:

$$
\begin{pmatrix}
N' \\
N
\end{pmatrix}_N
\begin{pmatrix}
\phi' \\
n' \\
\nu
\end{pmatrix}_N
$$

(3.36)

It is straightforward to determine the $U(1)$ quantum numbers in this model once one realizes that the $Y''$ charge of the $U(1)^3$ case (subsection(a)) is associated with the $T_3$ generator of $SU(2)_N$ when $T_3 = 0$. Therefore, one of the $U(1)$’s can be chosen to be the standard Weinberg-Salam $U(1)_Y$ and the $U(1)_{Y'}$, quantum numbers can be obtained by taking the limit $T_3 = 0$ in the $U(1)^3$ case. Thus, $Y$ and $Y'$ coincide with the quantum numbers of the group $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{Y'}$ discussed in Section 2.

If we consider only the survivors which are $SU(3)_c \times SU(2)_L \times U(1)_Y$ singlets, the only possible choice is the doublet

$$
\begin{pmatrix}
\bar{N} \\
\bar{N}'
\end{pmatrix}_N
$$

(3.37)

However, whether these survivors are present or not, the constraints imposed on the scalar potential by the $SU(2)_N$ symmetry are too stringent to allow a complete breaking of the gauge symmetry to $SU(3)_c \times U(1)_{em}$.

The $SU(2)_N$ symmetry provides relationships between some of the Yukawa couplings in Eq. (2.5). In particular, the term $\lambda_{ij}(\phi_j \phi_i N_k + \delta^j_ig_j N_k)$ appears. Our assumption that flavor changing neutral currents are absent excludes this term if $N'$ gets a vev. In this case, the $g$ quark would be massless, a phenomenological disaster.

If $N'$ does not get a vev, then $\bar{N}'$ must do so in order to allow the symmetry to be broken to $SU(3)_c \times SU(2)_L \times U(1)_Y$. In this case, we can analyze the constraints from the scalar potential. Since $\bar{N}$ and $\bar{N}'$ only couple through nonrenormalizable terms, $m^2_N$ and $m^2_{N'}$ are positive. Since $\bar{n'}$ is nonzero, we have

$$
m^2_{N'} = -\frac{g^2_{1N}}{4}(\bar{n'}^2 - h^2 + \bar{n'}^2) - \frac{5g^2_{1N}}{36}(4h^2 + h'^2 - 5\bar{n'}^2 + 5\bar{n'}^2) > 0
$$

(3.37)

where $\bar{n'}^2 \equiv n^2 - \bar{n'}^2$. If $\bar{n'}^2 > 0$, then this implies that $\bar{n'} < 110$ GeV, which results in an additional light $Z$ boson which is phenomenologically unacceptable. (The mass matrix is given in Appendix B.) The case $\bar{n'}^2 < 0$ is disposed of by using the constraint that $m^2_{N'} > 0$. Hence, the $SU(2)_N$ case cannot be reconciled with low energy phenomenology.

(d) Other Models

We now briefly discuss the other possible groups with no intermediate scale. If $G$ is $SU(3)_c \times SU(2)_L \times U(1) \times SU(3)_R$, we will encounter the same problem with the mass of the $W_R$ as occurred in the case of the $SU(2)_R$ model and so this
model is eliminated. Models in which $SU(2)_L$ is embedded in $SU(3)_L$ without an intermediate scale predict that $g_2(M_W) = g_3(M_W)$ which is clearly ruled out.

The models with extended color groups were discussed by Dine et al. [9]. If they lack an intermediate scale, most are eliminated because they predict proton decay via gauge bosons in the extended color group at a disastrously fast rate. The models based on $SU(4) \times SU(2)_L \times U(1)_A$ (rank 5) or $SU(4) \times SU(2)_L \times U(1)_A \times U(1)_B$ (rank 6) can avoid this disaster. Consider the former model. The decomposition of the 27 under $SU(4) \times SU(2)_L \times U(1)_A$ is as follows:

$$
\Phi[27] = A(4, 2, \frac{1}{\sqrt{6}}) + B(1, 2, -\frac{2}{\sqrt{6}}) + B'(1, 2, -\frac{2}{\sqrt{6}}) + C(6, 1, -\frac{2}{\sqrt{6}})
$$

$$
+ D(4, 1, \frac{1}{\sqrt{6}}) + D'(4, 1, \frac{1}{\sqrt{6}}) + E(1, 1, \frac{4}{\sqrt{6}})
$$

where the notation $X(x, y, z)$ indicates that $X$ transforms as an $(x, y)$ representation of $SU(4) \times SU(2)$ and has $U(1)_A$ charge equal to $z$. With the normalization given, all the coupling constants are equal at the scale $1/R$. The particles can be assigned as follows:

$$
A \ni Q, H
$$

$$
B \ni H'
$$

$$
B' \ni L
$$

$$
C \ni u', g
$$

$$
D \ni d', N
$$

$$
D' \ni g', N'
$$

$$
E \ni e'.
$$

The superpotential, suppressing generation indices, is

$$
W = \lambda_1(QH u'^c + Qg) + \lambda_2(QH'^c + HH') + \lambda_3(u'^c d'^c g' + g'^c N + g d'^c N') + \lambda_4 H' L e'.
$$

