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Strong wind forcing of the ocean

A dissertation submitted in partial satisfaction of the requirements for the degree Doctor of Philosophy in Oceanography

by

Sarah E. Zedler

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Daniel L. Rudnick
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2007
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The dissertation of Sarah E. Zedler is approved, and it is acceptable in quality and form for publication on microfilm:

Co-Chair

Co-Chair

University of California, San Diego

2007
I would like to dedicate this thesis to all my advisors: Peter Niiler and Detlef Stammer, my parents, and my sister, all of whom have endured me through the (sometimes arduous) process of preparing this thesis with patience and (exemplary) forebearance. I am grateful to have the opportunity to thank them formally for their dedication to this project.
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PUBLICATIONS


PRESENTATIONS


ABSTRACT OF THE DISSERTATION

Strong wind forcing of the ocean

by

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Research that seeks to characterize the relevant features of the ocean response to strong wind forcing, as models evolve towards reality, is instrumental to the development of parameterizations that could be used in future hurricane forecast models.

This thesis investigates several aspects of the ocean’s response to strong wind forcing. The studies were conducted using a high-resolution ocean model with sub-gridscale parameterizations of shear-instability type vertical mixing and a unique dataset collected during and after the passage of Hurricane Frances to the east of the Caribbean Leeward Islands in 2004.

Simulations conducted for a storm moving eastward at a range of constant speeds in various initial ocean environments, showed that the temperature change decreases with increasing storm speed, although the maximum kinetic energy input is generated when the storm residence time is equal to the near-inertial period. This upholds a previous result conducted using a model where vertical mixing depended on wind stress magnitude.

The simulated kinematic response to the hurricane included currents at near twice the inertial frequency, which in the mean field were attributed about
equally to horizontal, and vertical, advection of horizontal momentum. During the
storm passage, the wind stress curl imparted linear and non-linear components to
the vertical velocity fields of comparable magnitude.

In simulations where a warm or cold core eddy was forced with a uni-
form wind stress, a vertical velocity circulation was established in the mean field.
Expanding on results from previous studies, it is shown that the maximum verti-
cal velocity in this mean field increases approximately linearly with the maximum
vorticity in the cyclogeostrophically adjusted current fields in the eddy.

Finally, a simulation initialized with in-situ measurements of temperature
in the upper 200m, and observation based wind field, was used to choose the drag
coefficient parameterization (as a function wind speed) that provides the closest
statical match between simulated and observed temperature and near-inertial cur-
rent fields. Our results argue for a drag coefficient that saturates at $1.5 \times 10^{-3}$. 
I

Introduction

Over the past decade, the average intensity of hurricanes has increased dramatically (Emanuel, 2005; Holland and Webster, 2006). Although the underlying causes for the observed increase in intensity are currently under debate (Michaels et al., 2006), climate models predict that hurricanes will be more intense in an atmosphere with higher concentrations of $CO_2$ (Knutson et al., 2001). Operational hurricane forecast models remain poor predictors of storm intensity, but would likely be improved if they were modified to include more realistic parameterization of air-sea interactions (Bender and Ginis, 2000). An essential step towards achieving this goal, is to develop an understanding of the dynamic and thermodynamic responses of the the ocean to strong wind forcing. Considering the logistical difficulties posed for collection of data underneath a hurricane, progress in this field has relied heavily on ocean models. As ocean models evolve to become more realistic, it is important to carefully consider how the results found using the previous generation of models might be affected. As new data sets become avail-
able, it is important to incorporate them into the existing body of knowledge, and to use them to in conjunction with models to validate and improve parameterizations. As ocean general circulation models improve in their resolution of advective and mixing processes, these could well be used to quantitatively simulate observations obtained from field studies. Our study used the Massachusetts Institute of Technology General Ocean Circulation Model as a tool to simulate the full three dimensional response of the ocean to realistic hurricane forcing of realistic ocean areas.

Several features of storm response have been identified in the observational database that can serve as tests for ocean model verity. Key amongst these is the cool wake left behind a storm, visible in sea surface temperature satellite imagery (Nelson, 1996, 1998). For rapidly moving hurricanes, the cold wake is asymmetrical across the track, with colder temperatures on the right hand side of the storm (Price, 1981). Strong currents have been observed in the mixed layer of this wake that rotate in and out of convergence and divergence at near-inertial frequencies, pumping the temperature surfaces in the thermocline (Brooks, 1983; Shay et al., 1990; Dickey et al., 1998). The mixed layer currents typically have e-folding decay timescales of 5-10 days (Zedler et al., 2002). A net upwelling of isotherms can also exist throughout the water column (Brink, 1989). Large amplitude surface waves are generated, predominantly in the right front quadrant of the storm (Wright et al., 2001; Black et al., 2007).

In a theoretical, simplified modal configuration, Greatbatch (1984) identified the storm translational Rossby number

$$R_{os} = \frac{U_s}{fL}$$

as one basic parameter that determines amplitude and spatial structure of the
horizontal and vertical velocities; \( R_{os} = 1 \) is a condition of resonant forcing for the generation of near inertial currents. In this equation, \( U_s \) is the translational speed of the storm, \( f \) is the Coriolis parameter, and \( L \) is the radius of maximum hurricane winds. Greatbatch (1984) performed a suite of sensitivity experiments with linear, nonlinear without mixing, and nonlinear with mixing versions of a numerical model with high horizontal resolution and 14 layers in the vertical to verify his theoretical scaling of the ocean response as a function of \( R_{os} \). For the nonlinear model with mixing, he showed that the maximum entrainment velocity decreased with increasing Rossby number. He used the vertical turbulent mixing model of Kraus and Turner (1967), that parameterizes the entrainment velocity as proportional to the three halves power of wind stress magnitude. This assumption produces symmetric vertical mixing under a symmetric hurricane and the right biased asymmetry of the ocean response is due the horizontal advective processes. Price (1981) showed that if vertical mixing is allowed to be produced by vertical shear of horizontal currents, a right biased ocean storm results from this shear being the largest on the right hand side of the hurricane. This suggests that entrainment is very model dependent and may have been overestimated by Greatbatch modeling studies on the left side of the hurricane. This leaves the following question:

- What are the implications for the dependence of the upper ocean temperature response on \( R_{os} \), when turbulent mixing is parameterized to depend on the vertical shear of the ocean response, instead of the amplitude of the surface stress?

Using a 1.5 layer model, Greatbatch (1985) attributed temporal asymmetry in the thermocline vertical velocity (implying a net upwelling in the temporally averaged response) to nonlinear horizontal advection of horizontal momentum. His
model did not include vertical advection of horizontal momentum. In a high resolution, fully non-linear three-dimensional model, Niwa and Hibiya (1997) indicated the presence of vertical velocity fluctuations at near twice the inertial frequency. They performed a bispectral analysis to demonstrate that non-linear resonant triad wave-wave interactions occurred in their model between large scale near-inertial waves and small scale waves at near twice the inertial frequency. Two questions are suggested for further investigation:

- What are the contributions of vertical advection of horizontal momentum relative to those from horizontal advection of horizontal momentum, when it is resolved in more modern models?

- Are there other non-linear processes at play in creation of the temperature and velocity structure of a hurricane wake?

All current hurricane forced ocean models leave behind a strong response in the near-inertial internal wave train. Vertical propagation of mixed layer near-inertial waves can be modified through interaction with background low-frequency mesoscale (Kunze, 1985; vanMeurs, 1998). During the passage of a hurricane over a loop core eddy in the Gulf of Mexico, the temperature wake was advected by the eddy velocity field after the storm passage (Jacob and Shay, 2003). Generally, in theoretical treatments, relative length scales of the near-inertial wave and the mesoscale feature, in addition to the relative vorticity associated with the mesoscale, determine whether the near inertial waves are reflected by, or disperse around, background mesoscale features (Klein and Treguier, 1995; Klein and Smith, 2001). In a model of an eddying channel forced by a travelling hurricane-type wind stress, near-inertial energy was found in the deep ocean, concentrated underneath the center of warm-core, anticyclonic features (Zhai et al., 2005). In
this calculation, the mixed layer was not allowed to deepen, and may be a definitive parameter for interaction with ocean mesoscale (Klein and Treguier, 1995). For steady wind forcing of an anticyclonic eddy on an f-plane, Lee and Niiler (1998) demonstrated that in addition to trapping of near inertial energy at depth, a vertical velocity circulation established in the temporally averaged field. They were unable to perform stable numerical simulations for the full range of eddy sizes and amplitudes present in the ocean (Benitez-Nelson et al., 2007). There are a number of fully non-linear simulations of relatively simple initial ocean mesoscale states forced by strong winds, that can be done before a comprehensive analysis of near inertial current/mesoscale interactions in a hurricane wake is completed. Therefore we ask several precursory questions:

- Can stable solutions be generated of wind forced warm and cold core eddies for a range of observed eddy Rossby and Burger numbers?
- How do non-linear interactions between mesoscale and near-inertial waves vary as a function of Rossby number, when the eddy is forced by a spatially uniform strong wind?

Recent laboratory studies and field measurements suggest that the ocean stress drag coefficient saturates at high wind speeds, but disagree at what wind speed, and at what value, the drag coefficient levels off (Powell et al., 2003; Donelan et al., 2004; Black et al., 2007). The wind stress leaves behind a wake of near inertial currents and modified temperature field, both of which were sampled in early September of 2004 by a unique, hurricane experiment, CBlast, during the passage of Hurricane Frances to the north-east of the Caribbean Leeward Islands chain. This is the most extensive ocean response data set that has ever been collected underneath a hurricane (Black et al., 2007), consisting of (among
other observation platforms) an array of floats and drifters fitted with temperature sensors. The position of a subset of the drifters was additionally tracked by GPS, for which accurate velocity estimates could be calculated. This leads us to ask:

- How well can the MIT/OGCM simulate the observed temperature and current response underneath Hurricane Frances?
- To what extent can we use the MIT/OGCM to constrain an estimate of the drag coefficient parameterization with wind speed underneath Hurricane Frances?

The research plan, as reported in Chapters 2-4, was to apply both the MIT/OGCM as well as concepts from more simple models to investigate several aspects of the ocean’s temperature and dynamic response to a hurricane. With the general knowledge gained in these more theoretical simulations, we use a sequence of realistic simulations of the effects of Hurricane Frances as a function of the assumed drag coefficient (Chapter V). Both the observed temperature and velocity response of Frances will be used to select the best drag coefficient for forcing the MIT/OGCM.

We have organized this work into the following themes. Chapters II and III are focused on a parameter study of MIT/OGCM as a function of Rossby number $\mathcal{R}_o$, and simulations of non-linear processes in a hurricane wake. Chapter IV discusses the non-linear effects in eddy Rossby number parameter space for eddies that are forced by a constant wind. This response in the future will be compared to Eddy Opal under the constant action of the Trade Winds near the Hawaiian Islands (Benitez-Nelson et al., 2007). Chapter V presents observations in quantitative comparisons between simulated and measured temperature and current fields produced by Hurricane Frances, as well as analysis of the drag coefficient
parameterization in the model, that best matches the measured response.
II

The thermodynamic open ocean response to a hurricane: Parameters and Scales

II.1 Introduction

Current operational hurricane forecast models are poor predictors of storm intensity (Bender and Ginis, 2000). In order to sustain a mature hurricane, the heat released from the ocean surface must equal or exceed the energy dissipated as the wind drags on the sea surface (Emanuel, 1986). The energy exchange at the sea surface is large. Momentum fluxes typically generate near-inertial currents on the order of $1m/s$ and extract heat fluxes from the warm sea surface on the order of $1000W/m^2$. During the storm passage, the near-inertial current shear-instability driven mixing and the heat fluxes lost at the surface cause a decrease in upper
ocean temperature, which in turn reduces the magnitude of the evaporative flux to the storm. This positive feedback mechanism can result in a significant reduction of storm intensity (Chang and Anthes, 1979). Including parameterizations of this feedback mechanism in forecast models requires a thorough understanding of the upper ocean thermodynamic response to wind and heat flux forcing. Of specific interest, is to know how the post-storm sea surface temperature change is partitioned between surface heat flux losses and turbulent vertical mixing and entrainment in models with the best available heat and momentum flux forcing, what parameters are important for setting that response, and any expected regional differences.

In models and observations, entrainment has been identified as the dominant process for lowering the sea surface temperature (Price, 1981; Martin, 1982), with latent heat flux from the sea surface accounting for the remainder (10 – 30%) of the surface cooling. In models, this basic result has been shown to be relatively insensitive to choice of turbulent mixing scheme, despite differences in the predicted amount of water that is entrained (Large et al., 1994; Zedler et al., 2002; Jacob and Shay, 2003). The strength of the storm forced near-inertial current, which scales the vertical shear at the base of the mixed layer and therefore is related to the amount of Richardson-type shear instability mixing, is a function of latitude, storm translational speed, and storm size (Greatbatch, 1983). The balance of heat flux forcing driven and entrainment driven sea surface temperature should also be sensitive to the balance of heat and wind stress forcing. Fresh water fluxes (precipitation) may also play a compensating role to heat fluxes for modifying the density field.

In previous modeling studies of the ocean’s temperature response to a hurricane, the best estimate of hurricane wind speed was converted to stress using
a drag coefficient proportionality constant that is a linear function of wind speed (Large and Pond, 1981), and a constant transfer coefficient for estimating latent heat flux (Zedler et al., 2002; Jacob and Shay, 2003). Precipitation fluxes have been omitted entirely from these studies (Price, 1981, e.g.). In a Carnot-cycle type model of a mature hurricane (Emanuel, 1995), the ratio of the transfer coefficient for latent heat to the drag coefficient for wind stress, is estimated to obtain a value between 1.2 and 1.5 for steady state maintenance. Estimates of surface values of the drag coefficient from fitting wind speed profiles as measured by GPS dropsondes to the surface (Powell et al., 2003), and direct in-situ measurements of the drag coefficient (Black et al., 2007), suggest that the drag coefficient saturates at high wind speed, likely because of changes in the sea state (Lundquist, 1999). These estimates are consistent with direct measurements of the drag coefficient in a wave tank (Donelan et al., 2004). The wind speed at which the drag coefficient saturates, departing from the linear drag coefficient parameterization of Large and Pond (1981), occurs around 15m/s or 25m/s. Since hurricanes can sustain winds as high as 80m/s, this suggests that in previous studies of the ocean’s response to a hurricane, the drag coefficient has been significantly overestimated. If, as is suggested in Emanuel (1995), the latent heat flux scales as the drag coefficient, this further suggests that the balance of heat flux to wind stress forcing used in studies such as Zedler et al. (2002); Jacob and Shay (2003) should be changed.

Considering these factors, an aim of this thesis is to investigate the sensitivity of the ocean’s surface and subsurface response to surface forcing using a model with a sophisticated turbulent mixing scheme for different regions characterized by latitude and upper ocean stratification conditions. Using parameterizations for heat flux (Emanuel, 1995), wind stress (Powell et al., 2003; Donelan et al., 2004; Black et al., 2007), and precipitation Lonfat et al. (2004), we reassess
the importance of surface forcing on the mixed layer temperature.

II.2 Model and Methodology

II.2.A Model

All our experiments are based on the MIT OGCM, implemented with $\frac{1^\circ}{6}$ horizontal resolution and 40 layers in the vertical (5 m in the top 100 m; 4500 m total depth) in a 3900x4400 km domain (a region with extent 40° by 70° in latitude and longitude) and run with a free surface on a sphere. As described in detail by Marshall et al. (1997b,a), the MIT OGCM ocean model is based on the Navier Stokes equations for an inviscid, Boussinesq, hydrostatic, and incompressible fluid with a free surface and is fully described in the literature (see also Adcroft, 1995). The version of the code used here includes the “KPP” turbulence closure (Large et al., 1994) and sub-grid scale eddy parameterizations (Gent and McWilliams, 1996). Values for the background vertical viscosity and diffusivity were chosen to be $1 \times 10^{-3} m^2/s$ and $3 \times 10^{-5} m^2/s$, respectively. Horizontally, coefficients for biharmonic form of viscosity and diffusivity had amplitudes of $2 \times 10^{11} m^4/s$ and $1 \times 10^{10} m^4/s$, respectively. Boundary conditions were specified for a flow in a zonal channel with closed, no-slip side walls to the north and south, and periodic conditions in the east-west direction. In all experiments, the bottom topography is flat, so the model results are independent of zonal domain choice, which is therefore not distinguished between basins in this document.
II.2.B Experiments

Various simulations were performed which differed in their external forcing conditions, their initial stratifications, and their central latitude (i.e., in their inertial period). A summary of all experiments is provided in Table II.1 Initial temperature and salinity profiles for these experiments are shown in Figure II.1. They represent conditions for the western subtropical North Atlantic, the Caribbean, and the eastern Pacific, but are prescribed as horizontally uniform initial fields over the entire model domain (respective meridional domains are 10 – 50°N, 5°S-35°N, and 5°S-35°N). Accordingly the initial flow field is at rest. For the mid-Atlantic simulations (label A in the figures), initial conditions for temperature and salinity were constructed following Zedler et al. (2002) and are typical for the Sargasso Sea in late summer, with warm temperatures of 28°C at the surface, and a shallow seasonal thermocline at about 20m depth. For the Pacific and Caribbean simulations (labels P and C), the initial temperature and salinity profiles were obtained by averaging all available WOCE data that fell within the indicated latitude-longitude box during the periods April 2-May 19, 1989 and August 15-September 3, 1997, respectively. The observed profiles, available with 1m resolution, were interpolated to the model vertical grid.

