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Essays on Liquidity, Monopolistic Competition, and Search Frictions

DISSERTATION

submitted in partial satisfaction of the requirements  
for the degree of

DOCTOR OF PHILOSOPHY

in Economics

by

Mario Rafael Silva

Dissertation Committee:
Professor Guillaume Rocheteau, Chair  
Professor William Branch  
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2017
DEDICATION

To my parents, who have always been extremely patient and encouraging, helped ignite my passion in economics, and inspired me with their perseverance.
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I study the interactions between liquidity constraints, monopolistic competition, and search frictions for product markets, labor markets, and credit markets. Monopolistic competition is especially important for three different reasons. First, there is an externality that links the demand of firms to the state of the economy. Second, under free entry, the product space is influenced by policy and interacts with liquidity constraints. Third, monopolistic competition generates markups, which can augment other wedges and thereby interact with liquidity constraints.

The first chapter considers the role played by endogenous variety and monopolistic competition in the long-run transmission of monetary policy. The combination of free entry and product variety gives rise to both an intensive margin (quantity of particular good) and extensive margin (extent of variety), and search frictions imply that firm entry involves a congestion externality. Inflation generally reduces variety. Under constant-elasticity-of-substitution (CES) preferences, firms are inefficiently small, with the inefficiency increasing in product differentiation and the extent of search frictions. The Friedman rule, which involves contracting the money supply at the rate of time preference, is the best policy under CES preferences. In contrast, with variable elasticity of demand, inflation can increase firm size, reduce markups, and raise welfare, even though output is lower. Under CES preferences,
the welfare cost of inflation is high; moreover, it increases monotonically with the markup and is higher with endogenous variety than with a fixed variety alternative.

The second chapter departs from the dramatic growth of revolving credit since 1970 relative to both consumption and consumer credit. Importantly, revolving credit primarily determines short-run household liquidity and comoves positively with product variety. I augment the Mortensen-Pissarides model with an endogenous borrowing constraints and free entry of monopolistically competitive firms. Unemployment is amplified from a two-way feedback: higher debt limits encourage firm entry and raise product variety (the entry channel), and greater variety makes default more costly and thereby raises the equilibrium debt level (the consumption value channel). I compare the model to a counterfactual economy in which either channel is shut down and find that mean amplification exceeds 50%. Furthermore, only the model economy generates a procyclical response of the credit-to-consumption ratio, as observed in the data.

The third chapter examines the role of corporate finance and imperfect competition in the pass through of monetary policy to the real lending rate and its transmission into investment. Monopolistically competitive entrepreneurs can finance investment opportunities using bank-issued credit or money. They seek loans in an over-the-counter market where the terms of the contract (loan size, interest rate, and down payment) are negotiated subject to pledgeability constraints. I investigate pass through of the policy rate to the real lending rate and its transmission to output and investment, taking into account the interplay of (1) heterogeneous financial frictions from limited enforcement and (2) aggregate demand externalities from monopolistic competition. Whereas returns to scale or product diversity are not important for the pass through, the former substantially affect the transmission of policy to investment and output. Furthermore, financial frictions interact positively with demand complementarities from monopolistic competition. Greater dispersion of financial frictions reduces investment and output and also increases transmission unevenly across the range of nominal policy rates,
having a maximal effect at about a policy rate of 9%.
Chapter 1

Introduction

My research examines the general equilibrium interplay between liquidity constraints, monopolistic competition, and search frictions for labor and monetary macroeconomics. It can be distilled into four themes. The first concerns the link between payment capacity and aggregate demand in the economy. The second theme is positive mutual feedback between debt limits and product variety, and is the subject of the second chapter. The *entry channel* links higher debt limits through firm entry via stimulation of product demand; the *consumption value channel* associates firm entry to more debt limits by enlarging the product space, increasing the value of consumption and the incentive to repay loans, and thereby increasing debt limits. The third theme is the recurring use of monopolistic competition, which rationalizes imperfect substitutability of goods and markups, price setting by firms, provides a natural treatment of product variety, and gives rise to aggregate demand externalities. Fourth, I study two mechanisms of monetary policy. The first channel associates higher inflation to lower markups by reducing consumption and taste for variety. This effect can reduce firm congestion in the economy and improve welfare. The second channel concerns pass through from the nominal interest rate to the real interest rate. Lower nominal interest rates lower entrepreneurs cost of holding liquid assets. By holding more liquid assets,
entrepreneurs can make larger down payments and obtain lower interest rates in loans from banks.

The first chapter shows that taste for variety and monopolistic competition shape the long-run transmission of monetary policy. Taste for variety give rise to markups, which create a wedge on the intensive margin. Inflation amplifies the wedge on the intensive margin and also reduces the amount of firms, generating inefficiency on the extensive margin as well. In general, the welfare costs of inflation are high and are increasing with firm entry and the price markup. I derive a formula for optimal firm size, which depends on technology, taste for variety, search frictions, and preference shocks in the goods market. I also show that the Friedman rule–deflation at the rate of time preference–is generically efficient with preferences which exhibit constant elasticity of substitution (CES); however, under additively separable preferences, inflation can reduce sellers market power and increase firm size, generating deviations from the Friedman rule and reducing the welfare costs of inflation. Calibrating parameters based on targeting money demand and examining 30% markups, the welfare cost of inflation is 5.7% without firm entry and 7.97% with firm entry. Generally, the welfare costs of inflation are high (comparable to bargaining solutions in the goods market), are rise with firm entry (at various markups), and increase in the price markup.

The second chapter examines the business cycle comovement of unemployment, unsecured credit, and product variety. A major motivation is the fact that the Great Recession exhibited a large and persistent surge in unemployment, a sharp drop in unsecured credit, and a decrease in the aggregate number of firms, which proxy for product diversity. In order to study the labor market implications of endogenous household debt and consumption variety, I augment the Mortensen-Pissarides model with aggregate demand externalities arising from endogenous borrowing constraints (via limited commitment) and free entry of monopolistically competitive firms offering specialized goods. The model explains the strong negative correlation between unemployment rate and credit and the positive relationship be-
between credit and product variety. The key feature is mutual feedback between debt limits and product variety: greater debt limits encourage firm entry and raise product variety (the entry channel), and more varieties increase the opportunity cost of default and thus raise the equilibrium debt level (the consumption value channel). The mechanism amplifies credit and productivity shocks, increasing the volatility of unemployment, debt-to-consumption, and product variety. I showcase amplification in two experiments: the first compares two economies with diverse levels of product diversification, one which generates 5% markups and one which generates 40% markups, and the second compares a free entry economy to one with a fixed mass of sellers. I find that under both experiments, unemployment responds about 50% more strongly on impact. Debt-to-consumption is largely acyclical under low markups or a fixed product space but is procyclical under the two experiments. Given that labor search models exhibit highly nonlinear dynamics, and since the model has occasionally binding constraints, I compute the rational expectations equilibria using the parameterized expectations algorithm.

In the third chapter, I develop a general equilibrium model of firms’ investment and financing options to understand the channels through which the policy rate of central banks affects the use of internal finance, lending, output, and investment in the presence of monopolistic competition and associated aggregate demand externalities.

Entrepreneurs meet banks in an over-the-counter market and negotiate terms (loan size, interest rate, and down payment) subject to limited enforcement constraints, so that only a fraction of investment return is pledgeable. Search frictions and bilateral negotiation helps account for the intensive margin of bank credit (loan size) and the extensive margin (number of loans). I consider heterogeneity of financial frictions across entrepreneurs, and investigates how it interacts with the aggregate demand externality from monopolistic competition to affect the pass through of the policy rate to the real lending rate and its transmission to output and investment. Heterogeneity enables us to capture the cross section of transmission, with
smaller (more constrained firms) curtail investment more sharply under high policy rates. It also removes artificial kinks and discontinuities by allowing the proportion of constrained firms to vary continuously with policy, and allows us to analyze mean-preserving spreads of financial frictions.

The model is calibrated to match the three-month Treasury bill rate, its spread with the prime lending rate, the loan acceptance ratio of small businesses, and the semi-elasticity of money demand. Whereas neither returns to scale or product diversity have a major effect on the pass through, returns to scale strongly affects the transmission of policy to investment and output, and interacts positively with the extent of financial frictions. A mean-preserving spread of pledgeability coefficients reduces investment and output and strengthens transmission over a range of nominal interest rates. Pass through is largely unaffected.
Chapter 2

New Monetarism with Endogenous Product Variety and Monopolistic Competition

2.1 Introduction

The Lagos-Wright environment provides a tractable analysis of monetary equilibria across a wide variety of market structures. In doing so, it facilitates the integration of models that explicitly describe the frictions which make money useful and the rest of macroeconomics. I argue that introducing endogenous variety and monopolistic competition matter for the long-run transmission of monetary policy. First, taste for variety determines the substitutability of goods, which in turn pins down the markup. A higher markup reduces buyers’ gains from trade and hence the marginal benefit of holding money. Consumers thereby underinvest in money holdings, which magnifies the welfare costs of inflation. Second, inflation affects firms’ incentives to enter the market, which in turn impacts the size of the product space. I label
these respective channels the rent-sharing effect and entry effect.

Endogenizing variety via monopolistic competition enables me to study how inflation impacts the amount of a good exchanged (the intensive margin); the size of the product space (the extensive margin); and, depending on preferences, markups. I refer to the last effect as the markup channel. Syverson (2007) provides evidence that markups are lower in larger markets, which motivates the study of how inflation can affect markups.

There are three main contributions. First, I derive optimal measures of firm size, which generally depend on production costs, taste for variety, search frictions, and preference shocks in the goods market. With constant-elasticity-of-substitution (CES) preferences, firms are inefficiently small. Intuitively, firm entry creates congestion on existing firms and reduces equilibrium production. From hereon out, I use “size” interchangeably with production quantity.

Second, under variable elasticity of demand the Friedman rule—which is optimal under CES preferences—can be suboptimal. This happens because with variable elasticity of demand, inflation can reduce buyers’ desire to diversify their consumption basket, thereby diminishing sellers’ market power and increasing their production. Lower markups increase buyers’ surplus, which mitigates the cost of higher inflation. Provided that the reduction in average cost is large enough, welfare increases. Firms can either be smaller or larger than the optimal size.¹

Third, I show that love for variety and the extensive margin matter for the welfare cost of inflation. At 30% markups under CES preferences, the welfare cost of 10% inflation is 5.71% of output without firm entry and 7.97% with entry. With no taste for variety, these figures are 1.23% without entry and 1.48% with entry. This study thus obtains welfare costs of

¹However, if optimal firm size is smaller than equilibrium firm size, then the number of firms (and varieties) at the optimum exceeds the equilibrium level.
inflation comparable to those that arise under bargaining. Nevertheless, some important differences with bargaining emerge. In contrast to the solutions of Rocheteau and Wright (2009) and Dong (2010), which endogenize the frequency of trade, the welfare cost of inflation rises monotonically with price markups. This monotonicity arises from the fact that higher markups imply a more important entry effect.

This paper draws on the extensive literature on monopolistic competition and the New Monetarist framework. Dixit and Stiglitz (1977) formalized monopolistic competition with product variety in a tractable way. Monopolistic competition is used even more broadly and has realistic features: firms set prices and goods are imperfect substitutes. It also provides a natural setting for studying economies of scale, and hence for exploring the tradeoff between increasing variety and lowering average production cost.

New Monetarist theory, in turn, emphasizes frictions (e.g. limited commitment and anonymity) that render money essential. The use of matching frictions facilitates this purpose. The latest generation of models adopts the structure of Lagos and Wright (2005), which renders divisible money tractable. While many market structures have been studied (e.g. Rocheteau and Wright (2005)), relatively little attention has been paid to endogenous variety and mo-

\footnote{In competitive search equilibrium, which combines price posting and directed search, welfare costs of inflation tend to be much lower. For instance, Rocheteau and Wright (2009) find that 10\% inflation costs about 1.1\% of output. These low welfare costs arise from the fact that competitive search ensures competitive pricing in equilibrium. In contrast, Ennis (2008), who features price posting but with undirected search, finds that welfare costs of inflation are high, but this result relies on providing all the bargaining power to sellers. With monopolistic competition, instead, the division of surplus depends endogenously on the taste for variety and quantity of trade.}

\footnote{Thereafter, product variety has been incorporated into business cycle theory (e.g. Shleifer (1986), Caballero et al. (1996), Bilbiie et al. (2012)); growth theory (e.g Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1998)); and international trade (e.g. Krugman (1979) and Melitz (2003)). Monopolistic competition is used even more broadly, encompassing the fields above as well as the New Keynesian literature. On the empirical front, Broda and Weinstein (2010) use the AC Nielsen Homescan Database (1994–2003) to show that net product creation and growth in total sales closely correlate. The AC Nielsen Homescan Database is constructed by providing scanners to 55,000 households who scan in purchases of every good with a barcode. The sample is demographically balanced and includes 23 cities in the U.S. The sectors primarily include grocery, drugstore, and mass merchandise. The expenditure on these sectors amounts to 40\% of expenditure in the Consumer Price Index. Furthermore, since the novelty of a product is measured at the household rather than the store level, the inclusion of a new good in the sample genuinely represents an increase in variety at the household level.}
nopolistic competition.

In this environment, each period is subdivided into a decentralized market (DM) with Dixit-Stiglitz preferences, monopolistic competition, and search frictions, followed by a centralized market (CM). In the DM, a continuum of goods is produced by monopolistically competitive firms and sold to consumers. In the CM, a general good is consumed and produced. With one variety per seller, the extensive margin directly measures the amount of variety available in the economy.

The paper is organized as follows: Section 2.2 reviews the related literature. Section 2.3 presents the CES model with entry, following Dixit and Stiglitz (1977). In contrast to the findings of Shi (1997) and Rocheteau and Wright (2005), the Friedman rule maximizes equilibrium welfare. Section 2.4 examines equilibrium under additively separable preferences, which give rise to variable markups. Here the Friedman rule is not generally optimal; rather, inflation can increase firm size, reduce markups, and raise welfare. Section 2.5 analyzes the welfare costs of inflation using a compensated measure. Section 2.6 concludes. The appendices provide proofs, variations of the model, additional derivations, numerical checks, and a summary of the optimality of the Friedman rule for a number of models.

2.2 Related Literature

The Lagos-Wright environment has been a fertile setting for studying the role of different market structures. Rocheteau and Wright (2005) examine bargaining, Walrasian price taking, and competitive search in such a framework. Under competitive equilibrium, which is closest to the environment here, the Friedman rule implies efficiency along the intensive margin but not the extensive margin. The latter holds only under a Hosios-like condition.

\footnote{It is compatible with different types of bargaining, such as proportional bargaining (Aruoba et al. (2007)); mechanism design (Hu et al. (2009)) for the purposes of normative analysis; competitive search (Rocheteau and Wright (2005, 2009)); auctions; and Walrasian price taking.}
which the congestion externality balances out the thick market externality.

The closest paper is by Laing et al. (2007), who use monopolistic competition and multiple matching between a continuum of buyers and sellers. Search frictions arise in terms of limited consumption variety, as they do here. Time can be allocated to either supplying labor, searching for goods, or enjoying leisure. If leisure and goods are sufficiently substitutable, a “hot potato effect” arises, wherein inflation increases labor, search effort, and output. Yet there are three important differences between that paper and this one: (1) Laing et al. (2007) do not consider endogenous variety and the welfare consequences thereof; (2) their study is restricted to constant elasticity of demand; and (3) they do not use the Lagos-Wright alternating market structure, which is the dominant paradigm for monetary analysis. Thus, they cannot analyze the impact of inflation on the intensive margin, markups, or the size of the differentiated goods sector. A further consequence is that their framework does not readily accommodate ex post heterogeneity, which would arise from preference shocks in the goods market, search frictions in the labor market, or other sources.\footnote{Moreover, their conclusion that inflation can increase labor, search effort, and output presupposes no congestion in search effort. In the appendix, I show in an extension of my basic model that the hot potato effect vanishes with search effort and congestion.}

Another related approach is that of Aruoba and Schorfheide (2011). This paper studies optimal inflation, examining its effects on relative price distortions caused by nominal rigidities and the reduction of the rate of return on money. The centralized market features a New Keynesian environment, and the decentralized market has bilateral matches with monetary exchange. The model is fully estimated with Bayesian methods. The authors find that the optimal inflation rate lies between $-1\%$ and $-2\%$. They do not, however, report the welfare costs of large inflation, such as 10%. Because they keep search frictions in the decentralized market and nominal rigidities in the centralized market, these frictions do not interact on the same trade margin. Finally, the measure of sellers is fixed and there is no taste for variety, so the model cannot address the effects of inflation on the amount of variety and firm size.
Both of these are of primary concern here.

A comparable paper studying inflation in a New Keynesian setting is by Khan et al. (2003). The main difference is that they generate money demand via the costly use of credit in the retail sector rather than via search frictions. Similar to Aruoba and Schorfheide (2011), they find that deflation is optimal, but not to the level of the Friedman rule. In this environment, near price stability is optimal in response to real and nominal shocks.

Dong (2010) explores the welfare consequences of product variety in response to inflation in a search model with bilateral meetings, but does not incorporate taste for variety. Instead, consumers desire particular goods according to a preference shock. The amount of available varieties depends on investment in capacity by each firm. With sellers stocking greater variety, consumers are more likely to obtain the desired good. Though this probabilistic interpretation captures one aspect of variety, it implies that product variety is only valuable because of search frictions\(^6\), leaving out the role of tastes.

### 2.3 Constant Elasticity of Substitution (CES) Preferences

#### 2.3.1 Environment

The environment builds on the New Monetarist model by Rocheteau and Wright (2005) and Lagos and Rocheteau (2005). Time is discrete and the horizon is infinite. Each period has two subperiods: a decentralized market with Dixit-Stiglitz preferences and monopolistic competition (DM) followed by a centralized market (CM). In the decentralized market a

\(^6\) Dong (2010) also considers an extension in which consumers enjoy the ideal variety at \(\delta u(q)\) with \(\delta > 1\) and other varieties at \(u(q)\). But the consumer is barred from exploiting the concavity of utility function by consuming multiple varieties even though the seller has multiple goods in stock.
continuum of goods (DM goods) are produced by monopolistically competitive firms and sold to consumers. In the CM, agents are endowed with a technology that allows them to produce one unit of the general good with one unit of labor. Both the DM goods and general goods cannot be stored across periods. There is a measure 1 of buyers and measure $\mu$ of sellers.

Buyers and sellers differ in their preferences and production possibilities. During the CM both have the ability to produce and wish to consume. In the DM, buyers want to consume but cannot produce whereas sellers are able to produce but do not wish to consume.

The period utility function of the buyer is given by

$$U^b(x, h, q_j) = \psi u \left[ \left( \int_{j \in [0,1]} q_j^{-\eta} \, dq_j \right)^{-1\over \eta} \right] + U(x) - h$$

where $u(0) = 0, u'(0) = \infty, u'(q) > 0$, and $u''(q) < 0$ for $q > 0$. Similarly, $U(\cdot)$ satisfies $U(0) = 0, U'(0) = \infty, U'(x) > 0$, and $U''(x) < 0$ for $x > 0$. The term $\psi$ represents a Bernoulli preference shock: $\text{Prob}(\psi = 1) = \sigma, 0 < \sigma < 1$ and $\text{Prob}(\psi = 0) = 1 - \sigma$. Moreover, $\eta \in (1, \infty)$ and represents the (constant) elasticity of substitution of a particular variety.\(^7\)

The period utility function of seller $j$ is

$$U^s(x, h, q_j) = -c(q_j) + U(x) - h$$

where $c(0) = c'(0) = 0, c'(q) > 0, c''(q) \geq 0$ for $q > 0$.

There is multiple matching in the DM: each buyer meets a measure of sellers, and each

\(^7\)The restriction $\eta > 1$ is necessary for demand to be elastic (so that marginal revenue of firms is positive). As $\eta \to \infty$, the DM goods become perfect substitutes with each other. Also, note that the CM good is analogous to the outside sector in Dixit and Stiglitz (1977), subject to the restriction of quasilinearity.
seller produces goods for a measure of buyers. Each buyer matches with a set of measure \( \alpha(\kappa) \leq \mu \) of sellers (and varieties), where \( \kappa = \mu/\sigma \). The matching friction is interpreted as follows: buyers go to a shopping mall with a mass of vendors, which supply a subset of the available varieties in the economy. The measure of buyers serviced by each firm is given by \( \sigma \alpha(\kappa)/\mu = \alpha(\kappa)/\kappa \). The probability that a given buyer matches with a particular seller is \( \alpha(\kappa)/\mu \). I require that \( \lim_{\mu \to 0} \alpha(\kappa)/\mu = 1 \). By L’Hôpital’s rule, this requires \( \alpha'(0) = \sigma \).\(^8\)

\( \alpha = \sigma = 1 \) leads to Dixit and Stiglitz (1977) with CES preferences as a special case. \( \sigma < 1 \) and \( \eta = \infty \) leads to Rocheteau and Wright (2005) with competitive pricing and a fixed set of sellers.

Agents are anonymous and there are no forms of commitment or public memory that would render money inessential. Fiat money is costless to produce, intrinsically useless, perfectly divisible, and storable. The ex ante division between buyers and sellers together with anonymity rule out double coincidence of wants, generating an essential role for money. The gross growth rate of the money supply is constant over time and equal to \( \gamma: M_{t+1} = \gamma M_t \). New money is injected (or withdrawn if \( \gamma < 1 \)) by lump-sum transfers (or taxes). These transfers take place during the CM and without loss of generality they go only to buyers.\(^9\)

2.3.2 Equilibrium

I focus in the remainder of the paper on stationary equilibria where aggregate real balances are constant: \( \phi_t M_t = \phi_{t+1} M_{t+1} \). I rewrite the buyer’s preferences as

\[
U^b(\bar{q}, x) = \psi u(\bar{q}) + U(x) - h
\]

\(^8\)A general function that satisfies these conditions is \( \alpha(\kappa) = \sigma \kappa \varepsilon^{1/b}/[(\kappa^b + \varepsilon)^{1/b}] \). The elasticity is given by \( \varepsilon/(\kappa^b + \varepsilon) \).

\(^9\)Quasilinear utility in the CM implies no wealth effects from the lump-sum transfer and thereby makes the allocation of transfers immaterial.
where $\overline{q}$ is defined as

$$\overline{q} = \left[ \int_0^{\alpha} q_j^{\frac{n-1}{n}} dj \right]^{\frac{n}{n-1}}$$

Consider first the problem of a buyer holding $z$ balances when entering the CM. Buyers receive a lump sum transfer equal to $T = \phi_t(M_{t+1} - M_t)$. In order to hold $z'$ next period, the buyer must accumulate $\gamma z'$, where $\gamma$ is the gross inflation rate $\frac{\phi_t}{\phi_{t+1}}$. The consumer allocates real balances and transfers into spending on the general good and savings for the following DM. Hence, the CM value function $W(z)$ satisfies

$$W(z) = \max_{x', x, h \geq 0} \left[ U(x) - h + \beta V(z') \right] \quad \text{s.t.} \quad x + \gamma z' = h + z + T$$

Substituting the constraint, the value function can be rewritten as

$$W(z) = z + T + \max_{x \geq 0} \{ U(x) - x \} + \max_{z' \geq 0} \{ \beta V(z') - \gamma z' \}$$

As is standard in Lagos-Wright models, the value function is linear in $z$ and that the choice of real balances $z'$ is independent of $z$. The first order condition with respect to $h$ after substituting the constraint for $x$ yields $U'(x^*) = 1$. Note that $T = z(\gamma - 1)$, and from the budget constraint $h^* = x^*$.

The DM value function, $V(z)$, can be expressed as

$$V(z) = \max_{q_j} \left\{ \sigma u(\overline{q}) + \sigma W \left( z - \int_0^{\alpha} p_j q_j dj \right) + (1 - \sigma)W(z) \right\}$$
such that

\[ \int_0^\alpha p_j q_j dj \leq z \]  

(2.4)

If \( \gamma > \beta \), then money is costly to hold and the constraint \( \int_0^\alpha p_j q_j dj \leq z \) binds.

Exploiting the linearity of \( W(\cdot) \), rewrite (2.3) and (2.4) as

\[ V(z) = \max_{q_j} \{ \sigma [u(q) - z] + W(z) \} \]  

(2.5)

subject to \( \int_0^\alpha p_j q_j dj = z \)

The envelope condition gives

\[ V'(z) = \sigma \left[ u'(q) \left( \frac{q}{q_j} \right)^{1/\eta} \frac{1}{p_j} - 1 \right] + 1 \]  

(2.6)

Using \( V'(z) = \gamma / \beta \) and rearranging provides the inverse demand function for good \( j \):

\[ \left( 1 + \frac{i}{\sigma} \right) p_j = u'(q) \left( \frac{q}{q_j} \right)^{1/\eta} \]  

(2.7)

in which \( i = \gamma / \beta - 1 \) is the opportunity cost of holding real balances. The rate \( i \) can be interpreted as the interest rate on an illiquid bond.

Thus, the buyer equates \( (1 + i/\sigma)p_j \), the cost of acquiring the good adjusted for inflation and idiosyncratic uncertainty, with the marginal benefit of the good. As (2.7) holds for any \( j \),

\[ \frac{q_j}{q_i} = \left( \frac{p_i}{p_j} \right)^{\eta} \]

where \( \eta \) represents the elasticity of substitution and elasticity of demand.
Let us now turn to the maximization problem of the monopolistic competitor in the decentralized market. Each firm $j$ produces a unique type $j$ and quantity $q_j$ for a given consumer. A measure $\alpha(\kappa)/\kappa$ of consumers matches with the firm. Hence, each firm produces $\frac{\alpha(\kappa)}{\kappa}q_j$ overall, which I denote $q_s(j)$.

The firm’s value function in the CM is analogous to that of the buyer:

$$W_s(z) = \max_{z',x,h \geq 0} \{U(x) - h + \beta V(z')\} \tag{2.8}$$

such that $x + \gamma z' = h + z$. As before, (2.8) collapses to $W_s(z) = z + \max_x \{U(x) - x\}$

$$+ \max_{z' \geq 0} \{\beta V(z') - \gamma z'\}.$$ 

The value function in the DM satisfies

$$V_s(z) = \max_{p_j,q_s(j)} \{-c[q_s(j)] + W[z + p_j q_s(j)]\} \tag{2.9}$$

in which $p_j$ satisfies the inverse demand function given by (2.7). Using the linearity of $W(\cdot)$, the problem is equivalent to

$$\max_{q_s(j),p_j} \{p_j q_s(j) - c[q_s(j)]\} \tag{2.10}$$

subject to (2.7). Thus, even though firms carry money from the DM period to the CM period, their choices over prices and quantities are static.

From (B.1) and (2.7) the problem of the firm is strictly concave and thereby admits a unique solution $q_s(j)$. Prices thus satisfy

$$p_j = \frac{\eta}{\eta - 1} c'[q_s(j)] \tag{2.11}$$

There is a constant markup of price to marginal cost that depends negatively on the elasticity.
of substitution (positively on product differentiation).\textsuperscript{10} Perfect competition is the limiting case as $\eta \to \infty$.\textsuperscript{11} Each firm faces the same problem, so $q_s(j) = q_s$ for all $j$. Also, given identical production, $\bar{q} = q\alpha(\kappa)^{\eta/(\eta-1)}$, and $p_j = p$ for all $j$:

$$p = \frac{u'(\bar{q})}{1 + i/\sigma} \alpha(\kappa)^{\frac{1}{\eta-1}}$$  \hspace{1cm} (2.12)

As a result, the DM output level solves

$$\frac{\alpha(\kappa)^{1/(\eta-1)} u'[\alpha(\kappa)^{1/(\eta-1)}\kappa q_s]}{c'(q_s)} = \left(\frac{\eta}{\eta - 1}\right) (1 + i/\sigma)$$  \hspace{1cm} (2.13)

Sellers can choose to enter the market in the following DM period by paying a flow cost $a$ in the current CM. The cost must be paid each CM period for continued entry. The opportunity cost of entry is thus $k = (1 + \rho)a$, where $\rho$ is the discount rate $(1 - \beta)/\beta$. The free entry condition is

$$k = q_sp - c(q_s)$$  \hspace{1cm} (2.14)

It is useful to define the mapping $\Gamma(q_s) = [k + c(q_s)]/[q_sc'(q_s)]$, which expresses the ratio of average to marginal cost as a function of firm quantity. $\Gamma(q_s)$ is decreasing everywhere, tends to $\infty$ as $q_s \to 0$, and tends to 0 as $q_s \to \infty$. Note that (2.14) can be reexpressed as

$$\Gamma(q_s) = \frac{\eta}{\eta - 1}$$  \hspace{1cm} (2.15)

I define equilibrium for the model with entry.

**Definition 2.1.** A CES equilibrium is a pair $(q_s, \mu)$ that solves (2.13) and (2.15).

\textsuperscript{10}Nonlinear pricing schemes, if feasible, are more profitable than linear pricing (i.e. Stiglitz (1977)) and are efficient. The simplest case is the two-part pricing rule $p(q) = A + c'(q)q$, where $A$ is the consumer surplus at the efficient level $q^\ast$. One problem with such pricing rules is that they provide consumers an incentive to pool purchases of goods into as few purchases as possible and resell.

\textsuperscript{11}The second order condition is $2p''(q_s) + q_sp'''(q_s) - c''(q_s) < 0$, which simplifies to $\eta > 1$. 
Equation (2.15) states that in equilibrium average cost is a constant markup over marginal cost. This condition determines $q_s$. Equation (2.13) describes a markup relationship that determines $\mu$ given $q_s$. The equilibrium can thus be computed recursively.

Equilibrium can be characterized in terms of a price curve and function
\[
\lambda(q_s) = \alpha(\kappa)^{1/(\eta-1)}u'(\alpha(\kappa)\frac{1}{\eta-1}\kappa q_s),
\]
which describes marginal utility in terms of the production of each firm.

The inefficiency along the intensive margin can be analyzed in terms of the ratio of marginal utility to marginal cost, which I label the marginal markup. Table 2.1 compares the marginal markup in setting with proportional bargaining, generalized Nash, and additively separable preferences à la Zhelobodko et al. (2012).\textsuperscript{12} Here $\Theta(q) = \theta u'(q)/[\theta u'(q) + (1 - \theta) c'(q)]$ is the buyer’s share of surplus under Nash and satisfies $\Theta'(q) < 0$. Furthermore, $r_u(q) = -u''(q)/u'(q)$ is the inverse of the elasticity of demand under additively separable preferences, and the net markup in equilibrium. Both bargaining mechanisms feature rent sharing externalities—inefficiencies arising from the fact that the surplus of some party is not a monotonic transformation of the total surplus. Moreover, both externalities vary with the level of trade and hence the interest rate.\textsuperscript{13} The rent sharing externality is constant with CES preferences, yet varies with more general preferences that allow the demand elasticity to change.

\textsuperscript{12}The derivation is sketched in Appendix A.3.2.

\textsuperscript{13}However, the rent sharing externality disappears with proportional bargaining as $i \to 0$ since the buyer’s surplus is monotonic in the level of trade. Yet inefficiency persists in generalized Nash because consumer surplus is non-monotonic.
To analyze (2.13), it is helpful to formally define the marginal utility of a particular variety as a function of the measure of active sellers. For fixed \( q_s \), let \( \lambda(\kappa) = \alpha(\kappa)^{1/(\eta-1)}u'[\alpha(\kappa)^{1/\eta}\kappa q_s] \). \( \lambda(\kappa) \) captures the marginal utility of a particular variety as a function of the measure of sellers (relative to \( \sigma \)), where the production of each variety is fixed at \( q_s \).

Increasing the measure of sellers has two effects: buyers consume a greater overall quantity of goods but also greater variety of goods. The first effect reduces the marginal utility of a good and the second increases it. Which effect dominates depends on the elasticity of marginal utility, the elasticity of the matching function, and the elasticity of substitution. The exact relationship is given by

\[
\epsilon_{\lambda}(\kappa) = \frac{1}{\eta - 1}\epsilon_{\alpha}(\kappa) + \epsilon_{u'}(\overline{q}) \left[ \frac{1}{\eta - 1}\epsilon_{\alpha}(\kappa) + 1 \right]
\]  

(2.16)

I examine \( \lambda(\kappa) \) in the case of CRRA preferences.

**Example 2.1.** Let \( u(\cdot) = q^{1-\varepsilon}/(1 - \varepsilon) \). Then \( \lambda(\kappa) = \alpha(\kappa)^{1/\eta-1}\kappa^{-\varepsilon}q_s^{-\varepsilon} \). Hence, \( \epsilon_{\lambda}(\kappa) = \frac{1}{\eta - 1}\epsilon_{\alpha}(\kappa) - \varepsilon \). Since \( \epsilon_{\alpha}(\kappa) \) is bounded above by 1, \( \epsilon_{\lambda}(\kappa) \leq \frac{1 - \eta \varepsilon}{\eta - 1} \). As \( \varepsilon \to 0 \) (utility becomes linear in the composite good), \( \epsilon_{\lambda}(\kappa) \to \epsilon_{\alpha}(\kappa)/(\eta - 1) \).

As the measure of sellers increase, there is diminishing marginal utility for the marginal good unless there is sufficiently high taste for variety and/or a high enough elasticity of the
marginal utility of the composite good.

If $\eta \varepsilon < 1$, then the behavior of the matching function depends on $\mu$, as the following example shows.

**Example 2.2.** If $\alpha(\kappa) = \kappa/(1+\kappa)$, then $\epsilon_\lambda(\kappa) = \frac{1-\varepsilon}{\eta-1} - \frac{1}{1+\kappa} - \varepsilon$, which is positive for $\mu$ sufficiently low if $\eta \varepsilon < 1$.

I state results on comparative statics of equilibrium:

**Proposition 2.1.** *The comparative statics are provided by Table 2.2.*

Table 2.2: Comparative statics

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\overline{q}$</th>
<th>$q_s$</th>
<th>$p$</th>
<th>$\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) $\lambda'(\kappa) &lt; 0$</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$i$</td>
<td>↓</td>
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<tr>
<td>$k$</td>
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<tr>
<td>$\eta$</td>
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<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\overline{q}$</th>
<th>$q_s$</th>
<th>$p$</th>
<th>$\mu$</th>
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</thead>
<tbody>
<tr>
<td>(b) $\lambda'(\kappa) &gt; 0$</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$i$</td>
<td>↑</td>
<td>0</td>
<td>0</td>
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</tr>
<tr>
<td>$k$</td>
<td>−</td>
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<td>$\eta$</td>
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### 2.3.3 Social Welfare

The general good and the Dixit Stiglitz goods are produced using different technologies. The efficient choice of the CM good is $x^*$, which is the equilibrium quantity. Hence, the social planning problem can focus on the DM. The formal social planning problem is to choose quantities of variety $j$ for consumer $i$ $q_{i,j}$ to maximize aggregate welfare net of production costs, weighing each individual equally. Let $\Psi$ denote the set of buyers which has a preference shock (of measure $\sigma$), $A_i$ denote the set of sellers whom buyer $i$ contacts (of measure $\alpha(\kappa)$), and $B_j$ denote the set of buyers who approach seller $j$ (of measure $\alpha(\kappa)/\kappa$). The social
planning problem is thus given by

\[ \Omega(q_{i,j}) = \max_{q_{i,j}} \left\{ \int_{i \in \Psi} u \left[ \left( \int_{A_i} \eta_{i,j}^\eta \, dj \right)^{\frac{\eta}{\eta-1}} \right] di - \int_0^\mu c \left( \int_{B_j} q_{i,j} \, di \right) dj \right\} \] (2.17)

Given the convexity of the cost function, it is socially optimal for each firm to produce the same quantity \( q_s \). Furthermore, given the concavity of \( \bar{q} \) with respect to \( q_j \), it is optimal to have \( q_j = q \) for all \( j \). Given concavity of \( u(\cdot) \) it is optimal to set \( \bar{q}_j = \bar{q} \) for all \( j \). Noting that aggregate costs are given by \( \mu c(q_s) \), and writing \( \bar{q} = \alpha(\kappa)^{\frac{1}{\eta-1}} \kappa q_s \) the social welfare can be defined simply as a function of \( q_s \) and \( \mu \):\(^{14}\)

\[ W(q_s, \mu) = \sigma u[\alpha(\kappa)^{1/(\eta-1)} \kappa q_s] - \mu [c(q_s) + k] \] (2.18)

This is the utility of DM market consumption of buyers minus the production cost of the sellers and their entry cost. The first order conditions are

\[ [q_s] \quad \alpha(\kappa)^{\frac{1}{\eta-1}} u'[\alpha(\kappa)^{1/(\eta-1)} \kappa q_s] = c'(q_s) \] (2.19)

\[ [\mu] \quad \Gamma(q_s) = \frac{\epsilon_\alpha(\kappa)}{\eta - 1} + 1 \] (2.20)

where \( \epsilon_\alpha(\mu) \) is the elasticity of the matching function with respect to the measure of sellers. Note that the elasticity is decreasing and bounded above by 1.\(^{15}\)

Equation (2.19) equates the marginal utility to marginal cost of each variety. Equation (2.20) characterizes the optimal scale of production\(^{16}\), which depends on how effectively new

\(^{14}\)The quantities \( \mu \) and \( q_s \) are sufficient statistics for aggregate welfare. Though types are permanent, welfare by type is not very interesting: the per-period profits of each seller are pinned down by free entry: \( k = pq_s - c(q_s) \). Therefore, neither inflation or taste for variety has an effect on firm-level welfare.

\(^{15}\)This follows easily from the concavity of \( \alpha(\cdot) \). If \( f : \mathbb{R}^+ \to \mathbb{R} \) is increasing, differentiable, and concave, then \( f'(x) \leq \frac{f(x)}{x} \), so that \( \frac{xf'(x)}{f(x)} \leq 1 \), or \( \epsilon_f(x) \leq 1 \). Furthermore, \( xf'(x)/f(x) \) is decreasing with \( x \).

\(^{16}\)As \( ? \) stressed, the optimal point of production does not generally minimize average cost, because of the
sellers match with buyers. In particular, the optimal ratio of average cost to marginal cost equals the elasticity of the matching function with respect to the measure of sellers divided by $\eta - 1$, which reflects the taste for variety, plus unity. Note that for $\alpha(\kappa) = \kappa$ the right hand side equals $\frac{\eta}{\eta - 1}$. Also, as $\eta \to \infty$, the socially optimal scale minimizes average cost; under economies of scale, then the optimal measure of firms shrinks to zero, as fixed costs are positive.

