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Intertemporal Price Speculation and the Optimal Current-Account Deficit: Reply and Clarification

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Abstract

In the model of Obstfeld (1983), a country hurt by a temporary shift in its terms of trade, whether the shift is infinitesimal or not, always runs a temporary current-account deficit. Temporary rises in relative export prices always cause surpluses in the model. This note derives these results within an analysis that clarifies how temporary terms-of-trade shocks affect the consumption-based real interest rate on external debt and, hence, the current account.

I gratefully acknowledge support from the National Science Foundation and from a Ford Foundation grant to CIDER at the University of California, Berkeley.
My 1983 paper claimed that a transitory adverse shift in a country's terms of trade pushes its current account to a temporary deficit, one driven, in part, by an accompanying fall in the expected real interest rate paid on external debt.

In his comment on my paper, Amartya Lahiri agrees that my current-account results hold for the infinitesimal price changes I analyzed. He demonstrates, however, that a large fall in the relative price of a country's initial exports can occasion a current-account surplus.

This note shows that while Lahiri's outcome is a mathematical possibility in my 1983 model, it can arise there only when the assumed fall in relative export prices is so large that it actually benefits the exporting country through a complete (though perhaps temporary) reversal of its direction of trade. Lahiri's example thus is irrelevant to the question of how temporary trade setbacks affect the current account (as well as being of questionable empirical relevance). My model does not allow even a large temporary rise in relative export prices to cause a deficit.

My discussion here proceeds by developing a more transparent exposition of my 1983 model, one that, incidentally, shows more explicitly than did the original analysis how the dynamics of utility following a transitory terms-of-trade shock are driven by real interest rate effects. The analysis is perhaps of more general interest in that it clarifies some results in the recent literature on the intertemporal approach to the current account.

Sections I and II below explore real interest rate effects, while section III derives the central result that temporary terms-of-trade setbacks always cause the current account to deteriorate in the Obstfeld (1983) model.

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1 I am grateful to Lahiri for raising this issue in the original version of his comment (Lahiri 1994).
I. A benchmark case with a constant consumption-based real interest rate

The changes in real interest rates I discussed in my 1983 paper arose from the assumption of international loan contracts denominated in the importable good and carrying an exogenously fixed own rate of interest. That assumption, made with the experience of dollar-borrowing developing countries in mind, implies that an expected improvement in a country's terms of trade—a rise in the relative price of its exports—lowers the effective expected real interest rate the country faces in the world capital market (other things equal). The reason: an expected rise in the relative price of exports lowers the value of loan repayments in terms of the domestic consumption basket (provided some exports are indeed consumed).

As noted in my 1983 paper (p. 143) and in Frenkel and Razin (1987, p. 168), in a perfect-foresight setting interest rates on differently indexed bonds must be linked by an interest parity condition involving the expected change in the terms of trade. Thus, the small-country analyses of this section and the next differ fundamentally through the choice of the "world" interest rate that is being held constant as the terms of trade vary. (In this section it is the rate on consumption-indexed debt, in the next the rate on debt indexed to imports.) Different choices for the "world" interest rate that is taken to be exogenous, given also the exogenous path of the terms of trade, imply different exogenous paths for the consumption-based real interest rate.\(^2\)

It is important to keep in mind that the difference in assumed real interest rate paths, rather than the numeraire for assets and liabilities per se, is

\(^2\)Ostry (1988) provides a small-country analysis in which the own interest rate on exports is exogenous. Obviously, a general-equilibrium setup would be superior to the present small-country setup in allowing one to explore comovements in intratemporal and intertemporal relative prices.
what causes different economic behavior, given the economy's starting wealth.

Importantly, however, the denomination of external debt also determines starting wealth. The model assumes an initial steady state and perfect foresight except for an initial unanticipated terms-of-trade shock such that interest parity fails ex post. Thus, the way debt is indexed to unexpected changes in the terms of trade determines the economy's starting wealth relative to its pre-shock wealth.

In understanding the real interest rate effect that arises when the own rate of interest on imports is fixed—the "intertemporal price speculation" of my paper's title—it is illuminating to start with a benchmark case in which the effect is absent. In that case, modeled originally by Svensson and Razin (1983), bonds are indexed to the consumption basket rather than to one of the model's two goods, and it is the consumption-based real interest rate that is fixed by the world capital market.