All experiments presented here are forced with suddenly imposed, westward tracking idealized hurricane wind stress patterns translating at a constant speed in the absence of background flow (see Table II.1 for the relevant ways in which the experiments differ). We note that ramping the maximum wind stress linearly during the first two days of the simulation had insignificant effects (i.e. differences were everywhere ≤ 1%) on the current speed, temperature, and sea surface height anomaly fields, indicating that our results were negligibly affected by transient waves moving ahead of the storm associated with impulsive forcing.
The wind forcing (called NORMAL in Table II.1) had a symmetric pattern with a constant inflow angle of 30° as described by Equation A.1 in the appendix (see Price, 1983, for details). A modified wind stress forcing (called MODIFIED in Table II.1) was used to simulate the effects of using a drag coefficient suggested by Powell et al. (2003). Snapshots of NORMAL and MODIFIED translating wind stress patterns, and a cross section of the magnitude (at the central storm latitude) are shown in the bottom row left, middle, and right panels of Fig. II.2, respectively. In their study, GPS dropsonde measurements of the wind speed profile underneath a hurricane suggested that the drag coefficient parameterizations of Garratt (1977), Large and Pond (1981), and Yelland et al. (1998) are not valid for wind speeds larger than 25 m/s and that maximum wind stress values in a class 4 hurricane are capped at a value around $3\frac{N}{m^2}$ as described by Equation A.2 (see Figure II.2). Taking this into consideration, the experiments here that are forced with MODIFIED wind stress are representative of a weak Category I storm with a maximum drag coefficient of $2.3 \times 10^{-3}$, a strong Category I storm with maximum drag coefficient of $1.4 \times 10^{-3}$ or a moderate Category II storm with maximum drag coefficient of $1.5 \times 10^{-3}$.

We also forced some simulations with latent heat flux loss parameterized directly underneath the storm with zero background values following Equation A.6 (label MOD in Table II.1). In contrast with previous studies (e.g. Price et al. (1994); Jacob et al. (2000); Jacob and Shay (2003)) which assume a constant transfer coefficient for latent heat of $C_k = 0.0012$, we used the model of Emanuel (1995), where $C_k$ is parameterized as a linear function of the drag coefficient for wind stress $C_d$ (see Appendix B for details). Since the constant of proportionality is greater than one, and modelled drag coefficients are greater than 0.0012 for wind speeds above $25 \frac{m}{s}$ (Garratt, 1977; Large and Pond, 1981; Yelland and Taylor, 1996;
Table II.1: The simulation names, storm translational speed, maximum wind stress, type of wind stress, and heat flux forcing have column labels Code, $U_s$, $\tau_{\text{MAX}}$, $\tau$ type, and HFF, respectively. In this table, forcing conditions are distinguished by a two character code. In the text, a three character code is necessary to additionally distinguish the initial temperature and salinity profiles that were used. In the three character code used in the text, the first character is either A, P or C. These represent the mid-(A)lantic, (P)acific, and (C)aribbean, initial conditions with central grid latitudes of 31.16°N, 16.16°N, and 16.16°N, respectively.

<table>
<thead>
<tr>
<th>Code</th>
<th>$U_s$ (m/s)</th>
<th>$\tau_{\text{MAX}}$ N/m²</th>
<th>$\tau$ type</th>
<th>HFF</th>
</tr>
</thead>
<tbody>
<tr>
<td>M0/S0</td>
<td>5.6</td>
<td>3</td>
<td>MODIFIED</td>
<td>NONE</td>
</tr>
<tr>
<td>M1</td>
<td>5.6</td>
<td>2</td>
<td>MODIFIED</td>
<td>NONE</td>
</tr>
<tr>
<td>M2</td>
<td>5.6</td>
<td>2</td>
<td>MODIFIED</td>
<td>MOD</td>
</tr>
<tr>
<td>S1</td>
<td>2.0</td>
<td>3</td>
<td>MODIFIED</td>
<td>NONE</td>
</tr>
<tr>
<td>S2</td>
<td>3.8</td>
<td>3</td>
<td>MODIFIED</td>
<td>NONE</td>
</tr>
<tr>
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<td>6.8</td>
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<tr>
<td>S4</td>
<td>10.0</td>
<td>3</td>
<td>MODIFIED</td>
<td>NONE</td>
</tr>
</tbody>
</table>

Yelland et al., 1998; Powell et al., 2003), this results in enhanced heat flux losses for the equivalent wind stress. For example, in Jacob et al. (2000); Jacob and Shay (2003), Emanuel’s model predicts maximum latent heat flux of 3700 W/m² as opposed to the 1200 W/m² used, an increase of about a factor of three. This implies that heat fluxes may be more important than previously thought for their role in reducing the mixed layer temperature. One goal of this study is to reassess the importance of heat fluxes, by applying these forcing considerations. The label NONE in Table II.1 implies that no surface heat exchange was applied. No surface pressure forcing was used.
Figure II.1: (a) Mean temperature and (b) salinity profile data, used as horizontally uniform initial fields for the mid-Atlantic (31N, 64W), Pacific (9.5N, 105-135W) and Caribbean (14-17N, 64-66W) environments. See the text for details of how these initial profiles were obtained/constructed. (c) The structures of the first 3 dynamical modes for horizontal velocity, based on the mid-Atlantic stratification.
Figure II.2: (a) $\tau$ as a function of wind speed using various drag coefficients. (b) Snapshot of magnitude (contour) and (c) wind stress $\tau$ and $\tau_m$, respectively (subsampled). (d) Cross-section of wind stress profile at central storm latitude.
II.3 Results

A description of the basic features of the temperature and kinematic response in our model is followed by analysis of sensitivity of the upper ocean temperature response to new heat and precipitation surface flux forcing parameterizations and discussion of regional differences in the upper ocean temperature response as a function of Rossby number. In regions of the ocean with deep initial mixed layers and weakly stratified seasonal thermoclines such as the Caribbean, heat fluxes can account for 25% of the change in mixed layer temperature. Modifications to heat fluxes (Emanuel, 1995) and net freshwater fluxes (Lonfat et al., 2004) have comparable, competing effects for setting the mixed layer temperature change.

II.3.A Basic Response (A-M0/S0)

Several features of the ocean’s response to a hurricane that have previously been observed and modeled were reproduced in our simulations (e.g. Chang and Anthes, 1978; Price, 1981; Greatbatch, 1983). In the surface layer, near-inertial currents (Fig. II.3a) are accompanied by shear instability induced turbulent vertical mixing as parameterized by the KPP model and entrainment that result in lowering of shallow temperatures (by \( \approx 5^\circ C \)).

This sea surface temperature change is accompanied by net downward flux of heat below the initial mixed layer, calculated following

\[
q(h_m) = \int_{h_m}^{0} \rho_0 c_p \frac{T(t = ts) - T(t = 0)}{ts} dz'
\]  

(II.1)

where \( h_m = 23 m \) is the initial mixed layer depth, \( \rho_0 \) is the nominal density, \( T \) is the temperature, and \( ts \) is the storm residence time. This has a maximum (absolute)
value of $6.2 \times 10^3 \frac{m^2}{s^2}$, as shown in Figure II.3d. Put simply, warm layers above are vertically mixed with colder layers below. This generates an increase in seasonal thermocline temperatures while lowering sea surface temperature.

The fully developed thermodynamic and kinematic response occurs predominantly in the rear-right quadrant of the storm (Fig. II.3a), as found in previous studies (Chang and Anthes, 1978; Price, 1981; Greatbatch, 1983), but as we will see, the distance between the fully developed temperature wake and the center of the storm (in the reference frame of the storm) is a function of Rossby number. This suggests, that in the limit of slow moving, large storms propagating at low latitudes, tropical storm force winds may be sufficient to stabilize the water column, so that prolonged exposure to wind does not result in further mixing. In response to the net integrated horizontal current divergence field underneath the storm, there is a vertical velocity field that is initially upward through the entire depth of the water column due to a wind-forced divergence in the center of the storm Greatbatch (1983). Subsequently, the near inertial currents that are concentrated in the mixed layer pump the entire seasonal and main thermoclines up and down at the same frequency as they rotate in and out of partial convergence and divergence (Price, 1983). The vertical velocity spectrum in the wake has energetic inertial peaks at frequencies of about $f$ and $2f$. At a lower frequency, the near-inertial horizontal currents are sheared by the $\beta$-effect (D’Asaro et al., 1995). For a westward travelling storm, this results in counter-clockwise rotation of the initially north-south phase line connecting near inertial currents.

There is also temporal asymmetry in time between consecutive divergence-to-convergence and convergence-to-divergence, due to the along-track momentum terms simulated in Greatbatch (1983). Note that here, and elsewhere in the paper, whenever we refer to the along-track direction, we mean the direction that is anti-
parallel to the trajectory of the hurricane. Because our storm travels directly west, the along-track direction is equivalent to the east-component of momentum $u$ in this study. Greatbatch (1983) introduces the parameter $A = \frac{U_{MAX}}{U_s}$, where $U_{MAX}$ is the maximum current speed in the wake and $U_s$ is the storm translational speed, as a scale of the relative importance of the non-linear forcing terms ($u \frac{\partial u}{\partial z}, u \frac{\partial v}{\partial z}$) for generating such an asymmetry, where the horizontal divergence field becomes increasingly asymmetrical as $A$ transitions from a value of 0 to 1. For simulation $A = M0/S0$, $A = 0.26$ in support of this theory.

The initial mass divergence through the entire depth of the water column sets up a trough in sea surface height (Fig. II.3c) which geostrophically adjusts to a zonal flow of clockwise race-track like geostrophic current (with maximum surface value of $10 cm/s$), consistent with Shay and Chang (1997).

This balances a mean sea surface height anomaly trough of $8 cm$, that fluctuates at the near-inertial period (Shay and Chang, 1997). In our simulations, the maximum depth of this trough in sea surface height anomaly is a function of storm translational Rossby number as found previously (Geisler, 1970), but is negligibly affected by surface cooling (a few millimeters).

II.3.B Upper Ocean Temperature Response

A one-dimensional estimate of the change in mixed layer temperature due to entrainment can be made by iteratively solving the following equations

$$T_m(t) \frac{h(t)}{h(t) + \delta z} + T(h(t) + \delta z) \frac{\delta z}{h(t) + \delta z} = T_m(t + \delta t) \tag{II.2}$$

$$h(t + \delta t) = h(t) + \delta z$$

$$\Delta T_e(t) = T_m(t) - T_m(t = 0)$$
where $T_m(t = 0)$ is the temperature of the mixed layer depth $h$ using a $0.8^\circ$C criterion and $\delta z = 1m$. The change in temperature due to heat fluxes (Fig. II.4a) is shown for parameterizations of heat flux forcing as specified in Eq. A.6, with wind speed calculated inversely from the wind stress profile presented in Eq. A.1, assuming the drag coefficient parameterization of Large and Pond (1981, Eq. A.7).

In Eq. II.3, the heat flux forcing time series has been extracted along the center line of the storm track. The first parameterization for heat flux (label Standard $Q_t$ in Fig. II.4a), sets a constant transfer coefficient $C_e = 0.0015$. In the second parameterization, the enhanced heat (label Emanuel $Q_t$) flux (normalized by $c_d$) is presented in Fig. A.3. For surface forced mixed layers, the maximum decrease in SST that can be expected due to local surface cooling alone can be estimated according to the standard one-dimensional balance

$$\Delta T_q(t) = \int_0^t \frac{Q(t')}{\rho c_p h_m(t')} dt',$$

(II.3)

where we have assumed that the heat is mixed uniformly over the entire mixed layer depth, $h_m$. In this equation, $Q$ is the heat flux forcing at the surface, and both the density of sea water $\rho = 1027kg/m^3$ and the specific heat of water $c_p = 4184m^2/(s^2C)$ are set as constants. Using these two equations, the percent change due to heat flux losses can be estimated in a one-dimensional sense following

$$p(\delta z) = \frac{\Delta T_q(h + \delta z)}{\Delta T_q(h + \delta z) + \Delta T_e(h)} \times 100$$

(II.4)

We note here that these estimates are likely biased high because this one dimensional model neglects upwelling. Upwelling has been shown to significantly enhance the effect of entrainment Greatbatch (1985) during the storm passage. However, we use this one dimensional balance as a guide, in addition to presenting the more complete results from the full MIT model. The relevant conclusions that can be
made from this calculation are (1) the effect of entrainment alone for lowering temperature depends strongly on stratification (2) temperature changes induced by latent heat fluxes in previous studies (where a constant transfer coefficient was used) could be different by a factor of 2.

As can be expected, shallow initial mixed layers and enhanced surface cooling result in largest changes in mixed layer temperature, and for the initial stratification environments and wind speed timeseries used for forcing in this simple experiment, have magnitudes between 0.1 and 0.6°C (Fig. II.4a). The uncertainty in latent heat flux parameterization is sufficient to account for a significant difference in mixed layer temperature change (compare two curves in Fig. II.4a; Emanuel (1995)).

For these initial stratification environments, the extent of entrainment-induced $\Delta T_m$ is largest for shallow initial mixed layer depths coupled with strong stratification at the base of the initial mixed layer (Fig. II.4b). According to this simplified balance, for a weak hurricane, the surface heat flux accounts for 5, 9, and 45% of the total change in SST for the Atlantic, Pacific, and Caribbean simulations, respectively and the initial values of the mixed layer (Fig. II.4c), with the remainder due to entrainment. For the Caribbean environment, this estimate is higher than has been found in previous studies.

If the initial depth of the mixed layer is sufficiently large, and especially if this is coupled with vigorous upwelling into the thermocline such that the maximum shear is shifted several meters below the base of the mixed layer, shear induced turbulent entrainment can be much reduced at the mixed layer base (Fig. II.4c; Dohan (In prep.)), and surface cooling can become an important factor for setting $\Delta T_m$. We note, however, that these results depend on the magnitude of the transfer coefficient used in calculation of the heat fluxes, an issue of some debate.
for hurricane conditions (Emanuel, 1995; Powell et al., 2003; Donelan et al., 2004). The surface flux accounts for a significant percentage of temperature changes in the Caribbean for storms that move slowly.

Before addressing the additional effects of convective forcing, we present the temperature response due to entrainment alone. As expected, for a given storm speed, the maximum cooling (that typically appears on the right hand side of the storm Price (1981)) is a strong function of initial mixed layer depth and seasonal thermocline stratification, both of which vary regionally. Considering the cross-track temperature anomaly for the storm travelling at \( U_s = 5m/s \) (i.e., for simulations (A,C,P)-M0/S0), it is clear that mixing extends to 80-100m in all environments (see Fig. II.5).

However, the magnitude of mixed layer cooling (centered on the right hand side of the track) is largest in the mid-Atlantic, intermediate in the Pacific, and smallest in the Caribbean environments, respectively. This is what we would expect given the respective initial mixed layer depths of 23,45, and 60m, and strong, intermediate, and weak, seasonal thermocline stratification. These results are consistent with the simple one-dimensional model discussed earlier (compare Fig. II.4b).

Modifying the wind stress (Eq. A.2) in the absence of heat flux forcing (bottom panels of Fig. II.2) had significant effects on the thermodynamic and kinematic responses. Difference fields for the post-storm mixed layer temperature and horizontal currents were of order 0.2 – 0.4°C of relative net warming and 5 cm/s of relatively larger speeds, and asymmetrical across the track. Specifically, the wind mixed layer was not as deep on the right hand side of the wake for the lowered wind stress case (A-M1,C-M1,P-M1), as for the standard case (A-M0/S0,C-M0/S0,P-M0/S0). This was evident by the large amplitudes of the temperature
\(\Delta T \approx 0.6^\circ C, 0.3^\circ C, \text{ and } 1.2^\circ C,\) for A-M, C-M, and P-M simulations respectively) and current speed \(\Delta S \approx 15 \text{ cm/s}\) difference fields that occur between 60-80, 80-100, and 60-80 m depth, respectively. This demonstrates that the turbulent mixing parameterization in the model (KPP model) is sensitive to a maximum change of \(1 \frac{N}{m^2}\) over the inner 3° diameter of the storm (Fig. II.2e).

The effect of additional enhanced heat flux forcing (specified in Eq. A.6) of the modified wind stress case on the temperature response (Fig. II.6) is of similar magnitude to the effect modifying the wind stress had relative to the standard wind stress forcing.

Cooling at the surface accounts for a decrease of \(\approx 0.2 - 0.25^\circ C\) on the right hand side. This partially cancels the effect on the mixed layer temperature response of modifying the wind stress. The warming of \(\approx 0.2^\circ C\) centered at 70 m depth for simulation A-M2 results because of a KPP turbulent mixing parameterization where eddy diffusivity is enhanced for convectively unstable conditions due to non-local surface heat fluxes. This parameterization also explains the relative warming at \(\approx 100\) m in CM2, and at 80 m in P-M2, which are of smaller magnitudes. North of 32°N (for A-M2) and 17°N (for P-M2 and C-M2), both the latent heat flux and the cooling response diminish. On the left hand side, the mixed layer cools by \(\approx 0.3^\circ C\); the response is stronger here because the mixed layer is not as deep. With the exception of the Atlantic environment, the current speeds are negligibly affected. In this environment, the wind mixed layer is deeper when heat fluxes are applied (AM-2), with smaller currents in the upper 50 m and larger currents in the 50 – 100 m region. The temporal standard deviation in the current speed difference at a fixed longitude (AM-2 relative to AM-1) is a maximum of \(+ \approx 1 \text{ cm/s}\). For the Caribbean environment, sea surface heat fluxes accounted for 25% of the mixed layer temperature change.
We note that the buoyancy flux forcing due to precipitation could have a further compensating effect of reducing the cooling due to heat flux losses. To test the effects of these fluxes on the upper ocean temperature response, we forced our heat and wind modified mid-Atlantic simulation additionally with symmetrical profiles of rain taken from a compilation of TRMM satellite images, as presented in Lonfat et al. 2004. Specifically, we used rainfall profiles presented in their Figure 11 for category 1-2 and category 3-5 storms. The magnitude of the effect is about 0.1 – 0.15°C of net warming on the right hand side of the storm, so it is about a third to half the magnitude of the heat-flux induced temperature change (not shown). In other words, as dealt with in the KPP model, buoyancy forcing is significant. In the KPP model, the temperature field is affected by buoyancy forcing through the boundary layer depth (diagnosed by a bulk Richardson number), that is proportional to the diffusivity, in addition to setting the depth of maximum diffusivity.

