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THE 184-INCH CYCLOTRON

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ABSTRACT

A PIG type ion source was mounted in the center of the 184-inch cyclotron with extension lips placed on the dees to reduce the dee gap and aperture in the source region according to requirements established by calculations. The operation of the source and the nature of the cyclotron beam were observed for various conditions of dee voltage, dee bias, source position, and the time sequence of ion injection. A pulsewise increase of dee voltage at the ion injection time to achieve greater acceptance of ions was investigated. The restrictions imposed on the initial conditions for ions resulted in a decrease in the radial oscillation amplitude by a factor of 2 to 3. The beam current was comparable to that available from the conventional open arc source.
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INTRODUCTION

The introduction of ions into a frequency-modulated cyclotron is a simple
problem if no particular requirements are placed on the number of particles
required in the beam and upon their orbits. A conventional source is an open
arc, which produces a plasma of ions near the center of the cyclotron.
Particles on the surface of the plasma experience the electric gradient of
the radiofrequency voltage on the dee and are accelerated. In the presence
of the high magnetic field their paths become small spirals. After a number
of accelerations by a resonant-frequency voltage, the particles rotate at a
large enough radius to gain a sizable portion of the available dee voltage, and
are then considered captured. The effective origin of the particles does not
correspond to the center of the cyclotron. This results in initial radial
oscillation which persists throughout the accelerating process. The efficiency
of capture is not large and even though high enough for one particle may be
unsuitable for another type of ion in those cyclotrons which attempt to
accelerate protons, deuterons, He\(^3\)'s, and alphas.

An alternative to the open arc is an enclosed ion source with extensions
on the dees to provide the necessary electric gradient to accelerate particles
to sufficient energy to avoid striking the ion source after one turn. The
introduction of initial radial oscillations is avoided.

The production of ions in a confined region in the presence of a strong
magnetic field suggests the use of a discharge of the type in a Phillips
Ionization Gauge, PIG. The magnetic field strength at the source position
is 15,000 gauss.

The design of the source and its operation involves two classes of problems.
One includes the trajectory, dee-voltage requirements, source position,
electric field configuration, and the time sequence in the source operation.
The second is the mechanical design of the source and the creation and operation of a PIG discharge within the source. This paper discusses the first, and mentions the influence of the second. The limitations observed in the PIG discharge are discussed below.

**DESIGN OF THE SOURCE GEOMETRY**

Experience at this laboratory and others has shown that feelers—i.e., rods—maintained at dee voltage and extended from the dee until adjacent to the ion source, increase current and define the orbits of the particles in constant-frequency cyclotrons. When the gap is thus closed, the electric radiofrequency gradient is increased and the improvement follows.

In frequency-modulated cyclotrons the dee voltages are small compared to those in constant-frequency cyclotrons; this gives rise to new problems. The principal one is obtaining enough energy gain to clear the source structure and the feelers after one turn. Another problem is having sufficient vertical focusing. This focusing is electrical rather than magnetic near the center. Vertical defocusing is to be avoided.

These two conditions guide quite well the preliminary design of a source. Figure 1 shows the structure conceived. A scale model, five times the actual size, was made and was used in an electrolytic tank to obtain the electric field gradients. The minimum spacings across which voltage appears are limited by breakdown. A rule for a practical limit is that \( EV \leq 0.3 \text{ megavolt}^2/\text{ft} \) for voltage across a strong magnetic field and \( EV \leq 0.03 \text{ megavolt}^2/\text{ft} \) for voltages in the absence of a magnetic field.

