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Author
Steiner, Herbert.

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Ernest O. Lawrence Radiation Laboratory

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Herbert Steiner

October 1967

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Berkeley, California

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SPIN DEPENDENT EFFECTS IN \( \pi N \) AND \( NN \) INTERACTIONS

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INTRODUCTION

The spin dependence of elementary-particle interactions has on occasion been referred to by theorists as producing "inessential complications" in various calculations. Fortunately, a number of high energy physicists (including myself) have derived gainful employment in recent years from the source of these "inessential complications". In these lectures I would like to discuss some of the experimental and theoretical considerations that are pertinent to the study of spin-dependent effects in πN and NN interactions.

In the first part of this lecture I propose to outline the basic formalism and to discuss the experimental situation in πN Scattering at energies where resonance production is important. The emphasis in the second part will shift to the experimental and theoretical situation at high energies, and in particular we shall focus our attention on the "crucial" tests of high energy theories afforded by the study of various spin-dependent quantities in πN and NN scattering.

I. Spin-Dependent Effects in πN Scattering in Energy Region Where Resonances are Prominent.

A. Introduction: All of the possible types of experiments one can do by elastically scattering pions on nucleons can be summarized by the equation:

\[ \frac{I}{I_0} < \sigma_\mu > f = \sum_{\nu=0}^{3} D_{\mu \nu} < \sigma_\nu > i \]  

(I-1)

where \( I \) = the scattered intensity

\( I_0 \) = the scattered intensity when the initial state nucleon is unpolarized
\[ \sigma_\mu = (\sigma_0, \sigma_1, \sigma_2, \sigma_3) \]

where \( \sigma_0 \) is the (2x2) unit matrix \( \mathbb{1} \), and \( \sigma_1, \sigma_2, \sigma_3 \) are the three Pauli spin matrices. The subscripts \( f \) and \( i \) refer to the final and initial states respectively.

The operator \( D \) is sometimes referred to as the Depolarization Tensor. For example, \( D_{j0} \ j = 1,2,3 \) would refer to that experiment in which the \( 3 \) components of the vector polarization of the nucleon in the final state are determined when the initial state is unpolarized. Similarly \( D_{kl} \) would describe the experiment where the target proton is polarized along the "k" direction and measurement is made of the final proton's polarization in the "l" direction.

An explicit representation for \( D \) can be written if one chooses a specific form for the \( M \) matrix which acts on the initial state to produce the final state. (\( M \), which is a function of energy and angle, is an operator in the spin space of the nucleon.) \( D \) is related to the \( M \)-matrix by the equation

\[
D_{\mu \nu} = \frac{1}{2} \text{Sp}(M_{\nu} M^+_{\mu}) \ .
\]  

(I-2)

For example, let us choose the parity conserving form

\[ M = G \mathbb{1} + i H \sigma \cdot \hat{n} \]  

(I-3)

where \( G \) and \( H \) are functions of c.m. energy and angle, \( \mathbb{1} \) is the 2x2 unit matrix, \( \sigma \cdot \hat{n} \) is the component of the spin operator in the direction normal to the scattering plane, i.e.

\[
\hat{n} = \frac{\vec{k}_i \times \vec{k}_f}{|\vec{k}_i \times \vec{k}_f|}
\]
\( \vec{k}_i \) is the momentum of the nucleon in the c.m. before scattering.

\( \vec{k}_f \) is the momentum of the nucleon in the c.m. after scattering.

We will use a coordinate system such that \( \hat{n} \) is along the +y axis, \( \vec{k}_i \) is along the +z axis, and \( \vec{k}_f \) is in the x-z plane at an angle \( \theta \) with respect to the z axis (see Fig. 1).

![Diagram showing \( \vec{k}_i \), \( \vec{k}_f \), and the angle \( \theta \).]
Then the depolarization operator can be written

\[
D = \begin{pmatrix}
& 0 & y & z & x \\
0 & 1 & \alpha & 0 & 0 \\
y & \alpha & 1 & 0 & 0 \\
z & 0 & 0 & \gamma & \beta \\
x & 0 & 0 & -\beta & \gamma \\
\end{pmatrix}
\]

where \( I_0 = |G|^2 + |H|^2 \), \( I_0 \beta = 2 \text{Re} \, GH^* \), \( I_0 \alpha = 2 \text{Im} \, GH^* \), and \( I_0 \gamma = |G|^2 - |H|^2 \).

Note that there are no elements of \( D \) connecting the \((o,y)\) components with the \((z,x)\) components. This is a consequence of parity conservation in strong interactions which has been built into our form for \( M \). (I choose the rather odd order of labeling the elements of the matrix written above to show the block-diagonal nature of the \( D \)-matrix when parity is conserved.) Note also that \( \gamma = 1 \) means that there is no spin flip, whereas \( \gamma = -1 \) implies that there is only spin flip.
The various elements of the D-matrix can be directly related to the so-called Wolfenstein parameters. For example,

\[ P = D_{oy} \]  (Polarization Parameter)

\[ D = D_{yy} \]  (Depolarization Parameter)

These parameters have a relatively easy interpretation in terms of experiments. For example,

\[ D_{yo} = P = \alpha \]

\[ D_{yy} = D = 1 \]

\[ D_{xz} = -\beta \]

\[ D_{x'y'} = \gamma \sin \theta + \beta \cos \theta \]

where \( \uparrow \) indicates spin direction in \( x - z \) plane, \( \bigcirc \) indicates spin direction in \( y \)-direction (out of paper).