Following Greatbatch (1983) we use the storm translational Rossby number \( Ro_s = \frac{U}{fL} \) to organize the response of several features of the ocean (Greatbatch, 1983). In this equation, \( U \) is the translational storm speed, \( f \) is the local Coriolis parameter, and \( L \) is the scale of the storm (radius of maximum winds). This Rossby number represents the ratio of the relative vorticity in the storm winds \( \frac{U}{f} \) to the local inertial frequency \( f \). When the frequency of rotation of the wind vectors is exactly equal to the local inertial frequency (i.e. \( Ro_s = 1 \)), the ocean is forced resonantly. We hypothesized that when \( Ro_s = 1 \), near-inertial currents should be resonantly excited. Since the shear in near-inertial currents (normally maximum near or at the base of the mixed layer) scales roughly as the magnitude of the mixed layer current, and since most of the kinetic energy in the storm wake is concentrated at near inertial frequencies, we expect larger near-inertial current
amplitudes, to result in enhanced Richardson vertical shear instability mixing. To test this theory, we performed a set of experiments outlined in Table II.1, where \( Ro_s \) was varied between 0.2 and 2.8 for the Caribbean, Atlantic, and eastern Pacific environments. We estimate the rate of kinetic energy input (per unit meter in the along track direction) during the storm passage following

\[
f \times \int_0^{z_{KPP}} \int_{-y_L}^{y_L} 0.5 \rho_0 (u^2 + v^2) dy' dz'
\]  

(II.5)

where \( z_{KPP} \) is the KPP model diagnosed boundary layer depth (Large et al., 1994), \( y_L \) is the outer storm radius, the \( y \) axis is oriented in the cross-track direction, \( (u, v) \) are the current speeds, \( \rho_0 = 1027 \text{kg/m}^3 \) is the nominal density of water, and \( f = \frac{1}{T_i} \), where \( T_i \) is the inertial period and represents the timescale for the storm passage. In all three environments, the maximum value of the rate of kinetic energy input in the storm wake (evaluated from a snapshot of the area at 10 days; Eq. II.5), is largest when \( Ro_s = 1 \), as expected. However, the maximum mixed layer temperature change (Fig. II.7a) and maximum KPP modelled boundary layer depth (Fig. II.7b) both generally decrease as a function of increasing storm translational speed.

This emphasizes that the duration of the storm (residence time) is a dominant parameter for setting the extent of mixed layer deepening with maximum shear playing a secondary role (note the local maximum of \( \Delta T \) for the Atlantic environment in Fig. II.7b where \( Ro_s \approx 1 \)). The maximum mixed layer temperature changes in the Pacific and Atlantic environments are much larger than in the Caribbean, owing to the initially deep mixed layer and weakly stratified thermocline there. As the storm speed decreases, the Pacific wake displays a stronger trend towards cooling than the Atlantic. This is because the maximum boundary layer depths extends to the region just below the seasonal thermocline
in both cases (Fig. II.7b), and cold water from the top of the strongly stratified east Pacific thermocline is now entrained (Fig. II.1). The distance between the storm center and 85% – 88% of the maximum change in sea surface temperature as measured in the frame of the storm increases as a function of \( R_{o_s} \) (Fig. II.7d). Since the amount of latent heat released to the storm depends implicitly on the sea surface temperature, this should cap the heat release for slower moving, large storms, ultimately reducing the intensity of the storm. This also suggests that the temperature wake could be fully developed during the passage of the first half of the storm in the limit of large, slowly moving storms. The dependence of the relative wake position on Rossby number is a reflection that the timescale for mixing is set jointly by the entrainment rate (a strong function of \( f \)) and the storm residence time (\( \frac{L}{U_s} \)).

Overall, the modelled mixed layer temperature change in the storm wake was most sensitive to the modifying the wind stress, and least sensitive to the addition of precipitation. The magnitudes of mixed layer temperature change induced by each of these modifications to forcing (reducing wind stress in the band of maximum winds, adding enhanced heat fluxes and adding precipitation) were less than 0.5\(^\circ\). Wind stress modifications and precipitation induced reductions in the post storm mixed layer temperature change, while heat fluxes induced an enhanced post storm mixed layer temperature change. These results highlight the importance of correctly modelling forcing fields in simulations of the ocean’s response to a hurricane.

### II.3.C Temperature: Deep Response

At 600m depth there occur significant, net changes of temperature with the passage of the storm (Fig. II.5). For the storm translating at \( U_s = 5.5m/s \), the
largest temperature anomalies occur for mid-latitudes in the Caribbean and mid-Atlantic, with smaller values in the Pacific. These temperature anomalies reflect differences in the central storm latitude and the strength of the deep stratification (in increasing order of strength: P,C,A). In addition, near inertial and twice-inertial oscillations are present in the temperature anomaly timeseries (relative to initial stratification; Fig. III.1b). We find that the net change in temperature is due to vertical advection of the initial water mass by a linearized velocity \( w_E \) (Fig. III.1a). Following the theoretical formulation for \( \Delta T \) of Gill and Niiler (1973) we obtain:

\[
\Delta T(x, y, z, t) = - \int_{t-\Delta t}^{t} w_E(x, y, z, t') \frac{\partial T}{\partial z}(x, y, z, t') dt',
\]

(II.6)

where \( w_E \) is obtained from

\[
w_E(x, y, z, t) = - \int_{0}^{z} \left[ \frac{1}{f} \frac{\partial \zeta}{\partial t} + \frac{\beta}{f^2} \frac{\partial u}{\partial t} \right] dz' + \int_{0}^{z} \left[ \frac{1}{\rho_f} (k \cdot \nabla) \frac{\partial \tau}{\partial z'} + \frac{\beta}{f \rho_f} \frac{\partial \tau_{zz}}{\partial z'} \right] dz',
\]

(II.7)

The equation for \( w_E \) is derived by cross differentiating the momentum equations, solving for the linearized horizontal divergence in the flow field on a \( \beta \)-plane, and integrating cumulatively from the bottom of the domain (where \( w = 0 \)). Note that in derivation of this equation, we performed the transformation for the \( z \)-axis \( z' = H - z \) so that \( z' = 0 \) corresponds to the bottom of the ocean, and \( z \) in positive upward from the bottom. The internal shear stress \( \tau \) vector is estimated following

\[
(\tau_x, \tau_y) = (\nu \frac{\partial u}{\partial z}, \nu \frac{\partial v}{\partial z})
\]

where \( \nu \) is the eddy viscosity from the KPP code, and \( z \) is the vertical dimension. Here we have assumed that \( f \gg \zeta \) and \( w \frac{\partial u}{\partial z} \approx 0 \), and only retained the dominant terms in the balance. The assumption that \( f \gg \zeta \) is not valid at the surface where near inertial currents are strong, but is valid for the region where we make this calculation (i.e. \( \zeta \) decreases with depth, and \( \zeta \leq 0.05f \) everywhere below 275m). We note that \( \frac{w_E - w_{MIT}}{w_{MIT}} \leq 12\% \) everywhere, where \( w_{MIT} \) is the vertical velocity output from the model.
Eq. II.6 reproduces the net response and temporal variability of the temperature field favorably (compare Fig. III.1c with Fig. III.1b), emphasizing that in the deep ocean, \( w \frac{\partial T}{\partial z} \gg u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \). The differences between the MIT temperature field and the temperature field calculated using Eq. II.6 are due primarily to errors in \( w_E \) resulting from calculating relative vorticity on a C-grid, and secondarily to the omission of horizontal advection of the mean temperature field in Eq. II.6, of nonlinear and diffusivity terms in Eq. II.7.

Because the largest term in Equation II.7 results from the one proportional to \( \overrightarrow{\tau} \), it follows that the simplest model for the deep temperature response is Ekman upwelling in the base of the turbulent surface layer. In summation, after the storm passage, an upwelling signature in the deep ocean temperature (and hence pressure) field is left behind due to a net vertical displacement of the rotating particles by the vertical velocity field associated with the forcing of the storm and to which the currents adjust to a geostrophic balance.

Eq. II.6 implies a scaling of the net temperature change of the deep ocean due to a storm with a residence time \( \Delta t = \frac{T}{U_s} \),

\[
\Delta T^{MAX} \propto w_E^{MAX} \frac{\partial T}{\partial z} L \approx \frac{1}{\rho f^2} \frac{(\nabla \times \overrightarrow{\tau})^{MAX}}{U_s} \frac{\partial T}{\partial z}.
\]  

(II.8)

Here \( w_E \) is the vertical velocity forced from the upper layers. Eq. (II.8) suggests that the magnitude of deep temperature change should be large at low latitudes and for enhanced vertical temperature gradients. Formulating Eq. II.8 as a function of \( Ro_s \), we obtain

\[
\Delta T' = \frac{\Delta T f}{w_E^{MAX} \frac{\partial T}{\partial z}} = \frac{1}{Ro_s},
\]  

(II.9)

where \( \Delta T' \) represents the non-dimensional formulation of temperature. Underneath the storm center (at 750m), \( \Delta T' \) is roughly a function of \( \frac{1}{Ro_s} \), in support of
our hypothesis that the vertical velocity in the deep can be modelled as produced by Ekman upwelling (Fig. II.7e). This relationship suggests that fast moving, small storms travelling at high latitudes will generate a larger upwelling response, than slowly moving, large storms travelling at low latitudes. However, this scaling, which is most appropriate for a barotropic equivalent response that occurs under fast storms (Koblinsky and Niiler, 1982) does not extend for the slowest storms (P-S1 and C-S1).

II.4 Summary

The major results from this chapter are summarized as follows:

(1) Between the three environments chosen and in the absence of heat flux forcing, the maximum change in the near surface wake temperature was largest for the mid-Atlantic, moderate in the Pacific, and minimum in the Caribbean. The key parameters for setting the temperature change were central storm latitude, initial mixed layer depth, and thermocline stratification. The largest near-surface temperature changes occurred for the slowest storms, despite a trend of maximum integrated kinetic energy when the ocean was forced resonantly at its near inertial frequency (where $Ro_s = 1$). This does not contradict Greatbatch (1984), who found that maximum horizontal velocity levelled off as a function of increasing Rossby number because our quantity is integrated down to the boundary layer depth. We note here that we have omitted topography and background flows, which would have highlighted differences between the the three environments. This is beyond the scope of this study, which focuses on analyses and description of the ocean response as it depends on basic scaling parameters. We do agree with Jacob and Shay (2003), however, that the effects of topography and background flows could
significant alter some basic features of the response in more realistic simulations. Specifically, the presence of initial background flows can cause the storm induced inertial current response to be enhanced (reduced) in areas where they are in (out of) phase with the pre-existing flow (Pollard and R.C. Millard, 1970). An initial background field of relative vorticity could also affect the spatial distribution of inertial energy, causing trapping at depth in regions of negative relative vorticity Lee and Niiler (1998). Topography could aid the generation storm surges (Cooper and Thompson, 1989) and cause reflection and/or breaking of the deep internal waves.

(2) When forced with a modified Rankine vortex wind stress pattern (Powell et al., 2003) and enhanced heat flux forcing (Emanuel, 1995), heat fluxes accounted for 25% of the mixed layer temperature change for the Caribbean environment. This is on the high side of the 10 – 30% range bracketed by Jacob and Shay (2003). Forcing with an enhanced heat flux using the transfer coefficient of Emanuel (1995) relative to a constant transfer coefficient increased their contribution to the mixed layer temperature change by a factor of 2. The storm induced mixed layer temperature change was also sensitive to modifications of the wind stress pattern (Eq. A.2) and forcing by precipitation (Lonfat et al., 2004), which partially offset the effect of heat flux forcing.

(3) In the frame of the storm, the distance between the storm center and the wake is a linearly increasing function of $R_0$. Thus for large storms, moving slowly in high latitudes, the sea surface temperature wake is produced well ahead of the location where hurricane force winds arrive at the region. Tropical storm winds are sufficient to mix the seasonal thermocline vertically, thus the entire right hand side of the wind field should 'feel' the effects of the cold wake, a problem to be understood in ocean-atmosphere interaction models.
(4) The simplest model for the deep response can be conceptualized as upwelling due to the Ekman-type forcing of barotropic divergence during the storm passage due to the net input of relative vorticity, which is later supported by a geostrophic flow in balance with the sea surface depression, as found previously (Chang, 1985; Shay and Chang, 1997). We show additionally that the maximum normalized deep change in temperature roughly scales as $\frac{1}{f_0\sigma}$.

In this part of the thesis, we have extended the investigation of the upper ocean temperature response, which relates to the magnitude of heat flux release to the atmosphere and provided analysis of the deep response.

This relates to the the transfer of energy in the ocean from large scales and low frequencies, to small scales and high frequencies. Ultimately, understanding these features of the ocean hurricane response contributes to the improvement of storm prediction models, and adds to an evolving description of the features of the super-inertial internal wave spectrum and maintenance of the deep stratification of the ocean (i.e., the upwelling branch of the overturning thermohaline circulation).
Figure II.3: In each panel, the central storm latitude is indicated by a thick line at 31.16°N; the time the center of the storm passes this longitude is indicated by a cross. (a) Time series of temperature field from experiment A-M0/S0 with overlay of instantaneous horizontal current field, both at 27.5m depth. (b) Timeseries of vertical velocity at 27.5m depth. (c) Timeseries of sea surface height anomaly. (d) Net entrainment heat flux out of surface layer at 27.5 m depth during storm as a function of latitude.
Figure II.4: (a) Estimated mixed layer $\Delta T_p$ from heat flux forcing following Eq. II.3. Initial mixed layer depth (or other) intercepts for the Atlantic, Pacific, and Caribbean environments are indicated by square, triangle, and star symbols, respectively and on all panels of this figure. (b) Calculated change in mixed layer temperature due to linear vertical mixing as a function of change in mixed layer depth, $\Delta T_e$, following Eq. II.3 for three initial temperature profiles. (c) Projected percentage change in mixed layer temperature due to heat fluxes for the three environments as a function of change in mixed layer depth.
Figure II.5: Snapshots of temperature anomaly fields (relative to the initial stratification) for experiments M0/S0 at 310°E. The left (a,c,e) and right (b,d,f) columns show the shallow (0m-100m) and deep (300m-1700m) anomaly fields, respectively. Top row (a,b): A-M0/S0. Middle row (c,d): P-M0/S0. Bottom row (e,f): C-M0/S0.
Figure II.6: Snapshots of temperature anomaly fields for from experiment M2 relative to experiment M1. From top to bottom: For Atlantic, Caribbean, and Pacific environments. The storm is tracking westward; the center of the storm track is indicated by a black line.
Figure II.7: Top, middle, and bottom panels plotted against $U_s$, $Ro_s$, and $Ro_s$, respectively. (a) Maximum change in sea surface temperature and (b) maximum boundary layer depth. (c) Maximum depth integrated kinetic energy density to boundary layer depth normalized by local inertial period and (d) maximum distance between storm center and wake (criterion is 85% and 88% of maximum temperature change for lower and higher set of lines, respectively). (e) Maximum normalized temperature anomalies at 750m.
III

The kinematic open ocean response to a hurricane: Nonlinear Dynamics

III.1 Introduction

Internal waves in the ocean exist for range of time (from the local near-inertial frequency to the buoyancy frequency) and length scales (from microscales to hundreds of km). A large percentage of the annual large scale, near-inertial wave energy can be forced intermittently on short time scales during winter storm or summer hurricane passages (D’Asaro, 1985). A thorough understanding of the kinematic response to hurricanes, and especially non-linear mechanisms that transfer large scale motion (with frequencies near $f$) to smaller scales and higher frequencies, contributes to understanding of the observed internal wave spectrum (Garret-Munk) and the maintenance of deep stratification (Munk and Wunsch, 1998).
The presence of higher frequencies in the ocean response to a hurricane has been observed in numerical studies (Greatbatch, 1983; Niwa and Hibiya, 1997). Niwa and Hibiya (1997) used a 40 layer model ($\Delta z = 100m$), with constant eddy viscosity and eddy diffusivity in a linearly stratified ocean to demonstrate that non-linear wave-wave interactions occur with energy transfer from high modes at $f$ to low modes at $2f$. These non-linear wave-wave interactions occur as a result of horizontal advection of momentum, and, as Greatbatch (1983) showed in a 2-layer model, they are asymmetrical about the track and with time. In his 2-layer model, along track advection of momentum was responsible for temporal asymmetries in vertical velocity, forcing a gradual (rapid) transition from consecutive regions of downwelling (upwelling) to upwelling (downwelling). Cross track advection terms caused an offset of the location of maximum upwelling to the right hand side of the storm track.

Complementing the results of Greatbatch (1983) and Niwa and Hibiya (1997), we analyze the deep temperature response using a high resolution model with well resolved near surface mixing processes. Specifically, we show that vertical advection of momentum is equally important as previously identified along track momentum advection (Greatbatch (1983)) in nonlinear forcing of the deep response. We also demonstrate the net rectifying effect that nonlinear forcing has on the interior response.

### III.1.A Kinematic Response

At depth, the kinetic energy of the internal wave train extends farther to the left of the storm than to the right, but high concentrations occur in the upper 100m predominantly to the north (Fig. III.2a). On both sides of the storm, the wave train decays strongly as a function of depth, with a minimum at about 1250m,
where the first mode in the horizontal velocity has a zero crossing (Fig. II.1c). For
the kinetic energy spectrum calculated over the period domain of 0.112-114 hours,
between 70% to 90% of the variance is stored in frequency bands near $f$ (14-24
hours) and $2f$ (8.5-12.5 hours; Fig. III.2b). On the right hand side of the storm
near the surface, about 98% of the variance is stored in the near inertial frequency
band (Fig. III.2c). On the left hand side of the storm track, where the inertial
signal is not quite as strong, the variance in the near-inertial band contributes
30 – 70% of the total, and the twice near-inertial band, 20 – 50% (Fig. III.2d). We
note here that the total variance stored in frequency bands near $2f$ is about the
same magnitude on both sides of the storm. The percentage of variance stored in
these frequency bands is larger on the left hand side because the variance stored
in frequency bands near $f$ is proportionally smaller there.

