Title
CLASSIFICATION OF THE ENERGY LEVELS OF ODD-MASS NUCLEI IN THE HEAVY-ELEMENT REGION

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F. S. Stephens, Frank Asaro, and I. Perlman

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ABSTRACT

Most of the data available on energy levels in odd-mass nuclei of the heaviest elements has been summarized and evaluated. The observed levels have been classified according to the level diagrams calculated by Nilsson for nuclei with prolate spheroidal deformations. Qualitatively, the agreement between the data and the Nilsson diagrams is very good.
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INTRODUCTION

In the system of nuclei, regions of pronounced spheroidal deformation have been clearly recognized. In these regions, the Bohr-Mottelson unified nuclear model\(^1\) enjoyed an early great success in describing the energy levels and associated spectra, particularly in delineating the effects of collective modes of internal nuclear motion. One major area of applicability is in the heavy-element region delimited approximately by \(A > 225\).

More recently Nilsson\(^2\) and others\(^3,4\) have characterized the energy levels of a particle moving in an axially-symmetric but nonspherical potential. In the limit of high nuclear deformation Nilsson has found it possible to define independent particle states in terms of a set of quantum numbers not applicable to spherical nuclei. This picture of these states and the expected order of filling have already proved highly useful in correlating experimental information pertaining to the appropriate odd-mass nuclei.

It is the purpose of this communication to summarize, in terms of this system of classification, energy-level assignments in the heavy-element region. A number of the assignments to be listed have already been discussed in other publications, and for these, detailed justification will be omitted.

The Nilsson representations for neutron and proton states in the region \(A > 225\) are shown in the diagrams of Figs. 1 and 2. The energies at which particular states lie appear on the ordinate axis, but this scale will only be used here as a means of visualizing the order of filling of levels. The abscissa scale indicates the deformation of a prolate spheroid in terms of a parameter, \(\beta\), which also will not be used further. At zero deformation we see the typical shell-model level structure for a spherically symmetric potential. The \((2j+1)\)-fold degeneracy is seen removed as permanent deformation sets in, and at high
deformation the levels (now two-fold degenerate) are described in terms of a new set of quantum numbers indicated by the indices in the margins.

The origin of these numbers will not be described here (see Ref. 2) other than to define formally the nomenclature. At high deformation the projection of the particle angular momentum, \( j \), along the nuclear symmetry axis becomes a good quantum number, and this number, \( \Omega \), is the observed spin in the absence of rotational motion. The parity is designated (+) or (−) and depends upon the even or odd character of the principle quantum number, \( N \), the total number of nodes in the wave function. The numerical value of \( N \) is the first integer in the brackets and corresponds to the oscillator shell of the shell model. The second number in the brackets is \( n_z \), the number of nodal planes perpendicular to the symmetry axis; and the third number is \( \Lambda \), the component of the orbital angular momentum along the symmetry axis. We will also refer from time to time to the quantum number, \( K \), which represents the projection of the nuclear spin, \( I \), on the symmetry axis. For low-lying states, \( K \) and \( \Omega \) should be equal, and in the following discussions we use the two more or less interchangeably.

It should be pointed out that for a particular state, these asymptotic quantum numbers are fully descriptive only in the limit of high deformation. The absence of "purity" of these quantum numbers is not of concern here because (a) we shall not be dealing with the consequences of their impurity as they apply to transition selection rules, and (b) we are confining our attention to cases of large deformation where these numbers do serve as the best means of labeling different states.

An example will serve to show how this system of nomenclature operates and its application to specific assignments. Consider the proton states designated \( 5/2^+ \) [642] and \( 5/2^- \) [523] which lie close to each other at a nuclear deformation described by \( \delta = 0.25 \). (Author references for the assignments of these orbitals to the particular nuclear states will appear later in this paper.) By counting from proton number 82, it is seen that these levels come in at approximately proton numbers 93 and 95 which characterize the elements neptunium and americium. It is found that the state \( 5/2^+ \) [642] is undoubtedly the ground state of \( \text{Np}^{237} \), and \( 5/2^- \) [523] is the ground state of \( \text{Am}^{241} \). The term [642] defines the state as follows: It comes from the sixth oscillator shell as can
be seen by its connection to the 13/2 level in the spherically symmetric potential. The other 5/2+ states for \( N = 6 \) all lie much higher and none is shown in Fig. 2. The highest has \( n_z = 0 \) descending to the maximum value \( n_z = 1 \) which applies to the state under discussion. The value of \( \Lambda \) can differ from \( \Lambda \) only by \( 1/2 \) and takes an even or odd value as \( N - n_z \) is even or odd. As a means simply of designating states, the \( \Lambda \) quantum number is redundant as used here because there can only be one 5/2+ state which has \( N = 6 \) and \( n_z = 4 \).

The ground state of \( \text{Am}^{241}, \frac{5}{2}^- \) \([523]\) is seen to connect with the \( \frac{9}{2}^- \) state of spherical nuclei which is in the fifth oscillator shell. The highest-lying \( 5/2^- \) state possible for \( N = 5 \) has \( n_z = 0 \) and connects with the \( f_{5/2}^- \) state, the next lowest has \( n_z = 1 \) \((1512)\), and the one under discussion here has \( n_z = 2 \).

Since we have \( N - n_z = 3 \) (odd), \( \Lambda \) must be 3, i.e. \( N + 1/2 \).

Recapitulating, the asymptotic quantum numbers appearing in brackets in Fig. 2 serve to identify the doubly-degenerate states shown in the diagrams. Because each state is only two-fold degenerate, it would be expected to appear as an unpaired orbital in a ground state only for a single nucleon number. However, the same state would appear as excited levels for neighboring nuclei. One might also expect from time to time that the same configuration would repeat for ground states as the result of an appreciable change in deformation.

RESULTS

Tables 1 and 2 summarize assignments of Nilsson levels for odd-neutron and odd-proton configurations, respectively. In these tables the Nilsson levels are listed across the top in order of increasing energy (appropriate to the deformations shown by the broken lines in Figs. 1 and 2) and the numbers appearing in the tables are the energies of these states (in kev) relative to their ground states. The pattern is quite evident and shows that a particular assignment for the ground state of one nucleus appears in excited states of neighboring nuclei.

The method of coding the reliability of the assignments is by means of the parentheses enclosing the energies of the states. Absence of parentheses indicates rather conclusive evidence; a single set implies a tentative assignment based on substantial evidence; and double parentheses are used to signify that some evidence is available but that much information is yet lacking. For those entries which are underlined, the corresponding spin has been determined.
Fig. 1. Nilsson diagram for neutrons in the region $126 < N < 160$. The abscissa is the nuclear deformation (prolate), and the ordinate indicates the energy of the various levels. The dashed line indicates very roughly the deformations thought to pertain for most of the nuclei discussed.
Fig. 2. Nilsson diagram for protons in the region $82 \leq Z \leq 126$. The abscissa is the nuclear deformation (prolate), and the ordinate indicates the energy of the various levels. The dashed line indicates very roughly the deformations thought to pertain for most of the nuclei discussed.
Table 1. Odd-neutron level assignments

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</table>
directly, but it should be pointed out that this in itself does not constitute proof that the assignment is correct.