The survivor fields do not include a singlet under $SU(3)_c \times SU(2)_L \times U(1)_Y$ since the group is rank 5. Baryon number is violated via exchanges of the gauge bosons in $SU(4)$ which get mass when it is broken, but since lepton number is conserved in these interactions, the proton cannot decay via them. However, an analysis of the neutral gauge boson mass matrix (given in Appendix B) identical to that above, results in unacceptable values for the neutral gauge boson masses.

The model also has a rather low unification scale. The renormalization group equations for $SU(4)$ and $SU(2)$ are given by

$$
\frac{d\alpha_i}{dt} = \frac{\alpha_i}{2\pi} \beta_i
$$

with $\beta_3 = 6 - 3n_{27}$ and $\beta_4 = 12 - 3n_{27}$ where $n_{27}$ is the number of 27 representations. Since $\beta_4 = 0$ for the case $n_{27} = 4$, the model is allowed to have 4 generations. The scale $1/R$ is given by

$$
\ln(M_W/R) = \frac{\pi}{12} \left( \frac{1}{3\alpha_4(M_W)} - \frac{1}{\alpha_{em}(M_W)} \right)
$$

and

$$
\sin^2 \theta_W(M_W) = \frac{1}{4} + \frac{\alpha_{em}(M_W)}{3\alpha_4(M_W)}.
$$

By assumption, $SU(4)$ is unbroken to low energies, therefore $\alpha_4(M_W) = \alpha_3(M_W)$, which gives $1/R \sim 10^{13}$ GeV and $\sin^2 \theta_W(M_W) = .27$. Since the model is unified at this scale it seems difficult to avoid proton decay at a disastrous rate. Moreover, the large predicted value of $\sin^2 \theta_W$ is difficult to reconcile with current data. A similar problem occurs with any model based on $SU(4) \times G$.

* This implies that the model cannot have an intermediate scale.
4. MODELS WITH AN INTERMEDIATE SCALE

We now turn to the possibility that the group \( G \) could be broken at a scale \( M_I (M_W << M_I << 1/R) \) to some group \( G' \) which is either \( SU(3)_c \times SU(2)_L \times U(1)_Y \) or contains that group. We will consider the models based on a group \( G \) of rank 5 or 6. This group \( G \) necessarily contains \( U(1)_Y \times U(1)_Y' \). Since the fields which obtain a vev at the intermediate scale must be singlets under \( SU(3)_c \times SU(2)_L \times U(1)_Y \), we are left with \( N \) and \( N' \) as the only two possible candidates. These fields do not have opposite quantum numbers under \( U(1)_Y \cdot \)

Hence their vevs cannot cancel in the D terms and another field which is not a singlet under \( SU(3)_c \times SU(2)_L \times U(1)_Y \) will acquire a large vev in order to cancel it, breaking \( SU(3)_c \times SU(2)_L \times U(1)_Y \) at the scale \( M_I \).

This can be avoided if there are \( \overline{N} \) or \( \overline{N}' \), survivors, which restricts the rank of the group to six. If only one of them exists, \( G \) is broken to a rank 5 group \( G' \) at scale \( M_I \). Both of them exist only if \( G \) contains the group \( SU(2)_N \). In that case, \((N', N')\) is a doublet under \( SU(2)_N \) and one can use this symmetry to rotate the fields in order that the breaking occurs in, say, the direction of \( N \). Once again, \( G \) is broken to a rank 5 group \( G' \). Therefore, as noted by Dine et al., the gauge symmetry \( G' \) at low energy \( (<< M_I) \) is always larger than the standard model \( SU(3)_c \times SU(2)_L \times U(1)_Y \). It is on these extra degrees of freedom that we can perform an analysis similar to the one developed in the previous section and discriminate between the models.

The generation of an intermediate scale is not trivial. The only dimensionful parameters in the scalar potential are the soft mass terms. It is therefore natural that the ratios of vevs should be of the same order as the ratios of these masses. If this were true, then a large ratio of scales, \( M_I/M_W \), could only be produced by the mechanism which generated the supersymmetry breaking, an unlikely possibility. Dine et al.[9] proposed a way out of this dilemma. They pointed out that if there were a direction in the potential in which the quartic terms vanished, for example when \( \overline{n} \) and \( n \) are equal and the other vevs are zero, and if \( m_N^2 + m_{\overline{N}}^2 \) were negative, then the potential would be stabilized by the non-renormalizable terms in the superpotential which would yield \( M_I \sim \sqrt{M_W M_R} \).