**Kinematics: Shallow Response**

We have already qualitatively described the cross-track distribution of
oscillations near $f$ and $2f$. Here we discuss the role of non-linear processes in gen-
erating fluctuations near $2f$. Following Greatbatch (1983), we start by computing
from the MIT output the nonlinear terms in the momentum equations written here
as they appear in the model:

$$
\frac{\partial u}{\partial t} - fu + \frac{\partial P}{\partial x} + \frac{\partial (\kappa \partial u)}{\partial z} + F_x = -(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}) \quad (\text{III.1})
$$

$$
\frac{\partial v}{\partial t} + fv + \frac{\partial P}{\partial y} + \frac{\partial (\kappa \partial v)}{\partial z} + F_y = -(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z}),
$$

where $(u,v)$ are the east and north horizontal current components, $f$ is the Cori-
olis parameter, $P$ is the density normalized pressure, $\kappa$ is the eddy diffusivity as
calculated by the KPP mixing algorithm (Large et al., 1994), and $(F_x, F_y)$ are the
east and north components of the force due to horizontal diffusion of momentum.
as calculated in the model. The time average of the non-linear terms is largest underneath the storm swath (between 30 and 32.5°N), with a maximum in the seasonal thermocline (Fig. III.3a,b). There is also appreciable variance in momentum convergence above 50m. The depth of maximum variance increases from 60 to 120 m across the storm track from south to north, following the depth of the vertically mixed zone. The near-surface variance is intensified on the right hand side of the storm and is dominantly supplied by the along track advection term \( u \frac{\partial \zeta}{\partial x} \) (Fig. III.3c,d), with an appreciable but much smaller contribution from cross track advection \( v \frac{\partial \zeta}{\partial y} \) (Fig. III.3f,g). As discussed earlier, along-track advection was first identified as an important upper-ocean non-linear forcing term for divergence, or \( w \), by Greatbatch (1983). Our results concur with his findings. Inspection of the timeseries of the these flux convergences (Fig. III.3a) further reveals that their amplitude is concentrated at frequencies near \( 2f \). This effect is due to large-velocity amplitude waves with velocity fields predominantly at \( f \) multiply together in these terms, generating \( 2f \) frequency convergence.

Directly below the well-mixed region, the variance (also at \( 2f \);Fig. III.3i) is entirely due to vertical advection of horizontal momentum. This is due to the fact that on top of the seasonal thermocline, the vertical gradients in \( u \) and \( v \) are the largest. The vertical advection of horizontal momentum appears in one term of the nonlinear vorticity balance equation for \( w \) (the nonlinear version of Eq. II.7) as follows (see Lee et al. (1994)):

\[
w_{NL}(z) = - \hat{k} \cdot (\nabla \times \int_{\eta}^{z} \frac{1}{\zeta + f} w \frac{\partial \hat{u}}{\partial z'} dz') + ... \tag{III.2}
\]

where \( \eta \) denotes the sea surface height and the \( z \)-axis is negative downward, following a right hand coordinate system convention. The vertical advection of horizontal momentum is only large above 200m (Fig. III.3j); below this layer (i.e., after this
layer is integrated through in Eq. III.2), this term contributes significantly to \( w \) near frequency \( 2f \) against a backdrop of decreasing with depth \( w \)-terms near frequency \( f \). This term expresses how the predominant (linear) response for \( w \) at \( f \) interacts with vertical shear of horizontal currents (also predominantly at \( f \)) to produce nonlinear fluctuations at \( 2f \) in the vertically integrated curl.

Essentially, the four inertial day average fields for nonlinear terms are non-zero (Fig. III.3e,h,k). Not surprisingly, these have spatial patterns that are very similar to the root-mean square fields, with right-hand intensification for both \( u \frac{\partial u}{\partial x} \) and \( w \frac{\partial u}{\partial z} \) and small contributions from \( v \frac{\partial u}{\partial y} \). These terms force oppositely directed shallow overturning cells of vertical velocity (in the cross-track direction).

Below 200m, the momentum equations obey a linear balance to 2\% (not shown). Our interpretation is that the nonlinear processes are prevalent in the mixed layer and upper thermocline region, and the convergence of mass there produces via the vorticity conservation a linear mode at \( 2f \) throughout the water column.

**Kinematics: Non-Ekman forcing of Deep Response**

Given that advection of momentum is only appreciable near the surface, we view the water column as a top wind-driven layer (extending from the surface to \( \approx 200 \)m) with significant contributions from both linear and nonlinear processes forcing a linearly balanced lower layer (depths greater than 200m). We diagnose the effect of nonlinear forcing in the surface layer on the net response of the lower layer by separating the non-Ekman part of the temporal average of the vertical velocity field at the time dependent KPP model diagnosed boundary layer depth...
as follows:

\[ w_{EK2}(y, t) = \nabla \times \frac{\tau(t)}{\rho f(y)} \quad \text{(III.3)} \]
\[ w(y, t) = \frac{\int_1^{10} W_{MIT}(y, z_{KPP}, \zeta) d\zeta}{9} \]

\[ z_{KPP} = \text{KPP diagnosed depth of boundary layer depth}, \]

where the integral for \( w \) is over inertial periods, and the boundary layer depth is zero in the absence of background wind stress forcing. The (net) residual field for vertical velocity (Fig. III.4) is of comparable magnitude (with a maximum of \( \approx 2 \times 10^{-5} \text{ m/s} \)) and is positive there, so the non-Ekman part of the velocity imparts net positive vorticity to the flow in the lower layer (i.e., the depth region below 200m). It is also symmetrical across the track, and extends to about 2° latitude on either side. The residual vertical velocity field shows that there are net divergences above 200m offset on either side of the storm due to non-Ekman processes, which forces net convergence below. It represents the resulting \( w \) field from a variety of processes.

### III.2 Summary

The momentum equation has appreciable nonlinearities above 200m, maintaining a dominantly linear balance below. We have shown that both \( w \frac{\partial u}{\partial z} \) and \( u \frac{\partial w}{\partial z} \) contribute equally to non-linear forcing below the local mixed layer in addition to \( u \frac{\partial u}{\partial x} \) and \( u \frac{\partial v}{\partial y} \) as previously identified by Greatbatch (1983), predominantly at frequencies near 2\( f \). If there are nonlinear wave-wave interactions between high modes at \( f \) to produce waves with a frequency near 2\( f \) as found by Niwa and Hibiya (1997), they must occur in the upper 100m of the water column. We view the ocean as a two layer system, where the time rate of change of relative vorticity
Figure III.1: Linear vertical velocity and temperature anomaly timeseries (relative to initial stratification) for experiment $A - M0/S0$ at $31.16^\circ N, 330^\circ E$. (a) $w_E$. (b) Temperature anomaly from MIT model. (c) Advected timeseries using $w_E$. 
Figure III.2: (a) Total variance in zonal current component, $u$ (with inset of top 100m), (b) sum of fractional variances in $u$ at frequencies of $f$ and $2f$, (c) fractional variance in $u$ at frequency $f$ (bottom left panel), and (d) fractional variance in $u$ at frequency $2f$. 
Figure III.3: Latitude/time series (panels a,c,f,i or left col), depth/latitude standard deviations (b,d,g,j or middle col), and mean fields (panels e,h,k or right col) of time series of nonlinear forcing terms in momentum equations for north/south current component. Rows are lettered. (a-b) $\frac{\partial V}{\partial t} + fU + \frac{1}{\rho} \frac{\partial P}{\partial y} + \frac{\partial (\kappa \frac{\partial V}{\partial z})}{\partial z}$ (c-e) $w \frac{\partial V}{\partial x}$ at 27.5m. (f-h) $v \frac{\partial V}{\partial y}$ at 27.5m (i-k) $w \frac{\partial V}{\partial z}$ at 87.5m.
Figure III.4: Temporally averaged $W$ at the time dependent KPP model diagnosed boundary layer depth, the calculated $W_{EK}$ due to the Ekman divergence, and the difference field between the two $W - W_{EK}$, at a constant longitude and as a function of latitude.
in the upper layer (here, the depth region above 200m) has dominant frequencies from advection processes near $2f$, pumping the lower layer at those same frequencies (conveyed through the pressure gradients, which are felt throughout the water column).
IV

The interaction of mesoscale and steady wind driven flow

IV.1 Introduction

Wind forcing of the ocean can excite an energetic train of near inertial internal gravity waves which have very slow propagation speeds, as well as a train of relatively quickly propagating non-linear super-inertial waves. Once generated, travelling internal waves can break through interaction with other waves or rough topography. The more slowly moving, near-inertial wave characteristics are also modified through non-linear interactions with the mesoscale background field of relative vorticity (Kunze, 1985; Lee and Niiler, 1998). More specifically, one net effect of this interaction is the trapping of near-inertial energy at depth on the negative vorticity side of fronts, which is eventually dissipated (Kunze, 1985; Lee and Niiler, 1998; vanMeurs, 1998). If these processes occur at depth, they can contribute to the upwelling branch of the thermohaline circulation through maintenance of stratification of the main thermocline (Munk and Wunsch, 1998).
One effect of background relative vorticity is to change the local resonant frequency of rotation in the following way (Kunze, 1985):

$$f_{eff} \approx f_0 + \frac{\zeta}{2}$$  \hspace{1cm} (IV.1)

where $f_0$ is the Coriolis frequency, $\zeta$ is the vertical component of the relative vorticity, and $f_{eff}$ is the first order approximation to the non-linearly derived local resonant frequency. At a specific time and on an $f$-plane, this imposes spatial gradients in the local resonant frequency,

$$\nabla f_{eff} = \nabla \frac{\zeta}{2}$$  \hspace{1cm} (IV.2)

which set spatial variability in the size of permitted frequency bands of near-inertial, internal gravity waves $f_a$, $f_{eff} \leq f_a \leq N$ (Kunze, 1985) (here $N$ is the buoyancy frequency). This has the consequence that near-inertial waves generated in regions with relatively low $f_{eff}$ will be reflected when they reach regions of higher $f_{eff}$ (Kunze, 1985). This is true for northwards propagation of linear near-inertial internal waves on a $\beta$-plane (D’Asaro, 1989), and can easily be deduced by consideration of their dispersion and group velocity relationships (Eq. 8.4.13, 8.4.24, and 8.4.26 of Gill (1982)):

$$\omega^2 = \frac{f^2 m^2 + N^2 (k^2 + l^2)}{k^2 + l^2 + m^2}$$  \hspace{1cm} (IV.3)

$$c_{gH} = \frac{(N^2 - f^2)m(k^2 + l^2)^{0.5}}{(k^2 + l^2 + m^2)^{1.5}(f^2 m^2 + N^2 (k^2 + l^2))^{0.5}}$$

$$c_{gz} = \frac{(N^2 - f^2)m(k^2 + l^2)}{(k^2 + l^2 + m^2)^{1.5}(f^2 m^2 + N^2 (k^2 + l^2))^{0.5}}$$

$$\frac{c_{gz}}{c_{gH}} = \frac{-(k^2 + l^2)^{0.5}}{m}$$
where \((k,l,m)\) are the (east,north,vertical) wavenumbers, \(f\) is the local Coriolis parameter, \(\omega\) is the frequency of the wave, \(N\) is the buoyancy frequency, and \((c_{gH},c_{gz})\) are the horizontal and vertical components of group velocity. If we consider the case of \(\omega=f\), Eq. IV.3 clearly requires that \(k^2 + l^2=0\), and additionally that \((c_{gH},c_{gz})=(0,0)\). This implies that a linear northwards propagating near-inertial internal gravity wave on a \(\beta\)-plane is reflected southwards once it reaches a latitude with Coriolis parameter equal to it’s ambient frequency. In this circumstance, \(\frac{c_{gz}}{c_{gH}} \to 0\), suggesting that \(c_{gz}\) decreases at a faster rate than \(c_{gH}\), and that the wave has been rotated towards the horizontal. Similarly, in a standard anticyclonic eddy, which has a core of negative relative vorticity surrounded by a ring of positive relative vorticity, downward propagating internal gravity waves generated in the eddy core (with lowest \(f_{cff}\)) are internally reflected when they reach the positive relative vorticity barrier, and the near-inertial energy they carry is eventually trapped at depth, typically on timescales of several inertial periods (Kunze, 1985; Klein et al., 2004; Zhai et al., 2005).

In addition to the trapping of near-inertial energy at depth, eddy background flow can interact non-linearly with the wind-driven Ekman flow to generate a vertical velocity circulation (Lee and Niiler, 1998). In this chapter, we expand the analysis of Lee and Niiler (1998) as a function of relative size of the nonlinear terms in an eddy (which they could not do because of developing instabilities in their model). Additionally, we will discuss calculation of the vertical velocity using the vorticity equation. We will explain why the signal to noise ratio in calculation of the vertical velocity using the vorticity equation is \(\approx 1\). This prevented us from diagnosing which physical processes are responsible for generating the vertical velocity circulation that is set up in the wind-forced eddy (also a function of Rossby number). The vorticity calculation is a \(3-D\) extention of the analysis presented
in Lee et al. (1994), but for the case of an eddy, not a jet.

IV.2 Model Setup

The difference between the model setup for Chapter I/II and Chapter III are summarized below. The parameters that are not mentioned are the same. The model was configured on an $f$-plane (cartesian grid) for a 550 by 550 km domain. The internal model parameters of horizontal viscosity and horizontal diffusivity for temperature and salinity, and time step are presented in Table IV.1 for each horizontal discretization.

Simulations were initialized with cyclogeostrophically balanced horizontal velocity and temperature fields for a Gaussian cold core or warm core eddy following Lee and Niiler (1998):

$$V_\theta = V_0 \times \frac{r}{a}[exp\left[\frac{1}{\alpha^2}(1 - \frac{r^2}{a^2})\right]] \quad (IV.4)$$

$$T_i = T(z) \pm \frac{V_0 \lambda f a e x p(\lambda z)}{\alpha z} e x p\left[\frac{1}{2}(1 - \frac{r^2}{a^2})\right] + \frac{V_0 e x p(\lambda z)}{f a}[1 - \frac{r^2}{a^2}] \quad (IV.5)$$

with a bottom depth of 1000m or 4500m, and using either 40 or 60 vertical layers. In Eq. IV.5, $V_\theta$ is the azimuthal velocity, $V_0$ is the maximum velocity, $f$ is the Coriolis parameter, $\lambda$ is the vertical decay coefficient, $r$ is the radius, $a$ is the radius of maximum velocity, $T_i$ is the eddy perturbation temperature field and $T(z)$ is the horizontally uniform background temperature profile (initial fields are shown in Fig IV.1). Background salinity was set to a constant of 35.2ppt. The maximum horizontal velocity $V_0$, radius of maximum current, and Coriolis parameter $f$ (i.e., latitude) were varied to generate vortices with Rossby numbers

$$Ro_v = \frac{V_0}{fa} \quad (IV.6)$$
Table IV.1: The horizontal discretization ($\delta X/\delta Y$), horizontal viscosity ($v_h$), horizontal diffusivity for temperature and salinity ($\kappa_h$), and timestep ($\delta t$) as set in model, are tabulated above.

<table>
<thead>
<tr>
<th>$\delta X/\delta Y$ (km)</th>
<th>$v_h$ ($m^2/s$)</th>
<th>$\kappa_h$ ($m^2/s$)</th>
<th>$\delta t$ (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.4</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>0.8</td>
<td>40</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>1.2</td>
<td>60</td>
</tr>
<tr>
<td>5.5</td>
<td>10</td>
<td>2</td>
<td>120</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
<td>4</td>
<td>200</td>
</tr>
</tbody>
</table>

ranging over 0-0.55 for the cold core eddy ($0.1 \leq V_0 \leq 0.8 m/s$, lat=10°N or 17.5°N, $a = 330r60km$) and 0-0.3 for the warm core eddy ($0.1 \leq V_00.4$, lat=17.5°N, $a = 33km$). The initial temperature and velocity fields were adjusted to equilibrium for seven days before applying a horizontally uniform, constant wind stress of 0.1 N/m² to the east for an additional 11.5 days.

Averages of the fields during the wind forced period were taken over 3 near inertial periods (5 days at 17.5°N) to guide in choice of horizontal discretization. The average fields for temperature, current speed, and vertical velocity at 67.5m for a range of discretizations, clearly show that these quantities are aliased at 10km resolution and significantly noisy at 1km resolution, whereas there are only small differences between these fields at 2, 3, and 5.5 km resolution (see Figs. IV.2, IV.3, and IV.4). At 5.5km resolution, these fields are virtually insensitive to maximum depth and number of grid layers. Therefore, to conserve computer resources, we chose a resolution of 5.5km and 40 layers in the vertical, with a maximum depth of 1000m for our sensitivity studies in Rossby number $Ro_v$ space.
Figure IV.1: Initial temperature and velocity fields for cold core vortex. (a) Plan view of sea surface temperature field. (b) Plan view of surface current speed with overlay of current vectors. (c) East-west cross-section of vertical temperature structure of eddy. (d) East-west cross-section of vertical current speed structure.
Figure IV.2: Plan view of 52.5m temperature averaged over three inertial periods (days 4 to 9), for varying discretizations. Each panel is titled with the distinguishing features of horizontal discretization (dx), domain depth (Z\textsubscript{max}), and number of vertical levels (Nz). The timestep, horizontal viscosity, and horizontal diffusivity, are presented in Table IV.1. Note that in all simulations, dx = dy.
IV.3 Results

Near surface time averaged currents (from day 4, when the near inertial currents are fully established, through day 9; 3 inertial periods at 17.5°N) for both warm core and cold core eddies are asymmetrical, with stronger flow occurring on the side of the eddy with a component of flow to the south, in the direction of the mean Ekman forced current (i.e. 90 degrees to the right of the constant eastward wind forcing), as found in Lee and Niiler (1998) (see Fig. IV.5a,b; compare their Plate 2). The asymmetrical velocity field crosses temperature gradients, causing heat advection, as is apparent by the asymmetrical near-surface temperature fields. The destabilizing effect of near surface cold water adverting over warm water and resulting buoyancy-driven vertical mixing, as initially recognized for this situation by Lee and Niiler (1998), explains the relatively smaller temperature change in the cold core eddy. The a-cyclogeostrophic, non-linear Ekman part of the velocity and temperature fields representing non-linear interaction between the wind-driven (Fig. IV.5c,d, compare Plate 3 of Lee and Niiler (1998)) and background currents, show regions of convergence and divergence, and downwelling and upwelling, centered just outside the radius of maximum currents. In the model, the integrated nonlinear divergence and convergences in the horizontal velocity field force a strong vertical circulation that extends to around 200m (O(5 m/day); Fig. IV.5e,f compare Lee and Niiler (1998) Fig. 4).
Figure IV.3: Plan view of 52.5m current speed averaged over three inertial periods (days 4 to 9), for varying discretizations. Titles are constructed in the same way as for Fig. IV.2.
Figure IV.4: Plan view of 52.5m vertical velocity averaged over three inertial periods (days 4 to 9), for varying discretizations. Titles are constructed in the same way as for Fig. IV.2. Note the change in color bars in the top left and bottom left panels.
The maximum time dependent vertical velocities in the cold core eddy are on the order of \(10^{3} \frac{m}{m_{day}}\) and there are fluctuations in the magnitude at the near-inertial frequency evident in vertical and horizontal cross section timeseries (Fig. IV.6). The time mean secondary circulation cell for vertical velocity, shown in Fig. IV.5c, is also evident in this east-west cross-section timeseries. Other features of the response captured in this image include a small upward directed phase lines and therefore downward propagating energy.

