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Shear Layer Instabilities and Mixing in Variable Density Transverse Jet Flows

A dissertation submitted in partial satisfaction of the requirements for the degree Doctor of Philosophy in Aerospace Engineering

by

Daniel Robinson Getsinger

2012
ABSTRACT OF THE DISSERTATION

Shear Layer Instabilities and Mixing in Variable Density Transverse Jet Flows

by

Daniel Robinson Getsinger

Doctor of Philosophy in Aerospace Engineering

University of California, Los Angeles, 2012

Professor Ann R. Karagozian, Co-chair

Professor Owen I. Smith, Co-chair

This work describes the experimental characterization of the instabilities forming in the near-field of the variable density transverse jet shear layer, and mechanisms by which jet behavior and mixing may be controlled. Jets comprised of mixtures of gaseous nitrogen and helium are injected from a nozzle mounted flush with an injection wall, issuing into air crossflows at a constant jet Reynolds number of 1800. Jet-to-crossflow density ratio $S$ is varied between 1.00, representing the pure nitrogen jet, and 0.14, the pure helium jet, by changing the proportions of nitrogen and helium composing the jet fluid. Jet-to-crossflow momentum flux ratio is varied at incremental values between $\infty > J \geq 2$ at each value of $S$. The results of single-component hotwire measurements in the jet shear layer indicate that a transition to global instability likely occurs as $J$ is altered below approximately 10 and/or as $S$ is brought below approximately 0.45-0.40. This transition is characterized by several clear spectral features, including sharp spectral peaks and resistance to low level acoustic forcing for the globally unstable (self-excited) case, and broadband oscillations with high receptivity to applied forcing for the convectively unstable case. Evidence of a Hopf bifurcation is found under examination of the growth of the shear layer oscillation magnitude under variation of the control parameter $J$, as well in the response to pure-tone forcing near the shear layer’s natural mode frequency. These observations appear to link many previous independent studies of both equidensity transverse jets and low density free jets, which may become
globally unstable under alteration of $J$ and $S$, respectively. However, the dynamical character of the transition to global instability in the low density transverse jet displays differences under independent variation of $J$ and $S$, which may indicate the predominance of different modes. Particle image velocimetry is also used to expand upon these hotwire measurements using a less intrusive and planar technique, confirming the quantitative findings related to the shear layer behavior.

Planar laser-induced fluorescence measurements of jet fluid concentration in various cross-sections of the jet indicate that at large values of $J$ and $S$, the $Re_j = 1800$ transverse jet is highly asymmetric about its (often) assumed plane of symmetry. Mixing quantification shows that mixing may scale according to a normalized spatial coordinate $x/\sqrt{JD}$, and that lower density jets are less well mixed than their higher density counterparts at equivalent scaled downstream distances from the point of injection. Acoustic excitation of a globally unstable transverse jet under a variety of different square waveforms yields optimal mixing enhancement at a particular square wave duty cycle associated with highest penetration of individual vortex rings into the crossflow. Knowledge of the transverse jet shear layer’s stability characteristics remains an important factor in optimizing jet control.
The dissertation of Daniel Robinson Getsinger is approved.

Russel E. Caflisch
Jeff D. Eldredge
Owen I. Smith, Committee Co-chair
Ann R. Karagozian, Committee Co-chair

University of California, Los Angeles
2012
To my father, Rich, and to the life and memory of my mother, Cindy.
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<th>Definition</th>
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<tr>
<td>$C$</td>
<td>Concentration of jet fluid</td>
</tr>
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<td>$D$</td>
<td>Jet nozzle diameter</td>
</tr>
<tr>
<td>$DC$</td>
<td>Duty cycle of square wave forcing (pulse width/period)</td>
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<tr>
<td>$f_f$</td>
<td>Applied jet forcing frequency</td>
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<tr>
<td>$f_o$</td>
<td>Natural (fundamental) frequency of jet shear layer instability</td>
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<tr>
<td>$L/D$</td>
<td>Non-dimensional stroke ratio related to vortex ring formation</td>
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<tr>
<td>$J$</td>
<td>Jet-to-crossflow momentum flux ratio, $\rho_j U_j^2 / \rho_\infty U_\infty^2$</td>
</tr>
<tr>
<td>$J_{cr}$</td>
<td>Critical jet-to-crossflow momentum flux ratio at which bifurcation to a global mode occurs</td>
</tr>
<tr>
<td>$\dot{Q}$</td>
<td>Volumetric flow rate</td>
</tr>
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<td>$R$</td>
<td>Jet-to-crossflow velocity ratio, $U_j/U_\infty$</td>
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<tr>
<td>$Re_j$</td>
<td>Jet Reynolds number, based on mean jet velocity and nozzle diameter $D$</td>
</tr>
<tr>
<td>$Re_\infty$</td>
<td>Crossflow Reynolds number, based on freestream crossflow velocity and nozzle diameter $D$</td>
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<tr>
<td>$s$</td>
<td>Spatial coordinate along the center of the upstream shear layer (hotwire measurements), or along the scalar centerline (PLIF measurements)</td>
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<tr>
<td>$S$</td>
<td>Jet-to-crossflow density ratio $\rho_j/\rho_\infty$</td>
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<tr>
<td>$SMD$</td>
<td>Spatial mixing deficiency</td>
</tr>
<tr>
<td>$St$</td>
<td>Strouhal number based on diameter, $fD/U_j$</td>
</tr>
<tr>
<td>$T$</td>
<td>Period of acoustic forcing, or temperature</td>
</tr>
<tr>
<td>$U_j$</td>
<td>Mean jet velocity</td>
</tr>
<tr>
<td>$u'_{rms}$</td>
<td>Root mean square of the jet velocity perturbation</td>
</tr>
<tr>
<td>$U_\infty$</td>
<td>Freestream crossflow velocity</td>
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<tr>
<td>$U$</td>
<td>Unmixedness</td>
</tr>
<tr>
<td>$Y$</td>
<td>Jet fluid mass fraction</td>
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$x, y, z$  Downstream, transverse, and axial coordinates measured from jet orifice (see Fig. 1.1)

$\chi$  Mole fraction

$\theta$  Momentum thickness

$\tau$  Temporal pulse width of applied square wave jet excitation
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CHAPTER 1

Motivation and Background

The injection of a round jet into crossflow is a classical three-dimensional flowfield that is important in an extensive range of engineering systems, primarily those related to energy generation and propulsion. In any practical application, the dispersal (spread, penetration) of the jet as well as its mixing with the crossflow fluid are critical parameters to be predicted, measured and/or controlled (Karagozian, 2010). For instance, some of the very first scientific studies involving the transverse jet flowfield dating back to the 1930’s examined the issuing of chimney and smokestack plumes into the atmosphere to determine the resultant distribution of pollutants (Margason, 1993), a problem of significant impact on the environmental health of industrial areas. Of similar environmental concern is the efflux of liquid waste into streams and rivers.

Transverse jets are utilized for fuel injection in high-speed airbreathing engines such as ramjets or scramjets, where effective mixing of fuel and air in an extremely short residence time is required for efficient performance. Transverse injection schemes provide a means of rapid fuel spread and penetration, and the recirculation region forming upstream of injection results in sufficient flame-holding in many operational scenarios (Ben-Yakar et al., 1998). However, injection behind a rearward-facing step (Karagozian et al., 1996) or within or upstream of a cavity (Ben-Yakar, 2000) can provide enhanced mixing and flame-holding, and thereby sustained ignition and burning for improved combustion efficiency. Pulsed gas jets introduced just upstream of a rearward-facing step may also be used as an active control method for suppression of combustion instabilities (Choudhury et al., 1985).

Cooling of hot combustion products in gas turbine combustors is achieved using dilution air transverse jets. This application has motivated numerous experimental studies of the
resultant jet velocity and temperature fields for a single injector (Keffer and Baines, 1963; Kamotani and Greber, 1972), as well as various configurations of injector arrays (Holdeman, 1993). Introduced downstream of the combustion region and just upstream of the turbine inlet, controlled dilution jets can yield improved temperature pattern factor, prolonging the lifetime of turbine blades and enhancing engine performance (Vermeulen et al., 1992). Air may also be injected in the combustion region for tunable control of the fuel-air ratio and reduction of $NO_x$ and/or CO emissions.

Transverse jets are also used as vortex generators for control of boundary layer separation on turbine blades in aircraft gas turbine engines. Jets located on the leading edge of the blade may be actively controlled to promote boundary layer attachment at low turbine inlet Reynolds numbers, maintaining turbine efficiency (Bons et al., 2002). Such jets have the additional benefit of providing film cooling of the blade surface as they produce an insulating layer that protects the blade from hot combustion gases, thus allowing higher inlet temperatures and thermal efficiency. Pulsing of the jet has been shown to increase film effectiveness and lower heat transfer coefficient to the blade surface as compared with the steady jet (Ekkad et al., 2006).

Jets have long been used as thrusters for reaction control of rockets, missiles, and submarine vehicles, as well as for takeoff/landing transition of V/STOL aircraft such as the Harrier and the F-35 Lightning II. Recent development of the fluidic throat skewing technique for thrust vectoring promises significant reduction in the weight and mechanical complexity of aircraft nozzles (Miller et al., 1999; Yagle et al., 2001). Injection of control jets around the nozzle throat can achieve throat-area throttling, or, when injected in an asymmetric fashion, can effectively skew the sonic plane for subsonic vectoring, avoiding losses associated with shock-based fluidic vectoring. The penalty associated with bypass air requirements for these jets may be minimized by pulsing the jets via acoustic forcing (Vermeulen et al., 2004). Fluidic thrust vectoring can extend the flight envelope to otherwise unattainable angles of attack while reducing the necessary size, weight and drag of conventional aerodynamic control surfaces.
1.1 The Single-Phase Jet in Crossflow

The single-phase jet in crossflow (JICF) involves the perpendicular injection of jet fluid, characterized by velocity $U_j$, into a uniform crossflow of velocity $U_\infty$. The jet may be introduced via a pipe or nozzle mounted flush with or elevated above a wall. A schematic of the flush jet, including the dominant vortical structures resulting from the jet/crossflow interaction, is provided in Figure 1.1. As the jet fluid trajectory bends into the crossflow direction, the characteristics of the interaction in terms of the vorticity dynamics, shear layer stability, jet fluid penetration and spread, and scalar mixing are highly dependent upon several flow parameters. Among these are the jet-to-crossflow velocity ratio $R$, density ratio $S$, and momentum flux ratio $J$, defined as:

$$R = \frac{U_j}{U_\infty} \quad S = \frac{\rho_j}{\rho_\infty} \quad J = SR^2$$ (1.1)

respectively. For the equidensity JICF ($S = 1.0$), $J$ reduces to $R^2$. Additional parameters include the jet Reynolds number $Re_j$, which is based on jet velocity $U_j$ and diameter $D$, the crossflow Reynolds number $Re_\infty$, which is based on the crossflow velocity $U_\infty$ and the jet diameter $D$, and the scaled jet momentum thickness $D/\theta$, which varies azimuthally around the jet orifice due to the influence of the crossflow.

1.1.1 Transverse Jet Vortex Structures

The JICF interaction results in the formation of several inter-related types of coherent structures or vortex systems, as shown in Figure 1.1. These structures include: (a) the ring-like shear layer vortices which roll up on both the windward and leeward sides of the jet in the near-field; (b) the counter-rotating vortex pair (CVP) which dominates the jet cross-section in the far-field; (c) horseshoe vortices forming from the crossflow boundary layer upstream of the jet which wrap around the jet column and convect downstream; and (d) upright wake vortices which result from vorticity in the wall boundary layer. The CVP has been observed to be the dominant mechanism for entrainment of crossflow fluid into the jet, and its generation, growth, and turbulent breakdown is responsible for improved mixing of the JICF
Figure 1.1: Schematic of the transverse jet, introduced flush with respect to the injection wall, and relevant vortical structures. Adapted from Fric and Roshko (1994).

as compared with the free jet of equal $Re_j$ (Kamotani and Greber, 1972; Broadwell and Breidenthal, 1984; Karagozian, 1986; Smith and Mungal, 1998). These vortices are formed by the shearing of the jet fluid on the lateral edges of the jet, which effectively folds the cylindrical vortex sheet issuing from the jet orifice (Moussa et al., 1977; Kelso et al., 1996). While the CVP gives the jet its characteristic kidney shape in the far-field, its generation has been observed experimentally to occur very near to the jet orifice, and for certain flow conditions it may be initiated within the jet nozzle or pipe. In fact, vortex structures within the injection pipe, resulting from asymmetry of the fluid supply feed entering the pipe and/or separation of the pipe boundary layer, have been shown by Peterson and Plesniak (2004) to interfere with the CVP in either a constructive or destructive manner, depending upon their rotational sense. Thus the near-field interaction between the newly formed CVP and various other vortex systems, most notably the jet shear layer vortices, are thought to play a significant role in determining the trajectory and mixing characteristics of the jet (Cortelezzi and Karagozian, 2001).

The horseshoe vortex system forming just upstream of injection occurs in a similar fashion to boundary layer flow past a wall mounted cylinder. An adverse pressure gradient forms due to the blockage caused by the jet column, leading to boundary layer separation and
formation of vortices that convect around the jet and towards its wake. At low crossflow Reynolds number ($Re_\infty < 1000$), Kelso et al. (1996) observed that horseshoe vortices of the same sign of vorticity as the wall boundary layer extend downstream into the wake vortex system, while those of opposite sign lift away from the wall behind the jet and join with the CVP. Additionally, interaction with the roll-up of shear layer vortices on the windward side of the jet column, as well as with the wake vortex system, makes the formation of horseshoe vortices an unsteady, periodic process for certain flow regimes (Krothapalli et al., 1990). Kelso and Smits (1995) found that the horseshoe vortices can be steady, oscillating, or coalescing depending upon $R$ and $Re_\infty$, and that the unsteady types have either the same or double the oscillation frequency of the upright vortices in the wake.

The formation and dynamics of the wake vortex system was examined in detail by Fric and Roshko (1994) via smoke seeding of the jet and the wall boundary layer. While the jet wake is reminiscent of the von Kármán vortex street aft of a solid cylinder, its dynamics result from a different mechanism. Vortices are not shed from the jet boundary, as the crossflow does not separate from the lee side of the jet. Rather, eruption of the tornado-like upright vortices were found to be correlated with ‘separation’ events in the wall boundary layer just downstream of the jet. In this manner, boundary layer fluid is drawn upwards and into the jet, contributing vorticity to the CVP. This mechanism was extended by Smith and Mungal (1998) to explain their observation of the presence of jet fluid in the wake structures. The periodicity and definition of the wake structures seems to depend upon $R$, $Re_\infty$, and $\theta/D$.

### 1.1.2 Transverse Jet Shear Layer Instabilities

The roll-up of the transverse jet shear layer into ring-like vortex structures is typically attributed to a Kelvin-Helmholtz type instability which initiates near the jet exit (Kelso et al., 1996; Fric and Roshko, 1994). Visual evidence of periodic vortex roll-up and propagation on both the windward and leeward sides of the jet has led some to postulate that initially axisymmetric rings forming on the jet column become distorted by the crossflow, tilting
and stretching as they convect downstream (Kelso et al., 1996; Cortelezzi and Karagozian, 2001). The stretching of the ring-like vortices into loops on the leeside of the jet causes each subsequent structure to interact with its predecessor. This stretching begins near the location of CVP formation, lending credence to the assertion that the shear layer vortices contribute to the circulation of the CVP (Lim et al., 1998).

The large-eddy simulations of Yuan et al. (1999) identified several distinct shear layer vortex structures, providing an alternative to the previous ‘ring deformation’ interpretation of the shear layer roll-up. These structures include Kelvin-Helmholtz-induced ‘spanwise rollers’ on the windward and leeward sides of the jet, ‘hanging vortices’ on the lateral edges of jet which extend upwards and downstream, and ‘vertical streaks’ which result from perturbations in the spanwise rollers. In light of the later flow visualization experiments of Lim et al. (2001), these structures may be interpreted as parts of loop vortices of opposite vorticity content which form on the windward and leeward sides of the jet, rather than distinct vortices or distorted vortex rings. The formation of rings may be prohibited by the asymmetrizing effect of the CVP. Rather, the observed structures on each side of the jet are thought to be distinct, with lateral arms that merge with the CVP. At low $R$ conditions, these loop vortices have been found to form only on the windward side of the jet, and to undergo a change in vorticity from ‘jet-like’ to ‘wake-like’ in nature within several diameters of the jet orifice (Andreopoulos, 1985; Lim et al., 2001).