In the following paragraphs the assignments made in Tables 1 and 2 are discussed briefly. A number of known nuclei in the region have not been sufficiently investigated to warrant inclusion; in particular, a large proportion of known excited states have not been tabulated for want of sufficient information of the type needed to make meaningful assignments. In a sense, the general reliability of the model can be gauged both by the orderly sequence of assignments and by the fact that in no case were accurate data available that indicated an assignment significantly at variance with the expectations of the theory.

In the discussions which follow, the odd-neutron cases are considered first and are grouped according to element. Following these are the odd-proton nuclei, again grouped by element. The tables summarize the data in terms of increasing neutron or proton numbers.

### Odd Neutron Nuclei

**Th^{229}** (neutron number 139). The ground state of Th^{229} is assigned 5/2+ [633], which comes in at about the expected place for neutron number 139. This assignment will be seen to be the same as that for the ground state of U^{233} and was arrived at on the basis that the "favored alpha decay" of U^{233} leads to the ground state of Th^{229}. 5,6,7 Bohr, Fröman and Mottelson 6 were the first to give theoretical grounds for assigning identical structures to states that are connected by an alpha emission process whose rate obeys simple one-body alpha-decay theory. The ground-state transitions of even-even alpha emitters (both states 0+) provide the empirical basis for defining "favored" or "unhindered" alpha emission.

The reader is referred to the section below on U^{233} for the assignment of 5/2+ [633] for its ground state. The fact that Th^{229} and U^{233} both have the same ground-state configurations implies that the odd-particle state filled for Th^{229} is vacated when the particle becomes paired. Further information on low-lying states around this neutron number should help explain this repetition in configuration.
$^{239}$Th has a number of low-lying excited states defined by the alpha-spectrum of $^{233}U$. These levels, up to about 350 kev, include members of the ground-state rotational band, but in addition there are a few which undoubtedly consist of other intrinsic configurations. By analogy with $^{233}U$ the configurations $5/2^+$, $3/2^+$, and $1/2^+$ might be represented, but there is not enough information available to permit making assignments.

The states associated with neutron number 139 are discussed further under the section on $^{231}$Th.

$^{231}$Th (neutron number 141). The assignments for the levels of $^{231}$Th have already been reported in brief form by Pilger et al. and are based on studies of the alpha decay of $^{235}U$ and the beta decay of $^{231}$Th. Briefly, the arguments are as follows:

The ground state of $^{235}U$ is almost surely $7/2^-$, and its favored alpha decay goes to a level in $^{231}$Th at 390 kev which is, accordingly, given this assignment. At about 190 kev there is a group of levels (at least 3) which probably represents a highly compressed rotational band. The $7/2^-$ state at 390 kev drops to two or more members of this band by E1 transitions; therefore this band has odd-parity and spin $5/2$ or $7/2$ for the base state. The only Nilsson level near the $7/2^-$ state satisfying these conditions is $5/2^-$, $[752]$. It is interesting to note that alpha decay to this band is only slightly hindered and this is probably related to the fact that $7/2^-$, $[743]$ (ground state of $^{235}U$) and $5/2^-$, $[752]$ are structurally similar. The compressed nature of the band at about 190 kev is thought to be due to the close proximity and strong interaction of these two odd-parity bands. The effect referred to here was first worked out for another nucleus by Kerman.

The states at about 190 kev drop to the ground state band by E1 transitions, therefore the ground state has even parity and the most logical choice among the Nilsson levels is $5/2^+$, $[633]$. This is the same as the ground state of $^{233}U$ which is not unexpected because both have the same neutron number. Supporting evidence for this ground-state assignment comes from the spacing of members of the ground-state band and from the beta decay properties of $^{231}Th \rightarrow Pa^{231}$. 11

Recapitulating, the ground state of $^{231}$Th is the expected state, $5/2^+$, $[633]$, the state at about 190 kev is $5/2^-$, $[752]$ produced by creating a hole in a filled level (see Fig. 1), and the state at 390 kev is $7/2^-$, $[743]$ which is expected to be near because it will appear as the ground state for neutron number 143.
Th\textsuperscript{233} (neutron number 143). Th\textsuperscript{233} is a short-lived beta emitter about which very little is known. Nothing is known about its excited states, and the only information available concerning its ground state is that derived from its beta-decay properties.

By analogy with U\textsuperscript{235} and Pu\textsuperscript{237}, one might guess that the ground state is either 7/2\(-\) [743] or 1/2\(+\) [631]. The data of Freedman and co-workers\textsuperscript{12} indicate that an appreciable proportion of the decay of Th\textsuperscript{233} goes to the ground-state rotational band of Pa\textsuperscript{233} which is believed to have K = 1/2 as will be discussed in a later section. This information would seem to favor the 1/2\(+\) [631] assignment for Th\textsuperscript{233} if the choice rests between the two states mentioned.

Possible difficulty exists, however, because there also seems to be direct population of the 5/2\(+\) band of Pa\textsuperscript{233}, although this is by no means certain. In view of the paucity of information, no assignment is entered in Table 1. Still, it is interesting to note the relative positions of the above-mentioned two states in other nuclei with 143 neutrons. In Pu\textsuperscript{237}, the state 1/2\(+\) [631] is 145 kev above the 7/2\(-\) [743] ground state; in U\textsuperscript{235} the two states lie within 100 ev of each other; and if the above-mentioned evidence is significant, the 1/2\(+\) state becomes the ground state in Th\textsuperscript{233}.

U\textsuperscript{231} (neutron number 139). At present very little is known about the energy levels of U\textsuperscript{231} because information comes only from its electron-capture decay properties. Something possibly may be learned about its excited states when its alpha-emitting parent, Pu\textsuperscript{235}, can be studied.

