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Flexible Transmission Networks for Renewables Integration

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Flexible Transmission Networks for Renewables Integration

by

Jiaying Shi

A dissertation submitted in partial satisfaction of the
requirements for the degree of
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in the
Graduate Division

of the
University of California, Berkeley

Committee in charge:

Professor Shmuel S. Oren, Chair
Professor Javad Y. Lavaei
Professor Duncan S. Callaway

Spring 2018
Flexible Transmission Networks for Renewables Integration

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Jiaying Shi
Abstract

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by
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Doctor of Philosophy in Engineering – Industrial Engineering and Operations Research
University of California, Berkeley
Professor Shmuel S. Oren, Chair

Unlike conventional generators, the output of a renewable generator is only partially controllable and unpredictable. Consequently, its output level exhibits wide variability. The increasing amount of such kind of renewable resources in the generation mix of the electric power infrastructure poses new challenges to system operations. Due to the limitations of storage in current power systems, demand and supply of electricity must be matched instantaneously. Moreover, the flow of power is governed by Kirchhoff’s laws and is prone to congestion. In order to serve load reliably in power systems with large-scale renewable integration, extreme ramping requirements on conventional generation resources will be imposed. The system operators could mitigate those requirements by deploying more reserves or increased amount of flexible generation resources which could undermine both the economic and environmental objectives of deploying renewable resources. Instead of utilizing the flexibility provided by conventional generation, we will focus on exploiting flexibility from the transmission system to mitigate the uncertainty and variability of renewable generation for power systems day-ahead scheduling.

In this dissertation, we focus on harvesting flexibility from the transmission system to mitigate the uncertainty of renewable generation. There are two aspects of the such flexibility. First, we allow actively controlling the topology of the transmission network by switching on or off transmission lines as a recourse option. Second, the transmission lines can temporally adopt higher ratings to avoid the underutilization of the line capacity caused by the conventional conservativeness in line rating calculations. We adopt the two-stage stochastic unit commitment model for the power system day-ahead scheduling problem. Both transmission switching decisions and flexible line rating decisions are modeled as binary decisions in the second stage when renewable generation is realized. A scheme that decomposes an interconnected commercial system into zones is proposed to solve this large-scale stochastic mixed integer programming problem within a reasonable amount of time. In the sub-problem for each zone, conventional generators are separated into a set of slow-ramping generators and a set of fast-ramping generators. The commitment decisions of slow generators are first-stage decisions which are made before the realization of renewable generation. The commitment
decisions of fast generators and the dispatch decisions are second-stage decisions. Topology control decisions through switching on/off transmission lines and flexible line rating decisions are modeled as recourse actions. Such recourse actions provide the possibility of actively control the transmission system in response to the variability and the uncertainty of renewable generations. Comparing with deploying more flexible conventional generators which are expensive, flexible transmission network recourse incurs no additional cost other than the cost associated with the abrasions of breakers.

We provide demonstration studies based on the IEEE 118 system and a network representing the Central European system where there are over 650 conventional generators, 679 buses, and around 1000 transmission lines. We observe that the operating cost can be reduced by over 10% for the IEEE 118 system with purely topology control recourse. The cost reduction for the Central European system is over 3%, and substantial reduction is observed for some control areas with topology control recourse. The operating cost can be reduced if flexible line rating recourse is included.
To my dear parents,

Shujie Zhang and Yong-an Shi
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Chapter 1

Introduction

1.1 Overview

Renewable generation is the fastest-growing contributor to the electricity portfolio across the world. In the United States, different states adopt different Renewable Portfolio Standards (RPS). Taking California as an example, the RPS program requires investor-owned utilities, electric service providers, and community choice aggregators to increase procurement from eligible renewable energy resources to 33% of total procurement by 2020. However, the massive integration of renewable resources into the generation mix of the electric power infrastructure poses new challenges to system operations. In contrast with conventional generations, the renewable generation output is constrained by uncertain natural resources such as wind and solar. We can neither get accurate enough renewable generation predictions until at most few hours before delivery time. Moreover, renewable generation cannot be controlled so as to generate the desired amount of electricity. Moreover, the forecast accuracy of such generation resources is relatively low up to few hours ahead of the delivery time. Due to the limitation of large-scale storage in current power systems, demand and supply must be matched instantaneously. The flows of power throughout the electricity networks are governed by Kirchhoff’s laws and are prone to congestion. To serve the demand reliably with the presence of variability and uncertainty of renewable generations, more flexibility, which can be obtained from generation, transmission, and demand, is required.

Power systems are designed to have some inherent level of flexibility in operations to balance supply and demand at all times, such that unpredictable load changes and unexpected conventional generation failures would not affect the reliability of the system. Conventionally, the operational flexibility requirements are met by the deploying of a set of conventional controllable generators with enough headroom and ramping capability. For systems with large-scale renewable generation, resource uncertainty must be explicitly accounted for in the day-ahead unit commitment. This can be achieved by optimizing the unit commitment as a two-stage stochastic programming problem. Such optimization typically differentiates between slow ramping resources that must commit before the uncertainty of renewable re-
CHAPTER 1. INTRODUCTION

sources is realized, and flexible resources that provide recourse capability in response to
diverse realizations of renewable generation. Recourse actions serve as hedging mechanisms
that reduce the need for reserves provided by the early commitment of slow ramping gen-
erators. With large-scale renewable generation integration, the system operators need to
exploit more flexibility, which we define in this dissertation as the ability to deploy variable
resources to meet variable demand, from the system as recourse actions.

Flexibility can be obtained from generation, transmission, and demand of power systems.
On the generation side, more reserve from conventional units can be scheduled to ensure
that the planning of units’ output can withstand the uncertainty introduced by renewable
generation. However, this might undermine the goal of utilizing environmentally friendly
resources to supply electricity. It can also be costly to have more reserve from conventional
units. On the demand side, demand response and storage, which is still expensive at the
current stage, can also serve as flexible resources. Demand response can be modeled as
virtual generators on aggregator level and participate in the day-ahead market as flexible
resources. Extensive research has also been conducted how to incentivize energy consumers
to participate in demand response.

In this dissertation, we will focus on harvesting the flexibility from transmission systems
through topology control by switching on/off transmission lines and strategically changing
the rating of transmission lines, denoted as flexible line ratings. Switching a transmission
line off is equivalent to set the rating of a line to be zero and set the impedance of the
transmission line to be infinity. Thus, topology control can be considered as a special case
of flexible line ratings. Switching decisions and flexible line rating decisions are modeled as
second-stage recourse decisions in two-stage stochastic unit commitment problem. Practical
issues such as validating the stability of the system after switching off lines or altering line
ratings are out of the scope of the research. Here we are only focusing on optimizing the
recourse decisions. However, we note that if the switching of lines or the altering of line
ratings may undermine the stability of the system, we could put a constraint to exclude such
a line from the feasible region and solve it again.

1.2 Flexibility in Transmission System

How to make better use of the existing infrastructure of power system and to enable the
integration of large-scale renewable generation draws attention from both academia and
industry. Conventionally, transmission systems are considered as uncontrollable static assets
in the operations of power systems. However, in practice, system operators can and do change
both the topology of the network and the ratings of transmission lines as post-contingency
control actions. Such control actions are taken in an ad hoc way based on experiences of
system operators. By including such control actions in the scheduling and dispatching of
power systems, we can harness flexibility from existing transmission assets and operate the
system reliably with deep renewable penetration.
The transmission system can provide flexibility in three ways. First, the operators can alter the topology of the system through switching on/off transmission lines. In order to ensure mandate reliability, transmission networks are built to be redundant. Such redundancy can undermine the efficiency of generation scheduling and system operations. The optimal topology for different loading conditions varies. Co-optimizing the topology and the generation scheduling decisions will reduce the operation cost of the system. Moreover, the switching of transmission lines incurs no additional cost other than possible wear of the breakers, which is typically small comparing to the potential benefits. Second, they can also change the flow limit of transmission lines to allow generations with lower costs to be dispatched to mitigate the uncertainty. The flow limits of transmission lines are determined given extreme conservative ambient environment conditions and regardless of real-time current flow. It is possible to temporarily increase the flow limit of a small subset of transmission lines in order to reduce inefficient dispatching decisions caused by flow congestion, hence improve the efficiency of the whole system. Last, the impedance of transmission facilities could also be controlled using FACTS equipment. In contrast to the transmission switching and flexible line rating, which utilize existing transmission assets, this requires installing additional FACTS devices on transmission lines. In our research, we focus on utilizing the flexibility provided by transmission switching and flexible line rating.

1.3 Outline of the Dissertation

In Chapter 2, we have provided a thorough literature review of day-ahead scheduling models for power systems with large-scale renewable integration, transmission switching, and flexible line rating.

In Chapter 3, we have presented an overview of the power system day-ahead scheduling problem. In particular, we provide the formulation of both deterministic unit commit and stochastic unit commitment. Both stochastic unit commitment with topology control recourse (TCSUC) and stochastic unit commitment with flexible line rating recourse (FLRSUC) are based on the stochastic unit commitment model we present in Chapter 2.

In Chapter 4, we model topology control through transmission switching as a recourse action in the day-ahead operation of power systems with large-scale renewable generation resources. We prove that transmission switching could only reduce the linear objective value of direct current optimal power flow (DCOPF) when congestion exists. However, we also show, through a simple example, that unit commitment cost could be reduced by transmission switching even in the absence of congestion. To solve the stochastic unit commitment with topology control within reasonable computational time, we proposed a heuristic that first decomposes a practical system into zones and then solves the problem for each zone in parallel. The benefit of topology control recourse is demonstrated on IEEE 118 benchmark system and a network representing the Central European System.

In Chapter 5, we propose an optimization model that determines when and which line adopt higher ratings calculated based on anticipated weather conditions and loading con-
ditions in the recourse of a stochastic unit commitment problem. Flexible line ratings in the recourse help mitigate the uncertainty introduced by renewable generation and improve first-first stage commitment decisions. Numerical tests conducted on both IEEE 118 system and a network representing the Central European System demonstrate that with flexible line rating recourse, the expected operating cost will be substantially reduced.

In Chapter 6, we summarize all findings and provide concluding remarks.
Chapter 2

Literature Review

2.1 Introduction

The ideas of altering transmission network topology and transmission line ratings originated in the 1980s and the 1970s, respectively. Both are utilized by system operators as post-contingency corrective control actions. In this chapter, we present a thorough review of existing literature on these two topics. For transmission switching, we first go over early research on using it to relieve overloading and voltage violations after contingency occurs. We then focus on recent research on co-optimizing switching decisions and power system dispatching, scheduling, or planning decisions in order to achieve higher operational efficiency. For flexible line rating, we review how static line ratings are calculated in practice. Then we summarize existing literature on dynamic ratings.

2.2 Transmission Switching

The idea of transmission switching has been studied for decades. Transmission lines are switched on/off as preventive or corrective actions to enhance system security by relieving overload conditions [1]. In [2], the author proposed an approach to modeling the switching operations. He discussed the effects of switching operations and how it can be used as control methods in post-contingency operations. He also proposed a searching algorithm to solve the problem with various objectives such as minimizing loss or minimizing violations. In [3], the authors utilized transmission switching as corrective actions to deal with both load violation and voltage violation. They first reviewed the previous on relaxing switching decisions as continuous variables, and then formulate the problem in which switching decisions are modeled as discrete control variables. In [4], the authors proposed a series of methods that combines security analysis and optimal power flow to maintain N-1 reliability. Transmission switching serves as a corrective control action in their paper. In the most recent paper [5], the authors compared the results of contingency in day-ahead unit commitment, with and
without transmission switching. They showed that corrective transmission switching helped not only in post-contingency situations but also in achieving N-1-1 reliability.

Other than serving as a corrective control action or for the purpose of reducing system loss, transmission switching has also been studied as a method to harness flexibility from existing transmission systems to reduce the system operating cost. Fisher et al. [6] first proposed optimal transmission switching (OTS) in the context of DCOPF. In the proposed formulation, the on/off status of transmission lines is modeled as a binary variable. In their paper, they modified the $B - \theta$ formulation of DCOPF to include the switching decisions. The mathematical formulation of OTS in the paper is:

$$OTS: \min \sum_g c_{ng} P_{ng}$$

s.t. $\theta_n^{\text{min}} \leq \theta_n \leq \theta_n^{\text{max}}$ (2.1)

$P_{ng}^{\text{min}} \leq P_{ng} \leq P_{ng}^{\text{max}}$, for all generators $g$ at bus $n$ (2.2)

$P_{nk}^{\text{min}} z_k \leq P_{nk} \leq P_{nk}^{\text{max}} z_k$, for all lines $k$ at bus $n$ (2.3)

$\sum_k P_{nk} + \sum_g P_{ng} - \sum_d P_{nd} = 0$, for all bus $n$ (2.4)

$B_k (\theta_n - \theta_m) - P_{nk} + (1 - z_k) M \geq 0$, for all lines $k$ with endpoint $n$ and $m$ (2.5)

$B_k (\theta_n - \theta_m) - P_{nk} - (1 - z_k) M \geq 0$, for all lines $k$ with endpoint $n$ and $m$ (2.6)

$\sum_k (1 - z_k) \leq j$ (2.7)

where $P_{ng}$ is the output of generator $g$ connected to bus $n$, $P_{nk}$ is the line $k$ connected to bus $n$, and $P_{nd}$ is the demand $d$ at bus $n$. In OTS, $M$ is a large number and the number of open lines in the network is limited to be smaller than $j$. The $B - \theta$ formulation of OTS takes the flow limit of all transmission lines into consideration. All transmission lines can be switched in this formulation to achieve the optimal topology of the network. Transmission switching makes it possible to choose the best system topology so that the power generation is optimized on that topology.

The optimal transmission switching(OTS) problem can be considered as a relaxed version of DCOPF in the sense that we relaxed the constraint that forcing the on/off status of all lines to be on. Hedman et al. [7] extended the initial study on OTS and presented how transmission switching will affect nodal prices, load payment, generation revenues, cost, and rents, congestion rents, and flow gate prices. The paper also provided numerical results on IEEE 118 system in which transmission switching reduced the operation cost by 25%. In [8], the authors show that both the IEEE 118 system and RTS 96 system can be operated to satisfy N-1 requirement while reducing operation cost by optimizing transmission switching decisions in the DCOPF setting.
To solve the problem efficiently, researchers have developed heuristics to obtain near optimal switching decisions. Ruiz et al. proposed fast heuristics for \textit{OPF} with topology control instead of solving MIP directly [9, 10, 11]. The authors proposed the Power Transfer Distribution Factors (\textit{PTDF}) formulation in their papers and developed heuristics based on the \textit{PTDF} formulation of \textit{OTS}. Instead of directly modeling the switching decisions as removing or adding lines to the susceptance matrix, they proposed flow-cancelling transactions, an alternative approach that maintained the original topology of the system.

A power transfer that would cancel the flow on the interfaces between the rest of the system is applied in the flow-cancelling transactions approach. By introducing such virtual injections, lines are equivalent to be switched from the view of the rest of the system. To model the outage of line $k$, two virtual nodes $m'_k$ and $n'_k$ that are infinitely close to the terminal nodes $m_k$ and $n_k$ are introduced as shown in Figure 2.1. The transaction from $m'_k$ and $n'_k$ with magnitude $\nu_k$ which makes the impact of the transaction on the rest of the system is equivalent to the opening of line $k$. To satisfy this condition, we need to ensure the flow on the interface between link $k$ and the rest of the system to be zero. Let $\phi_{k}^{m'_k n'_k}$ denote the \textit{PTDF} factor, we have:

$$F_k - (1 - \phi_{k}^{m'_k n'_k})\nu_k = 0$$

By assuming that only a small set of transmission lines is constrained and monitored by the system operator, we can adopt the \textit{PTDF} formulation to reduce the number of decision variables and constraints. Let $\mathcal{M}$ be the set of monitored facilities, $f^\mathcal{M}_\tau$ and $\bar{f}^\mathcal{M}_\tau$ be vectors...
with transmission limits of monitored non-switchable facilities under topology \( \tau \). \( \Phi^M_\tau \) is denoted as the reduced shift factor matrix associated with monitored lines under topology \( \tau \), and \( \Phi^{MS}_\tau \) is the PTDF matrix of transactions between the terminal nodes of switchable lines with respect to lines in \( \mathcal{M} \) under topology \( \tau \). The PTDF – OTS can be formulated as:

\[
(PTDF – OTS) : \\
\min c^T_G P_G \\
s.t. \ 1^T (P_G - 1) = 0 \\
\quad P_G^{\min} \leq P_G \leq P_G^{\max} \\
\quad f^M_\tau \leq \Phi^M_\tau (P_G - 1) + \Phi^{MS}_\tau \nu_\tau \leq f^M_\tau \\
\quad \tilde{F}^S_\tau z \leq \Phi^S_\tau (P_G - 1) + (\Phi^{MS}_\tau - I) \nu_\tau \leq \tilde{F}^S_\tau z \\
\quad - M (1 - z) \leq \nu_\tau \leq M (1 - z) \\
\quad z \text{ is binary}
\]

The PTDF – OST formulation is equivalent to the \( B - \theta \) formulation in which the monitored set of facilities and the set of switchable lines are the same. The PTDF – OST formulation is significantly smaller than the \( B - \theta \) formulation if the cardinalities of \( \mathcal{M} \) and \( \mathcal{S} \) are both small. However, as the number of switchable lines increases, the time of solving the problem increases and it is not beneficial to use the PTDF – OTS formulation. Another issue of this formulation is that only a small subset of transmission lines are monitored.