The appearance of an intermediate scale could result in the generation of gaugino masses[15]. In this case, the analysis of the model at lower scales becomes more complicated. Large gaugino masses are a potential disaster since they cause radiative corrections which tend to increase the scalar mass-squared values at low energies making it difficult to break \( SU(2)_L \times U(1) \). The existing literature concerning gaugino masses is somewhat confused[13-15] and we shall delay a detailed discussion of them and of intermediate scale models to a future publication. In the remainder of this section we shall discuss the simplest case where \( G \) is \( SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_Y' \times U(1)_Y'' \) and shall continue to neglect gaugino masses.

If the survivor field is \( \overline{N} \), then the intermediate scale will be generated by having \( n = \overline{n} = O(M_I) \) (up to terms of \( O(M_W) \))[14]. Superpotentials of the type discussed in section 3 (see Eq. (34)) can be used. The Yukawa couplings \( \lambda_g \) and \( \lambda_H \) can be used to drive \( m_N^2 \) negative. The term \( \lambda_H H H' N \) is not allowed for at least one of the \((H, H')\) pairs since it will generate a mass of \( O(M_I) \) for them and render them unavailable to break the weak interactions. The unbroken symmetry is now \( SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_a \), where the quantum numbers of the relevant fields under \( U(1)_a \) are

\[
\begin{align*}
H : Y_a &= 2/\sqrt{6} \\
H' : Y_a &= -2/\sqrt{6} \\
N' : Y_a &= -5/\sqrt{6}.
\end{align*}
\]

The breakdown to \( SU(3)_c \times U(1)_a \) is now accomplished by vevs for \( H, H' \) and \( N' \). In the absence of neutrino masses \( N' \) will appear in the low energy potential only in the D terms and in a mass term. We shall assume that only...
one of the $H, H', N'$, denoted by $(H, H', N')$, obtain vevs. The relevant part of
the potential is
\[ V = m_H^2 |H|^2 + m_{H'}^2 |H'|^2 + m_N^2 |N'|^2 + \frac{1}{8} (g_1^2 + g_2^2) (|H|^2 - |H'|^2)^2 + \frac{g_2^2}{48} (2|H|^2 - 2|H'|^2 - 5|N'|^2)^2 + \sum_a \lambda_a^2 (|H|^2 |H'|^2) \]

where $g_a$ is the $U(1)_a$ coupling constant, and the $\lambda_a H H' N_a$ couple the $(H, H')$
pair to the $N$'s that have zero vevs.

Requiring that the potential have a minimum for nonzero $n', h$, and
using the constraint $m_H^2 < m_{H'}^2$, which holds in this case, we have
\[ n'^2 < \frac{6}{5g_2^2} \left[ \frac{1}{2} (g_1^2 + g_2^2) + \frac{1}{3} g_2^2 - \sum_a \lambda_a^2 \right] (h^2 - h'^2) < (300 \text{ GeV})^2 \]  

(4.3)

This results either in a shift in the mass of the $Z$ which is greater than 10 GeV or
in an extra massive neutral gauge boson lighter than 60 GeV (the mass matrix
is given in Appendix B).

If the terms $\lambda_4 g \phi' N$ are present, the $g$ quarks will obtain a mass of order
$M_I$ and will decouple from the low energy theory. The evolution of the strong
coupling constant is changed and it becomes possible to construct a model with
more than three generations. In a four generation model, we have no constraint
on the Yukawa coupling $\lambda_4 H L_4 N'$. There are now two cases to consider. If the
mass of the fourth generation $Q = 2/3$ quark ($u_4$) is much larger than that of
the $Q = -1/3$ quark ($d_4$), we will still have the constraint $m_H^2 < m_{H'}^2$, and the
bound of Eq. (4.3) becomes
\[ (\frac{5g_2^2}{6} - \lambda^2) n'^2 < \frac{1}{2} (g_1^2 + g_2^2) + \frac{g_2^2}{3} - \sum_a \lambda_a^2 (h^2 - h'^2). \]

(4.4)

This constraint is useless if $\lambda^2 > \frac{5g_2^2}{6}$ in which case we cannot eliminate the
model. The bound on $\lambda$ from the second derivatives of the potential is not

useful. Therefore, the fourth generation leptons must be heavy if this model is
to work. In the case where $d_4$ is much heavier than $u_4$ no definite conclusion
is possible. It is worth pointing out that any 3 generation model which breaks to
$SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_a$ will be eliminated by the above argument.