The vertical defocusing owing to an electrical field line's having a vertical component, which directs the particle away from the median plane, was avoided by surrounding the source with elements which paralleled it. The lines of force terminating on the source and the vertical bars were more nearly parallel to the median plane. This requirement is necessary for particles that are starting but rapidly becomes unimportant as the particles gain energy. After several turns the particles are beyond any influence of the vertical elements.
Fig. 1 Schematic of source structure.
The Lorentz equation was solved in the median plane by the UCRL differential analyzer. The equations in this plane are

\[
\frac{d^2 r}{d \phi^2} - r \left( \frac{d \theta}{d \phi} \right)^2 = \frac{\text{AE}_r}{\omega H_z} - r \frac{d \theta}{d \phi},
\]

\[
\frac{d^2 \theta}{d \phi^2} + \frac{2}{r} \left( \frac{dr}{d \phi} \right) = \frac{\text{AE}_\theta}{\omega r H_z} + \frac{1}{r} \left( \frac{dr}{d \phi} \right),
\]

where \( E_r = E_r(r, \theta, t) = E_0 \frac{\partial}{\partial r} \text{(Potential fcn}_r \text{)} \sin(\omega t + \delta) \),

\( E_\theta = E_\theta(r, \theta, t) = E_0 \frac{1}{r} \frac{\partial}{\partial \theta} \text{(Potential fcn}_\theta \text{)} \sin(\omega t + \delta) \),

and the time variable was changed to \( \phi \) by \( \omega t = \phi \) and \( \omega = \frac{e H_z}{M c} \), the resonance frequency at the source. The terms \( r, \theta \) are the polar coordinates of the particle; the origin of this system is the geometrical center of the cyclotron. \( H_z \) is the vertical magnetic field in the central region.

The trajectories obtained by the solution of these equations for the trial source by the differential analyzer gave the following information: (a) the required voltage, (b) the range of rf phase over which particles were accepted and accelerated, and (c) the restrictions on the orbits by the feelers and the source. These points are illustrated in Figure 2 and may be stated as (a) 17 kv or more, (b) about 35°, and (c) particles whose orbits are spirals about the center escape the source structure.

To provide for the optimum adjustment of the source position and to allow for the shifting of the dees and the dee extensions, under vacuum load and under magnetic field changes from turning on and off the field current, a positioning mechanism was provided for the ion-source tube. This mechanism provided for motions of about ± 0.5 inch parallel and normal to the dee edge, and rotation of the source tube on its own axis a total of 90°. The motions were controlled remotely. The source tube and this mechanism were mounted on the ion-source probe for insertion into the cyclotron for support, and for providing gas feed, electrical power, and metering.

1 Warren F. Stubbins, An Ion Source for the 184-inch Synchrocyclotron, University of California Radiation Laboratory Report no. UCRL-2078, Jan. 1953
Fig. 2 Computed trajectories for proposed source geometries.
The dee extensions were made of copper and clamped to the existing dee structure. The ion-source tube was made of tantalum and the end structure made of copper, blackened to aid in heat dissipation. Aluminum buttons served as emitters for the PIG discharge. This assembly is shown in Fig. 3.

VOLTAGE REQUIREMENTS IN STARTING PARTICLES

In synchrocyclotrons the dependence of the beam upon the radiofrequency voltage, the modulation rate, the electrode (dee) geometry, and other factors is very strong and involved. In order to achieve a good pickup of particles (and their acceleration to the desired energy) one wishes to arrange for optimum conditions. The extent to which one can provide these conditions is dependent upon the facilities of the cyclotron. The desirable arrangements and the means by which they were achieved on the 184-inch synchrocyclotron are discussed below.

The maximum dee voltage used in the 184-inch cyclotron when accelerating protons is about 15 kilovolts. This voltage is below the capacity of the oscillator, which can go to at least twice this value but is limited by the current that can be passed by the brushes on the rotating condenser. This current maximum occurs near the end of modulation cycle, however; the current near the start is one-half to one-third of this value.

The oscillator is pulsed on during the acceleration cycle and is cut off during the remaining time. In order for the oscillator to come on, it is necessary to remove a load on the oscillator caused by the free electrons in the vacuum chamber which dance along the magnetic field lines to the tune of the rf. This motion is called multipacting, and can involve considerable energy. A direct-current voltage is placed on the dee to sweep the electrons out of effective range and greatly reduce this load. This voltage is called dee bias, and is normally about 1500 volts.

The 184-inch cyclotron is constructed with a driven dee and a grounded dummy dee. These are separated by a gap of five inches. The center of the gap does not coincide with a diameter of the machine, however, and furthermore the electric field distribution is not symmetric over the gap but is offset from the gap center toward the driven dee. The effect is to make the electrical diameter from one to two inches from the diameter of the pole tips. Under these circumstances the open arc position for optimum
Fig. 3 Source and dee extensions assembled.
beam is determined experimentally and is about 3 inches from the cyclotron center.