The ground state would be expected to be 5/2\(-\) [752] (185-kev state in Th\textsuperscript{231}), 5/2\(+\) [633] (ground state of Th\textsuperscript{231} and U\textsuperscript{233}), or 3/2\(+\) [631] (an excited state in U\textsuperscript{233}). The 5/2\(+\) assignment can probably be ruled out because Th\textsuperscript{231} and U\textsuperscript{231} both decay to Pa\textsuperscript{231}, and there are selective differences in the levels occupied.\textsuperscript{11} Of the two remaining assignments, 5/2\(-\) [752] is slightly preferred on the basis that no population to the ground state, K = 1/2 band of Pa\textsuperscript{231}, could be detected in the U\textsuperscript{231} electron-capture decay. Table 3 and its discussion (see below) go into the beta-decay selection processes as related to the Nilsson levels, and it will be seen that the log-ft values for decay of U\textsuperscript{231} to the K = 1/2 band in Pa\textsuperscript{231} probably would be such as to have permitted the observation of this transition if U\textsuperscript{231} were 3/2\(+\) [631].
\( \text{U}^{233} \) (neutron number 141). The energy levels of \( \text{U}^{233} \) have recently been reviewed by J. O. Newton.\(^{13}\) The assignments suggested by him have been adopted (Table 1) although different conclusions were reached regarding the related subject of the \( \text{Pa}^{233} \) assignment which is discussed below. As seen in Table 1, the ground state of \( \text{U}^{233} \) has been assigned by Newton to \( 5/2^+ [633] \) (see also \( \text{Th}^{231} \)); a state at 313 kev is assigned to \( 3/2^+ [631] \) (which implies excitation of a particle from a previously filled level); and a state at 400 kev is labeled \( 1/2^+ [631] \). This does not mean that other expected levels (Table 1) are not present at even lower energies because it could easily be that such levels are not populated by the beta decay of \( \text{Pa}^{233} \).

Some light is shed on this subject by the recently investigated alpha branching of \( \text{Pu}^{237} \).\(^{14,15}\) The alpha group of highest energy so far observed (5.65 Mev) probably leads to the ground state of \( \text{U}^{233} \) or at least to a level near the ground state because this energy corresponds well with the total available energy calculated from closed decay cycles.\(^{14}\) The transition is rather highly hindered in agreement with the assignment of \( 7/2^- [743] \) for \( \text{Pu}^{237} \) (see below) and of \( 5/2^+ [633] \) for the ground state of \( \text{U}^{233} \) as made by Newton. The principal alpha group (or groups) leads to a level (or levels) some 300 kev higher and has a hindrance factor of 7. If the state at \( \sim 300 \) kev were the \( 3/2^+ [631] \) state observed from \( \text{Pa}^{233} \) decay, the hindrance factor seems much too low because, among other factors, the alpha wave would be limited to \( f > 3 \). This suggests that there is another level at \( \sim 300 \) kev. If it were the \( 7/2^- \) state (same as the parent, \( \text{Pu}^{237} \)), the hindrance factor is too high, hence this state might be \( 5/2^- [752] \) which is expected in this region, and seen at 185 kev in the analogous nucleus, \( \text{Th}^{231} \) (see above).

\( \text{U}^{235} \) (neutron number 143). The puzzle surrounding the low-lying energy levels of \( \text{U}^{235} \) has recently been solved.\(^{16,17}\) The principle (unhindered) alpha group of \( \text{Pu}^{239} \), which has spin 1/2, apparently led to the ground state of \( \text{U}^{235} \), which has spin 7/2 in contradiction to our major selection rules, but it was found that this favored alpha group instead went to the expected spin-1/2 state, which is an isomer lying less than 0.1 kev above the ground state. The Nilsson level assignments as shown in Table 1 have been discussed by Asaro and Perlman.\(^{16}\) Higher-lying levels populated by \( \text{Pu}^{239} \) alpha decay have also been seen,\(^{18,19}\) but there are not yet sufficient data to make assignments.
$^{237}\text{U}$ (neutron number 145). The ground-state spin of $^{237}\text{U}$ has not been measured, but it is almost certainly 1/2 or 3/2 because in its beta decay to $^{237}\text{Np}$ no state with spin higher than 3/2 is populated. Rasmussen et al. have suggested possible assignments 1/2- [501] and 1/2+ [631]. Because there is no evidence for the 1/2- [501] state in other nuclei in this vicinity, and since the log ft values can be reasonably well explained on the basis of the 1/2+ [631] assignment, we definitely prefer this latter one. It will be seen that two other nuclei with the same neutron number, $^{239}\text{Pu}$ and $^{241}\text{Cm}$, also have this configuration in their ground states.

The alpha decay of $^{241}\text{Pu}$ has been investigated and the favored group goes to a level in $^{237}\text{U}$ at 145 kev. The ground state of $^{241}\text{Pu}$ has been assigned 5/2+ [622] as will be discussed below, hence the $^{237}\text{U}$ level at 145 kev is ascribed to the same orbital. The gamma-ray de-excitation properties of this level support the idea that it has the same parity as the ground state. The transitions to the ground-state rotational band appear to be largely M1 (presumably K-forbidden). It will be seen that the same situation is found in $^{239}\text{Pu}$ in which the 5/2+ state appears at a somewhat higher energy. As has been mentioned, this state 5/2+ [622] shows up as the ground state for neutron number 147.

$^{239}\text{Pu}$ (neutron number 147). There is little experimental information on this short-lived beta emitter, but it might be expected in analogy to $^{241}\text{Pu}$ that the ground state is 5/2+ [622]. This assignment is consistent with the beta-decay properties of $^{239}\text{Pu}$ 23,24 in which a 5/2- state in $^{239}\text{Np}$ of 74 kev receives most of the beta population.

$^{237}\text{Pu}$ (neutron number 143). Although the spin of $^{237}\text{Pu}$ has not been measured directly, the assignments of the ground state to 7/2- [743] and a state at 145 kev to 1/2+ [631] are considered to be on rather firm ground. These states occur very close together in $^{235}\text{U}$ and result in an isomeric transition with a half life of 26.5 min. On the basis that the structure would repeat in $^{237}\text{Pu}$, Stephens et al. searched for and found an E3 isomeric transition, with a 0.18-sec half-life and energy as indicated, resulting from the alpha decay of $^{241}\text{Cm}$. The ground-state assignment 7/2- [743] has also been made by Hoffman and Dropesky from a study of the electron-capture decay of $^{237}\text{Pu}$ to $^{237}\text{Np}$. 
$^{239}$Pu (neutron number 145). The levels of $^{239}$Pu are populated following the decay of $^{238}$U, $^{239}$Th, $^{239}$Am, $^{241}$Cm, and $^{243}$Cm, and the Coulomb excitation of $^{239}$Pu itself. The level scheme is thus quite well worked out, and orbital assignments have been given by Hollander$^{28,32}$ and by Asaro et al.$^{30}$

There is a well-defined rotational band based upon the $K = 1/2$ ground state ($1/2^+ [631]$) which orbital appears as the first excited state in $^{237}$U and $^{235}$U as already discussed. The $7/2^+ [743]$ configuration which is the ground state for $^{235}$U and $^{237}$Pu occurs at 392 kev and the $5/2^+ [622]$ configuration which will appear as the ground state for neutron number 147, occurs here at 286 kev. These assignments have been discussed in detail,$^{27,25}$ but additional comment seems to be in order concerning a level at 512 kev assigned by Hollander et al.$^{27}$ as either $5/2^+$ or $7/2^+$. As pointed out$^{28,32}$ the only two assignments that seem reasonable are $5/2^+ [633]$ and $7/2^+ [624]$. The state $5/2^+ [633]$ would result from opening a filled level that appeared as the ground state for neutron number 141, and the $7/2^+ [624]$ would be a new level to appear for higher neutron numbers as the ground state.