In the case where $\overline{N}'$ is the survivor field it is not possible for both $N'$ and
$\overline{N}'$ to get vevs in the three generation model (see Eq. (3.9) and the following dis­
cussion). The presence of a fourth generation and the term $\lambda L_4 H N'$ is therefore
essential. Unfortunately, the $g$ quarks do not get mass from the vev of $N'$ and so
will have masses of order $M_W$. The Landau pole in the strong coupling constant's
evolution (see introduction) eliminates the model from further discussion. This
argument can be extended to any model in which the intermediate scale is to be
produced by $N'$ and $\overline{N}'$ and shows that such models are not viable. Furthermore,
gaugino masses generated at the scale $M_I$ do not affect this argument.
5. CONCLUSIONS

In this paper we have considered a class of low energy supersymmetric models based on superstring theory. Having made some general assumptions, we were able to demonstrate that these models are severely constrained by current phenomenology and, in particular, all those with only one scale, $M_W$, in addition to the compactification scale, are ruled out. The basic difficulty is that these models have an enlarged gauge group at low energy and that the extra massive gauge bosons which occur are too light to have avoided detection. In most of the models, the mixing between the $Z^0$ and one or more of the new bosons displaces the $Z$ mass sufficiently that its value no longer agrees with experiment.

In view of the rather strong conclusions of this paper, it is worthwhile to list and discuss our assumptions. They are:

(a) The supersymmetry breaking is manifested in the observable world via the appearance of masses for all the scalar particles which are equal when they are evaluated at the scale of compactification. This assumption is based on the idea that whatever mechanism is responsible for the supersymmetry breaking is "flavor blind". We have seen in section 2 that it may be possible to relax this assumption and still not be able to construct a viable model.

(b) There are no gaugino masses or "$A$" terms at tree level. These assumptions are coupled since if a gaugino mass is present an "$A$" term will be generated via radiative corrections. This assumption is critical to our analysis. The effect of gaugino masses will be investigated in a future publication. It is possible that when a scalar field gets a nonzero vev, gaugino masses will be generated. If this occurs it will not affect our analysis of models with no intermediate scale. The analysis of models with such a scale could be disrupted since such masses can contribute to renormalization group scaling at scales less than $M_f$.

(c) There are only three generations of quarks and leptons and the top quark's Yukawa coupling is larger than that of the bottom quark. In the case where the low energy group is $SU(3)_c \times G$ and there is no intermediate scale, there cannot be more than three generations if the QCD coupling is to be perturbative at all scales between $M_W$ and $1/R$. In the case of an intermediate scale, there can be four generations if enough quarks decouple at the intermediate scale. If the group is $SU(4) \times G$, four generations are allowed even without an intermediate scale but the model has other problems.

It is a reasonable assumption that the Yukawa coupling for the top quark is larger than that for the bottom quark. This assumption can be avoided if the potential is constructed so that $h/h'$ is very large. In this case, we cannot establish the critical inequality $m_H^2 < m_H^2$, and acceptable models may be possible. In models with four generations it is possible that the mass of the $Q = 2/3$ quark could be less than that of the $Q = -1/3$ quark so that a large ratio of vevs would not be required and the inequality could easily be evaded.

(d) Each component of the $27$ appears at most once in the particle spectrum. This follows from the assumption that the six-dimensional manifold has some definite properties[7], viz. there is only one independent harmonic $(1,1)$ form. This assumption is true in all currently known examples of manifolds[7,23]. Our conclusions could be evaded if there were an arbitrary number of survivor fields from the $27$('s).

(e) Symmetries are present which prevent flavor changing neutral currents in the presently observable sector. The constraints on tree level flavor changing neutral currents involving the light quarks are quite strong, but Higgs mediated flavor changing neutral currents may depend on small enough Yukawa couplings to be acceptable. If our assumption is relaxed, it may be possible to construct a model.

Since most of the difficulties arise from the presence of new gauge interactions, the simplest way to have the models agree with current data would be for the gauge symmetry to be broken to $SU(3)_c \times SU(2)_L \times U(1)_Y$ at $M_f$. In this case, the predictions of the models for accessible physics would be the same as those of models based upon the coupling of an $N = 1$ supersymmetric model to supergravity[16,24], unless the dynamics of the string theory were able to
predict relations between Yukawa couplings. However, in the class of models under discussion, this is not possible and the gauge group will always contain some extra factors. The constraints discussed in this paper are likely to cause difficulty for all such models.