In the recent research paper [12], the authors extended the optimal switching in real-time operations into the context of alternative current optimal power flow (ACOPF). They proposed a two-level iterative framework, in which a second order cone programming is solved to provide candidate optimal switching solution in the upper level and then the solution is screened to achieve AC feasibility in the lower level.

Transmission switching are also considered in other models for power systems. It has been shown that transmission switching can reduce the investment cost in power system expansion. In [13], the authors formulate the line capacity expansion problem as a two-stage stochastic programming problem, and switching decisions are made in the second stage with other operational decisions, while the investment decisions are made in the first stage. Their results showed that with transmission switching, the network could be augmented cheaper with respect to total cost including both investment cost and operational cost. Granelli et al. [14] studied utilizing transmission switching to conduct congestion management. The objective of the problem they formulate is to minimize the cost associated with overloading of transmission lines and transformers. The optimization of generation dispatch is not considered. In [15], the authors compared the congestion management cost in Germany given a high share of renewable generation with and without transmission switching. A transaction spot market model and a congestion management model are utilized to replicate the market regime. The results show that current congestion can be solved by re-dispatching.
generations and transmission switching. However, for future power systems, there is a need for improving the current congestion management regime to improve long-term efficiency.

In [16], the authors studied how transmission switching will change the optimal deterministic unit commitment as well as the optimal cost. The switching decisions and deterministic security-constraint unit commitment decisions are optimized iteratively. In this deterministic problem, the cost reduction is not as substantial as in optimal transmission switching on the IEEE 118 system. For day-ahead scheduling problems with large-scale renewable integration, more cost reduction is expected since the flexibility provided by transmission switching can mitigate the uncertainty of renewable generation which may lead to more efficient commitment decisions. We have shown in our previous paper [17] that transmission switching as a recourse action in response to realized uncertainty of intermittent renewable resources could mitigate such adverse variability and improve unit commitment efficiency in IEEE 118 system.

2.3 Line Ratings of Overhead Transmission Lines

Transmission owners and system operators determine static ratings for transmission facilities based on fixed meteorological and operating conditions. According to the PJM Transmission Operations Manual [18], three sets of thermal limits listed in a non-decreasing order are provided for all monitored equipment: normal limit, emergency limit (long-term and short-term limit) and load dump limit. System operators make dispatching decisions according to the normal limits. However, transmission facilities can stand for emergency limit within a prespecified period of time without violating the safety codes or jeopardizing the conductor. There are totally 16 sets of the three ratings provided for each monitored transmission facility. Eight ambient temperatures are used with a set for the night and a set for the day period. Transmission owners’ and the RTO’s security analysis programs must be able to handle all 16 sets of ratings and allow the operating personnel to select appropriate set for system operation. In this section, we will review how static line ratings are calculated, illustrate what is dynamic line ratings, and review how dynamic line ratings are incorporated into power system operations in existing literature.

Static Line Ratings

When calculating thermal-limited overhead transmission lines, the maximum current that can flow is determined by the maximum allowable temperature of the conductor. The temperature of a line should not be too high to avoid excessive sags and possible thermal damage to the conductor. Both IEEE [19] and Cigré [20] have standards to provide guidance for calculating the ampacity and the temperature of the bare overhead transmission lines. The heat exchange of overhead transmission lines is shown in Figure 2.2.

In both standards, the thermal behavior of conductors is modeled using a heat balance equation (HBE) which is used to model the fact that the heat gain of a conductor should
equal the heat loss at any time. The HBE can be expressed as:

\[ q_c + q_r + m \cdot C_p \frac{dT}{dt} = q_s + I^2 R(T) \]  \hspace{1cm} (2.17)

where \( q_c \) is the convection heat loss, \( q_r \) is the radiated heat loss, \( m \cdot C_p \) measures the thermal inertia of the conductor, \( q_s \) is the heat gain from the sun, and \( R(T) \) is the resistant of the conductor at temperature. This first order differential equation models how the temperature of a bare conductor responds to changes in the current and the ambient environment. There are two types of convection heat loss: natural convection and forced convection. Natural convection captures the phenomenon that cool air surrounding the heated conductor get heated and rises. It is equivalent to forced convection with a wind speed less than 0.2 \( m/s \), and has less power in cooling the conductor comparing to the forced convection. It’s mainly determined by the speed of the wind and the angle of the wind. The radiated heat loss models the energy transmitted by radiation to the surroundings when the conductors heated to a temperature above its surrounding air. The heated gain from sun radiation captures the amount of solar heat energy delivered to the conductor. This value of \( q_s \) is influenced by lots of factors such as the orientation of the conductor and the surface condition. The last term in the HBE is determined by the current flow on the conductor and the resistant of the conductor. The resistant of the conductor is a function of the power frequency, current, and the temperature. In the IEEE standard, a linear interpolation of AC resistance that only considers the influence of conductor temperature is utilized. The convection heat loss \( q_c \) and the radiated heat loss \( q_r \) are determined by the conductor’s surface temperature and steady stage ambient weather conditions. For steady state consideration, we can set \( \frac{dT}{dt} \) to be zero. Given the maximum allowable temperature and the ambient weather conditions, we can utilize the steady-state HBE to calculate a static thermal rating for the conductor. In this case, the heat gain caused by the current flows in the conductor is larger than when the current is below the thermal limit. Conventionally, in the practice of power system
planning, operations, and real-time control, as well as in most of the existing literature, static line ratings are utilized as parameters to model the capacity for lines.

The Cigré report [21] provides guides for the selection of weather conditions to calculate line ratings. One of the underlying assumptions in selecting the ambient weather condition is that conservative weather conditions are chosen to ensure safety. However, ambient conditions have substantial impacts on the thermal limit of transmission lines. For example, the static rating for ACSR Drake is 1047 A if the ambient temperature is 40°C. When the ambient temperature is 30°C, this value increased to 1139 A. The thermal rating is increased by around 9% when the line is rated for high operating temperature. If the line is rated for low operating temperatures, the impact of choosing ambient temperatures is more essential. For solar radiation, it is mostly assumed a condition of midday, clear sky perpendicular solar radiation, which captures the server radiation conditions. Wind speed and wind direction have an even larger impact on determining the line ratings. But these parameters are more difficult to estimate. References [22] and [23] suggest that meteorological conditions are correlated. High solar radiation induces local winds, and during high ambient temperatures, the opacity of the sky is significant which will reduce the solar radiation. According to [24], when calculating the thermal limitings, the 98% of the expected worst-case values are selected for these key environmental parameters. Furthermore, the assumptions suppose that adverse operating conditions all occur at the same time. Bucher [25] conducted a sensitivity analysis of the ampacity of the ACSR Drake transmission line with respect to different meteorological conditions. The results show that the influence of ambient conditions could be substantial. In some cases, the ampacity was more than twice as high as in the base case. Last but not least, the current flow on the conductor also contributes to heating the conductor. When computing the ratings, we always assume that the current are at its highest value. However, in practice, the current flow on the conductor can be much smaller than the limit. All these assumptions lead to a conservative estimation of the capacity of overhead transmission lines and underutilization of the capacity of overhead transmission lines. In power system operations, such conservative estimations of line ratings will introduce “artifical” network congestions and results in inefficient scheduling of generators.

**Dynamic Line Ratings**

In the HBE, the weather conditions influence both the heat gain and the heat loss of the conductor. The loading conditions of the line also have impact on the actual temperature of the transmission line. Since the static rating is determined based on very conservative assumptions of weather conditions, and for most of the time the lines are not operating at their ratings, the actual capacity of the line should be higher than the normal rating. Dynamic line ratings adapt the prevalent weather conditions, real-time conductor temperatures and actual loading of transmission lines.

Davis [26] first proposed dynamic ratings in 1977. Actual meteorological data and real-time conductor temperatures and line flow are used to calculate real-time ratings for overhead transmission lines. Since then, different aspects of dynamic rating has been studied. In [27],
the authors assessed reported an extensive one year multi-station power line study result which showed that transmission lines are underused by around 20%. Seppa [28] proposed a model for calculating the line rating based on the relationship between conductor’s temperature and its sag. In [29], the authors analyze the benefit of increasing transmission line capacity by real-time monitoring. The cost of installing monitoring equipment on transmission lines to implement real-time rating is compared with the cost of building new lines. The result shows that the cost for the real-time rating is only a fraction of the cost of investing a major physical upgrade. In recent literature [30] and [31], probabilistic forecast of dynamic ratings is studied.

In recent literature, researchers have explored how to include dynamic line ratings in both day-ahead and real-time operations to reduce the cost of transmission constrained power systems especially for those with large-scale renewable integration. Fu et al. [32] studied the impact of dynamic line rating on power systems. They quantified the extra power facilitated by dynamic line ratings. Results showed that with dynamic line ratings the test system could accommodate more renewable generation. In [33], the authors proposed a model to estimate real-time ratings and compared dynamic line ratings and static line ratings in an economic optimization simulation model. Their results showed that dynamic line rating facilitated wind generation integration. In [34], the authors included the heat balance equation in security constrained unit commitment. Representative scenarios of weather conditions are selected as parameters in the formulation. They utilize a convex approximation of the differential equation. In [35], the authors presented an approach on how to include dynamic line ratings in an N-1 secure dispatch optimization. The evolution of the conductor temperature is simulated using the basic Euler forward method, and the power flow is then guided by constraints on the conductor temperature rather than by the traditional static line ratings on power flow. In [36], the authors incorporate dynamic line ratings in security constrained economic dispatch. In the proposed approach, the real-time ratings are first calculated and then used to update the parameters in the security economic dispatch. In practice, Oncor is the first transmission owner that was able to integrate the dynamic ratings directly into the Electric Reliability Council of Texas’s (ERCOT’s) security constrained economic dispatch model [24].

According to the report of ENTSO-E, many TSOs use dynamic line rating in testing and operations with technologies. But, currently, dynamic line ratings are only used for information, alarms to the dispatchers and others. Further study is still required to fully incorporate dynamic line ratings in system operations and planning.
Chapter 3

Overview of Unit Commitment

3.1 Notations

This section lists the notation that is commonly used in this and the following chapters.

**Sets:**
- $T$: Set of time periods $1, 2, \ldots, 24$.
- $G$: Set of generators.
- $N$: Set of buses/nodes.
- $N(i)$: Set of buses/nodes that have transmission lines connected to bus $i$.
- $GF$: Set of fast generators.
- $GS$: Set of slow generators.
- $RG$: Set of renewable generators.
- $S$: Set of scenarios.
- $Z$: Set of control zones.

**Indices:**
- $t$: Time period indices; $t \in T$.
- $i, j$: Bus/node indices; $i, j \in N$.
- $g$: Generator indices; $g \in G$.
- $s$: Scenario indices; $s \in S$.

**Parameters:**
- $h_g$: Start-up cost of generator $g$.
- $k_g$: No-load cost of generator $g$.
- $c_g$: Fuel cost of generator $g$.
- $\rho_i$: Penalty cost of load shedding on bus $i$.
- $F_{ij}^{\text{max}}$: Flow capacity of line $ij$.
- $B_{ij}$: Susceptance of line $ij$.
- $\pi_s$: The probability of scenario $s$. 
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\[ P_{g_{\text{max}}} \] Maximal production level of generator \( g \).
\[ P_{g_{\text{min}}} \] Minimal production level of generator \( g \).
\[ R_{g}^{U} \] Maximal ramping up rage of generator \( g \).
\[ R_{g}^{D} \] Maximal ramping up rage of generator \( g \).
\[ \omega_{g_{\text{on}}} \] Minimal on-time of generator \( g \).
\[ \omega_{g_{\text{off}}} \] Minimal off-time of generator \( g \).
\[ \Gamma_{t} \] System reserve requirement at time \( t \).
\[ D_{i,t} \] Load on bus \( i \) at time \( t \).
\[ D_{i,t_{\text{net}}} \] Net load on bus \( i \) at time \( t \).

**Variables**

\[ u_{g,t,(s)} \] Commitment of generator \( g \) at time \( t \) (in scenario \( s \)).
\[ \sigma_{g,t,(s)} \] Start-up indicator of generator \( g \) at time \( t \) (in scenario \( s \)).
\[ P_{g,t,(s)} \] Production level of generator \( g \) at time \( t \) (in scenario \( s \)).
\[ \gamma_{g,t} \] Reserve of generator \( g \) at time \( t \).
\[ F_{ij,t,(s)} \] Active power flow on line \( ij \) at time \( t \) in scenario \( s \).
\[ \theta_{ij,t,(s)} \] Voltage angle of bus \( i \) at time \( t \) in scenario \( s \).
\[ r_{ij,t,(s)} \] On-off status of line \( ij \) at time \( t \) in scenario \( s \).
\[ L_{i,t,(s)} \] Load shedding on bus \( i \) at time \( t \) in scenario \( s \).

### 3.2 Introduction

The total load in power systems follows the cycles of human activities. The load during the daytime and early evening are usually higher than that of late night. We need adequate controllable generation in the system to follow the changing load given that large-scale storage system is still economically inefficient. There are various types of generators with significantly different characteristics distributed across the power network. For a conventional generator, it first transforms heat energy into kinetic energy of a turbine. Then, the kinetic energy will be transformed to electrical energy. Taking steam units as an example, to start a unit, it requires heating the boiler first. The turbine with large inertia also needs to speed up to generate electricity. It is impossible to shut down a steam unit and start it up within a short period of time. Moreover, due to fuel combustion stability and inherent steam generator design constraints, the minimum load is generally about 30% of the designed capacity. In power system day-ahead scheduling problems, we would like to balance the supply and demand with minimal cost. The unit commitment problem is formulated to schedule the start-up and shutdown of units in order to meet demand with the lowest cost, which can account for both the physical constraints of generators and line flow constraints of power systems. In this chapter, we will go over different formulations of the unit commitment problem without and with renewable generation. We will also review solution techniques of the unit commitment problem.
3.3 Unit Commitment without Renewable Generation

Two major types of decisions need be made in the day-ahead scheduling of power generation. First, the discrete decisions indicate the commitment of a certain unit at each point in time over the scheduling horizon. The unit commitment decisions are made taking capacity requirements, run-time constraints of units, and reserve requirement into consideration. The continuous decisions are the allocation of demand of power among the generating units. The unit commitment problem optimizes the discrete commitment decisions and continuous economic dispatch decisions to achieve the lowest cost while maintaining acceptable system reliability level.

Generation unit commitment problem is a multi-period mixed integer programming that is difficult to solve. In a generic model of unit commitment, binary variables represent the commitment, and continuous variables represent the production levels of units and network flow conditions such as voltage angle at each bus and active power flow on each line. The mathematical formulation of deterministic unit commitment (DUC):

\[
(DUC) : \min \sum_{t \in T} \sum_{g \in G} h_g \sigma_{g,t} + k_g u_{g,t} + c_g P_{g,t} \tag{3.1}
\]

s.t. \[
\sum_{i \in N, j \in N(i)} F_{ij,t} - \sum_{k \in N(i), r \in N} F_{ki,t} + \sum_{g \in G, g \in N(i)} P_{g,t} - D_{i,t} = 0, \forall i \in N, t \in T \tag{3.2}
\]

\[
F_{ij,t} - B_{ij}(\theta_{i,t} - \theta_{j,t}) = 0, \forall i, j \in N, t \in T \tag{3.3}
\]

\[
-F_{ij}^{\max} \leq F_{ij,t} \leq F_{ij}^{\max}, \forall i, j \in N, t \in T \tag{3.4}
\]

\[
P_{g,t} + \gamma_{g,t} \leq P_{g,t}^{\max}, \forall g \in G, t \in T \tag{3.5}
\]

\[
P_{g,t} \geq P_{g}^{\min} u_{g,t}, \forall g \in G, t \in T \tag{3.6}
\]

\[
\gamma_{g,t} + P_{g,t} - P_{g,t-1} \leq R_{g}^{U}, \forall g \in G, t \in T, t \geq 2 \tag{3.7}
\]

\[
P_{g,t-1} - P_{g,t} \leq R_{g}^{D}, \forall g \in G, t \in T, t \geq 2 \tag{3.8}
\]

\[
\sigma_{g,t} \geq u_{g,t} - u_{g,t-1}, \forall g \in G, t \in T, t \geq 2 \tag{3.9}
\]

\[
\sum_{\tau = t - \omega_{g}^{on} + 1}^{t} \sigma_{g,\tau} \leq u_{g,t}, \forall g \in G, t \geq \omega_{g}^{on} \tag{3.10}
\]

\[
\min(t + \omega_{g}^{off}, 24) \sum_{\tau = t + 1} \sigma_{g,\tau} + u_{g,t} \leq 1, \forall g \in G, t \in T, t < 24 \tag{3.11}
\]

\[
\sum_{g \in G} \gamma_{g,t} \geq \Gamma_{t}, \forall t \in T \tag{3.12}
\]

\[
u_{g,t} \in 0, 1, \sigma_{g,t} \in [0, 1], \forall g \in G, t \in T \tag{3.13}
\]

\[
P_{g,t}, \gamma_{g,t} \in R^{+} \tag{3.14}
\]

The objective of generation unit commitment as in (3.1) is to minimize the total generation cost in the scheduling time horizon. There are three components of costs in day-ahead...
generation scheduling: start-up cost, no-load cost, and incremental cost. Start-up costs are the cost of bringing the generator from shutdown conditions to a state that is ready to synchronize to the transmission systems. It is a fixed cost and is incurred once each time the unit starts, regardless the period of the operation. No-load cost is the fixed cost of generating electric energy. The incremental energy cost is the cost per $\text{MW} \cdot \text{h}$ to produce all of the energy segments above the minimum generation level. The incremental energy cost is calculated by summing the cost of each segment of energy in the unit’s incremental cost curve up to the generation level at a particular time period. The sum of incremental energy cost and the no-load cost is the hourly production cost of that generator. The total cost of a unit is the sum of the start-up cost and the total hourly production cost, which can be represented as below:

$$Total \ Generation \ Cost = Start-up \ Cost + \sum_{t \in T} \text{Hourly \ Production \ Cost} \quad (3.15)$$

For different types of units, the three components have different characteristics. For example, combustion generators have higher production cost while steam units have higher start-up cost.