My 1983 model, recast with this modification, goes as follows.

In a small open economy, the representative individual maximizes

\[
U_t = \int_0^\infty e^{-\delta t} \left( \frac{x_t^{1-\alpha}}{m_t} \right)^{1-\sigma} \frac{1}{1 - \frac{1}{\sigma}} \, dt, \quad \alpha \in (0,1), \ \delta, \ \sigma > 0,
\]

where \(\sigma\) (equal to \(1/R\) in my earlier paper's notation), is the elasticity of intertemporal substitution. Let \(q\) be the price of exports in terms of imports (equal to \(1/p\) in my earlier paper's notation, so that a fall in \(q\) is a worsening of the terms of trade). The minimum expenditure of imports needed to purchase a unit of the subutility index \(x_t^{\alpha} m_t^{1-\alpha}\) in (1) is given by

\[
Q = q^{\alpha}/\alpha^{\alpha}(1-\alpha)^{1-\alpha}.
\]
The index $Q$ is the economy's exact index of consumer prices measured in imports. If $z$ denotes total expenditure measured in imports, the indirect utility function corresponding to (1) is

$$\begin{align*}
U_0 &= \int_0^\infty e^{-\delta t} \frac{z_t/Q_t}{1 - \frac{1}{\sigma}} \, dt.
\end{align*}$$

The way to abstract from real interest rate changes is to assume that intertemporal trade is accomplished through a bond that quotes payoffs in terms of units of subutility $z/Q = x_m^{1-\alpha}$. Let $\rho$ be the (constant) instantaneous rate of interest on such bonds, so that $e^{-\rho t}$ is the price of time $t$ subutility in terms of subutility delivered at time $t+1$. Finally, let $c$ denote the level of bond holdings. Then, if the economy's endowment of its perishable export good is $y$, the intertemporal budget constraint is

$$\begin{align*}
\int_0^\infty e^{-\rho t} (z_t/Q_t) \, dt &= c_0 + \int_0^\infty e^{-\rho t} (q_t y_t/Q_t) \, dt.
\end{align*}$$

In line with the goal of obtaining a benchmark comparable to my 1983 model, I assume $\delta = \rho$.

Maximization of (3) subject to (4) yields the real expenditure function:

$$\begin{align*}
\frac{Z_0}{Q_0} = \delta \left[ c_0 + \int_0^\infty e^{-\rho t} \frac{q_t y_t}{Q_t} \, dt \right].
\end{align*}$$

One obvious feature of (5) is that real expenditure $x_m^{1-\alpha}$, and, hence, period utility, is expected to be constant over time, regardless of the behavior of
the terms of trade, \( q \). This constancy results from the constancy of the real interest rate, \( \rho \). Furthermore, since \( q/Q \) is proportional to \( q^{1-\alpha} \), an adverse terms-of-trade movement (fall in \( q \)) can never raise period utility.

In this setting, what is the effect of an unexpected temporary fall in \( q \), say, a fall from a constant level \( q \) to \( q' < q \) at time 0 that lasts until time \( T \)? To get at the "pure" effect of this change I assume \( y \) is constant. Using (5), the definition of \( Q \) in (2), and the current-account identity

\[
\dot{c} = \rho c + \frac{qy}{Q} - \frac{z}{Q},
\]

the initial current-account change is proportional to

\[
[(q')^{1-\alpha} - q^{1-\alpha}]y + \delta \left( \int_{0}^{T} e^{-\delta t} [(q')^{1-\alpha} - (q^{1-\alpha})] y dt \right) = e^{-\delta T} [(q')^{1-\alpha} - q^{1-\alpha}]y < 0.
\]

As \( T \to \infty \) (the case of a permanent shock) this temporary deficit goes to 0, a prediction peculiar to the constant time-preference case.

A deficit emerges irrespective of the size of the temporary terms-of-trade shock, of the sign of the initial foreign asset position, of the share of exports in consumption (\( \alpha \)), and of the size of the elasticity of intertemporal substitution (\( \sigma \)). It is worthwhile to notice why a fall in \( q \) can never raise lifetime utility, \( U_q \), here. Because they are indexed to utility, net foreign assets yields a utility payoff that is independent of the terms of trade. (If instead these assets were indexed to importables, their purchasing power over the consumption basket would rise when the terms of trade fell.)
the other hand, the real value of the export endowment falls when \( q \) falls. Thus, the consumer's lifetime consumption possibility set always shrinks and the consumer picks a constant path of instantaneous utility within this set.