In the timeseries of the horizontal cross section for vertical velocity, the lines of constant phase (which point in the same direction as the wavenumber vector; Fig. IV.6a) point away from the vortex center. Depth profile time series are shown 25km on either side of the eddy as indicated by the black (upwelling) and white (downwelling) lines in Fig. IV.6a, are shown in panels b and c, respectively. In these timeseries, the phase lines of vertical velocity slope upwards (compare Fig. 6a of Lee and Niiler (1998)). The evolution of the vertical velocity field as represented in this figure suggests that the phase velocity is directed towards the eddy’s core in the horizontal plane, and upwards in the vertical plane. This indicates that the energy propagates downwards and and away from the center of the vortex. This is consistent with the theory prediction for a wave propagating in geostrophic shear that regions of background positive relative vorticity do not act as a horizontal waveguide for trapping near inertial energy, as do regions of background negative relative vorticity (Kunze, 1985).

The non-linear wind forced vertical velocity at the base of the Ekman layer can be estimated on an \(f\)-plane following

\[
w_{EK} = w_0 - \int_{z_e}^{0} \frac{1}{\rho(f + \zeta/2)^{2}} \left( \frac{\partial \zeta}{\partial x} \frac{\partial \tau_y}{\partial z} + \frac{\partial \zeta}{\partial y} \frac{\partial \tau_x}{\partial z} \right) dz'
\]

(IV.7)

\[
\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}
\]
where $\zeta$ is the relative vorticity, $(\tau_x, \tau_y)$ are the east and north components of wind stress, $w_0$ is the surface vertical velocity, $z_e$ is the Ekman layer depth, and $f$ is the Coriolis parameter. Here we have assumed dominant balances of $(f + (\zeta/2))u_E = -\frac{\partial \tau_x}{\partial z}$ and $(f + (\zeta/2))u_E = \frac{\partial \tau_y}{\partial z}$ and invoked the incompressibility condition $\nabla \cdot \vec{u} = 0$ to solve for $w_{EK}$. We note here that we have simply replaced $f$ with $f_{eff}$, the modified frequency in the presence of background vorticity gradients (Kunze, 1985), in the traditional Ekman balance for horizontal velocity. These relationships can be presented as a function of the Rossby number $Ro_\zeta = \frac{\zeta}{f}$ (compare with Eq. IV.6) as follows:

$$u_E = \frac{2}{\rho f (2 + Ro_\zeta)} \frac{\partial \tau_x}{\partial z} \quad \text{(IV.8)}$$

$$v_E = -\frac{2}{\rho f (2 + Ro_\zeta)} \frac{\partial \tau_y}{\partial z}$$

$$w_{EK} = w_0 - \int_{z_e}^{0} \frac{1}{f \rho (1 + (Ro_\zeta/2))^2} \left( \frac{\partial Ro_\zeta}{\partial x} \frac{\partial \tau_y}{\partial z} + \frac{\partial Ro_\zeta}{\partial y} \frac{\partial \tau_x}{\partial z} \right) dz'$$

This suggests that the vorticity modified, wind forced horizontal velocity should decrease with increasing Rossby number, and that for low Rossby number background vorticity, the vertical velocity at the base of the Ekman layer is a function of the horizontal gradients in $Ro_\zeta$. For the Gaussian eddy horizontal velocity field set in Eq. IV.5 and the application of a steady, horizontally uniform eastward wind, Eq. IV.8 reduces to

$$w_{EK} = w_0 - \frac{3 Ro_v}{fa\lambda} \left[ \frac{1 - e^{-\lambda z_e}}{(1 + \frac{Ro_\zeta}{2})(1 + \frac{Ro_v e^{-\lambda z_e}}{2})} \right] \quad \text{(IV.9)}$$

where $\lambda$ and $a$ are defined when Eq. IV.5 is introduced, and this equation is scaled in terms of $Ro_v$ and $Ro_\zeta$. For the realistic case of low Rossby numbers (i.e. small $Ro_\zeta$), this relationship suggests that $w_{EK} \propto Ro_v$, or in otherwords,
the vertical velocity at the base of the Ekman layer increases in step with the relative size of the non-linear advection terms in the cyclogeostrophically balanced eddy. This theory is supported by the results of repeated numerical experiments, as verified in Fig. IV.7, where the theoretical curve showing $w_{EK}$ is generated using $Ro_c=Ro_v$, $f$ is the value at 10N, $Ro_v$ is varied from 0 to 0.55, and $z_e = 60m$. $z_e$ was set as the depth where the spatially and temporally (over 5 days) averaged horizontal current speed for the region outside the eddy reached 1% of it’s surface value. The other settings are the same as used for the majority of the numerical simulations. Interestingly, for the cyclonic (anticyclonic) eddy, the theoretical estimate for $w_{EK}$ is representative of the maximum vertical velocity for the downwelling (upwelling) cell. The respective upwelling (downwelling) cell is of significantly smaller magnitude, about half that value for $Ro_v = 0.5$.

**IV.4 Discussion**

The original research plan included calculation of the vertical velocity vorticity terms as presented in Eq. A.9 and its linearized form about the relative vorticity (Eq. A.11) in the Appendix. By taking the difference between these quantities, our goal was to estimate the contributions of nonlinear terms to the vertical velocity. However, Eq. A.11 does not represent the fully linear field for $w$ unless the input variables to the equations (e.g. $(u,v,w)$) themselves have also been linearized. The effort required to linearize the model output for input to Eq.
Figure IV.5: Plan view of temperature and velocity fields at 17.5m (and anomaly fields) averaged over 5 days during application of constant wind forcing for cold core (left column) and warm core (right column) eddies. (a) and (b) Contour of mean temperature field with overlay of mean velocity vectors. (c) and (d) Contour of temperature anomaly field (relative to initial condition) and velocity anomaly field (relative to sum of initial cyclogeostrophically balanced currents and linear Ekman flow. (e) and (f) East-west cross-section of average vertical velocity.
Figure IV.6: Vertical velocity timeseries underneath cold core eddy. (a) 67m north-south cross-section timeseries of vertical velocity, at central east-west position. (b) Depth-time profile of vertical velocity at central position in east-west direction, and 33km to the south of the center, of the cold-core eddy (demarcated by the thick black line in panel (a)). (c) Depth-time profile of vertical velocity at central position in east west direction, and 22km to the north of the center (as marked by the thick white line in panel a).
Figure IV.7: Maximum velocity of overturning cells as a function of vortex Rossby number, for both cold core and warm core eddies. Square markers indicate upwelling cell velocities, triangle markers represent downwelling cell velocities. Red and blue colors specify the eddy characteristics latitude and radius of maximum velocity (to define a particular Rossby number) as noted in the legend, and both show data for cold core eddies. The green color denotes that data is from a warm core eddy.
A.11 would introduce significant sources of error, and no less than tantamount to writing our own model. Therefore, we decided not to try and estimate the non-linear contributions to vertical velocity.

Nevertheless, calculation of the terms in the fully nonlinear vorticity equation (Eq. A.9) for vertical velocity would allow us to understand how the signal is partitioned among the various terms as a function of Rossby number. However, when first applying this calculation with an eddy present in the initial flow field, we discovered that the azimuthal symmetry in the initial eddy pressure and potential vorticity field in term \( w_B \) (see Eqs. A.12 and A.9) causes multiplicative errors which are on the order of the signal. The term \( w_B \) is not negligible, suggesting this is a poor calculation to make for studies involving eddies using models configured on a C-grid, which are designed to minimize errors in calculation of the divergence of horizontal velocity, not the relative vorticity.

For that reason, we elaborate on the problem posed by the azimuthal symmetry in the pressure and potential vorticity fields (Fig. IV.8a,b) for calculation of \( w_B \). At the initial timestep, the horizontal gradients in those fields are also symmetric (Fig. IV.8c,d,e,f), and it follows that any error in gradients \( \nabla B \) and \( \nabla \frac{1}{\zeta + I} \) will multiply in terms

\[
\frac{\partial B \partial \frac{1}{\zeta + I}}{\partial x \partial y} \quad \frac{\partial B \partial \frac{1}{\zeta + I}}{\partial y \partial x}.
\]

These product terms are

\[
\begin{align*}
\frac{\partial B \partial \frac{1}{\zeta + I}}{\partial x \partial y} &= \\
\frac{\partial B \partial \frac{1}{\zeta + I}}{\partial y \partial x} &= \\
\end{align*}
\]

where \( B \) refers to Bernoulli’s function as defined in Eq. A.10. Since the gradient fields of Bernouilli’s function and \( \frac{1}{\zeta + I} \) are parallel to one another, the difference between these product terms should be small relative to the size of the product terms (not necessarily insignificant relative to the size of the other terms in Eq.
IV.8).

For the initial velocity and temperature fields (Eq. IV.5), the partially analytical and numerically derived product terms from Eq. IV.11 are shown in the left hand and right hand columns of Fig. IV.9a-d at 67.5 m below the surface. Qualitatively, corresponding partially analytical and numerically derived terms have the same spatial pattern and are the same order of magnitude (in Fig. IV.9, compare panel a to b; and panel c to d). On visual inspection, the partially analytical (numerically derived) field represented in panel a (b) appears to be the negative of that in panel c (d). This turns out to be true in the partially analytical case, where the sum of fields in panels a and c (panel e) is effectively zero (i.e., several orders of magnitude smaller than the magnitude of the signal, $O(10^{-23})$. However, for the numerically derived solution, the sum of fields b and d, as shown in panel f, is $O(10^{-10})$ for the numerical calculation. When the numerically derived field is integrated vertically over the water column to obtain $w_B$ (shown in panel h), the numerical error is on the order of the $w_{NL}$. For the partially analytical solution, the error is several orders of magnitude smaller ($O(10^{-19})$, as expected (panel g).

Varying the numerical scheme for calculation of the relative vorticity gradients (Eq. IV.11) did not alter this result (Fig. IV.10). Relative to the analytical calculation of $w_B$, the error is large for a variety of slightly different calculation schemes (shown in plan view at 67.5 m for the initial timestep in Fig. IV.10b-d):

\[
\begin{align*}
\int_0^z \nabla \times \frac{\nabla B}{\zeta + f} dz' \\
\int_0^z \nabla \left( \frac{1}{\zeta + f} \right) \times \nabla B dz' \\
\int_0^z \frac{1}{(\zeta + f)^2} \nabla \zeta \times \nabla B dz'
\end{align*}
\]
This suggests that calculation of $w_B$ is better suited for models configured on a B-grid.

The error in the vorticity calculation for $w_{NL}$ is on the order of the signal it is designed to calculate (i.e., the vertical velocity field output from the model). This prevented us from diagnosing relative contributions of terms in Eq. A.9 to $w_{NL}$. However, we were still able to show that the strength of the mean vertical velocity circulation increases approximately linearly as a function of eddy Rossby number. The average vertical velocity cell as simulated here extends to depths on the order of a few hundred meters. In regions with an active field of mesoscale eddies forced by relatively constant wind, this could provide a mechanism to replenish nutrients or silicates to the euphotic zone. Example locations where this could be the case include the leeward side of the Hawaiian Islands in the Pacific Ocean, and Cabo Verde Islands in the Atlantic basin. Regions of strong divergence in the wind field are set up between the strong jets that develop between the islands, and the relatively calm winds in their shadow (Chavanne et al., 2002). The divergence in the wind field is on the order of the Coriolis parameter (both island chains are near $20^\circ$N), suggesting that the flow in eddies formed there can be strong, whereas the size is limited by the width of island separation. This argues for small scale eddies with large Rossby numbers. Opal serves as an example of such an eddy that formed off the coast of Hawaii in February of 2005, and had a Rossby number around $R_{O_v} = 0.24$ (Benitez-Nelson et al., 2007). Once formed, the eddies are forced by the relatively steady trade winds $O(10 m/s)$ to the west. In our simulations, the upwelling region of the mean vertical velocity circulation for a cold core eddy of this Rossby number when forced with wind stress to the west should be roughly equivalent to the downwelling region of the mean vertical velocity circulation for a cold core eddy forced with wind stress to the east. Therefore, our results estimate
that the strength of the vertical velocity cell in Opal is around $6m/d$. Depth cross-sections of silicate through Opal suggest a lopsided isotherm structure, where the isoline depths are relatively constant between 0 and 30km to the north side of the eddy, and increase 60m by 30km on the south side of the eddy. It is possible that the asymmetrical structure of the isotherm depth measured in Opal was enhanced by non-linear interactions between the wind driven near-inertial oscillations in the mixed layer and the cyclogeostrophically adjusted currents in the eddy. This would make the eddy a more efficient silicate pump. In our simulations, the width of the vertical velocity cell is about 80km in diameter, which is consistent with this observation. The maximum velocity in the vertical circulation is located at around 50m depth, which is close to where the isotherms level off on the right hand side of Opal (60m).

IV.5 Summary

Using a high resolution model, we generated stable solutions of warm and cold core eddies forced by a constant wind, with initial stratification representative of the Sargasso Sea and of the Caribbean, for eddies representing a range of Rossby numbers. We showed that the magnitude of the maximum 3-day average vertical velocity speed increased nearly linearly as a function of Rossby number for cold-core and warm core eddies, over Rossby number ranges of 0-0.5 and 0-0.3, respectively. This serves as a gauge of the strength of the non-linear interaction between the
Figure IV.8: Plan view of analytical terms used in calculation of Bernouilli term at the first time step at 47.5m. (a) Bernouilli function. (b) Inverse of potential vorticity. (c) Derivative of $\frac{1}{\zeta + f}$ in the east-west direction. (d) Derivative of Bernouilli function in the north-south direction. (e) Derivative of $\frac{1}{\zeta + f}$ in the north-south direction. (f) Derivative of Bernouilli funktion in the east-west direction.
Figure IV.9: Plan view of terms used in calculation of Bernouilli term at the first timestep for partially analytical solution (left hand column) and numerical solution (right hand column). (a) and (b) Analytically and numerically calculated \( \frac{\partial B}{\partial y} \times \frac{\partial \gamma}{\partial x} \) at 67.5m, respectively. (c) and (d) Analytically and numerically calculated \(-\frac{\partial B}{\partial x} \times \frac{\partial r}{\partial y}\) at 67.5m, respectively. (e) and (f) Analytical and numerically calculated \( \frac{\partial B}{\partial y} \times \frac{\partial r}{\partial x} - \frac{\partial B}{\partial x} \times \frac{\partial r}{\partial y} \) at 67.5, respectively. (g) and (h) Partially analytical and numerically calculated \( W_{delB} \) at 67.5m, respectively.
Figure IV.10: Plan view of $W_{deLB}$ term at 67.5m for initial conditions as calculated using a variety of numerical schemes (described in text). (a) Partially analytical. (b) Numerical method I. (c) Numerical method II. (d) Numerical method III.
near-inertial waves and the mesoscale eddies. We showed that the magnitude of the maximum vertical velocity cell over these Rossby number ranges is consistent with that predicted by theory. The strength of the overturning circulation for vertical velocity has implications for the efficiency of pumping of nutrients and/or silicates into the euphotic zone in high-nutrient low-chlorophyll, low-silicate regions such as off the island chains of Hawaii and Cabo Verde.

We also conclude, that calculating terms in the vorticity equation for vertical velocity is not appropriate for a C-grid.
V

Drag coefficient parameterization at high wind speeds

V.1 Introduction

At the sea surface, energy in a hurricane is introduced through the heat lost during evaporation, and dissipated as a shear stress through forcing of a wake of surface and internal waves. Estimating the amount of energy dissipated through shear stress at the sea surface is an important parameter for setting storm intensity (Black et al., 2007). The dissipation of wind energy at the sea surface is strongly influenced by the character of the surface wave field. For a developing wind forced wave field at low wind speed, the drag coefficient increases as waves form, modifying the sea surface geometry to include a component perpendicular to the wind speed. Waves are forced directly by the pressure of the impinging wind (i.e. the wind imparts a form drag), in addition to the shear stress over a hypothetically flat surface (Donelan et al., 2004). This creates a pressure difference between the windward and leeward faces of the wave that can distort the airflow over the wave
surface, thereby modifying the dissipation of energy. For winds below 25 m/s, the corresponding drag coefficient can be parameterized as a linear function of wind speed (Garratt, 1977; Large and Pond, 1981; Yelland and Taylor, 1996; Yelland et al., 1998). For strong winds with a very long fetch, the wave spectrum is broad and the phase speeds of the longest waves approach that of the wind itself. In the reference frame of the wind, these waves cannot supply additional drag on the sea surface, so that the drag is saturated. As waves continually break, the presence of resuspended sea spray from the crest of one breaking wave to the windward side of the next wave can have a smoothing effect on the geometry of the ocean surface perceived by the wind, thereby reducing the drag on the ocean (Lundquist, 1999; Donelan et al., 2004). By contrast, the presence of wind swell propagating in a direction opposite to, or at right angles to a wind, can enhance the drag on the ocean (Donelan et al., 1993). The way in which the wind forced wave field modifies the transfer of momentum to the ocean interior, especially in the presence of strong winds, is complex, and an area of ongoing research (Donelan et al., 1993; Powell et al., 2003; Donelan et al., 2004; Black et al., 2007).