Additionally, some researchers have questioned the instability mechanism that causes shear-layer vortex roll-up. Blanchard et al. (1999) determined that shear layer vortex generation was better described by the Landman and Saffman theory for a global elliptic instability of the CVP than by Kelvin-Helmholtz theory. Camussi et al. (2002) similarly linked the near-field instabilities to an instability of the CVP, accrediting shear layer vortex formation to a ‘waving of the jet flow’, or an oscillation of the jet curvature. However, the experiments of Blanchard et al. were performed for a rectangular jet orifice and Camussi et al. for very low Reynolds number ($Re_j < 500$), quite different flow conditions from the usual JICF configuration.

While the details of the formation mechanism and structure of the ring-like shear layer
vortices remains inconclusively determined, their interaction with the CVP and the implied influence on global jet structure and mixing is generally accepted. However, few quantitative studies of the natural evolution of the JICF shear layer have been undertaken until recent years. Two extensive surveys in this vein include the linear stability analyses (LSA) of Alves (2006) and the experimental work of Davitian (2008). The experimental work, incorporating both flush and elevated jets, focused on round jets composed of nitrogen injected into air crossflows, yielding equidensity mixing environments at \( Re_j = 2000 \) and 3000 and \( \infty < R < 1.15 \). As described in Megerian et al. (2007), the nature of the jet shear layer oscillation (as measured on the windward side of the jet) was found to differ from that of the free jet shear layer, and to undergo a stark transition as \( R \) was reduced below approximately 3.5 for the flush jet (see Figure 1.2 for comparison of shear layer spectra on either side of the transition). For \( R > 3.5 \), the instabilities initiated several jet diameters from the jet orifice, moving progressively closer as \( R \) was reduced. The ‘fundamental’ or ‘most amplified’ mode, \( f_0 \), exhibited frequency-shifting from higher to lower frequencies as the hotwire probe was moved downstream along the jet shear layer trajectory, and eventually deteriorated as the

Figure 1.2: Vertical velocity spectra in the equidensity transverse jet at two \( R \) conditions, one each above and below the apparent transition to global instability. Spectra are measured within the jet shear layer at varying distance \( s \) from the jet exit. Conditions correspond to \( Re_j = 2000 \). From Davitian (2008).
vortex pairing process resulted in the growth of a subharmonic mode at $f_0/2$. The presence of higher harmonics indicated the nonlinear nature of the instability. The LSA, performed for relatively large velocity ratios ($R > 4$) predicted variations in the Strouhal number ($St \equiv f_0D/U_j$) and growth rates of the instabilities that compare well with experimental data for flush jets in this regime (Alves et al., 2008).

When $R$ was reduced through a ‘critical’ velocity ratio range near $3.5 > R > 3.2$, the fundamental instability modes grew in magnitude, initiated closer to the jet exit, stabilized in frequency, and remained stronger at distances farther downstream, indicating a reduction in the degree of energy transfer to the subharmonic. In fact, evidence of the formation of subharmonics was removed as $R$ fell below approximately 2.5. A similar type of transition was found for the elevated JICF, but with the ‘critical’ $R$ range occurring at a lower value (around 1.2) due to the stabilizing influence of the vertical coflow exterior to the elevated nozzle. As a supplement to the natural instability analysis, Megerian et al. (2007) also observed the response of the jet to very low level acoustic forcing at frequencies $f_f \neq f_0$ (with imposed forcing amplitudes less than one percent of the mean jet velocity). While the low level forcing had little effect on the shear layer spectral character for $R$ values below the apparent transition, the forcing frequency was effectively amplified throughout the shear layer at higher $R$ values, overtaking the fundamental mode. Subsequent experiments by Davitian et al. (2010a) found that much stronger forcing amplitudes—as much as 30 percent of the mean jet velocity (depending upon $f_f$)—are required to overtake the shear layer’s naturally arising oscillations when $R$ lies below the transition range.

The work of Megerian et al. (2007) and Davitian et al. (2010a) provided a good deal of evidence indicating that the alteration observed in the spectral character of the JICF shear layer is due to a transition to global instability, or self-excited flow. This proposition is corroborated by the recent direct numerical simulations (DNS) and LSA by Bagheri et al. (2009), who found the jet to be “globally linearly unstable” at $R = 3$, the lone condition examined. In terms of local linear stability concepts (outlined in Huerre and Monkewitz (1990) under the assumption of weakly nonparallel flow), the shear layer of the equidensity jet appears to be convectively unstable everywhere at large $R$, and thus the flow acts as an
amplifier of random and/or imposed oscillations. As $R$ is lowered through the transition range, the evidence suggests the growth of a region of absolute instability, which eventually results in the contamination of the entire jet near-field by a single disturbance frequency and hence the sustainment of global instability. As described by Chomaz (2005), evidence of a nonlinear global mode in an infinite domain consists of a sharp disturbance front initiated at the upstream boundary of the absolutely unstable region, where the disturbance amplitude abruptly increases and remains high throughout the unstable region while oscillating at a singly amplified frequency. In this manner, the flow acts as an oscillator as opposed to an amplifier. Other flows that have been shown to undergo similar transitions include wake flows above a critical Reynolds number (Provansal et al., 1987; Hammond and Redekopp, 1997), countercurrent mixing layers above a critical velocity difference (Strykowski and Niccum, 1991, 1992), and, as will be described in section 1.3, low density axisymmetric jets below a critical jet-to-surroundings density ratio (Monkewitz et al., 1990; Kyle and Sreenivasan, 1993).

A list of commonly observed pieces of experimental evidence for transition to global instability in the transverse jet shear layer, as outlined by Davitian et al. (2010a), include (1) clear alteration of the shear layer spectral character in the form of strengthened oscillations with narrow peaks for global instability, representing pure tones with higher harmonics, (2) a dramatic shift in the Strouhal number associated with the initial instability as the governing flow parameter is varied through the critical value, (3) little response to low level excitation for the globally unstable flow, in contrast to significant response for the convectively unstable flow, (4) a significant increase in the strength of the disturbance amplitude as the critical value of the governing flow parameter is approached, in accordance with the characteristics of the Landau equation (Huerre and Monkewitz, 1990; Strykowski and Niccum, 1991), and (5) a reduction in the degree of energy transfer along the shear layer from the fundamental mode to its first subharmonic for the globally unstable flow, indicating an inhibition of vortex pairing. Additionally, as predicted by the theoretical work of Pier (2003) and shown experimentally for the low density free jet by Hallberg and Strykowski (2008), it was determined by Davitian et al. (2010a) that by exciting the shear layer at sufficiently high amplitude,
imposed upstream of the transition from convective to absolute instability, the self-excited oscillation could be overtaken. For several $R$ values within the globally unstable regime, the forcing amplitude required to overwhelm the global mode (to achieve ‘lock-in’) was found to be increase in proportion to $|f_0 - f_f|$. This lock-in dependence has been associated with a Hopf bifurcation to a global mode (Huerre and Monkewitz, 1990; Juniper et al., 2009). The identification of the transition to global instability and the analysis of the shear layer response to sinusoidal forcing have considerable implications for jet control.

1.2 Strategic Forcing of the Jet in Crossflow

Active forcing of the transverse jet has been attempted in many instances to elucidate the physical mechanisms of JICF vortex structure interaction, as well as to develop strategies for control of jet spread, penetration and mixing. Some of the more common forcing methods include the use of solenoid or spinning injection valves (Johari et al., 1999; Eroglu and Bredenthal, 2001; Narayanan et al., 2003), periodic squeezing of the jet fluid supply tube (Kelso et al., 1996; Camussi et al., 2002), and acoustic excitation (Vermeulen et al., 1992; M’Closkey et al., 2002; Shapiro et al., 2006; Davitian et al., 2010b). Many of the earlier studies yielded a widely ranging collection of jet responses with only preliminary understanding of the underlying physics. However, it was clear that jet penetration and mixing could be optimized by the selection of particular forcing frequencies. These optimal forcing conditions varied depending upon $R$ and $Re_j$ flow conditions, as well as whether enhancement of penetration or mixing was desired. For example, Narayanan et al. (2003) found that moderate amplitude sinusoidal forcing (less than 30% of the mean jet velocity) was adequate for control of jet behavior for $R = 6$ and $Re_j = 5000$. While penetration was enhanced for forcing frequencies well above the jet’s fundamental mode frequency, the best mixing was achieved at much lower frequencies, which yielded little penetration enhancement as compared to the unforced jet. In contrast to this response at a relatively high $R$ condition, the experiments of Kelso et al. (1996) for $R = 2.2$ ($Re_j = 13640$) and Camussi et al. (2002) for $R = 2$ ($Re_j = 200$) found that low level sinusoidal forcing (less than 10% of the mean jet velocity) at the funda-
mental mode frequency had little effect on the shear layer vortex structure or the overall jet behavior. Davitian et al. (2010a) have since argued that the variation in shear layer response to single-tone forcing is due to a transition to global instability that occurs as \( R \) is lowered past a critical value.

Full modulation of the jet, or partial modulation via square acoustic forcing waveforms, has been shown to yield improved jet penetration by the introduction of an imposed timescale to the flow: the temporal pulse width, \( \tau \). Independent variation of forcing frequency and duty cycle \( \alpha \), defined as the ratio of pulse width \( \tau \) to pulse period \( T \), has been undertaken over a large range of flow conditions for optimization of jet behavior. The experiments of Johari et al. (1999) for fully modulated jets at \( R = 5 \) and 10 showed that shorter injection times yield more compact vortex ring structures, and that reduced duty cycles diminish the interaction between the rings by increasing the distance between them. With this knowledge, the jet forcing waveform may either be tailored for enhancement of vortex ring pairing and merger for improved mixing, or for the reduction of ring interactions for improved spread and penetration. The penetration of the \( R = 5 \) jet was successfully extended by a factor of 5 by forcing at 1 Hz with duty cycle \( \alpha = 0.2 \). Johari (2006) later linked the optimal pulsing conditions for jet penetration with the ideas of Gharib et al. (1998) on the universal timescale of coherent vortex ring formation. This timescale may be associated with a nondimensional stroke ratio \( L/D \), defined as:

\[
\frac{L}{D} = \frac{1}{D} \int_0^\tau U_j dt
\]  

(1.2)

and is observed to be optimized in the range 3.6-4.5 for piston-generated vortex rings into ambient surroundings.

Systematic studies at UCLA on controlled transverse jets have involved the use of various open-loop compensation techniques to impose prescribed jet velocity waveforms via acoustic forcing. In the work of M’Closkey et al. (2002) and Shapiro et al. (2006), the use of a feedforward controller resulted in more accurate creation of distinct square-like pulses of desired width, as measured by a hotwire probe at the center of the jet orifice, compared with the uncompensated case. These experiments at \( R = 2.58 \) and 4.0 kept the root-mean-
square of the jet velocity amplitude, $U'_{j,rms}$, constant while varying the imposed forcing frequency and duty cycle, ensuring consistency of the excitation impulse delivered to the jet. Optimization of the imposed pulse width $\tau$ yielded significant enhancement of jet penetration and spread for both $R$ conditions, and was found to be most effective when applied at subharmonics of the shear layer’s fundamental mode frequency. Distinct, deeply penetrating vortical structures were observed under these controlled square wave forcing conditions, and for certain conditions a bifurcated jet structure was obtained.

Subsequent experiments by Davitian et al. (2010b) at $Re_j = 2000$ and $10 > R > 1.15$ compared the effects of sinusoidal and square wave forcing over a range of stroke ratios $L/D$ on jet penetration and spread. Informed by the evidence for transition of the shear layer to global instability, a two-pronged forcing strategy was recommended. For $R$ values above the apparent transition ($R \approx 3.3$ for the flush jet and 1.2 for the elevated jet), low to moderate sinusoidal forcing was deemed sufficient for significant alteration of jet behavior, with square wave forcing yielding little improvement. Below the transition, square wave forcing with temporal pulse widths resulting in $L/D \approx 3.1$ to 3.7 provided optimal penetration and spread, with visible improvement over sinusoidal forcing of equal $U'_{j,rms}$. This optimal $L/D$ range is in close agreement with the theory of Gharib et al. (1998). Additionally, while the temporal waveforms achieved by the compensated jet actuation system contained minor imperfections, the recent DNS by Sau and Mahesh (2010) suggest that minor imperfections make little difference in comparison with the perfect square waveform. Only a distinct upsweep and downsweep are required for generation of the discrete pulses of vorticity that lead to vortex ring formation.

1.3 Instability of the Variable Density Free Jet

The axisymmetric free jet in quiescent surroundings is a fundamental flowfield that has been studied for decades. Free jets are known to transition to turbulence through the formation of several distinct flow regions, which include the potential core, the mixing region adjacent to the potential core, a transitional region, and fully developed turbulent flow (Beer and
Chigier, 1972). Of great importance in the determination of jet mixing characteristics is the region leading up the breakdown of the potential core. In this near-field region, the spectral character of the velocity fluctuations within the equidensity free jet shear layer is consistent with its classification as a convectively unstable flow (Huerre and Monkewitz, 1990). Experiment indicates that the initial ‘shear layer’ mode observed at the jet exit gives way to the jet’s ‘preferred’ mode at the end of the potential core (Crow and Champagne, 1971; Kibens, 1981; Ho and Huerre, 1984). This ‘preferred’ mode frequency is simply a fraction of the ‘shear layer’ mode frequency—a result of the subharmonic resonances and vortex pairing/amalgamation processes undergone by the vortical structures convecting along the jet column. Vortex pairing has been identified as the dominant mechanism for the growth of a turbulent mixing layer, and as such its acceleration or inhibition may yield enhanced jet spread and mixing (Winant and Browand, 1974). The Strouhal numbers associated with the shear layer instability modes have been found to correlate with initial shear layer momentum thickness, $D/\theta$.

Jets of lower density than their surroundings may transition to global instability, depending on the jet-to-surroundings density ratio, $S$, amongst other parameters. Numerical studies have shown that a region of absolute instability can form in the near-field of a low density axisymmetric jet due to the Kelvin-Helmholtz instability (Monkewitz and Sohn, 1988; Nichols et al., 2007). An inflection point in the velocity profile at the jet exit suggests the onset of such an instability, which may be accelerated by the density gradient present in the shear layer. The resultant nonlinear global mode may be detected by a strong disturbance beginning at the upstream location of the onset of absolute instability (Lesshafft et al., 2006).

Experimentally, Sreenivasan et al. (1989) and Monkewitz et al. (1990) both observed global oscillations of the jet column for density ratios $S$ below approximately 0.6 (corresponding to Monkewitz et al.’s Mode II), while Monkewitz et al. also observed the onset of a second instability mode at $S = 0.7$ (their Mode I). These modes were characterized by sharp spectral peaks observed along the jet centerline, along with associated subharmonics and higher harmonics, as evidenced in Figure 1.3. Kyle and Sreenivasan (1993) subsequently
Figure 1.3: Pressure spectra as measured in the nearfield of a heated jet (density ratio $S = 0.47$), shown by the solid line, as compared with the spectra of the isothermal jet ($S = 1.0$), shown by the dashed line. From Monkewitz et al. (1989).

noted that their observation of the critical density ratio at $S_{cr} = 0.6$ was actually an upper limit for the range of flow conditions in their experiments. In fact, $S_{cr}$ was found to be altered under systematic variation of flow conditions. The value of $S_{cr}$ as well as the Strouhal number of the global instability was argued to scale with jet momentum thickness, $D/\theta$, while remaining independent of $Re$. However, Hallberg and Strykowski (2006) later developed a universal scaling of the global oscillation frequency, showing its dependence on $Re$, $D/\theta$, and $S$. Additionally, stability boundaries representing the variation of $S_{cr}$ were developed in the $(Re, D/\theta)$ plane. Despite their self-excited character, globally unstable free jets have also shown receptivity to ‘lock-in’ via acoustic forcing at or near their fundamental oscillation frequencies, in a manner expected for flows undergoing a Hopf bifurcation to a global mode (Sreenivasan et al., 1989; Hallberg and Strykowski, 2008; Juniper et al., 2009).

1.4 The Variable Density Jet in Crossflow

Given the established onset of global instability in shear layers of both the round free and transverse jets, it is of interest to characterize both the natural behavior and the forced response of the transverse jet as the jet density is altered from that of its surroundings. Some work in this regime has been undertaken over the years, including both non-reactive jets (Kamotani and Greber, 1972; Vermeulen et al., 1992) and jet flames (Huang and Wang,
Vermeulen et al. (1992) injected air jets into hot crossflows, finding that enhancement of the temperature field uniformity could be achieved by pulsing the jet at low frequencies. Kamotani and Greber (1972) analyzed the trajectory, spread and mixing of heated jets. They determined that while jet trajectories based on either maximum velocity or temperature scaled with momentum ratio, $J$, the temperature trajectories exhibited a slight dependence on the density ratio, $S$.

