Title
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Saturated Hole-coupled FEL Oscillator

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Suppression Of Mode-Beating In A Saturated
Hole-Coupled FEL Oscillator

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In a hole-coupled resonator, either empty or loaded with a linear FEL gain medium, the phenomenon of mode-degeneracy and mode-beating have been studied. When the magnitudes of the eigenvalues, derived from a linear analysis, are equal for two or more dominant eigenmodes, the system cannot achieve a stable beam-profile. We investigate this phenomenon when a saturated FEL is present within the cavity, thus introducing non-linearity. We use a three-dimensional FEL oscillator code, based on the amplifier code TDA, and show that mode-beating is completely suppressed in the nonlinear saturated regime. We suggest a simple, qualitative model for the mechanism responsible for this suppression.
I. INTRODUCTION

It is well known that for a linear system, there exist a set of eigenmodes and that a general solution of the system is a superposition of these eigenmodes. The same concept applies to a hole-coupled resonator, either empty or loaded with a linear FEL gain medium. It has been shown that in such a linear system [1, 2] (transverse) mode-beating may occur when the magnitudes of the eigenvalue of two or more dominant modes become degenerate. Mode beating should be avoided for stable FEL operation because when it happens that the cavity mode which is a superposition of the degenerate eigenmodes can not achieve a stable profile.

It has also been shown [1, 2] for the linear system that mode-beating can be avoided to some extent by either active or passive mode control. Passive mode control works by introducing additional diffraction loss to the higher order mode with intra-cavity apertures, thus restoring the dominance of the fundamental mode. However, passive mode control reduces the output coupling efficiency since it also introduces additional diffraction loss to the fundamental mode. Active mode control, on the other hand, works by introducing more gain to the fundamental mode due to the difference in the transverse overlapping factor with different modes, thus breaking mode-degeneracy.

Unlike the linear system where both passive and active mode control work independently on different eigenmodes, it is expected that, in the saturated regime, mode beating may be suppressed by another factor – the nonlinearity of the medium, which renders the principle of superposition inoperative and couples the different modes together.

Here we try to answer the question: does mode beating, which has a clear origin in linear theory, persist in the nonlinear regime? To this end we first consider a case where, with a cold cavity, there is mode degeneracy, and show that on introduction of an FEL interaction this degeneracy is suppressed. We than suggest a model for the mechanism of this action, and support the model with further simulations.
II. DETAILS OF THE SIMULATIONS

We have developed an oscillator code based on the amplifier code TDA of Tran and Wurtele [3]. This is a time-independent, three-dimensional, axisymmetric FEL code. It uses the nonlinear KMR equations [4] of motion for the electrons, and the paraxial wave equation for the radiation. To model an FEL oscillator we have implemented three segments of a Fresnel-Kirchoff integral [5] that propagates the radiation from the end of the wiggler, off two mirrors, back to the entrance of the wiggler. The electric field at the entrance to the wiggler is then used as input to TDA, and the process iterates for as many passes as are necessary. Further details of our code can be found in Ref. [6].

The system simulated consisted of a near-concentric optical cavity with a wiggler placed symmetrically at its centre. The radiation was initialized as a Gaussian, and followed for many passes. Electrons were injected afresh at every pass. Each simulation run continued until the total power was saturated and an equilibrium had been achieved; typically for thirty passes.

Parameters used in the simulation are shown in Table 1. The wiggler bore radius was chosen to be large enough so as not to clip the radiation profile. The hole size alone was varied to change the dynamics within the system, and thus change the mode-profile. In most of the comparisons shown below we consider the profile at the entrance to the wiggler. Obviously, the profile at the mirror with the hole, which may be of greater interest to designers, could be very different.

III. SUPPRESSION OF MODE-BEATING

In order to investigate mode beating, we had to choose parameters such that, as the hole radius was varied, the mode profile changed from fundamental to higher order. For the parameters of Table 1, we first performed cold-cavity simulations by varying the hole size from 2–4 mm. We found that the dominant mode at 2 mm was the fundamental
mode, which was smooth and near-Gaussian. At 4 mm the dominant mode was a higher-order mode with a zero and a secondary maximum. Mode beating, or mode competition was found to occur in the region between 3.0–3.3 mm. This is illustrated in Figure 1 by the discontinuity in the calculated hole coupling efficiency $\eta$.

We then ran simulations with our FEL oscillator code for the same parameters. For hole sizes of up to 3 mm we found the equilibrium mode-profile to be much the same as in the cold-cavity case. However, in the region where the cold cavity simulations indicated the presence of mode beating, we found no such phenomenon. The coupling efficiency $\eta$, in Figure 1, is continuous, suggesting the lack of mode beating. This behaviour persisted to the largest hole sizes we could go to, of around 4.3 mm. With bigger holes the gain within the wiggler could not compensate for the loss loss and the total power in the cavity decreased monotonically.