The marginal markup can be decomposed into a rent-sharing externality $\frac{\eta}{\eta - 1}$ and cost of holding balances $1 + \frac{i}{\sigma}$. Thus, the inefficiency along the intensive margin arises from (1) markups, (2) a positive nominal interest rate, and (3) $\sigma < 1$, which creates idiosyncratic uncertainty that is unresolvable until after acquiring money.

The Friedman rule sets $i = 0$: there is deflation at the rate of time preference. The Friedman rule is the optimal policy but does not attain the first best, due to the markup inefficiency. The asymptotic case $\eta \to \infty$ corresponds to Rocheteau and Wright (2005) with price taking. Moreover, there is a rent sharing externality that depends positively on the markup and which amplifies the welfare cost of inflation. Since the buyer’s share of the surplus is non-monotonic under monopolistic competition, the rent sharing externality does not vanish as $i \to 0$.

Figure 2.1: Equilibrium with entry

![Figure 2.1](value of variety)
Figure 2.1 illustrates equilibrium with two graphs. The first diagram plots average cost against marginal cost and indicates the equilibrium point $q^*_{es}$ together with $q^f_s$. The second graph takes $q_s$ as given and plots cost of holding real balances $\frac{\eta}{\eta-1}(1+i/\sigma)c'(q_s)$ against marginal utility as a function of the measure of sellers. The free entry condition $\Gamma(q_s) = \frac{\eta}{\eta-1}$ uniquely determines firm production independent of search frictions and inflation. Let $q^f_s$ define the scale at which average cost is minimized. The percentage deviation $(q^f_s - q_s)/q^f_s$ is higher with greater taste for product diversity.

Comparing (2.15) and (2.20), we see that as $\epsilon_\alpha(\kappa) \leq 1$, the ratio of average to marginal costs at the social optimum is less than or equal to the value at equilibrium. Hence $q^*_s > q^e_s$; Equality only holds if $\alpha(\kappa) = \kappa$. Note that in the general linear case $\alpha(\kappa) = A + B\kappa$, with the constraint that $0 < A \leq \kappa(1-B), \epsilon_\alpha(\kappa) < 1$.

**Lemma 2.1.** $1 < \Gamma(q^*_s) \leq \Gamma(q^e_s)$, so that $q^f_s > q^*_s \geq q^e_s$, with equality holding if and only if $\alpha(\kappa) = \kappa$.

Dixit and Stiglitz (1977) show that firm output and number of firms is the same between the constrained optimization problem of the social planner and equilibrium. Hence, the equivalence breaks down in a monetary economy with search frictions. The social planner optimally trades off both the cost savings of larger firms under scale economies with the additional variety provided from more and (smaller) firms. Unlike in Dixit and Stiglitz (1977), new firms cannot match seamlessly with customers, so the social planner takes into account how effectively matches are formed from entering sellers. Thus, the social marginal benefit of variety, $\epsilon_\alpha(\kappa)/(\eta - 1)$ is generally less than the private marginal benefit of variety $1/(\eta - 1)$, creating a distortion toward more and smaller firms.
By (2.13) and (2.15), necessary conditions for efficiency of equilibrium are

\[ \epsilon_s(\kappa) = 1 \]  \hspace{1cm} (2.21)
\[ \frac{\eta}{\eta - 1}(1 + i/\sigma) = 1 \]  \hspace{1cm} (2.22)

Equation (2.21) says that the elasticity of the matching rate with respect to the sellers equals unity. This property is a version of the condition in Hosios (1990), which balances the thick market and congestion externalities. The entry of a new firm increases the variety for consumers but reduces the buyers of other firms.

As the matching elasticity is strictly bounded above by unity, the Hosios condition cannot be satisfied. Similarly, as \( \eta > 1 \), the second necessary condition is inconsistent. The best policy, moreover, is the Friedman rule.

**Proposition 2.2** (Optimality of Friedman rule). The CES equilibrium is inefficient. The policy \( i = 0 \) maximizes welfare.

The intuition for the Friedman rule is straightforward. Firms impose a congestion on other firms upon entry, which produces a bias toward more and smaller firms, ceteris paribus. Inflation both leads to more inefficiency on the intensive margin, and reduces the number of firms. In order for aggregate welfare to have a chance at increasing, firms would have to become larger so as to reduce average costs. Inflation is useless as a tool for affecting firm size (and average costs) because firm size is pinned down by the (constant) markup. The optimality of the Friedman rule contrasts with the analysis of competitive equilibrium in Rocheteau and Wright (2005). The essential difference is that in Rocheteau and Wright (2005) there is a positive probability of sellers paying a fixed cost to enter and not matching with any buyers. This feature results in average cost exceeding marginal cost in equilibrium, so that firms operate below the average-cost-minimizing scale, even with no taste in variety.\(^\text{17}\)

\(^{17}\)Specifically, in Rocheteau and Wright (2005), the free entry condition is \( \alpha_s(n)[c'(q_s)q_s - c(q_s)] = k \), which can be rearranged as \( 1 = [\frac{k}{\alpha_s(n)} + c(q_s)]/[q_s c'(q_s)] < \Gamma(q_s) \), so that marginal costs lie below average costs.
In that setting, inflation can be useful by reducing the number of sellers and hence decreasing the probability of non-trade.

Hence, the welfare properties of inflation depend on the details of search frictions and preferences. The Friedman rule ceases to be optimal if inflation reduces markups in equilibrium, which can reduce average costs and attenuate the negative effects on the intensive margin. We indeed show in Section 2.4 that inflation can reduce average costs and markups, and I provide examples where deviations from the Friedman rule are optimal.

Equilibrium is not in general unique, as it depends on the crossing of $\lambda(\kappa)$ and the cost of holding real balances. I highlight the possibilities of uniqueness, nonexistence, and multiplicity using the functional forms $u(q) = q^{1-\varepsilon}/(1 - \varepsilon), c(q) = q^2/2$, and $\alpha(\kappa) = \kappa/(1 + \kappa)$.

Figure 2.2: Uniqueness, nonexistence, and multiplicity

Equilibrium is not in general unique, as it depends on the crossing of $\lambda(\kappa)$ and the cost of holding real balances. With $\eta \varepsilon > 1$, $\lambda(\kappa)$ slopes downward in Subplot (a). There is a single crossing between $\lambda(\kappa)$ and the cost of holding real balances. Subplots (b) and (c) consider $\varepsilon = 0.25, \eta = 3$ so that $\lambda(\kappa)$ is initially increasing. This means that the taste for diversity is stronger and that marginal utility is less elastic. In Subplot (b), $\lambda(\kappa)$ does not cross the adjusted price curve for the given costs of entry. In (c) preferences are the same, but costs of entry are lower, so that there is a double crossing of $\lambda(\mu)$ and adjusted prices. The two equilibria for this parameterization are

Figure B.3 depicts cases of uniqueness, nonexistence of equilibrium, and multiplicity. With $\eta \varepsilon > 1$, $\lambda(\kappa)$ slopes downward in Subplot (a). There is a single crossing between $\lambda(\kappa)$ and the cost of holding real balances. Subplots (b) and (c) consider $\varepsilon = 0.25, \eta = 3$ so that $\lambda(\kappa)$ is initially increasing. This means that the taste for diversity is stronger and that marginal utility is less elastic. In Subplot (b), $\lambda(\kappa)$ does not cross the adjusted price curve for the given costs of entry. In (c) preferences are the same, but costs of entry are lower, so that there is a double crossing of $\lambda(\mu)$ and adjusted prices. The two equilibria for this parameterization are
given by \((q_s, \mu) = (0.556, 0.133)\) and \((0.556, 1.591)\). In Appendix A.4, I show that the second derivative of the buyer’s objective function is negative at these two values, so the first order condition is sufficient.

There are thus a low-variety and a high-variety equilibrium in Figure B.3 (c). In the low variety equilibrium, the dearth of sellers keeps demand low. Given the low demand, sellers’ decision not to enter is optimal. In the high variety equilibrium, however, the taste for variety induces a higher demand, which is enough to sustain a greater measure of sellers. Social welfare is higher in the high-variety equilibrium, which can be determine by noting that the marginal utility curve at that point is higher than the cost of holding real balances. As sellers make zero profits in either equilibrium, the high-variety equilibrium Pareto dominates the low-variety equilibrium. A coordination problem arises: if sufficiently many sellers enter, the extra variety increases raises demand enough for the entry to be profitable. Yet, an individual seller has no incentive to enter unilaterally.

**Proposition 2.3** (Existence and Uniqueness). A CES equilibrium exists if \(\lim_{\kappa \to 0} \lambda(\kappa) \to \infty\) and \(\lim_{\kappa \to \infty} \lambda(\kappa) \to 0\). Equilibrium is unique if \(\lambda'(\kappa) < 0\) for all \(\kappa\).

The assumptions of Proposition 2.3 hold with \(\eta \epsilon \geq 1\).

**Corollary 2.1.** Given \(u(q) = q^{1-\epsilon}/(1-\epsilon)\), there is a unique CES equilibrium if \(\eta \epsilon \geq 1\).

### 2.4 Additively Separable Preferences

CES preferences have well-known problems, as discussed by Zhelobodko et al. (2012), among others. In particular, markups and prices are independent of firm entry and market size. Furthermore, the specification rules out any pass through from inflation to markups. Hence, I introduce variable elasticity of demand using additively separable preferences a là Zhelobodko et al. (2012).
2.4.1 Environment

The period utility function of the buyer is given by

\[
U^b(x, h, q_i) = \psi \int_0^{\Omega} u(q_i) \, di + U(x) - h
\]  

(2.23)

where \( u(0) = 0, u'(0) = \infty, u'(q) > 0, \) and \( u''(q) < 0 \) for \( q > 0 \). \( U(\cdot) \) satisfies the same conditions as \( u(\cdot) \). There is no change in the period utility function of the seller.

The concavity of \( u(\cdot) \) reflects taste for variety. Consumers prefer to spread consumption over all varieties than a small mass of varieties.\(^{18}\) Consumers' taste for variety can be measured from the relative love for variety (RLV)

\[
r_u(q) = -\frac{qu''(q)}{u'(q)} > 0
\]  

(2.24)

which is the familiar elasticity of marginal utility, or inverse of elasticity of substitution in the case \( q_i = q \ \forall i \). Preferences which display an increasing RLV mean that consumers perceive varieties as being less substitutable when they consume more. Preferences may also display a decreasing RLV and feature more substitutability with higher consumption. We assume \( r_u'(q) < 2 \ \forall q > 0 \), which makes the producers’ problem concave.

2.4.2 Equilibrium

The DM value function for the buyer is analogous to (2.5):

\[
V(z) = \max_{q_j} \left\{ \sigma \left[ \int_0^\alpha u(q_j) \, dq - z \right] + W(z) \right\} 
\]  

(2.25)

\(^{18}\)As Zhelobodko et al. (2012) point out, there is a formal equivalence between decision-making by consumers with taste for variety and the Arrow-Pratt theory of risk aversion.
subject to \( \int_0^\infty p_j q_j dj = z \).

As \( W(z) \) is the same as in the CES case, \( V'(z) = \gamma / \beta \) and the first order condition yields

\[
u'(q_j) = \left(1 + \frac{i}{\sigma}\right) p_j \tag{2.26}\]

As with CES preferences, the wedge between the marginal utility and price is \( 1 + i / \sigma \). An interior solution is guaranteed because of the Inada condition. Equation (2.26) implicitly defines a function \( p_i(q_i) \), which is strictly decreasing because \( u(\cdot) \) is strictly concave. The elasticity of the inverse demand \( \varepsilon_p(q) \) and the price elasticity of demand \( \varepsilon_q(p) \) are related to the RLV as

\[
\frac{1}{\varepsilon_q(p)} = \varepsilon_p(q) = r_u(q) \tag{2.27}
\]

Hence, the price elasticity of demand is just the reciprocal of the RLV, so that the RLV increases if and only if demand for a variety becomes less elastic with quantity (more elastic with price). Intuitively, consumers are less willing to substitute goods with higher consumption. Hence, as with CES preferences, one can speak interchangeably in terms of taste for variety, elasticity of substitution, and price elasticity of demand. Unlike with CES, however, each of these quantities depends on the consumption level \( q_j \).

Furthermore, suppose that there are is more variety available, and consumers spread out consumption among the greater set. Then \( q_j \) is lower for each variety \( j \), so that with increasing RLV, \( r_u(q_j) \) is lower for all \( j \). Hence, there is more substitutability between varieties. This reasoning affirms the intuition that the substitutability rises with the amount of variety, all else constant.

The producer \( j \) produces \( q_j \) for measure \( \alpha_s(\kappa) = \frac{\alpha(\kappa)}{\kappa} \) consumers. As before, let \( q_s(j) = \alpha_s(\kappa)q_j \). Using the same arguments as in the CES version, the producers’ problem is equiv-
\[ \max_{p_j, q_j} \{ p_j q_s(j) - c[q_s(j)] \} \]  

(2.28)

where \( p_j \) is given by (2.26). The solution equates marginal revenue \( \frac{u'(q)[1 - r_u(q)]}{1 + i/\sigma} \) to marginal cost \( c'(q_s) \). This step yields

\[ r_u(q_j) = \frac{u'(q_j) - (1 + i/\sigma)c'[q_s(j)]}{u'(q_j)} \]  

(2.29)

Notice that the right-hand side of (2.29) equals the net markup. Hence, a firm chooses \( q_s \) such that the markup equals the RLV. The first condition implies \( r_u(q) < 1 \), or that firms produce in the elastic region. We make the stronger assumption that \( r_u(q) \) is bounded below 1 for all \( q > 0 \).

The solution is unique provided the profit function is concave. Differentiating (2.29) with respect to \( q_j \), dividing by \( u'(q_j) \), and rearranging shows that the second order condition is equivalent to

\[ [2 - r_u'(q_j)]r_u(q_j) - [1 - r_u(q_j)]r_c[q_s(j)] > 0 \]  

(2.30)

where \( r_c = -qc''/c' \) is the negative elasticity of marginal cost and \( r_u'(q_j) = -qu'''(q_j)/u''(q_j) \) is the elasticity of the derivative of marginal utility. From hereon out, we assume (2.30) holds for all \( q > 0 \). By construction, \( r_c < 0 \), so that the second term in (2.30) is positive. \( r_u'(q_j) \) measures the convexity of demand for \( q_j \), so the second order condition requires that demand not be too convex. \( r_u'(q_j) < 2 \ \forall q \geq 0 \) is sufficient because the second term is nonnegative.

---

\(^{19}\) To be explicit, write the profit function \( \Pi(q) = \max_{\alpha} \left\{ \frac{u'(q)[1 - r_u(q)]}{1 + i/\sigma} \right\} \). Differentiating with respect to \( q \) yields \( \frac{u'(q)}{1 + i/\sigma} \left[ 1 + \frac{u''(q)q}{u'(q)} \right] \alpha_s - c'(q_s)\alpha_s \). Noting that \( \alpha_s \) cancels in each side and rewriting in terms of the RLV, we obtain (2.29).

\(^{20}\) Not to be confused with \( r_u'(q) \), the derivative of \( r_u(q) \).
Producers enter in the CM at cost $k$ up to the point until profits are zero:

\[ k + c(q_s) = p_j q_s \] (2.31)

Since each seller faces the same problem, which is concave and thereby admits a unique solution, $q_j = q \quad \forall i$.

**Definition 2.2.** A ZKPT equilibrium can be defined as a list of prices $p$, quantities $(q_s, q)$, and measure of sellers $\mu$ which satisfy

\[
\begin{align*}
\Gamma(q_s) &= \frac{1}{1 - r_u(q)} \\
u'(q) &= \frac{1 + i/\sigma}{1 - r_u(q)} \\
c'(q_s) &= \frac{c'(q_s)}{1 - r_u(q)} \\
p &= \frac{c'(q_s)}{1 - r_u(q)} \\
q_s &= \frac{\alpha(\kappa)}{\kappa}q 
\end{align*}
\] (2.32-2.35)

Since $r_u(q)$ is also the net markup, this means that the ratio of average to marginal costs equals the gross markup in equilibrium.

A ZKPT equilibrium can be described in $(p, q)$ space in terms of the following equations:

\[
\begin{align*}
p &= \frac{u'(q)}{1 + i/\sigma} \\
p &= \frac{c'[\phi(q)]}{1 - r_u(q)} 
\end{align*}
\] (2.36-2.37)

where the function $\phi(q) = \Gamma^{-1}\left[\frac{1}{1 - r_u(q)}\right]$ maps individual consumption to firm size and satisfies $\phi'(q) = r_u'(q)/(\Gamma'(\phi(q))(1 - r_u(q))^2) < 0 \iff r_u'(q) > 0$. Increasing RLV implies that $\varepsilon < 1$ is a sufficient condition.

\footnote{Note that, in the CES case $u(q) = q^{1-\varepsilon}/(1 - \varepsilon)$, $r_u = \varepsilon$, and $r_u' = 1 + \varepsilon$, so that $\varepsilon < 1$ is a sufficient condition.}
higher markups decrease firm size in equilibrium and that there is a negative relationship between individual consumption and firm size. Equation (2.36) is a demand curve, which depends on the cost of holding money, and Equation (2.37) is price setting rule, which reflects the markup directly and also indirectly via the effect on marginal costs expressed by $c'[\phi(q)]$.

Figure 2.3: Free entry equilibrium with variable markups

Existence and uniqueness of equilibrium holds generally, and is depicted in Figure 2.3. From the properties of $u(\cdot)$, the demand curve is decreasing and convex. Though Figure 2.3 depicts a linear price setting rule, in general the slope depends on the relative love of variety and convexity of the cost function.

**Proposition 2.4.** A ZKPT equilibrium exists and is unique.

Note that, in contrast to CES preferences, the Inada conditions on $u(\cdot)$ suffice for existence and uniqueness because there is a lack of a complementarity of varieties: having more varieties
does not increase the marginal utility of a particular variety. Hence, the marginal utility of
a variety approaches zero as the number of sellers approaches infinite without any further
assumptions.

The following proposition shows that higher interest rates (or entry costs) lead to lower
overall consumption and larger and fewer firms.

**Proposition 2.5.** If \( r_u'(q) > 0 \) in the neighborhood of equilibrium, then a small increase in
\( i \) leads to higher \( q_s \), lower \( q \), lower \( \mu \), and lower markups. Furthermore, a small decrease in
\( k \) leads to higher \( q \), lower \( q_s \), higher \( \mu \), and higher markups.

The presence of lower markups with fewer firms may seem counterintuitive. The reasoning
is as follows. A higher nominal interest rate makes it more costly to use money, lowering
consumption. Lower demand leads to lower sales and lower markups on those sales since the
taste for variety is lower. Hence, fewer firms enter. Lower markups lead to higher production
through the formula \( \Gamma(q_s) = 1/(1 - r_u(q)) \). Intuitively, demand is more elastic, which limits
the ability of firms to increase profits by scaling back production and charging higher prices.
Hence, firm size increases.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( q_s )</th>
<th>( q )</th>
<th>( r_u(q) )</th>
<th>( \mu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \uparrow i )</td>
<td>( \uparrow )</td>
<td>( \downarrow )</td>
<td>( \downarrow )</td>
<td>( \downarrow )</td>
</tr>
<tr>
<td>( \uparrow k )</td>
<td>( \uparrow )</td>
<td>( \downarrow )</td>
<td>( \downarrow )</td>
<td>( \downarrow )</td>
</tr>
</tbody>
</table>

In Figure 2.4, we consider a rise from \( i = 0 \) to \( i = 0.13 \) for \( u(q) = \frac{q^{1-\varepsilon}}{1-\varepsilon} + bq \), so that
\( r_u(q) = \frac{\varepsilon}{1+\varepsilon} \). This change generates a sharp decrease in \( q \) and mild decrease in \( p \). There is
a smaller change in \( q \) for a less elastic demand, which takes place with higher \( \varepsilon \) (Subplot B)
or a more negative \( b \) (Subplot C). In Subplot D we illustrate these effects by graphing the
relative love of variety for the different specifications in \( \varepsilon \) and \( b \). Higher \( \varepsilon \) raises both the
RLV and the rate at which it increases; a more negative $b$ has a modest effect on the RLV by itself but interacts with a higher $\varepsilon$ to both raise the RLV and its rate of increase.

Figure 2.4: A rise in the nominal interest rate

2.4.3 Social Welfare

The social welfare function can be written as

$$W(q_s, \mu) = \alpha(\kappa)u(q) - \mu[k + c(q_s)] \tag{2.38}$$
which has the following necessary conditions.\footnote{For (2.40) we obtain \( \alpha'(\kappa) u(q) + u'(q)[1 - \varepsilon_\alpha(\kappa)] q_s = k + c(q_s) \) using \( \frac{\partial q_s}{\partial q} = \frac{1 - \varepsilon_\alpha(\kappa)}{-\alpha(\kappa)} \). Then we divide both sides by \( q_s c'(q_s) \) and use the fact that \( u'(q) = c'(q_s) \) and rearrange in terms of \( \varepsilon_\alpha, \varepsilon_u, \) and \( \Gamma \).}

\begin{align*}
  u'(q) &= c'(q_s) \\
  \Gamma(q_s) &= \varepsilon_\alpha(\kappa) \left( \frac{1}{\varepsilon_u(q)} - 1 \right) + 1
\end{align*}

Without search frictions, note that \( \Gamma(q_s) = \frac{1}{\varepsilon_u(q)} \), or \( \varepsilon_{TC}(q_s) = \varepsilon_u(q) \), which is the result in Vives (1999). More generally, the optimal deviation of average costs from the minimum increases with a more concave utility function and a less concave matching function.

Comparing (2.39)-(2.40) to (2.32) and (2.33), we find the one-way comparison \( q^*_s < q^e_s \Rightarrow q^* > q^e \) and hence \( \frac{q^*_s}{q} < \frac{q^e_s}{q} \Leftrightarrow \mu^* > \mu^e \). Thus, as in Dixit and Stiglitz (1977) with variable elasticity of demand, if optimal firm size is smaller then the optimal number of firms is greater. Note that the relationship between \( q^*_s \) an \( q^e_s \) is indeterminate, because the former depends on the elasticity of utility and the elasticity of the matching function whereas the latter depends on the elasticity of demand. Compared to Dixit and Stiglitz (1977), however, \( q^e_s \) is higher if \( i > 0 \) because of the inflation wedge and \( q^*_s \) is lower because the social planner takes into account search frictions.

**Proposition 2.6.** The ZKPT equilibrium is inefficient.

The social planner wishes to produce a good up to the amount that marginal utility equals marginal cost and induce variety up to the point that the marginal social benefit equals the marginal social cost. If one abstracted from both congestion effects and the cost of holding money \((i = 0 \text{ and } \varepsilon_\alpha(\mu) = 1)\), efficiency would require both \( r_u(q) = 0 \) and \( r_u(q) = 1 - \varepsilon_u(q) \), which is impossible given the curvature of \( u(\cdot) \). Additionally, inefficiency also arises from congestion effects and the cost of holding real balances.
The Friedman Rule and Augmented HARA Preferences

For numerical analysis, we adopt the functional form used by Zhelobodko et al. (2010). Dubbed “augmented HARA”, it is an extension of HARA preferences consistent with either increasing or decreasing taste of variety. Preferences are given by

$$u(q) = \frac{1}{1-\varrho} \left[ (a + hq)^{1-\varrho} - a^{1-\varrho} \right] + bq$$

where $a \geq 0$, $h \geq 0$, $0 < \varrho < 1$. Additively separable CES arises with $a = b = 0$, and HARA arises with $b = 0$. Furthermore, $a > 0$ bounds the marginal utility at zero consumption.

At $a = 0$, RLV is given by

$$r_u(q) = \frac{\varrho h^{1-\varrho} + bq^\varrho}{h^{1-\varrho} + bq^\varrho}$$. The sign of $b$ determines whether RLV is increasing and $h$ and $\varrho$ are important for the magnitude of demand elasticity. Note that $h$ only affects the relative love for variety if $b \neq 0$. The expanded set of parameters is thus $(b, h, \varrho, A, k)$.

Using these preferences, I show examples in which the Friedman rule is not optimal, illustrated in Figure 2.5.

![Figure 2.5: Social welfare](image)

In this example, social welfare is maximized at $i = 0.0173$, $0.0361$, and $0.0303$ across subpanels a), b), and c), respectively. It is straightforward to check that output falls monotonically with inflation (see appendix). Thus, welfare can rise even though output falls. There are

---

23If $b = 0$, the coefficient of absolute risk aversion satisfies $-\frac{u''(q)}{u'(q)} = \frac{h\varrho^{-1}}{1+hq^\varrho}$, which is hyperbolic. Furthermore, $r_u(q)$ is decreasing for $a > 0$ and constant for $a = 0$. This explains the need to modify HARA to be consistent with increasing RLV.

---

34
two benefits of inflation in equilibrium. It reduces price markups, and it reduces congestion, 
in turn lowering average production costs. The reduction of price markups mitigates the 
inefficiency on the intensive margin. Variety and output both decrease, but if the congestion 
externality is sufficiently strong, welfare can increase.\textsuperscript{24} A lower level (subpanel b) or higher 
elasticity of costs (subpanel c) shift the optimal nominal interest rate higher.

It is instructive to compare this optimal deviation from the Friedman rule to that of compet-
itive equilibrium in Rocheteau and Wright (2005). Inflation there reduces entry of sellers and 
hence congestion, which in turn lowers average costs by increasing the probability that sellers 
match with buyers. Here, lower congestion does not reduce average costs directly. Instead, 
a higher interest rate depresses markups and hence average costs in equilibrium, leading to 
fewer and larger firms.\textsuperscript{25} In the present treatment, the reduction in average costs rests on 
variable elasticity of demand, whereas in Rocheteau and Wright (2005) it results from the 
matching technology.

Inflation thereby reduces sellers’ market power whenever \( r_u \) is increasing. The idea that 
inflation can reduce sellers’ market power is not new. In Diamond (1993) or in a basic New 
Keynesian model, inflation reduces markups with sticky prices, but in this environment the 
result arises from a fall in equilibrium taste of variety rather than nominal rigidities.

The ratio of average costs to marginal costs in the social optimum and equilibrium are given 
by \( \Gamma(q^*_s) = \frac{\varepsilon_\alpha(\mu)}{\varepsilon_u(q)} \) and \( \Gamma(q^e_s) = \frac{1}{1-r_u(q)} \). In words, the socially optimal quantity is uniquely 
determined by the elasticity of utility and elasticity of the matching, whereas the equilibrium 
quantity is determined by the elasticity of demand (or equivalently the RLV). In general, 
firms can be either too big or too small relative to the social optimum.

\textsuperscript{24}Recall that taste for variety is given by the RLV \(-u''(q)q/u'(q)\) which increases with \( q \) whenever \( B < 0 \). 
Since an increase in \( i \) reduces \( q \), it reduces equilibrium love of variety.

\textsuperscript{25}To better understand this, note from (2.32) that a lower markup is equivalent to a lower ratio of average 
to marginal costs: firms are fewer and larger. Average cost may increase or decrease depending on \( k \) and the 
convexity of \( c(\cdot) \). In the examples illustrated, however, they decrease, and the decrease is sufficient to make 
up for welfare losses on the intensive margin and from lower variety.
2.5 Measuring the Welfare Costs of Inflation

Computing the welfare cost of inflation quantifies the general equilibrium effects of taste for variety and monopolistic competition vis-à-vis bargaining and other pricing protocols. I estimate the money demand implied for each model with the empirical money demand from the United States. Following Lucas (2000), we set period length to a year and $\beta^{-1} = 1.03$. Given the assumption that $r = 3\%$ is consistent with zero inflation we ask: what is the percentage $\Delta$ of total consumption that individuals would be willing to sacrifice in order to be in the steady state with an interest rate of 3% instead of the steady state associated with $r$. I use three observables: money stock (M), nominal GDP (PY), and the nominal interest rate, which are taken from the dataset by Ireland (2009), compiled from several different sources. The time range is from 1900-2006. Empirical money demand is defined as $M/PY$. The interest rate $i$ is measured as the short-term commercial paper rate and $M_{1RS}$ is the measure of money demand. 26

I choose $c(q) = q^{\delta}/\delta$ at $\delta = 1.1$ as in Rocheteau and Wright (2009) and $u(q) = q^{1-\varepsilon}$. I also take $U(x) = A \ln(x)$ in the CM and the matching function $\alpha(\kappa) = \kappa/(1 + \kappa)$ and $\sigma = 1$. For the basic model, we normalize $\mu = 1$ so that $\alpha = \frac{1}{2}$.

I use the money demand data to match the first three moments of money demand for different values of $\eta$. Estimation is with respect to $(\varepsilon, A)$ in the basic model and $(\varepsilon, A, k)$ in the free entry model. 27

26 The M1RS aggregate is defined and constructed by Dutkowsky and Cynamon (2003) and and Cynamon et al. (2006) by correcting for a bias in M1 from the growth of retail deposit sweep programs, in which banks reclassified part of checkable deposits as money market deposits to avoid statutory reserve requirements.

27 Due to limited identifying information of money demand, we set $\sigma = 1$ in the calibration. This is not important for the welfare cost of inflation since the latter depends on the money demand and the distribution of surplus, and calibration scheme targets money demand. If one took other parameters as fixed, then idiosyncratic uncertainty would of course raise the welfare costs. However, some of the increase would be mitigated in the presence of a secondary financial market in which agents could transfer liquidity onto other individuals who wish to consume more, as discussed in Berentsen et al. (2007); Berentsen et al. (2014); and Geromichalos and Herrenbrueck (2016).
We calculate the cost $\Delta$ of 10% inflation, which corresponds to $r = 0.03$. The formulas for $\Delta$ are given by the following. In the basic model, sellers make equilibrium profits, which we distribute back to the buyers without loss of generality because of quasilinear utility. Hence, for the basic model, we compare utility net of production costs across interest rates. With entry, equilibrium profits are zero. Hence, we compare utility net of expenditures.

\[
\frac{q_{0.03}^{1-\varepsilon}}{1-\varepsilon}(1 - \Delta)^{1-\varepsilon} + A \ln(1 - \Delta) - \frac{q^{\delta}_{s,0.03}}{\delta} = \frac{\eta^{1-\varepsilon}}{1-\varepsilon} - \frac{q^{\delta}_{s,r}}{\delta}
\]

\[
\frac{q_{0.03}^{1-\varepsilon}}{1-\varepsilon}(1 - \Delta)^{1-\varepsilon} + A \ln(1 - \Delta) - \frac{\eta}{\eta - 1}\mu_{0.03}q^{\delta}_{s,0.03} = \frac{\eta^{1-\varepsilon}}{1-\varepsilon} - \frac{\eta}{\eta - 1}\mu_{r}q^{\delta}_{s,r}
\]

(2.41)

(2.42)

Table 2.4: Welfare costs of inflation: CES

<table>
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<tr>
<th>$\frac{\eta}{\eta - 1}$</th>
<th>$\Delta$</th>
<th>$\varepsilon$</th>
<th>$A$</th>
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<th>$\varepsilon$</th>
<th>$A$</th>
<th>$k$</th>
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<td>0.0105</td>
<td>0.0797</td>
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<td>0.312</td>
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</tr>
</tbody>
</table>

Table 2.4 shows the estimated welfare costs of inflation $\Delta$ along with elasticity of marginal utility $\varepsilon$ and CM consumption $A$. Note that $\eta \varepsilon > 1$, so that $\lambda(\mu)$ is decreasing everywhere and equilibrium is unique. Free entry makes a substantial difference in the estimated values of $\varepsilon$ and $A$ that best fit the money demand moments. Furthermore, there is little divergence in the welfare costs of inflation between the no-entry and free-entry cases until markups reach 30% in the decentralized market, at which point free entry is responsible for a full 2 percentage point increase with respect to the no entry case. The economic rationale is that
inflation reduces entry, which in equilibrium reduces both variety and also reduces the value of existing varieties due to the complementarity effect.

At $\frac{n}{n-1} = 1.0001$, taste of variety and markups are negligible, so the economy resembles Walrasian price taking. It is instructive to compare this to Rocheteau and Wright (2009), who find welfare costs of inflation at 1.54% with entry. The small difference is that the authors consider preferences with marginal utility bounded at zero and that they use nonlinear least squares rather than method of moments. The welfare costs of inflation are closer to bargaining. Rocheteau and Wright (2009) obtain a cost of 5.36% at a bargaining power $\theta = 0.3$, chosen to match a 10% markup. Here the associated welfare cost at 10% markup is 3.81%, which is lower than the bargaining estimate but much higher than the Rocheteau and Wright (2009) estimate at competitive equilibrium. The magnitudes are thus similar to bargaining outcomes of the literature, but rely instead on price posting. The results contrast sharply with Dong (2010), who obtains welfare costs of inflation of only 1.47% under price posting with varieties fixed and 1.52% with endogenous varieties.

Another point of comparison to the literature concerns extensive margin effects: the difference in the welfare impact of a policy variable from endogenizing the frequency (or density) of trade. Here, there are two major observations. First, at a given markup, free entry of sellers generates higher welfare costs of inflation–extensive margin effects are positive. Second, higher markups are associated with higher welfare costs of inflation.

The extensive margin consists of the number of sellers for Rocheteau and Wright (2009) and number of varieties for Dong (2010). In both these papers, the welfare costs of inflation are non-monotonic in the buyer’s bargaining power $\theta$. In the former case, there is a tradeoff between the thick market externality and congestion externality. With low bargaining power, endogenizing sellers raises welfare more by reducing congestion than it impairs it by reducing trading opportunities. With high bargaining power, the opposite is true. In the latter case, the tradeoff arises because of a double holdup problem: prior to trade, buyers and sellers
make a costly investment in holding money and capacity for varieties, respectively. Here the welfare cost is always higher with endogenous variety, and the difference is increasing with the markup. This is because low bargaining power of buyers is associated with high taste for variety, in which case the reduction in sellers is more costly.

There are two reasons we did not include computations for variable elasticity of demand in Table 2.4. First, the set of parameters to estimate \((\varepsilon, B, h, k, A)\) cannot be adequately pinned down from a markup target and the money demand. Second, with the augmented HARA preferences, there is a tradeoff between fitting markups and fitting money demand. An appropriate fit of money demand favors a choice of smaller values of \(\varepsilon\). However, in this case, markups tend to be too small (typically below 10%).  

Instead, we address a more restricted problem: for a given markup, are welfare costs of inflation lower than with CES preferences and entry? The answer to this question provides some evidence of the markup channel. We change the empirical strategy to using nonlinear least squares to obtain the best fit of money demand for these preferences. We fix \(k = 0.1\) and do some sensitivity analysis around the choice of \(k\). Table 2.5 reports the results. The average markup of the best-fitting curve is 9.4\%, and the estimated welfare cost of 10\% inflation is about 2.41\%, which is significantly lower than the 3.8\% under CES preferences with entry.

Table 2.5: Welfare costs of inflation: Additively separable preferences with entry

<table>
<thead>
<tr>
<th>Markup: ((1 - r_u(q))^{-1})</th>
<th>(\Delta)</th>
<th>(\varepsilon)</th>
<th>(B)</th>
<th>(A)</th>
<th>(h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.094</td>
<td>0.0241</td>
<td>0.0771</td>
<td>-0.251</td>
<td>10.27</td>
<td>1.64</td>
</tr>
</tbody>
</table>

28 The general problem is that high markups reduce the curvature of the model-implied money demand, making it nearly linear. This fact suggests a different functional form should be used; however, I am not aware of a suitable alternative that would ameliorate this problem.

29 The details of the money demand are described in A.3.3.

30 We try \(k = 0.12\), which yields a welfare cost of about 1.83\%, and \(k = 0.08\), which yields a welfare cost of 2.3\%.
2.6 Conclusion

This paper studies the role played by endogenous variety and monopolistic competition in the transmission of monetary policy. The integration of these ingredients into a New Monetarist model enables us to study the effects of inflation on the intensive and extensive margin with preferences that give rise to either constant or variable markups. Under CES preferences, firm size is inefficiently low, with the gap increasing in product differentiation and the extent of search frictions. The Friedman rule is optimal with CES preferences, but can be suboptimal with variable elasticity of demand. This can happen because higher interest rates increase firm size and lower markups with an increasing relative love for variety. This effect attenuates the inefficiency on the intensive margin and reduces average costs. Generally, the welfare costs of inflation are high and comparable to the bargaining outcome of Rocheteau and Wright (2009). A key difference, however, is that, given an endogenous measure of sellers, welfare costs of inflation are higher, and they monotonically increase with the markup. Inflation can only affect firm size with variable elasticity of demand, and will increase it with increasing RLV.
Chapter 3

Unsecured Credit, Product Variety, and Unemployment Dynamics

3.1 Introduction

Whereas the transition from barter to a monetary economy spanned thousands of years, and the use of checking deposits developed over hundreds of years, revolving credit increased from about 1% of consumption in 1970 to over 10% in 2009. This category also rose from 1.7% of consumer credit in 1970 to over 28.7% in 2013 (Durkin et al. (2014)).

For households with at least one credit card, available credit exceeds the amount held in checking and savings accounts and mainly determines what households can spend in the short run (see Fulford (2015)). Of most interest for business cycle purposes, revolving credit exhibits significant volatility. Fulford (2015) analyzes Equifax data from 1999-2013 and finds that 30% of individuals with credit access experience a change in debt limits between quarters. Conditional on a change in the debt limit, the mean percentage decrease is 29.4% and the mean per-

\footnote{Revolving credit is generally unsecured and defined in terms of a minimum payment, debt limit, and interest rate. The name comes from the fact that consumers can “revolve” the balance by paying only partially.}
percentage increase is 26.9%. Furthermore, revolving credit is volatile and comoves negatively with unemployment. The Great Recession, in particular, was characterized by a large and persistent surge in unemployment, a sharp drop in unsecured credit, and a decrease in the aggregate number of firms, which proxies for the extent of product variety.

As revolving credit is liquid and volatile, it is an important variable to consider in the analysis of business cycles. Accordingly, I augment the Mortensen and Pissarides (1994) unemployment model with an endogenous amplification mechanism consistent with the liquidity role of credit, its negative comovement with unemployment, and positive association with product variety. The mechanism also amplifies the effects of shocks on unemployment, which partially overcomes the unemployment volatility puzzle stressed by Shimer (2005). The multiplier relies on general equilibrium feedback between the credit market, the goods market, and the labor market; and can be decomposed into two channels. The entry channel links higher household debt limits to more product demand, firm entry, and variety. The consumption value channel links more product variety to a greater incentive to repay and higher debt limits.