In a hurricane, the seas are fetch limited because the winds are constantly changing direction. The strongest waves are generated in the right front quadrant, and obtain significant wave heights of 10s of meters (Wright et al., 2001; Black et al., 2007). There is the presence of ubiquitous resuspended sea spray, and breaking waves. In a theoretical energy budget calculation, it was concluded that if the drag coefficient increased linearly with wind speed, Category IV and V storms would require an unrealistic amount of heat (from evaporation) to be sustained against shear stress and form drag dissipation at the sea surface (Emanuel, 1995). This is one indication that the drag coefficient may level off at high wind speeds.

In-situ and laboratory measurements of the drag coefficient at high wind
speeds provide further evidence for this hypothesis. One estimate of the drag coefficient as a function of wind speed has been obtained from in-situ measurements of the vertical wind profile in a hurricane near the eyewall, using GPS dropwindsondes. Since 1997, GPS located dropsondes, i.e., pressure/temperature/humidity recording devices tethered to a parachute, have been routinely deployed at 1.5km or 3km above the sea surface near the eye wall of a hurricane. As the GPS dropwindsonde falls to the surface, a velocity profile can be reconstructed from the position information, which is transmitted to the aircraft at a frequency of 2Hz. Since the vertical speed of the dropsonde is typically 10-15 m/s, the corresponding vertical resolution is high (5-7.5 m). The accuracy of the calculated velocity is 0.5-2.0 m/s. Using 331 GPS dropsonde profiles collected in 15 hurricanes, that were binned by the average wind speed over the lower 500m in 5 groups of 10 m/s width (30-39 m/s, etc.), Powell et al. (2003) computed average wind speed profiles down to 10m above the sea surface. Assuming a logarithmic wind profile in the boundary layer for a neutrally stable atmosphere,

\[
U(z) = \frac{U_*}{k} \ln \left( \frac{z}{z_0} \right) \quad (V.1)
\]

\[
\tau = \rho U_*^2 = \rho c_d U_{10}^2
\]

Powell et al. (2003) estimated the roughness parameter \(z_0\) and slope using a least squares fit to each average profile, and from there, the drag coefficient. In Eq. V.2, \(U(z)\) is the wind speed at height \(z\), \(U_*\) is the friction velocity, \(k = 0.2\) is von Karman’s constant, \(\tau\) is the wind stress, \(c_d\) is the drag coefficient, and \(U_{10}\) is the wind speed at 10m above the sea surface. As a function of wind speed, the drag coefficient leveled off at 30 m/s, at a value much lower than would be predicted by extending the linear relationship of Large and Pond (1981) to values above 25 m/s.
A similar relationship established between the drag coefficient and wind speed in laboratory experiments, where a $15m \times 1m \times 1m$ wave tank, filled with 0.5m of water, was forced with winds ranging between 0 m/s and 50 m/s. For wind speeds below 26 m/s, the drag coefficient was calculated directly from velocity fluctuations using an x-film anemometer. These measurements were used to cross-calibrate an indirect method of estimating drag coefficient by applying a momentum balance on a small volume in the tank. The momentum balance method was used to estimate the drag coefficient for winds above 26 m/s. The two methods of calculating the drag coefficient were in good agreement for wind speeds below 26 m/s. In this experiment, the drag coefficient leveled off at a wind speed of 33 m/s, and remained approximately constant as the wind was increased to 50 m/s. One interpretation of why this occurred, is that the geometry of the wave surface was smoothed by breaking waves.

In-situ measurements of flight level velocity fluctuations were made in a hurricane using the Best Air Turbulence probe as part of the coupled boundary layers/air-sea transfer (hereafter referred to as CBLAST) program, for wind speeds ranging between 17 and 30 m/s (Black et al., 2007). The drag coefficient leveled off at wind speeds of 20 m/s.

These methods of drag coefficient parameterization concur that the drag coefficient levels off as a function of wind speed. They differ in the estimation of the critical wind speed and the value at which the drag coefficient levels off, which could additionally be a function of the surface wave field. Further research will be necessary to sort out these issues.

Additionally, each method has potentially significant errors. As acknowledged by Powell et al. (2003), the GPS dropwindsonde estimate could be biased low, because of the outward sloping shape of the hurricane eyewall with height
(owing to the decrease in the pressure gradient with height). Dropsondes are typically deployed just outside the eyewall at 1.5 or 3.0 km, and as they fall vertically towards the surface, it is possible that they move towards the center of the storm, or equivalently, towards regions of increasing winds. If this were a systematic bias, then the vertical wind shear measured by these platforms (and hence the drag coefficient estimate) would be biased low. By contrast, velocity fluctuation or momentum budget measurements taken in a wave tank (Donelan et al., 2004) are not subject to this potential error, but could be biased low simply because the waves generated in a controlled environment with a steady wind source may be smoother than those encountered in the real ocean. Furthermore the in-situ, direct measurements of drag coefficient taken by the BAT probe were at flight level, whereas for purposes of estimating the dissipation of energy by a storm at the sea surface, may not be accurately represented.

Another approach to generate the relationship between drag coefficient and wind speed is to adjust the wind stress forcing in an ocean model to minimize the difference between in-situ measurements of upper ocean temperature and velocities taken underneath a hurricane passage and the corresponding values extracted from a realistic simulation. Assuming that the (radial) wind speed profile $U_{10}$ is known, one can then estimate the drag coefficient relationship with wind speed from Eq. V.2. As a first step towards this effort, Sanford et al. (2007) has run a very high resolution model (10 km in horizontal, 5 m in upper 100 m of domain) in forward mode with upper ocean stratification measured by three profiling floats in Hurricane Frances, 2004, as part of the CBLAST experiment. The floats were programmed to profile between 0 and 200 m, collecting temperature, salinity, and current shear data. The wind stress used to force these model was computed using two different drag coefficient parameterizations following Large
and Pond (1981) and Powell et al. (2003). The conclusion was that the ocean response for temperature and vertically integrated velocity transport per unit meter, was consistent with the drag coefficient relationship of Powell et al. (2003), and was grossly overestimated when the wind stress was calculated using the drag coefficient relationship of Large and Pond (1981).

In the following body of work, we expand on the effort of Sanford et al. (2007), by extending a similar analysis using the MIT model and data collected from an array of 22 drifters fitted with thermistors at the surface and 9 additional floats (also programmed to profile between 0 and 200m) deployed ahead of Hurricane Frances. The specific goal of this research is to force a domain with realistic initial density stratification and the best wind speed product available for Hurricane Frances, and to determine the drag coefficient parameterization with wind speed, that resulted in the closest agreement between the simulated and measured temperature fields.

V.2 Data

On August 31, 2004, an array of 66 drifters and 12 floats were deployed from a C-130 aircraft in a region just to the northeast of the Caribbean island chain for an area bounded between 72-69E and 21.5-24N (Black et al., 2007). This deployment took place approximately one day ahead the passage of Hurricane Frances through the area on September 1, 2004. The instrument array consisted of 66 drifters drogued at either 15m or 100m, all fitted with sea surface temperature sensors. Of these, 17 had wind vanes and measured wind direction, 37 were fitted with surface pressure sensors, and 9 contained wind speed sensors. The 9 SOLO floats and 3 Em-APEX floats were programed to collect temperature and salinity
profiles between 0 and 200m depth at intervals of 4 and 2 hours, and at vertical resolutions of 4m and 2m, respectively. The Em-APEX floats additionally collected profiles of velocity shear, which are not used in this analysis Sanford et al. (2007). The surface position of the floats and drifters was tracked acoustically by the ARGOS satellite, which has an average position error of 350m. The 14 Minimmet drifters were additionally fitted with GPS receivers, but were only operational for part of the deployment period. This increases the position error to about 100m. The first several days of the drifter and float paths are overlaid on a 7-day AVISO composite sea surface height anomaly image centered on September 8, 2004, along with the path of Hurricane Frances in Fig. V.1.

The instrument array spanned an area about 200km wide on the right hand side of the storm. This allowed a cross section of the sea surface temperature wake to be sampled at approximately 50km intervals. The floats were positioned in two groups centered at 70W and 71W, respectively. There was a warm core mesoscale feature with a maximum sea surface height anomaly of 20 cm centered 100km to the northeast of the storm track; the drifters and floats followed the stream function of the mesoscale field closely at speeds of 20-40 cm/s (Fig. V.1). Estimates of the storm velocity from taking central differences of the best track (http://www.prh.noaa.gov/cphc/pages/glossary.php) and from the center positions of the storm as provided by the NOAA HWINDS only agree to within 1 cm/s, but both suggest that the storm was slowing down, from a speed of 7 m/s to 5.5 m/s as it approached the instrument array from the east. Storm speed is a relevant parameter for setting the sea surface temperature swath because slower storms produce enhanced turbulent mixing as discussed in Chapter II. Prior to the storm passage, the temperature field in the upper 200m, as documented by the floats, was roughly uniform over the sampling area. There was a pool of low salin-
V.3 Model and Methodology

V.3.A Model

The basic model setup for the experiments presented in this chapter are the same as in Chapter II; model details specific to this chapter are summarized below. The model was implemented with \( \frac{1}{12} \) degree resolution and 20 layers in the vertical (10m in the upper 100m, to 500m at the bottom) in a 45 by 30 degree domain just east of the Caribbean Island chain (88-43W,7-37N) and run with a free surface on a spherical grid. Values for the background vertical viscosity and diffusivity were chosen to be \( 1 \times 10^{-4} \) and \( 1 \times 10^{-6} \), respectively. Horizontally, coefficients for harmonic form of viscosity and diffusivity had an amplitude of \( 5 \times 10^9 \). The Caribbean Island chain was represented by a simple wall in the southeast corner of the domain extending across a straight line between \((-88W,24N)\) and \((58W,15N)\). Along the wall, a no-slip boundary condition was imposed. Boundary conditions were closed, but the domain was chosen to be large enough to minimize re-reflection of waves. For the region not covered by land, the topography is flat.

V.3.B Initial Conditions

The horizontally uniform, initial temperature and salinity fields were based on hydrographic measurements collected from the EMAPEX and SOLO profiling floats Fig. V.1 (0-200m) and from CTD casts taken during August, 2002
Figure V.1: Contour of AVISO 7-day composite image of sea surface height anomaly on September 8, 2004, with overlay of the track of the first several days of deployment of the 14 Minimet drifters, SOLO floats, and APEX floats.
(0m-ocean floor). The float data are described in Section 2 of this chapter. The 9 CTD casts were conducted as part of the World Ocean Circulation Experiment (http://woce.nodc.noaa.gov/wdu/). The float profiles collected on August 30, 2004 and CTD profiles collected on an east-west cruise track during August 12-20, 2002 were averaged into a mean profile for the upper (0-200m; Fig. V.2), and lower (200-2000m), regions of the ocean. Because there was a small offset between the WOCE and CBLAST profiles at 200m, the temperature and salinity were linearly interpolated for depth region 200m-600m between the two profiles. The composite profiles were then linearly interpolated to the model grid.

V.4 Results

For the sensitivity experiments outlined in this chapter, we used two different wind speed products based on in-situ observations, as listed in Table V.1: NOAA (HWINDS) and Morzel. A third experiment was performed to compare the temperature field from our model with those of Sanford et al. (2007). For most experiments, the drag coefficient parameterization shape function with wind speed is based on the laboratory measurements of Donelan et al. (2004), as presented in Eq. A.5 and shown in Fig. A.2. For the comparison with Sanford et al. (2007), we used the drag coefficient parameterization of Powell et al. (2003). The wind speed and drag coefficient fields are described more fully in the Appendix. In this section, we discuss the temperature response, current response, and comparison with Sanford et al. (2007) sequentially and separately.
Figure V.2: Left Panel. Average and standard deviation of temperature profile for each of 10 profiling floats from August 30, 2004; black thick line is ensemble average of all, used for the composite profile. Right panel. Same, for salinity.
V.4.A Temperature Response

The 12 profiling floats can be grouped according to their approximate distance from the storm track, as presented in Table V.2. Because the measured and simulated temperature response within each group was similar, we show time-series of floats 2, 7, 3, 5, and 9 as representative of the response at distances of 0, 50, 100, 150, and 200km away from the storm track, respectively. We chose these particular floats because their average position 1.5 days after the passage of Hurricane Frances through the instrument array are aligned along an axis that is nearly perpendicular to the storm wake (Fig. V.7).

Floats 2, 8, and 36 (group 1) were directly underneath the storm track, and exhibited the strongest upwelling during the storm passage, as well as the strongest vertical fluctuations of (thermocline) isotherms at the near-inertial period (Fig. V.3a). Mixed layer temperature decreased by a maximum of 2.3°C and isotherms in the thermocline fluctuated vertically by as much as 60m.

Floats 7, 4, and 33 were about 50 km to the right of the storm track underneath the band of maximum winds, and exhibited a large change in the mixed layer temperature and moderate vertical fluctuations of the isotherms (Fig. V.3b). The depth profiles of the temperature anomaly relative to the initial stratification for these floats are shown in Fig. V.4. The temperature change profiles are remarkably similar, and show cooling above scaled depth \( \zeta = \frac{z}{60 \text{m}} = -1 \) and below \( \zeta = -2 \) with net warming inbetween. If this were a linear system, an estimate of the heat lost at the surface could be made by integrating this temperature anomaly profile vertically over the interval \(-2 \leq \zeta \leq 0\), However, in the wake of Hurricane Frances, the non-linear effects that horizontal advection of horizontal momentum has on vertical heaving of the isotherms in the thermocline, (below \( \zeta = -1.5 \)) results in a longer time period of upwelling than downwelling of the isotherms there during an
inertial cycle (Greatbatch, 1983; Niwa and Hibiya, 1997). The effective net cooling this has in a one dimensional heat budget significantly offsets the apparent heat gain in the region directly below the mixed layer (namely, \(-1.5 \leq \zeta \leq -1\)). The error introduced by upwelling in the thermocline in the one-dimensional estimate of heat lost at the sea surface is comparable to the size of the signal, preventing a meaningful estimate of heat lost at the sea surface.

Floats 3, 10 and 34 were 100 km away from the storm track, and exhibited small changes in the mixed layer temperature and small vertical excursions of the isotherms (Fig. V.3c). Floats 5 and 9 were further than 150 km away from the storm track, and did not show large changes during the deployment period (Fig. V.3d). Float 6 was also at least 200 km away from the storm track, but was deployed further to the west. The somewhat enhanced response for sea surface temperature observed at this float may result because the storm was slowing down as it left the instrument deployment region (Fig. V.3e).

Simulated float timeseries are constructed by interpolating the model spatially and temporally (using a linear weighted average in time) at the exact ARGOS telemetered position of the actual floats (shown for experiment NCD1 as specified in Table V.1 in left hand column of Fig. V.3). In general, there is good agreement between simulated and actual temperature profile timeseries at the float locations. Specifically, the size of the isotherm fluctuations and the magnitude of mixed layer cooling are similar. The phases of the vertical fluctuations of the isotherms in the thermocline are approximately aligned in the simulated and measured float timeseries, suggesting that they are forced by the hurricane, instead of the lateral fluctuations of the float across sloped isotherms (that could be associated, e.g., with background mesoscale features). Specifically, the analyses presented in RELATING THESE RESULTS TO THE ANALYSIS PRESENTED IN CHAPTER II SUGGESTS
that the isotherm fluctuations in the thermocline can be interpreted to result from convergence and divergence of the near-inertial currents generated in the mixed layer (i.e., inertial pumping). In the model, the temperature change in the mixed layer is a result of Richardson-type shear instability driven turbulent mixing.

There are also higher frequency fluctuations in the measured temperature profiles, which are absent in the simulated fields. Two likely sources for this discrepancy are the different sampling method of the actual and simulated fields, and from the presence of mesoscale features in the actual field, not simulated in the model. Two major differences in the sampling methods are (1) the model output has been averaged over 1-hour intervals from 120s output, whereas the actual data are collected in real time and (2) the floats are advected horizontally by the mesoscale background flow during their vertical profile descent (i.e. travel at an angle to the vertical), whereas the simulated profiles are strictly vertical. Concerning point (1), the floats take \( \approx 30 \) minutes (0.1 m/s vertical velocity) to complete a vertical profile, and are therefore subject to noise introduced at frequencies higher than 1 cycle/hour. Significantly this is well below the local maximum buoyancy frequency for internal waves of 11 cycles/hour, which serves as the upper threshold frequency for excited internal waves. Concerning point (2), noise could also be introduced into the float measurement from sloping isopycnal surfaces associated with the the background mesoscale. We argue below that this is likely not a large source of error. From AVISO imagery, the surface current in mesoscale has moderate relative vorticity (magnitude near 0.1\( f \)). Therefore the isopycnal surfaces should not be steeply sloped. Assuming a background isotherm slope of 50m/100km, and considering an extreme horizontal speed of an eastward float relative to the westward translating mesoscale of 0.56 m/s, we estimate that the isopycnal depth associated with the mesoscale feature would change by about 4
m between 4-hour sampling intervals. This is smaller than the minimum vertical grid size used in the model (10m). Note that in this estimate, we assume a steady background mesoscale current of 0.5 m/s to the east and 0.06 m/s for the westward propagation speed of the mesoscale features. The estimate of 0.5 m/s is in the upper range of low pass filtered velocity estimates from the 15-m drogue drifers. As mentioned in Chapter III, the mesoscale could also modify the frequency of near-inertial motions, but the small magnitude of relative vorticity at the surface suggests that the frequency of near-inertial waves should be reduced by \( \approx 0.05f \). Note that this assumes that the mesoscale in a hurricane interacts with the near-inertial wave field induced by the storm, which may not be the case because the mixed layer is transitioned from summer-type stratification to fall type stratification during the storm passage (Klein and Treguier, 1995).

