Measurements of Branching Fractions and $CP$-Violating Asymmetries in $B^0 \to \pi^+\pi^-$, $K^+\pi^-$, $K^+K^-$ Decays

We present measurements of branching fractions and CP-violating asymmetries for two-body neutral $B$ meson decays to charged pions and kaons based on a sample of about $8 \times 10^6 \ U(4S) \to \ BB$ decays. From a time-independent fit we measure the charge-averaged branching fractions $B(B^0 \to \pi^+ \pi^-) = (4.7 \pm 0.6 \pm 0.2) \times 10^{-6}$, $B(B^0 \to K^+ \pi^-) = (17.9 \pm 0.9 \pm 0.7) \times 10^{-6}$, and the direct CP-violating charge asymmetry $A_{\pi\pi} = -0.102 \pm 0.050 \pm 0.016$ [−0.188, −0.016], where the ranges in square brackets indicate the 90% confidence intervals. From a time-dependent fit we measure the $B^0 \to \pi^+ \pi^-$ CP-violating parameters $S_{\pi\pi} = 0.02 \pm 0.34 \pm 0.05$ [−0.54, +0.58] and $C_{\pi\pi} = -0.30 \pm 0.25 \pm 0.04$ [−0.72, +0.12].

Recent measurements of the CP-violating asymmetry parameter $\sin2\beta$ reported by the BABAR [1] and Belle [2] Collaborations established CP violation in neutral B decays. These results are consistent with the standard model (SM) expectation based on indirect constraints on the magnitudes of the elements of the Cabibbo-Kobayashi-Maskawa (CKM) [3] quark-mixing matrix. However, a full test of the CP violation mechanism in the SM,
through a single complex phase in the CKM matrix, will require additional direct constraints on the angles ($\alpha$, $\beta$, and $\gamma$) of the unitarity triangle [4].

The time-dependent CP-violating asymmetry in the decay $B^0 \to \pi^+ \pi^-$ is related to the angle $\alpha$, and ratios of branching fractions for various $\pi \pi$ and $K \pi$ decay modes are sensitive to the angle $\gamma$. In this Letter we present results for branching fractions and CP-violating asymmetries in $B^0 \to \pi^+ \pi^-$, $K^+ \pi^-$, and $K^+ K^-$ decays [5] using a sample of $87.9 \pm 1.0 \times 10^6 \ B \overline{B}$ pairs. A detailed description of the BABAR detector is presented in Ref. [6], and more details on the analysis technique are given in Ref. [7], which describe our previous measurements of these quantities. Other measurements of the branching fractions and the charge asymmetry in $B^0 \to K^+ \pi^-$ have been performed by the CLEO and Belle Collaborations [8]. More recently, the Belle Collaboration reported a measurement of the time-dependent CP asymmetry in $B^0 \to \pi^+ \pi^-$ [9].

We reconstruct a sample of neutral $B$ mesons ($B_{\text{rec}}$) decaying to the $h^+ h^-$ final state, where $h$ and $h'$ refer to $\pi$ or $K$. Signal yields are determined with a maximum likelihood fit including kinematic, topological, and particle identification information. For the $K^\pm \pi^\mp$ components, the yield is parametrized as $N_{K^\pm \pi^\mp} = N_{K^\pi} (1 \pm \mathcal{A}_{K^\pi})/2$, where $N_{K^\pi}$ is the total yield and $\mathcal{A}_{K^\pi} = (N_{K^0 \pi^+} - N_{K^- \pi^-})/(N_{K^0 \pi^+} + N_{K^- \pi^-})$ is the CP-violating charge asymmetry. The asymmetry arises from interference between the $b \to s$ penguin and $b \to u$ tree amplitudes and is predicted [10,11] to be less than 20% in the standard model. However, a larger asymmetry could be induced by new particles, such as charged Higgs bosons or supersymmetric particles, contributing to the penguin amplitude.