The model features three key ingredients, two of which generate the mutual feedback. First, monopolistic competition with entry à la Dixit and Stiglitz (1977) (DS) gives rise to a positive association between variety and product demand. I use a large firm setup for the following reasons. First, downward sloping demand curves at the firm level renders firm size determinate and permits us to analyze both the intensive margin (quantity of a particular good) and the extensive margin (the product variety space). Second, with idiosyncratic destruction of both firms and workers, we can calibrate the model to match worker flows and the product destruction rate simultaneously. Third, by treating the entry of firms and posting of vacancies as separate margins, we can interpret the firm entry cost as a sunk product development cost, as do Bilbiie et al. (2012). Upon paying these costs, firms continue to operate until their product exogenously becomes obsolete.
The second component is endogenous borrowing constraints via limited commitment, as formalized by Kehoe and Levine (1993) (KL). Limited commitment means that creditors have some ability to observe the payment history of borrowers and punish by withholding future access to credit.\footnote{This feature differs from limited enforcement, in which lenders can seize a fraction of assets, through either collateral or a lawsuit. Limited commitment is reasonable for modeling uncollateralized credit. The role of legal enforcement seems small: Furletti (2003) provides evidence that banks sell non-bankrupt defaulting credit accounts for five cents on the dollar. Furthermore, Herkenhoff (2012) uses Equifax data to show that nearly 30\% of delinquent lines end up indefinitely in collection. The fact that collection agencies are willing to pay so little to purchase debt indicates that they expect to recoup only a small fraction of what is owed.} Though we opt to model credit constraints in terms of limited commitment, the mutual feedback loop would arise even with limited enforcement: in that case, greater incomes boost debt limits because more assets can be seized, which increases spending and stimulates the production of more variety.

The third model ingredient is a frictional labor market as in Mortensen and Pissarides (1994) (MP). Whereas the first two model ingredients are necessary and sufficient to generate the mutual feedback loop, the hiring channel links product demand to employment. More access to credit raises firm revenue and stimulates job creation, as in Bethune et al. (2015). The hiring channel also reflects the standard congestion externality in Mortensen and Pissarides (1994). A positive productivity shock raises hiring and market tightness, reduces the probability of filling a vacancy, and thereby raises hiring costs and marginal costs. Prices are a constant markup of marginal cost and hence rise one-for-one. Rising prices, in turn, dampen the quantities demanded by consumers. Thus, congestion induces upward pressure on prices which partially counteracts the direct negative effect of marginal costs from productivity shocks.\footnote{We verify in the simulated equilibria that productivity shocks have only a very small downward effect on prices: they directly lower marginal costs, but also spur firm entry and more credit, increasing hiring and marginal costs.} Therefore, this paper also models the implications of matching frictions on prices. Figure 3.1 summarizes the core components and causal pathway in the context of a total factor productivity shock. Similar reasoning clearly extends to shocks on preferences or credit.
The model successfully approximates the volatility of unemployment and credit in the data, as well as the negative relationship between these two variables. It also predicts the procyclicality of the credit-to-consumption ratio and the positive comovement of credit and product variety. This model thus helps overcome much of the unemployment volatility puzzle raised by Shimer (2005). Section 3.8 discusses these results in more detail and suggests further ways to improve the fit of the model.

We run two experiments. First, we compare the model economy to one with low product diversification and hence lower markups. Second, we compare the model economy to one with a fixed product space. In either case, the impulse response of unemployment in percentage points is on average 50% higher with a maximal difference of over 60%.

3.2 Literature Review

This paper relates to the empirical literature on product variety and credit constraints and several strands of the theoretical literature. We first discuss some of the most pertinent empirical work on credit and product variety.

Evidence abounds that unemployed households are credit constrained. For instance, Sullivan (2008) documents that households in the bottom decile of the asset distribution do not borrow
from unsecured credit markets in response to job loss. Using Canadian data, Crossley et al. (2013) find that a quarter of recent job losers could not borrow to increase consumption. Bethune (2014) shows that the series on consumer credit from the Flow of Funds Account is strongly procyclical and has a negative correlation with unemployment of 61.7%. The correlation between sales and credit is 64.3%, which motivates closer examination into the joint determination of employment, aggregate demand, and credit.

Product variety is both an important part of aggregate output and highly procyclical. Broda and Weinstein (2010) measure products at the product bar code level, covering 700,000 products purchased by 55,000 households. The covered sectors amount to 40% of expenditure on goods in the Consumer Price Index. Net product creation is strongly procyclical and primarily driven by creation rather than destruction. In particular, one percentage point in higher sales growth is associated with a rise in net product creation of 0.35 percentage points, and consumers devote 9% of annual purchases to new goods.

Moreover, this paper integrates four strands of the theoretical literature. The first body of work concerns limited commitment, which builds on Kehoe and Levine (1993) and Alvarez and Jermann (2000). Kocherlakota (2005) is the first to emphasize the role of record-keeping technologies in sustaining credit. Bethune et al. (2015) model the feedback between household unsecured debt and aggregate unemployment, allowing for the coexistence of credit and partially liquid assets. The aggregate state of the labor market affects the debt limit through the frequency of trade. Therefore, consumers are more likely to satisfy random consumption opportunities with more retailers in the market. The model explains a large fraction of the decrease in unemployment between 1978 to 2008, but can only explain a portion of

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4The data is provided by ACNielsen Homescan Database. ACNielsen provides home scanners to about 55,000 households, who scan the purchase of every good with a barcode. Households are a demographically balanced sample, spanning 23 U.S. cities.

5Furthermore, there is econometric evidence of significant welfare effects of product variety, motivating it as a potential source of amplification. In particular, Broda and Weinstein (2004) measure the growth in product variety from U.S. imports between 1972 and 2001 using extremely disaggregated data. They construct a price index from many estimated elasticities of substitution and quantify that the welfare gains from import variety growth are 2.8% of GDP.
unemployment after 2008. The mechanism explored here does not rely on search frictions in
the goods market but instead relies on the link between product variety and debt limits.

Second, this paper draws on the literature on unemployment and money/credit. Bethune
(2014) provides empirical evidence, based on the Survey of Consumer Finances, that the
unemployed face tighter borrowing constraints than the employed, and explores the business
cycle consequences thereof. He models the constraints in terms of limited enforcement,
estimates the innovations for both credit and productivity shocks, and compares the model-
implied series for unemployment, vacancies, GDP, and debt to the data. The model with
credit shocks matches the broad movements of these variables, with important exceptions.\textsuperscript{6}

Herkenhoff (2013) develops a general equilibrium model in which households search both for
jobs and opportunities to borrow, and can default on debt. The paper quantifies mecha-
nisms through which credit impacts unemployment over the business cycle, and constructs
aggregate time series for unemployed households’ access to credit and use of credit post-1970.
Households search for better paying but harder to find jobs, relying on credit should the effort
fail (self-insurance effect); and save less and incur more debt, which spurs them to find jobs
faster (wealth distribution effect). Credit growth generates longer and deeper recessions.

Third, this paper relates closely to several others which consider the role of firm entry and
monopolistic competition in propagating shocks. Bilbiie et al. (2012), for instance, augments
a real business cycle model with endogenous entry and variety. Expansions driven by pro-
ductivity shocks boost entry of sellers. The sluggish response of the number of producers
generates a new propagation mechanism. The model matches does at least as well as a tradi-
tional RBC model (and better in matching volatility of output and hours) and can also match
stylized facts related to entry, profits, and markups. Devereux et al. (1996) and Chatterjee
and Cooper (2014) also rely on procyclical product variety. None of these papers, however,

\textsuperscript{6}The experiment understates unemployment in the Great Recession and early 1980’s, and the fall in GDP
in the early 1980’s.
link firm entry to household credit or address unemployment. I abstract from a causal link between entry and markups.

Fourth, this model also contributes to the literature on coordination problems through the strategic complementarity between credit and product variety. Higher debt limits boost aggregate demand and generate more firm entry; the greater variety makes households more willing and able to repay their debt, rendering the initial rise in debt limits incentive compatible. Diamond (1982) studies a search model of the goods market with a thick market externality; the same fundamentals can generate either a high-trade or low-trade equilibrium. The paper by Schaal and Taschereau-Dumouchel (2016) is closer to the present treatment and generates a coordination problem in that firms in which firms wish to post more vacancies with higher employment. This type of aggregate demand externality arises from the fact that the demand for intermediate goods is an increasing function of output.\footnote{The introduction of some curvature in preferences or decreasing returns to scale should dampen these effects.}

3.3 Evidence on Credit, Product Variety, and Unemployment

First, we show the procyclicality of product variety from two data sources.\footnote{We also document the decline in the number of firms during the Great Recession in the appendix, based on the Census Statistics of U.S. Businesses.} We reprint Figure 1A from Broda and Weinstein (2010), which plots net product creation and growth in total sales, taken from the Nielsen Consumer Panel Dataset. Furthermore, much of the procyclicality of net product creation comes from creation rather than destruction.
Since I lack access to the raw data from Nielsen, I instead use international trade data to highlight the positive comovement between revolving credit and product variety. The data consist of the Tariff System of the United States (TSUSA) 8-digit data and the Harmonized Tariff System (HTS) 10-digit data between 1972 and 2006. We follow Broda and Weinstein (2004) by defining a variety to be a good-country pair.
Figure 3.2 shows that credit and product variety align well. In particular, the movements closely match in the early 1980’s, late 1990’s, and early 2000’s. There is evidence that unsecured credit Granger causes product variety. There is also some evidence that product variety Granger causes credit, but it is not significant at the 5% level for three lags.

Table 3.1: Granger Causality Tests

<table>
<thead>
<tr>
<th></th>
<th>lag= 2</th>
<th></th>
<th>lag= 3</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F-stat</td>
<td>$Pr(&gt; F)$</td>
<td>F-stat</td>
<td>$Pr(&gt; F)$</td>
</tr>
<tr>
<td>Variety → Credit</td>
<td>6.83</td>
<td>0.0038</td>
<td>2.72</td>
<td>0.066</td>
</tr>
<tr>
<td>Credit → Variety</td>
<td>8.73</td>
<td>0.0011</td>
<td>4.34</td>
<td>0.014</td>
</tr>
</tbody>
</table>

Second, we document the negative relationship between revolving credit and unemployment
during the Great Recession.

Figure 3.3 indicates that, from January 2008 to January 2010, unemployment rose from 5% to 10%, and the ratio of revolving credit to consumption fell from 10% to 9%. Though the decline in credit may not seem dramatic, it presents a significant departure from the prevailing upward trend: revolving credit relative to consumption increased had increased from about 1% in 1970 to 9% in 2009.

Third, Table 3.2 compares the moments of key variables: unemployment, consumption, market tightness, credit relative to consumption, and the price level, which we denote symbolically as \((u, C, \theta, d/C, P)\).

Most of the data we use is available through FRED. We use Real Gross Domestic Product (GDPC1), personal consumption expenditure on nondurables (PCND), and revolving credit (REVOLSL), number of unemployed (UNEMPLOY) and civilian noninstitutional population (CNP16OV). For vacancies, we use the help wanted index constructed by Barnichon (2010), which combines newspapers and online postings.

I consider the time period 1984-2014, and convert all series to a monthly frequency. The choice of time period reflects a compromise. Rising unsecured credit reflects an important structural
thermore, we divide consumption by population. I construct market tightness as the ratio of vacancies to unemployment, and transform each series by taking a proportional deviation from the mean: \( \tilde{Z} = (Z - \overline{Z}) / \overline{Z} \), as in Petrosky-Nadeau and Zhang (2016). Finally, I detrend with the Baxter-King filter, targeting a business cycle frequency between 6 and 32 quarters.

<table>
<thead>
<tr>
<th></th>
<th>SD</th>
<th>RSD</th>
<th>Cor(x,u)</th>
<th>Cor(x,d)</th>
<th>Cor(x, x-1)</th>
<th>Cor(x, x-2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u )</td>
<td>0.113</td>
<td>11.431</td>
<td>1.000</td>
<td>-0.322</td>
<td>0.995</td>
<td>0.979</td>
</tr>
<tr>
<td>( d )</td>
<td>0.044</td>
<td>4.426</td>
<td>-0.322</td>
<td>1.000</td>
<td>0.993</td>
<td>0.977</td>
</tr>
<tr>
<td>( d/C )</td>
<td>0.036</td>
<td>3.655</td>
<td>-0.021</td>
<td>0.878</td>
<td>0.992</td>
<td>0.974</td>
</tr>
<tr>
<td>( \theta )</td>
<td>0.204</td>
<td>20.615</td>
<td>-0.854</td>
<td>0.034</td>
<td>0.995</td>
<td>0.981</td>
</tr>
<tr>
<td>( P )</td>
<td>0.007</td>
<td>0.731</td>
<td>-0.421</td>
<td>0.567</td>
<td>0.982</td>
<td>0.941</td>
</tr>
</tbody>
</table>

Table 3.2: Moments of key variables: 1984-2014, monthly. Data is transformed by proportional deviations from the mean and detrended through a Baxter-King filter between 6 and 32 quarters.

Unemployment, market tightness, and credit are volatile series. There is a strong negative correlation between unemployment and credit. Table 3.2 also shows the well-known large negative association between unemployment and market tightness. The price level is slightly procyclical, but nevertheless features a negative correlation with unemployment and a positive correlation with credit. Lastly, all series are highly persistent.

### 3.4 Environment

Time is discrete and infinite: \( \{1, 2, 3, \ldots \} \). There exists a measure one of households and a large (endogenous) measure \( S_t \) of firms providing differentiated goods in a monopolistically competitive market. Households work, consume, and enjoy leisure; firms maximize profits change in the economy, and we wish to study the economic patterns once unsecured debt plays a prominent role. Nevertheless, because of the small number of business cycles, and to prevent an excessive influence from the Great Recession, we do not want to restrict the sample excessively toward the present.
and pay dividends. Firms have access to an interconnected monitoring technology, which renders household default public information. The only friction in the loan market is limited commitment. Each period is divided into three stages, as in Berentsen et al. (2011). In the first stage, firms can enter production, the aggregate shock is realized, and households and firms trade indivisible labor serves in a labor market (LM). In the second stage, households use credit to finance consumption goods in a monopolistically competitive sector (MM). In the third stage, households receive wages and an endowment of the numeraire good, settle their debts, and consume the general good. Figure 3.4 summarizes the timing.

The consumption good in the CM is the numeraire. Quasilinear utility eliminates the motive to smooth debt repayment, and enables us to assume without loss that the entire debt must be repaid in the CM. Each household is endowed with 1 unit of labor and has expected lifetime discounted utility of

\[ E_0 \sum_{t=0}^{\infty} \beta^t \left\{ l(1 - e_t) + \frac{AQ_t^{1-\varepsilon}}{1 - \varepsilon} + c_t \right\} \]  \hspace{1cm} (3.1)

in which \( Q_t \) is a Dixit-Stiglitz aggregator

\[ Q_t = \left( \int_{\Omega} q_{i,t}^\rho d_i \right)^{1/\rho} \]  \hspace{1cm} (3.2)

where \( 0 < \rho < 1 \) and \( \Omega \) is the range of varieties potentially desired by the consumer. A
measures the relative desirability of MM goods with respect to CM goods.\footnote{The preferences here can be regarded as a special case of Dixit and Stiglitz (1977) with the CM good playing the role of the outside good.} Moreover, $\beta$ is the discount factor, $l \in \mathbb{R}^+$ is utility from leisure or home production, $e_t \in \{0, 1\}$, and $q_{i,t}$ is consumption of variety $i$ at time $t$ in the MM, and $c_t$ is consumption of the CM good at time $t$.

The measure of matches between $U$ searching workers and $V$ vacancies are given by $m(U, V)$. The matching function $m(U, V)$ has constant returns to scale and is strictly increasing and concave in each argument. Furthermore, $m(0, V) = m(U, 0) = 0$ and $m(U, V) < \min\{U, V\}$.

The measure of searching workers is the previously unemployed $U_{t-1}$. The job finding probability $h(\theta_t)$ thus satisfies $m(U_{t-1}, V_t)/U_{t-1} = m(1, \theta_t)$, where $\theta_t = V_t/U_{t-1}$ is labor market tightness. The job filling probability for firms is given by $m(U_{t-1}, V_t)/V_t = m(1/\theta_t, 1) = f(\theta_t)$.

Matches formed in the LM break down at rate $s$ at the end of the CM. Additionally, firms exit the market with probability $\delta$. Each firm $i$ is large enough to eliminate uncertainty about labor flow. Employment, conditional on firm survival, satisfies $n_{i,t+1} = (1 - s)n_{i,t} + f(\theta_t)v_t$.

Posting vacancies costs $k$ units of output, so that $k/f(\theta)$ is the per-unit hiring cost.

Firms pay a sunk entry cost $\gamma$ in terms of the numeraire good at the beginning of the LM. Each worker produces $z$ units of the MM good, and there is also an endowment $z$ of the CM good each period. Thus, changes in $z$ proportionately scale the quantities of MM and CM goods. In the monopolistically competitive goods market, households match with a measure $S_t$ of retailers. Households trade in the MM through borrowing, but lack commitment to repay their debt. In order to sustain credit relationships, borrowers face a potential cost of default. With probability $\psi \in [0, 1]$, households’ default is publicly recorded. They cannot access the credit market this period but can do so in subsequent periods with probability $1 - \lambda$.

The following conventions compress notation. The symbol $E$ denotes expectations formed...
conditional on current information. A sufficient statistic for current information is the ag-
gregate state $\Theta = (u_{-1}, S_{-1}, z)$. In addition to productivity $z$, we need to track last-period
unemployment $u_{-1}$ and last-period measure of sellers $S_{-1}$ because the unemployment rate
affects the aggregate demand in the economy through the product variety space and because
the measure of sellers today will equal at least $(1 - \delta)S_{-1}$. Finally, I exploit the recursive
structure to omit time subscripts and use the prime symbol $'$ for next-period values.

3.5 Equilibrium

3.5.1 Centralized Settlement (CM)

Consider a household entering the CM in period $t$. The individual state of the household is
a pair consisting of employment status $e \in \{0, 1\}$ and debt $d$ from the previous MM. Section
3.5.3 shows that all households have the same debt limit $\overline{d}(\Theta)$, and, since they are identical,
hold debt $d(\Theta)$ in the MM. But, for now, we just regard $d$ as an individual state variable.
Let $W_e(d, \Theta)$ be the value function with state $(e, d, \Theta)$.

\[
W_e(d, \Theta) = \max_c \left\{ c + l(1 - e) + \beta \mathbb{E}[U_e(\Theta')] \right\} \quad (3.3)
\]

subject to

\[
c + d = z + ew + (1 - e)b + \Delta - T \quad (3.4)
\]

where $\Delta$ is the share of profits from firms. The expected value in the future labor market
depends on current employment status and the aggregate state. Equation (3.3) says that
households choose consumption $c$ in order to maximize lifetime utility subject to a budget
constraint. The budget constraint (3.4) says that the sum of consumption and debt repayment
equals income from labor and the profits of firms minus taxes $T$. Substituting the constraint (3.4) into the objective function (3.3) yields

$$W_e(d, \Theta) = z + ew + (1 - e)(l + b) + \Delta - T + \beta \mathbb{E}[U_e'(\Theta')]$$

(3.5)

which using the linearity in terms of $d$ can be expressed as

$$W_e(d, \Theta) = -d + W_e(0, \Theta)$$

(3.6)

The value of a household with no access to credit is

$$\tilde{W}_e(\Theta) = ew + (1 - e)(l + b) + \Delta - T + \beta \mathbb{E}[\tilde{U}_e'(\Theta')]$$

(3.7)

where $\tilde{U}_e(\Theta)$ is the LM value function of a household without access to credit with employment status $e$ and aggregate state $\Theta$.

### 3.5.2 Monopolistically Competitive Market (MM)

Each household is able to borrow $d \leq \tilde{d}$ from a competitive loan market, in which $\tilde{d}$ is determined in equilibrium by limited commitment. After securing the loan, the household meets with an endogenous measure of firms $S$. Let $V_e(\Theta)$ be the household lifetime utility upon entering the MM at date $t$ with employment status $e$. Let $Q = (\int_S q_i^e d_i)^{1/\rho}$, be the quantity index adjusted for the number of available varieties in a given period.

$$V_e(\Theta) = \max_{q, d} \left\{ A \frac{Q(S)^{1-\varepsilon}}{1-\varepsilon} + W_e(d, \Theta) \right\}$$

(3.8)
where
\[ d = \int_0^S p_i q_i \, di \] (3.9)
\[ d \leq \bar{d} \] (3.10)

The problem (3.8)-(3.10) features the following. First, the buyer can borrow up to \( d \leq \bar{d} \), which arises from limited commitment. Second, the sum of payments to firms consists of debt \( d \). If the household lacks access to credit, then \( d = \bar{d} = 0 \). We rewrite the problem using (3.6) as
\[ \max_{q_i} \left\{ \frac{A Q^{1 - \varepsilon}}{1 - \varepsilon} - \int_S p_i q_i \, di \right\} \] (3.11)

such that
\[ \int_0^S p_i q_i \, di \leq \bar{d} \] (3.12)

The condition (3.12) is the budget constraint, which depends on the endogenous debt limit.

Let \( \zeta \) be the corresponding multiplier on (3.12), which represents the marginal benefit of a one-unit increase in the debt limit. Then the first order condition with respect to \( q_i \) is
\[ [q_i] \quad A Q^{-\varepsilon} \left( \frac{Q}{q_i} \right)^{1 - \rho} - (1 + \zeta) p_i = 0 \] (3.13)

The demand \( q(p) \) satisfies
\[ q = \left( \frac{A Q^{1 - \rho - \varepsilon}}{(1 + \zeta) p} \right)^{1/(1 - \rho)} = \left( \frac{A S (1 - \rho - \varepsilon) / \rho}{(1 + \zeta) p} \right)^{1/\varepsilon} \]

under symmetry. The consumer is satiated (at level \( q^* \)) if \( \zeta = 0 \). In general, \( q \) (and \( q^* \)) increase in \( Q \) (and with symmetry \( S \)) if and only if \( \rho + \varepsilon < 1 \), which requires a high taste for variety.
relative to the elasticity of marginal utility. This latter feature is the aggregate demand externality introduced by Dixit and Stiglitz (1977) and Blanchard and Kiyotaki (1987) and emphasized by Schaal and Taschereau-Dumouchel (2016) in a labor search model.

Using (3.13), for any goods $i, j$, $q_i / q_j = (p_j / p_i)^{1/(1-\rho)}$. Hence, the elasticity of demand of any good is constant and equal to $\eta = 1/(1 - \rho)$, a standard result. The consumption decisions of a household can be summarized as $q = \min \{q^*, \bar{d}/(Sp)\}$. A defaulting household faces the same problem with $\bar{d} = 0$ and consumes zero. Hence, $\tilde{V}_e(\Theta) = \tilde{W}_e(\Theta)$.

**Labor Market**

Consider an employed household at the beginning of a period. Its lifetime expected utility is

$$U_1(\Theta) = (1 - s)(1 - \delta)V_1(\Theta) + [1 - (1 - s)(1 - \delta)]V_0(\Theta)$$ (3.14)

in which $\delta \geq \delta$ is the aggregate firm destruction rate. With probability $(1 - s)(1 - \delta)$, the firm is not exogenously destroyed and the household remains employed ($e = 1$). With complementary probability, the household loses its job and becomes unemployed ($e = 0$), in which case it cannot find another job until the next LM.

The expected lifetime utility of a household who is unemployed is

$$U_0(\Theta) = (1 - h)V_0(\Theta) + hV_1(\Theta)$$ (3.15)

where $h$ is the job finding probability. Using the values for $V_j$ and $V_0$, (3.14) and (3.15) can
be rewritten as

\[
U_1(\Theta) = A \frac{Q^{1-\varepsilon}}{1 - \varepsilon} - d + (1 - s)(1 - \delta)W_1(0, \Theta) + [1 - (1 - s)(1 - \delta)]W_0(0, \Theta) \tag{3.16}
\]

\[
U_0(\Theta) = A \frac{Q^{1-\varepsilon}}{1 - \varepsilon} - d + (1 - h)W_0(0, \Theta) + hW_1(0, \Theta) \tag{3.17}
\]

For brevity, we omit the value functions of households lacking credit access.

### 3.5.3 Incentive Compatibility

Consider a household with debt level \(d\) and employment status \(e\) in the CM. The incentive compatibility constraint is

\[
-d + W_e(0, \Theta) \geq \psi \tilde{W}_e(\Theta) + (1 - \psi)W_e(0, \Theta) \tag{3.18}
\]

in which \(\tilde{W}_e(\Theta)\) is the value of a household who is excluded temporarily from credit transactions. The left side of (3.18) is the expected lifetime utility of a household if it does not default, and the right-hand side is the expected lifetime utility of a household who defaults, which is recorded with probability \(\psi\).

An equivalent formulation of the incentive compatibility constraint is

\[
d \leq \bar{d} \equiv \psi[W_e(0, \Theta) - \tilde{W}_e(\Theta)] \tag{3.19}
\]

From (3.19), \(\bar{d}\) is independent of employment status. Replacing for \(W_e(0, \Theta)\) and \(\tilde{W}_e(0)\), we rewrite the debt limit as

\[
\bar{d} = \psi \beta \mathbb{E}[U_e'(\Theta') - \tilde{U}_e'(\Theta')] \tag{3.20}
\]

The appendix derives the following form of the debt limit:
Proposition 3.1. The debt limit \( \bar{d} \) satisfies the expectational difference equation

\[
\frac{\bar{d}}{\psi} = \beta \mathbb{E} \left\{ A Q^{1-\varepsilon} / (1 - \varepsilon) - S' q' + \lambda \frac{\bar{d}}{\psi} \right\}
\]

in which \( \lambda \) is the probability that a defaulting household is not able to reaccess the loan market. The quantity \( A Q^{1-\varepsilon} / (1 - \varepsilon) - S p q \) is the consumer surplus in the MM for households with access to credit. The Inada condition guarantees a \( q > 0 \) for which consumer surplus positive given sellers \( S \) and prices \( p \). Proposition 3.1 says that the present discounted value of credit access \( \bar{d} / \psi \) consists of the value of trade in the MM and the higher value of future credit lines.

### 3.5.4 Labor Market (LM)

**Firms**

A prospective entrant enters at cost \( \gamma \). The value function prior to entry satisfies

\[
J(\Theta) = \max \{-\gamma + \Pi(0, \Theta), 0\}
\]

in which \( \Pi(0, \Theta) \) is the value of a firm after entry at aggregate state \( \Theta \), with an initial set of 0 workers. Firms hire \( n \) units of labor to produce \( zn \) units of output. Posting a vacancy costs \( k \), \( f(\theta) \) is the vacancy filling probability, and matches are destroyed at rate \( s \). Firm-level employment transitions according to \( n' = (1 - s)n + f(\theta)v \). Hence, the value function for the firm after entry is

\[
\Pi(n, \Theta) = \max_{n', v} \{ p(zn')zn' - wn' - kv + \beta \mathbb{E} [(1 - \delta)\Pi(n', \Theta') + \delta J(\Theta')] \}
\]
subject to

\[ n' = (1 - s)n + f(\theta)v \quad (3.24) \]

\[ n' \geq (1 - s)n \quad (3.25) \]

in which \((3.25)\) reflects constraint \(v \geq 0\). In B.2.1, we show that the solution of the firm problem reduces to \(MR = MC\), for \(MR = \rho p\) for \(p = (1 + \zeta)AQ^{-\varepsilon}(Q/q)^{1-\rho}\). The expected marginal costs are

\[
MC = \frac{1}{z} \left\{ w + k\mathbb{E} \left[ \frac{1}{f(\theta)} - \frac{\beta(1 - \delta)(1 - s)}{f(\theta')} \right] \right\}.
\]

(3.26)

Contrary to Mortensen and Pissarides (1994), an interior solution exists for all \(k > 0\), which is depicted by Figure B.1 in the appendix. Additionally, if \(\rho + \varepsilon < 1\), then \(q\) is increasing in \(Q\), which implies hiring increases with aggregate demand.

Furthermore, the value of an entrant satisfies

\[
\Pi(0, \Theta) = pq - wn' - \frac{kn'}{f(\theta)} + \mathbb{E}\beta[(1 - \delta)\Pi(n', \Theta') + \delta J(\Theta')] \quad (3.27)
\]

The value \(\Pi\) of a firm equals period profits plus the continuation value. The profits are total revenues \(pq\) net of the wage bill \(wn'\) and vacancy posting costs \(kv\). The continuation value is \(\beta\mathbb{E} \{(1 - \delta)\Pi(n', \Theta') + \delta J(\Theta')\}\).

Free entry in each period implies \(J = 0\), provided that the measure of entrants \(\mu = S - (1 - \delta)S_{-1} > 0\).
\[
\gamma = \Pi(0, \Theta) \tag{3.28}
\]
\[
\Pi(n, \Theta) = \gamma + (1 - s) \frac{kn}{f(\theta)} \tag{3.29}
\]

Combining (3.27), (3.28) and (3.29) yields

\[
\gamma = pq - wn' - \frac{kn'}{f(\theta)} + \beta(1 - \delta)\mathbb{E}\left\{ \gamma + (1 - s) \frac{kn'}{f'(\theta')} \right\} \tag{3.30}
\]

We rearrange (3.30) in flow terms as \(\gamma[1 - \beta(1 - \delta)] = (p - MC)q\). Finally, using \(MC = \rho p\), the free entry condition reduces to

\[
\gamma(r + \delta) = (1 + r)(1 - \rho)pq \tag{3.31}
\]

Equation (3.31) provides a negative relationship between price and quantity such that firm-level sales are constant. In particular, \(S = \max\{(1 + r)(1 - \rho)d/\gamma(r + \delta)], (1 - \delta)S_{-1}\}\), highlighting the existence of path dependence from previous entry.

### 3.5.5 Wage Setting

Wages are determined according to a sharing rule which numerically coincides with Nash bargaining.\(^{11}\) The marginal surplus of hiring the \(n\)-th worker to the firm is

\[
J_n = MR[q(n)]z - w + \beta(1 - \delta)(1 - s)\mathbb{E}\left[ \frac{k}{f'} - \lambda_v \right] \tag{3.32}
\]

\(^{11}\)I avoid calling the wage determination Nash bargaining per se because the firm takes the wage as given in deciding the quantity of labor, whereas an explicit treatment would require subgame perfection and the Stole-Zwiebel bargaining outcome.
Furthermore, from (B.4) $J_n = \frac{k}{f} - \lambda V$ holds each period. The marginal surplus of the worker is $V_1(\Theta) - V_0(\Theta)$. The following result further characterizes the marginal surplus of the worker:

**Lemma 3.1.** The marginal surplus of a worker satisfies

$$V_1(\Theta) - V_0(\Theta) = w - (b + l) + [(1 - s)(1 - \delta) - h(\theta)]\beta \mathbb{E}[V_1(\Theta') - V_0(\Theta')]$$

(3.33)

**Proposition 3.2.** The wage $w$ solves

$$w = \chi \left\{ MR[q(n)] z + \beta h(\theta) \mathbb{E} \left[ \frac{k}{f(\theta')} - X_V' \right] \right\} + (1 - \chi)(b + l)$$

(3.34)

As in Mortensen and Pissarides (1994), the wage is increasing in marginal product, bargaining power, market tightness, and the outside option $b + l$. The novelty is that the wage depends on the marginal revenue of the good.

### 3.5.6 National Accounting

We define aggregate vacancies as the sum of the vacancies posted by incumbents and the vacancies posted by entrants:

$$V = \theta u_{-1}$$

(3.35)

Aggregate accounting profits are $\Delta = d - (1 - u)w - kV$, which are paid out as dividends. Aggregate consumption is $C = d + c$, where $c$ is composed of CM expenditures. The variable $c$ satisfies $c = uc_0 + (1 - u)c_1$, in which $c_0 = z + b - d + \Delta - T$, and $c_1 = z + w - d + \Delta - T$. We assume taxes are used to finance unemployment insurance each period, so that $T = ub$. 
Hence, \( c = z - d + (1 - u)w + \Delta^{12}. \) Substituting in \( \Delta, \)

\[
c = z - kV
\]

Thus, the sum of MM and CM expenditures satisfy

\[
C = d + z - kV
\]

### 3.6 Characterization of Steady State

The steady state of debt limit equation (3.21) is

\[
\frac{d}{\psi} = \beta\left(\frac{AQ^{1-\varepsilon}}{(1-\varepsilon)} - Spq + \lambda \frac{2}{\varepsilon}\right).
\]

Re-arranging, and replacing with the discount rate \( r = (1 - \beta)/\beta, \) we can rewrite the limited commitment constraint \( (r + 1 - \lambda)d = \Gamma(d), \) for

\[
\Gamma(d) = \psi\left\{\frac{AQ^{1-\varepsilon}}{(1-\varepsilon)} - Spq\right\}
\]

where the right hand side depends implicitly on \( d \) through \( q. \) Figure 3.5 shows the equilibrium debt limit in terms of the crossing of \( (r + 1 - \lambda)d \) and \( \Gamma(d). \)

\[12\] The outside option \( b + l \) affects the sensitivity of job creation.
The quantity \((r + 1 - \lambda)\bar{d}\) is the flow value to the household from having access to a line of credit of side \(\bar{d}\). \(\Gamma(\bar{d})\) is the flow cost of defaulting one’s debt if the debt limit for future MM trades is equal to \(\bar{d}\). This cost equals the probability of exclusion \(\psi\) multiplied by the surplus in the monopolistically competitive market. Credit is made available up to the point where it is ‘not too tight’: the flow cost of defaulting equals the flow value of credit.

Since \(q = \min\{\bar{d}/(Sp), q^*\}\), \(q = q^*\) implies \(\Gamma'(\bar{d}) = 0\). Equation (3.38) in general has a trivial solution \(d = 0\) and a unique positive solution. To see the latter, let \(g(\bar{d})\) be the ratio of the right hand side to the left hand side of (3.38), viewed as a function of \(\bar{d}\). Then

\[
\lim_{\bar{d} \to 0} g(\bar{d}) = \lim_{\bar{d} \to 0} \frac{\psi}{1 + r - \lambda} \left(\frac{AQ^{1-\varepsilon} - \bar{d}}{\bar{d}}\right) = \lim_{\bar{d} \to 0} \frac{\psi}{1 + r - \lambda} \left[AQ^{-\varepsilon} \left(\frac{Q}{q}\right)^{1-\rho} \frac{1}{Sp} - 1\right]
\]

\[
= \lim_{\bar{d} \to 0} \left[AQ^{-\varepsilon} S^{(1-2\rho)/\rho} / p - 1\right] = \infty
\]

Furthermore, \(\lim_{d \to \infty} g(\bar{d}) = 0\) since the right hand side of (3.38) is bounded above but the left hand side increases without bound. Using continuity of \(g\), there exists a \(d\) such that \(g(d) = 1\). From the fact that \(g'(\bar{d}) < 0\) for all \(\bar{d} \geq 0\), \(d\) is unique.
Using (3.38), we can also derive a simple closed form expression under which the first best is attainable. Suppose the household uses the entire debt limit, i.e. \( d = \bar{d} \). Then (3.38) is equivalent to \((r + 1 - \lambda)d/\psi = AQ^{1-\varepsilon}/(1 - \varepsilon) - d\). Substituting \(d = Spq\), we find that 

\[
1 + \zeta = (1 - \varepsilon)[(r + 1 - \lambda)/\psi + 1],
\]

so that

\[
q = \left[ \frac{AS^{(1-\varepsilon-\rho)/\rho}}{(1-\varepsilon)[(r + 1 - \lambda)/\psi + 1]^{1/\rho}} \right]^{1/\varepsilon} \quad (3.39)
\]

Comparing (3.39) to \(q^*\) we find that \(q < q^*\) if and only if \(1 \leq (1 - \varepsilon)((r + 1 - \lambda)/\psi + 1)\).

Thus, whether consumers are credit constrained in equilibrium depend only on the elasticity of marginal utility, rate of time preference, and monitoring probability \(\psi\). Lower discounting and a higher monitoring probability raise the incentive to repay in the future, and thereby raises debt limits and hence whether individuals are credit constrained. A higher elasticity of marginal utility reduces the first best consumption level of differentiated goods, and thereby makes it more likely that individuals are satiated. Notably, satiation does not depend on taste for variety \(\rho\), the level parameter \(A\), or labor market parameters.

We decompose the wedge between marginal utility and marginal cost into the product: 

\[
\frac{MU}{MC} = (1 - \varepsilon) \left[ \frac{r + 1 - \lambda}{\psi} + 1 \right] \frac{1}{\rho} \quad (3.40)
\]

The wedge is the product of credit frictions and the price markup. It increases with a higher rate of time preference \(r\), lower probability of being able to reaccess credit \(\lambda\), lower monitoring probability \(\psi\), lower curvature of preferences, and higher product diversity. It is instructive to compare (3.40) to the wedge in a pure-monetary economy with monopolistic competition considered in Chapter 1. The wedge in that environment is the product of the price markup and the nominal interest rate, which functions as a tax on real balances.

Equality of the inflow and outflow of sellers implies \(S = \mu/\delta\). The steady state condition for
marginal cost and wage are

\[
MC = \frac{1}{z} \left\{ w + k \left[ \frac{1 - \beta(1 - \delta)(1 - s)}{f(\theta)} \right] \right\} 
\]

\[w = \chi [MRz + \beta k \theta] + (1 - \chi)(b + l)\]  \hspace{1cm} (3.41)

which reduce to functions of \(\theta\) upon making the substitution \(MR = MC\).

Equalization of worker flows into and from unemployment implies the Beveridge Curve:

\[u = \frac{\delta + s(1 - \delta)}{h(\theta) + \delta + s(1 - \delta)}\]  \hspace{1cm} (3.43)

We define a steady state equilibrium as a list \((\theta, p, q, S)\) such that

\[q = \frac{A^{1 - r - \rho}/\rho}{(1 + \zeta)p}^{1/\rho}, \zeta = \text{max}\{0, (1 - \varepsilon)((r + 1 - \lambda)/\psi)\}\]  \hspace{1cm} (3.44)

\[p = \frac{1}{z\rho(1 - \chi)} \left[ \chi \beta k \theta + (1 - \chi)(b + l) + \frac{k[1 - \beta(1 - \delta)(1 - s)]}{f(\theta)} \right]\]  \hspace{1cm} (3.45)

\[\frac{Sq}{z} = \frac{h(\theta)}{s + \delta(1 - s) + h(\theta)}\]  \hspace{1cm} (3.46)

\[\gamma(r + \delta) = (1 + r)(1 - \rho)pq\]  \hspace{1cm} (3.47)

Equation (3.44) is the demand curve for differentiated goods, which depends on the measure of sellers, prices, and the debt limit. Equation (3.45) is the pricing curve, which relates market tightness and taste for variety to the price. Equation (3.46) reexpresses the Beveridge Curve to ensure consistency of production with employment. Finally, (3.47) is the free entry condition.
3.6.1 Comparative Statics

We consider two different ways of reducing equilibrium to two curves. The first method represents equilibrium in \((\theta, u)\) space, and is used to derive the comparative statics. The details are in the appendix. First, I state the comparative statics results and then use the representation of equilibrium in tightness-price space \((\theta, p)\) to highlight the possibility of multiplicity and amplification of changes in the monitoring probability \(\psi\).