To ensure reliable operations, several constraints must be satisfied in a unit commitment problem. There are two types of constraints. The first type is system-wide constraints and the second type is generator constraints. System-wide constraints include market-clearing constraints (3.2), power flow constraints (3.3), and line flow capacity constraints (3.4). Market-clearing constraints enforce that demand and supply are balanced at each node. Power flow constraints are DC power flow equations that represent line flows in terms of voltage angles at terminal buses and the susceptance of transmission lines. Line flow capacity constraints require that active power flow on transmission lines never exceed the limits, which are determined by thermal limits or stability requirements. Generator constraints consist of generator capacity constraints (3.5, 3.6), ramping up constraints (3.7), ramping down constraints (3.7, 3.8), on-off transition constraints (3.9), minimum on time constraints (3.10), minimum off time constraints (3.11), and reserve requirement constraints (3.12). The generation capacity constraints indicate that if at time $t$ a unit is committed, the sum of production level and reserve capacity should be less than or equal to the maximum production level. Moreover, the generation of committed units should be above the minimum production level. The ramping constraints ensure that the change of generation in each time period is within the ramping limits of generators. The on-off transition constraints link the start-up variables and the commitment decisions of generators. The minimum on/off constraints show that if a generator has been turned on or off, it should remain in the same status for a minimum period of time. The reserve constraints mean that the system reserve requirement is satisfied at each period of time. In the deterministic unit commitment model, the production level and the reserve capacity are non-negative continuous variables. The commitments of generators are binary variables. The start-up variables can be relaxed as continuous variables between 0 and 1.
3.4 Unit Commitment with Renewable Generation

The integration of renewable generation introduces variable supply to the system. In order to instantaneously balance the supply and demand in electricity in the system with renewable generation, we need to deploy conventional generation and other operational flexible resources efficiently. Various models have been studied for unit commitment of systems with renewable generations. In this section, we will go over three types of unit commitment models that take renewable resources into consideration. We will focus on how renewable generation is modeled and the differences in the decision-making process among the three models.

Deterministic Formulation

One of the simple approaches to incorporate renewable generation in unit commitment is to model exogenous requirements on excess generation and import of electricity from other regions in a deterministic formulation of unit commitment as described in [37] and [38]. Such a deterministic formulation of unit commitment with renewable generation is relatively easy to solve compared with the other two types of formulations. The authors simulated the day-ahead unit commitment using the data of ERCOT. The formulation is similar to that of the deterministic unit commitment problem without renewable generation. With renewable generation, the scheduled of renewable generation is modeled as a decision variable $g_{r,t}$, which is below the predicted renewable generation $\bar{g}_{r,t}$:

$$g_{r,t} \leq \bar{g}_{r,t}, \forall r \in RG, t \in T$$

This constraint means that the scheduled renewable generation can not exceed the available renewable generation but can be lower due to curtailment or some form of modulation. The uncertainty of renewable generation is modeled using a rating factor $\rho$, which is a predefined parameter ranging between 0 and 1. The realized renewable generation is the rating factor times the scheduled renewable generation.

$$\sum_{g \in G} \gamma_{g,t} \geq \Gamma_t + (1 - \rho) \sum_{r \in RG} \bar{g}_{r,t}, \forall t \in T$$

The reserve is enforced to be above a value plus the possible unrealized renewable generation as illustrated in inequality (3.17). In order to ensure reliable system operations, we should choose a small enough rating factor. However, this will impose a conservative reserve requirement.

Utilizing conservative reserve requirements in unit commitment problem to handle unexpected events such as load forecast error, generator outage, and transmission line outage is not a new technique in power system scheduling. There is extensive research on analyzing the levels of reserve requirements based on deterministic criteria since the 1960s. Anstine et al. [39] proposed a method of evaluating the reserve requirement to maintain a uniform level of risk in the day-to-day operation of an interconnected system with sufficient...
transmission capacity. Billinton et al. [40] proposed a probabilistic method to evaluate the system well-being indices and described how those indices could be used in assessing reserve requirements. The method incorporated the accepted deterministic criteria in the definition of “healthy” and “marginal” power system states. Gooi et al. [41] presented an algorithm that integrated a probabilistic reserve assessment with the unit commitment formulation. In the proposed algorithm, the generation can be scheduled to meet a given risk index. The optimal value of this risk index should be selected on the basis of a trade-off between the total schedule cost obtained from the unit commitment solution and the expected cost of energy not served which can be derived from the probabilistic reserve assessment. In [42], the authors presented a “well-being” framework that enabled us to consider both deterministic and probabilistic approaches in determining system reserve requirement. It provided a useful pedagogical framework for further development. These approaches are easy to implement in practice. However, it could be economically inefficient to mitigate uncertainty using flexible conventional generators as reserves. Moreover, with more renewable generation, the reserve determined by such approaches can be too conservative.

Robust Formulation

Robust optimization is a modeling framework for optimization under parameter uncertainty. It only requires moderate information, such as the expected value, the lower and upper bound of the parameter, about the underlying uncertainty. Available probabilistic information can be easily incorporated into the model. Moreover, the solution to robust optimization immunizes against all realizations of uncertainty within an uncertainty set. The robustness of the solution is consistent with one of the risk-averse goal of power system operation. Robust optimization has been applied in multiple areas in operations research and engineering practice.

In robust optimization, one of the most important steps is to build the uncertainty set of the stochastic parameter. The uncertainty set is a deterministic set in which the uncertain parameter can take any value. There are multiple ways of defining the uncertainty set. In [43] and [44], the wind generation uncertainty set is modeled as an interval bounded by the 0.95-quantile and the 0.05-quantile. The authors also introduced a cardinality budget to restrict the number of time periods in which the wind power output is far away from the predicted value:

\[ W = \{ w \in \mathbb{R}^{N \times T} : w_{i,t} = W_{i,t}^* + z_{i,t}^+ W_{i,t}^+ - z_{i,t}^- W_{i,t}^- , \sum_{t \in T} (z_{i,t}^+ + z_{i,t}^-) \leq \pi_i, \forall i \in N, t \in T \} \quad (3.18) \]

where \( W \) is the uncertainty set, \( w \) is an uncertainty parameter that is a vector consisting of \( |N| \times |T| \) components, \( W_{i,t}^* \) represent the predicted wind generation on bus \( i \) at time \( t \), \( W_{i,t}^+ \) is the 0.95 quantile, and \( W_{i,t}^- \) is the 0.05-quantile. The cardinality budget is \( \pi_i \), and \( z_{i,t}^+ , z_{i,t}^- \) are binary variables. When \( z_{i,t}^+ = 1 \), the wind generation on bus \( i \) reaches the 0.95-percentile.
Similarly, when \( z_{i,j} = 1 \), the wind generation takes the 0.05 percentile. When both values are 1 or 0, the wind generation is the predicted value. In this uncertainty set, there are three possible values for the wind generation on each bus.

Instead of only allowing the renewable generation on each bus at a time period to take 3 different values, [45] and [46] adopts a different definition for the uncertainty set. In their formulation, the renewable generation (or netload in the papers) on each bus can take value from an interval. A budget of uncertainty is imposed in the uncertainty to penalize extreme values of the uncertain parameter. If we adopt the same variables as in (3.18), we have the uncertainty set as:

\[
W = \{ w \in \mathbb{R}^{|N| \times |T|} : w_{i,t} \in [W_{i,t}^-, W_{i,t}^+], \sum_{t \in T} \frac{|w_{i,t} - W_{i,t}^*|}{(W_{i,t}^+ + W_{i,t}^-)} \leq \Gamma_i, \forall i \in N, t \in T \} \tag{3.19}
\]

Instead of utilizing binary variables to select which quantile of the parameters is in the uncertainty set, we adopt the budget of uncertainty constrain to penalize large width of the interval.

Lorca et al. proposed a dynamic uncertainty set for renewable generation in [47] to capture the temporal and spacial correlations of renewable generation. The formulation of the dynamic uncertainty set is:

\[
W = \{ w \in \mathbb{R}^{|N| \times |T|} : \exists \mu, \nu, \text{s.t.} \]
\[
w_{i,t} = \alpha_{i,t} + \beta_{i,t}\mu_{i,t} \tag{3.20}
\]
\[
\mu_t = \sum_{l=0}^{L} A_l \mu_{t-l} + B \nu_t, \forall t \in T \tag{3.21}
\]
\[
||\nu_t|| \leq \Gamma, \forall t \in T \tag{3.22}
\]
\[
\sum_{t \in T} ||\nu_t|| \leq \rho |T| \tag{3.23}
\]
\[
0 \leq w_{i,t} \leq w_{i,t}^{\text{max}}, \forall i \in N, t \in T \tag{3.24}
\]

where \( f \) and \( g \) account for deterministic seasonal components, \( L \) is the time lag, \( \Gamma \) represents a size parameter, \( \rho \in (0, 1] \) represents a budget over time periods, and \( w_{i,t}^{\text{max}} \) is the upper bound for renewable generation. The temporal and spatial correlations between renewable generation are captured by \( A_l \) and \( B \). In the formulation, \( \mu_t \) represents renewable generation after filtering out seasonality patterns \( f_{i,t} \) and \( g_{i,t} \). \( \nu_t \) is the residual uncertainty after temporal and spacial correlations are removed from \( \mu_t \). The size of the support of \( \nu_t \) is constrained by \( \Gamma \). The parameter \( L, A_l \) and \( B \) can be estimated from previous data. Comparing with the static uncertainty sets in which parameters in later time periods are independent of those in previous time periods, dynamic uncertainty set takes the temporal and spatial correlation into consideration. The dynamic uncertainty set is more complicated but it is still computationally tractable. In a unit commitment problem, where we need to dispatch
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generation for today, utilizing dynamic uncertainty set to model renewable generation is more suitable.

The generic compact formulation of robust unit commitment with renewable generation is:

\[(GRUC) : \min \sum_{t \in T} \sum_{g \in G} h_g \sigma_{g,t,s} + k_g u_{g,t} + c_g P_{g,t} \quad (3.26)\]

\[\text{s.t} \ (u, \sigma, \mathbf{P}, \mathbf{F}) \in \Delta(w), \forall w \in W \quad (3.27)\]

where \(u\) is the vector of commitment decisions, \(\sigma\) is the vector of start-up decisions, \(\mathbf{P}\) is the vector of generation dispatch decisions, and \(\mathbf{F}\) is the vector of line flows. All these decision variables are in the feasible set \(\Delta(w)\), where \(w \in R^{|N| \times |T|}\) is the vector of renewable generations. The feasible set is described by all constraints in \(DUC\). The difference is that the constraints need to be satisfied for all \(w \in W\). Thus, the commitment decisions are optimized using the worst case renewable generation in the uncertainty set.

In most of existing literature, researchers model the problem as a two-stage robust optimization problem or multi-stage robust optimization problem instead of adopting the \(GRUC\) directly. In [48], the authors formulated the problem as a two-stage robust optimization problem. The commitment decisions \(u\) and start-up decisions \(\sigma\) are first stage decisions, which are made day-ahead. The generation dispatch decisions \(\mathbf{P}\) and line flow decisions \(\mathbf{F}\) are second stage decisions. The first-stage decisions should be made to hedge against all possible renewable generation realizations which are unknown day-ahead. The second-stage decisions are made after the realization of renewable generation. In most of the existing literature [48, 49, 46, 47], the commitment decisions and the start-up decisions of conventional units are modeled as first-stage decisions while other decisions are modeled as second-stage decisions. The compact formulation of the problem is:

\[(TS - RUC) : \min( \sum_{t \in T} \sum_{g \in G} (h_g \sigma_{g,t,s} + k_g u_{g,t}) + \max_{w \in W} \min_{\mathbf{P} \in \Omega(u, \sigma, w)} \sum_{t \in T} \sum_{g \in G} c_g P_{g,t} ) \quad (3.28)\]

\[\text{s.t} \ (u, \sigma) \in \Delta \quad (3.29)\]

Notice that the worst case dispatch cost has a max-min form. The second term in objective (3.28) \(\min_{\mathbf{P} \in \Omega(u, \sigma, w)} \sum_{t \in T} \sum_{g \in G} c_g P_{g,t}\) minimize the dispatch cost in the second stage for fixed first-stage decisions \((u, \sigma)\) and renewable generation \(w\). Then it is maximized over the uncertainty set \(W\). Set \(\Omega(u, \sigma, w)\) defines the feasible region of generation dispatch, while \(\Delta\) determines the feasible region of first-stage decisions. All constraints for the second stage problem are linear. Thus, for given renewable realization \(w\) the inner minimization part:

\[C(w) = \min_{\mathbf{P} \in \Omega(u, \sigma, w)} \sum_{t \in T} \sum_{g \in G} c_g P_{g,t}\]

is a convex function of \(w\). Then, \(\max_{w \in W} C(w)\) need to maximize a convex function on a given set, which is in general NP-hard. Robust unit commitment with renewable generation
is much harder to solve than deterministic unit commitment. Another issue is that the
solution of robust optimization is always conservative since the problem is solved using the
worst-case parameters in the uncertainty set that rarely occurs. w

**Chance-Constrained Formulation**

The chance-constrained method is another approach to solving decision-making problems
under uncertainties. In the chance-constrained formulation, it is ensured that the probabil-
ity of meeting a certain constraint is above a given level. It is a probabilistic way of handling
probabilistic uncertainty. In this way, the feasible region is restricted so that the confi-
dence level of the solution is high enough. The generic formulation of chance-constrained
optimization problems is:

\[
\max_{x \in X} f(x) \quad (3.30)
\]

\[
s.t. \quad \Pr(g(x, w) \leq 0) \geq \alpha \quad (3.31)
\]

\[
h(x) \leq 0 \quad (3.32)
\]

For any given \( x \in X \), computing \( \Pr(g(x, w) \leq 0) \) can be difficult depending on function \( g \)
and the distribution of uncertain parameter \( w \). Moreover, even if function \( g(x, w) \) is convex
in \( x \), the feasible region defined by the chance constraint defined in (3.31) is not necessarily
convex. Thus, chance-constrained optimization problems are intractable in many cases.

In [50], the authors proposed a simple formulation of chance-constrained unit commitment
model, in which the constraints related to line flow are relaxed. There is no renewable
generation in the model. The load is considered as uncertain parameters, which can be
easily extended to the case with stochastic netload. The chance constraint is:

\[
\Pr\left(\sum_{g \in G} P_{g,t} \geq L_t, \forall t \in T\right) \geq \alpha
\]

where \( L_t \) is the total demand at time period \( t \). The constraints ensure that there is sufficient
generation in the system. The supply sufficiency constraints for different time periods are
linked in the chance-constraint. In the paper, the authors assumed the total demand is
normally distributed with mean \( \mu \) and covariance \( \Sigma \). Utilizing Boole’s inequality, we get a
strengthened form of the chance constraint:

\[
1 - \sum_{t \in T} \Pr\left(\sum_{g \in G} P_{g,t}\right) \geq 1 - \alpha^c
\]

where \( \alpha^c = 1 - \alpha \). With the normal distribution assumption, the chance constraint can be
transformed to deterministic constraints:

\[
\sum_{g \in G} P_{g,t} \geq \mu_t + z_{1-\alpha} \sigma_t
\]
and the problem can be solved as a deterministic unit commitment. In the proposed algorithm, the authors solved a sequence of deterministic unit commitment problem with updated \( z \) until the chance constraints are satisfied. Instead of arbitrarily choosing a threshold value for reserved, the model proposed in this paper ensured the probability of satisfying the balance of demand and supply subject to the joint distribution of renewable generation was above a threshold.