The modulation of the radiofrequency by the rotating condenser provides for the acceleration of particles to relativistic energies. In the 184-inch cyclotron the modulation rate vs radius of particle, or vs time, is different for protons than for deuterons and alphas. On the proton range the initial rate of frequency modulation is very slow and becomes quite rapid at the end of the acceleration process. For deuterons and alphas, the order is opposite. It has been shown that capture efficiency in starting ions is related to the threshold voltage, and the optimum capture occurs for a dee voltage twice the threshold voltage. The threshold voltage, however, is directly related to the modulation rate. The higher the modulation rate, the higher the dee voltage required. (A discussion of capture efficiency is given in Appendix I.)

The optimum voltage for capturing protons is thus quite low, since the capture takes place at the initial part of the modulation cycle where the modulation rate is very low. This condition is in conflict with the requirement that ions gain enough energy to clear the structure of the experimental source.

The features of the source that optimize the voltage and geometrical conditions are as follows: (a) The dee extensions are constructed and mounted so that the electric field is symmetric about the 0.5-inch gap between them, and the gap is centered about a diameter of the cyclotron. (b) The dee bias is arranged to be pulsed off after the start-up of the oscillator and before the time at which the arc is pulsed, and held off during the acceleration period.

The dee voltage is increased at the time of injection of ions, i.e., when the arc is pulsed, by a spike. This spike is produced by a pulse transformer that increases the plate voltage of the oscillator. The position of the pulse in the acceleration cycle is adjustable to precede, coincide with, or follow the time of the arc-pulse. The effect of this voltage increase upon ion acceptance is discussed below. The increased radiofrequency current during the spike does not exceed the permissible

\footnote{Bohm and Foldy, Phys. Rev. 72, 649 (1947).}
value because it occurs early in the modulation cycle. The radiofrequency voltage envelope with and without the spike is shown in Fig. 4.

The effect of modulation rate upon particle acceptance is studied by varying the rotor speed of the rotating condenser and by varying the dee voltage, amplitude, arc position, and spike position in the modulation cycle.

**SOURCE PERFORMANCE**

The factors to be determined about the ion-source performance are:

(a) the efficiency of ion capture for various particles,
(b) the influence of the source on particle dynamics throughout the acceleration cycle,
(c) the dynamic behavior of the source under different conditions of cyclotron operation,
(d) the determination of the optimum operating conditions.

These factors are interdependent and can be only partially separated.

Three instruments were used in measuring the beam current. A direct measure of current is made by using the current-reading probe head. This is a copper block, of sufficient azimuthal length to stop the highest-energy particles, covered by an insulating coating and then by a conducting coating. The outer coating serves as a radiofrequency shield. The total thickness of the coatings is a few thousandths of an inch.

For the regions very near the source, i.e., within 20 inches, a thermocouple was used. The wires of the thermocouple directly stopped the beam. The heat generated was measured as an increase in temperature, and the relative current was determined by dividing the temperature increase by the particle energy.

The time characteristic of the beam was measured by observing the particles caused by the beam's being stopped by a probe or other elements of the cyclotron. A crystal detector was placed in a low-field region outside the vacuum chamber, pulses from which were observed and photographed on an oscilloscope. At the destruction of the beam, from hitting a probe, many pulses were observed. The time over which these pulses occur is a measure of the time spent in destruction of the beam. From the rate of expansion of the beam, the radial change between the initially observed pulses and the finally observed pulses is a direct measure of the radial extent of the beam. Thus the amplitude of radial oscillation is determined.
Fig. 4 Rf envelope with and without spike.
Because of the differences between protons and deuterons in the modulation rate during the early part of the acceleration period, the study of the source performance for both particles is informative.

Prior to the installation of the enclosed source, measurements were made on the proton beam from the conventional source. The effects of a spike at the ion acceptance time and the radial oscillation amplitude were measured.