II. Real interest rate changes when the own rate on imports is given

In contrast to the preceding benchmark case, there is the (arguably more realistic) case treated in my 1983 paper, in which bonds indexed to imports carry an exogenously given interest rate. Let \( b \) denote the level of such bond holdings and \( r \) the (constant) instantaneous rate of interest they offer, so that \( e^r \) is the price of imports at time \( t \) in terms of imports at time \( t + 1 \). Now the real interest rate defined in the last section need not be constant, even though \( r \) is. Denote by \( R_{0,t} \) the price of time 0 subutility in terms of time 0 subutility:

\[
(6) \quad R_{0,t} = (Q_0/Q_t)e^{rt}.
\]

The average instantaneous real interest rate between times 0 and \( t \) is just \( r + \log(Q_0/Q_t)/t \). *Ceteris paribus*, an expected rise in the price of exports relative to their current price [an expected terms-of-trade improvement, which raises \( Q_t \) in (6) relative to \( Q_0 \)] lowers the instantaneous real interest rate import-denominated bonds pay out between 0 and \( t \). Only if \( Q \) is expected to be constant does the real rate equal \( r \). As in my 1983 paper, \( r = \delta \) is assumed.

The intertemporal budget constraint now has the form

\[
(7) \quad \int_0^\infty e^{-rt} z_t \, dt = b_0 + \int_0^\infty e^{-rt} q_t y_t \, dt,
\]
and the first-order condition for maximizing (3) subject to (7) is

\[
(8) \quad \left( \frac{z_t}{Q_t} \right) = \lambda Q_t,
\]

where \( \lambda \) is a constant Lagrange multiplier equal to the marginal utility of imports. The solution for real consumption is

\[
(9) \quad \frac{z_0}{Q_0} = \frac{b_0}{Q_0} + \int_{0}^{\infty} e^{-r t} \left( \frac{Q_t}{Q_0} \right) Q_t \frac{q_t y_t}{Q_t} dt
\]

By the definition of the real interest factor (6), the preceding becomes

\[
(10) \quad \frac{z_0}{Q_0} = \frac{b_0}{Q_0} + \int_{0}^{\infty} R_{0,t}^{-1} \left( \frac{q_t y_t}{Q_t} \right) dt
\]

Consumption function (10) simply generalizes (5) to take account of a real interest rate that can stray away from \( \delta \). This variability arises now because expected changes in the terms of trade alter the real interest rate the small country faces.

Let's suppose again that the terms of trade, previously at \( q \), fall unexpectedly at time 0 to \( q' \), with reversion back to \( q \) at time \( T \). Again, \( y \) is
constant. Equation (9) shows that $z/Q$ will change from

\[
(11) \quad \frac{z_0}{Q} = \left( \frac{rb_0}{Q} + \frac{qy}{Q} \right)
\]

to

\[
(12) \quad \frac{z'_0}{Q'} = \frac{(1-e^{-rT})\left(\frac{rb_0}{Q'} + \frac{rq'y}{Q'}\right) + e^{-rT}\left(\frac{q}{Q'}\right)\left(\frac{rb_0}{Q} + \frac{qy}{Q}\right)}{(1-e^{-rT}) + e^{-rT}\left(\frac{q}{Q'}\right)^{1-\sigma}}
\]

Where it appears in (12), the ratio $Q/Q' > 1$ captures the real interest rate effect; without its presence, post-shock real expenditure would be simply a weighted average of the hypothetical expenditure levels when the terms of trade are permanently at $q'$ and $q$, respectively. The total real interest rate effect is the sum of the usual income, substitution, and wealth effects—the first two evident in the denominator of (12), the last in its numerator.