Fig. 2.
From the experimental viewpoint, every time you see a spin in the initial state it means a polarized target is needed. Anytime you see a spin in the final state it means an additional scattering is needed in order to analyze the polarization. Thus only $P$ can be determined by experiments involving a single scattering, and then only if a polarized target is used. This is the reason that most of the experimental effort up to now has focused on measurements of the polarization parameter.

One slight complication arises due to the fact that most high energy experiments up to now at least have been done not in the center-of-mass but in the laboratory system. The polarization components normal to the scattering plane are unchanged under transformation from center-of-mass to laboratory frames of reference. On the other hand, care must be taken in relating measurements in the lab involving polarization components in the plane of the scattering to the components of the depolarization tensor which is defined in the center-of-mass. In 1954 Wolfenstein introduced the parameters $A$ and $R$ to describe the change of polarization in the plane of the scattering of the incident particle in the lab. In $nN$ scattering where the incident projectile has spin zero it is convenient to introduce analogous parameters $A_{\text{recoil}}$ and $R_{\text{recoil}}$ which refer instead to the change of the target nucleon's polarization in the plane of the scattering. (See Fig. 3.)

\[
A_{\text{recoil}} = -\beta \cos (\phi - \phi_L) + \gamma \sin (\phi - \phi_L) \quad \text{(I-4)}
\]

\[
R_{\text{recoil}} = +\beta \sin (\phi - \phi_L) + \gamma \cos (\phi - \phi_L) \quad \text{(I-5)}
\]
Target polarization $\mathbf{A}$

Final analysis of polarization

Laboratory

Recoil

Center of mass

Fig. 3. Geometry for measurement of depolarization parameters, $R$ and $A$, using a polarized target.
Recently at Saclay a polarized target has been put into operation which allows the target protons to be polarized in the scattering plane.\(^2\) With this target measurements are now being started at CERN to measure \(A_{\text{recoil}}\) and \(R_{\text{recoil}}\). It is clear that measurements of \(I_0, P, A_{\text{recoil}},\) and \(R_{\text{recoil}}\) should allow one to evaluate \(|G|, |H|\) and their relative phase, and thus to determine the \(M\) matrix up to an overall phase.

B. **Experimental Considerations** - Although it is possible to measure the polarization parameter, \(P\), by analyzing the polarization of the recoil nucleon by rescattering it, it is more common these days to use polarized targets for this type of experiment. A large number of rather precise measurements of \(P\) have been made in recent years for both \(\pi^+p\) and \(\pi^-p\) scattering in the energy region between about 200 MeV and 12 GeV. The results of the experiments below about 2 GeV when combined with the wealth of elastic and charge-exchange differential-cross-section data have allowed various groups\(^3\)-\(^8\) to make meaningful phase-shift analyses of pion-nucleon scattering. The results of these analyses indicate that the structure of many pion-nucleon resonances is much more complex than had been thought previously. Several of these resonances, instead being a single resonant state, actually consist of 2 or 3 or even 4 different resonances, all with about the same resonant energy. We shall return to this point later. The higher energy experiments, on the other hand, have indicated significant spin dependent effects even at the highest energy (12 GeV) so far measured, and this fact has caused much speculation among devotees of Regge Polology and other high energy models.
Especially noteworthy in this regard are the beautiful $\pi^- p \to \pi^0 n$ polarization measurements made at CERN. We will defer the discussion of these results until the second part of this lecture.

These experiments have several common features. They all involve use of a polarized target. They all use rather complex arrays of detectors to identify the events in which a pion is elastically scattered from a free proton. Many of them use similar data reduction techniques. Let us briefly summarize the most relevant aspects of these techniques.

1. **Polarized Target:** Until now all these experiments have used "impure" targets, i.e. targets containing only a very small proportion of hydrogen. In fact, so far the target material has been a substance called LMN ($\text{La}_2\text{Mg}_3(\text{NO}_3)_{12} \cdot 24\text{H}_2\text{O}$) in which only $3\%$ of the weight of the target is due to free protons. These protons are polarized by the so-called "Dynamic Method", which involves use of high uniform magnetic fields ($\sim 20$ kG), low temperatures ($\sim 1$ K), microwaves to induce electronic transitions ($\sim 1$ watt at 70 GHz), and Nuclear Magnetic Resonance techniques to determine the polarization of the target. The important points for the high-energy physicist who uses these targets are that (a) the magnitude of the polarization of the free protons is typically about $60\%$, (b) the sign of the polarization can be reversed easily without reversing the magnetic field by simply shifting the frequency of the microwaves by about $0.2\%$, (c) these targets are typically about $3$ to $7$ cm in length and $1$ to $2$ cm in
diameter, (d) the density of the free hydrogen is about the same as that of pure liquid hydrogen, (e) too much radiation destroys the polarization in these targets, i.e. the polarization is decreased by a factor of ~2 after the target is irradiated by \(10^{12}\) protons/cm\(^2\). As an example we show a schematic drawing of the CERN Polarized Target\(^{12}\) in Fig. 4.