Figure 2 shows the mode profile, for the simulations with an FEL interaction, at three different hole sizes, at the entrance to and at the exit from the wiggler. One can see that, at the entrance, the change in mode profile is continuous over a large range of hole sizes. The lack of mode competition was also evidenced by the fact that in each case the number of passes required to reach equilibrium is roughly the same, around twenty passes. In the cold-cavity case, in the regime of mode beating, the system takes many passes, hundreds or thousands, to reach equilibrium, while outside it may take only a few tens of passes.

We emphasize that the equilibrium beam-profile can be called a mode in the sense that it is independent of the initial conditions. We have verified by changing the width of the initial Gaussian radiation by a factor of 25, that the equilibrium radiation profile is always the same.

The absence of mode beating in the nonlinear saturated regime has important implications for the design of FEL oscillator systems. It suggests that one could design
oscillators with larger hole sizes than hitherto believed on the basis of cold-cavity simulations, thus allowing for greater power outcoupling and wider wavelength tuning range, without sacrificing mode quality.

IV. SATURATION MECHANISM

From Figure 2 we see that while the mode-profile at the entrance to the wiggler changes with changing hole-size, the profile at the exit to the wiggler is smooth and hardly changes. This shows that the interaction between the electrons and the radiation has a tendency to suppress structure in the input mode-profile, thus producing a smoother profile as the radiation exits the wiggler. We suggest that it is this phenomenon that is responsible for suppression of mode-beating.

This observation suggests the following simple model. For simplicity let us assume that the transverse electron distribution is uniform and that the gain depends only on the local optical intensity. Where the optical intensity is low, the gain will be high, and vice versa. Consequently, by the time the radiation exits the wiggler, variations in the intensity will tend to be smoothed out.

Of course, true electron distributions are generally Gaussian. Nonetheless the basic mechanism described above will still hold. Any higher-order mode structure that the optical beam may have at the entrance to the wiggler will tend to be smoothed out, resulting in a Gaussian-like profile at the exit to the wiggler.

This conclusion is supported by Figures 2, where we see that while the profile at the entrance to the wiggler changes continuously with change in the hole size, the profile at the exit to the wiggler is more or less unchanged.

Of course, the model is very simple and qualitative. It ignores effects, such as diffraction, that lead to focusing or divergence of the optical mode through the wiggler, which differs from our assumption of parallel propagation of the optical and electron beams.
We plan to do more quantitative work on this model in the near future.

An immediate consequence of the model is that the equilibrium intra-cavity mode-profile is stabilized by the saturated gain medium, making it less sensitive to geometrical factors of the resonator, such as the hole size. Since the mode at the mode at the wiggler exit is insensitive to the input profile, there can be no mode competition.

It should be noted that once the radiation exits the wiggler and propagates in free space through the rest of the resonator, it will not, generally, maintain its smooth shape. Secondary maxima or other structure may develop at a consequence of diffraction at the hole and the mirrors.

V. CONCLUSIONS

We find that the phenomenon of mode beating is completely suppressed when the FEL interaction within the cavity is taken into account. This has important implications in the design of FEL oscillator systems, because it suggests that larger hole sizes can be used than had hitherto been supposed, allowing for the outcoupling of more power.

We have suggested a simple, qualitative model to explain how this comes about. It suggests that as a consequence of saturation within the FEL the equilibrium beam-profile coming out of the wiggler is largely smooth and Gaussian, irrespective of what the profile looks like at the entrance to the wiggler.

VI. ACKNOWLEDGEMENTS

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REFERENCES


FIGURES

FIG. 1. Plot of hole-coupling efficiency as a function of hole size for the cold-cavity case and with an FEL interaction. The discontinuity in the curve for the cold-cavity case is indicative of mode-beating and mode-switching.

FIG. 2. Mode profiles (with an FEL interaction) at three different hole sizes; (a) at the entrance to, and (b) at the exit from, the wiggler. The mode profile at the entrance changes continuously with hole-size. The profile at the exit, however, is smooth and remains almost unchanged. In all three cases the gain was at least a factor of two below the small-signal gain, ensuring that the system is nonlinear.
TABLES

**TABLE I. Parameters used in the simulation**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Wiggler parameter ($a_w$)</td>
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<tr>
<td>Wiggler length ($L$)</td>
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<tr>
<td>Wiggler period ($\lambda_w$)</td>
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</tr>
<tr>
<td>Wiggler bore radius</td>
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<td>Normalized beam emittance ($\epsilon_n$)</td>
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<td>Beam radius ($r_b$)</td>
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<td>Beam energy ($\gamma$)</td>
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<td>Beam current ($I$)</td>
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<td>Optical wavelength ($\lambda$)</td>
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<tr>
<td>Radius of cross-section of mirrors</td>
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<td>Separation between the mirrors</td>
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<td>Number of radial grid points</td>
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<tr>
<td>Number of longitudinal integration steps</td>
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</tr>
</tbody>
</table>
Figure 1
Figure 2a
Figure 2b

Profiles at exit from wiggler

Field amplitude (arb. units)

Radial distance (mm)

1.6 mm
2.6 mm
3.6 mm