Figure 5 shows the small effect of the spike on protons captured from the conventional source and accelerated to 21 inches. As seen in Fig. 6, discussed below, the effect of increased voltage of the spike is expected to cause a reduction in accepted particles. In Fig. 5, however, this is not observed. An explanation is that there are two competing processes. The one, discussed in Appendix I, causes a reduction and the other, the stronger electric field at the plasma surface, increases the number of particles drawn into starting orbits. That these two effects are nearly compensating is surprising.

To determine the advantage or difficulty of temporarily increasing the voltage at injection time, the beam current was measured as a function of the rotor speed. The optimum operating voltage is determined directly by the modulation rate, which is set by the rotor speed. The voltage was held constant in order that capture of ions from the plasma might not differ electrically. Figure 6 shows the results for two related conditions. Because an increase in voltage is similar in effect to a decrease in rotor speed in rejecting ions, one may expect a loss of beam when voltage is increased. The advantage of increased voltage may be seen by comparing the maximum current in both cases of Fig. 6. It is concluded that the use of a momentarily increased dee voltage to enable particles to clear the source structure is permissible in the operating range of the 184-inch cyclotron.

Figure 7 shows the variation of current with rotor speed when the experimental source is used. The same effect as for the conventional source is observed. The dee voltage of 17 kv (observed on the dee voltage meter) is the combined 15-kv dee voltage and the pulse, which carries the voltage up to 21 kv peak.

Figures 8 and 9 show, respectively, the proton current and the deuteron current vs source rotation, each for two values of dee voltage. In Fig. 9
Fig. 5 Beam current vs arc delay.
Fig. 6 Rotor speed vs proton current from conventional source.
DEE VOLTS = 17 kv
MAX. ROTOR SPEED = 600 RPM

Fig. 7 Rotor speed vs beam current (protons) at 26".
Fig. 8 Proton current vs source rotation at 26°.
Fig. 9 Deuteron current vs source rotation.
the 12-kv curve was taken with a voltage spike, while the 8-kv data did not have a spike. The position of 0° corresponds to the source slit pointed normal to the driven dee. The optimum corresponds to the calculated angle in the studies of source design. The optima are broad because of a relatively large slit in the source tube. The source tube was rotated on its own axis. Only one vertical element on the driven dee was used in the actual source, while two were considered in calculating the source performance. Thus Fig. 2 shows two pins, while Fig. 9 shows only one. Higher dee voltage broadens the optimum.

Figure 10 shows proton current vs arc position for the experimental source. The arc position indicates the time in the modulation cycle at which the arc is fired— in this case, the time at which the PIG discharge is initiated. The sharp maximum obtained, when the voltage spike is adjusted in time to give the optimum current at each datum point, indicates a unique acceptance condition, since one does not expect a resolution in time sharper than the duration of the arc. Normal operation of the discharge was from 100 to 150 microsec in duration.

Figures 11 and 12 show the increases in proton and deuteron currents, respectively, from the experimental source with the increase in dee voltage. The leveling off of the proton current confirms the loss of beam discussed above. The continuing increase of deuteron current is expected because of the high modulation rate at the start of the deuteron acceleration, again according to the discussion above and in the appendix. Figure 13 shows an increase of deuteron current as the spike voltage is increased. The greater capture and successful acceleration on the first few turns, owing to the increasing gradients, account for the increase, as in Fig. 12. The dee voltage was 12 kv, with the spike from 0 to 4 kv being added at the arc time. Normal operation on the deuteron range is with about 8 kv dee voltage, where optimum current is obtained from the conventional source.