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APPENDIX A

In this Appendix, we calculate the $U(1)$ quantum numbers and the normalizations of the $U(1)$ coupling constants in the $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_Y'$, $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_Y'$, and $SU(3)_c \times SU(2)_L \times U(1)_L \times SU(2)_R \times U(1)_R$ models. Since these models contain $SU(3)_c$, it is convenient to consider the $SU(3)_c \times SU(2)_L \times U(1)_L$ subgroup of $E_6$. The 27 of $E_6$ decomposes into three representations of the subgroup as follows

$$\Phi[27] = [3, 1, 3] + [3, 1, 3] + [1, 3, 3].$$  (A.1)

The matter fields are assigned to these representations as follows.

$$[3, 1, 3] \ni Q(3, 2), g(3, 1).$$  (A.2)

Here, the notation $X(\alpha, \beta)$ indicates that $X$ transforms according to the $(\alpha, \beta)$ representation of $SU(3)_c \times SU(2)_L$. $Q$ is the usual quark doublet, and $g$ is a new charge $-1/3$ quark.

$$[3, 1, 3] \ni u^c(3, 1), d^c(\overline{3}, 1), g^c(\overline{3}, 1)$$  (A.3)

and

$$[1, 3, 3] \ni L^c(1, \overline{2}), H^c(1, \overline{2}), H(1, \overline{2}), e^c(1, 1), N(1, 1), N'(1, 1).$$  (A.4)

Here, $L^c$ and $H^c$ are the conjugates of the usual doublets of $SU(2)_L$, $(e_L^c)$ and $(H_L^c)$, $H$ is the second Higgs doublet $(H_R^c)^c$, $e^c$ is the right-handed electron and $N$ and $N'$ are singlets.

The coupling of the matter fields ($\psi$) to the gauge bosons (indicated by $A_\alpha$) is as follows,

$$i \frac{g_6}{2} \overline{\psi} A_\alpha T^\alpha \psi$$  (A.5)

where the $T^\alpha$'s are the generators normalized so that $Tr T^a T^b = 2 \delta^{ab}$ and $g_6$ is the $E_6$ coupling constant which is equal to the $SU(3)_c$ coupling constant ($g_3$) at the compactification scale. The gauge bosons of the various $U(1)$'s are orthogonal combinations of the gauge bosons $A_\alpha, A_\beta$ and $A_\gamma$ which are associated with the generator of $SU(3)_c$ which commutes with $SU(2)_L$ and the two diagonal generators of $SU(3)_R$. A convenient basis is

$$T^a = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$T^\beta = \frac{1}{\sqrt{3}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$T^\gamma = \begin{pmatrix} 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$  (A.6)

The coupling of any of the matter fields to one of the gauge bosons ($A_\alpha, A_\beta$ and $A_\gamma$) can now be read off. For example, the quark doublet ($Q$) is a singlet under $SU(3)_c \times SU(2)_L \times U(1)_L$ basis, so its charges with respect to $A_\alpha, A_\beta$ and $A_\gamma$ are $0, 0$ and $0$, respectively.

We now consider each model separately. In the $SU(3)_c \times SU(2)_L \times U(1)_L \times U(1)_R$ basis, the gauge boson corresponding to the $U(1)_L$ must be $A_0$. The $U(1)_R$ gauge boson, however, can be a linear combination of $A_\beta$ and $A_\gamma$. As discussed by Witten[7], it must correspond to the generator,

$$T^V = \frac{1}{\sqrt{3}} \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = \frac{1}{2} (\sqrt{3} T^\gamma + T^\beta).$$  (A.7)

The generator orthogonal to $T^V$ is,

$$T^W = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = \frac{1}{2} (\sqrt{3} T^\beta - T^\gamma).$$  (A.8)

We can rotate to the basis $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_Y'$, where the Weinberg-Salam gauge group appears as a factor. The physical gauge bosons $A_Y$ and $A_Y'$, are linear combinations of $A_\alpha$ and $A_Y$,

$$A_Y \equiv A_\alpha \cos \phi + A_Y \sin \phi$$

$$A_Y' \equiv -A_\alpha \sin \phi + A_Y \cos \phi.$$  (A.9)

Here, $A_Y$ is defined in terms of $A_\beta$ and $A_\gamma$ analogously to Eq. (A.7).
Since the couplings of the fermions to the gauge bosons $A_0$, $A_\beta$, and $A_\gamma$ as well as $A_Y$ are known, it is trivial to find the normalization of the coupling constants and the $Y'$ quantum numbers. As an example, consider the couplings of the $u$ quark to the $U(1)_L$ gauge boson $A_0$. The coupling of the left-handed $u$ quark to the $U(1)_L$ gauge boson $A_0$ is given by

$$i \frac{g_0}{2} \bar{\psi}_{uL} A_0 \psi_{uL} = i \frac{g_0}{2} \bar{\psi}_{uL} (A_Y \cos \phi - A_Y' \sin \phi) \psi_{uL}. \tag{A.10}$$