The data available for non-reactive low density transverse jets remains scarce, and is need of a systematic analysis of the effect of various flow parameters on the jet behavior in terms of spread, penetration and mixing, as well as the spectral character of the shear layer. Moreover, just as there had been no prior systematic studies of the shear layer instabilities associated with the equidensity transverse jet, there have been no instability studies performed for the low density case. It is not clear, for example, whether the velocity ratio $R$ is the most critical factor for transition to global instability, if it occurs, as for the equidensity transverse jet, or if the density ratio $S$ is more important, as for the low density free jet. Another possibility would be the jet-to-crossflow momentum flux ratio $J$, incorporating both velocity and density effects. This dissertation describes an evaluation of the transition to global instability in the jet shear layer in the range of density ratios $1.0 > S > 0.14$ and momentum flux ratios $\infty > J > 2.0$. Quantification of scalar mixing using planar laser-induced fluorescence of seeded acetone and velocity fields using particle image velocimetry is explored, in addition to an analysis of the receptivity of the globally unstable transverse jet to acoustic excitation.
CHAPTER 2

Experimental Apparatus

2.1 Transverse Jet Wind Tunnel

The low speed “blower”-type wind tunnel configuration used for the transverse jet experiments is shown in Fig. 2.1. A centrifugal blower driven by an adjustable speed electric motor introduced the crossflow air and was isolated from the tunnel by flexible ducting for minimization of mechanical vibration. The crossflow was conditioned by several screens and a honeycomb flow straightener section before entering the test section via a 9:1 area ratio contraction at freestream velocities $U_\infty$ of up to approximately 6.5 m/s (turbulence intensity less than 1.5%). The test section was 12 cm x 12 cm in cross-section and 30 cm in length, and was spray painted black on all surfaces with barbeque grill paint to minimize reflections. A quartz window was mounted to the top surface for laser sheet introduction, and plexiglass optical access windows were mounted to the sides. During hotwire measurements, one side of the test section was modified with a cut-out wall fitting that enabled probe introduction and traversal. For free jet measurements, the top quartz window was removed, allowing the jet to exhaust unhindered into the laboratory. One additional tunnel section of equal dimensions was attached downstream of the test section, followed by a 1 ft$^3$ cubic box with an exhaust duct attached to the top. This exhaust duct was routed to the main building exhaust system. A 3.5” square quartz window was mounted on the rear side of the exhaust box, allowing for optical access of the test section viewing upstream in the $-x$ direction.

The jet fluid was composed of varying proportions of nitrogen and helium, which were regulated by Tylan (model FC-260, (1) 0-5 NLPM N$_2$ and (1) 0-5 NLPM He, 1% calibration accuracy, 0.2% repeatability) and Sierra Instruments (model C100L, 0-50 NLPM He, 1%
Figure 2.1: Variable density transverse jet wind tunnel, and associated data acquisition and jet excitation apparatus. One additional tunnel section, of identical dimensions, was situated downstream of the test section shown.

calibration accuracy, 0.2% repeatability) mass flow controllers and mixed upstream of the nozzle assembly by combining the individual gas flows in a passive mixing chamber. In this manner, jet-to-crossflow density ratios $S$ between 1.00 (pure nitrogen) and 0.14 (pure helium) were available. After exiting the mixing chamber, the gas mixture passed into a plexiglass pipe via four symmetrically oriented inlet fittings before reaching the nozzle. Beneath this pipe was a plexiglass plenum section housing the jet actuator, used to apply longitudinal acoustic excitation. In the initial hotwire studies described in Chapter 3, the actuator was a 4” loudspeaker. The controlled jet excitation signal was amplified and delivered to the speaker via a stereo receiver, with the incoming waveform generated by a Matlab Simulink model and output by a dSPACE 1104 DSP data acquisition board. In later experiments involving acetone PLIF and PIV, described in Chapter 4, the speaker was replaced by a lightweight piston device for improvement of the frequency response of the actuation system. The piston was actuated by a modal shaker (Ling LVS-100) which moved axially in line with the jet injection axis. The shaker-piston actuator was controlled using MATLAB’s XPC.
Target application. The jet nozzle was brought to an approximately 4 mm exit diameter by a fifth order polynomial contraction, resulting in a nearly top-hat exit velocity profile with a thin boundary layer (see Section 3.1).

An experimental diagnostic using an acoustic waveguide was used to verify jet fluid density for various mixtures of helium and nitrogen, based on measurement of the speed of sound. This procedure is described in Canzonieri et al. (2009). As noted above, helium and nitrogen were introduced into a mixing chamber via separate flow controllers. By controlling the ratio of helium to nitrogen volumetric flow rates, the overall density of the mixture, $\rho_j$, could be determined on the basis of the mixture volumetric flow rate. Verification of the estimated gas densities for different proportions of helium and nitrogen flow rates into the mixer device was accomplished using a one-dimensional acoustic waveguide. The waveguide was bounded by a speaker at one end and a planar reflector at the other end, and was continuously filled with the gas mixture supplied from the mixer device. For a fixed distance between the speaker and reflector, standing acoustic waves could be set up in the waveguide, depending on the frequency of excitation applied by the speaker and the speed of sound in the gas mixture. Pressure transducers within the waveguide allowed determination of the frequency corresponding to standing waves (with either a pressure node at the center or a pressure antinode), and thus comparison with a theoretically predicted frequency enabled determination of the actual speed of sound and hence gas density. Jet mixture densities in the range $1.00 \geq S \geq 0.55$ were measured and determined to be within a maximum error of 1.5% with respect to their theoretical values, assuming thermally perfect component gases.

The jet Reynolds number $Re_j$ for all present experiments was held constant at 1800, based on bulk jet velocity $U_j$ determined by the specified mass flow rate of the jet fluid, the jet fluid density $\rho_j$, and the mixture viscosity as determined by the Wilke formulation (Wilke, 1950). Given the skewing of the velocity profile at the exit plane due to crossflow effects, the bulk jet velocity is used here rather than the centerline or top-hat velocity, which are usually used in studies of axisymmetric jets. The jet Reynolds number was selected to be very close to that used in previous studies of the equidensity transverse jet at UCLA (Megerian et al., 2007; Davitian et al., 2010a).


2.2 Hotwire Measurements

Velocity quantification and related spectral measurements corresponding to fluctuations in vertical (z-component) velocity were obtained using a single component, boundary-layer type hotwire probe (Dantec 55P15). A triple-axis platform comprised of linear stages allowed for probe traversal in three dimensions with an accuracy of 1 µm. For all transverse jet results shown herein, the hotwire probe entered the wind tunnel from the positive $y$ direction, enabling quantification of the upstream shear layer characteristics with a minimum blockage of the jet itself and minimum interaction with the jet via vortices shed from the crossflow passing over the probe holder. The hotwire probe output was delivered to a Dantec constant temperature anemometer module, and then to a dynamic signal analyzer (HP 35665A) which ensemble averaged 40 individual acquisitions for each measurement of the shear layer spectrum.

A calibrated hotwire signal was only possible in equidensity flows, since the hotwire responds to variation in the thermal transport properties of the fluid. When velocity quantification in the equidensity jet was desired for measurement of the initial momentum thickness, the hotwire was calibrated with respect to the wind tunnel crossflow using a pitot-static probe and two differential pressure transducers, one with a range of 0-3” H$_2$O (0-750 Pa) and the other with a range of 0-0.25” H$_2$O (0-60 Pa) for improved accuracy at low velocities. The pitot-static probe was used simultaneously for calibration of the freestream crossflow velocity as a function of the blower frequency.

2.3 Planar Imaging Diagnostics

While the hotwire measurements (to be described in Chapter 3) in the variable density transverse jet proved useful in illustrating the nature of the transverse jet shear layer in terms of its global stability under variation of $J$ and $S$, its intrusive nature and limited spatial domain motivated the use of planar imaging as an additional flow diagnostic. The techniques used were planar laser-induced fluorescence (PLIF) of acetone seeded into the jet
2.3.1 General Setup

A schematic of the general setup for planar imaging in the transverse jet is shown in Figure 2.2. The excitation source for both PLIF and PIV was a dual cavity Q-switched Nd:YAG laser (Litron Nano L PIV), which was equipped with both second and fourth harmonic generation for simultaneous collinear output at 532 nm and 266 nm. Each laser cavity had an energy output of approximately 35 mJ/pulse at 266 nm and 120 mJ/pulse at 532 nm in an 8 ns FWHM period. While the maximum laser repetition rate was 15 Hz, in general practice the laser was operated at no greater than 2 Hz, mainly to avoid ablation of the barbeque grill paint used to reduce reflections. However, low repetition rates also helped to ensure statistical independence of the images. Timing of the laser pulses as well as image acquisition were achieved using a PC equipped with an external Programmable Timing Unit and LaVision’s DaVis 8.1 control software.
The beam was formed into a thin, divergent sheet and directed into the test section by a combination of two spherical lenses, a turning mirror, and a \( f = -10 \) mm cylindrical lens, entering the test section through the upper quartz window. Rotation of the cylindrical lens by 90° allowed for measurements to be made in either the \( y = 0 \) jet centerplane or in constant-\( x \) cross-sections in the \( yz \) plane. The wind tunnel was mounted to a traversing platform, which could be precisely moved in the crossflow (\( x \)) direction by a lead-screw/stepper motor device in order to make measurements at varying \( yz \) cross-sections without altering the optical setup.

2.3.2 PLIF

Optical techniques are attractive options for fluid diagnostics due to their non-intrusive nature. They have been utilized for centuries in a multitude of evolving forms, perhaps most famously in line-of-sight methods such as shadowgraph and Schlieren, which make use of the variation in the refractive index within a flowfield due to nonuniformities in fluid density. More recently, planar methods have been developed which involve interactions between a thin light sheet and a naturally occurring or purposefully introduced molecule in the flowfield. Such methods avoid integration along the line of sight and may be combined with high-speed imaging systems to provide instantaneous snapshots from which fluid properties may be determined. Among such techniques, fluorescence-based flowfield imaging has become an extremely popular method for species measurement in both reacting and non-reacting flows. A light source is used to excite flowfield tracer molecules to higher electronic energy states. The subsequent collapse of the molecules back to their ground states is due in large part to spontaneous emission, a process which results in significant advantages in imaging sensitivity over Rayleigh or Raman scattering methods, which involve elastic scattering or virtual states, respectively. Imaging of fluorescence emission is particularly suited to high-speed flows due to its short lifetime (on the order of ns) relative to that of phosphorescence emission (on the order of \( \mu s \)). A thorough review of the development of Planar Laser-Induced Fluorescence (PLIF) techniques for purposes of reacting flow diagnostics is given by Hanson et al. (1990).
Selection of a tracer molecule is a critical part of the design of a PLIF system, as it determines the required hardware for both excitation and imaging of the flowfield. Common choices include naturally present species, such as NO or OH in flame studies, or seeded species, such as rhodamine or fluorescein dyes, I₂, acetone, or biacetyl. In non-reacting gaseous flows, acetone (CH₃ – CO – CH₃) is a natural choice, in part due to its mild toxicity and low cost. More importantly, it has a high vapor pressure, enabling high seeding concentrations, and absorbs over a broad wavelength range (225-320 nm), allowing a large selection of excitation sources. Emission occurs over a lifetime of less than 4 ns, and has adequate spectral separation from the wavelength of excitation, extending from 300-500 nm when excited at 266 nm. This allows instantaneous flowfield imaging with simple optical filtering to image only the emission spectrum. The use of acetone as a molecular tracer in planar laser-induced fluorescence is now an extensively utilized technique for concentration, temperature and pressure measurements in gaseous flows (Lozano, 1992; Smith and Mungal, 1998; Thurber et al., 1998; Su and Mungal, 2004). For isothermal flows in which the excited fluid medium is optically thin, fluorescence intensity is proportional to both the laser fluence (assuming absorption saturation is not reached) and the acetone concentration. For a comprehensive review of acetone photophysics and applications for fluid diagnostics, see Lozano et al. (1992) or Lozano (1992).

For our experiments involving only acetone PLIF (without simultaneous PIV), the laser beam was directed to a set of two 266 nm dichroic mirrors prior to reaching the sheet forming optics, which separated the wavelengths and removed the majority of the 532 nm light. Approximately 10% of the 266 nm beam energy was then deflected by a 3 mm thick UV grade fused silica window and directed to a pyroelectric joulemeter (Newport 818E-10-50-S) for pulse-to-pulse energy monitoring and recording to a PC via USB. The thickness of the laser sheet, as determined by traversing a razor blade through the sheet and measuring the full width at half the maximum (FWHM) of the transmitted energy, ranged between approximately 500-700µm in the camera’s field of view.

Fluorescence images were captured by a 14-bit CCD camera (LaVision Imager proX) with 1600x1200 pixel resolution, equipped with an external image intensifier (LaVision IRO).
to boost the signal-to-noise ratio (SNR) due to the low emission levels associated with the relatively weak laser fluence. The intensifier optics imposed a circular aperture on the rectangular CCD array, resulting in a sensitive area approximately 1500 pixels in diameter centered on the CCD. The remaining pixels were masked in image postprocessing. Two different imaging lenses were used to focus on the illuminated fields of view: a Sigma AF 90 mm at f/2.8 for the $y = 0$ centerplane, and a Nikon 200 mm f/4.0 for the $yz$ planes, each with a bandpass optical filter to isolate the fluorescence signal. For centerplane imaging, the camera was located on one side of the tunnel test section, viewing the jet through a plexiglass window. The resulting field of view was approximately 108 mm in diameter (limited to 86 mm vertically) with a resolution of 72 $\mu$m per pixel. When imaging cross-sectional slices of the jet in the $yz$ plane, the camera was moved around to the rear of the wind tunnel, viewing the fluorescence through the quartz window on the end of the crossflow exhaust box. These images had a 100 mm diameter field of view (80 mm vertical extent) with a 67 $\mu$m per pixel resolution. The actual resolution of the PLIF measurements in each case was therefore limited by the thickness of the laser sheet. The image intensifier was triggered by a PC and programmable timing unit, and was gated at 200 ns, assuring capture of the fluorescence and rejection of acetone phosphorescence, which occurs over a much larger timescale (approximately 200 $\mu$s). With a temporal spacing between the two laser pulses set to 50 ns, the fluorescence signal captured by the camera was doubled from that of a single pulse with negligible motion blurring (70 mJ/pulse combined).

Acetone seeding was accomplished by bubbling the $N_2$/He mixture comprising the jet fluid through a 7” tall x 7.5” diameter chamber, filled with about 4.5” of liquid acetone. The gas was diverted to the seeder via a tee junction into two inlet tubes, which were submerged in the liquid acetone and attached to sintered spray nozzles. The nozzles forced the gases to bubble upwards through the acetone, leaving the chamber through an exit tube at the top. Seeder pressure was measured by a pressure transducer (Omega PX409-015G5V, 0-15 psig, 0.08% BSL calibration accuracy) and temperature by a Type T thermocouple (Omega, with Analog Devices 2B50A Isolated Thermocouple Conditioner, 1°C max uncertainty), allowing for the acetone concentration in the exiting jet fluid to be determined by consideration
of the temperature dependence of acetone vapor pressure (given by Lozano (1992)), under the assumption that the carrier gas flow became saturated with acetone vapor. The seeder temperature was maintained constant throughout data acquisition runtimes by using a refrigerated recirculator to flow water through copper coils submerged in the liquid acetone, ensuring that a constant seeding density was achieved. The temperature was altered between jets of different jet-to-crossflow density ratios $S$ for control of the acetone seeding density (ranging between 12-20°C, corresponding to acetone concentrations ranging between about 11-24% by volume). However, the length of tubing between the exit of the seeder and the jet injection nozzle was sufficient for the jet fluid mixture to reach room temperature prior to entering the test section. The impact of the acetone vapor on mixture density and viscosity, estimated by the Reichenberg method (Poling et al., 2001), which takes into account the polarity of acetone, was included in the formulation of the required flow rates of each gas to meet desired values of $S$ and $J$ while maintaining a constant value of $Re_j = 1800$ (and thereby constant $D/\theta$ for the jet boundary layer in the absence of crossflow). Representative flow conditions utilized during acetone PLIF imaging are given in Table 2.1.

Post-processing of the PLIF images included a set of standard corrections to account for bias errors and shot-to-shot fluctuations in order to convert pixel values into jet fluid concentration. These corrections dealt with camera dark noise, background light and reflections, non-uniformity of the laser sheet energy profile, laser sheet attenuation due to absorption, pulse-to-pulse energy fluctuations, and the flat-field response of the imaging lens/intensifier/camera combination.