2. Detectors

One of the main problems associated with the use of these targets is how to isolate clearly those scattering events coming from the free protons from the more copious background arising from the interactions of the incident beam with the heavier nuclei in the target. In the case of elastic scattering, where there are two stable particles in the final state, kinematic constraints such as coplanarity and correlations between the angles of the scattered particles are usually sufficient to make a clear distinction between these types of events. The detection scheme used at Berkeley (see Fig. 5.) is typical of many that have been used. Basically it consists of a large number of overlapped scintillation counters above and below the beam to define the directions of the outgoing particles. A coincidence between counters above the beam line and those below the beam line define an event. For elastic events a definite correlation exists between the "up" counters and the "down" counters which define the polar and azimuthal angles. For the inelastic and quasi-elastic processes this correlation is washed out (mainly because of
Fig. 4. Schematic drawing of the polarized target used by the CERN group.12
Fig. 5. Side view of experimental arrangement used at Berkeley to measure the Polarization parameter in elastic $\pi^+p$ scattering.
the Fermi momentum of the target nucleons). For example in Fig. 6 we show how the elastic events stand out of the background when a correlation is made between one of the upper counters and all of the coplanar down counters. By careful choice of counter geometries one can achieve peak to background ratios ranging from 1 to more than 10 depending on the relative cross sections of the elastic and the background events. In principle it is also possible to isolate the elastic events by measuring both the direction and momentum of only one of the particles. This method has been used (e.g. see references 13,14) but the separation so far achieved has been significantly worse than in the coincidence method, due to the fact that there is one less kinematic constraint imposed.

The question naturally arises: Why use LMN when pure hydrogen exists? There is no basic reason why pure hydrogen cannot be polarized; however, there are some practical difficulties. The method usually proposed is called the "brute force" method because it only involves use of very low temperature, say $T \sim 0.01^\circ$, and very high but not necessarily very uniform magnetic fields, say $H \sim 10^5$ gauss to make the Boltzmann factor $\exp \left( \frac{\mu_p H}{kT} \right)$ as large as possible ($\mu_p$ is the magnetic moment of the proton). Then, if pure ortho-hydrogen is used the protons would have a thermal equilibrium polarization $P \sim \tanh \left( 10^{-7} \frac{H}{T} \right) \approx 80\%$.15 ($H$ is in gauss, $T$ is in degrees Kelvin.) All of these conditions (i.e. very low temperature, high magnetic fields, separation of pure orthohydrogen) have
Fig. 6. Coincidence rates between a given counter in the upper array ("up") with each of the counters in the lower array ("down"). The peaks correspond to elastic scatterings from the free protons in the polarized target. The dashed curve shows the normalized coincidence rate when a "dummy" target which contains no free hydrogen is substituted for the LMN crystals.

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been achieved separately, but up to now no one has put them all together to make a pure highly polarized hydrogen target. A few words of warning before you all rush back to your laboratories to try to build such a device--molecular hydrogen at low temperature prefers to exist in the pure parahydrogen form ($J=0$), and any orthohydrogen ($J=1$) converts to parahydrogen at the rate of about 1%/hr with a conversion energy of about $10^{-2}$ e.v. per molecule. The very large amount of heat thus produced causes serious problems in systems which are supposed to maintain very low temperatures. Furthermore there is some question about how long it would take a system at temperatures like 0.01 K to come to thermal equilibrium under a "brute force" technique. The time constant is likely to many days (though probably not in the case of orthohydrogen).

Practical considerations involving relaxation times and difficulty of injecting a sufficient concentration of paramagnetic impurities into pure orthohydrogen have so far stymied efforts to apply the "Dynamic Method" to pure hydrogen. On the other hand the free protons in substances like $\text{C}_2\text{H}_5\text{OH}$, glycerol, and others which contain significantly more hydrogen than LMN have been successfully polarized by the dynamic method, and it seems likely that these targets will supplant the LMN targets in many future experiments. Furthermore, methane ($\text{CH}_4$) appears to be an excellent target material for the "brute force" method.
One more aspect of the experimental method used in these experiments deserves comment. In several of these experiments a small computer on-line has been of very great use. Not only can it digest the information from the large number of counters (often > 100) quickly, but it can also present up-to-date summaries of various interesting sub-samples of the data. In this way one can continuously check the performance of the system both from a technical point of view and with respect to the physics results being obtained.

3. Results

The polarization parameter, $P$, is related to the scattered intensity and the polarization of the target by the equation

$$P(\theta) = \frac{1}{|P_T|} \frac{I^+(\theta)-I^-(\theta)}{I^+(\theta)+I^-(\theta)}$$  \hspace{1cm} (1-6)$$

where $I^+(\theta)$ and $I^-(\theta)$ are the intensities of the pions scattered at an angle $\theta$ from protons which are polarized in the "+" and the "-" directions respectively, and $|P_T|$ is the magnitude of the polarization of the target. A typical result for $P(\theta)$ for $\pi^+p$ scattering is shown in Fig. 7. These measurements cover essentially the complete angular interval and are statistically quite accurate. Measurements of this type exist at many energies at closely spaced intervals for both $\pi^+$ and $\pi^-$ in the energy range $0.2 \leq T_\pi \leq 2$ GeV. (Fig. 8) These measurements have been combined with differential- and total-cross-section results, including those for charge exchange scattering, in extensive programs to determine the $\pi N$ phase shifts and absorption parameters.3-8
Fig. 7. The polarization parameter, $P(\theta)$ vs $\cos \theta^*$ for $\pi^- P$ scattering at 1.352 GeV/c as measured at Berkeley.
MOMENTA (0.6-2.6 GeV/c) AT WHICH POLARISATION EFFECTS HAVE BEEN MEASURED IN \( \pi^- p \) SCATTERING

- **EANDI et al.**
- **B: BAREYRE et al.** (Double scattering)
- **DUKE et al.**
- **SUWA et al.**
- **CHAMBERLAIN et al.**
- **THIS EXPERIMENT**

\[
P_{\pi} \text{ GeV/c}
\]