We show the comparative statics for the two cases: (I) \(\rho + \varepsilon > 1\) and (II) \(\rho + \varepsilon < 1\) and \(2\rho + \varepsilon - \rho\varepsilon - 1 > 0\).

**Proposition 3.3.** Suppose \(\chi < (1 - \delta)(1 - s)\) and (for Case I) \((2\rho + \varepsilon - \rho\varepsilon - 1)[s + \delta(1 - s) + 1]\frac{\kappa\beta}{z(1-\chi)} > \rho + \varepsilon - 1\). Then Table 3.3 provides the comparative statics.

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The comparative statics of \(A\) and \(\psi\) are identical, and coincide with those of \(z\) except possibly for \(n\) and \(p\). Productivity shocks, however, give rise to several possibilities. If \(\rho + \varepsilon > 1\), then
a positive productivity shock involves a net reduction of prices and in firm size (though the total number of firms unambiguously increases). Otherwise, provided that $2\rho+\varepsilon-\rho\varepsilon-1 > 0$, then prices increase due to the very strong feedback between credit and aggregate demand that raises marginal costs. Similarly, a rise in vacancy posting costs $k$ directly raises marginal costs and prices. In case (b), demand falls sufficiently that prices actually decrease.

### 3.6.2 Amplification and the Role of the Aggregate Demand Externality

In this section, we describe equilibrium in price-tightness space in terms of the pricing rule $p(\theta)$ and an inverse demand curve $ID(\theta)$. This formulation helps us visualize the aggregate demand externality emphasized in Schaal and Taschereau-Dumouchel (2016).

We use (3.44) and (3.47) and write the inverse demand as a function of $\theta$:

$$ID(\theta) = \left( \frac{A}{1+\zeta}\right)^{1/(1-\varepsilon)} \left[ \frac{(1+r)(1-\rho)}{(r+\delta)\gamma} \right]^{\frac{\varepsilon}{1-\varepsilon}} S(\theta)^{\frac{1-\varepsilon-\rho}{\rho(1-\varepsilon)}}$$  

with the associated functions $d(\theta) = zp(\theta)\frac{h(\theta)}{s+\delta(1-s)+h(\theta)}$ and $S(\theta) = d(\theta)^{\frac{(1+r)(1-\rho)}{\gamma(r+\delta)}}$. A steady-state equilibrium reduces to a pair $(\theta, p)$ satisfying (3.44) and (3.45). The following lemma derives graphically useful properties of the curve $p(\theta)$.

**Lemma 3.2.** $p(\theta)$ is increasing, convex, and bounded below by $\left[1/(z\rho)\right][(1-\chi)(b+l)+k[1-\beta(1-\delta)(1-s)]].$

Consider a reduction in $\psi$. Provided that households are credit constrained, $\zeta$ is decreasing in $\psi$, and hence $p$ is increasing in $\psi$. First, we analyze the case $\rho+\varepsilon < 1$. 

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Figure 3.6 shows two economically interesting facts. First, the solid inverse demand curve $ID(\theta)$ crosses the price curve $p(\theta)$ twice, generating two equilibria. Second, an increase in $\psi$ shifts the inverse demand curve up from $ID(\theta)$ to $ID(\theta)'$. The size of the shift from $ID(\theta)$ to $ID(\theta)'$ depends directly on $\rho$: it vanishes as $\rho \to 1$, and becomes infinitely large as $\rho \to 0$. This increase can be divided graphically into two blocks. The first block only considers the direct effect of the increase in $\psi$ on aggregate demand, omitting the dependence on $\theta$. This block consists of the dark-shaded polygon, with price and market tightness at the counterfactual levels of $\theta'_2$ and $p'_2$. The light-shaded polygon considers the additional impact of the dependence of inverse demand on market tightness. This extra effect reflects the change from the counterfactual price-tightness pair to $\theta_2$ and $p_2$. The aggregate demand externality thus increases with the curvature of aggregate demand. Multiplicity of equilibria likewise requires this curvature.

Multiplicity has the following economic interpretation. If households anticipate few varieties in the future, they are less willing to repay their debt, which reduces the size of debt limits. Lower debt limits reduce product demand, which rationalizes low entry of firms. Firm hiring and hiring costs fall, and so do prices. Multiplicity arises from a similar reason as Schaal and Taschereau-Dumouchel (2016) that firms have a greater desire to hire workers if output is
larger. The aggregate demand interacts with endogenous borrowing constraints to raise debt limits. Multiplicity does not require increasing returns to matching or production (Diamond (1982)), shopping externalities (Kaplan and Menzio (2015)), or the use of sentiments to determine wages (Farmer (2012)).

3.6.3 Decomposing the Amplification Mechanism: Partial Equilibrium Analysis

I define restricted equilibria to isolate the effects of the entry channel and consumption value channel. In the first, we treat $S$ as a parameter, and do comparative statics with respect to $S$, and in the second we treat $\bar{d}$ parametrically and do comparative statics as well. Each case is a partial equilibrium exercise with respect to the general model, which enables us to isolate both blocks of the two-way feedback. In both cases, we use the calibration discussed in Section 4.7.

First, we fix $S$, defining a fixed-seller equilibrium:

**Definition 3.1.** A fixed-seller equilibrium is a triple $(\theta, p, q)$ satisfying (3.44), (3.45), and (3.46), with $S$ taken as a parameter.

Raising $S$ is analogous to raising the quality of the composite good. In Figure 3.7, we show that increasing the measure of sellers raises the amount of feasible debt, leading to more output, more vacancies, and hence lower unemployment, higher market tightness, and higher real wages. Higher market tightness pushes marginal costs up, raising prices. In particular, a 25% increase in varieties reduces the unemployment rate from 9.3% to 6.3%, and generates a

13 Multiplicity may not be robust to several realistic extensions. First, households have access to means of payment besides secured credit, particularly liquid assets and unsecured credit, which in practice would limit the kind of unraveling requisite for a high-unemployment equilibrium. Second, with variable elasticity of demand, there may be an adjustment along the markup that restores uniqueness, as in Silva (2017). Third, realistic forms of heterogeneity often suffice to eliminate multiplicity. In particular, Schaal and Taschereau-Dumouchel (2016) do so by considering heterogeneity in vacancy posting costs.
modest uptick in wages. There is also a small rise in the debt-consumption ratio level, from about 0.21 to 0.23.

Next, we turn to the special case with an exogenous debt limit \( d \leq \overline{d} \). The debt limit satisfies

\[
d = \min \left\{ \overline{d}, Sp \left( \frac{AS^{1-\rho-\xi}}{\rho} \right)^{\frac{1}{2}} \right\}
\]

We define equilibrium for this special case.

**Definition 3.2.** An exogenous-debt-limit equilibrium consists of \((\theta, p, q, S, d)\) such that \(d = Spq\) and (3.45), (3.46), (3.47), and (3.49) hold.

Next, we do comparative statics with respect to the debt limit \( \overline{d} \), where the lower endpoint of
\( \bar{d} \) is chosen to match the prevailing debt level of \( S \) in the special case with exogenous sellers, and \( \gamma \) is chosen so that the measure of sellers coincide.

Figure 3.8 illustrates the effects of increasing \( \bar{d} \) on the price level, unemployment rate, market tightness, the number of sellers, real wages, and debt-to-consumption. Through the entry channel, the measure of sellers increases, and there is a dramatic drop in unemployment from roughly 12% to just over 5%. The debt-to-consumption ratio increases from 0.20 to 0.26.

![Figure 3.8: Comparative Statics with Respect to Exogenous Debt Limit](image)

**3.7 Baseline Calibration**

The calibration borrows from Shimer (2005), Bethune et al. (2015), Bilbiie et al. (2012), and Petrosky-Nadeau et al. (2014), but also has some original elements. The frequency is monthly. We target a risk free rate of 4%, as in Livshits et al. (2007), yielding \( \beta = 0.9967 \).
The matching function is of the urn-ball form $m(U,V) = V(1 - \exp(-B/\theta))$, which ensures job finding and vacancy filling probabilities bounded between 0 and 1.\textsuperscript{14} The match efficiency parameter $B$ and destruction rates $s + \delta$ are jointly determined by targeting the job finding probability, the unemployment rate, labor market tightness, and the product destruction rate. The target unemployment rate is 6.1\%, the average during the time period, and the monthly job finding probabilities are $h = 0.45$ and $f = 0.71$. This implies a market tightness $\theta = h/f = 0.634$. We set $B$ to match the job finding probability: $h = \theta(1 - \exp(-B/\theta))$. The job destruction rate satisfies $s(1-\delta)+\delta = f \frac{w}{1-u} = 0.0398$. I follow Bilbiie et al. (2012) in setting $\delta = 0.025$, which implies a 10\% annual product destruction rate. Hence, $s = 0.0148$.

The remaining labor market parameters are vacancy posting costs $k$, the value of unemployment benefits $b$, and the value of leisure $l$. These quantities can be readily obtained once we solve for the ratio of the wage to marginal revenue product $\text{WMRP}$. This quantity, in turn, can be obtained from a target of 3\% of the vacancy costs the wage bill, following the evidence in Silva and Toledo (2009): $VAC = kV/(w(1-u)) = 0.03$. We can write the vacancy posting costs in two ways:

\begin{align*}
    k &= VAC\rhozp\text{WMRP}(1-u)/V \quad (3.50) \\
    k &= \rhozp(1 - \text{WMRP})f/(1 - \beta(1 - \delta)(1 - s)) \quad (3.51)
\end{align*}

Equating these yields $\text{WMRP}[1 - \beta(1 - \delta)(1 - s)] = \frac{V(1-\text{WMRP})F}{(1-u)VAC}$, which can be solved to yield $\text{WMRP} = 0.9675$.

We solve for prices using by using a target for the ratio of unsecured credit to consumption $d/C$. The empirical target for $d/C$ is the ratio of new monthly charges on credit cards and

\textsuperscript{14}There is a simple microfoundation for the urn-ball matching function (e.g. Petrongolo and Pissarides (2001)). Suppose each of $U$ workers sends an average of $B$ applications at random to $V$ vacancies. A job is filled provided that each vacancy receives at least one application, with workers chosen randomly provided there are two or more offers. The probability that each vacancy receives a worker’s application is $B/V$, and hence the probability that a vacancy does not receive any applicants is $(1 - B/V)^U$. Hence, the total number of matches is $M = V[1 - (1 - B/V)^U]$, which is well approximated for large $V$ as $V(1 - \exp(-BU/V)$, or $V(1 - \exp(-B/\theta))$. 

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charge cards, provided by the Survey of Consumer Finances and total monthly household expenditures as reported by the Consumer Expenditure Survey (CEX). The ratio \( d/C \) can be written as \((1 - u)pz/((1 - u)pz + z - kV)\). Substituting for \( k \), we obtain

\[
\frac{(1 - u)pz}{(1 - u)pz + z - \rho z(1 - WMRP)fV/(1 - \beta(1 - \delta)(1 - s))}
\]

After obtaining \( p \), we solve \( w = WMRP\rho z p \) and \( k = VACw(1 - u)/V \) and set \( b + l = 0.85w \).

For positive analysis, it is not necessary to separately identify \( b \) and \( l \).

Given \( p, k, b + l \), we solve the pricing equation \( p = \frac{1}{z\rho(1 - \chi)} \{ \chi\beta k\theta + (1 - \chi)b + k[1 - \beta(1 - \delta)(1 - s)]/f \} \) for \( \chi \), yielding \( \chi = 0.215 \).

We set \( \lambda = 0.25^{18} \), targeting the evidence in Cohen-cole (2009) that 75% of defaulting households are able to obtain unsecured credit within 18 months.\(^{15}\)

To target the firm entry costs \( \gamma \), we use evidence from Aguirregabiria and Mira (2007) that entry costs in the restaurant industry are 35.7% of annualized profits. As we use a monthly frequency, we set \( \gamma = 0.357 \times 12\Delta \), where \( \Delta = (1 - u)(pz - w) - kV \). From the free entry condition (3.47), we solve for \( q \), and solve for \( S \) via (3.46). Using \( 1 + \zeta = \max((1 - \varepsilon)((r + 1 - \lambda)/\psi + 1), 1) \) and \( S, p, q \), we obtain \( A \) from (3.44). The targets are not sufficient to pin down \( \varepsilon \) and \( \psi \). We fix baseline values of \( \varepsilon = 0.15 \) and \( \psi = 0.25 \) and in the appendix also report the results for \( \varepsilon = 1 - \rho \), shutting down the aggregate demand externality.

\(^{15}\)This model does not have default in equilibrium, so we do not technically match the percentage of individuals who re-obtain unsecured credit. Instead, we are match the implied beliefs about future credit access, should one default.
### Calibration Targets

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Economic interpretation</th>
<th>Source/target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>0.1318</td>
<td>Level parameter of MM utility</td>
<td>Equilibrium conditions</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.9967</td>
<td>discount factor</td>
<td>risk-free rate=4%</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>0.15</td>
<td>Composite elasticity</td>
<td>Fixed</td>
</tr>
<tr>
<td>$\rho$</td>
<td>1/1.4</td>
<td>Taste for variety parameter</td>
<td>40% retail markup</td>
</tr>
<tr>
<td>$b + l$</td>
<td>0.186</td>
<td>Leisure utility</td>
<td>$(b + l) = 0.85w$</td>
</tr>
<tr>
<td>$s$</td>
<td>0.0214</td>
<td>Worker separation rate</td>
<td>$0.0148, s = (hU/(1 - U) - \delta)/(1 - \delta)$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.008</td>
<td>Firm death rate</td>
<td>10% annual product destruction rate</td>
</tr>
<tr>
<td>$\chi$</td>
<td>0.233</td>
<td>Bargaining power of worker</td>
<td>Pricing equation</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.3662</td>
<td>Firm entry cost</td>
<td>35.7% of annualized profits</td>
</tr>
<tr>
<td>$k$</td>
<td>0.159</td>
<td>Vacancy costs</td>
<td>Vacancy costs 3% of wage bill</td>
</tr>
<tr>
<td>$B$</td>
<td>0.785</td>
<td>Labor matching efficiency</td>
<td>Solution of $\theta(1 - \exp(-B/\theta)) = H$</td>
</tr>
<tr>
<td>$\bar{z}$</td>
<td>1</td>
<td>Mean labor efficiency</td>
<td>(normalization)</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.25</td>
<td>Monitoring probability</td>
<td>Fixed</td>
</tr>
</tbody>
</table>

### 3.8 Dynamic Stochastic Equilibrium and Results

The dynamic stochastic general equilibrium is a list of quantities $\{q_t, \bar{d}_t, p_t, \theta_t, u_t\}_{t=1}^T$ satisfying (B.38), (B.39), (B.40), (B.41), (B.42), and (3.31), together with the stochastic process for productivity $z_t$:

$$\log(z_t) = \rho_z \log(z_{t-1}) + \varepsilon_{z,t}$$

$$\varepsilon_{z,t} \sim N(0, \sigma^2)$$

The appendix enumerates (B.40)-(B.42). Following Petrosky-Nadeau and Zhang (2016), we use $\rho_z = 0.9895^4$ and $\sigma = 0.0068$, converting the weekly targets to monthly.
We compute the equilibrium using the parameterized expectations algorithm, discussed in the appendix. The idea is to approximate the conditional expectations as polynomial functions of the state $\Omega$. Then, one can recover the other endogenous variables, checking the nonnegativity constraint on vacancies and that credit constraints bind. We use a global solution algorithm for several reasons. First, Petrosky-Nadeau and Zhang (2016) show that the labor search model has strong nonlinearities: log linearization understates the mean and volatility of unemployment and overstates the volatility of market tightness. Second, the use of a global solution algorithm handles occasionally binding constraints, which arise from the nonnegativity constraint on vacancies ($V_t \geq 0$), debt saturation ($d_t \leq d^*_t$), and nonnegativity constraint on entrants ($S_t \geq (1-\delta)S_{t-1}$). Third, global solution methods enable us to quantify the extent to which amplification depends on the size of the shock.

We present the model moments in Table 3.4.

<table>
<thead>
<tr>
<th></th>
<th>SD</th>
<th>Cor($x,u$)</th>
<th>Cor($x,d$)</th>
<th>Cor($x,x_{-1}$)</th>
<th>Cor($x,x_{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>0.0783</td>
<td>-0.947</td>
<td>0.988</td>
<td>0.964</td>
<td></td>
</tr>
<tr>
<td>$d$</td>
<td>0.0311</td>
<td>0.947</td>
<td>0.964</td>
<td>0.927</td>
<td></td>
</tr>
<tr>
<td>$d/C$</td>
<td>0.0061</td>
<td>-0.992</td>
<td>0.974</td>
<td>0.940</td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.145</td>
<td>-0.938</td>
<td>0.958</td>
<td>0.919</td>
<td></td>
</tr>
<tr>
<td>$P$</td>
<td>0.0009</td>
<td>0.999</td>
<td>-0.956</td>
<td>0.984</td>
<td>0.956</td>
</tr>
</tbody>
</table>

Table 3.4: Moments of key variables simulated from model, monthly. Data is transformed by proportional deviations from the mean.

Unemployment and credit are moderately less volatile than in the data: 0.0783 compared to 0.113 for unemployment and 0.0311 compared to 0.044 for credit. The standard deviation of market tightness is 0.145 in the model, compared to 0.204 in the data. Unemployment and credit have an extremely high negative correlation ($-0.947$), compared to $-0.32$ in the data. The extremity of the correlations arises naturally from the fact that credit is the only payment instrument. The presence of alternate payment instruments (money, claims on firm profits) should moderate the correlations substantially. The ratio of credit to consumption is procyclical in the model, but the standard deviation 0.0061 is much smaller than the level in the data (0.036). This discrepancy arises
primarily from the fact that consumption in the model is too volatile. The presence of a savings instrument, which again can take the place of money or claims on firm profits, can likely overcome much of this issue.

Figure 3.9 plots the distribution of aggregate vacancies and entry. Whereas the nonnegativity constraint on vacancies does not bind, the constraint on entry binds 21% of the time. The path dependence reflected by this constraint is quantitatively significant.

![Figure 3.9: Distribution of Vacancies and Entry](image)

The most striking gap between the model and data concerns the price level. The price level is highly positively correlated with unemployment in the model and negatively correlated with credit, whereas the the empirical correlation with unemployment is $-0.421$ and with credit is $0.567$. This finding highlights the problem of relying entirely on productivity shocks for aggregate disturbances. Allowing for preference shocks to $A$ and credit shocks $\psi$ would induce a positive association between credit and prices and a negative association between these two variables and unemployment. For instance, Bai et al. (2012) study a macroeconomic model in which product demand has a productive role. They allow for both preference and productivity shocks and estimate the volatility and autoregressive parameters of each by Bayesian means, finding that technology shocks account for less than 18% of fluctuations in output and the measured Solow residual. Further evidence against
the primacy of technology shocks is that aggregate consumption Granger causes TFP, whereas TFP
does not Granger cause aggregate consumption.

To recap, the model succeeds in generating volatile unemployment, credit, and product vari-
ety; a negative correlation between unemployment and credit; and procyclical ratio of credit-to-
consumption. However, the lack of a savings and alternate payment instrument exaggerates several
of the correlations and causes excessive consumption volatility. Finally, though the model has
internal propagation that generates positive feedback between prices and credit, the reliance on
productivity shocks makes prices comove positively with unemployment. The upshot for future
research is to permit multiple shocks and alternate forms of savings and payment.

To highlight endogenous amplification of the model, we conduct two major counterfactual experi-
ments. The first compares impulse responses from the baseline economy to an alternate economy
in which lower product diversification only gives rise to 5% markups. The second compares the
responses in the model economy to one with a fixed mass of sellers, rendering the product space
exogenous. For simplicity, we assume that a fraction $\delta$ of firms exit each period and are mechani-
cally replaced with entrants such that the total mass of firms $S$ is constant. This trick allows us to
maintain the parameterization of the baseline model and set $S$ to the steady state value.

In a nonlinear model, the notion of an impulse response is ambiguous, because of the importance
of (1) history of realizations of shocks, (2) future realizations of shocks, and (3) size of the impulse.
Following Koop et al. (1996), we define the generalized impulse response of variable $Y_t$ at period $n$,
impulse $\nu_t$ and history $\omega_{t-1}$:

$$GI_y(n, \nu_t, \omega_{t-1}) = \mathbb{E}(Y_{t+n}|\nu_t, \omega_{t-1}) - \mathbb{E}(Y_{t+n}|\omega_{t-1}) \text{ for } n = 0, 1, \ldots \quad (3.52)$$

The impulse response traces the expected change in the variable $Y_t$ given the impulse $\nu_t$ and history
$\omega_{t-1}$. We take steady state as the history and compute the expectation through a Monte Carlo
procedure of drawing future innovations and averaging out the response.
### 3.8.1 Experiment 1: Role of Taste for Variety

We analyze the percentage response of variables from a one-standard deviation and six-standard-deviation productivity shock, comparing the results for high taste of variety ($\rho = 1/1.4$) and low taste for variety ($\rho = 1/1.1$). We know that unemployment and market tightness can be highly nonlinear from Petrosky-Nadeau et al. (2014). One major reason is that job creation is forward looking and therefore very sensitive to a persistent drop in productivity. Second, the vacancy filling probability does not rise very much in recessions, thereby not lowering the hiring costs very much. We show this feature is even more pronounced with firm entry and product diversification. In Figure 3.10, we consider a one-standard deviation shock to total factor productivity.

![Graphs showing response to a unit standard deviation productivity shock: markup comparison](image)

Figure 3.10: Response to a unit standard deviation productivity shock: markup comparison
Unemployment responds much more sharply under a higher taste of variety than a lower taste of variety, with a peak difference of about 0.11 percentage points compared to over 0.07 percentage points. This feature results from the fact that the mutual feedback loop between credit and product variety is stronger with greater diversification. The initial jump in unemployment rate increases as fewer sellers enter the economy, reducing the amount of varieties and hence the value of consumption goods. Debt-relative-to consumption actually rises with 5% markups, but responds by nearly over \(-0.15\%\) with higher product diversification. With low product diversification, the mutual feedback between credit and product variety is weak, and the fall in credit is less than the general fall in consumption because a big part of consumption consists of the endowment of the numeraire.

Figure 3.11 illustrates the impulse response of a six-standard deviation TFP shock, highlighting the role of nonlinearity.
Figure 3.11: Response to a six-standard deviation productivity shock: markup comparison

The nonlinear response is particularly important for unemployment. Unemployment peaks at 0.72 percentage points under high product diversification compared to 0.47 percentage points under low product diversification. The impulse response of unemployment is on average over 50% greater, and has a maximal difference of over 60%. The nonlinearity is evident form the fact that the peak responses under the large shock are about seven times the peak response of unemployment to a one-standard-deviation shock. This finding suggests that financial propagation mechanisms become increasingly important as the size of shocks rises.
3.8.2 Experiment 2: Role of Free Entry

We assess the role of free entry in amplification by comparing simulated data to a version of the model with a fixed mass of sellers. An equilibrium is a list of quantities \( \{q_t, d_t, p_t, \theta_t, u_t\}_{t=1}^T \) satisfying (B.38), (B.39), (B.40), (B.41), and (B.42). Instead of \( S \) being determined according to the free entry condition, it is now parametrically fixed.

![Graphs showing response of unemployment, market tightness, and debt relative to consumption](image)

Figure 3.12: Response of a unit standard deviation productivity shock: no-entry comparison

Figure 3.12 reveals that the amplification is very similar in terms of the entry margin compared to the product diversification margin. This property arises from the fact that entry of sellers matters entirely through its role on the value of the consumption basket. We similarly verify the role of a
nonlinear impulse response by comparing to a six-standard deviation shock.

Figure 3.13: Response of a six-standard deviation productivity shock: no-entry comparison

The amplification is very similar to that which arises from product diversification.

### 3.9 Conclusion

We introduce endogenous borrowing constraints and monopolistic competition into a search and matching model of the labor market. First, the model explains the volatility of unemployment and credit, the negative correlation between unemployment and credit and positive correlation between
credit and product variety. The ratio of credit to consumption is procyclical, though much less volatile than in the data. Second, the mutual feedback loop amplifies shocks on unemployment, debt-to-consumption, and product variety. We illustrate the mechanism by comparing impulse responses in two experiments: the first examined economies with high versus low levels of product diversification, and the second compared a free entry economy to one with a fixed mass of sellers. Sufficiently high markups in the first experiment and free entry of sellers in the second are essential for debt-to-consumption to be procyclical. Third, we find that impulse responses of unemployment increase in the size of the shock.
Chapter 4

Corporate Finance, Monetary Policy, and Aggregate Demand

4.1 Introduction

I develop a general equilibrium model of firms’ investment and financing options to help understand the channels through which policy interest rates affect the use of internal finance, lending, output, and investment in the presence of monopolistic competition and aggregate demand externalities. The aggregate demand externality implies that entrepreneurs’ investment increases with output—and hence with the investment of other entrepreneurs.

The setup closely follows Rocheteau et al. (2016). Entrepreneurs stochastically generate ideas for an investment project, search for banks, and, if successful, bargain over the terms of the loan in an over-the-counter market, as in Wasmer and Weil (2004).\footnote{See Silveira and Wright (2010) for a treatment of the market for ideas, in which it is profitable for innovators to specialize in the origin of ideas and for entrepreneurs to specialize in their commercialization.} The dimensions of the loan are the size, down payment, and interest rate. The terms of the contract are subject to limited enforcement, with only a fraction of the returns being pledgeable, as in Holmström and Tirole (1998) or Kiyotaki
Credit in the model features both an intensive margin (loan size) and extensive margin (acceptability of loan applications), which accords well with actual corporate credit markets.

The dynamic general equilibrium approach to liquidity builds on the New Monetarist framework surveyed in Nosal and Rocheteau (2011). The lending rate is an intermediation premium, which arises from frictions characteristic of over-the-counter financial markets, as discussed by Lagos and Rocheteau (2009). Blanchard and Kiyotaki (1987) stresses the aggregate demand externality of monopolistic competition, which several recent papers examine in other contexts. Schaal and Taschereau-Dumouchel (2015) generate a coordination problem of investment with demand complementarities and variable capacity utilization. Using the global game approach to select among the multiple equilibria, they find that sufficiently large shock may prevent firms from coordinating on high output. Schaal and Taschereau-Dumouchel (2016), similarly, examine the role of strategic complementarity of hiring decisions in amplifying shocks.

Allowing entrepreneurs to self-insure against idiosyncratic risk by holding money is in line with Campello (2015)’s discussion of corporate liquidity management. Bates et al. (2009) run empirical tests and find that the precautionary motive can explain the increase in firms’ cash holdings from 1980-2006. Moreover, Mach et al. (2006) examine the Survey of Small Business Finance (SSBF) in 2003 and find that 95% of businesses have a liquid asset account. The SSBF also indicates that firms use money, bank, and trade credit as alternative means of financing investments. 34.3% of small businesses have a credit line and 60.1% have access to trade credit. In the model, we simplify by only considering internal finance and bank credit. Firms’ cash holdings provide insurance, should they not be able to secure a loan, and enables them to increase down payments and pay lower interest rates. If the enforcement constraint binds, then holding more cash also increases the size of the loan more than one-for-one.

The use of search and bilateral negotiation helps account for the intensive margin of bank credit (loan size) and the extensive margin (number of loans). The presence of both margins is consistent with Bernanke et al. (1999) and Holmström and Tirole (1998) endogenize financial frictions in terms of a principal agent problem with costly state verification, and Rocheteau et al. (2016) provides an alternative rationalization in terms of limited commitment.
with the evidence in the Joint Small Business Credit Survey (2014). Of survey participants applying for loans, 33% received what they requested, 21% receive less, and 44% were denied.

The concern with financial frictions, monopolistic competition and monetary policy relates the New Keynesian literature with a financial accelerator: the treatment in Bernanke et al. (1999) relies on negative comovement between the external finance premium (the difference between the cost of funds raised externally and opportunity cost of funds internal to the firm) and the net worth of potential borrowers. Stochastic financial returns, limited liability of entrepreneurs, and costly state verification give rise to a moral hazard problem: lower net worth in recessions worsens the agency problem, raises the external finance premium and real interest rates, and depresses investment. Bernanke et al. (1996) presents empirical evidence for this channel: they proxy credit access with firm size and find that small manufacturing firms experience larger variation in sales, inventories, and debt in a downturn (‘flight to quality’). Monacelli et al. (2011) study a different effect in which firms use debt strategically to lower the bargaining power of workers, reducing the wages and boosting employment. Thus, downturns can persist even though firms have significant liquidity on hand.

As Bernanke et al. (1999), the mechanism here also generates a pass through from the use of internal finance and the real lending rate. However, internal finance is also used to insure against idiosyncratic uncertainty in matching, and the monetary policy pass through does not depend on sticky prices. Finally, we go beyond Bernanke et al. (1999) by considering heterogeneity of financial frictions, which gives rise to distributional effects both on the level of investment and output as well as its transmission, and explains how financial bottlenecks spill over even to unconstrained firms.

This paper also relates to a literature in growth theory that examines the amplification of microeconomic distortions. Jones (2011), in particular, studies how linkages and complementarity of goods

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3See Ćorić (2011) for a review of the financial accelerator literature. The authors consider different formulations depending on whether the asymmetric information problem is solvable at some cost, and also whether there is a principal agent problem between owners and managers. They also discuss the empirical evidence.

4Sticky prices are not relevant for steady state effects of monetary policy. Nevertheless, because entrepreneurs have price setting power in this framework, one could incorporate sticky prices in a business cycle version of this model to study short-run effects.
amplifies such distortions and provides a more plausible explanation of large income differences across countries. The typical distortions considered include those on capital and labor (Hsieh and Klenow (2009)), taxes, corruption, and other political economy factors. Here, limited enforcement of loans is the salient distortion, which amplifies the effects of monetary policy and interacts with the aggregate demand externality from monopolistic competition.

Incorporating heterogeneity of financial frictions across entrepreneurs serves three purposes. First, it enables us to study how pass through and transmission depend on firm type. Gertler and Gilchrist (1994) provide evidence that size proxies for capital market access find that a negative monetary policy shock induces a bigger decline in sales, inventory, banks loans, and the ratio of bank loans to sales than do large firms. Ehrmann et al. (2003) find a similar result using a structural vector autoregression using business survey data on German manufacturing firms. The theory here both supports size as a proxy for capital market and access as well as an asymmetric transmission across firm size. Second, it incorporates distributional effects of investment and output. Mean-preserving spreads of financial frictions hurt investment and output because of both the concavity of firms’ revenue and the fact that, for unconstrained firms, further increases in pledgeability have no effect. Third, heterogeneity eliminates kinks in the effects of policy from transitioning to a region in which financial frictions either bind on all firms to one in which they bind on no firms.

We describe the general environment, but then discuss special versions of the model before turning to the most general version. First, we analyze an economy with entrepreneurs subject to heterogeneous financial frictions who can only borrow externally. Second, we add internal finance but allow for perfect enforcement of loans. The latter enables us to highlight the direct role of the aggregate demand externality on pass through and transmission. In both these cases, we can solve for economy-wide aggregates analytically. Third, we turn to the general model, which features internal and

---

5. These results are robust to the inclusion of real GNP growth, inflation, and the Federal Funds rate as controls.

6. Firms are clustered into five buckets by size. The baseline model includes money growth, industrial production, short-term interest rate, producer price inflation, and a measure $\Delta_{ij,t}$ which measures the difference in business conditions between firms in bucket $i$ and bucket $j$. The author imposes several standard cointegration relationships as well as that monetary policy does not affect inflation and output within the same month.
external finance and heterogeneity of entrepreneurs in terms of financial constraints. Section 4.7 describes the calibration strategy; the corresponding parameter values are used for the preceding figures throughout the article. For the sake of comparison, the appendix analyzes the case with limited enforcement under a representative entrepreneur.

4.2 Environment

Time is denoted by \( t \in \mathbb{N}_0 \). Each period is divided into two stages. In the first stage, there is a Walrasian market for capital goods and an over-the-counter market for banking services (loan provisions and means of payment) with search and bargaining. In the second stage, there is a frictionless centralized market where agents settle debts and trade final goods and assets. The capital good \( k \) is storable across stages but not across periods. A numeraire consumption good \( c \) is produced and traded in the CM. Good \( c \) is not storable. Figure 4.1 illustrates the timing.

<table>
<thead>
<tr>
<th>STAGE 1</th>
<th>STAGE 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perfectly competitive</td>
<td>Production of intermediate goods</td>
</tr>
<tr>
<td>capital market</td>
<td>subject to monopolistic competition</td>
</tr>
<tr>
<td>OTC banking market</td>
<td>Production of final good subject to perfect</td>
</tr>
<tr>
<td></td>
<td>competition</td>
</tr>
<tr>
<td></td>
<td>Debt settlement/choice of real balances</td>
</tr>
</tbody>
</table>

Figure 4.1: Timing

There are four types of agents labeled by \( \{e, s, b, f\} \). These consist of entrepreneurs (\( e \)), who require capital for investment projects; suppliers (\( s \)), who produce the capital; banks (\( b \)), who finance the acquisition of capital by entrepreneurs as explained below, and final goods producers (\( f \)).
measure of entrepreneurs is normalized to one. All agents have linear preferences $U(c, h) = c - h$, where $c$ is the consumption of the numeraire and $h$ is hours of work. Whereas suppliers, banks, and final goods producers are homogeneous, entrepreneurs are indexed by the pledgeability coefficient $\chi_j \in [0, 1]$.

Entrepreneurs can produce intermediate goods by converting the supplier’s primary good one-for-one into capital: $y = k$. Entrepreneurs acquire $k$ in Stage 1 and bring it to Stage 2 for production. Second, they can also produce $c$ using their own labor according to a linear technology, $c = h$. Suppliers produce $k$ in Stage 1 with a linear technology, $k = h$. Banks cannot produce $c$ or $k$.

Figure 4.2 details the production chain in which entrepreneurs obtain primary input from suppliers to produce intermediate goods, which are then purchased by perfectly competitive final good firms into a final good.

![Production chain diagram](image)

**Figure 4.2: Production chain**

Entrepreneurs face two types of idiosyncratic uncertainty: with probability $\lambda$, they receive an investment opportunity, as in Kiyotaki and Moore (1997); with probability $\alpha$, they can access financing from banks, as in Wasmer and Weil (2004). Thus, the probability that an entrepreneur has an investment opportunity and is matched with a bank is $\alpha \lambda$. With probability $\lambda(1 - \alpha)$, an entrepreneur has an investment opportunity but no access to a bank.

Next we discuss the enforcement technology in Stage 2 on debts incurred by entrepreneurs in the DM. Consider an entrepreneur with $k$ units of capital goods and liability $l > 0$ toward banks. Following production, the entrepreneur can default on his debt and walk away with part of the
returns of his capital. I assume no trade credit is possible, and that banks can enforce payment up to $\chi_j$ of the value of the investment project. The parameter $\chi_j$ reflects both portability and tangibility of capital and legal institutions (e.g., the judicial system).

Limited enforcement can generate a demand for liquid assets. Here, we focus on outside fiat money. Fiat money is storable and evolves according to $A_{m,t+1} = (1 + \pi)A_{m,t}$. The price of money in terms of the numeraire is $q_{m,t}$. In a stationary equilibrium, $q_{m,t} = (1 + \pi)q_{m,t+1}$, so $\pi$ is the inflation rate. We assume $\pi > \beta - 1$. The nominal interest rate $i$, defined from the Fisher equation $1 + i = (1 + r)(1 + \pi)$, is thus positive.

### 4.2.1 Problem of the Entrepreneur

Consider an entrepreneur at the beginning of Stage 2 with $k$ units of capital goods from Stage 1 and financial wealth $\omega$ denominated in numeraire. Financial wealth is composed of real balances $a^e_m$ net of obligations. The entrepreneur’s lifetime expected utility solves

$$W^e(k, \omega) = \max_{c, h, \tilde{a}_m} \{c - h + \beta V^e(\tilde{a}_m)\}$$

s.t. $c = f(k, Y) + h + \omega + T - (1 + \pi)\tilde{a}_m$ (4.2)

in which $f(k, Y)$ is the revenue from $k$ units of capital and aggregate demand $Y$, which we derive in Section 4.3. The total surplus is $S^e + S^b = f(k, Y) - k$. Substituting $c - h$ into $W^e$ yields

$$W^e(k, \omega) = f(k, Y) + \omega + T + \max_{\tilde{a}_m} \{- (1 + \pi)\tilde{a}_m + \beta V^e(\tilde{a}_m)\}$$

(4.3)

$W^e$ is linear in total wealth, and the choice of real balances $\tilde{a}_m$ is independent of $(k, \omega)$.

Similarly, the CM lifetime expected utility of a supplier or bank, $j \in \{b, s\}$ with wealth $\omega$ is

$$W^j(\omega) = \omega + \max_{\tilde{a}_m} \{- (1 + \pi)\tilde{a}_m + \beta V^j(\tilde{a}_m)\}$$

(4.4)
We next consider the problem of a supplier at the beginning of Stage 1:

\[
V^s(\bar{a}_m) = \max_{k \geq 0} \{-k + W^s(\bar{a}_m + q_k k)\} \tag{4.5}
\]

A supplier produces \(k\) at a linear cost in exchange for a payment \(q_k k\). If the market for capital goods is active, \(q_k = 1\) and \(V^s(\bar{a}_m, \bar{a}_g) = W^s(\bar{a}_m)\).

The portfolio choice solves

\[
\max_{\bar{a}_m} \{-(1 + \pi)\bar{a}_m + \beta\bar{a}_m\} \tag{4.6}
\]

Provided \(\pi > \beta - 1, \bar{a}_m = 0\). Suppliers hold no real balances as they have no liquidity needs.