For each raw fluorescence image, the total signal in each pixel, $S_i(x,z)$, is the sum of

<table>
<thead>
<tr>
<th>$S$ (NLPM)</th>
<th>$Q_{N_2}$ (NLPM)</th>
<th>$Q_{He}$ (NLPM)</th>
<th>$\chi_{acetone}$</th>
<th>$U_j$ (m/s)</th>
<th>$U_\infty@J = 20$ (m/s)</th>
<th>$T_{room}$ (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>3.002</td>
<td>1.545</td>
<td>0.2383</td>
<td>5.789</td>
<td>1.294</td>
<td>21.5</td>
</tr>
<tr>
<td>0.70</td>
<td>2.055</td>
<td>4.948</td>
<td>0.1962</td>
<td>8.962</td>
<td>1.677</td>
<td>23.0</td>
</tr>
<tr>
<td>0.55</td>
<td>1.485</td>
<td>7.849</td>
<td>0.1613</td>
<td>11.865</td>
<td>1.968</td>
<td>21.0</td>
</tr>
<tr>
<td>0.35</td>
<td>—</td>
<td>13.807</td>
<td>0.1135</td>
<td>19.797</td>
<td>2.619</td>
<td>21.0</td>
</tr>
</tbody>
</table>

Table 2.1: Flow conditions used during acetone PLIF imaging to create transverse jets at incremental values of density ratio $S$ while matching $Re_j = 1800.$
the acetone fluorescence $F_i(x, z)$, the camera dark noise level $B_{i,dark}(x, z)$, and the background light from the room and laser pulse reflections, $B_{i,light}(x, z)$, where the subscript $i$ refers to individual images and $(x, z)$ refers to spatial variation throughout the image. The fluorescence may be solved for according to Equation 2.1:

$$F_i(x, z) = S_i(x, z) - B_{dark}(x, z) - B_{light}(x, z)$$

(2.1)

Here, the overbar refers to a temporally averaged background image. The fluorescence intensity $F_i(x, z)$ can then be related to the actual concentration of acetone $C_i(x, z)$ in each pixel volume (voxel) by accounting for temporal and spatial variation in the laser sheet energy as well as the impact of absorption, as described by Equation 2.2:

$$F_i(x, z) = bE_i \beta_i(x, z)C_i(x, z)$$

(2.2)

$E_i$ represents pulse-to-pulse fluctuations in laser energy, and $\beta_i(x, z)$ is a function describing non-uniformities in fluorescence due to both the laser sheet profile and the attenuation of the laser sheet by the absorbing medium. The constant $b$ is related to the quantum efficiency of acetone fluorescence under 266nm excitation, as well as the efficiency of the collection optics.

Accounting for the coefficients in Equation 2.2, the basic processing routine for correction of camera signal into jet concentration is the following:

$$C_i(x, z) = k \frac{S_i(x, z) - B(x, z)}{L(x, z) - LB(x, z)}$$

(2.3)

In this expression, $k$ is a constant selected to give 100% concentration of jet fluid in the jet’s potential core. $B(x, z)$ is an average background image which in our experiments was the mean of 100 images taken with the crossflow on and laser flashing, but no acetone in the test section. Both the raw fluorescence images $S_i(x, z)$ and the average background image had an average camera dark noise image subtracted - the mean of 30 images taken with a lens cap on the camera. $S_i(x, z)$ and $B(x, z)$ were also each multiplied by the ratio of the pulse energies in each to the mean pulse energy of all of the raw fluorescence images, in order to account for pixel intensity variations due to pulse-to-pulse energy fluctuations (as measured by a reference joulemeter). A white image correction was applied to the result, accounting
for the spatial non-uniformities in the light reaching the CCD array from a uniform flat-field. This white image was obtained by averaging 100 images of an LCD screen placed slightly out of focus so as to blur any minor surface emission imperfections. An image of a target containing several rows of equidistant dots (5mm spacing) placed in the plane of the laser sheet was also used to warp the images, accounting for any slight angular offsets between the object plane and the measurement plane. This warping was performed using a camera pinhole model, and the known spacing between the dots provided a reference for the physical size of each pixel in the object plane.

$L(x, z)$ and $LB(x, z)$ are images of the laser sheet and the laser sheet background, respectively. The average laser sheet image $L(x, z)$ is the mean of 100 images taken with the test section sealed on both ends and filled uniformly with acetone-seeded nitrogen. A separate background image for the laser sheet, $LB(x, z)$, was required due to a slight alteration in the laser sheet reflections when the test section was sealed. This mean laser sheet background was the mean of 50 images taken with the test section sealed yet devoid of acetone vapor. Normalization by $L(x, z) - LB(x, z)$ removed non-uniformities associated with the laser sheet profile, on a mean basis (shot-to-shot fluctuations remained part of the uncertainty of the measurement). In practice however, the laser sheet image was modified to include the impact of absorption.

Along a ray from a point light source, absorption is described by the Beer-Lambert law:

$$E(x) = E_0 e^{-\sigma nx} = E_0 e^{-\alpha x}$$  \hspace{1cm} (2.4)

$\sigma$ is the absorption cross-section and $n$ the number density of acetone, which may be combined as $\alpha$, the absorption coefficient. $\alpha$ can be directly measured from the mean laser sheet image itself, which contains a large degree of absorption (image intensity drops through the image by about 50-70% along a ray). However, the divergence of the laser sheet in our experiment adds a slight complication, as the absorption may not simply be tracked along a column of pixels, but rather must be weighted across several pixels according to the angle of expansion of the sheet at each pixel. To determine the angle of sheet expansion, a secondary laser sheet image was taken with a sharp-edged obstacle in the beam path. As
a result, strong gradients in signal intensity were created along two rays of light from the divergent laser sheet’s “point source”. Edge detection was used to trace these rays back to the source, which, once known, allows the for the calculation of the angle of expansion of the ray passing through each pixel in the image. Next, a series of rays are traced through the original laser sheet image, and the absorption coefficient $\alpha$ is computed as the mean value from a least-squares fit of the Beer-Lambert law to each of the rays. Using this measured value of $\alpha$, the laser sheet image was then corrected for absorption using a marching, weighted scheme described by Smith (1996), which takes into account the sheet divergence. After the sheet image is corrected, the result is an image of the laser sheet profile without absorption. To remove the absorption from each individual background-subtracted image of acetone fluorescence in the transverse jets, the absorption-corrected laser sheet image was modified to include the impact of absorption from the sheet passing through the jet (see Smith (1996)). Normalization by this resultant ‘modified’ laser sheet thereby corrects the individual jet images for both spatial variation in the laser sheet energy as well as absorption by acetone seeded in the jet.

Several steps in the correction process are shown in Fig. 2.3. In Fig. 2.3(a), the raw fluorescence image $S_i(x, z)$ shows clearly the impact of background light and laser sheet absorption, noticeable particularly in the jet’s potential core. In Fig. 2.3(b), the background light and camera dark noise are removed, and in (c) both the flat-field and image warping corrections are applied. Lastly, Fig. 2.3(d) shows the same image after normalization by the modified laser sheet, which results in improved signal in the far-field of the jet as well as a uniform potential core. The image corrections through step (c) are performed using LaVision’s DaVis 8.1 software, while the laser sheet and absorption corrections utilize a MATLAB script.

Correction of the images via Equation 2.3 included multiplication by a constant $k$ to ensure that the jet concentration at the point of injection is equal to 100%. For images in the jet centerplane, each image contained a reference signal level, taken as the mean of a square region of pixels located in the potential core of the jet. Each individual centerplane image was normalized by its potential core reference level. However, for images acquired in
the $yz$ plane downstream of injection, there was no such reference level in the images. Instead, the correction factor $k$ was determined by comparing the common line of data between the ensemble-averaged and normalized centerplane image and the ensemble-averaged $yz$ plane images acquired at the same flow conditions. This same factor $k$ was then used to correct each instantaneous $yz$ plane image to the potential core reference concentration (and therefore did not account for slight variations in acetone seeding level or laser sheet profile variation from shot to shot).
2.3.3 PIV

Particle image velocimetry (PIV) has become a standard optical technique for quantitative flow visualization and planar velocity measurement in fluids since its initial development in the 1980’s. The basic principle of operation is that small seed particles are introduced into the flowfield, sized appropriately to follow the flow with negligible slip. A double-pulsed laser, with output formed into a thin sheet, is used to illuminate the flowfield twice at a prescribed time increment, while a CCD camera captures the resultant Mie scattering of the seed particles in a separate frame for each pulse. Each image pair is separated into smaller interrogation windows, which are spatially cross-correlated using FFT algorithms to estimate the most probable displacement of the particles in each window. This displacement and the known time increment provide a velocity field. An excellent review of the fundamentals of PIV as well as the current challenges and development of advanced methods can be found in Adrian and Westerweel (2011).

PIV measurements in our transverse jet experiments utilized the same double-pulsed Nd:YAG laser and sheet-forming optics used for the acetone PLIF measurements. However, for PIV, the dichroic mirrors used to isolate the 266 nm wavelength, as well as the beamsplitter/joulemeter used as a pulse energy reference, were both removed, allowing the 532 nm light to be the dominant excitation source. The thickness of the 532 nm laser sheet was measured to be approximately 1-1.5 mm within the illuminated area. Seeding of the crossflow was achieved using a commercial fog machine (Pea Soup Rocket), with particles introduced into the fan of the crossflow blower. The jet fluid was seeded by diverting part of the injection line through a TSI particle generator containing DEHS oil, which allowed for fine-tuning of the seeding density via a needle valve controlling the seeding line flow rate. The seeded and unseeded portions of the injectant were recombined at a tee fitting well upstream of the nozzle assembly.

Our experiments utilized a steroscopic PIV system, which involved the use of two cameras viewing from different directions (see Figure 2.2). This method allows for reconstruction of all three components of velocity within the plane of the laser sheet (referred to as a 2D3C...
technique), and thus removes bias errors arising from the motion of particles normal to the illuminated plane. The flowfield was imaged by two 14-bit cross-correlation CCD cameras (LaVision Imager proX, 1600x1200 pixel resolution), separated by a 60° angle (about the z-axis) and oriented at a 90° light scattering angle. Each PIV camera was equipped with a Nikon 60 mm lens at f/8.0, a 532nm narrowband filter, and a Scheimpflug lens mount, which was used to tilt the CCD sensor so as to retain focus over the entire image domain (alleviating the effect of the small depth of field). Their fields of view were mapped into physical coordinates via a 3rd order polynomial model using calibration images of a three-dimensional target (LaVision Type #7) placed as close as possible to the plane of the laser sheet. To account for any slight misalignment between the placement of the target and the illuminated plane, a self-calibration method on the particle images (Wieneke, 2005) was used to correct the image mapping model prior to the reconstruction of the velocity fields.

Velocity fields were calculated by a commercial code (LaVision). Raw particle images were first mapped into identical physical coordinates, and then pre-processed by subtracting a sliding background (8x8 pixels) and min/max filtering (5x5 pixels) for contrast enhancement. A multi-pass stereo correlation was selected, involving 2 passes at a 64x64 pixel interrogation window size followed by 4 passes at 32x32 pixels, each using a Gaussian weighting function on the interrogation windows. The windows were overlapped by 75% to increase vector yield. The final vector field provided a 204x147 grid with 162 µm grid spacing. The size of the vector grid $\Delta x_u$ relative to the jet diameter was therefore $\Delta x_u / D \approx 26$, while the size of the interrogation window $\Delta x_w$ relative to the jet diameter was $\Delta x_w / D \approx 6.5$. The time between laser pulses, $\Delta t$, was 25 µs. Post-processing at each step of the multi-pass algorithm deleted vectors with low correlation values, applied a median filter, replaced erroneous vectors by interpolation, and applied a 3x3 smoothing filter.
CHAPTER 3

Variable Density Transverse Jet Shear Layer Instabilities

Sections 3.2 and 3.4 of this chapter are taken with slight modification from the article “Shear layer instabilities in low-density transverse jets”, published in the journal Experiments in Fluids (Getsinger et al., 2012).

3.1 Mean Transverse Jet Characteristics and Scaling

The hydrodynamic stability of a parallel shear flow, considered in a local sense (i.e. at a particular streamwise station), is determined by the local mean velocity and density profiles upon which disturbances are imposed. More precisely, it is determined by the streamwise impulse response of the base flow to forcing—whether or not the flow will return to its original mean state after application of a localized perturbation. In studies of the near-field of the round axisymmetric free jet shear layer, variation of the mean velocity profile at the jet exit has been performed to link local stability properties with the global behavior of the shear layer. For instance, the LSA of Monkewitz and Sohn (1988) and the nonlinear simulations of Lesshafft et al. (2006) showed that a region of absolute instability may form within the heated (or low density) free jet shear layer under particular combinations of jet exit velocity and density profiles, and that a nonlinear global mode may result when the region of absolute instability reaches a sufficiently large streamwise extent. Experimental work by Kyle and Sreenivasan (1993) and Hallberg and Strykowski (2006) showed that the global stability of the low density free jet is dependent upon both Re and the scaled jet momentum thickness, $D/\theta$, as measured at the jet exit. Thus, in undertaking an experimental analysis of the
stability of the transverse jet shear layer, it is of interest to quantify the jet exit velocity profile of the equivalent free jet.

The mean and fluctuating velocity components as measured as close as possible to the jet orifice \((z/D = 0.1)\) are shown in Fig. 3.1 for various jet-to-crossflow momentum flux ratios \(J\) at constant \(Re_j = 1800\). Measurements were taken in nitrogen jets (jet-to-crossflow density ratio \(S = 1.00\)), which allowed for calibration of the hotwire in air with negligible error in the resultant velocity measurement. The resulting scaled momentum thickness of the initial free jet shear layer was \(D/\theta = 70\), computed by integrating the velocity profile from the centerline to the radial location at which the velocity reached 20% of the centerline value. For this measurement, the hotwire was temporarily oriented with the probe holder parallel to the jet axis and the wire parallel to the jet exit plane to minimize the effect of probe intrusion. This scaled momentum thickness is known to be important in the frequency scaling and stability properties of the free jet, and was held constant by Reynolds number similarity under variation of the jet-to-crossflow density ratio \(S\) between 1.00 (pure nitrogen)
and 0.14 (pure helium). For the combination of \( D/\theta = 70 \) and \( Re_j = 1800 \) as examined here, the studies of Hallberg and Strykowski (2006) suggest that \( S_{cr} \) should lie between 0.27 and 0.50 (although it should be noted that direct comparison would require defining \( Re_j \) in our work by the free jet’s centerline velocity rather than bulk velocity, which would alter it from 1800 to approximately 2000). Reynolds number similarity ensured identical results for lower density jets of equal \( J \) and \( Re_j \).

Trajectories of the upstream shear layer were determined from uncalibrated hotwire voltage profiles in the \( y = 0 \) plane at varying elevations in \( z \) above the jet exit. These probe traversals were performed at all combinations of jet-to-crossflow momentum flux and density ratio \((J,S)\) at which shear layer spectral measurements were desired. While uncalibrated hotwire voltages cannot provide the loci of velocity inflection points, as are usually used to determine shear layer trajectories, the voltage profiles can clearly show the limits of the potential core and the upstream extent of the shear layer, allowing for an estimate of the trajectory of the shear layer ‘center’ (denoted by the spatial coordinate direction ‘\( s \)’). Trajectories are shown in Fig. 3.2a for jets ranging from the free jet \((J \rightarrow \infty)\) to \( J = 2 \). These trajectories were obtained via power-law curve fits to the shear layer ‘center’ locations obtained from all density ratio cases at each momentum-flux ratio. When the data are scaled by momentum-flux ratio, as shown in Fig. 3.2b, a collapse to a power law of the following form is acquired:

\[
\left( \frac{z}{JD} \right) = 0.98 \left( \frac{x}{JD} \right)^{0.54}
\]

(3.1)

Scaling of the jet trajectory by \( J \) in similar power-law type forms has been found by several researchers to provide a reasonable collapse of far-field jet centerline trajectories as defined by various flow parameters: Pratte and Baines (1967) in terms of maximum velocity, Kamotani and Greber (1972) in terms of maximum temperature, and Smith and Mungal (1998) in terms of maximum jet fluid concentration. While this trajectory law appears to track all jet shear layers thus far examined with reasonable accuracy, all hotwire measurements in the shear layer were located via the \( J \)-specific curve-fits from Fig. 3.2a.
3.2 Spectral Character of the Variable Density Transverse Jet Shear Layer

Hotwire voltage spectra were acquired along the jet shear layer (following the ‘s’ coordinate) for values of the jet-to-crossflow density ratio in the range $1.00 \geq S \geq 0.14$ and jet-to-crossflow momentum flux ratios $\infty > J \geq 5$, revealing the dependence of the shear layer stability on independent manipulation of these flow parameters. While vertical velocity was the measure of interest, the hotwire could not be calibrated due to the variation of jet fluid density throughout the shear layer. A hotwire placed within the variable density jet shear layer responds to both fluctuations in velocity and local jet fluid concentration. However, the uncalibrated hotwire signal contains identical frequency content to its calibrated counterpart, and spectral measurements taken within the shear layer were very similar in frequency to measurements taken just upstream of the shear layer, where the hotwire is exposed only to crossflow fluid. Thus the frequency content of the hotwire voltage spectra may be considered representative of the fluctuations of vertical velocity in the jet shear layer, with uncertainty.
associated only with spectral magnitude. Comparison of spectral magnitude amongst varying $J$ values at constant jet-to-crossflow density ratio $S$ may be considered valid, but comparison of magnitude between different density ratio cases would not be justified.