In [51], the author modeled the day-ahead scheduling problem of power systems with wind generation as a two-stage chance constraint problem. The chance-constraint in the formulation guaranteed that a large portion of wind generation at each time period would be utilized with a high probability. The objective and constraints are similar to that of DUC. The dispatched wind generation at each time period \( t \) for wind generator \( g \in GW \) is a decision variable \( P_{g,t} \). Given a utilization rate \( \beta \) and the probability of violating the chance-constraint \( \alpha \), there are three types of chance constraints:

- Total wind power utilization of the entire time horizon is above \( \beta \):
  \[
  \Pr\left( \beta \sum_{t \in T} \sum_{g \in GW} w_{g,t} - \sum_{t \in T} \sum_{g \in GW} P_{g,t} \leq 0 \right) \geq 1 - \alpha
  \]

- The wind power utilization rate in each time period \( t \) is above \( \beta \):
  \[
  \Pr\left( \beta \sum_{g \in GW} w_{g,t} - \sum_{g \in GW} P_{g,t} \leq 0 \right) \geq 1 - \alpha, \forall t \in T
  \]

- The joint probability of enforcing utilization rate \( \beta \) is higher than \( 1 - \alpha \):
  \[
  \Pr\left( \beta \sum_{g \in GW} w_{g,t} - \sum_{g \in GW} P_{g,t} \leq 0, \forall t \in T \right) \geq 1 - \alpha
  \]

Among all three types of constraints, the first one is the most relaxed one, while the third one is the most restrictive one assuming the wind generation is normally distributed. In the paper, a combined sample average approximation algorithm is proposed to solve the problem efficiently. In contrast with modeling the balance of supply and demand as a chance-constraint, this paper utilizes chance-constraints to ensure the utilization of renewable generation. This means in the formulation, balance of demand and supply on each node is guaranteed and line flow capacity constraints can be enforced. However, due to congestion, higher utilization of renewable generation might is not equivalent to lower operating cost. Enforcing higher utilization rate of renewable generation might lead to the commitment of units with higher cost.

**Stochastic Programming Formulation**

Stochastic programming is another widely used framework for modeling optimization problems that involve uncertainty. In robust optimization, we optimize decisions given only the
CHAPTER 3. OVERVIEW OF UNIT COMMITMENT

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bounds of uncertain parameters. In stochastic programming, we assume that the distribu-
tion of the uncertain parameters is known or can be estimated accurately enough. Given the
probability distributions of uncertain parameters, the decisions are optimized based on an
expected objective.

In power system scheduling and operations, the output of renewable generators is consid-
ered as non-controllable. However, the supply and demand on each bus need to be balanced
simultaneously at each time period. The commitment and dispatch decisions of conventional
generators are made to minimize the operations cost subject to physical constraints. In the
United States, the wholesale energy market comprises distinct day-ahead and real-time pro-
cesses. Different types of decisions for different types of generators are made at different
stages. For unit commitment problems for power systems with renewable generation in day-
ahead market, it is natural to model the optimization problem as a two-stage stochastic
optimization model. The first-stage decisions are made based on the estimated renewable
generation scenarios, while second stage decisions are made after the realization of renewable
generation. The first-stage decision making problem cannot depend on future observations.
For all possible realizations of uncertain renewable generation, the second-stage problem
should be feasible given the first-stage decisions. By allowing decisions in the second-stage
after observing the uncertain parameters, it provides a hedging mechanism that helps avoid
over-conservative first-stage decisions. In order to solve stochastic programming problems
numerically, we typically assume we can use a finite set of possible realizations to accurately
approximate the probabilistic distribution.

In two-stage stochastic unit commitment for power systems with renewable generation,
the conventional generators are typically divided into a set of fast generators and a set of slow
generators as in [52]. Slow generators are committed in the first-stage while fast generators
are committed in the second-stage. All dispatch decisions are made in the second stage. The
solution to stochastic unit commitment (SUC) should be feasible for all possible scenarios
of renewable generation. To guarantee this, a typical strategy is to introduce load shedding
and/or renewable generation curtailment. The formulation of SUC is:

\[ (SUC) : \min \sum_{t \in T} \left( \sum_{s \in S} \pi_s \left( h_g \sigma_{g,t,s} + k_g u_{g,t,s} + c_g P_{g,t,s} + \sum_{g \in GS} c_g P_{g,t,s} + \sum_{i \in N} \rho_i L_{i,t,s} \right) \right) \]

\[ + \sum_{g \in GS} (h_g \sigma_{g,t} + k_g u_{g,t}) \]

s.t.

\[ \sum_{i \in N, j \in N(i)} F_{ij,t,s} - \sum_{k \in N(i), i \in N} F_{ki,t,s} + \sum_{g \in G, g \in N(i)} P_{g,t,s} \]

\[ - D_{i,t,s}^{\text{net}} + L_{i,t,s} = 0, \forall i \in N, t \in T, s \in S \]

\[ F_{ij,t,s} - B_{ij}(\theta_{i,t,s} - \theta_{j,t,s}) = 0, \forall i, j \in N, t \in T, s \in S \]

\[ -F^{\max}_{ij} \leq F_{ij,t,s} \leq F^{\max}_{ij}, \forall i, j \in N, t \in T, s \in S \]

\[ P^{\min}_{g,t} u_{g,t} \leq P_{g,t,s} \leq P^{\max}_{g} u_{g,t}, \forall g \in GS, t \in T, s \in S \]

\[ P^{\min}_{g,t} u_{g,t,s} \leq P_{g,t,s} \leq P^{\max}_{g} u_{g,t,s}, \forall g \in GF, t \in T, s \in S \]
The objective (3.33) is to minimize total expected cost. In (3.33), $\rho_i$ is the penalty cost for load shedding. $L_{i,t,s}$ is the amount of the shed load on bus $i$ at time period $t$ in scenario $s$. By introducing this variable, we ensure that for extreme renewable generation scenarios, the problem is still feasible. The remaining constraints are similar to those in DUC. In contrast to the deterministic formulation, there is no system-wide reserve requirement in SUC since we model extreme realizations of renewable generations are included in the discrete scenarios. In SUC, reserve requirements are modeled endogenously. In [53], the authors compared two approaches for committing reserves: stochastic unit commitment and a hybrid approach of scenario-based security-constrained commitment. Results showed that the stochastic unit commitment model outperformed the security-constrained model in terms of expected cost by reducing minimum load, startup and fuel costs. Lots of methods have been applied and developed to solve stochastic unit commitment problems efficiently. We will review them in detail in next section.
3.5 Algorithms to Solve \textit{SUC}

Stochastic unit commitment is a stochastic integer programming problem. It combines two types of mathematical programming problems which by themselves are computationally intractable. In literature, various approaches have been applied to optimize the expected operating cost in \textit{SUC} include Benders’ decomposition, Lagrangian relaxation, augmented Lagrangian methods, and progressive hedging methods. In this section, we will briefly review and compare literature on each method.

\textbf{Benders’ Decomposition}

Benders invented this method in his paper [54] and applied this algorithm to mixed-integer programming problems. The mixed-integer problem is split into a pure integer programming problem and a continuous variable linear programming problem. Each subproblem is then solved iteratively to recover the solution to the original problem. After the decomposition, each subproblem is easier to solve than the original problem. Slyke and Wets [55] first applied Benders’ decomposition to stochastic linear programming problems that take the advantage of the special block structure of the problem. Another name for this method, L-shaped method, is also based on this special block structure of the deterministic equivalent form where the uncertainty is approximated by a discrete set of scenarios. In the solution procedure, two types of cuts are added to the subproblems successfully: feasibility cuts and optimality cuts. Feasibility cuts exclude first-stage solutions that make the second stage infeasible. Optimality cuts are determined based on the dual of the second-stage subproblem. They provide lower bounds for the second-stage value function. Laportel and Louveaux [56] modified the classical L-shape method to solve stochastic integer programming problems with binary first-stage variables and mixed-integer second-stage variables with complete recourse. The proposed integer L-shape method is a finite exact algorithm for many stochastic integer programming problems.

\textbf{Lagrangian Relaxation and Augmented Lagrangian Relaxation}

Lagrangian relaxation is a relaxation method that approximates a difficult constrained optimization problem by a simpler problem. In Lagrangian relaxation, hard constraints are relaxed, and violations of these hard constraints are penalized using a Lagrange multiplier. When applying Lagrangian relaxation to stochastic programming problems that use a discrete set of scenarios to model the uncertainty, we can decouple the problem by scenario after relaxing the non-anticipativity constraints. Augmented Lagrangian method is similar to penalty method that penalizes violations of relaxed constraints. The difference of augmented Lagrangian method from general penalty method is that augmented Lagrangian method adds another term which is designed to mimic a Lagrange multiplier. Consider a
two-stage stochastic programming problem:

\[
(SP_0) \max f(x) + \sum_{s \in S} \pi_s g_s(y_s) \tag{3.51}
\]

\[
\text{s.t. } (x, y_s) \in Q_s \tag{3.52}
\]

where \(x\) is the first-stage decision, \(S\) is the set of scenarios, \(y_s\) is the second-stage decision, and \(Q_s\) defines the feasible region. The above formulation is equivalent to the following formulation with non-anticipativity constraints:

\[
(SP_1) \max \sum_{s \in S} \pi_s (f(x_s) + g_s(y_s)) \tag{3.54}
\]

\[
\text{s.t. } (x_s, y_s) \in Q_s \forall s \in S \tag{3.55}
\]

\[
x_s = \sum_{k \in S} \pi_k x_k, \forall s \in S \tag{3.56}
\]

where constraint (3.56) enforces the first stage decision is scenario independent. In Lagrangian relaxation, we can relax constraint (3.56):

\[
(LRSP) \max \sum_{s \in S} \pi_s (f(x_s) + g_s(y_s)) + \sum_{s \in S} \lambda_s^T (x_s - \sum_{k \in S} \pi_k x_k) \tag{3.57}
\]

\[
\text{s.t. } (x, y_s) \in Q_s \tag{3.58}
\]

The problem can then be decomposed to single scenario subproblems. In augmented Lagrangian relaxation, the problem is transformed to:

\[
(LRSP) \max \sum_{s \in S} \pi_s (f(x_s) + g_s(y_s)) + \sum_{s \in S} \lambda_s^T (x_s - \sum_{k \in S} \pi_k x_k) + \frac{\mu_s}{2} ||x_s - \sum_{k \in S} \pi_k x_k||^2 \tag{3.59}
\]

\[
\text{s.t. } (x, y_s) \in Q_s \tag{3.60}
\]

Notice that with the cross product in the squared term, it is non-separable across scenarios and as such this does not help reduce the problem size.

Carpentier et al. [57] applied augmented Lagrangian relaxation in unit commitment problem with random disturbances. Augmented Lagrangian technique provides satisfactory convergence properties to the decomposition problem. However, the authors only took the continuous generation scheduling decisions into consideration. No discrete commitment decisions are optimized. Nowak et al. [58] proposed a multi-stage stochastic programming formulation for the weekly unit commitment problem for hydro-thermal generation system with uncertain demand. In the proposed model, the operations of units are coupled by the load balancing constraints and the spinning reserve requirement constraints. The proposed model is a large-scale mixed integer stochastic programming problem, which cannot be solved
directly using linear programming solvers. The authors applied Lagrangian relaxation to this
problem and decoupled the operations of units by relaxing the load balancing constraints and
the reserve requirement constraints. The dual problem is optimized by applying a proximal
bundle method. The decoupled single unit subproblems are then solved using stochastic
dynamic programming. The authors designed heuristics to find feasible first-stage decisions
and an economic dispatch problem are solved to obtain near-optimal second-stage decisions.
Comparing with decomposition with respect to scenarios, decomposition with respect to
units has more subproblems and weaker convergence performance. In [52], the authors also
utilized Lagrangian relaxation to solve the SUC problem. The authors presented a parallel
algorithm based on Lagrangian relaxation. In the proposed parallel implementation of
Lagrangian relaxation, both first-stage problems and second-stage problems allow integer
decision variables.

Progressive Hedging

Progressive hedging algorithm is first introduced by Rockafellar and Wets in 1991 [59]. In
progressive hedging, the expected value term in constraint (3.56) is approximated by the
value at the previous iteration of the algorithm. Hence, the problem can be decomposed
by scenarios. The authors showed in their original paper that for convex cases, progressive
hedging algorithm converges to an optimal solution. In iteration \( i \), we need to solve:

\[
(PH - i) \max \sum_{s \in S} \pi_s (f(x_s^{(i)}) + g_s(y_s^{(i)}) + \lambda_s^{(i)}^T (x_s^{(i)} - \\
\sum_{k \in S} \pi_k x_k^{(i-1)}) + \frac{\mu}{2} \| x_s - \sum_{k \in S} \pi_k x_k^{(i-1)} \|^2)
\]

\[
s.t. (x_s^{(i)}, y_s^{(i)}) \in Q_s
\]

(3.61)

The algorithm proceeds as follows:

**Step-0** Set \( i = 0, \lambda^i = 0 \), and compute \( x_s^{(i)} = \arg \min_{x,y \in Q_s} f(x) + g_s(y) \) for all scenario \( s \).

**Step-1** Set \( \hat{x} = \sum_{s \in S} \pi_s x_s^{(i)} \), \( i \leftarrow i + 1 \), and compute \( \lambda_s^{(i)} = \lambda_s^{(i-1)} + \mu (x_s^{i-1} - \hat{x}) \)

**Step-2** For each scenario \( s \): \( x_s^{(i)} = \arg \min_{x,y \in Q_s} f(x) + g_s(y) + \lambda_s^{(i)}^T x + \frac{\mu}{2} \| x - \hat{x} \|^2 \).

**Step-3** If \( x \) has not converged, go to **Step-1**.

The progressive hedging algorithm decompose the stochastic programming problem by scen-
arios. It then solves subproblems of each scenario. The non-anticipativity is restored
through an iterative update framework of multiplier \( \lambda \). In this algorithm, \( \mu \) is a hyperpa-
rameter that is chosen before running the algorithm. It is well-known that this parameter
can critically influence the performance of progressive hedging.
Progressive hedging algorithm can solve nonlinear stochastic problems. However, the convergence guarantee is founded on convex cases. There are several papers on improving the performance of progressive hedging for problems with integer decision variables. In [60], the authors proposed to utilize variable specific penalty parameter $\mu$. To resolve the slow convergence problem that has been observed even when the parameter $\mu$ has been configured appropriately, Watson and Woodrull [61] proposed extensions of progressive hedging to improve the performance of this algorithm on mixed-integer stochastic programming problems. In their paper, they introduce problem-independent and parameter-free heuristics that can select $\mu$ for different decision variables. They also explored aggressively fixing variables to accelerate convergence.

In 1996, Takriti et al. [62] first introduced progressive hedging in solving stochastic unit commitment problem. They apply the algorithm and solve the 168-hour stochastic unit commitment problem of a system with about 100 thermal units. The algorithm converges after over 100 iterations. Goez et al. [63] compared three different solution methods for stochastic unit commitment problem. They showed a test case in which progressive hedging had cycling problem and suggested that Lagrangian relaxation is a better choice. The test cases are relatively small in this paper. There are only 32 thermal units and no run time statistics are reported.

In [64], the authors proposed a scenario-based decomposition algorithm based on progressive hedging. Numerical test results showed that the proposed algorithm could achieve a near optimal solution within a reasonable time on commodity computing platforms. In their formulation of $SUC$, the on/off status of thermal units is first stage decisions. All remaining variables are second-stage decisions. Thus, the first-stage decisions are pure binary, while the second stage decisions are continuous. In the original paper [59], $\mu$ is a scalar in the original paper that proposed progressive hedging. But we can have different parameters for different decision variables. In [64], the authors utilized variable specific $\mu$ for different first-stage decisions. A scaling factor is also introduced for all $\mu$ to tune this parameter. In this paper, the authors also allow early stops for subproblems in the first several iterations to reduce the total run time.

Other Algorithms

Other algorithms have been developed to solve such two-stage mixed-integer stochastic programming problems.

In [65], Ahmed proposed a scenario decomposition algorithm for stochastic programming problems with pure binary first-stage decision. The proposed algorithm explores solutions to the subproblems as candidate primal feasible solutions to the original problem. The explored solutions are then cut-off for future considerations in all subproblems to improve the efficiency. As other scenario decomposition algorithms, the algorithm can be implemented in parallel, and the subproblems can be solved by different processors other. For two-stage stochastic integer programming problems with complex second stage feasible regions, these steps require solving many single scenario hard problems and can present a significant bot-
In the following paper [66], the authors presented improvements and asynchronous implementation of the algorithm in [65]. They developed a lower bound for the objective of the problem with the nonantipativity constraints relaxed, and then checked the lower bound against the upper bound to terminate the evaluation early. They explored to add optimality cuts to first-stage solutions which are originally proposed in [67]. They proposed a master/worker asynchronous implementation of the scenario decomposition algorithm to deal with cases in which there exists significant variability in scenario solve times or solution evaluation times. The asynchronous implementation can reduce the idle time of worker processors. In the implementation, the master processor determines the current overall state of the algorithm and assign computation jobs to worker processors.

In [68], Zou et al. adopted the dynamic programming formulation of a multistage stochastic unit commitment problem. They developed a new type of decomposition algorithm based on Stochastic Dual Dynamic Integer Programming (SDDIP) to solve such a problem. In their formulation of the multistage unit commitment problem, they use binary approximations of dispatch decisions to fit in the framework of stochastic dual dynamic integer programming. Auxiliary variables are also introduced for minimum running time and minimum down time constraints to make the current stage only depend on the previous stage. Enhancements to SDDIP are also studied to improve the performance of the algorithm.

In [69], the authors proposed improvements to the integer L-shape method. They first introduced a modification in which the linear relaxation and mixed-integer subproblems are solved alternatively in the second-stage to avoid time-consuming exact evaluations. Comparing with approximating the second cost-function using the linear relaxation and iteratively adding cuts, the proposed mechanism to speed up the convergence. A general framework for generating optimality cuts to better approximate the second-stage cost function is also presented.