<table>
<thead>
<tr>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
<th>1.2</th>
<th>1.4</th>
<th>1.6</th>
<th>1.8</th>
<th>2.0</th>
<th>2.2</th>
<th>2.4</th>
<th>2.6</th>
</tr>
</thead>
</table>

Fig. 8. Summary of momenta where measurements of the polarization parameter, \( P(\theta) \) have been made for \( \pi^- p \) elastic scattering. This figure was taken from the report of K. S. Heard, C. R. Cox, J. C. Sleeman, P. J. Duke, R. E. Hill, W. R. Holley, D. P. Jones, J. J. Thresher, F. C. Shoemaker, and J. B. Warren, presented at The Heidelberg International Conference (1967).
C. Phase Shift Analyses:

1. Formalism:

The partial-wave decomposition of the scattering amplitudes can be written

\[ G = \frac{1}{k} \sum_{\ell=0}^{\infty} \left[ (\ell+1)T_{\ell+} + \ell T_{\ell-} \right] P_{\ell}(\cos \theta) \]  \hspace{1cm} (I-7)

\[ H = \frac{1}{k} \sum_{\ell=1}^{\infty} \left( T_{\ell+} - T_{\ell-} \right) P_{\ell-1}(\cos \theta) \]  \hspace{1cm} (I-8)

where \( \ell \pm \) stands for \( j = \ell \pm 1/2 \) and

\[ T_{\ell\pm} = \frac{\eta_{\ell\pm} e^{i \delta_{\ell\pm}} - 1}{2i} \]

are the partial wave amplitudes.

\( \eta_{\ell\pm} \) is the absorption parameter (\( \eta_{\ell} = 1 \) corresponds to no absorption, \( \eta_{\ell} = 0 \) corresponds to complete absorption)

\( \delta_{\ell\pm} \) is the phase shift for the state \( j = \ell \pm 1/2 \)

The partial wave amplitudes \( T_{\ell\pm} \) are conveniently represented in graphical form by an Argand diagram (see Fig. 9)

Fig. 9.
A simple Breit-Wigner resonance can be written

\[ T_e = \frac{x}{\epsilon - 1} \]  

(\text{I-9})

with \( x = \frac{\Gamma_{\text{elastic}}}{\Gamma_{\text{total}}} \) (the elasticity of the resonance which is not the same as absorption parameter \( \eta \))

and

\[ \epsilon = \frac{E_{\text{Res}} - E}{\Gamma/2} \]

(\text{I-10})

\[ E_{\text{Res}} = \text{the Resonance Energy} \]

\[ E = \text{the energy} \]

\[ \Gamma = \text{the total width of the resonance} \]

In this representation such a resonant amplitude would describe a circle moving counterclockwise as the energy \( E \) increases. When \( E = E_{\text{Res}} \) the resonant amplitude is pure imaginary. Thus, if there is no background, a resonant amplitude will have \( \phi = 0^\circ \) or \( 90^\circ \) depending on whether \( x < 1/2 \) or \( x > 1/2 \) (see Fig. 10)

\[ \text{Re } T_e = \frac{xc}{\epsilon^2 + 1} \]  

(\text{I-11})

\[ \text{Im } T_e = \frac{x}{\epsilon^2 + 1} \]

\[ T_e = |T_e|e^{i\phi} \]  

(\text{I-12})

\[ |T_e| = \frac{x}{\sqrt{\epsilon^2 + 1}}, \quad \tan\phi = \frac{1}{\epsilon} \]
Fig. 10. The elastic-scattering amplitude $T_e$ in the complex plane
(a) For pure elastic scattering ($\eta = 1$), $T_e$ lies on the unitary circle. If the amplitude is resonant, the circle represents a resonance with elasticity $x = 1$. (b) Resonant amplitude for $x = 0.5$. (c) Resonant amplitude for $x < 0.5$. Notice that at resonance $\delta = 0^\circ$. 

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Often a resonance amplitude is superposed on some background. In that case the resonant circle will not originate at the origin but somewhere else within the unitary circle. Other factors such as for example energy-dependent widths will further distort the picture of a smooth circle.

2. Method

A few words about how the phase shifts are actually determined. We have seen that the various experimental measurements can be directly related to the scattering amplitudes $G$ and $H$. These in turn can be expressed in terms of phase shifts. Thus it is possible to write the experimental observables in terms of phase shifts. Of course in principle there are an infinite number of partial waves involved and therefore an infinite number of phases. The usual approximation is to terminate the phase shift expansion at some $\ell = \ell_{\text{max}}$. Typically $\ell_{\text{max}}$ is 4. The procedure is to calculate the observables in terms of the phase shifts, and to compare these calculated values to the experimentally observed ones. A computer is used to minimize the quantity

$$\chi^2 = \sum_{\text{all observables}} \left| \frac{Q_j^{\text{exp}} - Q_j^{\text{calc}}}{\sigma_j^{\text{exp}}} \right|^2$$

(I-13)

where $Q_j$ is the value of the $j^{\text{th}}$ observable and $\sigma_j$ is the error associated with the measurement of the $j^{\text{th}}$ observable.
In the analysis of $\pi N$ scattering undertaken by our group at Berkeley we had to minimize $\chi^2$ in a 41 dimensional phase shift space. The 41 parameters break down as follows:

$$(2 \text{ Isotopic Spin States}) \times (9 \text{ Angular Momentum States}) \times (\text{An } \eta \text{ and a } 8 \text{ for each state}) = 36$$

In addition we used a normalization parameter for each of the 5 types of experiments used in the analyses i.e. $\frac{d\sigma}{dn}(\pi^+ p \rightarrow \pi^+ p)$, $\frac{d\sigma}{dn}(\pi^- p \rightarrow \pi^- p)$, $\frac{d\sigma}{dn}(\pi^0 p \rightarrow \pi^0 n)$, $P(\pi^+ p \rightarrow \pi^+ p)$ and $P(\pi^- p \rightarrow \pi^- p)$.