Drifter data show that the sea surface temperature decreases by a maximum of 2°C on a timescale of about a day, which is the approximate residence time of the storm. (a subset of 22 drifter timeseries for sea surface temperature is shown in Figs. V.5 and V.6; locations at 1.5 days after the storm passage are shown in Fig. V.7). Note that although we show sea surface temperature timeseries for the raw data, applying a 1-day smoothing filter does not make a significant difference in the estimate of temperature change from the initial condition. Additionally, some drifter timeseries were biased a few tenths of a degree, but this does not impact temperature difference computations. There are examples of drifters that agreed very well with the simulated drifters, e.g. drifter 54. There are also examples where the agreement is poor, e.g., drifter 62.

Starting with the assumption that the measured temperatures and the wind speed field for Hurricane Frances as represented by NOAA HWINDS are within a certain degree of accuracy, we forced the model with a range of wind
stress profiles with maximum values varying by about 50% (described in the Appendix; Fig. A.2 shows the drag coefficient parameterizations), to determine which produced the best agreement between the sea surface temperature change by the actual and simulated drifters. Our measure of best fit was the standard deviation in the ensemble of time-averaged in-situ post-storm sea surface (drifters) or upper ocean (floats) temperature anomaly (relative to the initial value). The temporal average was taken over an interval of a day, centered half a day after the storm passage. This was determined individually for each timeseries. For the floats, the temperature change was additionally averaged over the upper 20m. Since the post-storm water column was well mixed to much deeper depths (maximum of 120m), the depth averaged temperature change did not add significantly to the error. A scatterplot of the mean change of temperature is shown for the actual versus simulated drifters from experiments NCD1 and NCD9 in Fig. V.8. The error crosses plotted around each point show the 90% confidence interval in the mean. For both drag coefficient parameterizations NCD1 and NCD9, the temperature changes are overestimated by the model, but by differing amounts. The standard deviation about the actual temperature change is half that for simulation NCD1, 0.4°C, as it is for NCD9, 0.8°C. Clearly, the simulated and measured temperature field agree more favorably in simulation NCD1 than in NCD9. This argues for a drag coefficient that saturates at high wind speeds, as opposed to one that increases indefinitely (Large and Pond, 1981), in agreement with Sanford et al. (2007). Further, within the error of the model, it suggests an that the upper limit on the value to which the drag coefficient asymptotes is closer to 1.5 × 10^{-3}, in agreement with the calculations of Powell et al. (2003), than 2.3 × 10^{-3}, as found in wave tank experiments (Donelan et al., 2004). This is an original result. The availability of additional data enabled us to constrain the drag coefficient to a smaller range
of saturation values than was achieved by Sanford et al. (2007). We note that
this basic result was not changed when we modified the structure of the eye by
changing the radius of maximum winds from 15km to 45km, thereby increasing
the wind stress curl. The eye structure was not well resolved in the available wind
speed products. For simulations MCD1 and MCD9, the standard deviations of the
temperature change about the data were 0.4°C and 0.8°C, respectively.

As mentioned above, our results suggesting that the drag coefficient satu-
rates at high wind speeds are consistent with the findings of Sanford et al. (2007).
As a further assessment, we show a direct comparison of cross-track sections of sea
surface temperature using the MITOGCM implemented with the KPP turbulent
mixing parameterization (Large et al., 1994) and a three-dimensional version of
the PWP (Price et al., 1986, 1994) in Fig. V.9 (experiment JPP in Table V.1).
The simulated sea surface temperature wakes are similar, with the KPP exhibiting
0.2°C more cooling than the PWP for the maximum response, and approximately
0.1°C more cooling on the right hand side of the storm. This result is consistent
with previous turbulent mixing model intercomparison studies, which also found
that the KPP mixes more deeply than the PWP (Large and Crawford, 1995; Zedler
et al., 2002). This implies that the best agreement between measured and simu-
lated sea surface temperature for the KPP model should occur at lower, but not
necessarily statistically significant, wind stress (and hence drag coefficient) val-
ues, than the PWP. These models were set up with identical initial conditions and
wind stress forcing fields, for a storm moving directly north on an f-plane at 22°N.
This lends confidence, that if we set up an identical sensitivity experiment for a
model initialized with the PWP turbulent mixing parameterization, we would get
a similar result.

As a sensitivity case to demonstrate the lower bound, we forced simulation
NCNST using a constant drag coefficient of $1.2 \times 10^{-3}$, i.e., the value for wind speeds below 11 m/s from Large and Pond (1981). The temperature change scatter plot for NCNST is shown in Fig. V.10. For this simulation, while it is true that the standard deviation about the measured temperature change is statistically indistinguishable from that for NCD1, it underestimates the temperature change for the region underneath the band of maximum winds (e.g., floats 33 and 7; drifters 52 and 71).

V.4.B Near-Inertial Currents

The Minimet drifters were fitted with GPS sensors and drogued at 15m, and the tracks are shown in Fig. V.11. The drifter array roughly followed the streamline velocity field of the low frequency mesoscale, and covered most of the area where the model registered a response in the storm induced temperature field. Some of the drifters, notably 40, 37, and 39 crossed the region of maximum simulated temperature response as they were advected by the low frequency mesoscale. Many traced cyclodds at the near inertial frequency (e.g. drifters 30 and 35). For this array of drifters, we calculated band passed velocity estimates using simple differencing over the frequency range $0.5f_{22N} \leq 1.5f_{22N}$, where $T_{22N} = \frac{1}{f_{22N}} = 1.33 cpd$ to remove low-frequency fluctuations likely associated with mesoscale. For comparison with the simulated drifters, we band pass filtered the velocity fields as output by the model for the same frequency interval. The timeseries of high frequency filtered current amplitude are shown for the simulated and actual drifters in Fig. V.12. In general, the model overestimates the near-inertial current speeds estimated from the data. The near-inertial component of current speed generated in a hurricane wake is a function of both time (e.g. for Hurricane Felix, mooring measurements of current speeds at 45m had e-folding times of several days Zeëller
et al. (2002)) and space. These factors undoubtedly influence temporal fluctuations in filtered current speed amplitudes (e.g., for drifter 30, 31, and 32). With the exception of drifter 31, near inertial simulated and actual drifter speeds decreased significantly between days 8 and 10. This may have to do with the decay of the near inertial wave train. For drifters 39 and 40, the filtered speed amplitude decays more quickly in the actual data than in the simulated drifter. Both of these floats were advected clockwise around a low Rossby number region of negative vorticity, which in some situations has been shown to enhance the vertical propagation of near inertial energy (vanMeurs, 1998). An alternate explanation for the difference between the simulated and actual fields is of course that the wind stress we used to force the model with was simply too high. There are timeseries that show excellent agreement between the simulated and actual near-inertial speed, e.g. for drifter 33 and there are timeseries that agree rather poorly, e.g. for drifter 40. The least favorable agreement between simulated and measured drifter speeds is arguably for drifters 28 and 38, both of which were located on the left hand side of the track, where the inertial response should be smaller, and where we did not accurately represent the topography of the Caribbean Island chain.

The maximum near-inertial current speed in the data is plotted as a function of the maximum near-inertial current speed from the model in Fig. V.13 for experiments NCD1 and NCD9. The relationship is nearly linear, with the standard deviation about the data for NCD1 of 0.07°C, and for NCD9 of 0.15°C, respectively. Especially when considered in conjunction with the result for the simulated temperature field, this argues for a drag coefficient parameterization that saturates at high wind speeds at a value closer to $1.5 \times 10^{-3}$ than $2.3 \times 10^{-3}$. In experiment NCD9, the temperature change induced by the storm, as well as the amplitude of the near inertial currents left behind (after a significant proportion of
Table V.1: The simulation names, spatial wind speed structure, drag coefficient parameterization with wind speed, storm translational speed, and storm track have column labels Code, Storm Structure, $C_d$, $U_s$, and Storm heading, respectively. The spatial structure of the wind fields NOAA (HWIND), Price, and Morzel are described in the appendix. The Donelan drag coefficient parameterization is presented in Eq. A.5, where $A \times 10^{-3}$ is the value to which the drag coefficient asymptotes, minus $1.2 \times 10^{-3}$. The Powell drag coefficient parameterization is exactly that used in Sanford et al. (2007). NHC best track refers to the best track for Hurricane Frances as reported on the National Hurricane Center website (http://www.nhc.noaa.gov/2004frances.shtml; interpolated linearly in time at at 240s intervals), and NOAA best track refers to the path from the timeseries of wind speed provided by NOAA HWINDS (described in the appendix).

<table>
<thead>
<tr>
<th>Code</th>
<th>Storm Structure</th>
<th>$C_d$</th>
<th>$U_s$</th>
<th>Storm heading</th>
</tr>
</thead>
<tbody>
<tr>
<td>NCD1</td>
<td>NOAA</td>
<td>Donelan, $A=0.3$</td>
<td>Variable</td>
<td>NOAA best track</td>
</tr>
<tr>
<td>NCD9</td>
<td>NOAA</td>
<td>Donelan, $A=1.1$</td>
<td>Variable</td>
<td>NOAA best track</td>
</tr>
<tr>
<td>NCNST</td>
<td>NOAA</td>
<td>$1.2 \times 10^{-3}$</td>
<td>Variable</td>
<td>NOAA best track</td>
</tr>
<tr>
<td>MCD1</td>
<td>Morzel</td>
<td>Donelan, $A=0.3$</td>
<td>6.5 m/s</td>
<td>NHC best track</td>
</tr>
<tr>
<td>MCD9</td>
<td>Morzel</td>
<td>Donelan, $A=1.1$</td>
<td>6.5 m/s</td>
<td>NHC best track</td>
</tr>
<tr>
<td>JPP</td>
<td>Price</td>
<td>Powell</td>
<td>5.5 m/s</td>
<td>directly north</td>
</tr>
</tbody>
</table>

Kinetic energy from the initial storm pulse has been converted to potential energy through Richardson-type shear instability mixing), are both overestimated by a factor of two larger than in simulation NCD1. Note here that because the drifters did not register the maximum current speed response during the storm residence period, we infer the magnitude of the response in our data set by considering a combination of the temperature change induced by the storm, and the amplitude of the near-inertial currents that are left behind.
Figure V.3: Temperature profiles measured by SOLO floats deployed 1-2 days prior to the passage of Hurricane Frances (right column) and simulated temperature profiles extracted from the model domain at the equivalent (x,y,t) positions (left column). The mean positions of the floats 1.5 days after the passage of Hurricane Frances (relative to the simulated sea surface temperature wake) are shown in Fig. V.7. (a)-(e) show fields from SOLO floats 2,7,3,5, and 6 respectively. These are spaced at about 50km intervals at distances to the right of the storm track from 0-200km, respectively.
Figure V.4: Vertical profiles of the temperature anomaly relative to the initial pre-storm condition, as measured by floats 4, 7, and 33, which registered the strongest response for $\Delta SST$ and were located underneath the band of maximum winds. Before the difference was taken, the post- and pre-storm profiles were averaged over one inertial period to minimize the effect of upwelling. The nondimensional depth coordinate has been normalized by the average depth of the mixed layer during the period of mixed layer entrainment (i.e., the storm residence period), so that $\zeta = 1$ coincides with $\Delta T = 0$. 
Figure V.5: Sea surface temperature timeseries measured by drifters deployed 1-2 days prior to the passage of Hurricane Frances (real time and 1-day running averages) and simulated temperature profiles extracted from the model domain at the equivalent (x,y,t) positions. The mean positions of the drifters 1.5 days after the passage of Hurricane Frances (relative to the simulated sea surface temperature wake) are shown in Fig. V.7. (a)-(f) show timeseries from drifters 51-62, sequentially.
Figure V.6: Sea surface temperature timeseries measured by drifters deployed 1-2 days prior to the passage of Hurricane Frances (real time and 1-day running averages) and simulated temperature profiles extracted from the model domain at the equivalent (x,y,t) positions. The mean positions of the drifters 1.5 days after the passage of Hurricane Frances (relative to the simulated sea surface temperature wake) are shown in Fig. V.7. (a)-(f) show timeseries from drifters 62-72, sequentially.
Table V.2: Float number and approximate distance from storm track 1.5 days after storm passage through region.

<table>
<thead>
<tr>
<th>Group</th>
<th>Float ID</th>
<th>Approx. Distance from storm track</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2,8,36</td>
<td>0km</td>
</tr>
<tr>
<td>2</td>
<td>4,7,33</td>
<td>50km</td>
</tr>
<tr>
<td>3</td>
<td>3,10,34</td>
<td>100km</td>
</tr>
<tr>
<td>4</td>
<td>5,6</td>
<td>150km</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>200km</td>
</tr>
</tbody>
</table>

V.5 Conclusions

An array of profiling floats and drifters documented the temperature and upper ocean current speed before, during, and after the passage of Hurricane Frances through the deployment region. The 10-m wind speed field of Hurricane
Figure V.7: Simulated sea surface temperature wake as forced by NOAA HWINDS and using drag coefficient parameterization cd1 and average position of floats and drifters over 1 day centered at 1.5 days after the passage of Hurricane Frances.
Figure V.8: Scatter plot of 1-day average changes in sea surface temperature (relative to initial condition) as measured by drifters and floats, as a function of that simulated in the model for NCD1 and NCD9. The error bars are the 90% confidence intervals for the mean.
Figure V.9: Cross section of change in sea surface temperature averaged over one inertial period for experiment JPP, using the MITOGCM implemented with the KPP turbulent mixing parameterization (Large et al., 1994) and the Price et al. (1994) model implemented with the PWP algorithm for turbulent mixing (Price et al., 1986). The storm was translating on an $f$-plane directly north, and the cross-section of temperature is averaged over an inertial period.
Figure V.10: Scatter plot of 1-day average changes in sea surface temperature (relative to initial condition) as measured by drifters and floats, as a function of that simulated in the model for NCNST. The error bars are the 90% confidence intervals for the mean.
Figure V.11: Contour of sea surface temperature change in the model for the last timestep with an overlay of the AVISO September 8, 2004 composite image for sea surface height anomaly and the MINIMET drifter tracks. The MINIMET drifters were fitted with GPS receivers. The path of Hurricane Frances is also shown.
Figure V.12: Timeseries of current speed filtered in the near-inertial band from the actual and simulated Minimet drifters with tracks shown in Fig. V.11. The velocity was calculated using simple differencing of the GPS recorded positions. (a)-(n) Timeseries for drifters number 27-40, sequentially and respectively.
Figure V.13: Scatter plot of maximum 15-m current speed filtered in the near-inertial band, for actual and simulated drifters. Results are shown for simulations NCD1 and NCD9.
Frances known to some accuracy from a combination of in-situ measurements (e.g. sea level pressure, direct measurements of wind speed from a C-130 aircraft). This provided the opportunity to infer the wind stress field for Hurricane Frances (and hence the drag coefficient parameterization at high wind speeds) that provided the most favorable agreement between simulated and measured current and temperature fields. When forced using a variety of wind stress patterns, agreement between the simulated and measured fields argued for a drag coefficient parameterization that saturated at high wind speeds as suggested by Powell et al. (2003); Donelan et al. (2004); Black et al. (2007), over one that increases with wind speed (as suggested by an extension of Large and Pond (1981) to wind speeds higher than 25 m/s). Furthermore, our results suggest constrain this value to be closer to $1.5 \times 10^{-3}$ than $2.3 \times 10^{-3}$ for Hurricane Frances, in agreement with the drag coefficient estimates of Powell et al. (2003). Using a drag coefficient saturation value of $2.3 \times 10^{-3}$ for wind stress conversion resulted in overestimation of the wind stress field a factor of two larger than for the low wind stress case.

For simulations that were initialized and forced with identical conditions, the MITOGCM model implemented with the KPP turbulent mixing scheme generated a very similar temperature response to the the three dimensional model of (Price et al., 1994) implemented with the PWP turbulent mixing parameterization. This suggests that our results are not critically dependent on choice of turbulent mixing scheme.
VI

Summary and Outlook

In the introduction, we outlined a number of research questions that could be explored with simulations of the MIT/OGCM response to hurricane strength forcing. In this chapter, we provide a summary of the results of our investigations and suggest areas of future work.

- What are the implications for the dependence of the upper ocean temperature response on $Ro_s$, when turbulent mixing is parameterized to depend the vertical shear of the ocean response, instead of the amplitude of the surface stress?

When the turbulent mixing is parameterized using Richardson Number dependent shear-instability mixing that can potentially model the resonant response, the maximum change in sea surface temperature and maximum depth of mixing decrease as a function of Rossby number (equivalently, storm speed). The maximum cross-track and depth integrated kinetic energy to the boundary layer is largest when the ocean is resonantly forced by the winds ($Ro_s = 1$). These results are independent of initial density stratification.
During the storm passage, there is a period of vigorous entrainment and rapid sea surface temperature change followed by a period of much slower changes in sea surface temperature. Interestingly, in the frame of reference of the storm, the distance between the storm center and the location where the transition between these two regimes occurs, increases approximately linearly with $Ro$. This can be explained to result from the shape of the density profile. As the mixed layer progressively deepens, the thermocline stratification weakens. Because the mixed layer is deeper, more kinetic energy must be transferred to the currents in order to generate the horizontal current shear at the base of the mixed layer that is sufficiently large to cause a low Richardson number that is parameterized as a condition of large vertical diffusion. At the same time, the sea surface temperature changes generated are smaller. This is both because the thermocline stratification is weaker, and because the mixed layer is deeper. The locations where inertial currents and shears are small are those where the wind stress rotates opposite the inertial motion in the ocean. Essentially the wind stress reduces the currents and this acts to reduce the amount of shear and mixing in the wake. Our results also show that for very slow storms propagating at high latitudes, tropical winds ahead of the hurricane would be sufficient to generate a temperature ”wake” in front of the hurricane. The position of the temperature wake relative to the storm center is relevant for setting storm intensity. Because cooled water underneath the storm evaporates less readily, less latent heat is available to fuel the storm (Chang and Anthes, 1979).