Of these, the choice of the $7/2^+$ assignment seems preferable to us. As will be seen, $5/2^+ [622]$ (which is the 286-kev level in $^{239}$Pu) and $7/2^+ [624]$ are found in $^{241}$Pu and $^{245}$Cm (see Table 1), and the spacings between these levels are similar, as might be expected because the energies of these orbitals have a rather flat dependence on nuclear deformation (see Fig. 1). A similar spacing also occurs in $^{239}$Pu between the state $5/2^+ [622]$ and the one of 512 kev, hence this may be taken as some evidence that this latter state is $7/2^+ [624]$.

Also the absence of radiation from the 512-kev state to the ground state is nicely explained by the $7/2^+ [624]$ assignment, although even for $5/2^+ [633]$ the E2 transition to the ground state might not compete favorably with M1 transitions to the $5/2^+ [622]$ band.

$^{241}$Pu (neutron number 147). The ground-state spin of $^{241}$Pu has been measured as $5/2^+$. The $5/2^+ [633]$ orbital was the ground state for neutron number 141, so the ground state of $^{241}$Pu is almost surely $5/2^+ [622]$ (see Fig. 1). The only information on excited states is that obtained from the alpha decay of $^{243}$Cm.$^{31,35}$ The favored alpha group leads to a state 172 kev above ground. Another state of 53 kev higher energy has been observed and interpreted as the first member of the rotational band based upon the 172-kev state.$^{35}$
This spacing suggests that the band has \( K = 7/2 \) or higher. The parity is fixed as even from the observation that the 172-kev state decays to the ground-state band by M1 transitions. In particular, the transition to the 5/2+ ground state is definitely M1. This fact not only fixes the parity of the 172-kev state, but also is consistent with the spin assignment of 7/2. The only Nilsson level in this region with these properties is \( 7/2+ [62h] \), and the assignment is considered to be reasonably certain.

\[ \text{Pu}^{243} \] (neutron number 149). Information is available only on the ground state of \( \text{Pu}^{243} \) and is derived from the decay of this isotope to \( \text{Am}^{243} \), \( 36,37,38 \). States in \( \text{Am}^{243} \) having spins 5/2, 7/2, and probably 9/2 seem to receive direct beta population from \( \text{Pu}^{243} \) (see discussion of \( \text{Am}^{243} \) ) so that a spin of 7/2 for \( \text{Pu}^{243} \) seems most reasonable. Because this coincides with the expected Nilsson level, 7/2+ [62h], this assignment is given to \( \text{Pu}^{243} \).

\[ \text{Cm}^{241} \] (neutron number 147). The ground state of \( \text{Cm}^{241} \) is almost surely the same as that of \( \text{Pu}^{239} \) (also with 145 neutrons), 1/2+ [63l]. This assignment was made by Stephens et al.\textsuperscript{25} and the arguments will be reviewed briefly here. \( \text{Cm}^{241} \) decays by electron capture to \( \text{Am}^{241} \), \( 39,40 \) and although there are low-lying 5/2+ and 5/2− states, neither of these is directly populated. Instead decay takes place to states that seem to have spins of 3/2 or 1/2.

As already mentioned when \( \text{Pu}^{237} \) was discussed, the favored alpha decay of \( \text{Cm}^{241} \) populates a 0.18-sec isomeric state of \( \text{Pu}^{237} \) which drops to the ground state by an E3 transition.\textsuperscript{25} The evidence is excellent that the isomeric state has the assignment 1/2+ [63l], hence the ground state of \( \text{Cm}^{241} \) is almost surely the same.

There is no information at present on the excited states of \( \text{Cm}^{241} \), although applicable data might be obtained from studies of the alpha decay of \( \text{Cf}^{245} \).

\[ \text{Cm}^{243} \] (neutron number 147). The ground state of \( \text{Cm}^{243} \) might be expected to be 5/2+ [622], the same as \( \text{Pu}^{243} \). There is some experimental evidence for this assignment from the study of the alpha spectrum of \( \text{Cm}^{243} \), \( 29,30 \). The favored alpha group populates a level in \( \text{Pu}^{239} \) at 266 kev and the assignment of 5/2+ [622] has been made for this state.\textsuperscript{29,30}

Excited states of \( \text{Cm}^{243} \) are known from the electron-capture decay of \( \text{Bk}^{243} \), but no information is available that is suitable for making assignments.

\[ \text{Cm}^{245} \] (neutron number 149). As seen in Fig. 1, the expectations for neutron number 149 are either 7/2+ [62h] and 9/2− [73h]. In a brief earlier report resulting from the study of the alpha decay of \( \text{Cf}^{249} \), Stephens et al.\textsuperscript{41}
made the 7/2+ [624] assignment to the ground state of Cm$^{245}$, while the 9/2- [734] orbital appeared at a level 394 keV above ground. At 255 keV above ground, the orbital 5/2+ [622] reappeared through the opening of a filled level. (See Cm$^{243}$ and Pu$^{241}$ where this orbital represents the ground state). Because no discussion was presented in that report, a brief account of these assignments and one other will be given here.

The states of Cm$^{245}$ have been studied through the electron-capture decay of Bk$^{245}$ and the beta decay of Am$^{245}$ as well as from the alpha decay of Cf$^{249}$. In addition, the alpha decay of Cm$^{245}$ has been studied, and as has been already discussed under Pu$^{241}$, the 7/2+ [624] assignment is likely for the Cm$^{245}$ ground state.

Am$^{245}$ presumably has spin and parity 5/2- or 5/2+, and in its decay to Cm$^{245}$ populates both the ground state and a level at 255 keV. Bk$^{245}$, which we believe to have spin 3/2, does not populate the ground state of Cm$^{245}$, but only the 255-keV level. This 255-keV level decays only to the ground state of Cm$^{245}$ (not to higher members of the ground-state rotational band) by an M1 transition. Furthermore, four members of the rotational band based on the 255-keV level have been observed in the alpha decay of Cf$^{249}$, and the spacing and alpha population of these states suggest a spin 5/2 for the 255-keV level. All these data rather strongly suggest a spin and parity of 5/2+ for the 255-keV level, and its assignment as the 5/2+ [622] Nilsson level seems almost certain.