This can be difficult even for a large modern high speed computer.

3. Results

Most of the groups involved in the phase shift business use slightly different methods to obtain their results. These differences concern mainly the extent to which assumptions about variation of phase shifts with energy are put into the analysis a priori. The main features of the various phase shift analyses are summarized in Table I.

Lovelace in his report at the Heidelberg Conference summarized some of the main conclusions with regard to possible resonant states of the $\pi N$ system below 2 GeV. These results are based primarily on the very detailed analysis made by the CERN group. The resonance parameters, which I copied from the blackboard during the talk of Lovelace, should not be considered final in the sense that some of the numbers will undoubtedly change.
Table I. Summary of main features of various phase shift analyses.

<table>
<thead>
<tr>
<th>Group</th>
<th>Method</th>
<th>Input Assumptions</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Livermore</td>
<td>Energy dependent</td>
<td>$\delta_n = k^{2f+1} \sum_{n=0}^{n_{\text{max}}} A_n k^n + \text{Breit-Wigner Resonances}$</td>
<td>First found $P_{11}(1400)$ &quot;Roper&quot; resonance. Assumptions about energy dependence of phases tends to bias against finding new resonances. Results published.</td>
</tr>
<tr>
<td>Saclay</td>
<td>Energy independent</td>
<td>none</td>
<td>Up to 1.6 GeV. Many solutions at each energy. Energy Continuation made by making smooth connection between phase shifts at different energies. First found complex resonant structure in regions of (1512) and (1688) resonances. Work completed.</td>
</tr>
<tr>
<td>Hawaii</td>
<td>Energy independent</td>
<td>Assumed no resonances</td>
<td>Wanted to see if existing data could be satisfactorily fit with non-resonant amplitudes. Found solution which is in reasonable agreement with experimental observations. However, results disagree with spin-flip dispersion relations. Work completed.</td>
</tr>
</tbody>
</table>

(Continued on next page)
Table I. Summary of main features of various phase shift analyses. (Con't)

<table>
<thead>
<tr>
<th>Group</th>
<th>Method</th>
<th>Input Assumptions</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>CERN$^7$</td>
<td>Essentially energy independent analysis but with dispersion relation input.</td>
<td>Energy Continuation made with help of dispersion relations for partial wave amplitudes.</td>
<td>Most sophisticated analysis. Up to 2 GeV. They check self-consistency of dispersion relation input. Results could be slightly biased because solutions are forced to be in accord with dispersion relation input. Have found 18 resonant states. See Table II. Results published.</td>
</tr>
</tbody>
</table>
slightly and some of the resonant states may even go away or new ones appear. The point is that there are many resonances—in fact Lovelace claims to see 18—which were discovered through the phase shift analysis method. These results are summarized in Table II. Some of the Argand diagrams on which these conclusions are based are shown in Figs. 11 to 13. Everyone of the amplitudes shown resonates at least once. The low partial wave amplitudes, especially, show rather curious behaviors and more detailed experimental information is needed before the behavior of the $S_{31}$, $S_{11}$ and $P_{11}$ amplitudes can be considered to be reliably established. The analyses which determined these quantum numbers are based in large measure on the detailed polarization measurements described above. The uniqueness of these solutions is not yet completely established, and it would be very desirable to obtain information on the polarization parameter in charge exchange scattering, as well as on the $A_{\text{recoil}}$ and $R_{\text{recoil}}$ parameters in elastic $\pi^+ p$ and $\pi^- p$ scattering in order to further clarify the situation. Of these, the measurement of $P$ in $\pi^- p \rightarrow \pi^0 n$ seems to be the experiment of greatest interest.

At momenta above about 2 GeV/c phase shift analyses become cumbersome because of the very large number of partial waves which must be considered. Nevertheless polarization and cross section measurements in this energy region have been used to help establish
Table II. Baryon States. Taken from talk of Lovelace at the Heidelberg Conference, September 1967.

<table>
<thead>
<tr>
<th>Wave</th>
<th>Mass</th>
<th>$\Gamma_{\text{tot}}$</th>
<th>$\Gamma_e/\Gamma_{\text{total}}$</th>
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<td>1235.8</td>
<td>123.7</td>
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<tr>
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<td>$F_{37}$</td>
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<td>.387</td>
</tr>
<tr>
<td>$G_{17}$</td>
<td>2250</td>
<td>300</td>
<td>.3</td>
</tr>
<tr>
<td>$H_{3,11}$</td>
<td>2423</td>
<td>275</td>
<td>~.1</td>
</tr>
</tbody>
</table>

Probable Resonances

<table>
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<tr>
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<th>Mass</th>
<th>$\Gamma_{\text{tot}}$</th>
<th>$\Gamma_e/\Gamma_{\text{total}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{11}$</td>
<td>1535</td>
<td>155</td>
<td>.28</td>
</tr>
<tr>
<td>$D_{13}$</td>
<td>1872</td>
<td>163</td>
<td>.16</td>
</tr>
<tr>
<td>$F_{35}$</td>
<td>1915</td>
<td>324</td>
<td>.175</td>
</tr>
<tr>
<td>$P_{31}$</td>
<td>~2025</td>
<td>~330</td>
<td>~ .3</td>
</tr>
<tr>
<td>$P_{33}$</td>
<td>2140</td>
<td>~330</td>
<td>~ .25</td>
</tr>
</tbody>
</table>