- What are the contributions of vertical advection of horizontal momentum relative to those from horizontal advection of horizontal momentum, when it is resolved in more modern models?
• Are there other non-linear processes at play in creation of the temperature and velocity structure of a hurricane wake?

Vertical advection of horizontal momentum contributes equally with horizontal advection of horizontal momentum to non-linear pumping of the thermocline in general and at near twice the inertial frequency \((2f)\). Most of the signal at \(2f\) is concentrated in the upper layers of the post-storm seasonal thermocline. If wave-wave resonant triad interactions occur in the wake (as suggested by Niwa and Hibiya (1997)), they must be generated in the upper layers of the post-storm seasonal thermocline region.

Non-linear effects of vorticity and momentum advection of the upper layers reproduce the upwelling signal that is felt throughout the water column. This non-linear interaction has a magnitude very comparable to the "linear" Ekman-type upwelling produced directly by the surface winds.

• Can stable solutions be generated of wind forced warm and cold core eddies for a range of observed eddy Rossby and Burger numbers?

• How do non-linear interactions between mesoscale and near-inertial waves vary as a function of Rossby number, when the eddy is forced by a spatially uniform strong wind?

We generated stable solutions of warm and cold core eddies for a range of eddy Rossby and Burger numbers representative of the observed range in the ocean. As characterized by the strength of the vertical velocity circulation that is generated in the time mean, the strength of the velocity cell increases linearly as a function of Rossby number, in agreement with a scaled estimate using a simplified theoretical model.
We also showed, that in the calculation of the vertical velocity using the vorticity equation, noise introduced due to the eddy symmetry is comparable in magnitude to the sum of terms contributing to the total calculated vertical velocity signal. This has the implication that an after the fact calculation of vorticity balance from the solution of velocity is not appropriate for an eddy initialized on a C-grid.

- How well can the MIT/OGCM simulate the observed temperature and current response underneath Hurricane Frances?

- To what extent can we use the MIT/OGCM to constrain an estimate of the drag coefficient parameterization with wind speed underneath Hurricane Frances?

There was a reasonable quantitative agreement between simulated and measured float and drifter temperature fields. The model simulated realistic time variations of the isotherm surfaces in the thermocline that were in phase and of similar amplitude to the measured fluctuations by the floats. In general, the model overestimated the change in sea surface temperature by an average value of 25% of the net change. Clearly, even under the center of the sea surface temperature wake, the heat budget was not one-dimensional. As shown in Sanford et al. (2007), incorporation of upwelled water from below the mixing layer was important. High frequency current timeseries estimated from simulated current speed drifter profiles and actual drifter GPS difference fields were also in general good agreement.

For an identical setup representative of Hurricane Frances (Sanford et al., 2007), our simulated temperature field was in general agreement with a model used by Price et al. (1994) for many hurricane simulations. The largest differences (38%) were found in the shoulders of the wake.
In our sensitivity study for Hurricane Frances, we found that better statistical fit between the measured and simulated surface temperature and current fields was obtained for simulations where the drag coefficient saturated at a value closer to $1.5 \times 10^{-3}$ than $2.3 \times 10^{-3}$. This is agreement with the in-situ measurements from GPS dropwindsondes Powell et al. (2003) than the laboratory measurements made by Donelan et al. (2004).

Importantly, we were able to constrain the drag coefficient parameterization in Hurricane Frances within a specified range. The results are encouraging, and the research problem warrants further investigation. One form this effort could take would be to assimilate the temperature and current data into the model, while treating the wind stress field as an adjustable parameter. The logical first step would be to perform twin experiments, where "data" extracted from a forward run are assimilated into an adjoint version of the model to determine how much data is necessary to constrain the wind stress to a specified accuracy. If we have sufficient data for reconstruction of the temperture and current response for Hurricane Frances, we could then constrain an adjoint version of the Frances data and calculate the drag coefficient paramterization as a function of wind speed. If that is not the case, the twin experiment would provide us with the necessary information to design a sampling program with sufficient data for making this calculation. Ultimately, these results would be useful for making forecasts of hurricane intensity.
A

Supplementary Formulae

A.1 Wind Stress

Wind stress for the hurricane simulations presented in Chapters I and IV was parameterized in a variety of ways, depending on the purpose of the experiment.

A.2 Rankine Vortex

The wind stress equation (NORMAL in Table IV.1) used to force simulations presented in Chapter I was based on Price (1981) as (using cylindrical coordinates):

\[
\begin{align*}
\tau_\theta &= \tau (r/R) & r < R \\
\tau_\theta &= \tau (1.2 - 0.2r/R) & R \leq r \leq 6R \\
\tau_\theta &= 0 & r > 6R \\
\tau_r &= -0.3\tau_\theta & all r
\end{align*}
\]  

(A.1)
where $\tau_\theta$ and $\tau_r$ are the azimuthal and radial components of the wind stress, $\tau$ is the maximum wind stress, and $R$ is the radius of maximum winds. The standard forcing field used for sensitivity experiments is a modified form of Eq. A.1 with a band of constant maximum wind stress between two radii was created (called MODIFIED in Table IV.1) according to

$$\tau_{ref} = \sqrt{\tau_\theta^2 + \tau_r^2}$$

$$\tau_\theta = 2 \times \frac{\tau_{ref}}{\tau_{ref} \geq 2}$$

$$\tau_r = 2 \times \frac{\tau_{ref}}{\tau_{ref} \geq 2}$$

The modified radial profile of wind stress increases linearly to $2 \frac{N}{m^2}$ outward from the storm center. The maximum value is essentially capped over a radial band before decreasing linearly to zero (instead of rising to a maximum of $3 \frac{N}{m^2}$; see Equation A.2 for details). Parameters were chosen such that the above two wind models represent Hurricane Felix conditions which passed over the Bermuda Testbed Mooring in August of 1995 (i.e., $R=45$ km, Dickey et al., 1998). On the Saffir-Simpson scale for hurricanes, a storm that has a maximum wind stress of $2N/m^2$ is representative of a weak Category I storm (maximum winds of 33-42 m/s) with maximum drag coefficient $2 \times 10^{-3}$, a strong Category I storm with drag coefficient $2.3 \times 10^{-3}$, or a moderate Category II (maximum winds of 42-58 m/s) storm with maximum drag coefficient $1.55 \times 10^{-3}$. These equivalencies are based on the fact that the Saffir Simpson maximum wind speed applies to wind speed values measured at about 300m above sea level. The average shape profile from generated from an ensemble of GPS dropsonde measurements deployed near the eyewall at 1.5 or 3.0 km above the sea surface suggests that the corresponding surface values of wind speed are about 80% of the maximum (Powell et al., 2003).
A.2.A Hurricane Frances: From In-Situ Drifter Measurements (Morzel)

For some experiments in Chapter III, the domain was forced with a wind stress field based on Hurricane Frances travelling along the track of Hurricane Frances at 6.5 m/s (Fig. A.1). For the region outside of the radius of maximum winds, the radial model wind field was fitted to an exponential decay assuming a gradient wind balance, using the drifter measurements for pressure, wind speed, and wind direction, and is fully described on Jan Morzel’s web site (http://www.cora.nwra.com/ morzel/drifters.frances.html). The equations for wind speed are provided in radial coordinates:

\[ v(R) = 53.86 \times e^{-0.0054R} + 2.86 \]  \hspace{1cm} (A.3)
\[ u(R) = 13.27 \times e^{-0.0035R} - 3.33 \]

\[ R_{MAX} \leq R \leq 20R_{MAX} \]

where \( R \) is the radius in \( km \), and \( R_{MAX} = 40km \) the radius of maximum winds (Fig. A.1). Inside the radius of maximum winds \( (0 \leq R \leq R_{MAX}) \), an exponential was fit to match the outer solution at \( R_{MAX} \). For the region \( 20R_{MAX} \leq 22.5R_{MAX} \), the speed was ramped linearly to zero.

The wind stress field was then generated following

\[ \tau_x = \rho_aC_dS_{10}U_{10} \]  \hspace{1cm} (A.4)
\[ \tau_y = \rho_aC_dS_{10}V_{10} \]
\[ S = (U^2 + V^2)^{0.5} \]

where \( C_d \) is the nondimensional drag coefficient, \((U,V)\) are the east and north components of the wind speed at 10m height above sea level, and \( \rho_a = 1.28kg/m^3 \) is
the nominal density of air (Gill, 1982). The drag coefficient parameterization with wind speed was modeled as a smooth function that asymptoted to a maximum value at \( S = 30 \text{ m/s} \), following the in-situ and laboratory measurements of Powell et al. (2003), Donelan et al. (2004), and Black et al. (2007). The shape function was modeled as a hyperbolic tangent

\[
A10^{-3} \times \left[ \tanh((S_{10} - 25)/5) \times 0.5 - \tanh(-25/5) \times 0.5 + 1.2 \right] \tag{A.5}
\]

where \( A \) is a constant parameter in the range (0.3, 1.1) (see Fig. A.2).

### A.2.B Hurricane Frances: NOAA HWIND (NOAA)

Some simulations were forced with wind stress based on the NOAA HWIND wind speed analysis product, which is based on a combination of flight level in-situ measurements of wind speed and data collected from surface drifters Powell et al. (1998) and is publically available at 6-hour increments during the passage of Frances (http://www.aoml.noaa.gov/hrd/data-sub/wind2004.html; see Fig. A.1). Before converting the field to wind stress following Eq. A.5, the wind speed field and the position of the center of the storm were linearly interpolated using a temporal weighted average 240 second intervals, the timestep of the storm. This wind stress field is asymmetrical, with the strongest winds in the front right quadrant.

### A.2.C Frances: For Comparison with Price Model (Price)

A symmetrical wind stress field based on was computed following (Sanford et al., 2007) for intercomparison studies between the MIT OGCM model, which uses the KPP turbulent mixing scheme, and the PWP hurricane code and turbulent mixing scheme. This forcing field was translated at a constant speed of 5.5 m/s
to the north, and is very similar to the mean radial profiles from the Morzel and NOAA HWIND products (Fig. A.1).

## A.3 Heat Fluxes

The latent heat fluxes are parameterized, following Doney et al. (1998), as

$$Q_{lat} = \rho_a L_v C_k U_{10} (q_a - q_s)$$  \hspace{1cm} (A.6)

where the relative humidity is set to 80%, a typical value for tropical climates (Emanuel, 1987). Variables $q_a$ and $q_s$ are the specific humidity at 10m and just above the sea surface. The difference in specific humidity is calculated using equations presented in chapter 3.1 of Gill (1982). The latent heat of vaporization ($L_v$) is set as a constant $L_v = 2.45 \times 10^6$ J/kg. In Equation A.6, $U_{10}$ is interpolated from a wind stress/wind speed curve using a Yelland et al. (1998) drag coefficient,

$$c_d = 10^{-3} \times (0.5 + 0.071 \times U_{10}) \hspace{0.5cm} U_{10} \geq 6$$
$$c_d = 0.5 \times 10^{-3} \hspace{1.5cm} 0 \leq U_{10} \leq 6$$ \hspace{1cm} (A.7)

and the bulk formula wind stress relationship (Doney, 1996), as specified in Equations A.8:

$$\tau_x = -\rho_a c_d U_{10}^2 \sin \theta$$
$$\tau_y = -\rho_a c_d U_{10}^2 \cos \theta$$ \hspace{1cm} (A.8)

where $\tau_x$ and $\tau_y$ are the east and north components of the wind stress, $U_{10}$ is the wind speed at 10m, $c_d$ is the drag coefficient as formulated by Yelland et al. (1998), $\rho_a = 1.22 kg/m^3$ is the density of air, and $\theta$ is the direction from which the winds come and is measured clockwise from the north.

The transfer coefficient, $C_k$, is parameterized as linearly proportional to the drag coefficient $C_d$ following the model of Emanuel (1995), instead of being set
as a constant as in previous hurricane modelling studies (Price et al. (1994); Jacob et al. (2000); Jacob and Shay (2003)). Emanuel’s parameterization for the drag coefficient resulted from an energy budget analysis for steady state maintenance of a hurricane (Emanuel, 1995). The ratio of the latent heat transfer coefficient to the drag coefficient ($\frac{C_k}{C_d}$) was determined to range between 1.2 (label Emanuel Guideline for Sustainability in Fig. A.3a) and 1.5, for a mature hurricane. In his model, a hurricane could not form if $\frac{C_k}{C_d} \leq 0.75$. Clearly, for a constant transfer coefficient of $C_k = 0.0015$ (used in Price (1981); Zedler et al. (2002); Jacob and Shay (2003)) and several drag coefficient parameterizations, the heat fluxes are lower than expected for a hurricane to form (Fig. A.3a). The transfer coefficient consistent with the model of Emanuel (1995, i.e., $C_k = c_d \times 1.35$) shown in Fig. A.3b as a function of wind speed for several drag coefficient parameterizations. If Emanuel (1995) is correct, this suggests that hurricane heat fluxes used to force the ocean in previous modeling studies investigating the relative contributions of latent heat release and entrainment for lowering the sea surface temperature Price (1981); Jacob and Shay (2003); Zedler et al. (2002) were overestimated by at least 60%. In this study, we find that increasing the transfer coefficient as specified in Emanuel (1995) does not significantly alter the major results of these studies. Entrainment remains the dominant mechanism for lowering sea surface temperature.

Wind speed $U_{10}$ is calculated from the standard mid-Atlantic simulation (A-M0/S0) using Equations A.8, and drag coefficients $c_d$ are obtained using a linear fit to measurements made by Powell et al. (2003).
A.4 Vorticity Equation for Vertical Velocity

To derive the relevant equation for \( w \), we cross differentiate the momentum equation and integrate them vertically from the bottom (where \( w = 0 \)) to a depth \( z \) to derive a diagnostic equation for vertical velocity following Lee et al. (1994):

\[
w_{NL}(z) = k \left[ - \nabla \times \int_{0}^{z} \frac{1}{\zeta + f} \frac{\partial \vec{u}}{\partial t} \text{d}z' - \nabla \times \int_{0}^{z} \frac{1}{\zeta + f} \nabla B \text{d}z' \right] (A.9)
\]

\[
- \nabla \times \int_{0}^{z} \frac{1}{\zeta + f} \frac{w}{z} \frac{\partial \vec{u}}{\partial z} \text{d}z' + \nabla \times \int_{0}^{z} \frac{1}{\zeta + f \rho} \frac{\partial \vec{\tau}}{\partial z} \text{d}z' + \nabla \times \int_{0}^{z} \frac{1}{\zeta + f} \nabla \vec{F} \text{d}z'.
\]

Here \( \zeta \) is the vertical component of the relative vorticity, \( \vec{u} \) is the horizontal velocity vector with east and north components \((u,v)\),

\[
B = \frac{u^2}{2} + \frac{v^2}{2} + \frac{P}{\rho} \quad (A.10)
\]

is Bernouilli’s equation (with pressure \( P \)), \( \vec{\tau} \) is the turbulent shear vector with surface components northward and eastward, \((\tau_x, \tau_y)\), \( \vec{F} \) is the force due to horizontal diffusion of momentum as calculated from the model, and \( \rho \) is the density. Upon linearization, where \( f \gg \zeta \) and \( \frac{\partial u}{\partial z} \approx 0 \), eq. (A.9) yields:

\[
w_L(z) = - \int_{0}^{z} \left[ \frac{1}{f} \frac{\partial \zeta}{\partial t} + \frac{\beta}{f} \frac{\partial u}{\partial t} \right] \text{d}z' + \int_{0}^{z} \left[ \frac{1}{\rho f} \left( k \cdot \nabla \times \frac{\partial \vec{\tau}}{\partial z} \right) + \frac{\beta}{\rho f^2} \frac{\partial \tau_x}{\partial z} \right] \text{d}z' (A.11)
\]

\[
- \int_{0}^{z} \left[ \frac{\beta}{\rho f^2} \frac{\partial P}{\partial x} \right] \text{d}z' + \int_{0}^{z} \left[ \frac{1}{f} \left( k \cdot \nabla \times \vec{F} \right) + \frac{\beta}{f^2} \tau_x F_x \right] \text{d}z'.
\]

In this equation, \( \beta = \frac{d\tau}{dy} \). Eqs. (IV.9) and (A.11) can be re-written as

\[
w_{NL}(z) = -w_d \frac{\partial u}{\partial t} - w_B - w_d \frac{\partial u}{\partial z} + w_d \frac{\partial u}{\partial z} + w_F. \quad (A.12)
\]
and

\[ w_L(z) = -w^L_{dU/dt} + w^L_{d\theta/dz} - w^L_B + w^L_F. \]  \hspace{1cm} (A.13)

and the difference between these latter two eqs. can be use to estimate the non-linear component of vertical velocity as \( w_{NL} = w - w_L \) and also term by term.
(Various) Mean Radial Wind Speed Profiles for Frances

Spatially Averaged Noaa H*WIND, 9/1/04
wind speed modeled from drifters (J.Morzel)
std. profile from J.Price (based on Frances)

Figure A.1: Radial wind speed profiles used to generate wind stress for various experiments: as used for the intercomparison with (Sanford et al., 2007), a radial average of the wind speed from the NOAA HWIND product (and standard deviation), and from the wind speed product based on drifter measurements.
Figure A.2: Drag coefficient relationships as a function of wind speed from the wave tank laboratory experiments of Donelan et al. (2004) and from an extension of Large and Pond (1981) to wind speeds above 25 m/s, along with our shape function for several values of constant $A$ in Eq. A.5.
Figure A.3: Top panel. Ratio of latent heat transfer coefficient $C_k = 0.0015$ to drag coefficient $C_d$, where $C_d$ is based on projections of standard drag coefficient relationships to wind speeds above $25 \text{ m/s}$ and on in situ measurements of the wind speed profile (Powell et al., 2003). Bottom panel. Transfer coefficient derived from drag coefficient, assuming $\frac{C_k}{C_d} = 1.2$. 
References


Dohan, K., In prep.: Mixing in the transition zone during two storm events. For submission to *Journal of Physical Oceanography*.