The favored alpha decay of Cf$^{249}$ populates a rotational band whose base level is 394 keV above the ground state of Cm$^{245}$. This level decays by transitions that appear to be E1 to the 7/2 and 9/2 members of the ground-state rotational band, with no detectable branching to the 5/2+ [622] band at 255 keV. No branching to this (394-keV) level was observed in the decay of either Am$^{245}$ or Bk$^{245}$ (spins 5/2 and 3/2). From these data we conclude that the spin and parity of the 394-keV level is 7/2- or 9/2-, with 9/2- somewhat more likely (see Fig. 1). An alpha-gamma angular-distribution measurement was made to distinguish between these two choices, and the results, while not absolutely definitive, also favored the 9/2- spin. We thus conclude that the 394-keV level (and hence the ground state of Cf$^{249}$) is very likely the 9/2- [734] Nilsson level. The only other level observed in Cm$^{245}$ is one at about 630 keV populated in the decay of Bk$^{245}$. This level decays by a single gamma ray to the 5/2+ [624] state and
therefore presumably has low spin. We have very tentatively assigned it as the 
$1/2^+$ [631] state which comes in as the ground state for neutron number 145.

$\text{Cf}^{245}$ (neutron number 147). Chetham-Strode and co-workers\textsuperscript{46} have
obtained evidence that the alpha-decay of $\text{Cf}^{245}$ leads predominantly to the
ground state of $\text{Cm}^{241}$, and we can say that the transition is unhindered or
only slightly hindered. In the absence of other information, it might be
inferred that the ground state of $\text{Cf}^{245}$ is the same as $\text{Cm}^{241}$, $1/2^+$ [631]. This
assignment is possible but not expected. Because there is no evidence bearing
on this assignment other than the observation cited, we have not made an entry
in Table 1.

$\text{Cf}^{249}$ (neutron number 151). It has already been suggested under the
$\text{Cm}^{245}$ discussion that $\text{Cf}^{249}$ ground state has the assignment $9/2^-$ [734]. Consistent
with this assignment is the value log $\text{ft} = 7.0$ for the beta decay of $\text{Bk}^{249}$ \textsuperscript{47}
since $\text{Bk}^{249}$ is thought to have spin and parity $7/2^+$ (see Table 3).

**ODD-PROTON NUCLEI**

It has been seen (Table 1) that the same level repeats for ground states of nuclei having the same (odd) neutron number. This is a reflection of the
adequacy of the model employed. Similar behavior for unpaired protons allows
grouping of isotopes of each odd element for discussion. The applicable Nilsson
diagram is shown in Fig. 2, and the summary of assignments in Table 2.

**Actinium** (proton number 89). It should perhaps be pointed out at the
start that the lack of apparent rotational structure in the actinium (and, for
that matter, the protactinium) isotopes has for some time been noted, and
initially we felt that these isotopes were outside the region of stable spheroidal
deformation and could not be described by the strong-coupling approximation of
the unified nuclear model. We have recently concluded that this absence of
simple rotational bands is probably due rather to the many anomalous $K = 1/2$
rotational bands in this region, and the influence of these bands through
rotational-particle coupling on the $K = 3/2$ and even in some cases, the $K = 5/2$
bands in this vicinity. The assignments made for the actinium (and protactinium)
isotopes are based on this conclusion, and therefore are perhaps somewhat less
certain than those made in regions where the unified nuclear model is certainly
applicable.
The energy levels of Ac\textsubscript{227} have been studied following the beta decay of Ra\textsubscript{227},\textsuperscript{48,49} and also rather thoroughly following the alpha decay of Pa\textsubscript{231}.\textsuperscript{50-56} The levels of Ac\textsubscript{225} have thus far been studied only as populated by the beta decay of Ra\textsubscript{225},\textsuperscript{57,58} although some information from the alpha-decay of Pa\textsubscript{229} is being obtained.\textsuperscript{59} For this reason considerably more is known about the energy levels in Ac\textsubscript{227}, and this isotope will be discussed first.

The favored alpha decay of Pa\textsubscript{231} seems to populate a level in Ac\textsubscript{227} at 330 kev. Because the ground state of Pa\textsubscript{231} is believed to be the 3/2 member of the K = 1/2 rotational band based on the state, 1/2- [530], this assignment is also given to the 330 kev level in Ac\textsubscript{227}. It should be emphasized that the state at 330 kev has spin 3/2 according to this assignment but is properly designated by the K quantum number of the orbital which is 1/2. An unambiguous designation would be 3/2, 1/2- [530], showing that it is the I = 3/2 member of the K = 1/2 band. The I = 1/2 and I = 7/2 members of this band are also presumably seen in Ac\textsubscript{227} at energies of 356 and 386 kev respectively. The alpha populations from Pa\textsubscript{231} are in good agreement with those expected for favored decay to this band.

The ground state of Ac\textsubscript{227} has a measured spin of 3/2 and is probably connected with the 330-kev level by what appears to be an M2 transition. The ground state must therefore have even parity, and because the two states both have spin 3/2 there is implied a strong retardation of the permitted E1 transition. The most likely 3/2+ assignment is 3/2+ [651] (see Fig. 2).

Between about 25 and 125 kev above the ground state of Ac\textsubscript{227} there are at least six levels observed, most of which can be shown to have odd parity if the two previous assignments are correct. There is no apparent rotational structure among these levels, but, as will be pointed out, this does not necessarily mean that these states all have different intrinsic configurations. Under conditions which could apply here, there can be severe distortions in level spacings in a rotational band.

Returning to the assignments of these levels, it will be noted from Fig. 2 that aside from 1/2- [530] the only odd-parity states in the vicinity of 3/2+ [651] are those connected with the h\textsubscript{9/2} orbital: 1/2- [541], 3/2- [532], and 5/2- [523]. The positions of these levels for proton number 89 are not known, but it will be seen that the state 5/2- [523] comes in as shown in Fig. 2 as the ground state for proton number 95. In order for it to lie below 1/2- [530] (at 330 kev in Ac\textsubscript{227}) the nuclear deformation would have to be considerably less
for this state in Ac$^{227}$ than that indicated by the dashed line in Fig. 2. In this regard, the spin $5/2$ for Pa$^{229}$ suggested by Hill$^{59}$ (see section on protactinium) would make this assumption not unreasonable. From the foregoing arguments we might expect the $3/2$- [532] and $5/2$- [523] levels to lie closest to the $3/2^+$ [651] ground state of Ac$^{227}$ and suggest tentatively that a group of levels around 27 kev be assigned the orbital $3/2$- [532] and those around 110 kev, $5/2$- [532]. The apparent absence of rotational structure is attributed to the displacement of levels by the mechanism discussed by Kerman.$^{10}$ However, these assignments do not seem definite enough for inclusion in Table 2.