(Continued on next page)
Table II. Baryon States. Taken from talk of Lovelace at the Heidelberg Conference, September 1967. (con't)

<table>
<thead>
<tr>
<th>Wave</th>
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<th>$\Gamma_{tot}$</th>
<th>$\Gamma_e/\Gamma_{total}$</th>
</tr>
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<tr>
<td>$P_{11}$</td>
<td>1920</td>
<td>320</td>
<td>~.18</td>
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<tr>
<td>$F_{17}$</td>
<td>2030</td>
<td>270</td>
<td>~.15</td>
</tr>
<tr>
<td>$H_{19}$</td>
<td>2300</td>
<td>?</td>
<td>?</td>
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</tbody>
</table>

Resonance Interpretation in Doubt

<table>
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<tr>
<th>Wave</th>
<th>Mass</th>
<th>$\Gamma_e/\Gamma_{total}$</th>
</tr>
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<tbody>
<tr>
<td>$D_{33}$</td>
<td>1716</td>
<td>~.17</td>
</tr>
<tr>
<td>$D_{35}$</td>
<td>~2026</td>
<td>~.2</td>
</tr>
<tr>
<td>$P_{13}$</td>
<td>~2100</td>
<td>~.28</td>
</tr>
</tbody>
</table>
Fig. 11. Argand diagram for the $S_{11}$ amplitude found by the Berkeley group. 8
Fig. 12. Argand diagram for the P11 found by the Berkeley group.
Fig. 13. Argand diagram for the $F_{37}$ amplitude found by the Berkeley group.\cite{8}
the quantum numbers of the $\Delta(1920)$ as $F_{37}^{17}$ and the $N^*(2190)$ as $G_{17}^{18}$. The conclusions in each case are based on a detailed examination of the coefficients of the Legendre expansions for $I_0$ and $I_0^P$, i.e.

\begin{align}
I_0 &= \sum_{\ell=0}^{\ell_{\text{max}}} A_{\ell} P_{\ell}(\cos \theta) \quad (I-14) \\
I_0^P &= \sum_{\ell=1}^{\ell_{\text{max}}} B_{\ell} P_{\ell}^1(\cos \theta) \quad (I-15)
\end{align}

In this part of the lecture I have tried to show how detailed measurements of spin-dependent effects in low and medium energy pion-nucleon scattering have contributed to our understanding of these processes. In the next part we will examine how various polarization measurements provide interesting tests of high-energy theories.
II. Spin-Dependent Effects in \( \pi N \) and NN Scattering at High Energies*

It has been tacitly assumed by most physicists that the relative importance of spin-dependent amplitudes decreases as \( E \to \infty \). This expectation is based partly on intuition, partly on wishful thinking, and partly in their belief in various theoretical models. For example, if high energy elastic scattering is due to the diffraction of the incident wave by a strongly absorptive target (black disk) no spin-dependent effects are expected.\(^{19} \) There are other models--primarily Regge Pole models--which make definite predictions about the dependence on energy of various polarization effects. Many of these effects are expected to vanish at high energy, but often for reasons other than the fact that there are no spin-dependent amplitudes. I am referring here to constraints imposed by phase conditions and by factorization. In this part of the lecture we shall discuss how polarization experiments at high energy can be used to test the predictions of these theoretical models.

Before proceeding further we must specify what we mean by spin-dependent amplitudes. As Phillips\(^{19,20} \) points out the definition is somewhat ambiguous. It depends on the representation chosen to specify the scattering matrix, \( M \).

We can write

\[
M = G + i H \hat{\sigma} \cdot n
\]  

or equivalently

\[
M = f_1 + f_2 \hat{\sigma} \cdot \hat{k} \cdot \hat{s} \cdot \hat{k}
\]

where

\[
G = f_1 + f_2 \cos \theta
\]

\[
H = f_2 \sin \theta
\]

*Excellent review articles dealing with this subject may be found in references 19, 20, and 27.
It is also possible to express $M$ in terms of helicity amplitudes. In this case the spin of each particle is quantized along its directions of motion, instead of being quantized along some fixed axis in the center-of-mass system. The amplitudes $f_{++}$ and $f_{-+}$ are the helicity non-flip and the helicity flip amplitudes appropriate to pion-nucleon scattering.

$$f_{++} = (f_1 + f_2) \cos \theta / 2, \quad f_{-+} = -(f_1 - f_2) \sin \theta / 2 \quad (II-2)$$

A third representation for $M$ involves use of Dirac Spinors i.e.,

$$M = -A + iB \gamma_\mu \sigma^\mu, \quad \sigma^\mu = k_{\mu} i \mu$$

again it is possible to relate the non-flip amplitude, $A$, and the flip amplitude $B$ to the amplitudes in terms of the other representations.

The point is that usually spin-independence is associated with the vanishing of $H$, $f_{-+}$, or $B$. These definitions are not equivalent, except at $t=0$ where they vanish in any case. What we want to investigate now are the theoretical predictions and the experimental consequences for the behavior of these amplitudes at high energies.

Rarita, et al. have made predictions for $A_{\text{recoil}}$ and $R_{\text{recoil}}$ at 20 GeV/c for $\pi^-p$ scattering as a function of $t$ using a Regge Pole model which fits well existing data, but with the further assumption that the various types of spin-flip amplitudes vanish. These predictions are shown in Fig. 14.