An entrepreneur’s lifetime expected utility at the beginning of the DM is

\[
V^e(\bar{a}_m, \bar{a}_g) = EW^e(k, \bar{a}_m - \psi - \phi) \tag{4.7}
\]

The entrepreneur purchases \(k\) at total cost \(\psi = q_k k\) and compensates the bank for its intermediation services with payment \(\phi\). The total payment \(\psi + \phi\) is subtracted from the entrepreneur’s financial wealth in Stage 2. If the entrepreneur does not receive an investment opportunity, then \(k = 0\). If the entrepreneur is not matched with a bank, then \(\phi = 0\).

The choice of real balances solves

\[
\max_{\bar{a}_m} \{-(1 + \pi)\bar{a}_m + \beta[f(k, Y) + \bar{a}_m - \psi - \phi + T]\} \tag{4.8}
\]

Multiplying by \((1 + r)\), and using the Fisher equation \(1 + i = (1 = r)(1 + \pi)\), the portfolio problem reduces to

\[
\max_{\bar{a}_m \geq 0} \{-i\bar{a}_m + \mathbb{E}[f(k, Y) - k - \phi]\} \tag{4.9}
\]

where \((k, \phi)\) is a function of \(\bar{a}_m\).
4.3 Pure Bank Credit

We solve the profit maximization of the final goods firm and use it to derive the revenue function \( f(k,Y) \) of entrepreneurs, which determines the return on the investment project.

The quantity index is

\[
Y = \left[ \int_0^1 y_i^\sigma \, di \right]^{\frac{1}{\sigma}} \quad \sigma < \gamma \quad \gamma < 1 \tag{4.10}
\]

where \( i \in [0,1] \) denotes the identity of an individual entrepreneur, \( y_i \) denotes the input produced by the entrepreneur in a match with a supplier, and \( 1/(1-\sigma) \) is the elasticity of substitution between input factors. The parameter \( \gamma \) is effectiveness through which increases in the production of intermediate goods generate an increase in the consumption bundle: \( Y \) is homogeneous of degree \( \gamma \).

The final goods firm takes the output price \( P \) and input prices \( \{P_j|j \in [0,1]\} \) of the intermediate goods firms to maximize profit:

\[
\max_{y_i \geq 0} \left( PY - \int_0^1 P_i y_i \, di \right), \tag{4.11}
\]

The first order condition yields the individual demand curve

\[
y_i = \left( \frac{P \gamma}{P_i} \right)^{\frac{1}{1-\sigma}} Y^{\frac{\gamma-\sigma}{\gamma(1-\sigma)}} \tag{4.12}
\]

We normalize the final output price \( P = 1 \). Provided that \( \sigma < \gamma \), the demand of input \( i \), \( y_i \), depends positively on the output of other entrepreneurs, \( Y \). This positive dependence of \( y_i \) on \( Y \) is an aggregate demand externality and provides a channel that amplifies credit frictions. We will see that financial frictions on a subset of entrepreneurs reduces aggregate output and hence reduces the demand for other entrepreneurs’ inputs.

Rearranging (4.12) provides the inverse demand curve \( P(k,Y) \). An entrepreneur can transform \( k \)
units of primary input into $y$ units of secondary input, so the total revenue from acquiring $k$ units of primary input from a supplier is

$$f(k,Y) \equiv P(k,Y)k = \gamma Y^{\frac{\gamma - \sigma}{\gamma}} k^\sigma. \quad (4.13)$$

The revenue function in (4.13) is Cobb-Douglas but with endogenous total factor productivity $\gamma Y^{(\gamma-\sigma)/\gamma}$. The function $f$ satisfies $f(0,Y) = 0$, $\partial f/\partial k|_{k=0} = \infty$, and $\partial f/\partial k > 0 > \partial^2 f/\partial k^2$.

The bank can credibly promise a payment to the supplier and enforce payments up to the limit implied by $\chi$. The bank extends a loan to the entrepreneur by crediting a deposit account in his name for the amount $l$. The deposit claims are liabilities of the bank that can be transferred from the entrepreneur to the supplier in exchange for $k$. In Stage 2, the supplier redeems the claim on the bank for $\psi$, and the entrepreneur settles his debt by returning $\psi + \phi$ to the bank. The terms of the loan contract is a pair $(\psi,\phi)$ with $\psi = k$ determined through bilateral negotiation. The surpluses are

$$S^e \equiv W^e(k,\omega^e - k - \phi) - W^e(0,\omega^e) = f(k,Y) - k - \phi \quad (4.14)$$

$$S^b \equiv W^b(\omega^b + \phi) - W^b(\omega^b) = \phi \quad (4.15)$$

The bargaining problem is

$$\max_{k,\phi} [f(k,Y) - k - \phi]^{1-\theta} \phi^\theta \quad \text{s.t.} \quad \chi f(k,Y) \geq k + \phi \quad (4.16)$$

so that the total surplus is $S^e + S^b = f(k,Y) - k$. Figure 4.3 depicts the frontier in the contract space and utility space, taken with minimal modification from Rocheteau et al. (2016). The quantity $k^*(Y) = \arg\max_k [f(k,Y) - k]$ maximizes the bilateral surplus.$^8$ The maximum surplus of the bank is $\chi f(\hat{k},Y) - \hat{k} \leq f(\hat{k},Y) - \hat{k}$, where $\hat{k}$ solves $\chi \partial f/\partial k = 1$. The bargaining solution of $k \in [\hat{k},k^*]$. Suppose $k < \hat{k}$. Then a small increase in $k$ increases both the total surplus and the maximal surplus

$^8$Unlike in Rocheteau et al. (2016), monopolistic competition does not generate the efficient level of investment even in the absence of financial frictions. Dixit and Stiglitz (1977) show that monopolistic competition with free entry is constrained efficient under CES preferences.
of the bank. Thus, there is some higher intermediation fee that would raise surplus for both parties.

The lending rate \( r = \phi/k \) is the ratio of interest payment \( \phi \) to the size of the loan \( k \):

**Proposition 4.1.** The liquidity constraint binds if and only if

\[
\chi < \chi^* \equiv \frac{(1 - \theta)k^* + \theta f(k^*, Y)}{f(k^*, Y)} = (1 - \theta)\sigma + \theta
\]  

(4.17)

If \( \chi > \chi^* \), the solution is \( k = k^* \) and

\[
k = k^*
\]

(4.18)

\[
\phi = \theta [f(k^*, Y) - k^*]
\]

(4.19)

If \( \chi < \chi^* \), then there is a unique \((\phi, k) \in \mathbb{R}_+ \times [\hat{k}(Y), k^*(Y)]\), which solves

\[
\phi = \chi f(k, Y) - k
\]

(4.20)

\[
\frac{k}{f(k, Y)} = \chi \frac{\partial f(k, Y)}{\partial k} - \theta
\]

\[
(1 - \theta) \frac{\partial f(k, Y)}{\partial k}
\]

(4.21)
The cases can be combined as

\[
k = \min \left\{ 1, \left[ \frac{\chi}{(1 - \theta)\sigma + \theta} \right]^{\frac{1}{1 - \sigma}} \right\} (\sigma \gamma)^{1/(1 - \sigma)} Y^{\frac{\gamma - \sigma}{\gamma(1 - \sigma)}}
\]

(4.22)

\[
\phi = \min \{ \chi f(k, Y) - k, \theta[f(k, Y) - k] \}
\]

(4.23)

\[
r = \theta \left( \frac{1 - \sigma}{\sigma} \right)
\]

(4.24)

According to (4.22), the investment decision of an individual entrepreneur depends on the credit conditions of other entrepreneurs in the economy. If a shock reduces financial access for some mass of firms, then entrepreneur \(i\) will reduce his own investment provided that \(\gamma > \sigma\).

The pledgeability constraint is more likely to bind with a higher \(\sigma\) and \(\theta\). Greater bargaining power induces banks to charge higher interest rates on smaller loans. More substitutability of goods raises the target level of investment, increasing the likelihood that the pledgeability constraint binds. Given the symmetry of product differentiation and homogeneity of bargaining power, the real lending rate is the same for all firms. In particular, pledgeability coefficients of firms and the returns to scale do not affect the lending rate.

The following corollary summarizes comparative statics of investment.

**Corollary 4.1.** If \(\chi < \chi^*\), then the constraint binds and the solution \(k\) is increasing and continuous in \(\chi\), with \(k(0) = 0\) and \(k(\chi^*) = k^*\). Furthermore, \(\frac{\partial \log k}{\partial \theta} = -1/[(1 - \theta)\sigma + \theta] < 0\), \(\frac{\partial \phi}{\partial \theta} > 0\), and \(\frac{\partial \phi}{\partial \chi} > 0\).

Let \(G(\chi_b)\) denote the cumulative distribution of pledgeability coefficients with density \(g(\chi_b)\). From (4.10), aggregate output is

\[
Y = \left[ \int_0^1 \alpha \lambda k_i^\sigma di \right]^\frac{2}{\sigma}
\]

(4.25)

since each entrepreneur \(i\) has an opportunity to invest with probability \(\alpha \lambda\).
Substituting (4.22) into (4.25), we can write

\[ Y = \left[ \alpha\lambda(\sigma\gamma)^{\sigma\gamma/(\gamma(1-\sigma))} \right]^{\gamma/\sigma} F(\sigma, \theta)^{\gamma/\sigma} \]  \hspace{1cm} (4.26)

where

\[ F(\sigma, \theta) = \int_0^{(1-\theta)\sigma+\theta} \frac{\chi}{(1-\theta)\sigma + \theta}^{\sigma/(1-\sigma)} dG(\chi) + 1 - G[(1-\theta)\sigma + \theta] \]  \hspace{1cm} (4.27)

\( F \) is a weighting factor that scales output according to the mass of entrepreneurs that are constrained. If \( \chi_i \geq (1-\theta)\sigma + \theta \) \( \forall i \), then \( F = 1 \). In general, \( F < 1 \). \( F \) satisfies \( \frac{\partial F}{\partial \sigma} < 0 \) and \( \frac{\partial F}{\partial \theta} < 0 \).

Using integration by parts, we can express \( F \) more succinctly as

\[ F(\sigma, \theta) = 1 - \frac{\sigma}{1-\sigma} \left[ 1 - \frac{1}{(1-\theta)\sigma + \theta}^{(1-\sigma)/\sigma} \int_0^{(1-\theta)\sigma+\theta} G(\chi)\chi^{(2\sigma-1)/(1-\sigma)} \, d\chi \right] \]  \hspace{1cm} (4.28)

As \( \sigma \to 1 \) (allowing \( \gamma \to 1 \)), \( F \to 0 \), since the first integral approaches zero and \( (1-\theta)\sigma + \theta \to 1 \). As \( \sigma \to 0 \), \( F \to 1 \), which is especially clear from (4.28). Also, \( F(\sigma, 1) = \mathbb{E}(\chi^{\sigma/(1-\sigma)}) \), so that \( F(1/2, 1) = \mathbb{E}(\chi) \). Next, we solve for both \( Y \) and aggregate investment \( K = \mathbb{E}(k) \) in terms of \( F(\cdot) \):

\[ Y = \left[ (\alpha\lambda)^{(1-\sigma)/\sigma} \sigma\gamma F(\sigma, \theta)^{(1-\sigma)/\sigma} \right]^{\gamma/(1-\gamma)} \]
\[ = (\sigma\gamma)^{\gamma/(1-\gamma)} [\alpha\lambda F(\sigma, \theta)]^{\gamma/(1-\gamma)}/[\sigma(1-\gamma)] \]  \hspace{1cm} (4.29)

\[ K = \alpha\lambda F(\sigma, \theta)(\sigma\gamma)^{1/(1-\sigma)} Y^{\sigma/(1-\sigma)} \]
\[ = \alpha\lambda(\sigma\gamma)^{\gamma/(1-\gamma)}(\sigma\gamma)^{1/(1-\sigma)} [\alpha\lambda F(\sigma, \theta)]^{\gamma/(1-\sigma)}/[\sigma(1-\gamma)] \]  \hspace{1cm} (4.30)

which depend on the entire distribution of pledgeability coefficients through \( F \). Note that as \( \gamma \to 1, \gamma/(1-\gamma) \to \infty \), and \( Y \to 0 \) since the expression within the brackets of (4.29) is below 1. Furthermore, as \( \sigma \to 0, Y \to 0 \). Figure 4.4 is a surface plot of \( \sigma \) and \( \theta \) and showcases the sensitivity of \( F(\cdot) \) and \( Y \) with respect to \( \sigma \), compared with \( \theta \).\footnote{For this example and for the general model we use \( \chi \sim Beta(a, b) \), in which \( I(\cdot) \) is the incomplete beta function. In general
\[ F(\sigma, \theta) = 1 + \frac{B(a + \frac{\sigma}{1-\sigma}, b)}{B(a, b)} I(a + \sigma/(1-\sigma), b) - I(a, b) \]  \hspace{1cm} (4.31)}
From these expressions, it is clear that mean investment and aggregate demand have the same semi-elasticity with respect to a change in $G(\cdot)$ or $\theta$.

**Proposition 4.2.** Aggregate demand and mean investment have the same elasticity with respect to $\theta$ and functions of the distribution of $\chi$.

As the returns to scale increases, a one-unit change in average pledgeability has a bigger effect on investment.

**Definition 4.1.** A bank credit equilibrium is a list $(k_i, r, Y)$ satisfying (4.22), (4.24), and (4.29).

From the uniqueness of $Y$ in (4.29) and the uniqueness of $k_i$ given $Y$ from Proposition 4.1, it follows that equilibrium exists and is unique.

**Proposition 4.3.** There is a unique bank credit equilibrium.

The appendix derives some more comparative statics, illustrating how the elasticity of output with respect to $\alpha$, $\lambda$, and $\theta$ increases with the aggregate demand externality.
4.4 Internal and External Finance

We now let entrepreneurs accumulate cash in Stage 2 to finance investments in Stage 1 of the following period. Salient features of internal finance are that it is an immediate funding source, has no explicit interest payments, and can be used regardless of whether entrepreneurs are matched with a bank. The presence of internal finance is key for our analysis because monetary policy affects the cost of holding real balances and thus the lending rate and investment. We refer to the effect of the policy rate on investment as transmission and to the effect of the policy rate on the lending rate as pass through, in line with Rocheteau et al. (2016).

We first characterize the outside option of an entrepreneur. Given an investment opportunity but no access to a bank, feasibility requires $k \leq a^e_m$ and the surplus from investing is

$$\Delta_m(a^e_m) = f(k_m, Y) - k_m$$

where $k_m = \min\{a^e_m, k^*\}$ (4.32)

The function $\Delta_m(a^e_m)$ is increasing and strictly concave for all $a^e_m < k^*$ with $\Delta_m'(a^e_m) = \frac{\partial f}{\partial k} - 1 > 0$.

Suppose next that the entrepreneur is in contact with a bank. Then the terms of the contract specify (1) the investment level $k$, (2) the down payment $d$, and (3) the bank’s fee $\phi$. The surplus from a bank loan is thus $f(k, Y) - k - \phi - \Delta_m(a^e_m)$. Accordingly, $(k, d, \phi)$ solves

$$\max_{k, d, \phi} [f(k, Y) - k - \phi - \Delta_m(a^e_m)]^{1-\theta} \phi^\theta \quad \text{s.t.}$$

$$k - d + \phi \leq \chi f(k, Y) \quad (4.34)$$

$$d \leq a^e_m \quad (4.35)$$

Lemma 4.1 characterizes the solution of the bargaining given $a^e_m$ and $Y$ in terms of a threshold of retained earnings $a^*$, and is analogous to Lemma 1 in Rocheteau et al. (2016).

One important difference, however, is that Rocheteau et al. (2016) defined transmission into investment and output, respectively, in terms of absolute deviations $\partial Y / \partial i$ and $\partial K / \partial i$, whereas we will focus on percentage deviations of these variables from the initial state in which $i = 0$. The reason is that with monopolistic competition $k^*$ is a function of $Y$ and thus depends on the entire set of parameters; thus, absolute comparisons cannot be made because the initial state is different. For Rocheteau et al. (2016), $k^*$ does not depend on $\alpha, \lambda, \sigma, \theta$, or $\chi$, so that such comparisons are appropriate.
Lemma 4.1. There is an $a^*$ characterized by

$$(\chi^* - \chi)f(k^*, Y) = (1 - \theta)a^* + \theta f(a^*, Y)$$

satisfying $a^* < k^*$, in which $a^* > 0$ if and only if $\chi < \chi^*$. Moreover, if $a_m^e \geq a^*$, then

$$k_c = k^*$$

$$\phi^* = \theta [f(k^*, Y) - k^* - \Delta_m(a_m^e)]$$

(4.36)

(4.37)

(4.38)

If $a_m^e \leq a^*$, then $(\phi, k^*) \in \mathbb{R}_+ \times (\hat{k}, k^*)$ solves

$$\frac{a_m^e + \chi f(k_c, Y) - k_c}{(1 - \chi) f(k_c, Y) - a_m^e - \Delta_m(a_m^e)} = \frac{\theta}{1 - \theta} \frac{1 - \chi \frac{\partial f}{\partial k_c}}{1 - \chi (1 - \chi) \frac{\partial f}{\partial k_c}}$$

$$k_c + \phi = a_m^e + \chi f(k_c, Y)$$

(4.39)

(4.40)

If $a_m^e < a^*$, then Equation (4.39) uniquely determines $k_c$ given $a_m^e$, and Equation (4.40) solves for $\phi$ given $k_c$ and $a_m^e$.

Lemma 4.2. If the liquidity constraint binds, then $\frac{\partial k_c}{\partial a_m^e} > 0$, $\frac{\partial [a_m^e + \chi f(k_c, Y)]}{\partial a_m^e} > 1$, and $\frac{\partial k_c}{\partial \theta} < 0$.

Thus, accumulating reserves increases financing capacity by more than one since pledgeable output increases, and investment rises. Furthermore, a higher bank bargaining power reduces investment.

The lending rate is $r \equiv \phi/(k_c - a_m^e)$ is

$$r = \begin{cases} 
\frac{\theta [f(k^*, Y) - k^* - \Delta_m(a_m^e)]}{k^* - a_m^e} & \text{if } a_m^e \in [a^*, k^*] \\
\frac{\chi f(k_c, Y)}{k_c - a_m^e} - 1 & \text{if } a_m^e < a^*
\end{cases}$$

(4.41)

It immediately follows that $\partial r/\partial a_m^e < 0$ for all $a_m^e \in [a^*, k^*]$ and $r \to 0$ as $a_m^e \to k^*$. If the liquidity constraint binds, then there is no guarantee that $\partial r/\partial a_m^e < 0$. In particular, for $\theta = 1$, then $r = [\chi f(\hat{k}, Y)]/(\hat{k} - a_m^e)$, so that $\partial r/\partial a_m^e > 0$. In this case, bringing more assets finances
the same level of investment with a smaller loan bearing a higher interest rate. Thus, a priori, the pass through $\partial r/\partial i$ need not be positive, and its sign and magnitude depends closely on the bank bargaining power.

The entrepreneur’s choice of real balances solves

$$\max_{a_e^m \geq 0} \{-ia_e^m + \lambda(1 - \alpha)\Delta_m(a_e^m) + \alpha\lambda\Delta_c(a_e^m)\} \tag{4.42}$$

where $\Delta_c(a_e^m) \equiv f(k_c, Y) - k_c - \phi$ takes the following form:

$$\Delta_c(a_e^m) = \begin{cases} 
(1 - \theta)[f(k^*, Y) - k^*] + \theta\Delta_m(a_e^m) & \text{if } a_e^m \geq a^* \\
(1 - \chi)f(k_c, Y) - a_e^m & \text{otherwise}
\end{cases}$$

There are three cases. If $a_e^m > k^*$, then the entrepreneur finances $k^*$ without resorting to bank credit and he appropriates the full gains from trade. If $a_e^m \in [a^*, k^*)$, he can still finance $k^*$, but only by using bank credit. Finally, if $a_e^m < a^*$, then the liquidity constraint binds and the entrepreneur’s surplus equals the non-pledgeable output net of his real balances. However, under the standing assumption that $i > 0$, entrepreneurs always choose $a_e^m < k^*$, since the marginal benefit of an additional unit of real balances is zero at $k^*$ but the marginal cost is positive. Thus, we can just consider the last two cases and use $k_m = a_e^m$.

Aggregate demand satisfies

$$Y = \lambda(1 - \alpha)\left[\int_0^1 k_{i,m}^\sigma di\right] + \lambda\alpha\left[\int_0^1 (k_{i,c})^\sigma di\right]^{\gamma/\sigma} \tag{4.43}$$

A monetary equilibrium with internal and external finance is a list $(k_c, a_e^m, r, Y)$ that solves (4.33), (4.41), (4.42), and (4.43).

The first order condition for real balances can be written as
\[ i = \lambda [1 - \alpha (1 - \theta)] \left[ \partial f / \partial k_m - 1 \right] \]

\[ \frac{i}{\lambda} + 1 = (1 - \alpha) \frac{\partial f(k_m, Y)}{\partial k_m} + \alpha (1 - \chi) \frac{\partial f(k_c, Y)}{\partial k_c} \frac{dk_c}{da} \]

if \( a^e_m > a^* \) \hspace{1cm} (4.44)

otherwise \hspace{1cm} (4.45)

If \( a < a^* \), then \( \frac{\partial k_m}{\partial \lambda} < 0, \frac{\partial k_m}{\partial \alpha} < 0 \) and \( \frac{\partial k_m}{\partial \theta} > 0 \). where the curve \( h(a_m^e, k_c, \chi, Y) = \frac{\partial k_c}{\partial a_m^e} \)

which relates the effect of higher assets on the bargained level of investment \( k_c \) of an entrepreneur with coefficient \( \chi \) and aggregate demand \( Y \). Appendix C.1.5 provides the full expression. The latter two conditions show how conditions in the credit market affect money demand. As loans are less accessible or more costly, entrepreneurs hold more cash to compensate. Provided that \( \theta > 0 \), entrepreneurs hold cash even if \( \alpha = 1 \) in order to reduce the rents held by bankers. The next proposition shows that coexistence of cash and credit is robust.

**Proposition 4.4 (Coexistence).** For all \( i > 0 \) and \( \chi > 0 \), if \( \lambda (1 - \alpha) > 0 \) or \( \lambda \theta > 0 \), then equilibrium features coexistence of cash and credit.

Money exists in equilibrium provided that the insurance motive, \( \lambda (1 - \alpha) > 0 \), or strategic motive, \( \lambda \alpha \theta > 0 \), is active. There is a general motive to use credit if \( i > 0 \), either to finance more investment if the household is liquidity constrained, or to reduce interest payments otherwise.

In Rocheteau et al. (2016), there is a threshold \( \bar{i} \) such that for \( i < \bar{i} \) policy affects the component of investment financed internally but not that financed through banking. The aggregate demand externality eliminates this dichotomy: smaller internal finance reduces aggregate demand, which reduces entrepreneurs’ revenue and thereby decreases investment financed through banking. This fact holds even with homogeneity of firms and perfect enforcement, which we analyze next.
4.5 Pass Through with Perfect Enforcement

We consider the special case of perfect enforcement \( \chi_j = 1 \) for each entrepreneur \( j \). Doing so enables us to characterize many properties analytically and derive an upper bound of output that is useful as a benchmark in the more general version.

The solution is \( k_c = k^* \) with \( \phi = \theta[f(k^*) - k^* - \Delta_m(a^e_m)] \). The equations describing the nominal interest, real lending rate, investment, and output are

\[
\begin{align*}
    i &= \lambda[1 - \alpha(1 - \theta)] \left[ \frac{\partial f(a_\ell, Y)}{\partial a_\ell} - 1 \right] \\
r &= \theta \left\{ [f(k^*, Y) - k^*] - [f(a^e_\ell, Y) - a^e_\ell] \right\} \\
k^* &= [\sigma \gamma Y^{(\gamma-\sigma)/\gamma}]^{1/(1-\sigma)} \\
Y &= [\lambda(1 - \alpha) a^\sigma_m + \lambda \alpha k^* \gamma / \sigma]
\end{align*}
\]

The system in this form does not have a recursive representation. The variables \( a^e_\ell, r, \) and \( k^* \) each depend on \( Y \), and \( Y \) in turn depends on \( k_m \) and \( k_c \). However, we can solve for \( Y \) in closed form and sequentially back out the other variables.

First, we use (4.46) and (4.48) to characterize \( a^e_\ell \) as a fraction of the optimal investment \( k^* \):

\[
\begin{align*}
a^e_\ell(Y) &= \Upsilon(i)^{1/(1-\sigma)}[\sigma \gamma Y^{(\gamma-\sigma)/\gamma}]^{1/(1-\sigma)} \\
    &= k^* \Upsilon(i)^{1/(1-\sigma)}
\end{align*}
\]

for

\[
\Upsilon(i) = \left[ \frac{\lambda(1 - \alpha(1 - \theta))}{i + \lambda(1 - \alpha(1 - \theta))} \right].
\]

Thus, \( a^e_\ell \) is the product of \( k^* \) and a dampening factor \( \Upsilon(i)^{1/(1-\sigma)} \), which depends on the availability of investment opportunities, search frictions, bargaining power, the cost of holding real balances, and product differentiation. Higher \( \lambda \) raises investment opportunities and thereby induces cash
holdings. Higher $\alpha$ reduces the insurance motive of cash and therefore reduces money demand. Higher bargaining power of banks raises cash holdings through the strategic motive in negotiation. Finally, greater product diversity stimulates cash holdings by raising market power and thus the value of the investment project.

Using (4.50), it is straightforward to solve for mean investment:

$$K = \lambda k^*[(1 - \alpha)Y(i)^{1/(1-\sigma)} + \alpha]$$

(4.52)

Combining (4.50), (4.48), and (4.49) we obtain $Y$ in closed form (details in appendix):

$$Y = (\sigma \gamma)^{\gamma/(1-\gamma)} \lambda^{\gamma(1-\sigma)/\sigma(1-\gamma)} (1 - \alpha)Y(i)^{\sigma/(1-\sigma)} + \alpha \right)^{\gamma(1-\sigma)/\sigma(1-\gamma)}$$

(4.53)

We now provide a convenient description of equilibrium:

**Definition 4.2.** An equilibrium with internal and external finance under perfect enforcement is a list $(k^*, a^e_m, r, Y)$ satisfying (4.48), (4.50), (4.47), and (4.53).

We can solve for equilibrium recursively. We first obtain $Y$ through (4.53) and use it to back out $k^*$ with (4.48). With $k^*$ and $Y$, it is straightforward to obtain $a^e_m$ and then $r$. Thus, existence and uniqueness of equilibrium follows.

**Proposition 4.5.** An equilibrium with internal and external finance under perfect enforcement exists and is unique.

The following lemma characterizes aggregate revenue $R = \lambda [\alpha f(k_c, Y) + (1 - \alpha) f(k_m, Y)]$ and aggregate welfare. Aggregate welfare is the sum of the surpluses of the final goods producers, intermediate goods producers, and banks:

$$W = Y - \int_0^1 f(k_i, Y) di + \int_0^1 [f(k_i, Y) - k_i - \phi_i] di + \int_0^1 \phi_i di$$

(4.54)
Lemma 4.3. Aggregate revenue $R$ and welfare $W$ satisfy the following:

\begin{align*}
R &= \gamma Y \\
W &= Y - K
\end{align*}  

(4.55) \quad (4.56)

It is helpful for comparative statics to do a first-order approximation of $\log Y$ at $i = 0$: \( \log Y = \log Y|_{i=0} + i \frac{\partial \log Y}{\partial i} \bigg|_{i=0} \).

Proposition 4.6 shows the semi-elasticity of aggregate demand, financed capital, internal finance, and revenue with respect to the nominal interest rate.

Proposition 4.6. The semi-elasticity of money demand satisfies

\[
\frac{\partial \log a_m}{\partial i} = -\frac{\lambda[1 - \alpha(1 - \theta)]}{(1 - \sigma)[i + \lambda(1 - \alpha(1 - \theta))]} \left[ \frac{(\gamma - \sigma)(1 - \alpha) Y(i)^{(2\sigma - 1)/(1 - \sigma)} Y(\alpha) + 1}{(1 - \sigma)Y(i) + \alpha + 1} \right] \tag{4.57}
\]

Furthermore, the first order approximation of the pass through from the interest rate to aggregate demand, financed capital, internal finance at $i = 0$ satisfies

\[
\begin{align*}
\frac{\partial \log Y}{\partial i} \bigg|_{i=0} &= -\frac{\gamma(1 - \alpha)}{(1 - \gamma)\lambda[1 - \alpha(1 - \theta)]} \\
\frac{\partial \log k_c}{\partial i} \bigg|_{i=0} &= -\frac{\gamma - \sigma}{(1 - \sigma)(1 - \gamma)\lambda[1 - \alpha(1 - \theta)]} \\
\frac{\partial \log a_m}{\partial i} \bigg|_{i=0} &= -\frac{1}{(1 - \sigma)\lambda[1 - \alpha(1 - \theta)]} \left[ \frac{(\gamma - \sigma)(1 - \alpha)}{1 - \gamma} + 1 \right] \\
\frac{\partial \log f(k_c, Y)}{\partial i} \bigg|_{i=0} &= \frac{\partial \log k_c}{\partial i} \bigg|_{i=0}
\end{align*}
\]

(4.58) \quad (4.59) \quad (4.60) \quad (4.61)

The semi-elasticity of output is increasing in the returns to scale, decreasing in access to external finance, decreasing in the probability of financing an investment project, and decreasing in the bargaining power of banks. Intuitively, investment financed by banks does not respond to interest rate changes, whereas cash-financed investment does. More opportunities for investment (\( \uparrow \lambda \)) or higher bank bargaining power (\( \uparrow \theta \)) stimulates money demand and consequently reduces transmission. The semi-elasticity of output does not, to a first order, depend on the product diversity $\sigma$. We shall see
that, as a result, the pass through from the nominal rate to the real rate is also independent of $\sigma$, to a first-order approximation.

The semi-elasticity of financed capital $k_c$ is positive provided that $\gamma > \sigma$; it depends negatively on $\sigma$ with a derivative proportional to $-(1 - \gamma)/(1 - \sigma)^2$, which approaches $-(1/(1 - \gamma))$ as $\sigma \to \gamma$.

Again, the aggregate demand externality induces a negative transmission to investment even for $i$ in the neighborhood of $i = 0$, in contrast to Rocheteau et al. (2016).

Define the external share of finance $ext_f = 1 - a_m/[(1 - \alpha)a_m + \alpha k_c]$. We also define average leverage

$$Lev = \alpha \lambda \frac{k_c - a_m^e + \phi}{f(k_c, Y) - [k_c - a_m^e + \phi]}$$

(4.62)

which is the ratio of debt to equity. Less financial constraints enable firms to have a higher leverage ratio, which is associated with less sensitivity to output.

**Proposition 4.7.** The external share of finance satisfies

$$ext_f = \frac{\alpha(1 - \Upsilon_{1/(1-\sigma)})}{\alpha(1 - \Upsilon_{1/(1-\sigma)}) + \Upsilon_{1/(1-\sigma)}}$$

(4.63)

A first order approximation in the neighborhood of $i = 0$ yields

$$ext_f \approx \frac{\alpha i}{(1 - \sigma)\lambda[1 - \alpha(1 - \theta)]}$$

(4.64)

Furthermore, the leverage ratio satisfies

$$Lev = \frac{\alpha \lambda[(1 - \theta)\sigma + \theta \Theta(i)](1 - \Upsilon(i)^{1/(1-\sigma)})}{1 - [(1 - \theta)\sigma + \theta \Theta(i)](1 - \Upsilon(i)^{1/(1-\sigma)})}$$

(4.65)

which is increasing in $\sigma$ since $\frac{\partial \Theta}{\partial \sigma} > 0$.

Thus, the external share of finance approaches zero as the nominal interest rate approaches zero.

We define the composite parameter $\Theta(i)$, which will be useful for characterizing the real lending
rate:

\[ \Theta(i) = \frac{1 - \Upsilon(i) \frac{1}{\sigma}}{1 - \Upsilon(i)^{1/(1-\sigma)}} \]

The following lemma helps us approximate the pass through to a first order.

**Lemma 4.4.** \( \lim_{i \to 0} \Theta(i) = \sigma \) and \( \lim_{i \to 0} \Theta'(i) = \frac{\sigma}{2\lambda[1-\alpha(1-\theta)]} \). As \( i \to \infty \), \( \Theta(i) \to 1 \).

**Proposition 4.8.** The real interest rate satisfies

\[ r = \theta \left\{ \frac{1}{\sigma} \Theta(i) - 1 \right\} \]  

(4.66)

As \( i \to 0 \), \( r \to 0 \), and as \( i \to \infty \), \( r \to \theta(1-\sigma)/\sigma \), the same value as under external finance alone.

The best first order approximation to \( r \) in the neighborhood of \( i = 0 \) is

\[ r = \frac{\theta i}{2\lambda[1-\alpha(1-\theta)]} \]  

(4.67)

Equation (4.67) coincides with the approximation in Rocheteau et al. (2016) without monopolistic competition and aggregate demand externalities. In particular, the real interest rate is independent of the returns to scale and, to a first-order approximation, independent of the elasticity of substitution between goods. Moreover, higher \( \theta \) has a direct effect raising interest rates for a given holding of real balances \( a^e_m \), and an indirect effect as entrepreneurs compensate by holding more real balances. However, the first effect predominates. As \( \alpha \) increases, entrepreneurs have less need to insure themselves, so they reduce their real balances. However, their disagreement point in negotiations is worse, so the real lending rate increases. Similarly, higher \( \lambda \) strengthens the outside option and lowers the real lending rate.

Though product diversity has no first-order effect on the real interest rate, in general the latter decreases with \( \sigma \) because, as competition rises, the profitability of investment diminishes and thus the real interest rate that banks can charge falls. The appendix compares the first-order approximation of the lending rate to the global solution and shows that it is generally an upper bound and
deteriorates fast for $i > 0.05$.

We conclude this section by numerically analyzing the pass through of the nominal interest rate on capital, internal finance, aggregate demand, the real interest rate, aggregate revenue, and external finance. Figure 4.5 examines the pass through for $\gamma = 0.8$ and $\gamma = 0.9$, and Figure 4.6 considers $\sigma = 1/1.4$ and $\sigma = 1/1.3$. I express the responses in the top three panels as percentage deviations from the initial value, which facilitates comparison.\textsuperscript{11}

![Graphs of Aggregate Investment, Aggregate Internal Finance, Output, Mean Lending Rate, Mean External Share of Finance, Mean Leverage for different values of $\gamma$.]

Figure 4.5: Pass Through of Nominal Interest Rate: different values of $\gamma$.

As we have seen from Proposition 4.6, a change in $\gamma$ has a first-order effect on investment and output, so the path for these series diverges rapidly. The same holds qualitatively for internal finance. As we have also seen, the real interest rate and the external share of finance respond the same for different values of $\gamma$. The path is identical for aggregate revenue and output.

\textsuperscript{11}Both the real interest rate, external share of external finance, and leverage are zero for $i = 0$ and are thereby already normalized in their respective ways.
Figure 4.6 validates the result that a change in $\sigma$ has no first order effect on aggregate demand. There is a moderate first order effect on investment and small first order effect on internal finance. More product differentiation increases the surplus and hence the real interest rate, but the effect is largely undone from the greater motive to hold real balances. We will see in the next section that, with heterogeneous financial frictions, transmission can actually rise with greater substitutability of goods, as the proportion of constrained firms increases. Similarly, as greater product diversity spurs entrepreneurs to hold more cash, it lowers the external share of finance.

We also compare the pass through from the nominal interest rate to loan sizes. A rise in $i$ has two effects. First, agents substitute away from internal into external finance, which tends to raise the loan size. Second, higher $\gamma$ implies a stronger aggregate demand externality and bigger decline in output, reducing the loan size. In the appendix, we show that for large $\gamma$, the latter effect eventually dominates, producing a hump shape, in the loan size, whereas for lower $\gamma$ the aggregate demand externality is weaker and the loan size increases monotonically.
4.6 Heterogeneous Entrepreneurs with Limited Enforcement

We now assume entrepreneurs possess and are heterogeneous in terms of financial constraints: \( \chi \sim Beta(a, b) \). There is a unique mapping from the mean and standard deviation of the distribution \((\mu, \sigma_\chi) \rightarrow (a, b)\) provided that \(\sigma_\chi^2 < \mu(1 - \mu)\).

If we combine Lemma 4.1 with (4.50), we obtain a closed form characterization of the threshold \(\chi^{**}\) at which the liquidity constraint binds.

**Lemma 4.5.**

\[
\chi^{**} = \theta + (1 - \theta)\sigma - \left\{ \theta \Upsilon(i)^{\sigma/(1 - \sigma)} + (1 - \theta)\sigma \Upsilon(i)^{1/(1 - \sigma)} \right\}
\]

The left hand side of 4.5 coincides with the expression under external finance. The right hand side is an adjustment for the availability of internal finance. As \(i \rightarrow \infty, \Upsilon \rightarrow 0\), and \(\chi^{**} \rightarrow \chi^*\). As \(i \rightarrow 0, \chi^{**} \rightarrow 0\), so that the liquidity constraint never binds. The intuition is that if there is no cost to internal finance, then entrepreneurs can always reach the desired level of investment. Also, note that if \(\sigma \rightarrow 1\), then entrepreneurs have no incentive to hold cash, and \(\chi^{**} = 1\), so that all firms are constrained.
Figure 4.7: $\chi^{**}$ as a function of the nominal interest rate

Figure 4.7 plots $\chi^{**}$ as a function of the interest rate together with the upper limit $(1 - \theta)\sigma + \theta$. An upward shift in $\sigma$ or $\theta$ reduces the profitability of entrepreneurs and shifts the $\chi^{**}$ curve upward. The threshold $a^*$ for the bargaining is decreasing in $\theta$, but it does not decrease fast enough so that the first best is attainable.

Money holdings $a_m(\chi)$ and bargained investment $k_c(\chi)$ are both a function of $\chi$, so that aggregate demand satisfies

$$Y = \lambda^{\gamma/\sigma} \left\{ \alpha \left[ (1 - G(\chi^{**}))k^{**} + \int_{0}^{\chi^{**}} k_c(\chi)^{\sigma} dG(\chi) \right] + (1 - \alpha) \int_{0}^{1} a_m^e(\chi)^{\sigma} dG(\chi) \right\}^{\gamma/\sigma} \quad (4.69)$$

The appendix provides a detailed discussion of the strategy to compute equilibrium. The determination of aggregate demand $Y$ involves a fixed point problem, which also requires numerical integration.