Very little is known about Ac$^{225}$ but there is one piece of information which suggests similarity to Ac$^{227}$. In Ac$^{227}$ there is a 27-kev El transition between an excited state of 27 kev and ground. In Ac$^{225}$ a 40-kev El transition is found and is the only prominent gamma transition following Pa$^{225}$ decay. This implies that there is a level in Ac$^{225}$ at 40 kev bearing the same relation to the ground state as the 27-kev level in Ac$^{227}$.

**Protactinium (proton number 91).** A considerable number of protactinium isotopes are known but substantial data are available for only Pa$^{231}$ and Pa$^{233}$. The energy levels of Pa$^{231}$ have been studied from the beta decay of Th$^{231}$, $^{11,60-62}$ U$^{231}$ electron capture, $^{11,63}$ alpha decay of Np$^{235}$, $^{64}$ and coulomb excitation of Pa$^{231}$ itself. $^{65}$ The states of Pa$^{233}$ have been studied in conjunction with Th$^{233}$ beta decay$^{12}$ and Np$^{237}$ alpha decay. $^{66,67}$

The ground-state spin of Pa$^{231}$ has been measured as $3/2$ and that for Pa$^{233}$ is deduced to be the same from the decay properties. Each is probably the $I = 3/2$ member of the $K = 1/2$ band, $1/2$- [530], which has already been mentioned in the discussion of Ac$^{227}$ where this state appears at 330 kev above ground. In Pa$^{233}$, the $I = 1/2$, $3/2$, $5/2$, and $7/2$ members have been observed, and these lie at $\sim 6$, $0$, $69$ and $56$ kev respectively. The structure and spacings in this anomalous rotational band are probably much the same in Pa$^{231}$.

In both Pa$^{231}$ and Pa$^{233}$ there is another intrinsic state at about 85 kev. Each drops to the $3/2$ and $7/2$ members of the ground state band by El transitions and has been assigned $5/2^+$ [642] partly on the basis of these gamma transitions and also because this level in Pa$^{233}$ receives the favored alpha transition from Np$^{237}$ decay and $5/2^+$ [642] is almost surely the ground state of Np$^{237}$. 
A somewhat uncertain assignment of \( \frac{3}{2}^+ \) \([651]\) has been made for levels in \( \text{Pa}^{231} \) and \( \text{Pa}^{233} \) at 166 kev and \(~200\) kev respectively. There is no obvious rotational-band structure based on either this state or the \( \frac{5}{2}^+ \) \([642]\) state. However, this would be expected, since such similar states lying so close to each other might be expected to interact in such a way as to distort the normal rotational-level spacings. Calculations are underway at present to see if the experimental level pattern can be reproduced, and the preliminary results seem favorable.

There is also some information available for assigning the ground states of \( \text{Pa}^{235} \) and \( \text{Pa}^{229} \). \( \text{Pa}^{235} \) might be expected to have the \( \frac{1}{2}^- \) \([530]\) band as its ground state in analogy to \( \text{Pa}^{231} \) and \( \text{Pa}^{233} \). This would be consistent with the observed decay of \( \text{Pa}^{235} \) without gamma-ray emission to \( \text{U}^{235} \), \(^{68}\) since a \( \frac{1}{2}^+ \) state lies within 0.1 kev of the ground state of \( \text{U}^{235} \). Also the log ft value for this decay is very similar to that observed for decay between these two states in other nuclei (see Table 3). Nevertheless, because the ground state of \( \text{U}^{235} \) has a spin of \( \frac{7}{2}^- \), almost any spin up to \( \frac{9}{2} \) would be possible for \( \text{Pa}^{235} \). An enlightening experiment would be a determination of whether the 26-min isomeric state (spin \( \frac{1}{2} \)) of \( \text{U}^{235} \) is populated in the decay of \( \text{Pa}^{235} \), but this has not yet been done.

Hill\(^{59}\) has suggested that the ground state of \( \text{Pa}^{229} \) is \( \frac{5}{2} \) on the grounds that states having spins of \( \frac{7}{2} \) and probably \( \frac{5}{2} \) are populated in the decay of this isotope to \( \text{Th}^{229} \), and also on preliminary evidence that a \( \frac{5}{2} \) rotational band in \( \text{Ac}^{225} \) receives the favored alpha decay of \( \text{Pa}^{229} \). If this is the case, the only two reasonable assignments would be \( \frac{5}{2}^+ \) \([642]\) or \( \frac{5}{2}^- \) \([523]\). It is not possible to make a clear choice between these assignments. We slightly prefer the latter, however, because it seems less likely (see Fig. 2) that the \( \frac{1}{2}^- \) \([530]\) state (ground states of \( \text{Pa}^{231} \) and \( \text{Pa}^{233} \)) would cross the \( \frac{5}{2}^+ \) \([642]\) state than the \( \frac{5}{2}^- \) \([523]\) state.

Neptunium (proton number 93). At several places in the previous discussions, special use has been made of observed El transitions in identifying states (see, for example, protactinium) because such transitions limit considerably the possible choices. For low energies, \(<100\) kev), El transitions are particularly easy to identify because the conversion coefficients are small and unique. In fact, very often a rough intensity measurement of the photon is sufficient for identifying the transition unambiguously as El. Three isotopes of neptunium,
Np$^{235}$, Np$^{237}$, and Np$^{239}$, all have prominent El transitions following alpha decay of their respective parents, and the assignments of the states involved for two of these (Np$^{237}$ and Np$^{239}$) have been previously made. These are $5/2^+$ [$642$] for the ground states and $5/2^-$ [$523$] for low-lying excited states. The arguments for these assignments are considered sound and will not be discussed further here. The energies may be seen in Table 2. The supporting evidence in Np$^{235}$ is not so extensive but the analogy in energy and intensity of the El transition is so close that these same assignments may be made with confidence. It may be mentioned that the electron-capture decay characteristics of Np$^{235}$ are consistent with the ground-state assignment.

Rotational states based upon these two intrinsic states have been identified in Np$^{237}$ and Np$^{239}$, but the only other well-studied intrinsic state in an odd-mass neptunium isotope is the 268-kev level of Np$^{237}$, which very likely has spin and parity $3/2^-$, and was assigned as the $3/2^-$ [$521$] state by Rasmussen et al. Although this assignment is certainly possible, we prefer that of $1/2^-$ [$530$] with the $3/2^-$ member of this band lying lowest, the same assignment made for the ground states of protactinium isotopes. This assignment is preferred for the following reasons: (1) an energy of 268 kev is already somewhat higher than might be expected for the $1/2^-$ [$530$] band on the basis of its position relative to the $5/2^+$ [$642$] state in the protactinium isotopes, yet it is quite unlikely that the $1/2^-$ [$530$] band could lie at lower energies in Np$^{237}$ and not have been detected from the beta decay of U$^{237}$. On the other hand, from the assignments made below for the americium and berkelium isotopes, it would be expected that the $3/2^-$ [$521$] level would lie at energies somewhat higher than 268 kev in Np$^{237}$. (2) De-excitation of the 267-kev level has been shown to have M2 decay in competition with El, and E2 with M1, which can be best explained by the $1/2^-$ [$530$] assignment, since this would involve K-forbidden restrictions for the dipole transitions. Neither of these arguments is very conclusive, however, so the preference for $1/2^-$ [$530$] over $3/2^-$ [$521$] is slight.