A thorough discussion of the Theory of Regge Poles will be presented
Fig. 14. $A_{\text{recoil}}$ and $R_{\text{recoil}}$ for some simple models at 20BeV/c: (a) $B=0$ (solid line,) (b) $H=0$ (dash-dot line, - - - - - - ) and (c) $f_{+}\neq 0$ (dashed line,-------). For $A_{\text{recoil}}$, (a) and (c) effectively coincide.
in other lectures. Here we shall be concerned with only those aspects which bear directly on questions involving spin-dependent effects.

The Regge Poles which can be exchanged in πP elastic scattering are the P (Pomeranchuk or Vacuum Pole), the P' and the ρ. There are two essential comments to be made regarding these poles: (1) Each one has a spin-flip and a non-spin-flip part (for definiteness let us use helicity flip, f⁺⁻, and helicity non-flip, f⁺⁺); (2) The phases of both the flip and non-flip parts of a given pole are the same; in fact, the signature factor

\[ f_1(t) = \frac{1 + e^{-i\pi \alpha(t)}}{\sin \pi \alpha(t)}, \]

completely specifies this phase for the ith pole. Thus, in the case where one pole is dominant the polarization parameter, P, which is proportional to \( \text{Im}(f⁺⁺f⁺⁻) \) will be zero.

Let us consider the charge exchange process \( \pi^- p \rightarrow \pi^0 n \). The only simple Regge Pole which can be exchanged in this reaction is the ρ. The differential cross section, \( d\sigma/dt \), as a function of t, shows a dip near \( t=0 \), and shows a minimum at \( t \approx -0.6 (\text{GeV}/c)^2 \). See Fig. 15. This behavior together with the small difference in the total \( \pi^+ p \) and \( \pi^- p \) cross sections, is well explained by Reggeized ρ exchange if one assumes a large spin-flip amplitude, ρ⁺⁻. Using this simple model one would predict that the polarization parameter in charge-exchange scattering at high energies should vanish. In fact it does not (see Fig. 16). The ρ pole must be interfering with something else in order to produce this polarization. It is not clear at present what
Fig. 15. $\pi^- + p \rightarrow \pi^0 + n$ differential cross sections at 5.9, 9.8, 13.3, and 18.2 BeV/c. Curves are predictions of Regge Pole fit of Rarita et al., UCRL-17523.
Fig. 16. Preliminary results of the Saclay-Orsay-Pisa collaboration\textsuperscript{9} to measure Polarization in $\pi^- p \rightarrow \pi^- n$ elastic scattering at 5.9 and 11.2 GeV/c.
this "something else" is—it may be a Regge cut, it may be secondary trajectories, it may be resonance tails—further experiments at higher energies are needed to clarify this situation.

What about polarization in π⁺p elastic scattering? The non-flip and flip amplitudes appear in the combinations

\[ f_{π⁺p}^{π⁺p} = \mathcal{P}_{++} + \mathcal{P}_{++}' + \rho_{++} \]  

\[ f_{π⁻p}^{π⁺p} = \mathcal{P}_{--} + \mathcal{P}_{--}' + \rho_{--} \]  

(II-4)  

(II-5)

Experimental measurements at 6, 8, 10 and 12 GeV indicate that these polarizations are positive for π⁺p and negative for π⁻p in momentum transfer region |t| ≤ 1 (GeV/c)² (Fig. 17). These facts suggest that perhaps the amplitude \( \rho_{--} \) (which is known to be large from the charge exchange analysis) is interfering with \( \mathcal{P}_{++} + \mathcal{P}_{++}' \); i.e.

\[ P \propto \text{Im} \ (\mathcal{P}_{++} + \mathcal{P}_{++}')^* \]

If this were a true picture of what was happening we would expect to find equal and opposite polarization in π⁺p and π⁻p scattering. In fact although the signs are indeed opposite, the magnitudes are not equal. Here again the most simple-minded model is not completely satisfactory and modifications are needed to bring the phenomenology into accord with the experimental facts.

We have seen that the fact that the phases of spin-flip and non-flip amplitudes for a single Regge Pole are the same makes it very difficult
Fig. 17. Results of Polarization measurements in \( p^+ p \) scattering at 6, 8, 10, and 12 GeV/c. Solid lines are fits obtained by Rarita et al.\textsuperscript{20}
to learn anything about the high-energy behavior of these amplitudes from measurements of the polarization parameter. In this respect measurements such as $A_{\text{recoil}}$ or $R_{\text{recoil}}$ (or some combinations of these) would be very valuable since they relate directly to $\text{Re} f^*_{++}$ and to $|f_{++}|^2 - |f_{--}|^2$. The first of such experiments is presently underway at CERN by a group from Saclay who is using a polarized target made with superconducting coils to measure polarization components in the plane of the scattering. Measurements of the polarization parameter, $P$, at high energies will be useful in clarifying the questions relating to interference between various poles (or between poles and cuts).

There is another aspect of Regge Pole theory which relates directly to the study of spin-dependent effects at high energy; namely, factorization. Consider the following diagram

![Diagram](image)

The contribution to the scattering amplitude from the $i^{th}$ Regge Pole can be written

$$T_{12-12} = \frac{g_0}{8\sqrt{g}} \eta_1(t) \eta_2(t) \xi(t) \left| \frac{g_0}{g_0} \right| \alpha(t)$$

(II-6)
where $\eta_1(t)$ and $\eta_2(t)$ are vertex functions characterizing the coupling of the Regge Pole to particles 1 and 2. When there is spin they are spin operators. The factorization property refers to the fact that the scattering amplitude can be written as shown with $\eta_1(t)$ and $\eta_2(t)$ appearing separately; i.e. Vertices 1 and 2 are uncorrelated.