Differentiation of (12) with respect to $Q/Q'$ shows that for small terms-of-trade deteriorations, the substitution and wealth effects of the concomitant real interest rate change dominate the income effect and thus tend to raise expenditure compared to what it would be absent these effects. This net positive effect of the CPI-based real interest rate is the one I alluded to in my 1983 paper. It is also easy to see from (12) why, as I claimed, a very brief terms-of-trade deterioration necessarily causes real expenditure—and, therefore, the instantaneous flow of utility—to rise on impact. Equation (12) discloses that, as $T \to 0,$
\[
\frac{Z^*}{Q^*} \rightarrow \left( \frac{Q}{Q^*} \right) \sigma \left( \frac{r b_0}{Q} + \frac{q y}{Q} \right) > \left( \frac{r b_0}{Q} + \frac{q y}{Q} \right) = \frac{Z_0}{Q}.
\]

It is obvious that this phenomenon derives entirely from the influence of the transitory terms-of-trade change on the relevant real interest rate.

To ascertain the impact of the shock on the current account, defined as

\[\dot{b} = rb + qy - z,\]

let \(\lambda'\) denote the permanent post-shock value of the multiplier in (8). If \(\lambda' > \lambda\), the pre-shock value, then, by (8), \(z/Q\), and, hence, utility, are lower once the terms of trade have reverted to their initial level. Since the final position is a steady state, the economy must have had a deficit and run down some of its net foreign assets between dates 0 and T. Notice that because \(\lambda'\) doesn't change at time T, (8) implies a discrete fall in instantaneous utility when the terms of trade rise from \(q'\) back to \(q\) on that date.

III. Can an adverse terms-of-trade shift cause a current-account surplus?

Lahiri points out that for a large enough fall in \(q\), \(\lambda\) can \textit{fall} when the temporary terms-of-trade change hits. As he observes, the possibility is implied by my 1983 paper's current-account equation [equation (9) in his note, and the unnumbered equation at the bottom of p. 142 in my paper] if \(\sigma = 1/R < 1\) and the stock of foreign assets is positive and sufficiently large. But this case can never result from a fall in \(q\) that is, in any meaningful sense, a terms-of-trade \textit{deterioration}. The following proposition about my 1983 model establishes my claim.
PROPOSITION. A surplus emerges in response to a temporary fall in the price of a country's initial export if and only if the economy attains higher momentary utility on every date as a result of the price-path change.

PROOF. Remember that a surplus emerges in the model if, and only if, $\lambda$ falls to $\lambda'$ when the shock occurs. A surplus between times 0 and T implies that instantaneous utility is higher from time T on. On date 0, however, $Q$ and $\lambda$ both fall if there is a surplus; so equation (8) shows that $(z/Q)^{-1/\sigma}$ must fall, too, i.e., that $z/Q$ must rise. Since $z/Q$ is constant between times 0 and T, $z/Q$ must therefore be higher on every date for a surplus to emerge. Conversely, if $z/Q$ higher is on every date, it is higher after T, so the economy must have been in surplus before T.

Behind the possibility that $\lambda$ falls is a phenomenon familiar in trade theory, one that motivates the customary focus on small terms-of-trade changes in assessing welfare effects. A large enough price change can reverse both the pattern of trade and the usual welfare effects. In the present setup, noncontingent foreign assets denominated in imports behave exactly like an exogenous endowment of the import good. If the economy holds a positive stock, a large enough rise in their price can induce the economy, at least for a time, to export the previously imported good and import the domestically produced good. The price change also can make the economy very wealthy. But this is hardly what one would call an adverse terms-of-trade movement.

That the economy benefit from the temporary fall in q is a necessary, but not sufficient, condition for the price change to cause a surplus (since the economy can benefit in the intertemporal sense without the flow of utility being higher on every date). If intertemporal substitutability is high enough that $\sigma > 1$, even a beneficial temporary fall in the price of the initial export good occasions a current-account deficit (because it induces a sharp
temporary spending binge for which the economy suffers later). This follows from equations (7) and (8), which show that an unexpected temporary fall in $q$ makes $\lambda' > \lambda$ whenever $\sigma > 1$.

A seeming implication of Lahiri's analysis is that a temporary rise in $q$ (an unambiguously favorable terms-of-trade movement) can induce a current-account deficit when $\sigma < 1$. In fact, this can never happen in the model. The logic of this section's Proposition shows that a deficit could occur only if momentary utility were lower on every date. But that outcome cannot describe the optimal response to a rise in relative export prices.\(^3\)

\[^3\text{One can also prove directly that any temporary rise in } q \text{ causes a surplus.}\]


OSTRY, J.D. "The balance of trade, the terms of trade, and real exchange rate." International Monetary Fund Staff Papers, December 1968, 35: 541-573.

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