Similarly, the right-handed $u$ quark couples to the gauge bosons as

$$i \frac{g_0}{2} \bar{\psi}_{uR} A_0 \psi_{uR} + i \frac{g_0}{2} \bar{\psi}_{uR} A_Y \psi_{uR} = -i \frac{g_0}{\sqrt{3}} \bar{\psi}_{uR} (A_Y \sin \phi + A_Y' \cos \phi) \psi_{uR}. \tag{A.11}$$

The couplings of $\psi_{uL}$ to the $U(1)_Y$ gauge bosons are known

$$i \frac{g_1}{2} \bar{\psi}_{uL} \frac{1}{2} A_Y \psi_{uL} + i \frac{g_1}{2} \bar{\psi}_{uL} (-\frac{4}{3} A_Y) \psi_{uL} \tag{A.12}$$

where $g_1$ is the $U(1)_Y$ coupling constant. Comparing Eqs. (A.10),(A.11) and (A.12), we find immediately

$$\frac{g_0}{\sqrt{3}} \cos \phi = \frac{3}{2} g_1$$

$$\frac{g_0}{\sqrt{3}} \sin \phi = \frac{2}{3} g_1 \tag{A.13}$$

and hence $g_0 = \sqrt{3} g_1, \cos \phi = \frac{1}{\sqrt{3}}, \text{and} \sin \phi = \frac{2}{\sqrt{3}}$ at $1/R$. The $Y'$ quantum numbers of $u_L$ and $u_R$ can now be read off from Eqs. (A.10) and (A.11). The $Y$ and $Y'$ quantum numbers of all the particles are given in Table (A.1).

In the $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_Y' \times U(1)_Y''$ model, we have

$$\begin{pmatrix} A_0 \\ A_Y \\ A_W \end{pmatrix} = \begin{pmatrix} c_1 & -s_1 c_2 & -s_1 s_3 \\ s_1 c_2 & c_1 c_3 - s_1 s_3 & c_1 s_3 + s_2 c_3 \\ s_1 s_2 & c_1 s_2 c_3 + s_2 s_3 & c_1 s_2 s_3 - c_2 c_3 \end{pmatrix} \begin{pmatrix} A_Y \\ A_Y' \\ A_Y'' \end{pmatrix}. \tag{A.14}$$

where $c_1 = \cos \theta_1, s_1 = \sin \theta_1$.

As previously, we calculate the couplings of the $u$ quark to the gauge bosons and obtain

$$\frac{g_0}{2 \sqrt{3}} c_1 = \frac{g_1}{6}$$

$$\frac{g_0}{\sqrt{3}} s_1 c_2 = \frac{2 g_1}{3} \tag{A.15}$$

so $s_1 c_2 = 2 c_1$. Similarly, from the couplings of $d$ quark to the gauge bosons, we find

$$\frac{g_0}{2 \sqrt{3}} c_1 = \frac{1}{6} g_1$$

$$\frac{g_0}{\sqrt{3}} s_1 c_2 = \frac{1}{3} g_1 \tag{A.16}$$

Combining these, we find that $s_1 = 0$ and $c_2 = 1$, thus $\tan \theta_1 = 2$ and $c_1 = 2/\sqrt{3}$. We then obtain the usual relationship, $g_0 = \sqrt{3} g_1$.

We can now relate the hypercharges $Y, Y', Y''$ to the original charges in the 27, viz. $Y_0, Y_\beta, Y_\gamma$ (Eq. (A.6)). The coupling of the matter fields to the gauge bosons at the unification scale (compare with Eq. (A.5)) is

$$\mathcal{L} = \frac{1}{2} g_1 (Y \bar{\psi}_\mu A_\mu^0 \psi + Y' \bar{\psi}_\mu A_\mu^0 \psi + Y'' \bar{\psi}_\mu A_\mu^0 \psi)$$

with

$$Y = \frac{1}{\sqrt{3}} Y_0 + \frac{1}{\sqrt{3}} Y_\beta + \frac{1}{\sqrt{3}} Y_\gamma$$

$$Y' = -\frac{1}{\sqrt{3}} (c - 3 \sqrt{2} s) Y_\beta + \frac{1}{\sqrt{3}} (c + 3 \sqrt{2} s) Y_\gamma$$

$$Y'' = \frac{1}{\sqrt{3}} (s + 3 \sqrt{2} c) Y_\beta + \frac{1}{\sqrt{3}} (s - 3 \sqrt{2} c) Y_\gamma. \tag{A.17}$$

where $c = \cos \theta_2, s = \sin \theta_2$. The $U(1)$ quantum numbers can now be read off from Eq. (A.6) and are given in Table (A.1). The angle $\theta_2$ refers to the arbitrariness in defining $Y'$ and $Y''$. If $\sin \theta_2 = 0$, for example, then the $Y'$ is the same as in the first model considered.
In this model, there is an ambiguity in determining which field is \( H' \) and which is \( L \) and also in defining \( N \) and \( N' \). We define \( N \) to be the field which couples to the \( g \) quarks in the superpotential. Having done this, the \( U(1) \) quantum numbers of \( H', L, \) and \( N \) are fixed in terms of \( \theta_3 \).