A review of the pertinent shear layer spectral characteristics may begin with the examination of free jet spectra, shown in Fig. 3.3; for these and subsequent spectral data, the hotwire entered the wind tunnel from the positive $y$–direction. Hotwire power spectra are shown at the left side of Fig. 3.3 on a spatial grid of $\Delta s/D = 1.0$. At right on a spatial grid of $\Delta s/D = 0.1$, with the ordinate axis representing spatial location, the abscissa represents the Strouhal number $St = fD/U_j$ or scaled frequency of oscillation, and the colormap represents spectral magnitude. The spatial evolution of the free jet shear layer instabilities at $S = 0.55$ (Fig. 3.3ab) is representative of the type of spectrum found for all density ratios $1.00 \geq S \geq 0.55$, and is similar to that documented in other studies of equidensity ($S = 1.00$) free jets (Petersen and Samet, 1988; Xu and Antonia, 2002). The instabilities were first detected at about 1-2 jet diameters from the jet orifice, with several weak, relatively indistinct spectral peaks arising. These near-field spectra contained peaks near $St = 0.55$ which may be considered to be the jet’s initial shear layer mode. As the hotwire was traversed farther downstream along the shear layer, nearing the end of the potential core, the initial mode gave way to the jet’s ‘preferred’ mode near $St = 0.49$, which may be interpreted as the passage frequency of vortex structures in the jet near-field. Weak higher harmonics and subharmonics were also apparent, indicating a slight degree of nonlinearity and vortex amalgamation in the free jet near-field. These types of spectra are typically associated with convectively unstable free jets. This classification holds when $S$ remains above a critical value, which is dependent upon $Re_j$ and $D/\theta$ (Hallberg and Strykowski, 2006). The instabilities were relatively weak and broadband here, and were not detected throughout the shear layer in the near field of the jet.

As the jet density was lowered further from that of its surroundings (see Fig. 3.3c-h) with an increase in jet velocity to maintain $Re_j = 1800$, the shear layer oscillations grew in magnitude and were initiated closer to the jet exit. At $S = 0.45$, strong side-lobe peaks formed at frequencies both above and below the shear layer mode frequency $f_0$. At $S = 0.40$,
Figure 3.3: Power spectra of the hotwire response at various $s/D$ along the free jet ($J \to \infty$), with oscillation frequency given in terms of Strouhal number $St$. Results are shown for various values of $S$ at constant $Re_j = 1800$. At left are power spectra on a course spatial grid, with each line color representing a different $s/D$ location. At right, a finer spatial grid is represented by spectral magnitude contours, with the magnitude colorbar (in dB) matched for each condition.
these side-lobes were eliminated, and a dominant, relatively pure-tone instability at St = 0.55 (and harmonics thereof) was detected throughout the shear layer. As mentioned, this type of transition in the natural spatial evolution of the variable density free jet shear layer spectra is indicative of a transition to global instability (Monkewitz et al., 1990). This process would correspond to the growth of a region of absolute instability in the shear layer, resulting from an alteration in the alignment of local streamwise velocity and density profiles. Upon reaching a sufficient size, this region of absolute instability results in the initiation of a nonlinear global mode and hence the predominance of a single frequency of oscillation throughout the entire jet near-field, along with related harmonics of the global mode frequency (Huerre and Monkewitz, 1990; Chomaz, 2005). In our particular combination of flow parameters and experimental configuration, the transition appeared to occur as the density ratio S was altered below the range of 0.45 to 0.40, which is within the range suggested in Hallberg and Strykowski (2006). The value of the global mode’s Strouhal number, if scaled with centerline velocity rather than mean velocity (altered from St = 0.55 based on the mean to St = 0.49 based on the centerline), is consistent with the range observed by Monkewitz et al. (1990) in heated air jets between 0.47 ≤ S ≤ 0.69.

Considering the free jet (J → ∞) spectral measurements as indicators of the effect of the jet-to-surroundings density ratio on the near-field shear layer instabilities, the jet-to-crossflow momentum flux ratio was lowered by introducing crossflows of varying freestream velocity U∞. For the equidensity (S = 1.00) jet, the resulting shear layer spectra obtained by systematically lowering J (corresponding to increasing the crossflow velocity) are shown in Figure 3.4. For J ≥ 12, the initial instability modes were first detected between 1-2 jet diameters from the nozzle exit and at frequencies corresponding to 1.00 ≥ St ≥ 0.70. These dominant instabilities may be referred to as the ‘fundamental’ modes of the transverse jets, and their frequencies of oscillation referred to as the fundamental frequencies, f0.

Lowering J from ∞ to 12 had the effect of initiating the instabilities closer to the jet exit and increasing the Strouhal number of the oscillations. Interestingly, moving downstream along the shear layer, the transverse jet’s fundamental mode (as well as its higher harmonics and subharmonic) exhibited shifting to lower frequencies and subsequent hopping
Figure 3.4: Power spectra of the shear layer instabilities at various \( s/D \) along the \( S = 1.00 \) transverse jet, with oscillation frequency given in terms of Strouhal number \( St \). Results are shown for various values of \( J \) at constant \( Re_j = 1800 \). The magnitude of the oscillations (in dB) are represented by spectral amplitude contours.
to higher frequencies in multiple stages, before eventually deteriorating in magnitude farther downstream. Such frequency shifting/hopping behavior is a result of the shear layer tone phenomenon, as examined in both planar and axisymmetric free jets by Hussain and Zaman (1978). The presence of the hotwire probe itself influences the jet shear layer, resulting in feedback that serves to alter the shear layer’s fundamental instability frequency in a manner that is dependent on the distance between the probe and the jet exit. In certain instances, moving the hotwire well upstream of the shear layer (in the \(-x\) direction) allowed for measurement of the shear layer instabilities with the effect of the shear layer tone eliminated. This resulted in removal of the feedback effect and the presence of an instability at a more constant frequency, within the band traveled by the shifting/hopping shear layer tone (in a similar fashion to the measurements of Hussain & Zaman). However, in most cases the detection of the instabilities was not possible without placing the hotwire very near to or within the shear layer, where the effect of the shear layer tone was predominant. Therefore, all measurements shown were taken with the hotwire placed in the originally intended shear layer ‘center’ trajectory, and the shear layer tone was viewed as a diagnostic of sorts in the determination of the shear layer’s local stability characteristics, as will be shown. Such frequency shifting was never observed for the free jet, irrespective of the density ratio \(S\).

At these relatively high momentum flux ratios, \(J \geq 12\), the decay of the fundamental mode amplitude as the hotwire was traversed downstream beyond about \(s/D = 3.0\) was accompanied by a strengthening of the subharmonic at \(f_0/2\), indicating the pairing of vortex structures along the upstream side of the jet column, and thus a transfer of energy from the fundamental to the subharmonic. As seen in other shear flows experiencing convective instability (Strykowski and Niccum, 1992), these traits in the spatial evolution of the \(S = 1.00\) transverse jet shear layer spectra suggest that the shear layer may be convectively unstable in the momentum flux ratio range \(J \geq 12\). Rather than strong, single-frequency global oscillations, the spectral content of the high momentum flux ratio equidensity transverse jet was relatively broadband and strongly dependent on measurement location, indicating the presence of the shear layer tone and a sensitivity to feedback. This type of characterization is identical to that made for the equidensity transverse jet at high jet-to-crossflow velocity
ratios $R \geq 3.2$ by Megerian et al. (2007), at higher Reynolds numbers, $Re_j = 2000$ and $3000$.

As the crossflow velocity was increased for $S = 1.00$, lowering $J$ below 12, the oscillations were altered quite strikingly. The fundamental shear layer mode continued to be initiated closer to the jet orifice with decreasing $J$, and, likewise, remained stronger at distances farther downstream. This resulted in a reduction in vortex pairing and thus a reduced magnitude of the subharmonic, until it became nearly indistinguishable through much of the shear layer at $J \leq 5$. The Strouhal number of the fundamental oscillation decreased as $J$ was lowered, after reaching a peak at $J = 10$, and the impact of the shear layer tone was reduced until the frequency-shifting behavior was altogether eliminated below approximately $J = 10$. At these lower momentum flux ratios, the oscillations were very strong and pure-tone in comparison with the higher momentum flux ratio cases, and were detected throughout the shear layer up to $s/D = 5.0$. This transitional behavior of the instability is similar to that of the free jet below $S_{cr} \sim 0.45 - 0.40$, and was also linked to the emergence of global instability in the $Re_j = 2000$ transverse jet shear layer by Davitian et al. (2010a), below $R_{cr} \sim 3.2$ ($J_{cr} \sim 10.2$). The spectra shown in Figure 3.4 are the beginnings of such evidence for the equidensity transverse jet at $Re_j = 1800$, and it appears that the transition to global instability may occur at a nearly identical value of $J$ as the $Re_j = 2000$ jet.

Equivalent hotwire measurements were made in transverse jets of reduced density. At consistent values of $\infty > J \geq 5$, with $Re_j$ held constant at 1800, the jet-to-crossflow density ratio $S$ was incrementally lowered. For example, the hotwire spectra obtained for $S = 0.55$ are shown in Figure 3.5. While the relative magnitudes and Strouhal numbers of the oscillations varied between density ratio cases (the magnitude variation likely owing to the inability to calibrate the hotwire signal for a true measurement of vertical velocity), the general character and transitional behavior of the shear layer instability displayed remarkable similarity in the range $1.00 \geq S \geq 0.55$ when viewed at consistent values of $J$. As momentum flux ratio was lowered through $J = 10$ at each density ratio $S$ (values of $S$ between 1.00 and 0.55 were examined but are not shown), the fundamental oscillations strengthened, became resistant to the jet edge tone, exhibited a distinct shift in Strouhal number with a peak at $J = 10$, persisted in larger spatial ranges in the shear layer, and transferred less energy to
Figure 3.5: Power spectra of the shear layer instabilities at various \( s/D \) along the \( S = 0.55 \) transverse jet, with oscillation frequency given in terms of Strouhal number \( St \). Results are shown for various values of \( J \) at constant \( Re_j = 1800 \). The magnitude of the oscillations (in dB) are represented by spectral amplitude contours.
Figure 3.6: Power spectra of the shear layer instabilities at various $s/D$ along the $S = 0.45$ transverse jet, with oscillation frequency given in terms of Strouhal number $St$. Results are shown for various values of $J$ at constant $Re_j = 1800$. The magnitude of the oscillations (in dB) are represented by spectral amplitude contours.

At $S = 0.45$ (Figure 3.6), just above the transition density ratio for the free jet, the beginning of a divergence from the $J_{cr} \sim 10$ transition was encountered. At $J = 41$, the shear layer tone effect associated with convective instability was observed as the hotwire was moved along the shear layer, until reaching $s/D = 1.5$. At this location, a strong peak at a lower frequency appeared, with $St$ near that of the free jet. This pure-tone oscillation was
dominant throughout the shear layer for $s/D \geq 1.5$, replacing the higher frequency, probe-induced mode. The frequency ‘jump’ was hysteretic, observed to occur closer to the jet exit ($s/D = 1.1$) when the hotwire was traversed in the negative $s$ direction (towards the jet exit). In this case, the feedback effect of the shear layer tone may have illuminated regions of varying response to localized forcing, i.e. the boundary of an absolutely unstable region. For $S = 0.45$ and $J < 41$, the shear layer tone was found to occur in a similar manner to the higher density ratio cases, and the jet appeared to transition to global instability between $J = 12$ and $J = 8$, albeit with stronger subharmonics occurring closer to the jet exit than for $S = 0.55$ and above.

For jet-to-crossflow density ratios below the value of $S_{cr}$ determined for the $Re_j = 1800$ free jet, such as $S = 0.25$ as shown in Figure 3.7, the hotwire spectra contained strong global oscillations for all values of $J$ examined. No shear layer tone effects associated with the probe were observed, based on measurements external to the shear layer by both a hotwire probe (located in the crossflow upstream of the jet) and a microphone (PCB Piezotronics Model 378C01, located in the jet fluid pipe upstream of injection). Several additional features were apparent in these low density ratio cases ($S < 0.55$) for high jet-to-crossflow momentum flux ratios ($J \geq 12$). Double-peaked subharmonics appeared in several cases (see Figure 3.6c and Figure 3.7c-d), which has been linked to $m = 1$ mode excitation in plane jet experiments (Huang and Hsiao, 1999). The asymmetry induced by disturbances of Fourier mode number $m \neq 0$ may have resulted in linear phase jittering between the instabilities at $f_0$ and $f_0/2$, leading to nonlinear parametric resonance and an apparent double peak with frequencies equally above and below $f_0/2$. Additionally, nondeterministic temporal switching between oscillations at $f_0$ and $f_0/2$ was commonly observed in these cases. Short-time single realizations of the hotwire spectrum revealed this behavior, which is not apparent in the ensemble-averaged spectra of Figures 3.4-3.7. Switching between the fundamental and subharmonic modes occurred over periods as long as 10-15 seconds to as short as several milliseconds, with higher magnitude oscillations at $f_0/2$ replacing lower magnitude oscillations at $f_0$, and vice versa. The mode switching phenomenon has been observed in both heated (Monkewitz et al., 1990) and equidensity (Corke et al., 1991) axisymmetric free jets, and
Figure 3.7: Power spectra of the shear layer instabilities at various $s/D$ along the $S = 0.25$ transverse jet, with oscillation frequency given in terms of Strouhal number $St$. Results are shown for various values of $J$ at constant $Re_j = 1800$. The magnitude of the oscillations (in dB) are represented by spectral amplitude contours.

may correspond to the influence of helical modes. Experiments involving multiple hotwires on lateral sides of the jet shear layer might shed light on the impact of higher order Fourier mode disturbances. At the present time we can only speculate on the origin of these observations, although it is interesting that they are only seen in the low density, high momentum flux ratio transverse jets.

The impact of the variation of $(J, S)$ conditions on the transverse jet shear layer, under
maintenance of constant \( R_e_j \), is most readily apparent in Figures 3.4-3.7 by the elimination of the shear layer tone and the growth of strong global oscillations below critical values of \( J \) and/or \( S \). The influence of this transition on the Strouhal number of the fundamental shear layer mode is shown in Figure 3.8. \( St \) is plotted against the square root of momentum flux ratio (also known as the ‘blowing ratio’) simply for purposes of enhancing the clarity of the trends. For \((J, S)\) combinations resulting in convectively unstable shear layers dominated by the shear layer tone, \( St \) associated with the dominant frequency \( f_0 \) was selected at a location \( s/D \) at which the fundamental mode frequency had reached a near-asymptotic value with traversal of the hotwire downstream. This method for selection of \( f_0 \) was deemed appropriate by consideration that a reduction in the frequency shifting with probe traversal in the ‘s’ direction indicates a weakening of the feedback effect, and a more accurate indication of the shear layer’s natural roll-up frequency. Of course, globally unstable transverse jets required no such selectivity in the measurement of \( f_0 \), as strong instabilities were detected with little to no variation in frequency along the shear layer trajectory.

For transverse jets of density ratios in the range \( 1.00 \geq S \geq 0.55 \) (Figure 3.8a), relative agreement in \( St \) values and hence data collapse was found as \( J \) was altered from \( \infty \) through
values as low as 2 or 5; crossflow speed limitations precluded reaching $J = 2$ conditions for $S \leq 0.80$. Free jet Strouhal numbers (based on the initial ‘shear layer’ mode rather than the ‘preferred’ mode) ranged between 0.52 and 0.62, generally decreasing with decreasing density ratio. As the crossflow velocity was increased at constant values of $S$, lowering the momentum flux ratio towards $J_{cr}$, the Strouhal number of the fundamental shear layer modes increased, reaching peak values near $J = 12$. Upon lowering the momentum flux ratio further, which transitioned the weak, shear tone-influenced instabilities to strong, discrete peaks, the values of $St$ decreased significantly. Similar trends were found for lower density ratios, $S \leq 0.45$ (Figure 3.8b), but the collapse of the data found at higher density ratios was lost as $S$ was brought below $S_{cr}$. These low density ratio cases exhibited maximum Strouhal numbers near $J = 12$ or 10 before dropping off at lower momentum flux ratios, but the peak $St$ decreased with decreasing $S$. This departure from the collapse of $St$ for low values of $S$ may result from the significant variation in the crossflow velocities required to match values of $J$ at constant jet Reynolds number $Re_j$. While Reynolds number similarity holds the free jet’s initial momentum thickness constant for all density ratios, the required variation of the crossflow boundary layer thickness upstream of the jet as $S$ is lowered for a single value of $J$ may in part be responsible for the loss of quantitative agreement in the dominant jet frequencies. However, even the free jet exhibited some degree of variation in $St$ as $S$ was altered between 1.00 and 0.14, without an easily distinguishable trend. Additionally, as the jet density is lowered beyond $S_{cr}$ at any particular value of $J$, the global mode may transition from a shear layer type to one more akin to the ‘jet column’ mode found in low density axisymmetric free jets, possibly resulting in a divergence in frequency scaling.