3.6 The Modeling of Renewable Generation for SUC

In power system control and operations, different types of models should be adopted for different studies. When we need to study the stability of system or design controllers, the dynamics of renewable generators should be taken into consideration. However, in the study of power system operations, dynamics of wind generators are always neglected, and static models are adopted. In this dissertation, we model the renewable generation using a discrete set of scenarios.

Extensive studies have been conducted on the modeling of renewable generation to represent the intermittency and variability better. Taking the modeling of wind generation as an example, Brown and his co-authors proposed to use time series models for wind speed modeling in 1984 [70]. In [70, 71, 72], the authors model the wind speed first and then obtain the forecasted wind power by mapping the predicted wind speed to wind power. To incorporate transmission constraints into unit commitment problem, Papavasiliou et al. [73] developed a multi-area wind production model. The model can capture the seasonal and
diurnal patterns of wind power production and account for the temporal and spatial correlations of the original dataset. The model consisted of removing seasonal and diurnal patterns from wind speed data, fitting the process to a parametric or non-parametric distribution, and fitting an appropriate time series model to the underlying noise to capture the temporal correlation. In [74], Lowery et al. studied the influence of forecast error statistics on the performance of a unit commitment to understand the priority of information required for a good decision to be made and the degree to which inaccuracy in error quantification alters the quality of the unit commitment.

Since stochastic unit commitment is a complex two-stage mixed integer programming problem, we also need to select representative scenarios in order to reduce the complexity of the problem. In [75], the authors first proposed scenario reduction algorithms for two-stage and multi-stage stochastic programming problem with convex feasible regions for first-stage decisions. In the proposed algorithm, scenarios are selected to minimize the Kantorovich functional that bounds the distance between two probability measures. In [76], the authors proposed a different algorithm that removed the scenario that causes the least change in the second-stage objective value. In [73], the authors proposed a scenario reduction algorithm inspired by importance sampling. The cost of a deterministic unit commitment problem is solved for each scenario. The probability of selecting a scenario is the probability of the occurrence of such scenario divided by a weighted factor computed from the deterministic unit commitment cost. When solving the associated term in the objective function is reweighted by the inverse of the weight factor to remove the bias. This algorithm ensures that the cost impact of all selected scenarios are of the same order of magnitude. Scenarios with small probabilities but might high influence of the first-stage decision can be selected. But since we reweight the selected scenario, the objective is still an unbiased estimator of the expected cost. We adopt the algorithm in this method to select scenarios in this dissertation.
Chapter 4

Stochastic Unit Commitment with Transmission Switching Recourse

4.1 Introduction

The output of renewable generators exhibits wide variability. The increasing amount of such renewable resources in the generation mix of electric power infrastructure brings new challenges to system operations. In order to serve load reliably in power systems, the system operators need to deploy more flexible resources in the system. In this chapter, we utilize the flexibility provided by actively altering the topology of the transmission network through transmission switching to mitigate the uncertainty of renewable generation. We model topology control through transmission switching as a recourse action in the day-ahead operation of power systems with large-scale renewable generation resources. We prove that transmission switching could only reduce the linear objective value of direct current optimal power flow (DCOPF) when congestion exists. However, we also show, through a simple example, that unit commitment cost could be reduced by transmission switching even in the absence of congestion. To solve the stochastic unit commitment with topology control within reasonable computational time, we proposed a heuristic that first decomposes a practical system into zones and then solves the problem for each zone in parallel. The benefit of topology control recourse is demonstrated on IEEE 118 system and a network representing the Central European System. In the IEEE 118 test case, our results show that with topology control recourse, the operation cost will be reduced. Moreover, to achieve a significant cost reduction, only a small fraction of lines need be switched off. In the central European system test case, we compare the costs of the network with different loading and renewable generation conditions. The cost reduction of the test system can reach 3.34% with heavy load and large-scale renewable generation while in a single zone the cost reduction can be above 7%.
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4.2 Formulation

In this thesis, we adopt the, so called, $B - \theta$ formulation of stochastic unit commitment to incorporate the switching decisions of transmission lines. Similarly as in [52], the set of conventional generators are split into two sets, one set of fast generators and one set of slow generators. Fast generators can be synchronized to or disconnected from the power network within a shorter period of time than slow generators. In the first stage, we optimize the commitment decisions of slow units. The commitment of fast generators are made in the second stage after uncertain renewable generation is realized. Other than controlling the commitment of fast generators and the dispatch of all conventional units, we also model the topology control decisions as recourse actions. The formulation of stochastic unit commitment with topology control recourse (TCSUC) is:

\[
(TCSUC) : \min \sum_{i \in T} \sum_{s \in S} \pi_s \left( \sum_{g \in GF} h_g \sigma_{g,t,s} + k_g u_{g,t,s} + c_g P_{g,t,s} + \sum_{i \in N} c_i I_{i,t,s} \right) \\
+ \sum_{i \in T} \sum_{g \in GS} (h_g \sigma_{g,t} + k_g u_{g,t}) \\
\text{s.t.} \\
\sum_{i \in N} \sum_{i \in N(i)} F_{ij,t,s} - \sum_{k \in N(i), i \in N} F_{ki,t,s} + \sum_{g \in G, g \in N(i)} P_{g,t,s} \\
- D_{i,t,s}^{\text{net}} + L_{i,t,s} = 0, \forall i \in N, t \in T, s \in S \\
- M_{ij}(1 - r_{ij,t,s}) \leq F_{ij,t,s} - B_{ij}(\sigma_{i,t,s} - \sigma_{j,t,s}) \leq M_{ij}(1 - r_{ij,t,s}), \\
\forall i, j \in N, t \in T, s \in S \\
- F_{ij}^{\text{max}} r_{ij,t,s} \leq F_{ij,t,s} \leq F_{ij}^{\text{max}} r_{ij,t,s}, \forall i, j \in N, t \in T, s \in S \\
P_{g,t,s}^{\text{min}} u_{g,t} \leq P_{g,t,s} \leq P_{g,t,s}^{\text{max}} u_{g,t}, \forall g \in GS, t \in T, s \in S \\
P_{g,t,s}^{\text{min}} u_{g,t,s} \leq P_{g,t,s} \leq P_{g,t,s}^{\text{max}} u_{g,t,s}, \forall g \in GF, t \in T, s \in S \\
P_{g,t,s} - P_{g,t-1,s} \leq R_g^U + \sigma_{g,t,s}(P_{g,t}^{\text{max}} - R_g^U), \\
\forall g \in GF, t \in T, t \geq 2, s \in S \\
P_{g,t,s} - P_{g,t-1,s} \leq R_g^U + \sigma_{g,t}(P_{g,t}^{\text{max}} - R_g^U), \\
\forall g \in GS, t \in T, t \geq 2, s \in S \\
P_{g,t-1,s} - P_{g,t,s} \leq R_g^D + (u_{g,t-1,s} - u_{g,t,s})(P_{g,t}^{\text{max}} - R_g^D), \\
\forall g \in GF, t \in T, t \geq 2, s \in S \\
P_{g,t-1,s} - P_{g,t,s} \leq R_g^D + (u_{g,t-1} - u_{g,t})(P_{g,t}^{\text{max}} - R_g^D), \\
\forall g \in GS, t \in T, t \geq 2 \\
\sigma_{g,t} \geq u_{g,t} - u_{g,t-1}, \forall g \in GS, t \in T, t \geq 2 \\
\sigma_{g,t,s} \geq u_{g,t,s} - u_{g,t-1,s}, \forall g \in GF, t \in T, t \geq 2
\]
CHAPTER 4. STOCHASTIC UNIT COMMITMENT WITH TRANSMISSION SWITCHING RECURSE

\[
\sum_{\tau=\omega_{on} g+1}^{t} \sigma_{g,\tau} \leq u_{g,t}, \forall g \in GS, t \geq \omega_{on}^g \quad (4.13)
\]

\[
\sum_{\tau=\omega_{on} g+1}^{t} \sigma_{g,\tau, s} \leq u_{g,t,s}, \forall g \in GF, t \geq \omega_{on}^g, s \in S \quad (4.14)
\]

\[
\min(t+\omega_{off}^g, 24)
\sum_{\tau=t+1}^{t} \sigma_{g,\tau} + u_{g,t} \leq 1, \forall g \in GS, t \in T, t < 24 \quad (4.15)
\]

\[
\min(t+\omega_{off}^g, 24)
\sum_{\tau=t+1}^{t} \sigma_{g,\tau, s} + u_{g,t,s} \leq 1, \forall g \in GF, t \in T, t < 24, s \in S \quad (4.16)
\]

\[
\sum_{ij \in M} (1 - r_{ij,t,s}) \leq J, \forall t \in T, s \in S \quad (4.17)
\]

\[
u_{g,t} \in \{0, 1\}, \sigma_{g,t} \in [0, 1], \forall g \in GS, t \in T \quad (4.18)
\]

\[
u_{g,t,s} \in \{0, 1\}, \sigma_{g,t,s} \in [0, 1], \forall g \in GF, t \in T, s \in S \quad (4.19)
\]

\[
r_{ij,t,s} \in \{0, 1\}, \forall i, j \in N, t \in T, s \in S \quad (4.20)
\]

In *TCSUC*, the objective function and the constraints related to conventional units are the same as that in *SUC*. The constraints related to line flows are modified to include switching decisions. The switching decision \(r_{ij,t,s}\) is a binary decision. Constraints (4.3) formulates the DC power flow equation of transmission lines. These constraints are relaxed when the line is switched off. Constraints (4.4) indicate that when \(r_{ij,t,s}\) is 1, line \(ij\) is on and the flow should be within the line rating. If \(r_{ij,t,s}\) is 0, line \(ij\) is off and the flow capacity constraints are relaxed. In (4.17), we limit the number of lines that can be switched in each scenario at each time period.

### 4.3 Transmission Switching in Optimal Power Flow and in Unit Commitment

Transmission switching in deterministic optimal power flow and unit commitment settings for IEEE test cases has been proved very effective in reducing the operating cost. Switching on/off transmission lines can divert flows in the system and relieve congestion or potential congestion in the system. Hence, generation decisions are able to be adjusted optimally to reduce operating cost. In this section, we will prove that in a single period *DCOPF*, transmission switching can only reduce the objective function value when congestion is present. However, we will also show, through an example that in a multi-period unit commitment, this is no longer the case due to the discrete nature of the optimization problem. In fact, transmission switching can enlarge the feasible region of commitment decisions and hence reduce total cost by reducing “potential” congestion.
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In a DCOPF setting, we formally prove that if there is no line congestion, changing the topology brings no benefits to the objective. The output of generators is denoted as a vector $P_G \in \mathbb{R}^N$. When there is no generation connected to a particular bus, the corresponding component is zero. The voltage angles of buses are represented using vector $\theta_N$. The active power flow of transmission lines is represented as $F_M$. The optimal power flow (OPF) problem is defined as:

$$\text{(OPF)} : \min c^T G P_G$$

subject to

$$\begin{bmatrix} I_{|N| \times |N|} & 0_{|N| \times |N|} \\ 0_{|M| \times |N|} & B_{|M| \times |N|} \end{bmatrix} \begin{bmatrix} P_G \\ \theta_N \\ F_M \end{bmatrix} = \begin{bmatrix} P_G \\ 0 \\ \theta_N \end{bmatrix}$$

(4.22)

$$P_G^{\min} \leq P_G \leq P_G^{\max}$$

(4.23)

$$-F_M^{\max} \leq F_M \leq F_M^{\max}$$

(4.24)

where $|·|$ represents the cardinality of a set, $A_{|N| \times |M|}$ is the incidence matrix. Matrix $B$ is a sparse matrix with non-zero components:

$$B_{l,i} = \frac{1}{x_l}, \quad B_{l,j} = -\frac{1}{x_l}$$

where $x_l$ is the impedance of line $l$ starting from bus $i$ ending at bus $j$. The optimal solution of OPF is $\bar{x}^T = [\bar{P}_G, \bar{\theta}_N, \bar{F}_M]$. Due to the optimality of $\bar{x}^T$, there is no feasible direction $d^T = [d^T_{P_G}, d^T_{\theta_N}, d^T_{F_M}]$ such that $c^T G d^T_{P_G} < 0$. When there is transmission switching, the operating cost will be less than or equal to the operating cost of OPF. Let us consider an OPF with optimal production $P_G$. There is no congestion, i.e. the line flow capacity constraints are not binding. The optimal production level $P_G^*$ of transmission switching problem OTS can be obtained by solving the following problem:

$$\text{(OTS)} : \min c^T G P_G$$

subject to

$$\begin{bmatrix} I_{|N| \times |N|} & 0_{|N| \times |N|} \\ 0_{|M| \times |N|} & B_{|M| \times |N|} \end{bmatrix} \begin{bmatrix} P_G \\ \theta_N \\ F_M \end{bmatrix} = \begin{bmatrix} P_G \\ 0 \\ \theta_N \end{bmatrix}$$

(4.26)

$$P_G^{\min} \leq P_G \leq P_G^{\max}$$

(4.27)

$$-F_M^{\max} \leq F_M \leq F_M^{\max}$$

(4.28)

where $\tilde{M}$ is the set of transmission lines after switching on/off lines. The OTS is essentially an OPF solved on a network with a different set of transmission lines.

Assume $c^T G P_G^* < c^T G P_G$. The feasible direction of OPF satisfies:

$$B_{|M| \times |N|} d_{\theta_N} = d_{F_M}$$

(4.29)

$$d_{P_G} + A_{|N| \times |M|} d_{F_M} = 0$$

(4.30)
we get
\[ d_{PG} = B' d_{\theta_N} \]  
where \( B' \) is the coefficient matrix of DC power flow equation and it is of rank \(|N| - 1\). We can delete the last row and the last row. We can first take the inverse of the reduced matrix to get \( d_{\theta_{|N|-1}} \) using \( d_{PG} \) and then find the last component of \( d_{\theta_N} \). Since the line flow capacity constraints in OPF is not binding, there exists a feasible direction for OPF satisfying \( c_{GT} d_{PG} < 0 \), which contradicts the optimality of \( \bar{P}_G \) when switching is not allowed. Thus, if there is no congestion in optimal power flow, transmission switching will not benefit the system. Based on this result, heuristics could be developed by monitoring only a subset of transmission lines and target the relieving line flow congestion in the OPF solution.

While the above result may seem intuitive, this is presented here in order to highlight the contrast with the multi-period unit commitment optimization where transmission switching could be beneficial even in the absence of congestion, as illustrated through the following simple example. It is shown that “potential” line flow congestion may lead to an optimal commitment decision, such that if we fix the binary commitment decisions, no congestion can be observed in the optimal solution. Switching on/off some of the lines can still reduce the cost.

<table>
<thead>
<tr>
<th>Generator</th>
<th>( G1 )</th>
<th>( G2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start-up Cost</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>No-load Cost</td>
<td>70</td>
<td>150</td>
</tr>
<tr>
<td>Production Cost (per Unit of Demand)</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Capacity</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Ramping Rate (up and down)</td>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Line</th>
<th>( L12 )</th>
<th>( L23 )</th>
<th>( L13 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow Capacity</td>
<td>8</td>
<td>30</td>
<td>30</td>
</tr>
</tbody>
</table>

Consider a 2-period deterministic unit commitment problem for a 3-bus system with two units as shown in Figure 4.1. The impedance of lines is also depicted on the figure. We consider only two time periods. The demand in the two periods is 18 and 24. Other related
parameters of generators and transmission lines are listed in Table 4.1 and Table 4.2, for brevity, the parameters are all measured as the ratio with respect to some base value.

The results are depicted in Figure 4.2 and Figure 4.3. Though there is no line congestion observed in the optimal solution of UC without transmission switching, the cost can be reduced in both periods by switching off $L_{12}$. From the optimal solution of UC with transmission switching, we can see that for both time periods it is optimal to commit only $G_2$. Without switching, $L_{12}$ would be congested if we only had $G_2$. Hence, only $G_1$ is committed in $t_1$. Since the binary decision is changed, the potential congestion could not be observed. An important implication highlighted by this example, which is significant for the main problem explored in this paper, is that heuristics based on relieving congestion on specific lines cannot be directly applied in this case. For stochastic unit commitment with transmission switching recourse, the problem is even more complicated.