The quantum numbers for the \( SU(3)_c \times SU(2)_L \times U(1)_Y \times SU(2)_R \times U(1)_{R'} \) model can be found in a similar manner and are given in Table (A.2).

\[
\begin{align*}
\text{APPENDIX B} \\
\text{We give here the neutral gauge boson mass matrices for some of the models discussed in Sections 3 and 4.}
\end{align*}
\]

(i) \( SU(3)_c \times SU(2)_L \times U(1)_Y \times SU(2)_N \times U(1)_{Y'} \)

This model is discussed in Section 3(c). Allowing for the presence of \( N \) and \( N' \) survivors (the cases of interest are obtained by deleting the appropriate terms), the mass matrix is

\[
M^2 = 
\begin{pmatrix}
\ell_0^2 (16\lambda^2 + \lambda'^2 + 25\tilde{n}^2) & \ell_0 \sqrt{3} \left( 5\lambda^2 - \lambda'^2 \right) & \frac{1}{2} g'_1 \sqrt{3} \left( g_2^2 + g_1^2 \left( \lambda'^2 - 4\lambda^2 \right) \right) \\
\ell_0 \sqrt{3} \left( 5\lambda^2 - \lambda'^2 \right) & \frac{1}{2} g'_1 \sqrt{3} \left( g_2^2 + g_1^2 \left( \lambda'^2 - 4\lambda^2 \right) \right) & -\frac{1}{2} g_2 \sqrt{3} \left( g_2^2 + g_1^2 \left( \lambda'^2 - 4\lambda^2 \right) \right) \\
\frac{1}{2} g'_1 \sqrt{3} \left( g_2^2 + g_1^2 \left( \lambda'^2 - 4\lambda^2 \right) \right) & -\frac{1}{2} g_2 \sqrt{3} \left( g_2^2 + g_1^2 \left( \lambda'^2 - 4\lambda^2 \right) \right) & M^2_{Z_0}
\end{pmatrix}
\]

where

\[
\begin{align*}
\tilde{n}^2 &= n^2 + \tilde{n}^2 + n'^2 + \tilde{n}'^2 \\
\tilde{n}'^2 &= n'^2 - n^2 + \tilde{n}'^2 - \tilde{n}^2
\end{align*}
\]

and \( M^2_{Z_0} \) is the mass of the \( Z \) boson in the standard \( SU(2)_L \times U(1)_Y \) case

\[
M^2_{Z_0} = \frac{1}{2} \left( g_2^2 + g_1^2 \right) \left( \lambda'^2 + \lambda^2 \right).
\]

The coupling constants are defined in Section 3(c) and are \( g_2, g_2N, g_1, g'_1 \) for \( SU(2)_L, SU(2)_N, U(1)_Y \) and \( U(1)_{Y'} \), respectively.

(ii) \( SU(4) \times SU(2)_L \times U(1)_A \)

This model is discussed in Section 3(d). Since it is of rank 5, we do not include \( N \) or \( N' \) survivors. The mass matrix reads:

\[
M^2 = 
\begin{pmatrix}
\frac{1}{8} g_4^2 \lambda^2 + \frac{1}{8} g_4^2 \lambda'^2 + \frac{1}{8} g_4^2 \left( n^2 + n'^2 \right) & \frac{1}{3} \sqrt{3} \left( g_4^2 \lambda^2 + \frac{1}{2} g_4^2 \left( \lambda'^2 - \frac{1}{2} g_4^2 \lambda^2 \right) \right) \\
\frac{1}{3} \sqrt{3} \left( g_4^2 \lambda^2 + \frac{1}{2} g_4^2 \left( \lambda'^2 - \frac{1}{2} g_4^2 \lambda^2 \right) \right) & M^2_{Z_0}
\end{pmatrix}
\]

where

\[
\begin{align*}
g_4^2 &= \frac{9}{10} g_4^2 + \frac{1}{6} g_4^2
\end{align*}
\]

Here, \( g_4, g_2 \) and \( g_A \) are the coupling constants of \( SU(4), SU(2)_L \) and \( U(1)_A \) re-
spectively and $g_1$ is the standard hypercharge coupling of the Glashow-Weinberg-Salam model. In this model, $g_1$ is expressed in terms of $g_4$ and $g_A$ by:

$$g_1^2 = \frac{g}{10} g_A^{-2} + \frac{1}{6} g_4^{-2}.$$  \hspace{1cm} (B.6)

(iii) Models with an intermediate scale $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{y'} \times U(1)_{y''} \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{y'}$