In addition to alterations in the frequency scaling of the fundamental oscillations under variation of $J$ and $S$, the dynamics of the upstream shear layer vortices and energy transfer between modes also appeared to be linked with these parameters. The streamwise evolution of the shear layer instabilities was examined by tracking the amplitudes of the fundamental and subharmonic modes. The magnitude of each was isolated by summing the spectral magnitude in a $\Delta St = 0.35$ band around the fundamental frequencies from Figure 3.8 and a $\Delta St = 0.175$ band around the subharmonics, which was sufficient to capture the full range
Figure 3.9: Spatial development of the $S = 1.00$ transverse jet shear layer’s fundamental and subharmonic instability modes, shown for various values of $J$ at constant $Re_j = 1800$. 

of frequency shifting associated with the shear layer tone for cases where both $J$ and $S$ were greater than their critical values. The development of these modes along the shear layer trajectory under variation of $J$ at constant $S = 1.00$ is shown in Figure 3.9. At $J = 41$, (Figure 3.9a) the fundamental mode grows exponentially beginning at $s/D=0.5$, before saturating near $s/D \approx 2.5 - 3.0$ and decaying further downstream. This decay of the fundamental is due to the interaction and pairing between vortex structures initially separated by the fundamental wavelength, hence the growth of the subharmonic and transfer of energy from the fundamental for $s/D \geq 3.9$. As $J$ was lowered, growth of the fundamental mode occurred nearer to the jet exit, and its eventual saturation and decay were delayed until further downstream distances. This resulted in a reduction in the degree of energy transfer to the subharmonic with decreasing $J$ (or increasing crossflow velocity), until eventually at $J = 5$ the subharmonic magnitude did not overtake that of the fundamental at all within $s/D \leq 5.0$.

A similar reduction in vortex pairing after an apparent bifurcation to a self-excited or
globally unstable flow was observed by Davitian et al. (2010a) in the $Re_j = 2000$ equidensity transverse jet shear layer with lowered $J$ values and by Strykowski and Niccum (1992) in an axisymmetric equidensity jet shear layer under increasing degrees of counterflow. Their interpretations maintain that either discretely imposed forcing or self-excitation of the shear layers promotes a more distinct spacing between neighboring vortex structures than that present in a convectively unstable shear layer responding to broadband laboratory noise. Hence, a self-excited shear layer would remain less susceptible to vortex pairing until further distances downstream of the jet exit. While this view seems to be upheld by the trends for the equidensity transverse jet in Figure 3.9, an interesting departure is apparent when viewing the impact of variation of the jet-to-crossflow density ratio $S$ at constant momentum flux ratio $J$. As shown in Figure 3.10 for $J = 20$, well above the critical momentum flux ratio $J_{cr}$, lowering the jet density causes the fundamental mode to saturate and transfer energy to the subharmonic closer to the jet exit. Scaling the spatial coordinate by the wavelength
of the fundamental instability (according to $\lambda = U_j/2f_0$) did not alter this observation, as the wavelength remained nearly invariant ($\lambda \approx 0.58D$, within about 2%) in the range of $S$ shown. Thus, for the transverse jet, an effect of lowering the jet density appeared to be that vortex pairing was enhanced, despite the fact that sufficiently lowering the jet density resulted in global instability. Transverse jets of densities well below $S_{cr}$ (e.g., see Figure 3.7) demonstrated strong subharmonics, and this has been well documented in low density free jet studies (Monkewitz et al., 1990; Kyle and Sreenivasan, 1993). Hence the dynamical character of the transition to global instability under independent variation of $J$ (or $R^2$ as in the equidensity transverse jet) and $S$ (as in the low density free jet) appear to be sufficiently different as to have an influence on the nature of the transition in the low density JICF. Understanding the independent roles of $J$ and $S$ on vortex dynamics in the transverse jet shear layer and their impact on jet mixing requires more detailed quantitative flow imaging, which will be discussed in Ch. 4.

3.3 Estimation of the Critical Flow Parameter for Bifurcation to a Global Mode

The evidence of a transition to global instability motivated an attempt to determine more precisely the critical $J$-value at which a global mode arises, and whether that critical value may vary slightly with $S$. A true determination of the critical parameter would require an analysis of the transient response of the flowfield to an impulsive alteration of the control parameter. Such an analysis was performed for the case of vortex shedding aft of a circular cylinder by Strykowski and Sreenivasan (1990), under an impulsive variation of the flow Reynolds number. Unfortunately the ability to create a step change in the transverse jet momentum flux ratio is quite difficult to perform experimentally. Rather, a simpler experiment was enabled by consideration of Landau’s nonlinear instability theory, as performed by Monkewitz et al. (1990) in the heated free jet and Strykowski and Niccum (1991) in the countercurrent mixing layer. As developed in detail by the text of Drazin and Reid (1981) and review paper of Huerre and Monkewitz (1990), the amplitude $|A|$ of the dominant or
global mode near critical conditions may be described by the forced Landau equation:

$$\frac{d|A|}{dt} = c_1|A| - c_2|A|^3 + \alpha$$  \hspace{1cm} (3.2)

where $c_1$ is the linear temporal growth rate and $\alpha$ the applied forcing. For the case $c_2 > 0$ and $J < J_{cr}$ in the absence of forcing, the saturation amplitude $|A|_0$ of the global mode has a dependence on the momentum flux ratio (considered to be the control parameter in our study) according to:

$$|A|_0 \approx c_3(J_{cr} - J)^{\frac{1}{2}}$$  \hspace{1cm} (3.3)

a condition referred to as a supercritical bifurcation. This theoretical form of the saturation amplitude lends itself to a reasonably simple experimental confirmation of the bifurcation and an estimate of the critical control parameter value.

Measurement of the saturation amplitude of the global mode was performed by successively placing the hotwire at three different spatial locations within the $Re_j = 2000$, $S = 1.00$ transverse jet shear layer, corresponding to $s/D = 0.5, 0.7$ and $1.0$. At each measurement location, the control parameter $J$ was varied and the calibrated hotwire signal recorded for high accuracy measurement of the root-mean-square of the velocity fluctuation, $u'_{rms}$. Fig. 3.3 shows that as $J$ was systematically reduced at fixed $s/D$, a significant increase in the square of the disturbance amplitude ($u'_{rms}/U_j$)$^2$ was observed. The linear amplitude dependence confirms the validity of Equation 3.3, and allows for estimation of $J_{cr}$ by extrapolation to $u'_{rms} = 0$, as shown by the dashed lines. Each measurement location provided a different estimated $J_{cr}$ using this method, which is inconsistent with Landau’s theory. This issue was also observed and commented on in the countercurrent mixing layer experiments of Strykowski & Niccum. Since the total spatial amplification of localized disturbances increases along the jet shear layer, which is not accounted for under the assumption of zero forcing in the Landau equation, there may be increasing errors in the estimate of $J_{cr}$ when measurements are made at larger $s/D$. However, the linear relationship seen in Fig. 3.11 provides strong evidence that the transition in the transverse jet shear layer is due to a supercritical Hopf bifurcation. Similar results, not shown here, were obtained for jets of various jet-to-crossflow density ratio at $Re_j = 1800$. Thus, the best method for estimation
Figure 3.11: The saturation amplitude of velocity perturbation in the transverse jet shear layer, scaled by mean jet velocity and measured at fixed spatial locations $s/D = 0.1, 0.5, & 1.0$, for a range of jet-to-crossflow momentum flux ratios, $J$. Conditions correspond to $S = 1.00$ and $Re_j = 2000$. The linear relationship between $(u_{rms}^*/U_j)^2$ and $J$ can be extrapolated to locate $J_{cr}$ at $u_{rms}^* = 0$.

of $J_{cr}$ remains a careful study of the spatial evolution of the shear layer spectra. As such, $J_{cr}$ is thought to be approximately, or slightly below, 10 for all jet-to-crossflow density ratios thus far considered.

### 3.4 Shear Layer Response to Single-Tone Acoustic Forcing

An experiment to aid in the characterization of the transverse jet shear layer instabilities was performed by exciting the shear layer with low level acoustic forcing. Low amplitude acoustic excitation has been utilized by Raman et al. (1994) in the axisymmetric equidensity free jet and by Megerian et al. (2007) in the $Re_j = 2000$ and $3000$ equidensity transverse jets to examine the spatiotemporal evolution of small applied perturbations. Such experiments can be used to identify the naturally arising global response of the shear layer as being due to regions of convective or absolute instability in the sense of local linear concepts, per the reviews of Huerre and Monkewitz (1990) and Chomaz (2005). A convectively unstable shear flow will act as an amplifier of external noise, allowing periodic disturbances to grow and
convect downstream. It will return to a steady base flow in the absence of perturbations. In contrast, an absolutely unstable flow will allow periodic disturbances to grow and spread both upstream and downstream, eventually 'contaminating' the entire flowfield and inducing global oscillations. An absolutely unstable flow will hence become self-excited once initially perturbed, and will continue to undergo global oscillations after the applied forcing is halted, remaining resistant to external noise. Thus, it is suggested that a global mode arises due to a finite region of absolute instability in the flow.

Megerian et al. (2007) found that the application of low level forcing at frequencies $f_f$ both below, at, and above the fundamental frequency $f_0$ provided an enlightening piece of evidence for the transition to global instability below $R \sim 3.2$. In the variable density transverse jet shear layer, a similar experiment was undertaken. Transverse jets at various $(J, S)$ conditions were excited at forcing amplitudes corresponding to less than than 1% of the mean jet velocity, as measured by the hotwire at $s/D = 0.1$. The frequency of forcing was maintained at $f_f = 0.8f_0$ amongst all cases shown, so as to examine the impact of forcing off of the natural instability frequency or any related harmonic thereof. Forcing at $f_f = 0.4f_0$ and $1.2f_0$ was also performed, with similar results.

The shear layer spectra at various spatial locations $s/D$ are shown for the $J = 20$ transverse jet in Fig. 3.12, with Reynolds number matched at $Re_j = 1800$ for four different density ratio cases: $S = 1.00, 0.45, 0.35$ and 0.14. The forced and unforced spectra may be compared at each $s/D$ location. Note that maintaining an equivalence in forcing level as a percentage of mean jet velocity between jets of different densities required larger excitation levels in terms of the hotwire voltage response for lower density (and higher $U_j$) jets, which is apparent from the perturbations in the initial shear layer at $s/D = 0.1$. For the equidensity jet (Figure 3.12a), the applied forcing frequency $f_f$ was clearly visible throughout the shear layer. At $s/D = 2.0$ the applied frequency coexisted with the jet’s fundamental mode, and induced a strong nonlinear interaction made apparent by the presence of numerous higher harmonics and interharmonics of both the forced and fundamental modes. By $s/D = 3.0$, the applied forcing became the dominant mode, completely subjugating the shear layer’s fundamental mode. This type of ‘lock-in’ to an applied forcing frequency $f_f$ sufficiently far
Figure 3.12: Power spectra (amplitude in dB) as measured at various spatial locations in the $J = 20$ transverse jet shear layer at several different jet-to-crossflow density ratios $S$, with the corresponding $s/D$ labeled. Black lines represent the unforced jet spectra, while red lines represent the spectra when the jet is forced at low amplitude with $f_f \approx 0.8 f_0$. 

(a) $S = 1.00$

(b) $S = 0.45$

(c) $S = 0.35$

(d) $S = 0.14$
Figure 3.13: Power spectra (amplitude in dB) as measured at various spatial locations in the \( J = 8 \) transverse jet shear layer at several different jet-to-crossflow density ratios \( S \), with the corresponding \( s/D \) labeled. Black lines represent the unforced jet spectra, while red lines represent the spectra when the jet is forced at low amplitude with \( f_f = 0.8f_0 \).
from \( f_0 \) and at very low amplitude is expected of a convectively unstable flow. At \( S = 0.45 \) (Fig. 3.12b), the applied forcing also created a stark departure from the shear layer’s natural spectral character, resulting in ‘lock-in’ to the forcing frequency by \( s/D = 2.0 \). Lowering the density ratio to \( S = 0.35 \), as in Figure 3.12c (now just below the free jet transition value \( S_{cr} \sim 0.45-0.40 \)), produced a condition where lock-in was no longer achieved under equivalent low-level forcing. The forced and natural instability modes appeared to compete in the jet near-field but the natural mode remained dominant. In the \( S = 0.14 \) case, the applied forcing was only visible in the spectrum obtained at \( s/D = 0.1 \). This dramatic alteration in the receptivity to applied forcing under variation of \( S \) at constant, high momentum flux ratio \((J = 20)\) corroborates the apparent transition to global instability. This transition appeared to scale with \( J \) at constant values of \( S \), but may also be achieved at constant \( J \) by altering \( S \) to a sufficiently low value.

At a momentum flux ratio \( J = 8 \), below the critical momentum flux ratio for transition to global instability, the resultant shear layer spectra under equivalent forcing conditions revealed reduced convection of the applied disturbances as compared to the high momentum flux ratio case, as shown in Fig. 3.13. While the forcing frequency \( f_f \) was apparent throughout the shear layer in the equidensity case (Figure 3.13a), its amplitude remained small in comparison to the fundamental mode, and the forced spectra were otherwise nearly indistinguishable from the unforced case. In the lower density shear layers (Figures 3.13bc), the effect of the applied forcing was barely visible by \( s/D = 2.0 \), and was completely engulfed by noise at further distances from the jet orifice. For the \( S = 0.14 \) jet (Figure 3.13d), the applied forcing was seen at the jet exit \((s/D = 0.1)\), but had no effect upon the rest of the shear layer, as one would expect for a flow that is self-excited or globally unstable. Although well short of conclusive, the evidence gathered from this experiment seems to support the possible transition to global instability near \( J_{cr} \sim 10 \) for jets in the range \( 1.00 \geq S \geq 0.55 \), and near \( S_{cr} \sim 0.45-0.40 \) for all values of \( J \). The slight disparity in amplification of the applied forcing between the equidensity and low density cases also suggests that the low density transverse jet may behave more like an absolutely unstable flow.

Several experiments in low density axisymmetric jets have focused on the frequency de-
pendence of the critical amplitude of imposed periodic forcing at which the globally unstable jet will lock in to the forcing frequency (Sreenivasan et al., 1989; Hallberg and Strykowski, 2008; Juniper et al., 2009). A linear dependence of this critical amplitude on \(|f_f - f_0|\) is observed by Juniper et al. (2009), suggesting a Hopf bifurcation to a global mode in their helium jet. Experiments in the \(Re = 2000\) equidensity transverse jet by Davitian et al. (2010a) involve a similar ‘control map’ representation of the lock-in of the low-\(R\) transverse jet to imposed forcing. Their results are reminiscent of the expected ‘\(V\)’ shape for a Hopf bifurcation. However, a coarse frequency and amplitude grid in these experiments precludes a close evaluation of this relationship when forcing at frequencies very near to the shear layer’s fundamental.

The \((J, S) = (5, 0.55)\) transverse jet was examined in this context. At this condition, the jet-to-crossflow density ratio \(S\) is above the transition value for the free jet, but the jet-to-crossflow momentum flux ratio \(J\) is low enough to cause the jet shear layer to be globally unstable. An example of the response of the jet shear layer (defined by hotwire measurements at \(s/D = 2.0\)) under variation of the imposed forcing amplitude is shown in Figure 3.14a. Here, the forcing amplitude was quantified as the pressure fluctuation filtered to \(f_f\) normalized by the jet dynamic head, as measured by a microphone located outside the near-field of the free jet of identical \(S\) and \(Re_j\) (approximately 2.5D from jet centerline). In this manner, the frequency response of the speaker/plenum/nozzle actuation system was taken into account, and the amplitude of forcing was considered as a perturbation to the initial shear layer. In the case shown, lock-in of the jet to \(f_f\) was achieved between normalized forcing amplitudes of 0.126 and 0.138. In a similar manner to the results of Juniper et al. (2009), many spectral peaks related to both \(f_0\) and \(f_f\) were observed at lower forcing amplitudes, as the competition between the natural and imposed frequencies resulted in coupled nonlinear oscillator behavior. As shown in Figure 3.14(b), the dependence of the critical forcing amplitude for lock-in on the imposed forcing frequency was linear with \(|f_f - f_0|\). As \(J\) was increased to 8 and subsequently to 10 while maintaining \(S = 0.55\), the same linear dependence was observed, but with the ‘\(V\)’ shape opening up to allow effective lock-in of the shear layer at lower amplitudes of forcing. The global instability was clearly
Figure 3.14: (a) Power spectra (amplitude in dB) as measured at $s/D = 2.0$ in the $(J, S) = (5, 0.55)$ transverse jet shear layer at several normalized forcing amplitudes. The dashed lines represent the spectra of the forced shear layer, and the solid line represents the unforced shear layer spectra. Forcing is applied at $f_f = 2528$ Hz and the fundamental frequency is $f_0 = 2248$ Hz. (b) Forcing amplitude at which lock-in to the forcing frequency is achieved, as a function of forcing frequency $f_f$, for several values of $J$ at $S = 0.55$.

weakened as $J$ was increased towards $J_{cr}$, until even exceedingly low levels of forcing could be made effective in controlling the shear layer’s oscillation frequency.
CHAPTER 4

Scalar and Velocity Field Measurements

In this chapter, the results of PIV and acetone PLIF measurements in what is often assumed to be the symmetry plane of the transverse jet ($y = 0$) are presented, in addition to acetone PLIF measurements in several $yz$ cross-sections downstream of the jet. PIV measurements were confined to within approximately 6 jet diameters of the jet exit, enabling comparison of near-field shear-layer characteristics with those obtained via hotwire anemometry (described in Chapter 3). Acetone PLIF measurements were made over a larger field of view (approximately 20x20 jet diameters) in order to observe global jet structure and quantify mixing under independent variation of both $J$ and $S$, as well as under controlled acoustic forcing.