4.4 Framework for Solving $TCSUC$ for Commercial-Scale Power Systems

Regional transmission organizations, like ENTSO-E in Europe and PJM in the US, coordinate the transaction of electric power through different regions or zones subject to different regulatory frameworks. Taking ENTSO-E as an example, each zone in the interconnected system has its own transmission system operator (TSO). Cross-border electricity exchange is cleared in the day-ahead market without considering renewable generation. Inspired by the current operation of this large-scale power system, we propose a framework to solve stochastic unit commitment with transmission switching recourse ($TCSUC$) for commercial-scale interconnected power systems, in which TSOs of each zone co-optimize generator commitment and dispatch decisions as well as transmission line switching decisions. In the proposed framework shown in Figure 4.4, the problem we solve in each step is a two-stage stochastic programming problem. Each zone first solves a stochastic unit commitment ($SUC − PE$) problem without transmission switching.
Figure 4.2: UC solution without transmission switching
Figure 4.3: UC solution with transmission switching
\( (SUC - PE) : \)
\[
\begin{align*}
\min & \sum_{t \in T} \sum_{s \in S} \pi_s \left[ \sum_{g \in GF} (h_g \sigma_{g,t,s} + k_g u_{g,t,s} + c_g P_{g,t,s}) + \sum_{g \in GS} c_g P_{g,t,s} + \sum_{i \in N} \rho_i L_{i,t,s} \right] \\
& + \sum_{t \in T} \sum_{g \in GS} (h_g \sigma_{g,t} + k_g u_{g,t}) \\
& + \sum_{t \in T} \sum_{s \in S} \pi_s \left( \sum_{i \in N_i, j \notin N_i} F_{ij,t,s,k_i,j} + \sum_{i \notin N_i, j \in N_i} F_{ij,t,s,k_i,j} \right) \\
\text{s.t.} & (u^z_{GS}, \sigma^z_{GS}) \in \Delta_{GS} \\
& (u^z_{GF,s}, P_{G,s}, L_s) \in \Delta_{GF}(W^z_s, u^z_{GS}), \forall s \in S
\end{align*}
\] (4.32)

The last term of the objective represents the cost/revenue for exchanging electricity with other zones. The vector \( F^z_z \) represents line flow within the zone \( z \) while \( F^z_c \) represents line flows between zones. For brevity, we do not list all constraints for SUC. The first constraint represents the on/off transition, minimum up time, and minimum down time constraints of slow units. \( \Delta_{GF}(W^z_s, u^z_{GS}) \) represents the feasible set of the second stage decisions that depend on the renewable generation scenarios and the first-stage decisions. All constraints are linear. In \( SUC - PE \), the inter-zone flows are modeled as electricity imports and exports.

Figure 4.4: Framework for solving TCSUC for commercial-scale interconnected system

The binary commitment decisions are submitted to the interconnected system coordinator. The coordinator has access to the model and the data of each zone. It then solves a stochastic economic dispatch (SED) problem in the day-ahead market for the entire system.
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The SED is a two-stage stochastic programming problem.

\[
(SED) : \min \sum_{t \in T} \sum_{s \in S} \pi_s (\sum_{g \in GF} c_g P_{g,t,s} + \sum_{g \in GS} c_g P_{g,t,s} + \sum_{i \in N} \rho_i L_{i,t,s}) \tag{4.35}
\]

\[
\text{s.t. } (P_{G,s}^z, F_{s}^z, L_{s}^z) \in \Delta(W_s^z, F_c^z), \forall s \in S, z \in Z \tag{4.36}
\]

In SED, the commitment generators are fixed as the solution of SUC − PE. SED optimize the expected operation cost of the entire system. \(\Delta(W_s^z, F_c^z)\) defines the feasible region. In \(\Delta(W_s^z, F_c^z)\), decision variable \(F_c^z\) is the first-stage decision and it does not depend on scenarios. It is equivalent to utilizing \((P_{G,s}^z, F_{s}^z, L_{s}^z) \in \Delta(W_s^z, F_c^z)\) for all scenarios and zones and then forcing \(F_c^z = F_c\). In SED, we have market clearing constraints, line flow constraints, line flow capacity constraints, generation capacity constraints, and ramping constraints. The SED is a stochastic linear programming problem. Different time periods are coupled by the ramping constraints. Different scenarios are coupled by the cross-border line flows.

The cross-border flows are first stage decisions forced by non-anticipativity constraint since this value is settled in day-ahead before the realization of uncertainty; generation dispatches are second stage decisions. Finally, the cross-border flows are broadcast to zones, and each TSO solves TCSUC with the cross-border flows fixed. For each renewable generation scenario, the cross-border flows are same as the solution in SED. The formulation of TCSUC is similar to that of SUC − PE other than line flow constraints and flow capacity constraints. The proposed model decomposes the large-scale mixed integer programming problem into sub-problems with fewer decision variables and significantly reduce the complexity of solving the problem. The sub-problems can be solved in parallel, and the decomposition can be fit into the coordination framework of interconnected power systems. For TCSUC, we solve the problem of each zone for all scenarios simultaneously with a good warm start to reduce the solve time. The warm start solution is created following two steps: we first solve SUC; we then solve the topology control for each scenario with the commitment decisions fixed in SUC. The warm start solution of binary decision variables is composed of the solutions of the two steps.

Since we fix the cross-border flows between zones and only allow the switching of lines within a zone, the solution to the decomposed model is sub-optimal. For the central European system that we used in the numerical test with ten scenarios, there are over 400,000 binary variables without decomposition. Even if we use scenario-based decomposition method such as Progressive Hedging, there are around 80,000 binary variables in each sub-problem. For an interconnected practical power system with thousands of buses, such heuristics that reduce the complexity of solving the problem is essential. Given that real-time coordination between zones is still limited in the current European market, we think it is reasonable to model power exchanges between zones as day-ahead pre-fixed values instead of decisions, and it is appropriate to leave the switching decisions in the zonal level where detailed transmission system configuration information is accessible.
4.5 Numerical Tests on IEEE 118 System

We conduct numerical tests on an IEEE 118 system as shown in Figure 4.5. There are 118 buses, 186 transmission lines, four slow generators, and 15 fast generators in the test system. The data of the system and the costs of the generators are the same as those in [77]. In our test, wind speed and wind power data of three wind farms in Wyoming are obtained from NREL Western Wind Resources Dataset [78]. In the dataset, each turbine icon represents a site consisting of ten 3MW wind turbines. For each wind farm, we randomly picked one site and used it to represent the location. The wind generation is simulated using the method presented in [79]. We generate 1000 scenarios of 24 hours wind power output using Monte Carlo simulation. The scenario reduction is implemented based on the algorithm in [52]. The 10 selected scenarios for one of the wind farms are shown in Figure 4.6. The total wind generation ranges between 0MW and 300MW. We connected the wind generation to 3 buses: B26, B80, B100.

In the test, TCSUC is implemented in Java. The problem is solved using CPLEX through concert technology with Java. The test is conducted on a laptop with an Intel Core i7 2.6 GHz CPU and 12 GB RAM. The same computing plat form is utilized in the numerical test in this and the next chapter. When the gap tolerance is set to be 5%, and the default setting of CPLEX is adopted, the program does not terminate after 8 hours. This indicates that this MIP cannot be solved directly within a reasonable time. One of the approaches that can help reduce the solution time is to provide CPLEX a good warm-start.

The objective function of TCSUC is the sum of operation cost over 24 hours. Assuming a switching solution can reduce the cost of the 24-hour unit commitment problem, it should
reduce the operation cost for some 1-hour sub-problem. To obtain a start solution for switching decision, we can solve the optimal power flow problem with topology control for 1 hour with the heaviest net-load. Here, netload is the actual demand minus the wind generation. To obtain warm-start for unit commitment decisions, we can solve the unit commitment without transmission switching. Normally, the system is more congested when the netload is heavier. In this case, the switching of transmission lines can relieve congestion and increase the output of cheaper generation. Combining the warm-start for unit commitment decisions and the warm-start for line switching decisions will yield the warm-start for the problem.

We conducted nine numerical tests, in which different numbers of lines are allowed to be switched off, in order to explore whether topology control recourse will benefit the day-ahead operations of power system with wind generation. For brevity, we name the cases “TCSUC − x”, where “x” stands for the maximum number of lines that can be switched off. For example, TCSUC − 5 means a stochastic unit commitment with topology control recourse and at most five lines can be switched off in each scenario at each time. TCSUC − ∞ represents the case where there is no limit on the number of lines that can be switched. An SUC is also implemented whose objective value serves as a reference value. The warm-start of switching solutions obtained by solving optimal power flow with topology control are listed in Table 4.3.

Costs for ten cases are compared to demonstrate that topology control recourse can mitigate the variability of wind generation and reduce the operation cost. The time limit for the CPLEX was set to be 30 minutes. The maximum optimality gap was configured to be 5%. If a solution within the given optimality gap cannot be found within 30 minutes, a feasible solution with the best objective value is regarded as the solution. The cost of the 10 cases is depicted in Figure 4.7 Cost reductions defined as the percentage difference of costs with and without topology control recourse for ten cases are shown in Figure 4.8.

From the results, we can see that the total cost can be reduced significantly with topology
CHAPTER 4. STOCHASTIC UNIT COMMITMENT WITH TRANSMISSION SWITCHING RECURSE

Table 4.3: Generator parameters of the 3-bus system

<table>
<thead>
<tr>
<th>Case</th>
<th>Start Switching Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$TCSUC - 1$</td>
<td>132</td>
</tr>
<tr>
<td>$TCSUC - 2$</td>
<td>132, 136</td>
</tr>
<tr>
<td>$TCSUC - 3$</td>
<td>132, 136, 153</td>
</tr>
<tr>
<td>$TCSUC - 4$</td>
<td>132, 136, 151, 153, 163</td>
</tr>
<tr>
<td>$TCSUC - 5$</td>
<td>132, 136, 148, 153, 161, 162</td>
</tr>
<tr>
<td>$TCSUC - 6$</td>
<td>63, 132, 136, 148, 153, 161, 162</td>
</tr>
<tr>
<td>$TCSUC - 7$</td>
<td>126, 132, 136, 146, 151, 153, 157, 165</td>
</tr>
<tr>
<td>$TCSUC - 10$</td>
<td>126, 132, 136, 146, 151, 153, 157, 165</td>
</tr>
<tr>
<td>$TCSUC - \infty$</td>
<td>1, 10, 14, 25, 28, 31, 57, 63, 66, 77, 79, 86, 96, 103, 110, 111, 132, 136, 146, 151, 153, 161, 165, 184</td>
</tr>
</tbody>
</table>

Figure 4.7: Cost for different test cases
control recourse. Moreover, to achieve the cost reduction, we do not need to switch many lines. When at most three lines are allowed to be switched off, the cost reduction is above 12%. Switching off more lines may not benefit the system much more if we set a limit on the time for solving the problem. For instance, when at most four lines can be switched, the best solution CPLEX can find within 30 minutes is not better than the solution of $TCSUC - 3$. The highest cost reduction is achieved in $TCSUC - 10$. We note that the cost reduction in $TCSUC - \infty$ where there is no constraint on the number of lines that can be switched is lower than the cost reduction in $TCSUC - 3$ to $TCSUC - 10$. An explanation for this anomalous result is that when starting from the given solution, relaxation induced neighborhood search (RINS) or solution polishing may be invoked to improve the given warm-start solution. In $TCSUC - \infty$, the solution to $OTS$ is more targeted on that load profile. It is sensitive to the change of loads and the uncertainty of wind generation in the system. Switching off some line in one load profile may increase the cost in other load profiles. Thus, in $TCSUC$, the warm-up solution cannot be generalized well to other scenarios. However, the solution to $OTS$ with a constraint on the number of lines to switch tends to find lines that are in common with the switching solutions to other load profiles. They can be generalized to other scenarios effectively.

With topology control recourse, the system could achieve lower cost as compared to generic stochastic unit commitment. The objective of $SUC$ has four components: production cost for generators, start-up cost for generators, no-load costs for generators and the penalty cost for load shedding in the system. The cost reduction could be broken down into four parts based on the structure of the objective of $SUC$. First, given the commitment of generators, in each hour, the scheduling of production can be regarded as an $OPF$ with transmission switching. The production of generators with low production costs could increase while the production of generators with high production costs could decrease given they are all
on, which leads to lower production cost for the whole system. Second, with transmission switching, the number of on-off status change of generators could be lowered, which reduces the startup cost. Third, if the production of some generators with high no-load cost is low, the production of that generator can be shifted to other generators that are on to reduce no-load cost. Fourth, the average load shedding in each hour is lower. All four parts are observed in the test results.

The production costs of fast generators are shown in Figure 4.9. In $TCSUC - 10$, the production of fast generators in scenario 1 hour 18 with and without topology control recourse is shown in Figure 4.10. From Figure 4.9 and Figure 4.10, when there is topology control recourse, some of the production of Generator 15, Generator 18 and Generator 19 with higher production costs are shifted to Generator 9 and Generator 10 with lower production costs.

Figure 4.11 shows the commitment decisions of Generator 6 and Generator 8 in $SUC$. Figure 4.12 shows the commitment decisions of Generator 6 and Generation 8 in $TCSUC –$
10. In \textit{SUC} where there is no switching, Generator 8 with higher start-up cost will be turned on twice. In \textit{TCSUC} – 10, the number of on-off status change of Generator 8 reduces to 1 while the number of on-off status change of Generator 6 with lower start-up cost will increase to 2. The numbers of on-off status change of other generators in this scenario are the same. Thus, the start-up cost in this scenario is reduced.

![Figure 4.11: On-off status changes of Generator 6 and Generator 8 in SUC](image1)

![Figure 4.12: On-off status changes of Generator 6 and Generator 8 in TCSUC](image2)

The no-load costs of fast generators for all scenarios in \textit{SUC} and \textit{TCSUC} – 10 are calculated and compared. The results are shown in Figure 4.13. From the results, we can see that with topology control recourse, the no-load cost in all scenarios can be reduced.
We plot the average total system load shedding defined as the expected load shedding for the whole system in each hour in SUC and TCSUC − 10 in Figure 4.14. From the results, we can see that except for hour 8, the average total load shedding in TCSUC − 10 is higher than that in SUC. Hence, the penalty cost for load shedding in SUC will be higher than that in TCSUC − 10 given that we use uniform penalty cost for different buses.

In TCSUC, wind generation scenarios are selected to represent the uncertainty. The purpose of reducing the number of scenarios is to reduce the complexity and make the problem easier to solve. In order to evaluate the performance of the switching policy generated using the reduced scenario set, we test the model on a larger set of scenarios. In the evaluation, the switching decisions of lines are restricted on the set of the switching solution
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to $TCSUC - 10$, which means only lines that have been switched off in $TCSUC - 10$ are allowed to be switched in the evaluation process. The commitment of slow generators is fixed as the solution to $TCSUC - 10$. Since the switching solution is constrained on a set, the result of the evaluation process provides a lower bound for the cost reduction when there is no restriction on switching decisions. 1000 wind generation scenarios are generated using Monte Carlo simulation. In all 1000 tests, when there is topology control recourse, the total cost is less than when there is no transmission switching. The average total cost is reduced by 12.9% with topology control recourse. This result shows that topology control in the recourse could help reduce the cost of the system with wind generation.

Progressive hedging is also applied to solve $TCSUC$ for the IEEE 118 system. The configuration of the system is the same, but we did not limit the number of lines that can be switched. For $TCSUC$, the first-stage decision variables are binary decision variables. The quadratic term in the original problem can be expressed as:

$$\frac{\mu}{2} \| x_s^{(i)} - \hat{x} \|^2 = \frac{\mu}{2} x_s^{(i)} - \mu (\hat{x} - \frac{1}{2} \mathbb{1})^T x_s^{(i)} + \frac{\mu}{2} \hat{x}^T \hat{x}$$

since $x_s^{(i)} x_s^{(i)} = \mathbb{1}^T x_s^{(i)}$ for binary variables.

![Figure 4.15: Convergence of progressive hedging](image)

Other than enhancements mentioned in Watson’s paper, to reduce the running time for IEEE 118 case, I also use the following two techniques. The first technique is to allow a larger optimality gap for early iterations. The purpose of allowing a larger optimality gap is that we can get a feasible solution to each single scenario problem first and then enforce the first stage variables to converge. The second technique is that we use the solution to the previous iteration as a warm start for the current iteration. By doing this, we actually assume that The optimality gap for each sub-problem is set to be 4% for first two iterations.
and the time limit for each sub-problem is set to be 6 minutes. The algorithm converges after seven iterations. The estimated time for solving the problem in parallel is 42 minutes. The cost is reduced by 10.1% with topology control recourse. This figure shows the 1-norm error of first-stage decisions in each iteration. We can see that progressive hedging converges after seven iterations. In the last iteration, the 1-norm error is zero, which shows that the first-stage decisions are same among different scenarios.

4.6 Numerical Tests on Central European System

In this section, we conduct numerical tests on a network representing the Central European system [80]. We compare the costs for each zone with and without topology control recourse in two settings of renewable generation and loading conditions. In the first setting (Case 1), we use base values for both loads and renewable generations. In the second setting (Case 2), we increase the load by 10% and increase the renewable generation by 5% to create more congestion in the system.

There are 679 buses, 667 conventional units, 1036 transmission lines and 1437 renewable units in our test system. The interconnections between different countries within and outside the Central European system are shown in Figure 4.16.

Figure 4.16: Central European System
There are seven countries in the network. Detailed information on the grid of those countries is listed in Table 4.4. Among all seven countries, Germany has the largest number of buses, lines, and units. The peak load of Switzerland and Luxembourg is much larger than the maximum generation capacity (Max. Gen. Cap.). We need to combine the two countries with other interconnected countries as zones to balance the load and generation. There are nine lines connecting Switzerland and France and five lines connecting Switzerland and Germany. Switzerland is connected more tightly with France than Germany. When we solve the problem, we take France and Switzerland as a single zone. Similarly, we also consider Belgium and Luxembourg as a zone. In our numerical tests, we decompose the system into 5 zones.