In order to extract the CP asymmetry parameters in $B^0 \to \pi^+ \pi^-$ decay, we examine each event in the $B_{\text{rec}}$ sample to determine whether the second $B$ meson ($B_{\text{tag}}$) decayed as a $B^0$ or $\overline{B}^0$ (flavor tag) and reconstruct the difference $\Delta t$ between the proper decay times of the $B_{\text{rec}}$ and $B_{\text{tag}}$ decays. The decay rate distribution $f_\pm(f_-)$ when $h^+ h^- = \pi^+ \pi^-$ and $B_{\text{tag}} = B^0(\overline{B}^0)$ is given by

$$f_\pm(\Delta t) = \frac{e^{-|\Delta t|/\tau}}{4\tau} \left[ 1 \pm S_{\pi\pi} \sin(\Delta m_d \Delta t) \right] + C_{\pi\pi} \cos(\Delta m_d \Delta t),$$

(1)

where $\tau$ is the mean $B^0$ lifetime and $\Delta m_d$ is the mixing frequency due to the eigenstate mass difference. The parameters $S_{\pi\pi}$ and $C_{\pi\pi}$ are defined as

$$S_{\pi\pi} = \frac{2 \text{Im} \lambda}{1 + |\lambda|^2} \quad \text{and} \quad C_{\pi\pi} = \frac{1 - |\lambda|^2}{1 + |\lambda|^2},$$

(2)

and vanish in the absence of CP violation. If the decay proceeds purely through the $b \to u$ tree amplitude, the complex parameter $\lambda$ is given by

$$\lambda(B \to \pi^+ \pi^-) = \left( \frac{V_{ub} V_{td}}{V_{ub} V_{td}} \right) \left( \frac{V_{ud} V_{tb}}{V_{ud} V_{tb}} \right).$$

(3)

In this case $C_{\pi\pi} = 0$ and $S_{\pi\pi} = \sin 2\alpha$, where $\alpha = \arg[-V_{ub} V_{td}^*/V_{ud} V_{tb}^*]$. In general, the $b \to d$ penguin amplitude modifies both the magnitude and phase of $\lambda$, so that $C_{\pi\pi} \neq 0$ and $S_{\pi\pi} = \sqrt{1 - C_{\pi\pi}^2} \sin 2\alpha_{\text{eff}}$, where $\alpha_{\text{eff}}$ depends on the magnitudes and relative strong and weak phases of the tree and penguin amplitudes. Several approaches have been proposed to obtain information on $\alpha$ in the presence of penguins [10,12].

The event selection and $B_{\text{rec}}$ reconstruction used in this analysis are similar to those used in Ref. [7]. Hadronic events are selected based on charged particle multiplicity and event topology. Candidate $B_{\text{rec}}$ decays are reconstructed from pairs of oppositely charged tracks forming a good quality vertex, where the $B_{\text{rec}}$ four-momentum is calculated with the pion mass assumed for both tracks.

Signal decays are identified kinematically using two variables, the difference $\Delta E$ between the center-of-mass (c.m.) energy of the $B_{\text{rec}}$ candidate and $\sqrt{s}/2$, and the beam-energy substituted mass $m_{\text{ES}} = \sqrt{(s/2 + p_1 \cdot p_B)/E_1^2 - p_B^2}$, where $\sqrt{s}$ is the total c.m. energy, and the $B_{\text{rec}}$ momentum $p_B$ and the four-momentum of the initial state $(E_1, p_1)$ are defined in the laboratory frame. For signal decays $\Delta E$ and $m_{\text{ES}}$ are Gaussian distributed with resolutions of 26 MeV and 2.6 MeV/$c^2$, respectively. For $\pi^+ \pi^-$ decays $\Delta E$ peaks near zero, while for decays with one or two kaons the $\Delta E$ peak position is parametrized as a function of the kaon momenta in the laboratory frame, with an average shift of $-45$ MeV and $-91$ MeV, respectively. The distribution of $m_{\text{ES}}$ peaks near the $B$ mass. We require $5.20 < m_{\text{ES}} < 5.29$ GeV/$c^2$ and $|\Delta E| < 0.15$ GeV.