Proton current from the experimental source vs radius is shown in Figs. 14 and 15. For the small radii the current drops very rapidly, with only a small fraction being accelerated to high energies. The effect of rotor speed is seen to be large; the higher the speed the greater the final acceptance. A greater final acceptance should be achieved with a different modulation pattern. The energy for radii shown in Fig. 14 is given by
Fig. 10 Proton Current vs arc position at 26".
Fig. 11 Dee voltage vs proton current at 26".
Fig. 12 Deuteron current vs dee voltage with spike at 26".
Fig. 13 Deuteron beam current vs spike voltage at 26".
Fig. 14 Proton current vs radius.
Fig. 15 Proton current vs radius.
\[ E \approx 0.07 r^2 \], where \( E \) is in Mev and \( r \) in inches. For larger radii, the current drops stepwise at particular points. The most striking effect of the experimental source is shown in Fig. 16, where the radial oscillation amplitude is plotted vs radius for both the conventional and the experimental sources. The well-defined starting conditions cause a decrease in the maximum radial oscillation amplitude by a factor of 2 to 3. This measurement was made using the scintillation counter mentioned above. This technique does not allow an evaluation of the distribution of the radial amplitudes of particles. It is believed that a reduction in the maximum amplitude of the sort observed implies a change in the distribution, giving a distribution more skewed toward small amplitudes than the distribution of the conventional source.

**PRODUCTION OF IONS IN THE EXPERIMENTAL SOURCE**

Unsteadiness in operation and an examination of the parts of the source after the four-day period of use established that the PIG discharge was not operating favorably. Figure 17 shows the aluminum buttons that serve as cathodes in the PIG discharge, and their copper holders. These elements fit in the conical sections of the source as seen in Fig. 3. The button diameter is that of the inside of the tube, and its position is directly in line with the tube. Bombardment of these buttons by the gas ions produces electrons which are repelled and travel along the magnetic field lines up the tube. In this manner the electrons ionize the gas, which then is pulled out of the source tube by the penetration of the rf electric fields through the slit.

The gas was introduced into the source through a small tube attached close to the junction of the lower cone and the source tube, at the position of the lower aluminum button. No gas was injected through the upper cone. The cathode shown on the right of Fig. 17 was in the lower cone, and the aluminum button is seen to have completely eroded away, as has a portion of the copper holder. The upper cathode is seen nearly untouched, and has the same appearance as when installed. Thus it is established that the discharge was concentrated in the lower cone and only a small fraction of the ions must have been formed in the region of the slit. This is an error in design and suggests an improvement of the beam intensity by a correction of the error.
Fig. 16 Radial oscillations amplitude vs radius.
Fig. 17 Discharge cathodes in PIG source.
Even under this unfavorable operation, the currents were comparable to those from the conventional source. Table I shows the currents from the different sources.

Table I

<table>
<thead>
<tr>
<th>Source</th>
<th>Proton</th>
<th>Deuteron</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Scattered Deflected Circulating</td>
<td>Circulating</td>
</tr>
<tr>
<td>Conventional Source</td>
<td>$0.7 \times 10^{-8}$ amp $0.3 \times 10^{-7}$ amp $0.2 \times 10^{-6}$ amp</td>
<td>$0.5 \times 10^{-6}$ amp</td>
</tr>
<tr>
<td>Experimental Source</td>
<td>$0.28 \times 10^{-8}$ amp $0.26 \times 10^{-8}$ amp $0.9 \times 10^{-7}$ amp</td>
<td>$0.4 \times 10^{-7}$ amp</td>
</tr>
<tr>
<td>Ratio</td>
<td>0.25</td>
<td>0.09</td>
</tr>
</tbody>
</table>

'Scattered current' refers to that which was scattered to an outer radius as a means of deflecting particles. "Deflected" means the current moved to an outer radius by means of the pulsed electrical deflector. "Circulating" current is that measured at 80 inches within the stable radii of the cyclotron. The others were measured at 83 inches. The circulating beam does not go beyond 81 inches. The small deflected current is expected, since the deflector is adjusted in normal operation for a beam which has a large amplitude of radial oscillation. A study showed small amplitudes would not be efficiently deflected.

CONCLUSIONS

It may be concluded that if a source is used in which suitable and well-defined starting conditions are established, the amplitude of radial oscillation of the beam in a synchrocyclotron may be substantially less than that obtained from conventional ion sources. One may thus obtain a sharper energy spectrum of the beam.

The modulation rate is a strong factor in beam capture, and suggests that improvement or increase of accelerated beam may be achieved by optimizing the modulation pattern.

The rapid loss of current at small radii indicates that the source is productive of ions and does not limit the current.