Figure 4.8 illustrates the effects of a higher standard deviation of the enforcement technology $\sigma_\chi$ with the mean fixed. The concavity of the revenue function makes output decrease with a higher variance: a given increase in $\chi$ for a firm near the first best level of investment changes investment less than the same decrease for a firm farther away from the first best level. Furthermore, for firms with $\chi > \chi^{**}$, investment levels do not change other than through the aggregate demand externality.
The real lending rate eventually decreases because declining output lowers the surplus of trading opportunities between entrepreneurs and the bank.

Figure 4.8: Effect of a mean-preserving spread on output and the real lending rate. Output is in log deviations.

Figure 4.9: Pass Through of Nominal Interest Rate: different values of $\sigma$

Product diversity does not affect the monetary transmission and pass through substantially. Though a lower $\sigma$ strengthens the aggregate demand externality, there are two countervailing effects. Greater product diversity spurs entrepreneurs to hold more real balances, which weakens transmission. Second, greater product diversity lowers $\chi^{**}$ and hence the measure of constrained firms, which
further weakens transmission. This last channel only arises with heterogeneous financial frictions. Consequently, transmission can actually be stronger with more substitutable products, though the effect is generally weak.

Figure 4.10 illustrates the effects of the policy rate on outcomes of interest under different levels of returns to scale. There is a first-order effect of the policy rate on output without any compensating difference in the ability of entrepreneurs to raise external funds. Thus, there is a major change in the transmission of monetary policy (and hence the level of internal finance), but no effect on the pass through to the real interest rate.

Figure 4.10: Pass Through of Nominal Interest Rate: different values of $\gamma$

Figure 4.11 showcases the financial multiplier: a higher policy rate reduces investment and output more with lower mean pledgeability (higher financial frictions) and generates a higher pass through to the mean lending rate. More financial frictions, unsurprisingly, imply a smaller increase in the mean external share of finance and the mean leverage.
Figure 4.11: Pass Through of Nominal Interest Rate: different values of $\mu$

Figure 4.12 illustrates the interaction between the financial multiplier and the aggregate demand externality. Increasing average pledgeability (decreasing financial frictions) positively affects aggregate investment and output, alongside the external share of finance. A higher $\gamma$ implies a stronger aggregate demand externality, leading to a stronger demand for the investment of each entrepreneur. Notably, the change in the leverage ratio does not depend on $\gamma$.

Note that there are two competing effects on aggregate internal finance. Higher pledgeability reduces entrepreneurs needs to accumulate cash, but it also raises output and, though the aggregate demand externality, investment opportunities. These greater investment opportunities raise money demand. For a sufficiently high $\gamma$, the second effect predominates.
Figure 4.12: Effects of higher pledgeability: different values of $\gamma$

Figure 4.13 depicts the effects of the policy rate under higher variation of the pledgeability coefficients, with the mean held constant. A mean-preserving of liquidity constraints affects the transmission or pass through non-uniformly, achieving a maximal effect on investment and output at about $i = 9\%$. Moreover, it attenuates the increase in the external share of finance. Greater dispersion of financial frictions implies an increase in the share of unconstrained firms, for which there is no change in the external share of finance other than through the aggregate demand externality. However, there is now a larger share of severely constrained firms, for which the external share of finance falls, leading to a decline in the overall external share of finance. Consequently, a mean-preserving spread of the pledgeability distribution negatively affects investment and output, reduces the external share of finance, and increases transmission over some range.
Finally, the theory provides a rich cross section of transmission of monetary policy with respect to pledgeability coefficients. Figure 4.14 shows the dependence of investment on firm’s pledgeability coefficient, for percentiles 10, 25, 50, and 90. This illustration justifies the empirical strategy of identifying firms size with capital market access after controlling for various non-financial variables, as by Gertler and Gilchrist (1994).
Figure 4.14: Cross section of transmission by pledgeability coefficient. Investment is expressed in percentage deviations from the level associated with $i = 0$.

### 4.7 Calibration

We calibrate using a strategy similar to Rocheteau et al. (2016). The sample period is 1958-2007. We interpret $i$ as the 3-month Treasury bill secondary market rate, which averages 5.4% for the sample period. Rocheteau et al. (2016) find that the associated real rate is $\rho = 2\%$ using the method by Hamilton et al. (2016).

We focus exclusively on the special case with perfect enforcement. We use the semi-elasticity of money demand as a target for calibration. Lucas (2000) estimates a semi-elasticity of $-7$ from the data, which is consistent with the estimate using data from Mulligan (1997). We interpret $\alpha$ as the probability that a loan application is accepted. The 2003 Survey of Small Business Finance reports that this ratio is 0.9. The target for the real lending rate in the data is the average difference between the prime lending rate and the 3-month T-bill rate. We solve for the semi-elasticity of money demand (4.57) and the real lending rate (4.47) as a system to obtain $\lambda$ and $\theta$. Table 4.1 lists the parameter values of the calibrated model.
Table 4.1: Parameterization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
<th>Calibration Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>0.8000</td>
<td>Fixed</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.9000</td>
<td>loan application acceptance rate</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.8841</td>
<td>semi-elasticity of money demand</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.7143</td>
<td>40% markup</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.3954</td>
<td>real lending rate</td>
</tr>
<tr>
<td>$i$</td>
<td>0.0540</td>
<td>3-month T-bill rate (nominal)</td>
</tr>
</tbody>
</table>

4.8 Conclusion

This paper studies the interaction of an aggregate demand externality from monopolistic competition with corporate finance and monetary policy. Entrepreneurs have random idiosyncratic investment opportunities, and match with banks in an over-the-counter market subject to heterogeneous limited enforcement. Monetary policy influences retained earnings and hence the size of investment and real lending rate. The aggregate demand externality influences transmission—though not the pass through to the real lending rate—and interacts strongly with financial frictions. Heterogeneous financial frictions implies cross-sectional dispersion of transmission, with monetary policy yielding bigger effects on smaller, more constrained firms, which is consistent with Gertler and Gilchrist (1994). Finally, greater dispersion of financial frictions reduces output and the effects of monetary policy on the external share of finance and raises pass through and transmission for intermediate values of the policy rate.
Chapter 5

Future Work

The research in this thesis regarding liquidity, aggregate demand externalities, and search frictions invites several promising extensions. A limitation is that neither of the models focused on consumers accommodates coexistence of cash and credit, though the treatment of corporate finance and monetary policy in Chapter 4 does. The model in Chapter 2 is a pure currency economy, and the model in Chapter 3 is a pure credit economy. These restrictions are important. The welfare costs of inflation are generally lower in the present of alternate payment instruments. Furthermore, households can use liquid assets to at least partially offset decreases in debt limits. As we noted in Chapter 3, unsecured credit in particular is both the primary short-run determinant of households’ liquidity and is also volatile. Thus, there is a risk motive for holding cash, which, together with the inflation motive for holding credit, can motivate households to hold both, even in the absence of constraints on acceptability by vendors. It is especially fruitful to combine the coexistence of cash and credit with endogenous variety.

The model of mutual feedback presented in Chapter 3 has several restrictions worth noting. We already hinted at the first: the reliance purely on credit for trade and absence of a savings technology exaggerates several correlations in the model, particularly between credit and market tightness, and causes excessive volatility of consumption. Therefore, a recommendation for future work is to allow households to save by purchasing mutual funds of firms, which would introduce another asset with
a liquidity role and hopefully moderate out the correlations. It would also be sensible to endogenize production of the general good as well. Finally, CES preferences are somewhat restrictive, for the reasons noted in Chapter 2, and it makes sense for the elasticity of substitution to adjust with respect to the measure of varieties.

There are also a number of extensions of the model which can further bolster amplification. First, Bethune (2014) provides evidence and develops a model in which employed households are able to borrow more than unemployed households. The addition of some enforceable debt would automatically generate this feature. Second, Petrosky-Nadeau et al. (2014) provide evidence from the American Time Use Survey that search intensity is procyclical. It is straightforward to endogenize search intensity in this model, which would rise with the value of consumption and hence the size of the product space. Hopefully, the ideas developed in this thesis will stimulate more research on the comovement of credit, product variety, and unemployment.

It also important, of course, to test the mutual feedback empirically, particularly the consumption value channel. I would like to combine data on variety at the individual level (which can be done by the Nielsen Consumer Panel Dataset), with data on consumer credit. Since individual consumer credit would almost certainly be unavailable, I would likely have to aggregate up to a suitable metropolitan area, and then use the fixed effects model from panel data econometrics to identify the effect from product variety to credit.

Furthermore, this dissertation abstracts from the effects of the wealth distribution on individual decision-making. In particular, whereas Chapter 3 emphasizes the liquidity role of credit, it abstracts from its insurance role of among consumers facing idiosyncratic employment shocks, and the determination of credit interest rates. Here, I instead propose to study the insurance role of credit among risk-averse consumers subject to idiosyncratic employment shocks. In particular, I am interested in how debt limits vary with the cross section of wealth, which also links them to employment status, and the implications for the number of firms and differentiated goods. New work would thus build financial frictions into the framework of Krusell et al. (2010), which itself nests Mortensen and Pissarides (1994) and Aiyagari (1994).
The treatment of corporate finance, monetary policy, and demand complementarities in Chapter 4 can be extended to encompass more instruments of monetary policy as well as more dimensions of financial frictions. First, we can incorporate conventional monetary policy through purchases and sales of bonds, as in either Rocheteau et al. (2016) or Rocheteau et al. (2014), which would also generate a zero lower bound and endogenize liquidity traps. Second, given the importance of frictions on banks themselves (the so-called lending channel), we can model the role of unconventional monetary policy. Under such policy, the central bank lends directly to private credit markets. The potential need for such policy arises if a crisis disrupts financial intermediation, as occurred during the Great Recession. Gertler and Karadi (2011) compute the optimal unconventional response in which financial intermediaries face endogenous balance sheet constraints. Gertler and Kiyotaki (2015), in turn, allow for the possibility of bank runs by allowing bank assets to be less liquid than deposits. These extensions would go for in bridging economic theory with the increases set of levers that central banks have utilized since 2008.
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Appendix A

Appendix for Chapter 2

A.1 Proofs

A.1.1 Proposition 2.1

Proof. I examine the comparative statics of equilibrium.

Consider an increase in $k$. Equation (2.15) shows that the quantity sold by sellers to all consumers, $q_s$, is increasing. Hence, $p = \frac{\eta}{\eta - 1} c'(q_s)$ is increasing as well.

Consider now an increase in $\eta$. This implies a lower markup. In order for the right hand side of (2.15) to remain constant, $q_s$ must rise. As we have seen, a rise in $q_s$ does not imply a fall in $\mu$. To see the effect on $p$, write $p = \frac{k + c(q_s)}{q_s}$, for which

$$\frac{\partial p}{\partial q_s} = \frac{q_sc'(q_s) - [k + c(q_s)]}{q_s^2}$$

which is positive for $k$ sufficiently low and negative for $k$ sufficiently high. Hence, it is ambiguous.

Suppose $i$ increases. Then the left hand side of (2.13) must increase. $q_s$ is determined independently
of \( i \), so that \( \mu \) must adjust. As we have seen, the effect on \( \mu \) depends on the sign of \( \lambda'(\mu) \).

To complete the analysis with respect to \( \mu \) we consider the cases \( \lambda'(\kappa) > 0 \) and \( \lambda'(\kappa) < 0 \). Suppose \( \lambda'(\kappa) < 0 \). Then \( \mu \) falls whenever the cost of holding real balances increases. As already shown, increases in \( i \) or \( k \) shifts the cost of holding real balances up. Suppose instead that \( \lambda'(\kappa) > 0 \). Then \( \mu \) increases whenever the cost of holding real balances decreases.

Finally, consider \( \eta = \alpha(\kappa)^{1/(\eta-1)}\kappa q_s \). If \( \lambda'(\kappa) < 0 \) then an increase in \( i \) decreases \( \mu \) and hence \( \eta \) and otherwise increases it. The effect of \( k \) on \( \eta \) is indeterminate. If \( \lambda'(\kappa) < 0 \), then higher \( k \) increases \( q_s \) but the decrease in \( \mu \) can be made arbitrarily small or great by the elasticity of marginal cost. Similarly, an increase in \( \eta \) is ambiguous. It increases \( q_s \) but has ambiguous effects on the price, and hence on \( \mu \). \qed

### A.1.2 Proposition 2.2

**Proof.** Consider a monetary authority who takes \( q_s \) as given from (2.15) and chooses \( \mu \) to maximize social welfare. Then we check that the corresponding \( i \) from (2.13) is never satisfied with \( i > 0 \). The planner solves

\[
\max_{\mu} \left\{ u(\alpha(\kappa)^{1/(\eta-1)}\kappa q_s) - \mu [c(q_s) + k] \right\}
\]

given \( q_s \) satisfying (2.15). The first order condition is

\[
u'[(\alpha(\kappa)^{1/(\eta-1)}\kappa q_s)][\frac{1}{\eta - 1}\alpha(\kappa)(2-\eta)/(\eta-1)\alpha'(\kappa)\kappa + \alpha(\kappa)^{1/(\eta-1)}] = \frac{c(q_s) + k}{q_s} \tag{A.1}
\]

In general, (A.1) is not sufficient. Let \( \mu^* \) denote the welfare maximizing root and \( \kappa^* = \mu^*/\sigma \). Using (2.13) and (2.15) we write

\[
\frac{c(q_s) + k}{q_s} = \frac{\alpha(\kappa^*)^{1/(\eta-1)}u'(\alpha(\kappa^*)^{1/(\eta-1)}\kappa^* q_s)}{1 + i/\sigma}
\]
Hence we can rewrite (A.1) in terms of the interest rate as

\[
(1 + \frac{i}{\sigma}) \left[ \frac{1}{\eta - 1} \alpha (\kappa^s)^{(2-\eta)/(\eta-1)} \alpha' (\kappa^s) \kappa^s + \alpha (\kappa^s)^{1/(\eta-1)} \right] = \alpha (\kappa^s)^{1/(\eta-1)}
\]

This simplifies to

\[
1 + \frac{i}{\sigma} = \frac{\eta - 1}{\epsilon_\alpha (\kappa^s) + \eta - 1} < 1
\]

This is a contradiction because \( i \geq 0 \). Hence, \( i = 0 \) maximizes social welfare.

\[
\text{\[A.1.3\] Proposition 2.3}
\]

\textbf{Proof.} We have already shown that there is a unique \( q^*_s \) satisfying \( \Gamma(q^*_s) = \eta/(\eta - 1) \). Given \( q^*_s \), let \( P = (1 + i) \frac{n}{\eta - 1} c'(q^*_s) \), the price adjusted for the cost of holding real balances. It suffices to show that there exists \( \mu \) such that \( \lambda(\kappa) = P \). The two Inada conditions on \( \lambda(\kappa) \) ensure that there exists \( \mu^*, \mu^{**} > 0 \) for which \( \lambda(\kappa) > P \) for \( \mu < \mu^* \) and \( \lambda(\kappa) < P \) for \( \mu > \mu^{**} \). By continuity, there exists \( \mu^{***} \) such that \( \lambda(\mu^{***}) = P \). Moreover, if \( \lambda(\kappa) \) is decreasing everywhere, then equilibrium is unique.

\[
\text{\[A.1.4\] Proposition 2.4}
\]

\textbf{Proof.} First, we show that there is a unique crossing of demand and the price setting rule. It suffices to show that marginal revenue minus marginal cost

\[
\frac{u'(q)[1-r_u(q)]}{(1+i/\sigma)} - c'[\phi(q)] = 0
\]

for unique \( q \). From the second order condition (2.30), this quantity is always decreasing. Hence, there is at most one zero.

Furthermore, from the Inada condition, \( \lim_{q \to 0} u'(q)[1-r_u(q)] = \infty \) and \( \lim_{q \to \infty} u'(q)[1-r_u(q)] = 0 \). Since \( c'(q_s) \geq 0 \forall q_s > 0 \), an equilibrium exists from the Intermediate Value Theorem. This defines unique values \( p^* \) and \( q^* \). In turn, \( q^*_s = \phi(q^*) \) and \( \mu^* \) is defined implicitly from \( q^*_s = \frac{\alpha(\mu^*/\sigma)}{\mu^*/\sigma} q^* \)
A.1.5 Proposition 2.5

Proof. An increase in $i$ shifts demand downward and does not change the price setting rule, resulting in lower $p$ and $q$. Hence, markups $r_u(q)$ are lower. $q_s = \phi(q)$ is higher. The measure of sellers satisfies $\frac{\alpha(\kappa)}{\kappa} = \frac{\phi(q)}{q}$. The right hand side is higher, so that $\mu$ is lower.

Lower $k$ implies a rightward shift of the price setting curve: consider $\phi(q) = \Gamma^{-1}(1/(1 - r_u(q)))$. Write $\phi'(q) = \frac{1}{\Gamma'(1/(1 - r_u(q)))}$ and note that $\Gamma'(q_s)$ is higher with lower $k$. Since $\phi'(q) < 0$ a lower right hand side implies higher $q$. Equilibrium $p$ decreases and $q_s = \phi(q)$ is lower due to the higher $q$. Thus, $r_u(q)$ and markups are higher. $\frac{q_s}{q} = \alpha(\mu)/\mu$ is lower, so more sellers enter. 

A.1.6 Proposition 2.6

Proof. From (2.39)-(2.40) and (2.36)-(2.37), necessary and sufficient conditions for equilibrium to implement the social optimum are $\frac{1+i/\sigma}{1-r_u(q)} = 1$ and $1 - r_u(q) = \frac{\varepsilon_u(q)}{\varepsilon_u(\kappa)}$. This requires $i = 0$, $r_u(q^*) = 0$ and $\varepsilon[u(q^*)] = \varepsilon[\alpha(\mu^*/\sigma)]$. But $r_u(q) = 0$ is inconsistent with firm optimization. Hence, equilibrium is inefficient.

A.2 Variations of the CES Model

A.2.1 Basic Model: Fixed Mass of Sellers

We now consider $\mu$ to be a fixed parameter, so that $\kappa = \mu/\sigma$ is also a parameter. Analogues of equations (2.13) and (2.11) hold in this setting; hence, we can define equilibrium as follows.
**Definition A.1.** A (steady state) equilibrium is a pair \((q_s, p)\) satisfying

\[
\frac{\alpha(\kappa)^{1/(\eta-1)}u'[\alpha(\kappa)^{1/(\eta-1)}\kappa q_s]}{c'(q_s)} = \frac{\eta}{\eta - 1} \left(1 + \frac{i}{\sigma}\right)
\]

(A.2)

\[
p = \frac{\eta}{\eta - 1} c'(q_s)
\]

(A.3)

Equation (A.2) determines the equilibrium level of \(q_s\).

**Proposition A.1 (Existence and uniqueness).** There is a unique equilibrium.

Proof. Define the left hand side of (A.2) as \(G(q_s)\). Because \(u'(\cdot) < 0, c'(\cdot) > 0\), \(G(q_s)\) is decreasing. Due to the Inada conditions, \(G(0) = \infty\) and \(G(\infty) = 0\). Hence, there is a unique value \(q_s^*\) for which \(G(q_s^*) = \frac{\eta}{\eta - 1} \left(1 + \frac{i}{\sigma}\right)\). Given \(q_s^*, p^*\), is uniquely defined by (A.3).

Equilibrium can be characterized in terms of a price curve and function \(\lambda(q_s) = \alpha(\kappa)^{1/(\eta-1)}u'(\alpha^{\frac{1}{\eta-1}}\kappa q_s)\), which describes marginal utility in terms of the production of each firm.

![Figure A.1: Equilibrium](image)

The following result is immediate.

**Proposition A.2.** An increase in \(i\) leads to a decrease in equilibrium values of \(q_s, \bar{q}, q\)
By making money more costly to hold, a higher interest rate decreases $q_s$, which has a one-to-one relationship with $q$ and $\overline{q}$. In order to study comparative statics with respect to $\kappa$, we derive from (A.2) the following relationship between elasticities$^1$:

$$\varepsilon_{qs}(\kappa)[\varepsilon_u'(-) - \varepsilon_c'(-)] = \frac{i\kappa}{\mu + i\kappa} - \frac{1}{\eta - 1}\varepsilon_u(\kappa)[1 + \varepsilon_u'(\cdot)]$$

(A.4)

Since $\varepsilon_u'(-) - \varepsilon_c'(-)$ is negative, $\varepsilon_{qs}(\kappa)$ is negative if and only if the right hand side is positive. Since the elasticity of the matching function is at most one, a sufficient condition for the right hand side to be positive is

$$\frac{i\kappa}{\mu + i\kappa} - \frac{1}{\eta - 1}(1 + \varepsilon_u') > 0$$

(A.5)

In the case of CRRA preferences, (A.5) reduces to

$$\frac{i\kappa}{\mu + i\kappa} + \frac{\eta\varepsilon - 1}{\eta - 1} > 0$$

Hence, $\eta\varepsilon \geq 1$ implies $\varepsilon_{qs}(\kappa) < 0$: firm quantity decreases with more sellers and increase with a greater percentage of active buyers.

**Proposition A.3.** Suppose $u(q) = q^{1-\varepsilon}/(1-\varepsilon)$. Then $\eta\varepsilon > 1$ implies that $q_s$ decreases with $\kappa$ (an increase in $\mu$ or decrease in $\sigma$). This condition is tight in that for $\eta\varepsilon$ arbitrarily close to 1 from below, there exist matching functions $\alpha$ and values of $\mu, \sigma, i$ for which $q_s$ increases with respect to $\kappa$.

**A.2.2 Endogenous Search Intensity**

**Environment**

I extend the basic model by endogenizing buyers’ search for sellers, abstracting away from preference shocks ($\psi = 1$). The purpose is to understand how shopping effort and output varies in response to interest rates and compare to Laing et al. (2007). Laing, Li, and Wang use a flexible labor market and

$^1$I use the convention $\epsilon_f(x) = \frac{\pi f'(x)}{f(x)}$
find that with enough substitutability of labor and goods in preferences, higher inflation increases labor, search effort, and output if taste for variety is high enough. In contrast to their model, I consider the congestion effect of search. I find that though it is possible for effort to rise in response to interest rates, it requires the taste for variety to be implausibly high relative to the elasticity of the marginal utility of consumption. Effort typically falls, with a magnitude that increases with taste for variety and decreases with the elasticity of marginal utility. Hence, the hot potato effect emphasized by Laing et al. (2007) is not robust to the presence of congestion in search.

The measure of overall matches is given by $\alpha(\overline{e})$, where

$$\overline{e} = \int_0^1 e(i) di$$  \hspace{1cm} (A.6)

The measure of varieties of the DM good that a buyer can purchase is given by $\alpha_b = e\alpha(\overline{e})/\overline{e}$. Since buyers can match up with at most measure $\mu$ sellers, $\alpha(\overline{e}) \leq \mu$. The measure of buyers for each seller is $\alpha_s = \alpha(\overline{e})/\mu$. I assume $\lim_{e \to \infty} \alpha_s(e) = 1$.

The aggregate good is a function of the measure of contacted sellers, and hence a function of effort

$$\eta(e) = \left( \int_0^{e\alpha/\overline{e}} q_{j}^{\eta-1} dq_{j} \right)^{\frac{\eta}{\eta-1}}$$

The disutility of effort is given by $\psi(e)$, where $\psi'(e) > 0, \psi''(e) > 0$ and $\psi(0) = \psi'(0) = 0$. The buyer’s period utility function becomes $U^b(x, h, \overline{q}, e) = u(\overline{q}) - \psi(e) + U(x) - h$.

Though we do not explicitly introduce a labor-leisure tradeoff as do, we proxy one via the convexity of $\psi(e)$.
**Equilibrium**

The feasibility constraint on the transfer of real balances \( \int_0^{e \alpha / \bar{e}} p_j q_j dj \leq z \) is binding and the buyer’s problem is

\[
\max_{q_j, e} \left\{ -(1 + i) \int_0^{e \alpha / \bar{e}} p_j q_j dj - \psi(e) + u(q) \right\}
\]

The first order condition with respect to \( q_j \) is

\[
(1 + i)p_j = u'(q) \left[ \frac{q_j}{q} \right]^{1/\eta}
\] (A.7)
as before.

The first order condition with respect to effort is:

\[
u'(q) \left( \frac{\eta}{\eta - 1} \right) \left[ \frac{q_j}{q} \right]^{1/\eta} q_j \frac{\alpha}{\bar{e}} - \psi'(e) - (1 + i)p_j q_j \frac{\alpha}{\bar{e}} = 0
\]

This can be simplified using (A.7) into

\[
\psi'(e) = \frac{(1 + i)p_j q_j \alpha / \bar{e}}{\eta - 1}
\] (A.8)

Equation (A.8) says that the marginal cost of effort is equal to the marginal utility of effort. Note that (A.8) reflects a ‘hot potato’ effect. A higher interest rate raises the cost of holding unused real balances and thereby raises search intensity among buyers. However, this is a partial equilibrium effect. I later show that in general equilibrium effort is likely to fall due to congestion.

The interpretation of (A.8) is as follows. An extra unit of effort enables the buyer to access \( \alpha / \bar{e} \) extra sellers and hence \( \frac{\alpha}{\bar{e}} q_j \) extra units. These extra units cost \((1 + i)p_j\) each, which equals their marginal benefit by (A.7). Since the right hand side is positive and does not depend on \( e \), and since \( \psi'(e) > 0 \) with \( \psi'(0) \) and \( \psi'(\infty) = \infty \), there is a unique value of \( e \) that solves (A.8). As all buyers
face the same problem, $e = e$.

The problem of seller $j$ is identical to the basic case except that now $q_s(j) = \alpha_s(e)q_j$. Similarly, as buyers face the same problem with a unique solution, $q_j = q$ for all $j$ and $\bar{q} = \alpha^{n/(\eta-1)}q$. I am now ready to define equilibrium.

**Definition A.2.** An equilibrium for the model with endogenous search intensity is a list $(q_s, e)$ such that

$$\frac{\alpha(e)^{1/(\eta-1)}u'[\alpha(e)^{1/(\eta-1)}\mu q_s]}{e'(q_s)} = \frac{\eta}{\eta - 1} (1 + i)$$

(A.9)

$$e\psi'(e) = \frac{\eta}{(\eta - 1)^2} e'(q_s)\mu q_s(1 + i)$$

(A.10)

Equilibrium can be described in terms of two curves relating $e$ and $q_s$. Equation (A.9) is the marginal markup condition. (A.10) describes the effort as a function of firm output. Specifically, there is a linear relationship between the semi-elasticity of effort cost and the semi-elasticity of production cost. I prove existence of equilibrium.

**Proposition A.4** (Existence and uniqueness). There exists a unique equilibrium.

**Proof.** Fix $e$ in $(0, \infty)$. This fixes $\alpha(e) \in (0, 1)$. By the standard argument, tending $q_s \to 0$ tends the left hand side of (A.9) to $\infty$, and tending $q_s \to \infty$ tends the left hand side to 0. By the intermediate value theorem, there is a (positive) $q_s \to \infty$ such that (A.9) holds. Given that the left hand side is decreasing in $q_s$, this value is unique. By the implicit function theorem, this defines a differentiable function $q_s = f(e)$ such that $f'(e) < 0$. Moreover, using the fact that as $e \to \infty, \alpha/\mu \to 1$, $\lim_{e \to \infty} f(e)$ is given by the unique value $q_s^\infty$. Also, $\lim_{e \to 0} f(e) = \infty$.

From (A.10), define the function

$$g(e) = e\psi'(e) - \frac{\eta(1 + i)}{(\eta - 1)^2} e'[f(e)]\mu f(e)$$

It suffices to show that there is a unique $e > 0$ such that $g(e) = 0$. As $f'(e) < 0, g'(e) < 0$. Furthermore, as $e \to \infty, f(e) \to q^\infty$, so that $g(e) \to \infty$. Second, note that $g(e) \to -\infty$ as $e \to 0$. 


As $e \to 0$, $f(e) \to \infty$. By the intermediate value theorem, there is a unique $e^* > 0$ such that $g(e^*) = 0$. 

I derive a necessary and sufficient condition for effort to fall in equilibrium with respect to interest rates. I take the logarithmic transforms of (A.9) and (A.10), totally differentiate with respect to $1 + i$, and rearrange:

$$
\epsilon_e (1 + i) = \frac{1 + \epsilon_u'}{[1 + \epsilon_c'(qs)]} \frac{1}{\eta - 1} \epsilon_e (\alpha) [1 + \epsilon_u'(\eta)] - [1 + \epsilon_u'(e)] [\epsilon_c'(qs) - \epsilon_u'(\eta)]
$$

(A.11)

**Example A.1.** Suppose $\alpha(e) = e/(1 + e)$, $u(q) = q^{1-\varepsilon}/(1 - \varepsilon)$ for $0 < \varepsilon < 1$, $c(q) = cq$ for $c > 0$, $\psi(e) = \psi e$ for $\psi > 0$. Then the sign of $\epsilon[e(1 + i)]$ is given by the sign of $\frac{(1-\varepsilon) - \varepsilon(\eta-1)}{(\eta-1)(1+e)}$. This is clearly negative for $e$ sufficiently high. Note that at $e = 0$, the relevant sign is that of $(1-\varepsilon) - \varepsilon(\eta-1)$. In particular if $\eta\varepsilon > 1$, then effort falls at a higher nominal interest rate.

I next examine the sensitivity of search intensity to inflation. Let $c(q) = q, u(q) = q^{1-\varepsilon}/(1 - \varepsilon)$, $\psi(e) = e^2/2$, and $\alpha = e/(1 + e)$. Table A.1 summarizes search intensity under different combinations of $\varepsilon$ and $\mu_p$ under the Friedman rule and 10% inflation, and computes the logarithmic differences between the two.

<table>
<thead>
<tr>
<th>$\mu_p$</th>
<th>i=0</th>
<th>i=0.13</th>
<th>% A</th>
<th>i=0</th>
<th>i=0.13</th>
<th>i=0</th>
<th>i=0.13</th>
<th>i=0</th>
<th>i=0.13</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>1.2</td>
<td>1.3</td>
<td>1.4</td>
<td></td>
<td>1</td>
<td>1.2</td>
<td>1.3</td>
<td>1.4</td>
</tr>
<tr>
<td>0.2</td>
<td>0.179</td>
<td>0.131</td>
<td>0.0552</td>
<td>0.00345</td>
<td>0.133</td>
<td>0.0895</td>
<td>0.0310</td>
<td>0.00103</td>
<td>-0.296</td>
</tr>
<tr>
<td>0.3</td>
<td>0.233</td>
<td>0.248</td>
<td>0.222</td>
<td>0.176</td>
<td>0.199</td>
<td>0.208</td>
<td>0.182</td>
<td>0.139</td>
<td>-0.158</td>
</tr>
<tr>
<td>0.4</td>
<td>0.262</td>
<td>0.315</td>
<td>0.329</td>
<td>0.322</td>
<td>0.237</td>
<td>0.284</td>
<td>0.294</td>
<td>0.285</td>
<td>-0.0975</td>
</tr>
</tbody>
</table>

The main finding is that search intensity decreases with inflation the most under higher taste for variety as measured by the gross markup $\mu_p$ and lower elasticity of marginal utility $\varepsilon$. Moreover, the relationship between search intensity and taste for variety is ambiguous: there is a direct effect.
to want to search more, but there is also higher markups and congestion. Under \( \varepsilon = 0.3 \) and \( \varepsilon = 0.4 \), it first increases and then decreases whereas for \( \varepsilon = 0.2 \) it is monotonically decreasing. The non-monotonicity arises with higher \( \varepsilon \) from the fact that households have lower incentive to boost search effort under higher taste for variety when the marginal utility of the aggregate good diminishes more.

**Social Optimum**

I write the (constrained) social planning problem. From an analogous argument to the basic model, symmetry simplifies the social planning problem into a choice of \( e \) and \( q_s \) so as to maximize the following welfare function:

\[
W(e, q_s) = u[\alpha(e)^{1/(\eta-1)} \mu q_s] - \mu c(q_s) - \psi(e) \tag{A.12}
\]

The first order conditions are given by

\[
\alpha(e)^{1/(\eta-1)} u'(\bar{q}) = c'(q_s) \tag{A.13}
\]

\[
\mu q_s \alpha^{(2-\eta)/(\eta-1)} \alpha'(e) u'(\bar{q}) = (\eta - 1) \psi'(e) \tag{A.14}
\]

Equation (A.13) says that marginal utility equals aggregate marginal costs. Equation (A.14) says that the marginal cost of effort equals the marginal cost of the induced production. Taking the ratio of (A.14) and (A.13) and rearranging, we obtain

\[
\mu q_s c'(q_s) \varepsilon \alpha(e) = (\eta - 1) e \psi'(e) \tag{A.15}
\]

Equation (A.15) characterizes the optimal matching elasticity in terms of the marginal cost of effort, the marginal cost of production, and taste for variety scaled by the relative measure of sellers to buyers. Comparing (A.15) with (A.10), we note that implementing the social optimum would
require

\[ \epsilon_\alpha(e) = \frac{\eta}{\eta - 1} (1 + i) \]

which is impossible, as the elasticity of the matching function is bounded above by 1. The following proposition is immediate.

**Proposition A.5.** Equilibrium is inefficient.

### A.3 Additional Derivations

#### A.3.1 Consumer Surplus

The real balances used for all purchases for one consumer is

\[ z(q_s) = \frac{\eta}{(\eta - 1)} c'(q_s) \alpha(\kappa) q = (\eta/(\eta - 1)) \kappa c'(q_s) q_s \]

Since \( z = \phi M \), the value of money is given by

\[ \phi = \frac{(\eta/(\eta - 1)) c'(q_s) \alpha(\kappa) q}{M} \]  (A.16)

The consumer surplus is given by

\[ \Omega(q_s) = \sigma u[\alpha(\kappa)^{n-1} \kappa q_s] - (\sigma + i) z(q_s) \]  (A.17)

Using \( z'(q_s) = (\eta/(\eta - 1)) \kappa [c''(q_s) q_s + c'(q_s)] \), we find

\[ \Omega'(q_s) = \kappa [\sigma u'(\bar{\eta}) \alpha(\kappa)^{n-1} - (\sigma + i) \mu_p c'(q_s)] - (\sigma + i) \mu_p \kappa c''(q_s) q_s \]  (A.18)

which is proportional to

\[ \sigma u'(\bar{\eta}) \alpha(\kappa)^{n-1} - (\sigma + i) \frac{\eta}{\eta - 1} c'(q_s) - (\sigma + i) \frac{\eta}{\eta - 1} c''(q_s) q_s \]  (A.19)
which is positive for small \( q_s \) and negative for sufficiently large \( q_s \). Using (A.2) and rearranging, it is clear that the change in consumer surplus in equilibrium is proportional to \(- (\sigma + i) \mu_p c''(q^e_s) q^e_s < 0\)

**Lemma A.1.** Consumer surplus is decreasing in equilibrium if and only if costs are strictly convex.

Lemma (A.1) shows that consumer surplus is generally nonmonotonic. The non-monotonicity depends on the convexity of costs, just as in Walrasian price taking. This contrasts with generalized Nash bargaining, where real balances are a weighted average of the utility of the buyer and costs of the seller, and where the weights are a function of the marginal utility and marginal cost depending on the bargaining power. With Nash bargaining, the buyer’s surplus is falling in equilibrium even if costs are linear. This difference in the type of non-monotonicity does not depend on taste for variety.

We now consider the entry of sellers on the change in consumer surplus. Since \( \Gamma(q^e_s) = (\eta/\eta - 1) \),

\[
\Omega'(q^e_s) = -\mu^e (\sigma + i) \mu_p c''(\Gamma^{-1}(\mu_p)) \Gamma^{-1}(\mu_p)
\]

(A.20)

Noting that \( \Gamma^{-1}' \left( \frac{\eta}{\eta - 1} \right) = \frac{1}{\Gamma(q_s)} > 0 \), and \( \Omega'(q^e_s) \) is more negative, provided \( \lambda' (\mu^e / \sigma) < 0 \). Higher markups increase \( q_s \), which interact with convex costs to make the change in consumer surplus more negative unless the measure of sellers decreases sufficiently. The behavior of consumer surplus is sensitive to firm entry in two ways: it depends directly on the measure of sellers, and it depends on quantities provided by each firm given by (2.15).

**A.3.2 Marginal Markups**

**Generalized Nash Bargaining**

Under generalized Nash bargaining, real balances can be expressed as \( z_\theta(q) = [1 - \Theta(q)] u(q) + \Theta(q) c(q) \), where \( \Theta(q) = \frac{\theta u'(q)}{\theta u'(q) + (1 - \theta)c'(q)} \) is the buyer’s share of the match surplus. The optimal
consumption problem of the buyer in the DM is

$$\max_{q \in [0, q^*]} \left\{ -iz(q) + \sigma[u(q) - z(q)] \right\}$$  \hspace{1cm} (A.21)

which has solution

$$\frac{u'(q)}{z'_{\theta}(q)} = 1 + \frac{i}{\sigma}$$  \hspace{1cm} (A.22)

Substituting \( z'_{\theta}(q) \), dividing both sides by \( c'(q) \), and rearranging yields the marginal markup.

**Proportional Bargaining**

Under proportional bargaining, as long as buyer spends all real balances in the DM period, then

$$z_{\theta}(q) = \theta c(q) + (1 - \theta)u(q).$$

The optimal consumption problem of the buyer in the DM can be written as

$$\max_{q \in [0, q^*]} \left\{ -iz(q) + \sigma\theta[u(q) - c(q)] \right\}$$  \hspace{1cm} (A.23)

This problem is concave provided \( \frac{\theta}{1 - \theta} > \frac{i}{\sigma} \), and has solution

$$\frac{u'(q) - c'(q)}{z'_{\theta}(q)} = \frac{i}{\theta\sigma}$$  \hspace{1cm} (A.24)

Substituting \( z'_{\theta}(q) \), dividing through by \( c'(q) \), and rearranging algebraically yields the marginal markup.
A.3.3 Money Demand Functions

Basic Model

Money demand is defined as the ratio of aggregate money balances to aggregate nominal output. For this model, this can be written as

\[ L = \frac{z}{z + A} \]  

(A.25)

using \( \sigma = 1, \kappa = 1 \) and where the quantity of the CM good is \( A \). I obtain \( z = \frac{\eta}{\eta - 1} c'(q_s)q_s = \frac{\eta}{\eta - 1} q_s^{\delta} \).

Using \( q_s = \left[ \frac{\alpha^{(1 - \varepsilon)/(\eta - 1)}}{\eta^{1 - \varepsilon}} \right]^{\frac{\eta}{\eta - 1}} \), \( z = \frac{\eta}{\eta - 1} \left[ \frac{\alpha^{(1 - \varepsilon)/(\eta - 1)}}{\eta^{1 - \varepsilon}} \right]^{\frac{\eta}{\eta - 1}} \).