Identification of $h^+ h^-$ tracks as pions or kaons is accomplished with the Cherenkov angle measurement $\theta_\gamma$ from a detector of internally reflected Cherenkov light. We construct charge-dependent double-Gaussian probability density functions (PDFs) from the difference between measured and expected values of $\theta_\gamma$ for the pion or kaon hypothesis, normalized by the error $\sigma_{\theta_\gamma}$. The PDF parameters are measured in a sample of $D^+ \to D^0 \pi^+$, $D^0 \to K^- \pi^+$ decays, reconstructed in data. The typical separation between pions and kaons varies from $8 \sigma_{\theta_\gamma}$ at 2 GeV/$c$ to $2.5 \sigma_{\theta_\gamma}$ at 4 GeV/$c$.

We have studied potential backgrounds from other $B$ decays and find them to be negligible. Backgrounds from the process $e^+ e^- \to q\overline{q}$ ($q = u, d, s, c$) are suppressed by their topology. In the c.m. frame we define the angle $\theta_q$ between the sphericity axis of the $B$ candidate and the sphericity axis of the remaining particles in the event, and require $|\cos \theta_q| < 0.8$, which removes 83% of this background. For these particles we also define a Fisher discriminant $\mathcal{F} = 0.53 - 0.60 \times \sum_i p_i^* + 1.27 \times \sum_i |p_i^*| \cos(\theta_q^*)^2$ where $p_i^*$ is the momentum of particle $i$ and $\theta_q^*$ is the angle between its momentum and the $B_{\text{rec}}$.
thrust axis in the c.m. frame. The shapes of \( \mathcal{F} \) for signal and background events are included as PDFs in the maximum likelihood fit.

We use a multivariate technique [13] to determine the flavor of the \( B_{\text{tag}} \) meson. Separate neural networks are trained to identify primary leptons, kaons, and soft pions from \( D^* \) decays, and high-momentum charged particles from \( B \) decays. Events are assigned to one of five mutually exclusive tagging categories based on the estimated mistag probability and the source of the tagging information (Table I). The quality of tagging is expressed in terms of the effective efficiency \( Q = \sum_k \epsilon_k (1 - 2w_k)^2 \), where \( \epsilon_k \) and \( w_k \) are the efficiencies and mistag probabilities, respectively, for events tagged in category \( k \).

Table I summarizes the tagging performance measured in a data sample \( B_{\text{flav}} \) of fully reconstructed neutral \( B \) decays to \( D^{(*)} \) (\( \pi^\pm \), \( \rho^\pm \), \( a_1^\pm \)). The assumption of equal tagging efficiencies and mistag probabilities for signal \( \pi^+ \pi^- \), \( K^+ \pi^- \), and \( K^+K^- \) decays is validated in a detailed Monte Carlo simulation. The background hypothesis has separate tagging efficiencies.

The time difference \( \Delta t \) is obtained from the known boost of the \( e^+e^- \) system and the measured distance between the \( z \) positions of the \( B_{\text{rec}} \) and \( B_{\text{tag}} \) decay vertices. A detailed description of the algorithm is given in Ref. [14]. We require \( |\Delta t| < 20 \) ps and \( \sigma_{\Delta t} < 2.5 \) ps, where \( \sigma_{\Delta t} \) is the error on \( \Delta t \). The resolution function for signal candidates is a sum of three Gaussians, identical to the one described in Ref. [13], with parameters determined from a fit to the \( B_{\text{flav}} \) sample (including events in all five tagging categories). The background \( \Delta t \) distribution is modeled as the sum of an exponential convolved with a Gaussian, with two additional Gaussians to account for tails. Common parameters are used to describe the background shape for all tagging categories. We find that 96\% of background events are described by an effective lifetime of approximately 0.7 ps.

We use an unbinned extended maximum likelihood fit to extract yields and CP parameters from the \( B_{\text{rec}} \) sample. The likelihood for candidate \( j \) tagged in category \( k \) is obtained by summing the product of event yield \( N_j \), tagging efficiency \( \epsilon_{j,k} \), and probability \( \mathcal{P}_{i,k} \) over the eight possible signal and background hypotheses \( i \) (referring to \( \pi^+\pi^- \), \( K^+\pi^- \), \( K^-\pi^+ \), and \( K^+K^- \) decays). The extended likelihood function for category \( k \) is

\[
L_k = \exp\left( -\sum_i N_i \epsilon_{i,k} \prod_j \left[ \sum_i N_i \epsilon_{i,k} \mathcal{P}_{i,k}(\vec{x}_j; \vec{\alpha}_i) \right] \right). \tag{4}
\]