<table>
<thead>
<tr>
<th>Table 4.4: System zonal information</th>
</tr>
</thead>
<tbody>
<tr>
<td>AT</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>Buses</td>
</tr>
<tr>
<td>Linew</td>
</tr>
<tr>
<td>Fast Units</td>
</tr>
<tr>
<td>Slow Units</td>
</tr>
<tr>
<td>Peak Load (MW)</td>
</tr>
<tr>
<td>Max. Gen. Cap. (MW)</td>
</tr>
</tbody>
</table>

We select 10 renewable generation scenarios from 120 simulated scenarios using historical data. Since our goal is to justify the benefit of topology control recourse in stochastic unit commitment, we want scenarios with high variance, so that they cover extreme realizations of renewable generation. The total renewable generation in each scenario of a day is plotted in Fig. 6. From the figure, we can see that renewable generation varies a lot. The highest renewable generation scenario can be as much as roughly ten times the lowest renewable generation scenario. The negative value of the renewable generation represents pumped hydroelectric storage. Taking the two largest zones FR+CH and DE as examples, the maximum penetration of renewable generation in the second setting among all scenarios and all time periods is 17.1% in FR+CH and 60.5% in DE. Details on how renewable generation is distributed in the system can be find in [80].

The test results of the two cases are listed in Table 4.5 and Table V. The solver we used is CPLEX. The optimality gap of SUC is set to be 0.5% for all zones. The optimality gap of TCSUC is set to be 4% for DE and FR+CH, and 2% for the other three zones to ensure all sub-problems can be solved within the similar amount of time. The lower bound of TCSUC is from solving a relaxation of the problem. Solutions with high optimality gap are feasible solution whose objective value is greater than or equal to the optimal one. We did not
choose a uniform gap for different zones in $TCSUC$, but the result is still comparable since the higher optimality gap induce a lower bound for cost reduction. The larger the optimality gap is in $TCSUC$, the more conservative the cost saving is. From the results, we can see that with topology control recourse, the total cost of the system will be reduced by 0.2944 million euros in Case 1 and 1.8794 million euros in Case 2. From the results, we can see that the total cost of Case 2 is higher than that of Case 1, but the percentage cost saving is 3.34% with topology control recourse which is much higher than 0.75% in the first case. The zone FR+CH has the largest cost saving with transmission switching recourse in both cases. In contrast, no cost saving is observed in the zone BE+LX in Case 2 and the cost saving in the zone representing NL is close to zero in Case 1.

In Case 2, the load on each bus is increased by 10%. Not only more generation is required...
Table 4.5: Case 1 test results

<table>
<thead>
<tr>
<th></th>
<th>SUC(MEUR)</th>
<th>TCSUC(MEUR)</th>
<th>Cost Saving(MEUR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AT</td>
<td>3.2257</td>
<td>3.2123</td>
<td>0.0134</td>
</tr>
<tr>
<td>BE+LX</td>
<td>3.2139</td>
<td>3.2119</td>
<td>0.0011</td>
</tr>
<tr>
<td>DE</td>
<td>15.1121</td>
<td>15.0078</td>
<td>0.1043</td>
</tr>
<tr>
<td>FR+CH</td>
<td>13.5106</td>
<td>13.3359</td>
<td>0.1711</td>
</tr>
<tr>
<td>NL</td>
<td>4.1957</td>
<td>4.1912</td>
<td>0.0045</td>
</tr>
<tr>
<td>Total</td>
<td>39.2571</td>
<td>38.9627</td>
<td>0.2944</td>
</tr>
</tbody>
</table>

Table 4.6: Case 2 test results

<table>
<thead>
<tr>
<th></th>
<th>SUC(MEUR)</th>
<th>TCSUC(MEUR)</th>
<th>Cost Saving(MEUR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AT</td>
<td>7.0057</td>
<td>6.8244</td>
<td>0.1813</td>
</tr>
<tr>
<td>BE+LX</td>
<td>6.2083</td>
<td>6.2083</td>
<td>0.0000</td>
</tr>
<tr>
<td>DE</td>
<td>14.2089</td>
<td>14.0540</td>
<td>0.1549</td>
</tr>
<tr>
<td>FR+CH</td>
<td>17.3961</td>
<td>16.0753</td>
<td>1.3478</td>
</tr>
<tr>
<td>NL</td>
<td>10.5475</td>
<td>10.3793</td>
<td>0.1682</td>
</tr>
<tr>
<td>Total</td>
<td>55.3665</td>
<td>53.5141</td>
<td>1.8521</td>
</tr>
</tbody>
</table>

to balance the demand, but also more congestion is created. Due to the congestion, more
expensive units have to be scheduled to meet the demand. Hence, the total cost increase is
high. Moreover, with 5% more renewable generation, more flexible units with higher costs
need to be deployed to mitigate the variability without topology control recourse. Thus,
topology control plays a more important role in Case 2. In the following analysis when we
compare the cost savings in different zones, we will focus on Case 2.

To understand why the percentage cost saving in FR+CH is above 7% while that of
BE+LX is zero, we will examine the loading conditions, ramping capabilities, congestion of
the two zones. We also define other metrics to analyze the results. The net load ramping
\((NLR)\) requirement is defined as:

\[
NLR^{Up}(t) = \max_{s \in S} (D_{s,t+1}^{net}) - \min_{s \in S} (D_{s,t}^{net})
\]

\[
NLR^{Down}(t) = \max_{s \in S} (D_{s,t}^{net}) - \min_{s \in S} (D_{s,t+1}^{net})
\]

\[
NLR(t) = \max\{NLR^{Up}(t), NLR^{Down}(t)\}
\]
Figure 4.18 provides a graphical illustration on how $NLR$ is defined. The $NLR$ measures the variability of renewable generations. Moreover, it is designed to capture the extremes among different scenarios, which reflects the extreme ramping requirements in the second stage that the operators need to consider when the first stage decisions are made. In the two-stage stochastic unit commitment, the first stage commitment decisions need to accommodate such variability in the second stage among different scenarios. For a system with high $NLR$ value, if the ramping capacity of fast units is small, more first-stage commitment decisions will be cut-off, and the number of feasible solutions will be smaller. Thus, the higher the value of $NLR$ is, the fewer the feasible slow generator commitments there are.

We also calculated the congestion rate for zone $z$ defined as:

$$CR_{z} = \frac{\sum_{s \in S} \pi_s \sum_{t \in T} \sum_{i,j \in N_z} \mathbb{1}(|F_{ij,t,s}| = F_{ij}^{\max})}{(#T)(#M_z)}$$

where $\#$ represents the cardinality of a set, $\mathbb{1} (\cdot)$ is the indicator function, $|\cdot|$ is the absolute value of a variable, and $F_{ij,t,s}$ is the line flow of the stochastic unit commitment without topology control recourse. This quantity represents the average percentage of lines congested per time period. It attempts to quantify how congested the network is.

Statistics of the two zones are listed in Table 4.7. We can see that the $NLR$ of BE+LX is much higher than the ramping capacity of slow units while the $NLR$ of FR+CH is closed to the ramping capacity of slow generators. Thus, in BE+LX, the variability of the renewable
generations is mitigated by fast units. Moreover, the $CR$ of FR+CH is much lower than that of BE+LX. The zone of BE+LX is more congested than FR+CH. Topology control recourse can change the commitment schedule by reducing potential congestions. However, if there are too many lines congested in a network, by switching on/off lines might not enlarge the feasible set of the first stage commitment decisions to provide a better solution.

Table 4.7: Loading, generation and congestion statistics of BE+LX and FR+CH

<table>
<thead>
<tr>
<th></th>
<th>BE+LX</th>
<th>FR+CH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Max Cap. of Slow Generators</td>
<td>13899.6</td>
<td>84952</td>
</tr>
<tr>
<td>Total Max Cap. of Fast Generators</td>
<td>3086.5</td>
<td>9730</td>
</tr>
<tr>
<td>Total Slow Ramping Cap.</td>
<td>1424.4</td>
<td>10034.7</td>
</tr>
<tr>
<td>$NLR$</td>
<td>2381.4</td>
<td>12809.4</td>
</tr>
<tr>
<td>Max. Net Load</td>
<td>12451.3</td>
<td>77447.2</td>
</tr>
<tr>
<td>Min. Net Load</td>
<td>9682.8</td>
<td>50682.3</td>
</tr>
<tr>
<td>$CR$</td>
<td>0.1916</td>
<td>0.0254</td>
</tr>
</tbody>
</table>

Figure 4.19: Cost comparison of slow units

The comparison of detailed cost information in zone FR+CH is shown in Figure 4.19 and Figure 4.20. To compare different cost components of slow units and fast units, we scale each component through dividing it by the corresponding value of $SUC$. The values in the figures represent the cost component in $TCSUC$ corresponding to that of $SUC$. From Fig. 8, we
can see that by including topology control as a recourse action, the first stage commitment decisions have been altered so that both start-up cost and no-load cost have been reduced. Moreover, almost the same amount of generation from slow units is dispatched in the second stage. But the average fuel cost of TCSUC is lower than that of SUC. The expected fuel cost of slow units is decreased with topology control recourse. Similarly, the expected start-up cost expected no-load cost, and expected fuel cost are all reduced. Around 1.5% less fast generation is dispatched in TCSUC. That reduction in the fast generation is covered by slow units with cheaper fuel costs. From the results, we can see that with topology control recourse in stochastic unit commitment, we can utilize the flexibility provided by switching on/off transmission lines to mitigate the variability introduced by renewable generation.

### 4.7 Summary

We have studied modeling topology control through transmission switching as a recourse in a two-stage stochastic unit commitment model for power systems with large-scale renewable generation. We analyzed how the switching decisions could affect the commitment decisions and the dispatching decisions in OPF and unit commitment. To solve TCSUC for practical system efficiently, we also proposed a decomposition heuristic. Numerical tests conducted on IEEE 118 system and a network representing the Central European System demonstrate that with topology control recourse, the expected operating cost will be reduced. Moreover, to achieve a significant cost reduction, only a small fraction of lines need be switched off. The flexibility provided by topology control allows the commitment of cheaper units in both stages in the stochastic unit commitment problem. But such flexibility is limited by other conditions of the system. We observed that, for heavily congested systems, the operating
CHAPTER 4. STOCHASTIC UNIT COMMITMENT WITH TRANSMISSION SWITCHING RECOURSE

cost could not be reduced significantly by purely introducing topology control recourse.

For future research, system contingencies traditionally monitored through N-1 security criteria should be included in the probabilistic scenarios so that they are accounted for in the first-stage decision of stochastic unit commitment. Further work is also needed for developing efficient scenario-based decomposition algorithms, accounting for switching cost and preventing cycling.
Chapter 5

Stochastic Unit Commitment with Flexible Line Rating Recourse

5.1 Introduction

The thermal ratings of overhead transmission lines are typically conservative, which leads to underutilization of transmission assets. In this chapter, we propose an optimization model that accounts for the inherent flexibility in line ratings. We determine, in a stochastic unit commitment framework, when and which line can and should adopt higher ratings (calculated based on anticipated weather conditions and loading) as part of the recourse actions. Flexible line ratings in the recourse help mitigate the uncertainty introduced by renewable generation and improve first-stage commitment decisions. Numerical tests conducted on both IEEE 118 system and a network representing the Central European System demonstrate that with flexible line rating recourse, the expected operation cost can be substantially reduced without degrading reliability.

5.2 Formulation

To incorporate dynamic line ratings into the operations of power systems, we need to have real-time measurements of the meteorological and operation conditions or accurate enough forecast for those conditions. However, it is costly to install sensors and communication systems with the operating center for all lines. Even if we have all required equipment installed, we still need weather forecast or use selected scenarios in day-ahead operations to adopt dynamic line ratings. If the weather forecast or selected scenario leads to a conservative line rating profile or if we simply utilize static ratings, it will result in conservative commitment decisions.

In this chapter, we propose to utilize flexible line ratings as a recourse action in stochastic unit commitment as illustrated in Figure 5.1. An acceptable higher rating $F_{ij}^{\max,\text{high}}$ than the normal static line rating $F_{ij}^{\max,\text{normal}}$ is calculated based on anticipated weather conditions.
and loading conditions. Based on the HBE, the conductor has thermal inertia so that the
temperature of the line will not increase immediately. By limiting the time $t_{ij}^H (\leq \hat{t}_{ij}^H)$ of
adopting high ratings and mandate normal or even lower ratings afterward for a certain
period of time $t_{ij}^N (\leq \hat{t}_{ij}^H)$, we actually allow the lines to be heated and then cool down. This
will avoid excessive sagging and thermal damage caused by high conductor temperature.

![Illustration of flexible line ratings](image)

Figure 5.1: Illustration of flexible line ratings

When flexible ratings are adopted in stochastic unit commitment for power systems with
intermittent renewable resources, we introduce second stage decisions regarding when to
allow the line ratings to be higher than the normal static ratings. Moreover, in this paper, we
also include the switching of transmission lines in the definition of flexible line ratings, which
means that the rating of a line is allowed to be zero when it is switched off. Hence, there are
at most three states of a transmission line: off, adopting normal rating or adopting higher
rating. Noted that the normal rating of transmission lines are calculated conservatively
assuming severe ambient conditions, we don’t require lower ratings after allowing higher
ratings. But such a state of utilizing a lower rating could be modeled in the same way and
easily incorporate in the proposed model. The three states of transmission lines are modeled
as recourse actions in our two-stage stochastic model. By allowing such recourse actions in
the second stage, we actually solve a relaxation of the stochastic unit commitment without
such recourse actions. Including flexible line ratings will render more aggressive first- stage
decisions feasible.

\[
(FLRSUC) : \quad \min \sum_{t \in T} \sum_{s \in S} \pi_s \left( \sum_{g \in GF} h_g \sigma_{g,t,s} + k_g u_{g,t,s} + c_g P_{g,t,s} + \sum_{g \in GF} c_g P_{g,t,s} + \sum_{i \in N} \rho_i L_{i,t,s} \right) \\
+ \sum_{t \in T} \sum_{g \in GS} (h_g \sigma_{g,t} + k_g u_{g,t}) \quad (5.1)
\]
CHAPTER 5. STOCHASTIC UNIT COMMITMENT WITH FLEXIBLE LINE RATING REcourse

\[ s.t. (u_{GS}, \sigma_{GS}) \in \Delta_{GS} \]
\[ (u_{GF,s}, P_{G,s}, L_s) \in \Delta_{GF}(W_s, u_{GS}, F_s), \forall s \in S \]
\[ r_{ij,t,s}^0 + r_{ij,t,s}^1 + r_{ij,t,s}^2 = 1, \forall ij \in M, t \in T, s \in S \]
\[ -M_{ij}(r_{ij,t,s}^1 + r_{ij,t,s}^2) \leq F_{ij,t,s} - B_{ij}(\theta_{i,t,s} - \theta_{j,t,s}) \leq M_{ij}(r_{ij,t,s}^1 + r_{ij,t,s}^2), \forall i, j \in N, t \in T, s \in S \]
\[ -F_{ij,t,s} \leq F_{ij}^{\text{max,normal}} r_{ij,t,s}^1 + F_{ij}^{\text{max,high}} r_{ij,t,s}^2, \forall i, j \in N, t \in T, s \in S \]
\[ F_{ij,t,s} \leq F_{ij}^{\text{max,normal}} r_{ij,t,s}^1 + F_{ij}^{\text{max,high}} r_{ij,t,s}^2, \forall i, j \in N, t \in T, s \in S \]

\[ \sum_{k=0}^{\min(\{T|t_i|^N\})} r_{ij,t+k,s}^2 \leq i_{ij}^N, \forall ij \in M, t \in T, s \in S \]

\[ \sum_{k=0}^{\min(\{T|t_i|^N\})} (1 - r_{ij,t+k,s}^2) \geq \min(\{T|t_i|^N\})(r_{ij,t,s}^2 - r_{ij,t-1,s}^2), \forall ij \in M, t \in T, t \leq |T| - 1, s \in S \]

\[ \sum_{ij \in M} r_{ij,t}^0 \leq r_{ij}^{\text{max}}, \forall t \in T, s \in S \]

\[ \sum_{ij \in M} r_{ij,t}^2 \leq r_{ij}^{\text{max}}, \forall t \in T, s \in S \]

\[ r_{ij,t}^0, r_{ij,t}^1, r_{ij,t}^2 \in \{0, 1\}, \forall ij \in M, t \in T, s \in S \]

In the above formulation, we minimize the expected operating cost including production cost, no-load cost, start-up cost and the penalty cost for load shedding, as expressed in SUC. For brevity, we adopt set notations to represent constraints related to conventional units. Set constraint (5.2) represents the on/off transition constraints, minimum up-time constraints and minimum down-time constraints of slow units. Set constraint (5.3) include the on/off transition constraints, minimum up-time constraints and minimum down-time constraints of fast units. It also includes the ramping constraints, generation capacity constraints and the market clearing constraints of all units. The statuses of transmission line \( ij \) are represented by binary decision variables \( r_{ij,t,s}^0, r_{ij,t,s}^1, r_{ij,t,s}^2 \). Transmission line \( ij \) is switched off when \( r_{ij,t,s}^0 = 1 \) at time \( t \) in scenario \( s \). It adopts normal rating at time \( t \) in scenario \( s \) if \( r_{ij,t,s}^1 = 1 \). If \( r_{ij,t,s}^2 = 1 \), the line utilizes a higher rating than its normal rating, which is computed using less conservative ambient parameters. At each time period, the line must be in one status which is achieved in constraint (5.4). Constraints (5.5) is the modified DC power flow. In (5.5), \( M_{ij} \) is a large enough number. Constraints (5.6) and (5.7) is the line flow capacity constraint. Similar as in TCSUC, the voltage angle of bus \( i \) and bus \( j \) are coupled only if the line \( ij \) is on. If \( r_{ij,t,s}^0 = 1 \), we have \( F_{ij,t,s} = 0 \). Otherwise, the flow is within the capacity of the line. When \( r_{ij}^2 \) equals to 1, the line flow can exceed the normal rating, but still capped by \( F_{ij}^{\text{max,high}} \). In the numerical tests, we have \( F_{ij}^{\text{max,high}} = 1.1F_{ij}^{\text{max,normal}} \). Constraint (5.8) limits the number of consecutive time periods that a line can adopt higher ratings. Constraint
(5.9) mandates normal rating or switched off for a certain amount of time after the higher rating is utilized. This constraint is only active when $r_{ij,t,s}^2 = 1$ and $r_{ij,t+1,s}^2 = 0$ since it is a binary variable. Constraint (5.10) and constraint (5.11) limit the number of lines switched off and the number of lines utilizing higher ratings in each time period and each scenario. For switching decisions, previous literature [6] and [17] shows that the marginal benefit of enabling additional lines to be switched decrease significantly after a small portion of lines is allowed to be switched off. For line rating decisions, by limiting the number of lines adopting higher ratings in each time period, we can enhance the reliability of the system. Moreover, by including these constraints in the model can limit the search space of the line status decisions hence reduce the complexity of solving \textit{FLRSUC}. Simulation and stability analysis should be conduct. Details on constraints related to generators can be found in equation (3.34) and equation(3.37) - equation(3.50).