In the range of the present proton operation it is possible to increase
the dee voltage at the injection time, when the modulation rate is low, and not have a net decrease of beam. In the deuteron range (initial modulation rate high) the increase of voltage at injection results in a linear increase of current.

The currents accelerated from the experimental source are comparable to those from the conventional source. Prior to any permanent installation of a source of this type, however, an improved internal PIG operation must be achieved.

ACKNOWLEDGMENTS

Mr. Robert Avery made the mechanical design of the source and dee extensions and directed their installation. Messrs. James Vale and Lloyd Houser and members of the cyclotron crew assisted in the installation and cooperated in the operation of the cyclotron during the tests. Messrs. John Barale, George Hampton, and Ivan modified the oscillator and arranged electronic equipment to provide the spike and various controls. Dr. Lawrence Ruby assisted in the experimental measurements. The efforts of these men are greatly appreciated, as are the interest and advice of Professor Robert Thornton.

This work was done under the auspices of the U.S. Atomic Energy Commission.
Appendix I

Qualitative Discussion on Requirement for Initial Low Dee Voltage in Synchrocyclotrons

A study of the voltage requirements on fixed-frequency cyclotrons may be applied to frequency-modulated cyclotrons to establish the necessity of low dee voltage in the latter.

In fixed-frequency cyclotrons, Figures 18 and 19 may be drawn:

The solid line represents the natural angular frequency of the particles as a function of energy, \( \omega = eH/Mc \). The decrease in \( H \) with radius, required for vertical stability, and the increase in \( m \) due to relativistic effects lead to a monotonic decrease in \( \omega \) with \( T \). \( T \) is the kinetic energy of the particle. The shape of the solid line in Fig. 18 is quite general, and is suitable for this discussion.

The term \( \omega_0 \) is the fixed rf angular frequency and matches the natural frequency of the particle at one radius or energy.

To observe the phase of the particle with respect to the radiofrequency voltage, consider Fig. 19:
The convention is that the phase is negative if the particle is ahead of the rf. Particles beyond \( \pm \pi/2 \) are decelerated and lost from the useful beam.

Three cases are of interest:

(a) accelerating voltage just equal to the threshold voltage, \( E = E_{th} \);
(b) accelerating voltage less than threshold, \( E < E_{th} \);
(c) accelerating voltage greater than threshold, \( E > E_{th} \).

Assume that the particle is started at \( 0^\circ \) in phase, i.e., in phase with rf (this is observed to be the condition achieved in ion-source studies), and is accelerated by the rf voltage. After one turn, the particle leads the rf by the phase error indicated by the difference of the intercepts of the lines in Fig. 19. This is a negative error according to our convention. The next turn the error will be less, since at a greater energy the separation between the two curves is less. The error per turn becomes less as the particle gains energy, until at the point \( k \) in Fig. 18 the error becomes zero. Beyond the point \( k \) in energy, the error accumulates in the opposite direction.

If the voltage is sufficient so that the particle reaches \( k \) in radius before it reaches \(-\pi/2\) in phase, then it continues to be accelerated to greater
Fig. 18 Angular frequency vs particle kinetic energy for fixed frequency cyclotron.
Fig. 19 Relative particle phase angle vs particle kinetic energy for fixed frequency cyclotron.
radii and the phase moves increasingly rapidly in the positive direction. At the value of $\phi = \pi/2$, the particle becomes decelerated, and so it is removed at this phase or before it by the deflection system of the machine.

In Figure 19, Case (a) shows the limit where $k$ and $-\pi/2$ are reached simultaneously. This is the minimum dee voltage that will bring particles to the design energy $\ell$; this is called the threshold voltage.

Case (b) shows a particle not gaining energy rapidly enough, and the error in phase is still accumulating negatively when the point $\phi = -\pi/2$ is reached. The particle is then decelerated and returns to the center of the cyclotron.

Case (c) is that for a dee voltage in excess of threshold. The particle quickly increases its energy and passes point $k$ before reaching $-\pi/2$. It continues to approach energy $\ell$ rapidly. The higher the dee voltage, the more rapidly the particle moves outward and the less phase error it has. In this case, the phase acceptance angle $\phi_{acc}$ is large and the beam current is large. See Fig. 19, Cases $c^1$ and $c^2$. Thus the statement, "the higher the dee voltage the greater the current," applies to fixed-frequency cyclotrons.