The probabilities \( \mathcal{P}_{i,k} \) are evaluated as the product of PDFs for each of the independent variables \( \vec{x} = \{ m_{\text{ES}}, \Delta E, \mathcal{F}, \theta_\ell^t, \theta_\tau^t, \Delta t \} \), where \( \theta_\ell^t \) and \( \theta_\tau^t \) are the Cherenkov angles for the positively and negatively charged tracks. We use separate PDF parameters for \( \theta_\ell^t \) and \( \theta_\tau^t \) to account for possible systematic differences. The total likelihood \( L \) is the product of likelihoods for each tagging category, and the free parameters are determined by maximizing the quantity \( \ln L \). The fitted sample contains 26,070 events.

Signal yields are determined from a fit excluding tagging or \( \Delta t \) information. There are 16 free parameters, including signal and background yields (6 parameters); \( K\pi \) asymmetries (2); and parameters for the background shapes in \( m_{\text{ES}} \) (1), \( \Delta E \) (2), and \( \mathcal{F} \) (5). Table II summarizes signal yields, total efficiencies, charge-averaged branching fractions, and \( A_{K\pi} \). The efficiency calculation we neglect possible effects due to final state radiation from the \( B_{\text{rec}} \) decay products. The significance of \( A_{K\pi} \) is 2.0, where significance is defined as the square root of the change in \( -2\ln L \) when \( A_{K\pi} \) is fixed to zero. These results are consistent with our previous measurements [7] and with measurements from other experiments [8]. For the decay \( B^0 \rightarrow K^+K^- \) we measure a yield of only \( 1 \pm 8 \) events and so compute a Bayesian 90\% confidence level (C.L.) upper limit on the branching fraction. Reference [7] gives a detailed description of the method used.

The dominant sources of systematic error on the branching fraction measurements are from possible fit bias (determined in large samples of Monte Carlo simulated events), uncertainty in track and \( \theta_\tau \) reconstruction efficiencies, and imperfect knowledge of the PDF shapes. The calculation of selection efficiencies using Monte Carlo simulated decays has been checked against control samples in data and residual uncertainties are included in the systematic error on branching fractions. For \( A_{K\pi} \) the systematic error is dominated by the \( \theta_\ell \) PDF shape and possible charge bias in track reconstruction. The total systematic error is computed as the sum in quadrature of the individual uncertainties.

Figure 1 shows distributions of \( m_{\text{ES}} \) and \( \Delta E \) after selecting on probability ratios to enhance the signal purity. The solid curve in each plot represents the fit projection after correcting for the efficiency of the additional selection (52\% for \( \pi\pi \), 79\% for \( K\pi \)).

The parameters \( S_{\pi\pi} \) and \( C_{\pi\pi} \) are determined from a second fit including tagging and \( \Delta t \) information, where the \( B_{\text{flav}} \) sample is included to determine the signal

<table>
<thead>
<tr>
<th>Category</th>
<th>( \epsilon ) (%)</th>
<th>( w ) (%)</th>
<th>( \Delta w ) (%)</th>
<th>( Q ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lepton</td>
<td>9.1 ± 0.2</td>
<td>3.3 ± 0.7</td>
<td>-1.6 ± 1.3</td>
<td>8.0 ± 0.3</td>
</tr>
<tr>
<td>Kaon I</td>
<td>16.6 ± 0.2</td>
<td>9.5 ± 0.7</td>
<td>-2.8 ± 1.3</td>
<td>10.7 ± 0.4</td>
</tr>
<tr>
<td>Kaon II</td>
<td>19.8 ± 0.3</td>
<td>20.6 ± 0.8</td>
<td>-5.3 ± 1.3</td>
<td>6.7 ± 0.4</td>
</tr>
<tr>
<td>Inclusive</td>
<td>20.1 ± 0.3</td>
<td>31.7 ± 0.9</td>
<td>-2.6 ± 1.4</td>
<td>2.7 ± 0.3</td>
</tr>
<tr>
<td>Untagged</td>
<td>34.4 ± 0.5</td>
<td></td>
<td></td>
<td>28.4 ± 0.7</td>
</tr>
</tbody>
</table>
parameters describing tagging information and the \( \Delta t \) resolution function. The \( \Delta t \) PDF for signal \( \pi^+ \pi^- \) decays is given by Eq. (1), modified to include \( w_k \) and \( \Delta w_k \) for each tagging category and convolved with the signal resolution function. We also take into account possible differences in reconstruction and tagging efficiencies between \( B^0 \) and \( B^0 \) mesons. The \( \Delta t \) PDF for signal \( K^+ \pi^- \) events takes into account \( B^0 \overline{B}^0 \) mixing based on the charge of the kaon and the flavor of the \( B \) meson.