**Americium (proton number 95).** The data on the americium isotopes come from the alpha decay of the berkelium isotopes$^{39,40}$ the electron capture decay of Cm$^{241}$, the beta decay of Pu$^{243}$, and the alpha decay of the americium isotopes, themselves.$^{51,73-78}$ Americium-241 and Am$^{243}$ have measured spins of $5/2^9,80$ and from studies of their alpha decay to Np$^{237}$ and Np$^{239}$, it is reasonably certain that their ground state is the level, $5/2^-$ [523].
The ground state of Am$^{239}$ is assigned as this same Nilsson state because its pattern of alpha decay seems to be quite similar to that of Am$^{241}$ and Am$^{243}$.

We will next turn to two excited states in Am$^{243}$, at 84 and 465 kev, observed following the beta decay of Pu$^{243}$. The 84-kev level decays by a prominent E1 transition to the ground state of Am$^{243}$ and by a very weak E1 transition to an ~40 kev level - presumably the first member of the ground-state rotational band. This fixes the spin and parity of the 84-kev level at 5/2+ or 7/2+. Assignment is made to $K = 5/2$, in particular 5/2+ [642], because the energy spacing with respect to the ground state, 5/2- [523], is similar to that seen in neptunium isotopes, except that the states are reversed. The 465-kev level decays by a predominantly E1 transition to the 84-kev level and to one, and possibly two, higher members of the rotational band based on the 84-kev level. Thus the parity of the 465-kev band is even, and the spin is probably 7/2 or 9/2, although 5/2 is also a possibility. The assignment 7/2+ [633] is consistent with these data and with the proposed 7/2+ spin of Pu$^{243}$. There is no other Nilsson level in the vicinity that seems to be satisfactory for this state.

The alpha decay properties of Bk$^{243}$, Bk$^{245}$, and Bk$^{247}$ have all been studied, and provide an interesting comparison of the levels in Am$^{239}$, Am$^{241}$, and Am$^{243}$. In each case the ground-state transition is highly hindered, with hindrance factors of 660, 450, and ~500 respectively. The lowest intrinsic excited states observed in each of the americium isotopes also have rather similar alpha-hindrance factors. These states lie at energies of 187, 206, and 84 kev (in order of increasing mass number) and their alpha-hindrance factors are 54, 37, and 68 respectively. This suggests the same Nilsson level is involved for each isotope, and because the 84-kev level in Am$^{243}$ has been assigned as the 5/2+ [642] state, we suggest this assignment for the other two levels as well. There is additional evidence in favor of these assignments. In both Am$^{239}$ and Am$^{241}$ the levels (187 and 206 kev) drop to the I = 5/2 and I = 7/2 members of the ground-state (5/2- [523]) rotational band by transitions which are probably E1 (although E2 cannot be ruled out). These are just the two levels to which we would expect decay from the 5/2+ [642] state. This argument, of course, hinges on whether or not the radiations observed are indeed E1. Furthermore, Cm$^{241}$ (spin 1/2+) has no observed population to the 206-kev level in Am$^{241}$, which is to be expected if the 5/2+ assignment of this level is correct. The differences in spacing of the 5/2+ [642] levels relative to the ground states in the three americium isotopes are rather large and will be discussed later.
A third state which seems to be populated systematically in the alpha decay of the berkelium isotopes lies at energies of 540, 480, and 265 keV in Am$^{239}$, Am$^{241}$, and Am$^{243}$, respectively. The hindrance factors to these states are 3.6, 2.3, and 4.2, respectively. In Am$^{239}$ the 540-keV state decays to the ground state by a transition whose multipolarity is uncertain. In Am$^{241}$, however, the 480-keV level is also heavily populated in the electron-capture decay of Cm$^{241}$, and decays to the ground state of Am$^{241}$ by a predominantly M1 transition (with apparently some E2 admixture). These data, together with the 1/2+ spin of Cm$^{241}$ and the 5/2- ground-state spin of Am$^{241}$, fix the spin and parity of the 480-keV level at 3/2-. In Am$^{243}$ the 265-keV level also decays to the ground state by a transition that has been shown to be predominantly M1. Thus the 540-, 480-, and 265-keV levels in the three americium isotopes seem to be quite similar, and probably all have spin and parity 3/2-. 

On this basis the Nilsson assignment for these levels is almost certainly either 1/2- [530] (3/2- member lying lowest) or 3/2- [521]. We slightly prefer the former assignment because the level seems to vary in energy relative to the 5/2- [523] state in about the same manner as the 5/2- [642] state. From the slopes of the levels on the Nilsson diagram this is more reasonable for the 1/2- [530] than the 3/2- [521] assignment.

The small hindrance factors for the alpha decay of berkelium isotopes to these states are worthy of note. It will be seen that the ground states of the berkelium isotopes are assigned 3/2- [521] and the states in americium I = 3/2 member of 1/2- [530]. The small hindrance factors are probably associated with the structural similarity of these orbitals. Also the 3/2- [521] level in the americium isotopes is probably not too far away in energy from this level, so that mixing of the type described by Kerman would further lower the hindrance factor.

The large variation in spacing between the 5/2+ [642] and 1/2- [530] states in the three americium isotopes relative to the 5/2- [523] ground state is somewhat puzzling. This is possibly due to a larger nuclear deformation in Am$^{239}$ and Am$^{241}$ than in Am$^{243}$ (or probably Am$^{245}$). The reason for larger nuclear deformations in Am$^{239}$ and Am$^{241}$ might be found in the neutron levels filling in this vicinity, which are the steeply down-sloping (deforming) 7/2- [743] and 1/2+ [631] orbitals. On the other hand, around Am$^{243}$ and Am$^{245}$ the much flatter (less deforming) neutron levels, 5/2+ [622] and 7/2+ [624], are filling.
The only other level observed in an americium isotope is the 630-kev level populated in Am$^{241}$ by Cm$^{241}$ electron-capture decay. This state decays both to the ground and the 480-kev states, and we have very tentatively given it the assignment, $3/2^-$ [521]. The assignment $3/2^+$ [651] is also possible but seems less likely because this state would be expected to decay to the $5/2^+$ [642] state, and no such gamma transition could be observed.