For frequency-modulated cyclotrons, Fig. 18 may be redrawn to give Fig. 20.

The modulation of the radiofrequency moves the points $k$ from small energy to large as the acceleration cycle proceeds.

A similar curve for Fig. 19 may be drawn for this case to give Fig. 21.

Case (a'). At the condition $\omega_0 = \omega_0'$ and $E' = E_{th}'$ the particle reaches $k'$ just at $-\pi/2$ and reaches $\ell'$ at $\pi/2$, and is decelerated. In this case $\ell'$ is short of the full energy of the cyclotron. (Note that $E_{th}'$ for this case is much smaller than in the fixed-frequency machine, since the energy gain before $\phi = -\pi/2$ is $k' \ll k$ for fixed-frequency cyclotrons.)

Case (b'). $E' < E_{th}'$; the particle does not reach $k'$ before $-\pi/2$ and is decelerated.

Case (c'). $E' > E_{th}'$; the particle gets beyond $\ell'$ but not to the full energy of the machine.

If the frequency modulation rate is low, i.e., $df/dt \sim 0$, then the point $k'$ does not move appreciably during the time of the particle existence in Case (c').
Fig. 20 Angular frequency vs particle kinetic energy for frequency-modulated cyclotron at different time.
Fig. 21 Relative particle phase angle vs particle kinetic energy for frequency-modulated cyclotron.
The time the particle exists to be further accelerated is given by the following: The number of turns the particle makes in going to its greatest radius is

\[ N = \frac{T \text{ at } \pi/2}{V_{av}} \]

where \( T \text{ at } \pi/2 \) is the kinetic energy when the particle starts to be decelerated, and \( V_{av} \) is the average energy gain per turn. For large dee voltage, \( V_{av} \approx E' \) for most of the time. Thus

\[ t = \tau_0 N \approx \frac{2\pi}{\omega_0} \frac{T \text{ at } \pi/2}{V_{av}} \]

Note: The error in phase accumulates at the rate given by the separation between the lines in Fig. 20, and is independent of the rate of energy gain. For Case (c') the rate of phase increase when the particle is beyond \( f' \) is large, and in a short time the particle arrives at \( \pi/2 \). For Case (a') the time is much longer for two reasons. One is that \( V_{av} < E' \) and the other is that the phase error accumulation is slower since the particle is not as far from \( k' \) as at the end of the path c'.

As the frequency is changed, \( k' \) moves toward \( k'' \), and if this motion is rapid enough, the accumulation of phase error becomes slower or even reverses direction if \( k'' \) exceeds the energy of the particle. Thus, for rapid frequency modulation, \( df/dt << 0 \), a high accelerating voltage is useful and necessary. But low voltage is required for low modulation rates.

For successful acceleration to large radii, the particle must be accelerated slowly enough to be retained during the initial phase oscillation. The modulation of the frequency alters the bounds of phase stability of Fig. 18b to the case for synchrocyclotrons wherein there is phase focusing. The optimum conditions now may be found corresponding to Bohm and Foldy. \( E_{th} \) is directly proportional to the rate of frequency modulation, \( E_{th} \propto df/dt \).

Figure 22 indicates the conditions as \( E \) varies for a given threshold voltage \( E_{th} \).

Case (a). \( E = E_{th} \), i.e., 1.0 on abscissa; the range of acceptable phases is so small that no particles are observed.

Case (b). \( E = 2E_{th} \), i.e., 0.5 on abscissa; the optimum condition exists.
Fig. 22 Relative accelerated beam vs dee voltage for a given threshold in a frequency-modulated cyclotron.
Case (c). $E \gg E_{th}$, i.e., near 0 on abscissa, most particles have been lost by passing $\pi/2$ before the frequency shift was sufficient to allow the phase error accumulation to be zero or reverse itself. The relatively flat top on the curve of Fig. 22 shows suitable dee voltages to be from 1.5 to 4 times the threshold voltage.