The point to be made here is that since the spin-dependence of these amplitudes comes only from the vertex functions the spin dependence is factorizable. This has the consequence that a given Regge pole couples to a nucleon in exactly the same way independent of whether it describes $\pi\Lambda$, $K\Lambda$, $p\Lambda$ or $\bar{p}\Lambda$ scattering. This spin-dependent coupling depends only on the 4-momentum transfer, $t$. When more than one pole is exchanged the simple factorization property is no longer true for the amplitude as a whole, although it is still valid for each pole separately. Let us examine some the experimental consequences:

Consider the general case of elastic scattering of two particles; e.g. $NN$, $\pi N$, $KN$, $\bar{N}N$. The following types of experiments are of interest:

- **Case (a):** No polarization initially, no polarization measured finally. Measurement $\frac{d\sigma}{dt}$, $\sigma_{\text{tot}}$.

- **Case (b):** One polarization either initially or finally. Measurement $P$

- **Case (c):** Particle 1 Polarized in $j$ direction, before scattering. Polarization of particle 1 measured
in k direction after scattering.  
Described by depolarization tensor, \( D_{kj} \)  
Measurements: \( D, R, A, \) etc.

(d) Particle 1 polarized in \( j \) direction before scattering. Polarization of particle 2 measured in \( k \) direction after scattering. Described by Polarization Transfer tensor, \( K_{kj} \)  
Measurements: \( D_T \) (or \( K_{NN} \)), etc.

(e) Particle 1 polarization in \( j \) direction and Particle 2 in \( k \) direction, initially (or finally). Described by Polarization correlation tensor, \( C_{kj} \)  
Measurements: \( C_{NN}, C_{KP} \), etc.

(f) Higher correlations, involving more than two polarizations.

Clearly in the case of \( \pi N \) and \( KN \) scattering only experiments (a), (b), and (c) are possible (the meson has no spin).

Factorization makes the following predictions (assuming one pole exchange only):

1. \( P_{\pi N} = P_{KN} = P_{NN} \) (\( = 0 \) because of phase rule)
2. \( D_{kj}(\pi N) = D_{kj}(KN) = D_{kj}(NN) \). In particular  
   \( D(\pi N) = D(KN) = D(NN) = 1 \)  
   \( A_{\text{recoil}}(\pi N) = A_{\text{recoil}}(KN) = A_{\text{recoil}}(NN) \)  
   and \( R_{\text{recoil}}(\pi N) = R_{\text{recoil}}(KN) = R_{\text{recoil}}(NN) \)
It will be interesting to see to what extent these predictions hold at energies accessible with the Serpukhov proton synchrotron.

One of the problems challenging the experimentalists at Serpukhov is how to produce the highly polarized beams of high energy protons needed to do some of these experiments. There is unfortunately no foolproof way. The following obvious possibilities exist:

(1) Install a source of polarized protons and accelerate these. Clearly this would involve major modifications of the existing injector. Furthermore there are likely to be sizeable depolarization effects caused by the oscillating transverse components of magnetic field as seen by the proton in its rest frame during the acceleration process. More detailed calculations are needed before the feasibility of this scheme is established.

(2) Produce polarized protons by scattering high energy protons from hydrogen (or other materials). Here again the experimental outlook is dim. Figure 18 shows the maximum polarization achieved in pp scattering as a function of energy. Very small polarizations are likely at 70 GeV. As mentioned above, the theoretical expectations are in accord with these results.
Fig. 18. Maximum Polarization observed in pp scattering as a function of proton lab energy.
(3) Hyperon decays--Because of parity violation in the decay of hyperons, the nucleons arising from the decay of hyperons are often strongly polarized. It may be possible to use this fact to produce low intensity beams of polarized nucleons. However, the experimental problems are difficult.

(4) Backward Scattering of π (or K) mesons from protons in a polarized target. In an earlier part of this lecture we have seen that the so-called D parameter of Wolfenstein must be equal to unity in πN scattering. Experimentally this means that the polarization of the nucleon before and after the scattering must be the same. Thus if a highly polarized nucleon could be knocked out of a polarized target its polarization component along the normal to the scattering plane will be unchanged. High energy polarized protons could be produced by πN elastic scatterings involving large momentum transfers. Unfortunately the cross sections are small so that the expected fluxes will be low.

(5) Charge exchange Scattering np → pn from protons in a polarized target. N. Byers has suggested that there may be appreciable polarization in high energy np charge exchange scattering, and that if this is true then this process could be used to produce high-energy polarized proton beams. At present there is no experimental evidence pro or con so that the feasibility of this method is not yet established.
There is one last remark to be made, and admittedly it falls into the category of wild speculation. It would be interesting to test parity and time-reversal invariance symmetries in processes involving very high momentum transfers at very high energies. These processes really probe the innermost structure of these interactions and the symmetry violations associated with weak interactions may manifest themselves in some of these processes. For example, it would be relatively straightforward to scatter high energy \( \pi \)-mesons from polarized protons at large angles, and to look for possible asymmetries in the plane of the scattering. To test time-reversal invariance at high energy one could for example compare the analyzing and polarizing power in pp scattering. These quantities can only be different if T is violated.

The study of spin-dependent effects at high energy offers many experimental and theoretical problems; hopefully the Serpukhov proton synchrotron will allow us to gain a better understanding of some of these phenomena.
REFERENCES

2. L. Van Rossum, private communication.


25. N. Ryers, private communication.


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