CES with Entry of Sellers

Real balances are given by \( z(q_s) = \frac{\eta}{\eta - 1} c'(q_s)\mu q_s = \frac{\eta}{\eta - 1} \mu q_s^{\delta} \). From the zero profit condition, \( q_s = \left( \frac{k}{\eta - 1 - \delta} \right)^{\frac{1}{\delta}} \). Moreover, \( \mu \) is implicitly characterized by \( \mu^\varepsilon q_s^{\delta + \varepsilon} = \frac{\alpha(\mu)^{(1 - \varepsilon)/(\eta - 1)}}{\eta^{1 - \varepsilon}(1 + i)} \). Here there is no explicit solution for money demand.

Additively Separable Preferences

Real balances in the economy are given by \( z(q_s) = \mu q_s \frac{c'(q_s)}{1 - r_u(q)} \). Taking \( c(q_s) = q_s \), money demand is given by \( L = \frac{z}{z + A} = \frac{\mu/r_u(q^*)}{\mu/r_u(q^*) + A/k} \), where \( q^* \) satisfies \( u'(q^*) = (1 + i)/(1 - r_u(q^*)) \) and \( \mu \) is defined implicitly through \( \alpha(\mu)/\mu = k[1 - r_u(q^*)]/r_u(q^*) \). We adopt the utility function \( u(q) = \frac{(hq)^{1 - \varepsilon}}{1 - \varepsilon} + bq \), so that \( r_u(q) = \frac{zh^{1 - \varepsilon}}{ht^{1 - \varepsilon} + bq^\varepsilon} \).

Variable Search Intensity

I obtain a closed form solution and obtain the money demand by restricting \( \psi(e) = e^2/2 \) and otherwise using the same specification as in the basic case. I obtain a similar expression for \( q_s \) as
in the basic model, except now as a function of $e$:

$$q_s = \left[ \frac{\alpha(e)(1-\varepsilon)/(\eta-1)}{\eta(1+i)} \right]^{1/\varepsilon}$$

where $e = \sqrt{\frac{\eta}{(\eta-1)^2} q_s (1+i)}$. Letting $e^*$ denote the equilibrium value of search effort, the money demand is given by

$$L = \left\{ 1 + A \left[ \frac{(\frac{\eta}{\eta-1})^{1-\varepsilon} (1+i)}{\alpha(e^*)(1-\varepsilon)/(\eta-1)} \right]^{1/\varepsilon} \right\}^{-1}$$

(A.26)

### A.4 Numerical Checks

#### A.4.1 Effects of Higher Interest rate on Output

We plot the effects of a higher interest rate on output, based on the examples in Section 2.4.3. Output decreases monotonically in each case.

![Figure A.2: Output as a function of the interest rate](image)

#### A.4.2 Second Order Condition for Multiple Equilibria with Entry

The second derivative condition of the buyer’s objective function is

$$u''(\bar{q})\alpha(\kappa) \frac{1}{\eta} + u'(\bar{q}) \frac{1}{\eta} \alpha(\kappa) \left[ \frac{q_j \alpha(\kappa) \frac{\eta}{\eta-1} - \eta}{q_j^2} \right] \leq 0$$

(A.27)
at equilibrium, which simplifies to

$$\epsilon w(q) + \frac{1}{\eta} [\alpha(\kappa)^{\frac{1}{\eta - 1}} - \alpha(\kappa)^{\frac{\eta}{\eta - 1}}] \leq 0$$  \hfill (A.28)

In the case of CRRA preferences, (A.28) reduces to

$$\alpha(\kappa)^{\frac{1}{\eta - 1}} - \alpha(\kappa)^{\frac{\eta}{\eta - 1}} \leq \eta \epsilon$$  \hfill (A.29)

Rewrite this condition as $$\alpha(\kappa)^{\frac{1}{\eta - 1}} - \alpha(\kappa)^{\frac{\eta}{\eta - 1}} \leq \eta \epsilon$$. Without loss of generality assume $$0 \leq \alpha(\kappa) < 1$$. Note that as $$\eta \to 1$$, the left hand side approaches zero, and the inequality is trivially satisfied. Moreover, as $$\eta$$ increases, the left hand side increases monotonically and approaches $$1 - \alpha(\kappa)$$, whereas the right hand side increases without bound. It is easy to check that this condition holds even for low $$\eta$$.

We verify numerically for $$\alpha(\kappa) = \frac{\kappa}{1 + \kappa}, \eta = 3, \epsilon = 0.25$$ as indicated in Figure A.3. There we plot $$h(\mu) = \alpha(\mu)^{\frac{1}{\eta - 1}} - \alpha(\mu)^{\frac{\eta}{\eta - 1}} - \eta \epsilon$$. Note that without loss of generality we can restrict attention to $$\mu$$ on $$[0, 1]$$, as $$h(\kappa)$$ is always negative for $$\mu > 1$$.

Figure A.3: Second order condition

A.5 Optimality of Friedman rule

Table A.2 summarizes the optimality properties of the Friedman rule under various market structures. Here, ‘first best’ refers to the implementation of the social planning problem. For the first four market structures, I refer to the analysis in Rocheteau and Wright (2005). The next two are
from the variety model in Dong (2010). The last three are the subject of this paper.

Table A.2: Optimality of Friedman rule

<table>
<thead>
<tr>
<th>Market structure</th>
<th>First best</th>
<th>Best policy</th>
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<tr>
<td>Generalized Nash bargaining</td>
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<td>Yes</td>
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<tr>
<td>Price taking</td>
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<tr>
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<tr>
<td>MC fixed sellers</td>
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<td>Yes</td>
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<tr>
<td>MC entry of sellers CES</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>MC entry of sellers ZKPT</td>
<td>No</td>
<td>No</td>
</tr>
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Appendix B

Appendix for Chapter 3

B.1 Additional Figures

B.1.1 Firm Hiring and Production

\[ MR = \rho p(q, Q) = \frac{\rho}{1 + \zeta} AQ^{-\epsilon} \left( \frac{Q}{q_i} \right)^{1-\rho} \]

\[ MC = \frac{1}{\beta} \left\{ w + kE \left[ \frac{1}{f(\theta)} - \frac{\beta(1-\delta)(1-s)}{f(\theta')} \right] \right\} \]

Figure B.1: Intersection of Marginal Revenue and Marginal Cost
B.1.2 Number of Firms

Figure B.2: (Log) Number of Firms from Statistics of U.S. Businesses, Census

B.2 Proofs and Derivations

B.2.1 Firm Problem

Combining equations (3.23) and (3.24) and reducing the choice variables to labor, we have

$$\Pi(n, \Theta) = \max_{n' \geq (1-s)n} \left\{ p(zn')zn' - w'n' - \frac{k[n' - (1-s)n]}{f(\theta)} + \beta \mathbb{E}[(1-\delta)\Pi(n', \Theta') + \delta J(\Theta')] \right\} \quad (B.1)$$

Note that the value function $\Pi$ is linear in past employment $n$, provided the nonnegativity constraint does not bind: $\Pi(n) = (1-s)kn/f(\theta) + \Pi(0)$.

The first order and envelope conditions of (B.1) are

$$MR(q)z + \lambda \nu = w + \frac{k}{f(\theta)} - \beta(1-\delta)\mathbb{E} \frac{\partial \Pi(n', \Theta')}{\partial n'} \quad (B.2)$$

$$\frac{\partial \Pi(n, \Theta)}{\partial n} = (1-s) \left[ \frac{k}{f(\theta)} - \lambda \nu \right] \quad (B.3)$$
in which $MR(q)$ is the marginal revenue of $q$ and $\lambda_v$ is the Lagrangian multiplier on the nonnegativity constraint on vacancies.

Combining (B.2) and (B.3), $n$ is determined by

$$MRz + \lambda_V = w + \frac{k}{f(\theta)} - \beta(1 - \delta)(1 - s)E \left[ \frac{k}{f'(\theta')} - \lambda'_{V} \right]$$

The equality in (B.4) arises from the fact that there is an Inada condition on marginal revenue, which we show. The marginal revenue $MR$ satisfies $MR = \rho p = \frac{n-1}{n}p$, with inverse demand $p = (1 + \zeta)AQ^{-\varepsilon} \left( \frac{q}{Q} \right)^{1-\rho}$. Note that $\lim_{q \to 0} MR_i = \infty$ and $\lim_{q \to \infty} MR_i = 0$.

The firm problem has the following interpretation. Hiring $n$ workers generates a marginal revenue product $MRz$, in which $MR$ is the marginal revenue and $z$ is the marginal product of the worker. The marginal cost of hiring consists of the extra wage bill and the expected realized vacancy cost. The former consists of the wage paid to the marginal worker as well as the wage change in the inframarginal workers. For the latter, hiring $n$ workers requires $f(\theta)v$ vacancies, so the effective hiring cost is $k/f(\theta)$. However, a fraction $(1 - s)$ of the hired workers remain employed the following period, which reduces expected vacancy costs by $(1 - s)E \frac{k}{f'(\theta')}$.

Provided the nonnegativity constraint on vacancies does not bind, the marginal cost satisfies

$$MC = \frac{1}{z} \left\{ w + kE \left[ \frac{1}{f'(\theta)} - \frac{\beta(1 - \delta)(1 - s)}{f'(\theta')} \right] \right\}.$$  

(B.5)

Thus, using (B.4) and (3.26), we see that the choice of $n$ solves $MR = MC$. Given that $MR$ is decreasing and satisfies the Inada conditions, there is a unique positive crossing between $MR$ and $MC$: firms are guaranteed to produce a positive amount even with a linear production technology and costs of opening vacancies. Hence, unlike Mortensen and Pissarides (1994), we do not need to impose restrictions on $k$ in order to guarantee production.

\footnote{Labor adjustment costs are linear and hence labor choice is independent of the current stock of workers.}
B.2.2 Wage and Marginal Cost as a Function of Market Tightness

The steady state condition for marginal cost and wage are

\[ MC = \frac{1}{z} \left\{ w + k \left[ \frac{1 - \beta(1 - \delta)(1 - s)}{f(\theta)} \right] \right\} \]

and

\[ w = \chi [MRz + \beta k \theta] + (1 - \chi)(b + l). \]

Using \( MR = MC \), and substituting, we obtain expressions for wages and marginal cost depending only on \( \theta \).

\[ MC = \frac{1}{z(1 - \chi)} \left\{ \chi \beta k \theta + (1 - \chi)(b + l) + k \left[ \frac{1 - \beta(1 - \delta)(1 - s)}{f(\theta)} \right] \right\} \quad \text{(B.6)} \]

\[ w = \frac{\chi}{1 - \chi} \left\{ \beta k \theta + k \left[ \frac{1 - \beta(1 - \delta)(1 - s)}{f(\theta)} \right] \right\} + b + l \quad \text{(B.7)} \]

B.2.3 Proof of Proposition 3.1

Let \( \Psi_e = \beta[U_e' - \tilde{U}_e'] \)

Note

\[ U_0(\Theta) = AQ^{1 - \varepsilon} \frac{1}{1 - \varepsilon} - d + (1 - h)W_0(0, \Theta) + hW_1(0, \Theta) \quad \text{(B.8)} \]

\[ \tilde{U}_0(\Theta) = \lambda[(1 - h)\tilde{W}_0(\Theta) + h\tilde{W}_1(\Theta)] + (1 - \lambda)[(1 - h)W_0(0, \Theta) + hW_1(0, \Theta)] \quad \text{(B.9)} \]

Further note

\[ W_e(0, \Theta) = ew + (1 - e)(l + b) + \Delta - T + \beta \mathbb{E}U'_e(\Theta') \quad \text{(B.10)} \]

\[ \tilde{W}_e(\Theta) = ew + (1 - e)(l + b) + \Delta - T + \beta \mathbb{E}\tilde{U}_e'(\Theta') \quad \text{(B.11)} \]

Hence, \( W_e(0, \Theta) - \tilde{W}_e(\Theta) = \Psi_e \). As a result, we can compute \( U_e' - \tilde{U}_e' \):

\[ U_e(\Theta) - \tilde{U}_e(\Theta) = \frac{AQ^{1 - \varepsilon}}{1 - \varepsilon} - d + \lambda[(1 - h)(W_0(0, \Theta) - \tilde{W}_0(\Theta)) + h(W_1(0, \Theta) - \tilde{W}_1(\Theta))] \]

\[ = \frac{AQ^{1 - \varepsilon}}{1 - \varepsilon} - d + \lambda \Psi_e \quad \text{(B.12)} \]

\[ = \frac{AQ^{1 - \varepsilon}}{1 - \varepsilon} - d + \lambda \Psi_e \quad \text{(B.13)} \]
Substituting for $U_e(a)$ and $\tilde{U}_e(\tilde{a})$, we have

$$\Psi_e(\Theta) = \beta E \left\{ \frac{AQ^{1-\varepsilon}}{1-\varepsilon} - d' + \lambda \Psi_e'(\Theta') \right\}$$

Since the last expression does not depend on the employment status, we drop the subscript. Thus, the debt limit is solely a function of the aggregate state variable.

### B.2.4 Derivation of Pure Credit Steady State

Provided the credit constraint binds, the steady state debt limit is

$$\frac{r + 1 - \lambda + \psi}{\psi} Spq = AQ^{1-\varepsilon}/(1-\varepsilon) \quad (B.14)$$

We rewrite $Q = S^{1/\rho}q = [(1-u)/n]^{1/\rho}q = (1-u)^{1/\rho}q = (1-u)^{1/\rho} \left[ \frac{(1+r)(1-\rho)p}{\gamma(r+\delta)} \right]^{1-\rho}$. 

Thus,

$$\frac{r + 1 - \lambda + \psi}{\psi} Spq = A \left\{ [(1-u)^{1/\rho} \left[ \frac{(1+r)(1-\rho)p}{\gamma(r+\delta)} \right]^{(1-\rho)/\rho} \right\}^{1-\varepsilon}/(1-\varepsilon) \quad (B.15)$$

Furthermore, $Spq=\frac{(1-u)^{1/\rho}}{q}pq = (1-u)zp$, so that

$$\frac{r + 1 - \lambda + \psi}{\psi} (1-u)zp = A \left\{ [(1-u)^{1/\rho} \left[ \frac{(1+r)(1-\rho)p}{\gamma(r+\delta)} \right]^{(1-\rho)/\rho} \right\}^{1-\varepsilon}/(1-\varepsilon) \quad (B.16)$$

This expression can be finally simplified into

$$\frac{r + 1 - \lambda + \psi}{\psi} [(1-u)^{1/\rho} \left[ \frac{(1+r)(1-\rho)p}{\gamma(r+\delta)} \right]^{(1-\rho)/\rho} \right\}^{1-\varepsilon}/(1-\varepsilon) \quad (B.17)$$
B.2.5 Proof of Lemma (3.1)

\[ V_1(\Theta) = AQ^{1-\varepsilon}/(1 - \varepsilon) - d + W_1(0, \Theta) \] (B.18)
\[ V_0(\Theta) = AQ^{1-\varepsilon}/(1 - \varepsilon) - d + W_0(0, \Theta) \] (B.19)

Hence, \( V_1(\Theta) - V_0(\Theta) = W_1(0, \Theta) - W_0(0, \Theta) \). Using equation (3.5),

\[ W_1(0, \Theta) - W_0(0, \Theta) = w - (b + l) + [(1 - s)(1 - \delta) - h(\theta)] \beta E[V_1(\Theta') - V_0(\Theta')] \] (B.20)

We thereby obtain a recursive relationship for \( V_1(\Theta) - V_0(\Theta) \):

\[ V_1(\Theta) - V_0(\Theta) = w - (b + l) + [(1 - s)(1 - \delta) - h(\theta)] \beta E[V_1(\Theta') - V_0(\Theta')] \] (B.21)

B.2.6 Proof of Proposition (3.2)

Nash bargaining over the marginal surplus yields

\[ \chi J_n = (1 - \chi)[V_1(\theta) - V_0(\Theta)] \] (B.22)

Note that \( V_1(\Theta') - V_0(\Theta') = \frac{\chi}{1-\chi}J_n(\Theta') = \frac{\chi}{1-\chi} \left[ \frac{k}{f'} - \lambda' \right] \).

Write

\[ V_1(\Theta) - V_0(\Theta) = w - (b + l) + \beta[(1 - s)(1 - \delta) - h(\theta)] \frac{\chi}{1-\chi} \left[ \frac{k}{f'} - \lambda' \right] \] (B.23)

Again using \( V_1(\Theta) - V_0(\Theta) = \frac{\chi}{1-\chi} \left[ \frac{k}{f'} - \lambda' \right] \), and plugging in for \( J_n \),

\[ \frac{\chi}{1-\chi} \left\{ MR(q)z - w + \beta(1 - \delta)(1 - s)E \left[ \frac{k}{f'} - \lambda' \right] \right\} \] (B.24)
\[ = w - (b + l) + \beta[(1 - s)(1 - \delta) - h(\theta)] \frac{\chi}{1-\chi} \left[ \frac{k}{f'} - \lambda' \right] \] (B.25)
Rearranging, we obtain the equilibrium wage.

**B.2.7 Representation of Equilibrium in \((\theta, u)\) Space**

In the spirit of the labor search literature, we reduce the representation of a steady-state equilibrium to a pair \((\theta, u)\) satisfying (3.43) and the following:

\[
\frac{r + 1 - \lambda + \psi}{\psi}[(1 - u)z]^{(\rho + \varepsilon - 1)/\rho}p(\theta) = A \left[ \frac{(1 + r)(1 - \rho)p}{\gamma(r + \delta)} \right]^{(1 - \rho)(1 - \varepsilon)/\rho} / (1 - \varepsilon) \quad (B.26)
\]

I refer to (B.26) as the cost push curve: it describes the equilibrium change in marginal costs associated with a change in unemployment. (B.26) implies a positive relationship between \(\theta\) and \(u\) provided that \(\rho + \varepsilon > 1\) (which entails \(2\rho + \varepsilon - \rho \varepsilon - 1 > \rho(1 - \varepsilon) > 0\)), or if \(2\rho + \varepsilon - \rho \varepsilon - 1 < 0\). Hence, just in the case \(\rho + \varepsilon < 1\) and \(2\rho + \varepsilon - \rho \varepsilon - 1 > 0\), there is a negative relationship between \(\theta\) and \(u\).

**Lemma B.1.**

- The cost-push curve is negatively sloped if and only if \([(1 - \rho)(1 - \varepsilon) - \rho] / (\rho + \varepsilon - 1) > 0\).

- A sufficient condition for the cost-push curve to be positively sloped is \(\rho + \varepsilon > 1\). In the knife-edge case \(\rho + \varepsilon = 1\), the cost-push curve pins down \(\theta\) uniquely, and the equilibrium is recursive.

- If \(\frac{\partial \theta}{\partial u} < 0\), a sufficient condition for concavity of the cost push curve is \((\rho + \varepsilon - 1) / [(1 - \rho)(1 - \varepsilon) - \rho] < 1\).

Thus, the cost-push curve is negatively sloped if and only if \(\rho + \varepsilon < 1\) and \(2\rho + \varepsilon - \rho \varepsilon - 1 > 0\). Figure (B.3) depicts multiplicity of equilibria for the baseline parameterization, and shows how this feature disappears with \(\varepsilon = 0.3\).
From (3.38), we know that there is always an inactive equilibrium in which \( \bar{z} = 0 \) and \( u = 1 \). The more interesting question concerns multiplicity of active equilibria. From Lemma B.1, multiplicity happens if and only if the cost push curve is negatively sloped. This fact, in turn, requires \( \rho + \varepsilon < 1 \), or that \( q_i \) to be increasing in \( Q \) (as in Schaal and Taschereau-Dumouchel (2016)). The latter property has the implication that an increase in output in the monopolistically competitive sector is associated with both more firms and larger firms. One of these produces a low unemployment rate and the other a high unemployment rate, in this case in excess of 40%.

### B.2.8 Proof of Proposition 3.3

It helps to rewrite the cost push curve as follows:

\[
\left[ (1 - \varepsilon) \left( \frac{r + 1 - \lambda + \psi}{\psi A} \right) \right]^\rho p(\theta)^{\rho(1-\varepsilon)+\rho+\varepsilon-1} \left[ \frac{\gamma(r + \delta)}{(1 + r)(1 - \rho)} \right]^{1+\rho\varepsilon-\rho-\rho} \frac{z^\varepsilon}{1+\rho\varepsilon-\rho} = (1 - u)^{1-\varepsilon-\rho}
\]

(B.27)

- **Case I:** \( \rho + \varepsilon > 1 \).

1. Suppose there is a rise in \( z \). This reduces \( p(\theta) \) for all \( \theta \) proportionately greater than the rise in \( z^\rho \), and thereby shifts the cost-push curve to the left. Hence, \( u \) is lower and \( \theta \) is higher. \( zp(\theta) \), which depends only on \( \theta \), is higher. Since \( zp(\theta)n \) is constant by the free
entry condition, \( n \) is lower. \( S = (1 - u)/n \) is higher as well. \( d = Spq \) can be rewritten as \( d = (1 - u)zMC/\rho \), which is higher. \( Q \) can be written as \( Q = [(1 - u)^\rho z]/[n^{1-\rho}] \), which rises since \( u \) falls, \( z \) increases, and \( n \) falls. Notice that \( p \propto [(1 - u)z]^{\rho(1-\rho)/\rho} \). Since the exponent is negative, so \( p \) falls and \( q \) rises.

2. Now consider an increase in \( k \). This raises \( p(\theta) \) and hence shifts the cost push curve to the right, lowering \( \theta \) and raising \( u \). However, \( p(\theta) \) remains higher. Thus, \( n \) is lower. The measure of sellers \( S = (1 - u)/n \) is lower. To see this, first write \( S = \frac{1-n}{\gamma(r+\theta)}(1+r)(1-\rho)zp(\theta) \), so that \( S \propto (1 - u)zp(\theta) \). From the equilibrium conditions, one can show that 

\[
(1 - u)zp(\theta) \propto (1 - u)^{\rho+\varepsilon}p(\theta)2^{\rho+\varepsilon-\rho\varepsilon},
\]

or that 

\[
[(1 - u)p(\theta)]^{1-\rho-\varepsilon} \propto p(\theta)^{\rho(1-\varepsilon)}. \]

As \( p(\theta) \) is higher and \( \rho + \varepsilon > 1 \), \( (1 - u)zp(\theta) \) is lower and hence \( S \) is lower. \( Q = S^{1/\rho}zn \) is lower as well. The debt level \( d = (1 - u)zp(\theta) \) is lower.

3. Consider an increase in \( \psi \). The shift of the cost push curve depends on \( \left( \frac{\psi}{r+\psi} \right) \rho \), which increases with \( \psi \). Thus, the cost push curve shifts to the left, so that \( \theta \) rises and \( u \) falls. \( p(\theta) \) rises, so that \( n \) falls. From \( S = (1 - u)/n \), \( S \) clearly rises. \( Q = S^{1/\rho}q \) can be rearranged as \( (1 - u)^{1/\rho}z/(n^{1-\rho}/\rho) \), which rises. Debt limits \( d = (1 - u)zp(\theta) \) rise, as \( p(\theta) \) is higher and \( u \) is lower.

4. Consider an increase in \( r \). From the cost-push curve, at a given \( u \), \( p(\theta) \) is lower. Provided \( p(\theta) \) weakly increases with \( r \), then \( \theta \) must decrease, so that the cost-push curve shifts to the right.

We can show that 

\[
\frac{\partial p}{\partial \theta} = \frac{1}{z(1-x)}(xk\theta - \frac{k(1-\delta)(1-s)}{f(\theta)}) = \frac{k\theta}{z(1-x)}[x - \frac{(1-\delta)(1-s)}{h(\theta)}]. \]

As \( h(\theta) < 1 \), this quantity is bounded above by 

\[
\frac{k}{z(1-x)}[x - (1-\delta)(1-s)] \propto [x - (1-\delta)(1-s)],
\]

which is negative provided that \( (1-\delta)(1-s) > x \). Hence, 

\[
\frac{\partial p(\theta)}{\partial r} = -\frac{\partial p(\theta)}{\partial \theta} \frac{1}{(1+r)^2} > 0.
\]

Given that the Beveridge curve is unaffected, \( \theta \) falls and \( u \) rises. As \( r \) is higher, it remains to show that equilibrium \( MC(\theta) \) is lower.

Using \( \gamma(r + \delta) = (1 + r)(1-\rho)zp(\theta)n \), \( n \) increases, as \( MC(\theta) \) is lower and \( r \) is higher. \( d = (1 - u)zp \) thus falls. Further, \( S = (1 - u)/n \) decreases. \( Q = (1 - u)^{1/\rho}z/(n^{1-\rho}/\rho) \) decreases from both the increase in \( u \) and rise in \( n \).

5. Consider an increase in \( \gamma \). The cost push curve shifts to the right, so \( \theta \) and \( p(\theta) \) fall,
and \( u \) increases. As before, from the entry condition \( n \) rises. \( d = (1 - u)zp \) thus falls, and so does \( S = (1 - u)/n \). \( Q = (1 - u)^{1/\rho}z/(n^{(1-\rho)/\rho}) \) decreases from both the decline in \( u \) and rise in \( n \).

6. Consider an increase in \( A \). The cost push curve shifts to the left, so that \( \theta \) increases and \( u \) decreases. \( p(\theta) \) rises, so that \( n \) falls. From \( S = (1 - u)/n \), \( S \) clearly rises. \( Q = S^{1/\rho}q \) can be rearranged as \( (1 - u)^{1/\rho}z/(n^{(1-\rho)/\rho}) \), which rises. Debt limits \( d = (1 - u)zp(\theta) \) rise, as \( p \) is higher and \( u \) is lower.

7. Consider an increase in \( s \). \( p(\theta) \) increases, inducing a rightward shift of the cost push curve. Furthermore, the Beveridge curve shifts to the right. The effect of the shift in the cost push curve is to raise \( u \) and lower \( \theta \). The effect of the rightward shift of the Beveridge curve is to raise \( u \) and \( \theta \). Thus, \( u \) clearly rises, but \( \theta \) is ambiguous.

Through implicit differentiation, we can provide a weak necessary and sufficient condition for \( \theta \) to fall. Rearranging the equilibrium conditions provides

\[
(2\rho + \varepsilon - \rho \varepsilon - 1) \log MC(\theta) = \delta + (1 - \rho - \varepsilon) \log \left( \frac{h(\theta)}{s + \delta(1 - s) + h(\theta)} \right)
\]

in which \( \delta \) is a function of parameters, not including \( s \). Totally differentiating (and using \( \frac{\partial MC}{\partial s} = \frac{k\beta(1-\delta)}{f(\theta)(1-\chi)} \)) yields

\[
(2\rho + \varepsilon - 1) \left[ \frac{MC'(\theta) \, d\theta}{MC(\theta) \, ds} + \frac{k\beta(1-\delta)}{f(\theta)(1-\chi)} \right] = (1 - \rho - \varepsilon) \left( \frac{p'(\theta) \, d\theta}{p(\theta) \, ds} - \frac{p'(\theta) \, d\theta / ds + 1 - \delta}{s + \delta(1 - s) + h(\theta)} \right)
\]

which can be further rearranged as

\[
\frac{d\theta}{ds} = \frac{(1 - \rho - \varepsilon) h(\theta)(1 - \delta) + h(\theta)[s + \delta(1 - s) + h(\theta)](2\rho + \varepsilon - \rho \varepsilon - 1) \frac{k\beta(1-\delta)}{f(\theta)(1-\chi)}}{(1 - \rho - \varepsilon) h'(\theta)[s + \delta(1 - s)] - (2\rho + \varepsilon - \rho \varepsilon - 1) h(\theta)[s + \delta(1 - s) + h(\theta)]MC'(\theta)/MC(\theta)}
\]

By presupposition, the sign of the denominator is negative, so the sign of \( d\theta/ds \) is the
opposite sign of the numerator, which has the same sign as

\[
1 - \rho - \varepsilon + [s + \delta(1 - s) + h(\theta)](2\rho + \varepsilon - \rho\varepsilon - 1) \frac{k\beta}{f(\theta)z(1 - \chi)}
\]  

(B.31)

We can derive a sufficient condition in terms of parameters for which (B.31) > 0. Consider the expression \(\frac{s + \delta(1 - s) + h(\theta)}{f(\theta)}\). This expression is strictly increasing in \(\theta\), and minimized as \(\theta \to 0\), in which \(h(\theta) \to 0\) and \(f(\theta) \to 1\). The expression converges to \(s + \delta(1 - s) + 1\). Thus, (B.31) is positive provided that \((2\rho + \varepsilon - \rho\varepsilon - 1)(s + \delta(1 - s) + 1) \frac{k\beta}{z(1 - \chi)} > \rho + \varepsilon - 1\).

Provided this condition is met, \(\theta\) decreases. To figure out the net effect on \(MC\), we see that the total derivative has the same sign as \((1 - \rho - \varepsilon)[h'(\theta)d\theta/ds[s + \delta(1 - s)] - h(\theta)(1 - \delta)]\), which is positive if \(d\theta/ds < 0\). Hence, \(MC\) increases, so \(n\) decreases. \(Q = (1 - u)^{1/\rho}z/(n^{1/\rho})\), which unambiguously decreases. The measure of firms satisfies \(S = (1 - u)/n\), which is ambiguous. Thus, we derive the comparative static of \((1 - u)/n\).

From the equilibrium conditions, \((\frac{1 - u}{n})^{1 - \rho - \varepsilon} n^{\rho(1 - \varepsilon)}\) is constant. Since \(n\) decreases, this means \((1 - u)/n\) decreases. Hence, \(S\) decreases. \(Q = S^{1/\rho}zn\) thus decreases as well. Writing \(d = \frac{1 - u}{n}nzp\), which decreases since \(1 - u\) decreases and \(np\) is constant from the free entry condition.

8. Consider an increase in \(\delta\). As before, the Beveridge curve shifts to the right, symmetrically as with an increase in \(s\). The cost push curve shifts to the right even more than in the case of \(s\). This is because a higher \(\delta\) both raises the marginal costs of existing firms and deters entry. Thus, \(u\) clearly rises, \(\theta\) is ambiguous, and the condition (B.31) is sufficient for \(\theta\) to fall.

- Case II: \(\rho + \varepsilon < 1\) and \(2\rho + \varepsilon(1 - \rho) > 1\).

1. Suppose there is a rise in \(z\). This reduces \(p(\theta)\) for all \(\theta\), and thereby shifts the cost-push curve upward. Hence, \(u\) is lower and \(\theta\) is higher. The rest is identical to Case I, except for \(p\) and \(q\). As \(p \propto [(1 - u)z]^{\frac{1 - \varepsilon - \rho}{\rho(1 - \varepsilon) + \rho + \varepsilon - 1}}, \) and the exponent is positive, \(p\) increases and

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1. Consider an increase in $q$. $p(\theta)$ increases, lowering $\theta$ and raising $u$. The rest follows as in Case I.

2. Now consider an increase in $k$. $p(\theta)$ rises and hence shifts the cost push curve downward, lowering $\theta$ and raising $u$. Note that the RHS of the cost push curve is lower, so that $p(\theta)$ remains lower than initially. Thus, $n$ is higher. The measure of sellers $S = (1 - u)/n$ is thus lower. $Q = S^{1/\rho}zn$ can be rewritten as $Q = (1 - u)z^\rho/(n^{1-\rho})$, which is lower as well. The debt level $d = Spq$ can be rewritten as $d = S\gamma(r + \delta)/(1 + \rho)(1 - \rho)$, which is lower since $S$ is lower.

3. Consider an increase in $\psi$. The shift of the cost push curve depends on $(\psi r + \psi)/(1 + \rho)$, which increases with $\psi$. Thus, the cost push curve shifts to the left, so that $\theta$ rises and $u$ falls. The rest follows as in Case I.

4. Consider an increase in $r$. From the cost-push curve, at a given $u$, $p(\theta)$ is lower. Provided $p(\theta)$ weakly increases with $r$, then $\theta$ must be lower, so that the cost-push curve shifts downward. As Case I, $\chi < (1 - \delta)(1 - s)$ is a sufficient condition.

Given that the Beveridge curve is unaffected, $\theta$ falls and $u$ rises. Since the RHS of (B.27) falls, so must the left hand side. Hence, $p(\theta)$ is lower. Using $\gamma(r + \delta) = (1 + r)(1 - \rho)zp(\theta)n$, $n$ increases, as $p(\theta)$ is lower and $r$ is higher. $d = (1 - u)zp(\theta)$ thus falls. Further, $S = (1 - u)/n$ decreases. $Q = (1 - u)^{1/\rho}z/(n^{1-\rho}/\rho)$ decreases from both the increase in $u$ and rise in $n$.

5. Consider an increase in $\gamma$. The cost push curve shifts to the right, so $\theta$ falls and $u$ increases. As before, from the entry condition $n$ rises. The rest follows identically as in Case I.

6. Consider an increase in $A$. The cost push curve shifts to the left, so that $\theta$ increases and $u$ decreases. The rest follows identically as in Case I.

7. Consider an increase in $s$. $p(\theta)$ increases at a given $\theta$, inducing a downward shift of the cost push curve. Furthermore, the Beveridge curve shifts to the right. The effect of the shift in the cost push curve is to raise $u$ and lower $\theta$. Unlike in Case I, the effect of the rightward shift of the Beveridge curve is to raise $u$ and lower $\theta$. Thus, $u$ clearly rises and $\theta$ falls. From $\rho + \varepsilon < 1$, the RHS of (B.27) falls. Hence, $p(\theta)$ also falls. Thus,
\( n \) increases. \( S = (1 - u)/n \) correspondingly decreases. \( d = (1 - u)zp(\theta) \) falls as well. \( Q = (1 - u)^{1/\rho}z/(n^{(1-\rho)/\rho}) \) decreases from both the increase in \( u \) and rise in \( n \).

8. Consider an increase in \( \delta \). As before, the Beveridge curve shifts to the right, symmetrically as with an increase in \( s \). The cost push curve shifts to the right even more than in the case of \( s \). This is because a higher \( \delta \) both raises the marginal costs of existing firms and deters entry. Thus, \( u \) rises, \( \theta \) decreases, and the condition (B.31) is sufficient for \( \theta \) to fall. As with a rise in \( s \), \( p(\theta) \) falls, since the RHS of (B.27) decreases and \((r + \delta)/(1 + r)\) rises in the LHS. The remainder is analogous to a rise in \( s \).

### B.2.9 Properties of Cost Push Curve

We can rearrange (B.26) as

\[
p(\theta) \frac{(1-\rho)(1-\varepsilon) - \rho}{\rho} = \mathcal{N}_0 (1 - u) \frac{\rho + \varepsilon - 1}{\rho}
\]

for the composite parameter \( \mathcal{N}_0 = (1 - \varepsilon) \frac{\rho + 1 + \lambda + \psi z^{(\rho + \varepsilon - 1)}p((1 + r)(1 - \rho)/(\gamma(r + \delta)))^{-(1-\rho)(1-\varepsilon)/\rho}}{A} > 0 \). Further rearrangement yields

\[
p(\theta) = \mathcal{N}_1 (1 - u) \frac{\rho + \varepsilon - 1}{(1 - \rho)(1 - \varepsilon) - \rho}
\]

for \( \mathcal{N}_1 = \mathcal{N}_0 \frac{\rho}{((1 - \rho)(1 - \varepsilon) - \rho)} \).

Implicit differentiation with respect to \( u \) yields

\[
\frac{\partial \theta}{\partial u} = -\frac{\mathcal{N}_1}{p'(\theta)} \frac{\rho + \varepsilon - 1}{(1 - \rho)(1 - \varepsilon) - \rho} (1 - u) \frac{\rho + \varepsilon - 1 + \rho - (1 - \rho)(1 - \varepsilon)}{(1 - \rho)(1 - \varepsilon) - \rho}
\]

(B.33)

Hence \( \frac{\partial \theta}{\partial u} < 0 \) if and only if \((\rho + \varepsilon - 1)/(1 - \rho)(1 - \varepsilon) - \rho > 0 \). The second derivative can be simplified to

\[
\frac{\partial^2 \theta}{\partial u^2} = \frac{\mathcal{N}_1}{p'(\theta)^2} \frac{\rho + \varepsilon - 1}{(1 - \rho)(1 - \varepsilon) - \rho} (1 - u) \frac{\rho + \varepsilon - 1 + \rho - (1 - \rho)(1 - \varepsilon)}{(1 - \rho)(1 - \varepsilon) - \rho} - 2 \left( (1 - u) \frac{\partial \theta}{\partial u} + p'(\theta) \left( \frac{\rho + \varepsilon - 1}{(1 - \rho)(1 - \varepsilon) - \rho} - 1 \right) \right)
\]

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A sufficient condition for concavity, provided \( \frac{\partial \theta}{\partial u} < 0 \) is \((\rho + \varepsilon - 1) / ((1 - \rho)(1 - \varepsilon) - \rho) < 1\). In general, this condition is stronger than necessary, but the necessary condition is a complicated relationship involving \( u \) and \( \aleph_1 \).