As discussed in Section 4, the only relevant case is the one where $\overline{N}$ and $\overline{N}'$ obtain nonzero vevs at the intermediate scale. The $U(1)_{a}$ quantum numbers of the low energy fields are then given in Eq. (4.1) and the mass matrix reads:

$$M^2 = \begin{pmatrix}
\frac{1}{4} (4h^2 + 4h'^2 + 5n'^2) \\
\frac{1}{\sqrt{6}} g_a \sqrt{g_2^2 + g_1^2 (h^2 + h'^2)} \\
\end{pmatrix}
\frac{M_2^2}{\sqrt{6} g_a \sqrt{g_2^2 + g_1^2 (h^2 + h'^2)}} \hspace{1cm} (B.7)
$$

where $g_a$ is the $U(1)_{a}$ coupling constant.

REFERENCES
Table A.1. Quantum numbers for the $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)' \times U(1)_Y$ model. These quantum numbers are given in terms of an arbitrary angle $\theta_3 (c_3 = \cos \theta_3, s_3 = \sin \theta_3)$. The quantum numbers for the $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)'$ model are found by taking $\theta_3 = 0$.

<table>
<thead>
<tr>
<th></th>
<th>$Y$</th>
<th>$Y'$</th>
<th>$Y''$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$</td>
<td>1</td>
<td>$+4/3 c_3$</td>
<td>$+4/3 s_3$</td>
</tr>
<tr>
<td>$H'$</td>
<td>$-1$</td>
<td>$+c_3/3 - \sqrt{5/3} s_3$</td>
<td>$+\sqrt{5/3} c_3 + s_3/3$</td>
</tr>
<tr>
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<td>0</td>
<td>$-5/3 c_3 + \sqrt{5/3} s_3$</td>
<td>$-5/3 s_3 - \sqrt{5/3} c_3$</td>
</tr>
<tr>
<td>$Q$</td>
<td>$1/3$</td>
<td>$-2/3 c_3$</td>
<td>$-2/3 s_3$</td>
</tr>
<tr>
<td>$g$</td>
<td>$-2/3$</td>
<td>$+4/3 c_3$</td>
<td>$+4/3 s_3$</td>
</tr>
<tr>
<td>$u^c$</td>
<td>$-4/3$</td>
<td>$-2/3 c_3$</td>
<td>$-2/3 s_3$</td>
</tr>
<tr>
<td>$d^c$</td>
<td>$2/3$</td>
<td>$+c_3/3 + \sqrt{5/3} s_3$</td>
<td>$-\sqrt{5/3} c_3 + s_3/3$</td>
</tr>
<tr>
<td>$g^c$</td>
<td>$2/3$</td>
<td>$+c_3/3 - \sqrt{5/3} s_3$</td>
<td>$s_3/3 + \sqrt{5/3} c_3$</td>
</tr>
<tr>
<td>$L$</td>
<td>$-1$</td>
<td>$+\sqrt{5/3} s_3 + c_3/3$</td>
<td>$-\sqrt{5/3} c_3 + s_3/3$</td>
</tr>
<tr>
<td>$e^c$</td>
<td>2</td>
<td>$-2/3 c_3$</td>
<td>$-2/3 s_3$</td>
</tr>
<tr>
<td>$N'$</td>
<td>0</td>
<td>$-5/3 c_3 - \sqrt{5/3} s_3$</td>
<td>$-5/3 s_3 + \sqrt{5/3} c_3$</td>
</tr>
</tbody>
</table>

Table A.2. $U(1)$ quantum numbers for the $SU(3)_c \times SU(2)_L \times U(1)_L \times SU(2)_R \times U(1)_R$ model.

<table>
<thead>
<tr>
<th></th>
<th>$\frac{Y_L}{\sqrt{2}}$</th>
<th>$\frac{Y_R}{\sqrt{2}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$</td>
<td>$-1$</td>
<td>$1$</td>
</tr>
<tr>
<td>$H'$</td>
<td>$-1$</td>
<td>$1$</td>
</tr>
<tr>
<td>$N$</td>
<td>2</td>
<td>$-2$</td>
</tr>
<tr>
<td>$Q$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$g$</td>
<td>$-2$</td>
<td>0</td>
</tr>
<tr>
<td>$u^c$</td>
<td>0</td>
<td>$-1$</td>
</tr>
<tr>
<td>$d^c$</td>
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</tr>
<tr>
<td>$g^c$</td>
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<td>2</td>
</tr>
<tr>
<td>$L$</td>
<td>$-1$</td>
<td>$-2$</td>
</tr>
<tr>
<td>$e^c$</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$N'$</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>
Figure 1.

A one-loop contribution to gaugino masses. The suffixes L,R refer to left and right helicity states. Solid lines are fermions and dashed lines are scalars.
Figure 1
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