4.1 PIV Measurements in the Jet Near-field

While the hotwire measurements described in Chapter 3 provided insight into the instability of the near-field jet shear layer and its receptivity to applied perturbations, its intrusive nature and limitation to a single component of velocity made the use of planar, optical diagnostics a natural progression. Velocity measurements were made in the near-field of the equidensity transverse jet using stereo PIV, selected to enable measurement of the in- and out-of-plane velocities. This third velocity component is often ignored (largely due to practical limitations of the imaging setup) when making symmetry plane PIV measurements in transverse jets (Hasselbrink, 1999; Su and Mungal, 2004). However, it may yield important information as to the interaction between vortex systems (Meyer et al., 2007), in particular in transverse jets where symmetry about the $y = 0$ centerplane may not be assumed (for examples, see Smith and Mungal (1998) and Kuzo (1995)).
Figure 4.1: Sample instantaneous vector field, zoomed in to show detail. The arrow lengths represent the relative magnitudes of the in-plane velocities $\sqrt{u_x^2 + u_z^2}$, while the background colormap represents the $\omega_y$ vorticity field (scaled by $U_j/D$). The final step in vector processing uses interrogation windows of 32x32 pixels with 75% overlap to increase vector yield.

Stereo PIV measurements were made in equidensity transverse jets between $J = 41$ and $J = 2$, corresponding to the same momentum flux ratios studied in the hotwire experiments. As an example of the measurement resolution, an instantaneous vector field, zoomed in to a small field of view around the jet column, is shown in Figure 4.1. Each vector was computed from a correlation involving an interrogation window of 32x32 pixels, with 75% overlap between windows. This degree of overlap was selected to increase the vector yield, and means that the size of the interrogation window was actually the distance between 5 adjacent vectors in the resultant fields. The in-plane velocity magnitude is represented by the vector length, while the out-of-plane velocity magnitude is absent from this example. The y-direction vorticity was computed from the in-plane components of velocity, and is plotted via a square pixel colormap at each vector grid point. For subsequent figures, the vectors themselves will be left absent due to their high density relative to the overall size of the images. Instead, the quantities of interest will be plotted as square pixel colormaps, as
\( \omega_y \) in this example.

### 4.1.1 Instantaneous Vorticity and Velocity fields

A direct comparison of the nature of shear layer vortex roll-up and pairing observed by the hotwire and the PIV system may be made via the hotwire voltage spectra (see Figure 3.4, corresponding to fluctuations in vertical velocity \( u_z \)) and the \( \omega_y \) vorticity fields obtained from PIV, shown in Figure 4.2. The agreement is excellent. For both the high and low momentum flux ratio cases, the locations along the upstream shear layer at which the initial onset of the instabilities became apparent in the hotwire spectrum corresponds well with the initial shear layer roll-up locations in the PIV measurements. In Figure 4.2(a), where \((J, S) = (41, 1.00)\), the initially laminar shear layer began to roll up on its upstream side at about \( z/D = 2.0 \) to 2.5, consistent with the initiation of the fundamental mode frequency \( f_0 \) in the hotwire measurements at about \( s/D = 2.0 \) (Figure 3.4(b)). Beyond \( z/D \approx 3.0 \), vortices of opposite sign on the upstream and downstream sides of the jet were observed to pair and merge, eventually breaking down into a turbulent free shear layer. This behavior was also evidenced by the growth of a subharmonic peak in the hotwire voltage spectrum (near \( f_0/2, St = 0.38 \)) at about \( s/D = 3.0 \). Similar agreement was found for the other momentum flux ratios in the convectively unstable regime, \( J = 20 \) and \( J = 12 \). At each of these successively lower momentum flux ratios, the location of shear layer roll-up and subsequent vortex pairing moved closer to the jet exit, and the spacing between adjacent vortices was reduced, consistent with increasing \( f_0 \) (Figure 3.8). At \( J = 8 \) (Figure 4.2), conditions at which the flow appeared to be globally unstable, little evidence of vortex pairing was seen. A distinct spacing between adjacent shear layer vortex structures was maintained until further distances downstream before vortex merging and turbulent breakdown took place, a result of the self-excitation of the jet shear layer at a pure-tone frequency. This spacing also increased with decreasing \( J \) for these globally unstable cases, again consistent with the trends in \( f_0 \) found from the hotwire measurements shown in Figure 3.8.

Observations of these instantaneous vorticity fields confirmed some of the major features
Figure 4.2: Sample instantaneous fields of y-direction vorticity $\omega_y$, normalized by $U_j/D$, at several values of $J$ in the equidensity ($S = 1.00$) transverse jet. These values of $J$ correspond to the same flow conditions as the hotwire measurements in Figure 3.4.

of the hotwire data in the equidensity transverse jet, without the impact of probe intrusion. Additional interesting features of the PIV measurements may be found by inspection of Figure 4.3, which shows representative instantaneous snapshots of the out-of-plane velocity,
Figure 4.3: Sample instantaneous fields of the out of plane velocity $u_y$, normalized by $U_j$, at several values of $J$ in the equidensity ($S = 1.00$) transverse jet. These images are taken from the same instants as the vorticity plots shown in Figure 4.2.

$u_y$, as the momentum flux ratio was altered. Starting with the highest value of $J$, 41, bands of opposing $u_y$ sign were observed, oriented at a declined trajectory relative to the bulk flow of jet fluid. As will be made clear by larger field of view acetone PLIF measurements later in
This chapter, these bands were caused by the presence of a lower, tertiary vortex separate from
the CVP, but with axis of rotation similarly aligned with the jet flow direction. Its rotational
sense was counterclockwise when viewed from downstream along its axis (corresponding to
negative $\omega_z$), and thus in the $y = 0$ centerplane it induced bands of $u_y$, positive at larger $z/D$
and negative closer to the tunnel wall. The formation mechanism for this vortex was unclear,
but it appeared to be initiated very near to the jet exit on the downstream side of the jet
column, and was in fact a mean flow feature of the jet cross-section. Similar tertiary vortices
were observed in transverse jets (with assumed top hat velocity profiles at injection) by Kuzo
(1995), at several high momentum flux ratios over a large range: 25, 100, 225, and 400. The
jet Reynolds number $Re_j$ was altered from about 2000 to 13,000 in Kuzo’s experiments, and
it was found that jets of each different value of $J$ had a critical Reynolds number above which
the tertiary vortices, and the asymmetric jet cross-sections they induced, gave way to the
more well known symmetric CVP. This critical Reynolds number was smaller for lower values
of $J$, leading to an attempted but indefinite scaling of the transition from asymmetry to
symmetry by the parameter $Re_j/R = Re_\infty$, the crossflow Reynolds number. Alterations in
the jet cross-sectional symmetry and the existence of tertiary vortices in the current work
will be detailed later in this chapter.

At $J = 20$ (Figure 4.3(b)), the bands of $u_y$ corresponding to a tertiary vortex were absent,
with only a few regions of relatively high velocity magnitude located in the jet shear layer
Corresponding to motions of the shear layer vortices. As $J$ was lowered to 12, structures
were observed in the jet wake, apparent by alternating, vertically oriented bands of positive
and negative $u_y$. Similar structures were observed in the stereo PIV measurements of Meyer
et al. (2007) at $J = 10.89$ and 1.69 ($Re_j = 7920$ and 3120, respectively), and were interpreted
as wake vortices with a strong interaction with the jet core. In our measurements, the
magnitude of these bands increased as $J$ was lowered further down to a value of 2, and their
structure became finer and less regular. A thorough modal analysis using Proper Orthogonal
Decomposition (POD), including measurements in $yz$ cross-sections of the jet, is suggested
for future efforts to analyze the dynamic flow features of the jet and interactions between
vortex systems as $J$ is altered through the transition to global instability.
4.1.2 Mean Streamlines, Vorticity and Velocity Fields

The mean flow patterns in the transverse jet centerplane under alteration of \( J \) are expressed in terms of two-dimensional streamlines in Figure 4.4. At each momentum flux ratio, crossflow fluid originating above approximately \( z/D > 0.6 \) became entrained into the jet rather than stagnating. Below this distance from the wall, the streamlines curved downwards, likely entering into either the horseshoe vortex system, a ‘hovering’ vortex as first observed by Kelso et al. (1996), or even entering into the nozzle itself, which has been observed both in experiments (Kelso et al. (1996)) and DNS (Muppidi and Mahesh (2005)) involving jets with pipe inflow. Limitations on the measurement distance from the wall due to laser light reflections precluded exploration of these lower vortex systems in this study. In the wake of the jet, fluid originating in the crossflow was drawn upwards and entrained into the jet by the influence of the CVP. The mean wake structure in \( J = 12, 8 \) and \( 5 \) (and possibly 2 as well below the limits of the field of view) showed a node, a commonly observed feature in pipe inflow experiments involving either flush or elevated injection (Kelso et al., 1996; Hasselbrink and Mungal, 2001b; Su and Mungal, 2004), although the node is usually found near the wall closer to the jet exit (smaller \( x/D \)). This node has been correlated with a high degree of two-dimensional divergence, thus implying out-of-plane motion, and is usually interpreted as the location of opposed flow of crossflow fluid having diverged around the sides of the jet (Kelso et al., 1996). The lack of such a node at \( J = 41 \) and \( J = 20 \) may be due to a lack of flow symmetry about the centerplane in these cases. Any asymmetry resulting in mean out-of-plane motions of course would indicate that streamlines computed from the in-plane velocity components do not represent actual flow streamlines. The presence of a tertiary vortex in the \( J = 41 \) jet would certainly create this sort of impact, and as will be detailed in the acetone PLIF results later in the chapter, asymmetry about the jet centerplane was significant in these higher momentum flux ratio cases.

Figures 4.5-4.11 present the mean velocity fields \( u_x, u_y \) and \( u_z \), as well as the mean in-plane vorticity field \( \omega_y \), at successively lowered values of \( J \) between 41 and 2. They have been ensemble averaged over 200 instantaneous vector fields. In each of the velocity field
plots, black zero-velocity contour lines are drawn to show the boundaries between positive and negative flow. At $J = 41$ and 20 (Figures 4.5 and 4.6), alternating regions of high and low $u_x$ occurred along the jet shear layer, as well as a ‘bulging’ of the vorticity field in these locations. This effect was the result of aliasing due to the constant acquisition rate of the PIV system, resulting in a tendency of vortices to appear at the same positions for a statistically large number of images. It is possible that with intermittent acquisition or perhaps even averaging over a larger number of samples, the velocity fields would better converge and this effect would be eliminated. In fact, at lower values of $J$, the convergence appears to have been achieved.

A large region of negative $u_x$ just downstream of the jet column was found at $J = 41$, extending about 6 jet diameters in $z$ along the jet and in $x$ along the wall. This region of reversed flow was found at each value of $J$, maintaining a similar shape while shrinking closer to the floor as $J$ is lowered. The position along the jet trajectory at which this region of reversed flow ended was concomitant with turbulent breakdown of the jet shear layer, as evidenced by a decline in the vertical velocity $u_z$ and dissipation of the vorticity sheet caused by increased fluctuations and out-of-plane motion. The $u_z$ fields exhibited a bifurcated structure with two peaks in the direction normal to the jet trajectory, one approximately along the bulk jet flow and the other at a lower trajectory. This is consistent with the measurements of Su and Mungal (2004) in pipe flow transverse injections, both flush with and elevated from the wall. These researchers linked this structure to the effect of the low-pressure jet wake drawing the jet trajectory towards the wall.

Mean out-of-plane velocities $u_y$ displayed largely random, low magnitude variations likely associated with the uncertainty of the technique, except for the $J = 41$ case. This case showed a mean structure consistent with the presence of a tertiary vortex, where the zero-velocity contour between the positive and negative bands of $u_y$ represents the approximate vortex line trajectory. However, as will be shown via PLIF imaging in $xz$ cross-sections, the tertiary vortex did not reside in the centerplane in the mean, but rather on the positive $y$ side of the jet.
Figure 4.4: Two-dimensional streamlines computed from the mean velocity fields at several values of $J$ in the equidensity ($S = 1.00$) transverse jet’s $y = 0$ centerplane.
Figure 4.5: Colormaps of the mean scaled velocity components $u_x$, $u_y$ and $u_z$, as well as the mean scaled vorticity field $\omega_y$, in the centerplane of the $J = 41$ equidensity transverse jet. In each of (a)-(c), the black lines denote the contours of zero velocity.
Figure 4.6: Colormaps of the mean scaled velocity components $u_x$, $u_y$ and $u_z$, as well as the mean scaled vorticity field $\omega_y$, in the centerplane of the $J = 20$ equidensity transverse jet. In each of (a)-(c), the black lines denote the contours of zero velocity.
Figure 4.7: Colormaps of the mean scaled velocity components $u_x$, $u_y$ and $u_z$, as well as the mean scaled vorticity field $\omega_y$, in the centerplane of the $J = 12$ equidensity transverse jet. In each of (a)-(c), the black lines denote the contours of zero velocity.
Figure 4.8: Colormaps of the mean scaled velocity components $u_x$, $u_y$ and $u_z$, as well as the mean scaled vorticity field $\omega_y$, in the centerplane of the $J = 10$ equidensity transverse jet. In each of (a)-(c), the black lines denote the contours of zero velocity.
Figure 4.9: Colormaps of the mean scaled velocity components $u_x$, $u_y$ and $u_z$, as well as the mean scaled vorticity field $\omega_y$, in the centerplane of the $J = 8$ equidensity transverse jet. In each of (a)-(c), the black lines denote the contours of zero velocity.
Figure 4.10: Colormaps of the mean scaled velocity components $u_x$, $u_y$ and $u_z$, as well as the mean scaled vorticity field $\omega_y$ in the centerplane of the $J = 5$ equidensity transverse jet. In each of (a)-(c), the black lines denote the contours of zero velocity.
Figure 4.11: Colormaps of the mean scaled velocity components $u_x$, $u_y$ and $u_z$, as well as the mean scaled vorticity field $\omega_y$, in the centerplane of the $J = 2$ equidensity transverse jet. In each of (a)-(c), the black lines denote the contours of zero velocity.
4.2 Acetone PLIF - Scalar Concentration Field and Jet Mixing Quantification

The overarching goal of the vast majority of experimental transverse jet research has been to enable accurate prediction and modeling of the distribution of jet fluid under various types of injection and crossflow conditions, and to inform methods of effective control. Scalar field measurements using PLIF and smoke visualization have been numerous, but the wide variety of flow conditions utilized in these works, including the variation of jet injection type (nozzle, pipe flow, etc.), velocity and momentum flux ratio, jet Reynolds number, crossflow boundary layer thickness, etc., have uncovered an enormous range of flow regimes which may result in significant alterations in jet behavior. The current work serves to add to this collection of data, analyzing the same alterations in $J$ and $S$ at constant, relatively low Reynolds number $Re_j$ that have been linked with a transition to global instability. The main points of interest are the alterations in jet structure and mixing, both as the flow conditions are varied and as different acoustic forcing waveforms are applied at a single flow condition.

4.2.1 Unforced Jet Centerplane Measurements

The initial set of PLIF measurements focused on the assumed symmetry plane of the transverse jet, the $y = 0$ centerplane. While PIV measurements in this plane required accurate resolution of vortex structure in the jet shear layer and therefore were limited to about about $6D$ surrounding the jet exit, the PLIF measurements were extended to a larger field of view, ranging to about $z/D = 20$ and $x/D = 18$. At each $(J,S)$ combination, 200 individual PLIF images were acquired, enabling the calculation of a mean concentration field and quantification of several common metrics related to jet structure and mixing: the jet penetration, spread, scalar centerline trajectory, and concentration decay.