5.3 Numerical Tests on IEEE 118 System

In this section, we compare the results of \textit{SUC} and \textit{FLRSUC} on the IEEE 118 system with the same configuration, load profile, and renewable generation scenarios. If we solve the stochastic unit commitment without flexible line ratings (SUC), the expected cost is $36507. When flexible line ratings are modeled as a recourse action (\textit{FLRSUC}), the expected cost is $29481 that is 19.2\% lower than the expected cost of SUC.

In addition to the laptop with an Intel Core i7 2.6 GHz CPU and 12 GB RAM, we also have a desktop with an Intel Core i7 2.6 GHz CPU and 32 GB RAM for numerical tests in this chapter. We compare the commitment of slow units. In the optimal solution of \textit{SUC} and \textit{FLRSUC}, only the scheduling of G17 is different. As shown in Figure 5.2, G17 stays off for 6 more hours in \textit{FLRSUC} than in \textit{SUC}. Figure 5.3 and Figure 5.4 show the cost comparison of slow units and fast units. In the two figures, we divide the cost components in \textit{FLRSUC} by the corresponding cost components of \textit{SUC}. The numbers in the figures show ratios of cost components in \textit{FLRSUC} to those of \textit{SUC}. The numbers of start-ups of slow units in \textit{FLRSUC} and \textit{SUC} are the same, so the start-up costs of slow units are the same. When flexible line ratings are allowed, less expected generation is provided by slow units. However, the average fuel cost of generation from slow units is reduced with flexible line ratings. Fast units generate more power in \textit{FLRSUC} with lower costs as shown in Figure 5.4. Since in this test case, most of the generation capacity is from fast units, fast units contribute more to the cost reduction.

To understand how flexible line ratings could influence the dispatch, we can take a part of the IEEE 118 test case containing 4 buses and 5 lines as an example. The topology of this part of the system is shown in Figure 5.5 The load connected to Bus 90 is much larger than that of other buses. Bus 89 and Bus 92 are connected to the rest of the network. Without flexible line ratings, the bottleneck of this part of the system is the line connecting Bus 89 and Bus 92.

From Figure 5.6 we can see that without flexible line ratings, the flow on line Bus92-Bus89
Figure 5.2: Commitment of slow unit G17

Figure 5.3: Cost comparison of slow units in IEEE 118 test case
Figure 5.4: Cost comparison of fast units in IEEE 118 test case

Figure 5.5: A part of the IEEE 118 test case
reaches the static rating for 10 hours. Due to this congestion, units with lower costs could not be dispatched. When flexible line ratings are included in the second stage as decisions, line Bus89-Bus 91 is off for 15 hours. Line Bus92-Bus89 adopts higher line ratings in two hours. The peak flow of line Bus92-Bus89 is around 230 MW, which is only 105% of the normal static rating. The congestion on line Bus92-Bus89 is relieved, and better dispatch is allowed with flexible line ratings.

In FLRSUC, a subset of wind generation scenarios is selected to represent the uncertainty in order to reduce the complexity and to make the problem easier to solve. To evaluate the performance of the first-stage commitment decisions generated using the reduced scenario set with or without flexible line ratings, we conduct an out-of-sample test of the model on a larger set of scenarios. In the evaluation, we fixed the first stage decisions as the optimal commitment of slow units in SUC/FLRSUC. We generate 1000 wind generation scenarios using Monte Carlo simulation. In the 1000 test cases, we solve SUC and FLRSUC and compare the costs. In all 1000 tests, when flexible line ratings are allowed in the second stage, the cost is less than when there are no flexible line ratings. The average cost reduction is above 18% with flexible line ratings. This means that flexible line ratings enables better slow unit commitment decisions in the first stage.

5.4 Numerical Tests on Central European System

We also test the idea of utilizing flexible line ratings as recourse actions on a network representing the Central European System as shown in Figure 4.16. There are 679 buses, 667 conventional units, 1036 transmission lines and 1437 renewable units in this test case. Similar as in the numerical test of TCSUC, the connections between the Central European System and other countries outside the system are modeled as fixed imports and exports. The loading conditions and the renewable generation profiles are the same as in the second test case in Chapter 4 Section 6.

We select ten scenarios to represent the uncertainty of renewable generation. There are around 1 million continuous decision variables and over 900,000 binary decision variables in FLRSUC. Even for a single scenario sub-problem, there are over 120,000 binary decision variables. To reduce the complexity of solving this problem, we adopt the heuristic in Chapter 4 Section 4. We decompose the system into 5 zones. In the sub-problem of the largest zone FR+CH, there are around 450,000 binary decision variables and over 500,000 continuous variables. The solution time for this zone is around 18 hours using CPLEX on a laptop with an Intel i7 CPU and 12GB memory.

We compare the cost of SUC, TCSUC (the second test case in Chapter 4 Section 6) and FLRSUC. The results are shown in Table 5.1. From the results, we can see that with flexible line ratings modeled as recourse actions, the operating cost can be further reduced than in TCSUC. Comparing with SUC, the cost reduction of FLRSUC for the entire system is around 4.5%. The zone FR+CH has the largest cost saving. The cost reduction is above 10%. In the remaining part of this section, we will take FR+CH as an example. In
Figure 5.6: Line flows in the part of the IEEE 118 test case in scenario 3
Table 5.1: Test results of Centural European System

<table>
<thead>
<tr>
<th></th>
<th>SUCEUR</th>
<th>TCSUCEUR</th>
<th>FLRSUCEUR</th>
<th>Cost Saving of FLRSUCEUR</th>
</tr>
</thead>
<tbody>
<tr>
<td>AT</td>
<td>7.0057</td>
<td>6.8244</td>
<td>6.7980</td>
<td>0.2077</td>
</tr>
<tr>
<td>BE+LX</td>
<td>6.2083</td>
<td>6.2083</td>
<td>6.1850</td>
<td>0.0233</td>
</tr>
<tr>
<td>DE</td>
<td>14.2089</td>
<td>14.0540</td>
<td>13.9496</td>
<td>0.2593</td>
</tr>
<tr>
<td>FR+CH</td>
<td>17.3961</td>
<td>16.0753</td>
<td>15.5977</td>
<td>1.7984</td>
</tr>
<tr>
<td>NL</td>
<td>10.5475</td>
<td>10.3793</td>
<td>10.3642</td>
<td>0.1833</td>
</tr>
<tr>
<td>Total</td>
<td>55.3665</td>
<td>53.5141</td>
<td>52.8945</td>
<td>2.472</td>
</tr>
</tbody>
</table>

BE+LX, no cost saving is observed in TCSUC. With flexible line ratings, the cost is slightly reduced.

Figure 5.7 shows an example of a transmission line that utilized higher ratings in two time periods. The line flow is above the normal static rating at time period 1 and time period 6. After exceeding the normal rating, the flow goes below the normal ratings, and the transmission line gets cool down. Figure 5.8 compares the average number of lines switched off per hour in TCSUC and in FLRSUC. In some of the scenarios, more lines are switched off with flexible line ratings while in other scenarios, fewer lines are switched off. The switching of transmission lines will re-dispatch the flow in the network. This may cause the overflow of some transmission lines after the switching and make the topology of the network infeasible in TCSUC. With flexible line ratings modeled as recourse actions, the program can optimally choose when and on which line higher ratings will be utilized. The overflow caused by switching might become feasible in this case, and the operating cost can be reduced. On the other hand, the switching decisions are co-optimized with the rating decisions. In the cases where increasing the ratings of line facilitate better commitment units and dispatch of generation, we might not need to switch off lines.

The comparison of detailed cost information in zone FR+CH is shown in Figure 5.9 and Figure 5.10. To compare different cost components of slow units and fast units, we scale each component, dividing it by the corresponding value of SUCEUR. The values in the figures represent the cost component in FLRSUCEUR corresponding to that of SUCEUR. From Figure 5.9, we can see that flexible line ratings in the second stage facilitate more aggressive first stage commitment decisions of slow units. The start-up cost, no-load cost, and expected fuel cost of slow units is reduced. The expected generation from slow units decreased by less than 0.1%. However, both the expected cost of slow units and the average fuel cost of slow units decreased by over 10%. Moreover, almost the same amount of generation from slow units is dispatched in the second stage. As shown in Figure 5.10, due to the aggressive first stage
Figure 5.7: Line flow of transmission line F-198 to F-194 in scenario 1

Figure 5.8: Average number of lines switched off per hour in TCSUC and FLRSUC in zone FR+CH
CHAPTER 5. STOCHASTIC UNIT COMMITMENT WITH FLEXIBLE LINE RATING RECOUERSE

Figure 5.9: Cost comparison of slow units in zone FR+CH

Figure 5.10: Cost comparison of fast units in zone FR+CH
decisions, the expected start-up cost of fast generators increase by around 8%. Moreover, the expected generation of fast units increases by 0.7%. Figure 5.11 shows start-up cost of fast units in each scenario of SUC and FLRSUC. The start-up cost is higher in FLRSUC in three out of total ten scenarios. The probability of those three scenarios is around 0.43 hence the expected start-up cost of fast units in FLRSUC is higher. The expected no-load cost and the expected fuel cost was reduced when flexible line ratings are included as recourse actions. The expected cost of fast units and the average fuel cost of fast units are also reduced.

5.5 Summary

We investigate the potential benefits of utilizing flexibility provided by transmission system through flexible line ratings. We present an optimization model for determining when and which line should be switched or utilize higher ratings in the recourse of a stochastic unit commitment problem. We found that substantial cost saving could be achieved with such flexible line rating recourse actions in numerical tests where the higher rating of lines is only 10% higher than the normal static rating. The cost reduction is above 19% in the IEEE 118 system while it is around 4.5% in a network representing the Central European System. Result analysis of both systems shows that flexible line rating recourse serves as a hedging mechanism against the uncertainty brought about by renewable generation and supports more aggressive first stage commitment decisions.
This chapter is a first step in analyzing the potential benefits of flexible line ratings. Future work will take several directions, including the design and analysis of heuristics, the study of the impacts of flexible line ratings on system reliability, and the cost or surplus allocation.
Chapter 6

Concluding Remarks

The integration of large-scale renewable generation in power systems requires more operational flexibility. In this dissertation, we study the potential of utilizing the flexibility provided by existing transmission assets to mitigate the variability of renewable generation in the day-ahead scheduling of power systems. We model the switching of transmission lines and the decision of utilizing higher ratings of transmission lines as recourse actions in stochastic unit commitment models. Decisions associated with flexible transmission systems are made in the second stage after the realization of renewable generation. With such recourse actions modeled in the second stage, more aggressive first stage decisions are enabled. We also present a decomposition heuristic that first decomposes an interconnected multi-area system into zones and solves the problem for each zone in parallel. In this chapter, we first summarize our conclusions and then discuss future perspectives.

6.1 Conclusions

Stochastic Unit Commitment with Topology Control Recourse

In contradiction to the perception that more transmission lines enable the dispatching of thermal units with cheaper costs, actively switching off transmission lines may relieve congestion or potential congestion and reduce the operating cost of power systems. Previous research has illustrated the benefits of topology control through switching on/off transmission lines in corrective control and reducing system loss. There is also extensive research on utilizing topology control to reduce the operating cost of real-time power system.

In this dissertation, we model topology control as recourse actions in a two-stage stochastic unit commitment model for power systems with renewable generation. We analyzed how the switching decisions could affect the commitment decisions and the dispatching decisions in OPF and unit commitment. To solve stochastic unit commitment with topology control recourse for practical system efficiently, we also proposed a decomposition heuristic. Numerical tests conducted on both IEEE 118 system and a network representing the Central
European System demonstrate that with topology control recourse, the expected operating cost will be reduced. Moreover, to achieve a significant cost reduction, only a small fraction of lines need be switched off. The flexibility provided by topology control allows the commitment of cheaper units in both stages in the stochastic unit commitment problem. But such flexibility is limited by other conditions of the system. We observed that, for heavily congested systems, the operating cost could not be reduced significantly by purely introducing topology control recourse.

Stochastic Unit Commitment with Flexible Line Rating Recourse

We also extend the recourse actions to include flexible line rating decisions. Other than switching on/off transmission lines after the realization of renewable generation, we also allow overhead high voltage transmission lines to utilize higher ratings than normal ratings under security constraints. To ensure security operation, we limit the number of a number of consecutive time periods of adopting higher line ratings. Moreover, we enforce the line to adopt normal rating after utilizing higher rating for a certain amount of time periods to let the transmission line cool down.

In this thesis, we present an optimization model for day-ahead scheduling of power systems with renewable generation. In the model, we co-optimize the scheduling of conventional units as well as the flexible transmission system decisions including switching decisions and line rating decisions. Flexible line rating recourse serves as a hedging mechanism against the uncertainty brought about by renewable generation and assists more aggressive first stage commitment decisions. Combining transmission switching with flexible line rating decisions, more flexibility from the transmission network can be harvested to mitigate the uncertainty of renewable generation. We conduct numerical tests on both the IEEE 118 benchmark system and a network representing the Central European system. Results show that substantial cost saving could be achieved with such flexible line rating recourse actions in numerical tests where the higher rating of lines is only 10% higher than the normal static rating. The cost reduction is above 19% in the IEEE 118 system while it is around 4.5% in a network representing the Central European System.

6.2 Future Perspectives

Breaker Cost and Maintenance

With topology control recourse, the transmission lines need to be switched more frequently. This will cause the wear of circuit breakers and leads to higher maintenance fees. Both should be considered as components of the cost of switching transmission lines. Comparing with the operation costs of conventional units, the circuit breaker cost is relatively small. In this dissertation, we did not include the cost of breakers. But we think that properly identifying the cost of switching transmission lines is a future research direction. Moreover, introducing
switching cost will reduce the symmetry of the transmission switching solution. We observed cycling behaviors of solutions while applying scenario-based decomposition algorithms. We think it would be a great direction of future research on utilizing different values of switching cost to break cycling in scenario-based decomposition algorithms.

**N-1 and Stability Check**

$N-1$ security is a mandatory requirement for practical operations. It might have a substantial impact on the switching decisions. However, $N-1$ is a robustness criterion for dealing with uncertainties which are appropriate in a deterministic setting as done in the paper [16]. In a stochastic unit commitment formulation, the $N-1$ reliability criteria are replaced by incorporating contingencies of monitored elements such as transmission lines and generators into the probabilistic scenarios that are used in the stochastic optimization. In [81], for instance, the authors proposed an importance sampling method that applies to select scenarios that include both renewable generation contingencies, network contingencies and generation contingencies. The scenarios are generated by taking the Cartesian product of the renewal generation scenarios with the system contingencies (that would have been considered by the $N-1$ criterion). In this approach, we assume that contingencies and renewable generations are statistically independent. The representative scenarios for stochastic unit commitment can then be selected by a scenario reduction method. In stochastic unit commitment, we are trying to optimize the first-stage decision such that it is robust enough for both extreme renewable generation scenarios and system contingencies. Hence, in our approach, transmission switching serves not only as a recourse action against uncertain renewable generations but also as a control action that helps ensure system security under system contingency. In theory, the first-stage decision should be better than without topology control recourse. In this dissertation, we have not included system contingencies in the probabilistic scenarios. Adding system contingencies to the scenario set is computationally challenging and is deferred to future work. As an alternative, one may introduce post optimization processing to evaluate the robustness of the solution with respect to specific contingencies and eliminate switching actions that do meet the criterion.

**Generation and Transmission Expansion**

Another future research direction is to investigate how to incorporate flexible transmission system recourse in the expansion of conventional generation and the transmission system for power systems with renewable generation. In such problems, different expansion plans are evaluated and compared to a set of demand and renewable generation profiles with fixed system topology and static line ratings. By allowing topology control and flexible line ratings, we might have cheaper investment plans for new conventional units and transmission lines. If topology control and flexible line ratings are modeled in the planning stage, more flexibility might be harvested from an existing configuration of the transmission system in the day-ahead scheduling stage. However, this will make the planning problem more complex and
harder to solve. More sophisticated algorithms are required to efficiently solve the planning problem.
Bibliography