**B.2.10 Proof of Lemma 3.2**

The only nontrivial statement is the convexity of \( p(\theta) \). Note that it suffices to show that \( 1/f(\theta) \) is convex. The second derivative of \( 1/f(\theta) \) is

\[
F(\theta) = \frac{2f'(\theta)^2 - f(\theta)f''(\theta)}{f(\theta)}
\]  

(B.35)

Using \( f(\theta) = 1 - \exp(-B/\theta) \), we have \( f'(\theta) = -B \exp(-B/\theta)/\theta^2 \) and \( f''(\theta) = B \exp(-B/\theta)(2\theta - B)/\theta^4 \) after simplification. Using these expressions, rewrite

\[
2f'(\theta)^2 - f(\theta)f''(\theta) = 2B^2 \exp(-2B/\theta)/\theta^4 - (1 - \exp(-B/\theta)) \frac{B \exp(-B/\theta)(2\theta - B)}{\theta^4}
\]

(B.36)

We can factor this latter expression:

\[
\frac{B \exp(-B/\theta)}{\theta^4} \{2B \exp(-B/\theta) - (2\theta - B)(1 - \exp(-B/\theta))\}
\]

(B.37)

It suffices to show that the expression in brackets \( G(\theta) \) is always positive. Figure B.4 shows that \( G(\theta) \) in brackets is positive, decreasing, convex, and has limits \( \lim_{\theta \to 0} G(\theta) \to \infty \) and \( \lim_{\theta \to \infty} G(\theta) = 0 \).
B.3 Dynamic Stochastic Equilibrium Equations

Equilibrium consists of a list of quantities \( \{q_t, a_t, p_t, \theta_t, u_t, S_t\}_{t=1}^T \) satisfying

\[
q = \min \left\{ \left( \frac{AS^{(1-\varepsilon-\rho)/\rho}}{p} \right)^{1/\varepsilon}, \frac{d}{Sp} \right\} \tag{B.38}
\]

\[
\frac{d}{\psi} = \beta \mathbb{E} \left\{ A(S^{1/\rho}p')^{1-\varepsilon}/(1-\varepsilon) - S'p'q' + \lambda \frac{d}{\psi} \right\} \tag{B.39}
\]

\[
\rho z p(1-\chi) = [\chi \beta h(\theta) - \beta (1-\delta)(1-s)] \mathbb{E} \left[ \frac{k}{f(\theta')} - \lambda_V \right] + (1-\chi)(b+l) + \frac{k}{f(\theta)} - \lambda_V \tag{B.40}
\]

\[
\frac{S q}{z} = 1 - u \tag{B.41}
\]

\[
u = [1-h(\theta)]u_{-1} + [1-(1-s)(1-\delta)](1-u_{-1}) \tag{B.42}
\]

\[
S = \max\{(1+r)(1-\rho)pq/\gamma(r+\delta), (1-\delta)S_{-1}\} \tag{B.43}
\]

\[
\theta u \lambda_V = 0 \tag{B.44}
\]

All other equilibrium quantities can be backed out in terms of the aforementioned variables. The wage is given by

\[
w = \chi \left\{ \rho z p + \beta h(\theta) \mathbb{E} \left[ \frac{k}{f(\theta')} - \lambda_V \right] \right\} + (1-\chi)(b+l). \]

The expectation operator is
equivalent to \( \frac{\rho z_p(1-\chi)(1-\chi)(b+l)-(k/f-\lambda V)}{\beta \chi h(\theta)-(1-\delta)(1-s)} \) leading to

\[
w = \chi \left[ \frac{\rho z_p[h(\theta) - (1 - \delta)(1 - s)] - h(\theta)(1 - \chi)(b + l) - (k/f(\theta) - \lambda V)}{\chi h(\theta) - (1 - \delta)(1 - s)} \right] + (1 - \chi)(b + l) \quad (B.45)
\]

The price index \( P = p(Spq/(Spq + z)) + z/(Spq + z) \) and the wage index is \( w_r = w/P \). Vacancies satisfy \( V = \theta u_{-1} \). Aggregate consumption is \( C = Spq + z - kV \), and profits \( \Delta = Spq - (1-u)w - kV \).
B.4 Histograms

We simulate 28,000 draws of data and plot the histogram for the price level, unemployment, market tightness, the measure of sellers, real wages, debt, consumption, debt relative to consumption, and aggregate profits.

Figure B.5: Histograms of simulations
B.5 Parameterized Expectations Algorithm.

We solve the dynamic, stochastic model using the parameterized expectations algorithm, which was introduced by Den Haan and Marcet (1990). The conditional expectations, which here arise in the job creation condition and in the debt limit, are a function of Θ. The strategy is to approximate these conditional expectations by a polynomial function of the state variables. Specifically, we represent the log of the conditional expectation with a polynomial in logs. Solving for the unknown conditional expectations function thus simplifies to calculating the polynomial coefficients. We start with an initial guess of coefficients for these functions, generate data from the model, and update the conditional expectations using nonlinear least squares. Consequently, the algorithm is interpretable as least squares learning.

Using moving bounds, proposed by Maliar and Maliar (2003), stabilizes the algorithm with respect to the initial choice of coefficients. These initial bounds are close to the steady state levels and are gradually removed until they no longer bind in subsequent iterations.

1. Consider state space \( \Theta_t = (N_{t-1}, S_{t-1}, z_t) \).

2. Consider the job creation condition:

\[
\frac{k}{f(\theta_t)} - \lambda_t^V = \rho z_t p_t - w_t + E_t \beta(1-\delta)(1-s) \left\{ \frac{k}{f(\theta_{t+1})} - \lambda_{t+1}^V \right\}
\]  

(B.46)

where \( \lambda_t^V \) is the Lagrangian multiplier on the nonnegativity constraint on vacancies. Rearrange the job creation condition as a function of prices and market tightness using the wage equation:

\[
\rho z_t p_t (1-\chi) = (1-\chi)(b+l) + \frac{k}{f(\theta_t)} - \lambda_t^V + [\chi \beta h(\theta_t) - \beta(1-\delta)(1-s)]E \left[ \frac{k}{f(\theta_{t+1})} - \lambda_{t+1}^V \right]
\]  

(B.47)

The debt level equals \( d_t = S_t p_t q_t = (1-u_t) z_t p_t \). So, \( p_t = d_t / [(1-u_t) z_t] \), which, using the law

\[\text{The rationale for using polynomial functions comes from the Weierstrass theorem, which asserts that for a continuous function } f(x) \text{ on } [a, b], \text{ there is a polynomial function } p_n(x) \text{ arbitrarily close to } f: \forall \varepsilon > 0 \forall x \in [a, b], |f(x) - p_n(x)| < \varepsilon.\]
of motion for employment

\[
\frac{\rho d_t(1-\chi)}{(1-s)(1-\delta)(1-u_{t-1}) + f(\theta_t)\theta_t u_{t-1}} = (1-\chi)(b+l) + \frac{k}{f(\theta_t)} - \lambda_t^V
\]  \hspace{1cm} (B.48)

+ [\chi \beta h(\theta_t) - \beta (1-\delta)(1-s)] \mathbb{E}\left[\frac{k}{f(\theta_{t+1})} - \lambda_{t+1}^V\right]  \hspace{1cm} (B.49)

3. Consider the debt limit equation:

\[
\frac{d_t}{\psi} = \beta \mathbb{E}\left\{A(S_{t+1}^{1/\rho} q_{t+1})^{1-\varepsilon}/(1-\varepsilon) - S_{t+1} p_{t+1} q_{t+1} + \lambda_d t + 1/\psi\right\}
\]  \hspace{1cm} (B.50)

4. Start at iteration \(i = 1\).

5. Simulate a time series of length \(T = 28,000 - 200\) as follows (burn-in of 200). Draw a length \(T\) of \(z_t\).

(a) Initialize \(S_1\) and \(N_1\) at steady-state values.

(b) Using the initialization \(\beta \mathbb{E}\left\{\Omega_{t+1} + \mathcal{G}_{t+1}/\psi\right\} = H(\Theta_t, \beta_2) = \exp(X_t^t \beta_2)\), we obtain the debt limit \(\mathcal{G}_t\). The polynomial generating vector \(X_t\) is

\[
\begin{bmatrix}
\log(z_t) & \log(N_{t-1}) & \log(N_{t-1})^2 & \log z_t \log(N_{t-1}) & \log(z_t) \log(S_{t-1}) & \log(S_{t-1}) \log(N_{t-1}) & \log(S_{t-1}) \log(N_{t-1}) \log(S_t) \log(z_t)
\end{bmatrix}
\]

(c) Use \(\mathbb{E}\left[\frac{k}{f(\theta_{t+1})} - \lambda_{t+1}^V\right] = G(\Theta_t, \beta_1) = \exp(X_t^t \beta_1)\)

(d) Conjecture that \(d_t^0 = \mathcal{G}_t, \lambda_t^V = 0\), and recover \(d_t = H_t \psi\)

(e) Restrict \(G_t\) and \(d_t\) to satisfy moving bounds: \(G_t \leq G_{t+1} \leq \overline{G}_t\) and \(d_t \leq d_{t+1} \leq \overline{d}_t\), where

\[
\begin{align*}
G_t &= \frac{k}{f(\theta_{ss})} \exp(-ai) \\
\overline{G}_t &= \frac{k}{f(\theta_{ss})} \exp(2 - \exp(ai)) \\
d_t &= d_{ss} \exp(-ai)
\end{align*}
\]

The parameter \(a\) controls the speed of moving the bounds and \(d_{ss}\) and \(\theta_{ss}\) are the steady
state values of debt and market tightness and recover $\theta_t$

(f) If $\theta_t < 0$, set $\theta_t = 0$ and use (B.48) to recover $\lambda_t^V$. 

(g) Define $V_t = \theta_t u_{t-1}$. 

(h) Let $f_t = 1 - \exp(-B/\theta_t)$ and compute $p_t$ from equation (B.47). 

(i) Calculate sellers: $S_t = \max\{(1 + r)(1 - \rho)d_t/(\gamma(r + \delta)), (1 - \delta)S_{t-1}\}$. 

(j) Compute first best debt level $d_t^* = S_t p_t \left(\frac{A S_{t+1}^1 - \rho}{p_t}\right)^{1/\varepsilon}$. 

(k) $d_t = \min\{d_t^0, d_t^*\}$. If $d_t = d_t^*$, recompute 5e-5j under $d_t^0 = d_t^*$. Iterate until $\text{loss} = (d_t - d_t^0)^2 < 1e - 5$. 

(l) Let $q_t = d_t / (S_t p_t)$. 

(m) Update unemployment: $u_t = [1 - h(\theta_t)]u_{t-1} + [1 - (1 - s)(1 - \delta)](1 - u_{t-1})$.

6. After simulating a series of length $T$, compute

$$y_1^t = \frac{k}{f(\theta_{t+1})} - \lambda_{t+1}^V$$

$$y_2^t = \beta \left\{ A(S_{t+1}^{1/\rho})^{1-\varepsilon}/(1 - \varepsilon) - S_{t+1} p_{t+1} q_{t+1} + \lambda^{d_{t+1}/\psi} \right\}$$

7. Recover $\beta_1^{\text{new}}$ and $\beta_2^{\text{new}}$ using nonlinear least squares:

$$\beta_1^{\text{new}} = \arg \min_1 \frac{1}{T} \sum_{t=0}^{T} |y_1^t - \exp(X_t' \beta_1)|^2$$

$$\beta_2^{\text{new}} = \arg \min_2 \frac{1}{T} \sum_{t=0}^{T} |y_2^t - \exp(X_t' \beta_2)|^2$$

8. Update $\beta_i$ as $\Gamma \beta_i^{\text{new}} + (1 - \Gamma)\beta_i$ for $i = 1, 2$, with $\Gamma = 0.9$.

9. Update iteration $i$ until $||b_1^{\text{new}} - b_1|| + ||b_2^{\text{new}} - b_2|| < 1e - 5$. 

After convergence, we obtain a time series on the remaining variables as follows: $u = 1 - N$, $C = d + z, d_c = d/C, w = \chi(pz + \beta hG) + (1 - \chi)(b + l), P = p(d/C) + (z/C), w_r = w/P$, and $\Delta = (1 - u)(pz - w) - kV$.
B.6 Monte Carlo Procedure for Generalized Impulse Responses

1. Fix the length of a series $T = 120$ and number of Monte Carlo iterations $N = 40$.

2. Draw matrix $E$ of normal random variables with standard deviation $\sigma_z$ of size $T \times N$.

3. Let $z_1 = \omega + E_{1,j}$ and $\hat{z}_1 = E_{1,j}$ for impulse $\omega$.

4. For $t = 2 : T$, define $z_t^j = \rho z_{t-1}^j + E_{t,j}$ and $\hat{z}_t^j = \rho \hat{z}_{t-1}^j + E_{t,j}$.

5. Let $Y(z^j)$ be the simulated series under sequence $z^j$ and $Y(\hat{z}^j)$ be the simulated series under sequence $\hat{z}^j$.

6. Compute the difference $Y(z^j) - Y(\hat{z}^j)$. Repeat for all $j$ and compute the mean response

$$ IRF = \frac{1}{N} \sum_{j=1}^{N} [Y(z^j) - \hat{Y}(z^j)] . $$

7. Calculate the impulse responses in percentage deviation from the unconditional mean of each series.

B.7 Robustness Checks

B.7.1 Parameterization

We consider two robustness checks with respect to parameterization. First, we increase $\varepsilon = 1 - \rho$, which makes individual demands $q$ independent of $Q$, and recalibrate the model. This change also shuts down the type of aggregate demand externality analyzed in Schaal and Taschereau-Dumouchel (2016). Table B.1 summarizes the results.
Table B.1: Moments of key variables simulated from model, monthly. Data is transformed by proportional deviations from the mean.

Unemployment and credit are moderately less volatile, but market tightness remains volatile. This change also makes the correlation between market tightness and credit near unity.
Appendix C

Appendix for Chapter 4

C.1 Proofs

C.1.1 Comparative statics with Pure Bank Credit

We can derive the following:

\[ \frac{\partial F}{\partial \theta} = \frac{\sigma}{1 - \sigma} \left\{ \frac{\sigma}{[(1 - \theta)\sigma + \theta]^1/(1 - \sigma)} \int_0^{(1 - \theta)\sigma + \theta} G(\chi)\chi^{(2\sigma - 1)/(1 - \sigma)} d\chi - (1 - \sigma) \frac{G[(1 - \theta)\sigma + \theta]}{(1 - \theta)\sigma + \theta} \right\} \]

(C.1)

\[ \epsilon_{Y,\alpha} = \epsilon_{Y,\lambda} = \frac{\gamma}{1 - \gamma} \frac{1 - \sigma}{\sigma} \]

(C.2)

\[ \epsilon_{Y,\theta} = -\frac{\gamma}{1 - \gamma} \frac{\theta F(\chi^*)^{-1}\sigma}{[(1 - \theta)\sigma + \theta]^1/(1 - \sigma)} \int_0^{(1 - \theta)\sigma + \theta} G(\chi)\chi^{(2\sigma - 1)/(1 - \sigma)} d\chi - (1 - \sigma) \frac{G[(1 - \theta)\sigma + \theta]}{(1 - \theta)\sigma + \theta} \]

(C.3)
C.1.2 Proof of Proposition 4.1

The first part of this argument parallels the proof of Proposition 1 in Rocheteau et al. (2016). If \( k + \phi \geq \chi f(k, Y) \) does not bind, then the solution to (4.16) maximizes the match surplus \( f(k, Y) - k \), while \( \phi \) shares the surplus according to the bargaining powers. The solution is thereby characterized by \( k = k^* \) and \( \phi = \theta[f(k^*, Y) - k^*] \). The pledgeability threshold \( \chi^* \) satisfies \( k^* + \theta[f(k^*, Y) - k^*] = \chi^* f(k^*, Y) \). Rearranging,

\[
\chi^* = \theta \frac{f(k^*, Y)}{f(k^*, Y)} + (1 - \theta) \frac{k^*}{f(k^*, Y)} = (1 - \theta)\sigma + \theta
\]

If the pledgeability constraint binds, then \( k \) solves

\[
k \in \arg \max (f(k, Y))^{1-\theta} [\chi f(k, Y) - k]^\theta
\]

The first order condition yields (4.20), and we recover \( \phi \) from the pledgeability constraint.

As in Rocheteau et al. (2016), the left hand side of (4.20) is increasing in \( k \) from 0 to \( \infty \), with limits obtainable by L’Hopital’s rule. The right hand side is decreasing for all \( k \in [\hat{k}, k^*] \). The right hand side evaluated at \( k^* \), \((\chi - \theta)/(1 - \theta)\), is smaller than the left hand side given \( \chi < \chi^* \). At \( k = \hat{k} \), the right hand side is \( 1/(\partial f/\partial k|_{k=\hat{k}}) = 1/\chi \), which exceeds the left hand side. Hence, there is a unique solution \( \hat{k} \in [\hat{k}, k^*] \) to (4.20).

Combining (4.23) with (4.18) and (4.20) yields (4.24).

C.1.3 Proof of Corollary 4.1

The only nontrivial part is the proof that \( \partial k/\partial \theta = (\sigma - 1)/[(1 - \theta)\sigma + \theta] < 0 \). First, rewrite (4.20) as

\[
(1 - \theta)\sigma = \chi \frac{\partial f}{\partial k} - \theta
\]
and totally differentiate with respect with $\theta$ to obtain

$$\frac{\partial k}{\partial \theta} = \frac{1}{\chi \partial^2 f / \partial k^2}$$

Plugging in for $\partial^2 f / \partial k^2$, we obtain

$$\frac{\partial k}{\partial \theta} = \frac{k^{1-\sigma}}{\chi \sigma (\sigma - 1) \gamma Y ((\gamma - \sigma)/\gamma)}$$

Plugging in for $k$ and simplifying, we obtain

$$\frac{\partial k}{\partial \theta} = \frac{\sigma - 1}{(1 - \theta) \sigma + \theta} < 0$$

### C.1.4 Proof of Lemma 4.2

Differentiate the right hand side of (4.39) with respect to $\theta$, obtaining:

$$\frac{1}{(1-\theta)^2} \left( \frac{1 - \chi \frac{\partial f}{\partial k_c}}{(1-\chi) \frac{\partial f}{\partial k_c}} - \frac{\theta}{1-\theta} \frac{(1-\chi) \partial^2 f / \partial k_c^2 \partial k_c}{[(1-\chi) \frac{\partial f}{\partial k_c}]^2} \right)$$

Differentiating the left hand side with respect to $\theta$ yields

$$-\frac{\partial k_c}{\partial \theta} \left\{ \frac{[(1 - \chi)f(k_c, Y) - f(a_m, Y)][1 - \chi \frac{\partial f}{\partial k_c}] + (a_m + \chi f(k_c, Y) - k_c)(1 - \chi) \frac{\partial f}{\partial k_c}}{[(1 - \chi)f(k_c, Y) - f(a_m, Y)]^2} \right\}$$

If $\frac{\partial k_c}{\partial \theta} \geq 0$, then the right hand side is positive but the left hand side is negative. Hence, $\frac{\partial k_c}{\partial \theta} < 0$. It is clear from inspecting the $h(a_m, Y)$ defined in (C.4) that $\frac{\partial k_c}{\partial a_m} > 0$. Consequently, $\frac{\partial [a_m + \chi f(k_c, Y)]}{\partial a_m} > 1$. 

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C.1.5 Effect of Money Holdings on the Bargained Level of Investment

The curve \( h(a_m, k_c, \chi, Y) = \frac{\partial k_c}{\partial a_m} \) specifies the change in investment with respect to real balances for an entrepreneur with coefficient \( \chi \), economic conditions \( Y \), assets \( a_m \), and investment level \( k_c \).

\[
- \left( \frac{\partial f}{\partial k_c} + \frac{\theta}{(1-\theta)(1-\chi)} \frac{\partial f}{\partial a_m} \right) \left( \chi \frac{\partial f}{\partial k_c} - 1 \right) = \frac{\partial f}{\partial k_c} + \left( a_m + \chi f(k_c, Y) - k_c \right) \frac{\partial f}{\partial k_c} \left( \frac{\theta}{(1-\theta)(1-\chi)} \right) \left[ (1-\chi)f(k_c, Y) - f(a_m, Y) \right] \left( \chi \frac{\partial f}{\partial k_c} - 1 \right) \left( 1-\chi \right) \frac{\partial f}{\partial k_c} \right]
\]

provided that \( a_m^c < a^* \), and is 0 otherwise.

C.1.6 Proof of Proposition 4.4

The entrepreneur’s problem over real balances is

\[
J(a_m^c, i, Y) \equiv -ia_m^c + \lambda(1-\alpha)\Delta_m(a_m^c) + \lambda \alpha \Delta_c(a_m^c)
\]

for \( Y > 0 \) given. Using \( \Delta_m'(0) = \infty \) and \( \Delta_c'(0) = \infty \) for all \( \theta > 0 \), there is a positive solution to \( \max_{a_m^c} J(a_m^c) \) if \( \lambda(1-\alpha) > 0 \) or \( \lambda \alpha > 0 \). We check that \( k_c > k_m \) for \( i = 0 \). If the liquidity constraint does not bind, then \( k_c = k^* \) and by 4.44 \( k_m < k^* \). If the liquidity constraint does not bind, then the bargaining problem implies

\[
(1-\chi)f(k_c, Y) - a_m^c - \Delta_m(a_m^c) > 0
\]

\[
\Leftrightarrow (1-\chi)f(k_c, Y) - f(a_m^c, Y) > 0
\]

\[
\Leftrightarrow k_c - a_m^c > 0
\]
C.1.7 Proof of Proposition 4.6

Take the logarithm of (4.49):

\[
\log Y = \frac{\gamma}{1 - \gamma} \log(\sigma \gamma) + \frac{\gamma(1 - \sigma)}{\sigma(1 - \gamma)} \log \lambda + \gamma(1 - \sigma) \log \left\{ (1 - \alpha) \left[ \frac{\lambda(1 - \alpha(1 - \theta))}{i + \lambda(1 - \alpha(1 - \theta))} \right]^{\sigma/(1 - \sigma)} \right\} + \alpha
\]

(C.5)

Differentiating (C.5) with respect to \(i\) yields:

\[
\left[ \frac{\gamma (1 - \alpha) \Upsilon(i)^{(2\sigma - 1)/(1 - \sigma)}}{(1 - \alpha) \Upsilon(i)^{(1 - \sigma)} + \alpha} \right] \left( \frac{-\lambda (1 - \alpha(1 - \theta))}{[i + \lambda(1 - \alpha(1 - \theta))]^2} \right)
\]

As \(i \to 0\), \(\Upsilon(i) \to 1\), and we obtain

\[
\frac{\partial \log Y}{\partial i} = -\frac{\gamma(1 - \alpha)/(1 - \gamma)}{\lambda[1 - \alpha(1 - \theta)]}
\]

Using (4.58) with (4.48) and (4.50) we can derive the semi-elasticity of money demand: Note

\[
\frac{\partial \log \sigma_m}{\partial i} = \frac{\partial \log k^*}{\partial i} + \frac{1}{1 - \sigma} \frac{\partial \log \Upsilon(i)}{\partial i}
\]

\[
= \frac{\gamma - \sigma}{\gamma(1 - \sigma)} \frac{\partial \log Y}{\partial i} - \frac{1}{1 - \sigma} \frac{\lambda(1 - \alpha(1 - \theta))}{(i + \lambda[1 - \alpha(1 - \theta)])^2}
\]

\[
= -\frac{\lambda[1 - \alpha(1 - \theta)]}{(1 - \sigma)[i + \lambda(1 - \alpha(1 - \theta))]^2} \left[ \frac{(\gamma - \sigma)(1 - \alpha)}{1 - \gamma} \frac{\Upsilon(i)^{(2\sigma - 1)/(1 - \sigma)}}{(1 - \alpha)\Upsilon(i)^{(1 - \sigma)} + \alpha} + 1 \right]
\]

Applying (4.58) yields (4.59) and (4.60). To generate the pass through on revenue, take the logarithm of (4.13):

\[
\log f(k^*, Y) = \log \gamma + \frac{\gamma - \sigma}{\gamma} \log Y + \sigma \log k^*
\]

Differentiating with respect to \(i\) and applying (4.58) and (4.59) yields (4.61).
C.1.8 Proof of Lemma 4.4

1. The quantity $\Theta(i)$ is an indeterminate form as $i \to 0$, so we differentiate numerator and denominator with respect to $i$:

\[
-\frac{\frac{\sigma}{1-\sigma} \Gamma(i)^{(2\sigma-1)/(1-\sigma)}}{\frac{1}{1-\sigma} \Gamma(i)^{\sigma/(1-\sigma)}}
\]

As $i \to 0$, $\Upsilon \to 1$, and hence any power of $\Upsilon$ approaches 1. Thus, $\Theta(i) \to \sigma$

2. By the quotient rule,

\[
\Theta'(i) = \Gamma'(i) \left(1 - \frac{\Gamma(i)^{1/(1-\sigma)}}{(1-\sigma)(\Gamma(2\sigma-1)/(1-\sigma))} + \frac{1/(1-\sigma)}{(1-\Upsilon(i))^{1/(1-\sigma)}}\right)
\]

First, calculate $\Gamma'(i) = -\frac{\lambda(1-\alpha(1-\theta))}{\lambda(1-\alpha(1-\theta))^{2}} \to -\frac{1}{\lambda(1-\alpha(1-\theta))}$ as $i \to 0$.

Second, the first part of the expression can be rewritten as

\[
\frac{\sigma}{1-\sigma} \frac{\Gamma(2\sigma-1)/(1-\sigma)}{1-\Upsilon(i)^{1/(1-\sigma)}}
\]

which is indeterminate. Hence, we can apply L’Hopital’s rule, obtaining

\[
-\frac{\frac{\sigma}{1-\sigma} \Gamma(i)^{(3\sigma-2)/(1-\sigma)}}{\frac{1}{1-\sigma} \Gamma(i)^{\sigma/(1-\sigma)}}
\]

which approaches $\sigma(2\sigma-1)/(1-\sigma)$ as $i \to 0$.

The second part of the expression can be written as the product

\[
\frac{1}{1-\sigma} \left(\frac{1 - \Gamma(i)^{\sigma/(1-\sigma)}}{1-\Gamma(i)^{1/(1-\sigma)}}\right) \left(\frac{\Gamma(i)^{\sigma/(1-\sigma)}}{1-\Gamma(i)^{1/(1-\sigma)}}\right)
\]

By L’Hopital’s rule, the second and third factors approach $\sigma$ as $i \to 0$, so that the
expression converges to $\sigma^2/(1-\sigma)$. Finally, the overall fractional expression approaches

$$\frac{\sigma(2\sigma - 1) - \sigma^2}{1 - \sigma} = -\sigma$$

Hence, $\Theta'(i) \to \frac{\sigma}{\lambda(1-\alpha(1-\theta))}$ as $i \to 0$.

### C.1.9 Derivation of Aggregate Demand Under Perfect Enforcement

Using (4.49) and (4.50), we can write

$$Y = \left[ \lambda(1 - \alpha)\sigma_m^\sigma + \lambda \alpha k^* \sigma \right]^{\gamma/\sigma} \quad \text{(C.6)}$$

$$= \left[ \lambda(1 - \alpha)k^* \sigma Y(i)^{\sigma/(1-\sigma)} + \lambda \alpha k^* \sigma \right]^{\gamma/\sigma} \quad \text{(C.7)}$$

$$= k^* \gamma \lambda^{\gamma/\sigma} [(1 - \alpha)Y(i)^{\sigma/(1-\sigma)} + \alpha]^{\gamma/\sigma} \quad \text{(C.8)}$$

$$\quad \text{(C.9)}$$

Then we substitute (4.48):

$$Y = [\sigma \gamma Y^{(\gamma-\sigma)/\gamma} (1-\sigma) \lambda^{\gamma/\sigma} [(1 - \alpha)Y(i)^{\sigma/(1-\sigma)} + \alpha]^{\gamma/\sigma} \quad \text{(C.10)}$$

$$= (\sigma \gamma)^{\gamma/(1-\sigma)} Y^{(\gamma-\sigma)/(1-\sigma)} \lambda^{\gamma/\sigma} [(1 - \alpha)Y(i)^{\sigma/(1-\sigma)} + \alpha]^{\gamma/\sigma} \quad \text{(C.11)}$$

Hence,

$$Y^{(1-\gamma)/(1-\sigma)} = (\sigma \gamma)^{\gamma/(1-\sigma)} \lambda^{\gamma/\sigma} [(1 - \alpha)Y(i)^{\sigma/(1-\sigma)} + \alpha]^{\gamma/\sigma} \quad \text{(C.12)}$$

Rearrangement yields (4.53).
C.1.10 Proof of Lemma 4.3

Compute

\[ R = \lambda [\alpha f(k^*, Y) + (1 - \alpha) f(a_m, Y)] \]
\[ = \gamma Y^{(\gamma - \sigma)/\gamma Y^{(\sigma - \gamma)/\gamma}}(\lambda \alpha k^* + \lambda (1 - \alpha) a_m^\sigma) \]
\[ = \gamma Y^{(\gamma - \sigma)/\gamma Y^{\sigma/\gamma}} \]
\[ = \gamma Y \]

which directly shows the result for aggregate revenue.

C.1.11 Proof of Proposition 4.7

Given \( ext_f = 1 - 1/[(1 - \alpha)a_m + \alpha k_c] \), the substitution of (4.48) yields

\[ 1 - \frac{1}{(1 - \alpha) + \alpha 1/(1 - \sigma)} \]

which can be algebraically rearranged as

\[ \frac{\alpha(1 - \Upsilon(i)^{1/(1 - \sigma)})}{\alpha(1 - \Upsilon(i)^{1/(1 - \sigma)}) + \Upsilon(i)} \]

Differentiating with respect to \( i \) yields

\[ -\frac{\alpha \Upsilon'(i)}{1 - \sigma} \Upsilon^{\sigma/(1 - \sigma)} \left\{ \frac{[\alpha(1 - \Upsilon^{1/(1 - \sigma)}) + \Upsilon^{1/(1 - \sigma)}] + (1 - \Upsilon^{1/(1 - \sigma)})(1 - \alpha)}{[\alpha(1 - \Upsilon^{1/(1 - \sigma)}) + \Upsilon^{1/(1 - \sigma)}]^2} \right\} \]  

(C.13)
As \( i \to 0 \), \( \Upsilon \to 1 \), so the expression in braces converges to 1. Using the limit of \( \Upsilon'(i) \) as \( i \to 0 \), we obtain

\[
\frac{\partial \text{ext}_f}{\partial i} = \frac{\alpha}{(1 - \sigma)\lambda[1 - \alpha(1 - \theta)]}
\]

For the second part, note that the leverage ratio can be rewritten by dividing through by \( k_c - a_m^e \) as

\[
\frac{\alpha \lambda(1 + r)}{f(k^*, Y)/(k^* - a_m^e) - (1 + r)} = \frac{\alpha \lambda(1 + r)}{\gamma Y(\gamma - \sigma)/\gamma k^*\sigma^{-1}/[1 - \Upsilon(i)^{1/(1-\sigma)}] - (1 + r)} = \frac{\alpha \lambda(1 + r)}{\gamma Y(\gamma - \sigma)/\gamma k^*\sigma^{-1}/[1 - \Upsilon(i)^{1/(1-\sigma)}] - (1 + r)} = \frac{\alpha \lambda[1 + \theta(1/\sigma \Theta(i) - 1)]}{(1 - \Upsilon(i)^{1/(1-\sigma)})} - [1 + \Theta(i)(1/\sigma \Theta(i) - 1)]
\]

Rearranging yields (4.65).

### C.1.12 Proof of Proposition 4.8

Using (4.47), (4.50), and (4.13), we can express the real interest rate as (4.66). From Lemma (4.4) and (4.66), it immediately follows that as \( i \to 0 \), \( r \to 0 \), and, as \( i \to \infty \), \( r \to \theta[(1 - \sigma)/\sigma] \). The first-order approximation of \( r \) in the neighborhood of \( i = 0 \) follows immediately from Lemma 4.4.
C.1.13 Proof of Lemma 4.5

From Lemma 4.1, we have that the threshold $a^*$ given bargaining power $\chi$ is

$$(\chi^* - \chi) = \frac{(1-\theta)a^* + \theta f(a^*, Y)}{f(k^*, Y)}$$

Now, we shift the point of view and find the minimum threshold $\chi^{**}$ consistent with the first best. Using $a^* = k^* \Upsilon(i)^{1/(1-\sigma)}$, we have

$$(\chi^* - \chi^{**}) = (1-\theta)\frac{k^* \Upsilon(i)^{1/(1-\sigma)}}{f(k^*, Y)} + \theta \Upsilon(i)^{\sigma/(1-\sigma)}$$

Using $\sigma f(k^*, Y)/k^* = 1$, we can rewrite this as

$$\chi^{**} = \chi^* - [(1-\theta)\sigma \Upsilon(i)^{1/(1-\sigma)} + \theta \Upsilon(i)^{\sigma/(1-\sigma)}] \quad (C.14)$$

C.2 Extra Figures

C.2.1 First-order Approximation of Lending Rate

We numerically assess the validity of the first-order approximation relative to the global solution.
The first-order approximation always lies above the global solution, and the inaccuracy becomes large after $i = 0.05$. We consider 20% markups ($\sigma = 0.83$) and 40% markups ($\sigma = 0.71$). The approximation is better for lower $\sigma$: greater market power pushes up the return on investment and hence the real lending rate, causing a smaller departure from the first order approximation.

C.2.2 Response of Loan size to Increases in Nominal Interest Rate according to Returns to Scale

Figure C.1: Loan size for different values of $\gamma$
Increases in the policy rate cause entrepreneurs to substitute external finance for internal finance. However, investment in general declines, with a feedback that depends positively on the aggregate demand externality. Lower investment reduces money demand. For $\gamma = 0.8$, loan size increases monotonically in the nominal interest rate, whereas for $\gamma = 0.99$, the loan size eventually declines.

### C.2.3 Effect of Bargaining Power on Pass Through and Transmission with Heterogeneous Entrepreneurs

Figure C.2 shows how changes in the bank bargaining bargaining power influences the effects of the policy rate. A higher $\theta$ increases the pass through and the external share of finance, but reduces transmission into aggregate investment and output. The intuition is that higher bargaining power induces entrepreneurs to hold more cash, which reduces transmission.

![Figure C.2: Effect of Nominal Interest Rate: different values of $\theta$](image-url)
C.3 Computational Strategy Under Heterogeneous Firms with Limited Enforcement

1. First, we simplify aggregate demand in a way that facilitates computation. Noting that $\chi > \chi^{**}$ implies $a^e_m = k^* \Upsilon(i)^{1/(1-\sigma)}$, we can write

$$ Y = \lambda^{\gamma/\sigma} \left\{ [1 - G(\chi^{**})]k^{*\sigma}[\alpha + (1 - \alpha)\Upsilon(i)^{\sigma/(1-\sigma)}] + \int_0^{\chi^{**}} [ak_c(\chi)^\sigma + (1 - \alpha)a^e_m(\chi)^\sigma]dG(\chi) \right\}^{\gamma/\sigma} $$

(C.15)

$$ Y = \lambda^{\gamma/\sigma} \left\{ [1 - I_{\chi**}(a, b)]k^{*\sigma}[\alpha + (1 - \alpha)\Upsilon(i)^{\sigma/(1-\sigma)}] + \int_0^{\chi^{**}} [ak_c(\chi)^\sigma + (1 - \alpha)a^e_m(\chi)^\sigma] \frac{\chi^{a-1}(1 - \chi)^{b-1}}{B(a, b)} d\chi \right\}^{\gamma/\sigma} $$

(C.16)

2. We define a mapping $L : \mathbb{R}^+ \rightarrow \mathbb{R}$ as follows. First goal is to evaluate

$$ I_1 = \int_0^{\chi^{**}} [ak_c(\chi)^\sigma + (1 - \alpha)a^e_m(\chi)^\sigma] \frac{\chi^{a-1}(1 - \chi)^{b-1}}{B(a, b)} d\chi $$

3. We approximate $I_1$ using the composite midpoint method. Divide $[0, \chi^{**}]$ into $N$ evenly spaced subintervals, of length $h = \chi^{**}/N$.

4. Choose sequence of $\chi_i$ on midpoints of subintervals: $\chi_i = (i - 1/2)h, i = 1, \ldots, N$

5. Compute vector $a^e_m(\chi)^\sigma$ using (4.42) and then compute vector $k_c(\chi)^\sigma$

6. Compute

$$ Q_N = h \sum_{i=1}^{N} [a_m(\chi_i)^\sigma + k_c(\chi_i)^\sigma] \frac{\chi_i^{a-1}(1 - \chi_i)^{b-1}}{B(a, b)} $$
7. Compute quantity \( I_2 = [1 - I^{**}_{\chi(a,b)}] k^* \sigma[\alpha + (1 - \alpha) Y(i)^{\sigma/(1-\sigma)}] \) and then \( Y_{new} = \chi^{\gamma/\sigma}(Q_N + I_2)^{\gamma/\sigma} \)

8. Return \((Y_{new} - Y)/Y\)

9. Find zero of function \( L(Y) \) in endpoints \((0, Y)\).

10. Compute vector \( a^e_m(\chi) \) and \( k_c(\chi) \)

11. Calculate \( k^* \) and then aggregate investment again using the composite midpoint rule.

12. Also calculate the following via numerical integration:

\[
K_{cons} = \int_0^{\chi^{**}} \lambda[\alpha k_c(\chi) + (1 - \alpha)a^e_m(\chi)] \frac{\chi^{a-1}(1 - \chi)^{b-1}}{B(a, b)} d\chi
\]

\[
a^e_{m, cons} = \int_0^{\chi^{**}} a^e_m(\chi) \frac{\chi^{a-1}(1 - \chi)^{b-1}}{B(a, b)} d\chi
\]

\[
Lev_{cons} = \int_0^{\chi^{**}} \left[ \alpha \lambda (k_c - a^e_m + \phi)/(f(k_c, Y) - (k_c - a^e_m + \phi)) \right] \frac{\chi^{a-1}(1 - \chi)^{b-1}}{B(a, b)} d\chi
\]

\[
r_{cons} = \int_0^{\chi^{**}} \left( \frac{a^e_m(\chi) + \chi f(k_c(\chi), Y) - k_c(\chi)}{k_c(\chi) - a^e_m(\chi)} \right) \frac{\chi^{a-1}(1 - \chi)^{b-1}}{B(a, b)} d\chi
\]

13. We recover mean investment, the real interest rate, and leverage as follows:

\[
K = [1 - G(\chi^{**})] k^* \left[ \alpha + (1 - \alpha) Y(i)^{1/(1-\sigma)} \right] + K_{cons}
\]

\[
\mathbb{E}(a^e_m) = [1 - G(\chi^{**})] k^* \beta Y(i)^{1/(1-\sigma)} + a^e_{m, cons}
\]

\[
\mathbb{E}(r) = [1 - G(\chi^{**})] \frac{\theta \left( [f(k^*, Y) - k^*] - [f(a^e_m, Y) - a^e_m] \right)}{k^* - a_m} + r_{cons}
\]

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C.4 Limited Enforcement with Homogeneous Entrepreneurs

I detail the computational strategy in the following steps. If \( \chi < \chi^{**} \), then the first best level of investment cannot be obtained. We solve for the triple \((a^e_m, k_c, Y)\) using (4.42), (4.49), and (4.39). Then we can obtain \( r \) from (4.40). Otherwise, the first best is achievable, and we solve equations (4.46)-(4.49) as before.

Figure C.3: Pass Through of Nominal Interest Rate: different values of \( \sigma \)
Figure C.4: Pass Through of Nominal Interest Rate: different values of $\gamma$

Figure C.5: Pass Through of Nominal Interest Rate: different values of $\chi$
Figure C.6: Effect of Pledgeability Coefficient: different values of $\gamma$