A total of 76 parameters are varied in the fit, including the values of \( S_{\pi\pi} \) and \( C_{\pi\pi} \) (2); signal and background yields (5); \( K\pi \) charge asymmetries (2); signal and background tagging efficiencies (16) and efficiency asymmetries (16); signal mistag fraction and mistag fraction differences (8); signal resolution function (9); and parameters for the background shapes in \( m_{ES} \) (5), \( \Delta E \) (2), \( f \) (5), and \( \Delta t \) (6). We assume zero events from \( B^0 \overline{B}^0 \) and \( K^+ \pi^- \) decays and fix \( \tau_{\overline{B}} \) and \( \Delta m_d \) to their world average values [15]. As a means of validating the analysis technique, we determine \( \tau \) and \( \Delta m_d \) in the \( B_{\text{rec}} \) sample and find \( \tau = (1.56 \pm 0.07) \) ps and \( \Delta m_d = (0.52 \pm 0.05) \) ps\(^{-1}\).

The combined fit to the \( B_{\text{rec}} \) and \( B_{\text{flav}} \) samples yields

\[
S_{\pi\pi} = 0.02 \pm 0.34 \text{(stat)} \pm 0.05 \text{(syst)} | [-0.54, +0.58],
\]

\[
C_{\pi\pi} = -0.30 \pm 0.25 \text{(stat)} \pm 0.04 \text{(syst)} | [-0.72, +0.12],
\]

where the range in square brackets indicates the 90% C.L. interval taking into account the systematic errors. The correlation between \( S_{\pi\pi} \) and \( C_{\pi\pi} \) is \(-10\%\). The signal yields determined in this fit are within 3% of the yields obtained from the time-independent fit. Systematic uncertainties on \( S_{\pi\pi} \) and \( C_{\pi\pi} \) are dominated by imperfect knowledge of the PDF shapes and possible fit bias. We also evaluate multiplicative systematic errors (0.015), which are calculated at 1 standard deviation and summed in quadrature with the additive systematic uncertainties. Figure 2 shows distributions of \( \Delta t \) for events with \( B_{\text{tag}} \) tagged as \( B^0 \) or \( B^0 \), and the asymmetry as a function of \( \Delta t \) for tagged events enhanced in signal \( \pi\pi \) decays.

In summary, we have presented updated measurements of branching fractions and \( CP \)-violating asymmetries in \( B^0 \rightarrow \pi^+ \pi^- \), \( K^+ \pi^- \), and \( K^+ K^- \) decays. These results are consistent with and supersede our previous measurements.
We do not observe large mixing-induced or direct CP violation in the time-dependent asymmetry of $B^0 \rightarrow \pi^+ \pi^-$ decays, as reported in [9].

We are grateful for the excellent luminosity and machine conditions provided by our PEP-II colleagues, and for the substantial dedicated effort from the computing organizations that support BABAR. The collaborating institutions wish to thank SLAC for its support and kind hospitality. This work is supported by the DOE and NSF (U.S.A.), NSERC (Canada), IHEP (China), CEA and CNRS-IN2P3 (France), BMBF and DFG (Germany), INFN (Italy), NFR (Norway), MIST (Russia), and PPARC (United Kingdom). Individuals have received support from the A.P. Sloan Foundation, Research Corporation, and Alexander von Humboldt Foundation.

*Also with Università di Perugia, I-06100 Perugia, Italy.

[5] Unless explicitly stated, charge conjugate decay modes are assumed throughout this paper.