**Berkelium (proton number 97).** The similarity of the alpha decay schemes of Bk$^{243}$, Bk$^{245}$, and Bk$^{247}$ has already been described, and thus we believe these three isotopes all have the same Nilsson level for their ground states. Information as to the identity of this level may be derived from the electron-capture decay of Bk$^{245}$ to Cm$^{245}$. This decay goes predominantly to a level assigned spin and parity $5/2^+$ in Cm$^{245}$, with no detectable branching to the $7/2^+$ (ground) or $9/2^-$ (394 kev) states. This suggests a spin of $3/2$ for Bk$^{245}$, although $5/2$ or $7/2$ would also be possible. From the Nilsson diagrams, either the $3/2^-$ [521] or the $7/2^+$ [633] level would be expected as the ground state of the berkelium isotopes, and on the basis of the above data, we definitely prefer the $3/2^-$ state.

Considerably more information is available about the energy levels of Bk$^{249}$, mostly from the alpha decay of E$^{253}$ [61,82]. The favored alpha decay of E$^{253}$ goes to the ground state of Bk$^{249}$, and four members of the ground-state rotational band have been observed to receive alpha population. This means that the ground-state configurations of Bk$^{249}$ and E$^{253}$ are identical. The spacing of this band gives the best agreement with a $7/2$ spin, but the rotational constant $(\hbar^2/2\alpha)$ is considerably smaller than usual for this region. However, for an even-parity odd-proton state in this region an abnormally small rotational constant is reasonable according to arguments given by Hollander,Smith, and Rasmussen in explaining the similarly small value for the $5/2^+$ [642] band in Np$^{237}$. [70] We have accordingly assigned the ground-state of Bk$^{249}$ to the $7/2^+$ [633] orbital. The beta decay of Bk$^{249}$ to Cf$^{249}$ is consistent with this assignment.

Alpha population is also observed to three members of a rotational band whose base level is 393 kev above the ground state of Bk$^{249}$. These levels decay by predominantly M1 transitions to the ground-state rotational band of Bk$^{249}$. The pattern of gamma rays de-exciting this band, and the pattern of alpha population to the band members suggests that this is the $5/2^+$ [642] band; however, the spacing of the levels is not in very good agreement with this conclusion, nor, in
fact, with any other reasonable spin. Nevertheless we have tentatively given this assignment to the 393-kev level, because the unusual spacing of the band can possibly be explained in terms of rotational-particle coupling to the 3/2+ [651] band. Another group of levels has been observed in the alpha decay of $^{253}$Er, and these seem to comprise a rotational band whose lowest observed level is at 84 kev above the ground state of $^{249}$Bk. The spacing of this band indicates that the 84-kev level has a spin of 7/2, and the rotational constant appears normal. Asaro et al. \( ^{31} \) have been able to show that the 84-kev level is followed by two prompt (<2 \( \mu \)sec) transitions, which are highly converted in the L shell. It is possible that the 84-kev level is the 7/2- [514] Nilsson state, which decays by an M2 transition to the 9/2+ member of the ground-state rotational band, and then on to the ground state. However, we feel that a somewhat more likely situation is that the rotational band is really based on the 3/2- [521] Nilsson state, but that the alpha decay to the 3/2 and 5/2 members of this band is obscured (as it certainly would be) by the intense alpha groups to the first two members of the ground-state rotational band. The two prompt transitions following the 84-kev level would then be those de-exciting this level to the 3/2- state, which subsequently decays by an unobserved (~10 kev) M2 transition to the ground state of $^{249}$Bk. A preliminary search has failed to reveal an isomeric state of $^{249}$Bk (the 10-kev level); however, we feel that the possibilities are not yet exhausted. Of course, one would like to find a low-lying 3/2 state in $^{249}$Bk because in the other berkelium isotopes the 3/2- [521] orbital represents the ground state.

\textbf{Einsteinium (proton number 99)}. The only einsteinium isotope for which sufficient data are available to make an assignment is $^{253}$Er. This isotope very likely has a ground-state assignment of 7/2+ [633] because favored alpha decay was observed to a state in $^{249}$Bk that was given this assignment.

\textbf{CLASSIFICATION OF BETA TRANSITIONS}

The beta transitions upon which the Nilsson assignments in Tables 1 and 2 are partially based are summarized in Table 3. Here the log-ft value and the parent nucleus are listed at the intersection of the column and row corresponding to the two levels that the beta transition connects. The log-ft values are only approximate, and in a few cases the value given very likely represents that
Table 3. Beta decay log-ft values

<table>
<thead>
<tr>
<th>Neutron state</th>
<th>Proton state</th>
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<tbody>
<tr>
<td>3/2+ [631]</td>
<td></td>
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<tr>
<td>5/2+ [652]</td>
<td></td>
</tr>
<tr>
<td>7/2+ [631]</td>
<td></td>
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<tr>
<td>5/2- [652]</td>
<td></td>
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<tr>
<td>7/2- [743]</td>
<td></td>
</tr>
<tr>
<td>1/2+ [631]</td>
<td></td>
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<tr>
<td>5/2+ [622]</td>
<td></td>
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<tr>
<td>7/2+ [624]</td>
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<tr>
<td>9/2- [734]</td>
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</tbody>
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<table>
<thead>
<tr>
<th>Neutron state</th>
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<tbody>
<tr>
<td>3/2+ [631]</td>
<td>1f, u (Pa$^{233}$ 7.1)</td>
</tr>
<tr>
<td>5/2- [652]</td>
<td>2f $^{231}$ &gt;7.3 $^{231}$ &gt;5.9</td>
</tr>
<tr>
<td>7/2- [743]</td>
<td>1f, u (Pu$^{237}$ &gt;6.8)</td>
</tr>
<tr>
<td>1/2+ [631]</td>
<td>1f, u Am$^{239}$ &gt;6</td>
</tr>
<tr>
<td>5/2+ [622]</td>
<td>2f $^{239}$ &gt;9.1 Am$^{239}$ &gt;7 (Bk$^{245}$ 7.3)</td>
</tr>
<tr>
<td>7/2+ [624]</td>
<td>1f, u $^{243}$ 6.1 (Pu$^{243}$ 5.5)</td>
</tr>
<tr>
<td>9/2- [734]</td>
<td>1f, u Bk$^{249}$ &gt;7.0</td>
</tr>
</tbody>
</table>
for decay to more than one member of the rotational band, rather than just the base level. These errors are probably not larger than a factor of 2 or 3 in the $ft$ value, however. The classification of each beta transition is given at the top of each group. These classifications are allowed (a) or first or second forbidden (1f or 2f), according to the Nordheim selection rules, and hindered (h) or unhindered (u) in the asymptotic quantum numbers of deformed nuclei according to the rules given by Alaga.\textsuperscript{83} The parentheses around the transitions indicate uncertainty in orbital assignment as in Tables 1 and 2, and in this case the notation on the beta transition is that of the least certain of the levels which it connects. The range of log-$ft$ values for the various transition types is similar to that found by Alaga\textsuperscript{83} for the spheroidal region $150 < A < 190$.

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