Figures 4.12 and 4.13 display sample instantaneous scalar fields and the mean scalar fields, respectively, for jet-to-crossflow density ratios $S = 1.00, 0.70, 0.55,$ and $0.35$, matched at $J = 41$ and $Re_j = 1800$. In each instantaneous image, the jet branched into one or more bifurcations within the range $x/D < 5$, including, for the $S = 1.00$ and 0.55 cases,
the same type of streamwise-oriented tertiary vortex structure evidenced by the near-field PIV measurements in the $S = 1.00$ transverse jet described in Section 4.1. This tertiary vortex corresponds to the bifurcation nearest the wall in the PLIF images. At the lowest density ratio in these experiments, $S = 0.35$, this lower, bifurcated part of the jet appeared to hold significantly larger concentrations of jet fluid and to persist farther downstream in the centerplane. The mean scalar field images in Figure 4.13 show clearly the bifurcated or trifurcated jet structure in the centerplane, which initiated slightly nearer the jet exit as $S$ was lowered. The increased dispersal of jet fluid into these lower, bifurcated regions of the lower density jets, and the resultant loss of vertical momentum in the core jet flow, also resulted in lower overall penetration of the jet into the crossflow. Clearly in each of these cases, the jet did not take on the usual kidney shaped cross-section in the far-field brought about by the effect of the CVP, but rather a form containing multiple concentration peaks along the $z$ direction. This will be discussed in more detail in the following section, in which the results of PLIF imaging in the $yz$ plane are presented.

In Figures 4.14 and 4.15, sample instantaneous and mean scalar field images are shown for $J = 20$ at each density ratio. Here the bifurcated structure found at $J = 41$ was no longer present, but the presence of thin filaments of jet fluid oriented perpendicular to the core jet trajectory became increasingly apparent. Both Smith and Mungal (1998) and Su and Mungal (2004) noted the presence of these filaments and attributed them to the drawing of jet fluid into the upright wake vortices, which burst upwards from the tunnel floor and curved slightly upstream and into the jet core by interaction with the CVP. As the momentum flux ratio was lowered further at to 8 and 5\(^1\) (Figures 4.16-4.19), several trends were evident, among them: (1) more jet fluid was drawn into the wake structures at lower values of $J$, (2) transition to turbulence and the resulting increase in jet spread occurred nearer the jet exit as either $S$ was reduced at constant $J$ or vice versa, (3) in general, lower density ratio jets retained higher concentrations at equivalent distances downstream of injection when compared at a fixed value of $J$.

From the ensemble-averaged concentration fields in Figures 4.13-4.19, scalar centerline

\(^1\)Additional data at $J = 12$, 10 and 2 are available in Appendix A.
trajectories were established by determining the locii of scalar maximum points. This definition of the jet trajectory, as an alternative to the locus of maximum velocity, or the streamline originating from the center of the jet exit, has been utilized in many instances and is usually found to penetrate less into the crossflow as compared to the center streamline. Penetration relative to the locus of maximum velocity is less clear, since as the velocity ratio is lowered, the maximum velocity at distances well downstream of injection may be found along a trajectory closer to the wake than to the bulk of the jet fluid, as briefly noted in section 4.1. To determine the scalar centerline points, corresponding to the green pixels in the example of \((J, S) = (20, 1.00)\) shown in Figure 4.20, an iterative procedure was used. Near the point of injection \((x/D < 1)\), scalar maxima were determined in constant \(z\) (horizontal) cross-sections, while further from injection constant \(x\) (vertical) cross-sections were used. The resulting points were least-squares fitted with a power law according to \(z/JD = A(x/JD)^m\), used to define the trajectory or ‘s’ direction. Further iterations determined scalar maxima by traversing along the \(s\) direction and finding maxima in the \(s\)-normal direction, re-computing the power law fit each time. Six iterations of this procedure were sufficient for the scalar maxima locus to converge adequately and a scalar centerline trajectory (blue line in Figure 4.20) to be determined.

Scalar centerline trajectories of the equidensity \((S = 1.00)\) transverse jet at several values of \(J\) are shown in Figure 4.21, with scaling attempted by normalization with \(D, RD\) (in this case equivalent to \(\sqrt{JD}\)), and \(JD\). None of these types of scaling were sufficient to collapse the data entirely, with \(RD\) and \(JD\) scaling giving equally inadequate agreement. The findings of Keffer and Baines (1963) using velocity trajectories and Smith and Mungal (1998) using scalar concentration suggest that in the near-field, \(JD\) scaling should give the best collapse. However, in those experiments, \(U_\infty\) was held constant and \(U_j\) increased to reach increasingly large values of the momentum flux ratio \(J\), resulting in a constant thickness of the wall boundary layer into which the jet is injected for all values of \(J\). Our experiments held \(U_j\) constant and altered \(U_\infty\) to alter \(J\) at a constant jet density, thus causing thinner crossflow boundary layers at lower values of \(J\). Trajectory scaling efforts by Muppidi and Mahesh (2005) found that inclusion of a dependence upon the crossflow boundary layer thickness
yielded improved collapse of the trajectories from their DNS data. Intuitively, a thicker crossflow boundary layer results in reduced impediment to the jet’s initial momentum and thereby a higher jet trajectory, which could in part explain the departure found in the simpler scaling of our data.

Trajectories at constant values of $J$ are presented in Figure 4.22, showing the dependence upon the density ratio $S$. Lower density jets in each case were increasingly deflected into the crossflow direction. While the crossflow boundary layers were indeed thinner for lower values of $S$ (due to the requirement to match $J$ at constant $Re_j$), the data were found to reach a reasonable collapse when scaled by $\sqrt{S}D$, shown for the $J = 20$ case in Figure 4.23. Development of a scaling law to compliment this finding has not yet been undertaken. However, the $\sqrt{S}$ dependence does appear in the scaling determined by Hasselbrink and Mungal (2001a) for the scalar centerline concentration in both the near and far-fields.

In Figure 4.24, the penetration and spread of the equidensity jet are shown, as defined by the locations at which the jet fluid concentration dropped below 5% of its value at injection. Penetration was quantified as the uppermost $z/D$ position reached by this threshold concentration level, while the spread $\delta$ was computed along the trajectory-normal direction. Jets of higher momentum flux ratio penetrated further into the crossflow and spread wider in the far-field. The latter trend was likely governed primarily by the increasing proximity of the jet trajectory to the injection wall.

Figure 4.25 shows the scalar concentration decay along the centerline trajectory, including attempted scaling of the $s$ coordinate with $RD$ and $JD$. The decay rate was altered slightly between different values of $J$ but appeared to vary between approximately $s^{-1}$, corresponding to free jet scaling, and $s^{-1.3}$, as found by Smith and Mungal (1998) in turbulent ($Re_j \geq 16600$) transverse jets at high momentum flux ratios in the range $25 \leq J \leq 625$. However, in the work of Smith and Mungal (1998), “branch points” at which the transverse jets transitioned from a $s^{-1.3}$ decay rate to a $s^{-2/3}$ (wake-like) scaling were found for $J \geq 100$. When plotted in $JD$ space, these branch points seemed to align at approximately $s/JD = 0.2$, although no branch point was found at $J = 25$, which was deemed to be part of a “different class of transverse jets” due to its strengthened interaction with the injection wall.
In contrast, Su and Mungal (2004) found that their $J = 32.5$ transverse jet exhibited $s^{-1}$ scaling before transitioning to an even more rapid decay rate. Neither of these behaviors were found in our equidensity measurements, but rather a constant trend at or slightly in excess of the free jet decay rate is observed at each value of $J$. At a given downstream distance along the jet trajectories ($D$ scaling), the higher momentum flux ratio jets retained higher concentrations, until near approximately $s/D = 20$, all jets besides $J = 41$ converged to similar concentration levels.

Jet penetration, spread, and centerline concentration decay under variation of the density ratio $S$ at constant values of $J$ are shown in Figures 4.26-4.29 ($J = 41$, 20, 8 and 5, respectively). In each case, the lower density jets penetrated less into the crossflow and had greater centerplane spread. At $J = 41$ (Figure 4.26(c)), the relationship between concentration decay levels and density alteration was non-monotonic, and the lowest density case, $S = 0.35$ seemed to transition to a reduced decay rate near $s^{-2/3}$, the same type of wake-like rate observed in high $J$ cases by Smith and Mungal (1998). This same type of transition is observed at $(J, S) = (20, 0.35)$ (Figure 4.27), resulting in the $S = 0.35$ jet maintaining higher centerline concentrations than the higher density cases, while also having the highest spread. At $J = 8$ and 5 (Figures 4.28 and 4.29), there was a clear alteration from $s^{-1}$ concentration decay scaling towards $s^{-2/3}$ as the density ratio was reduced from 1.00 to 0.35. There was also a monotonic trend in all three metrics shown, with lower density jets maintaining greater spread and lower concentration decay while penetrating less into the crossflow. Trends resulting in increased spread and concentration decay have both been commonly linked with improved mixing in the literature, yet interestingly here the trends in spread and decay with reduced density ratio were at odds in terms of their implications for mixing. Particularly in consideration of the asymmetry about the jet’s $y = 0$ centerplane that was evidenced from the PIV measurements at higher momentum flux ratios, any comparisons involving jet mixing should take into account the three-dimensionality of the jet structure. With this in mind, the imaging setup was altered to obtain PLIF cross-sections in $yz$ planes (normal to the crossflow direction), viewing the jet from downstream of its injection.
Figure 4.12: Sample instantaneous jet fluid concentration fields in the centerplane \((y = 0)\) of \(J = 41\) transverse jets at four different values of the jet-to-crossflow density ratio, \(S\), with \(Re_j\) fixed at 1800.
Figure 4.13: Mean jet fluid concentration fields in the centerplane ($y = 0$) of $J = 41$ transverse jets at four different values of the jet-to-crossflow density ratio, $S$, with $Re_j$ fixed at 1800.
Figure 4.14: Sample instantaneous jet fluid concentration fields in the centerplane \((y = 0)\) of \(J = 20\) transverse jets at four different values of the jet-to-crossflow density ratio, \(S\), with \(Re_j\) fixed at 1800.
Figure 4.15: Mean jet fluid concentration fields in the centerplane \((y = 0)\) of \(J = 20\) transverse jets at four different values of the jet-to-crossflow density ratio, \(S\), with \(Re_j\) fixed at 1800.
Figure 4.16: Sample instantaneous jet fluid concentration fields in the centerplane \((y = 0)\) of \(J = 8\) transverse jets at four different values of the jet-to-crossflow density ratio, \(S\), with \(Re_j\) fixed at 1800.
Figure 4.17: Mean jet fluid concentration fields in the centerplane \((y = 0)\) of \(J = 8\) transverse jets at four different values of the jet-to-crossflow density ratio, \(S\), with \(Re_j\) fixed at 1800.
Figure 4.18: Sample instantaneous jet fluid concentration fields in the centerplane ($y = 0$) of $J = 5$ transverse jets at four different values of the jet-to-crossflow density ratio, $S$, with $Re_j$ fixed at 1800.

(a) $S = 1.00$

(b) $S = 0.70$

(c) $S = 0.55$

(d) $S = 0.35$
Figure 4.19: Mean jet fluid concentration fields in the centerplane ($y = 0$) of $J = 5$ transverse jets at four different values of the jet-to-crossflow density ratio, $S$, with $Re_j$ fixed at 1800.
Figure 4.20: Scalar centerline trajectory in the $y = 0$ centerplane for the $(J, S) = (20, 1.00)$ transverse jet. Green data points show the locus of scalar maxima, while the blue line shows the power law curve fit.
Figure 4.21: Scalar centerline trajectories for the $S = 1.00$ transverse jet at various values of $J$, with $Re_j = 1800$. (a) Scaled by jet diameter $D$. (b) Scaled by $RD$. (c) Scaled by $JD$. 

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Figure 4.22: Scalar centerline trajectories for transverse jets at various jet-to-crossflow density ratios $S$. Each of (a)-(f) represents a different value of $J$, with $Re_j$ held constant at 1800.
Figure 4.23: Scalar centerline trajectories for $J = 20$ transverse jets at various jet-to-crossflow density ratios $S$, scaled by $\sqrt{SD}$.

Figure 4.24: Transverse jet scalar penetration (a) and spread in the trajectory-normal direction along the scalar centerline (b). Shown for the equidensity ($S = 1.00$) jet, at varying values of $J$. Both metrics are based on a jet fluid concentration threshold of 5%.
Figure 4.25: Maximum mean scalar concentration decay along the jet centerline coordinate, \( s \), for the equidensity transverse jet at several values of \( J \). (a) Scaled by jet diameter \( D \). (b) Scaled by \( RD \). (c) Scaled by \( JD \).
Figure 4.26: Transverse jet scalar distribution metrics at $J = 41$, $S = 1.00$, 0.70, 0.55 and 0.35. (a) Scalar penetration, 5% concentration threshold (b) Jet spread in the trajectory-normal direction along the scalar centerline, 5% concentration threshold. (c) Maximum concentration decay along the scalar centerline.
Figure 4.27: Transverse jet scalar distribution metrics at $J = 20$, $S = 1.00$, 0.70, 0.55 and 0.35. (a) Scalar penetration, 5% concentration threshold (b) Jet spread in the trajectory-normal direction along the scalar centerline, 5% concentration threshold. (c) Maximum concentration decay along the scalar centerline.
Figure 4.28: Transverse jet scalar distribution metrics at $J = 8$, $S = 1.00$, 0.70, 0.55 and 0.35. (a) Scalar penetration, 5% concentration threshold (b) Jet spread in the trajectory-normal direction along the scalar centerline, 5% concentration threshold. (c) Maximum concentration decay along the scalar centerline.
Figure 4.29: Transverse jet scalar distribution metrics at $J = 5$, $S = 1.00$, 0.70, 0.55 and 0.35. (a) Scalar penetration, 5% concentration threshold (b) Jet spread in the trajectory-normal direction along the scalar centerline, 5% concentration threshold. (c) Maximum concentration decay along the scalar centerline.
4.2.2 Unforced Jet $yz$ Plane Measurements

For each $(J, S)$ combination discussed in the previous section, cross-sectional measurements of the jet fluid concentration field were made at four positions: $x/D = 2.5, 5.5, 10.5$ and $15.5$. The ensemble-averaged scalar fields for each case are presented in Figures 4.30-4.45, along with the corresponding $y = 0$ centerplane field to aid in visually reconstructing the mean three-dimensional structure of these jets.\(^2\) Note that as in earlier figures, the $y = 0$ fields are plotted using a logarithmic colormap to improve the visibility of the jet structure in the far field. However, for the $yz$ plane fields, the colormap is linear, making any asymmetry about the $y = 0$ plane more obvious. Additionally, as mentioned in Section 2.3.2, the $yz$ plane fields lacked a reference level for $C_0$, the concentration at injection, that was present in each centerplane image. They were corrected for quantitative concentration levels in postprocessing by comparing the common line of data between the centerplane and cross-sectional images. In certain cases, particularly those involving significant asymmetries in the $yz$ plane and at larger $x/D$ (see $(J, S) = (41, 1.00)$ at $x/D \geq 5.5$ in Figure 4.30, for example), this comparison was poor enough that a quantitative correction of the $yz$ plane images could not be made. These images are still included here, but lack concentration level labels on the colormap scale. Slight variations in flow conditions from day to day (which were the timescales associated with capture of each set of PLIF images at a given density ratio and laser sheet position) seemed to have a greater impact on the high momentum flux ratio and highly asymmetric jets than the lower $J$ or $S$ (and globally unstable) jets. However, the general structure at a given $(J, S)$ condition was consistent over the course of several weeks - that is, an asymmetrical jet cross-section with higher concentrations on one particular side of the jet always had higher concentrations on the same side.

Beginning with $J = 41$ (Figures 4.30-4.33), significant asymmetry in the $yz$ scalar fields was found at density ratios of $1.00, 0.70$ and $0.55$. In each of those cases, a lower, tertiary vortex structure was observed, positioned below approximately $z/D = 5$ and always on the positive $y$ side of the jet centerplane. This lower vortex rotated in the clockwise direction

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\(^2\)Example instantaneous scalar fields may be found in Appendix C.
(negative $\omega_z$), drawing jet fluid away from the bulk of jet and resulting in a cross-sectional shape resembling a reverse question mark. The upper part of the jet appeared to have a kidney like shape resembling the usual CVP cross-section, except that it was highly skewed by the impact of the tertiary vortex. PIV measurements in these planes are recommended to analyze the relative strengths and positions of the vortex structures in the jet cross-section as well as the dynamics of their interactions.

At $S = 0.55$, shown in Figure 4.32, the lower vortex structure still induced a highly skewed scalar field, but it appeared to be separated from the rest of the jet, with no mean connecting structure. At $S = 0.35$ (Figure 4.33), the lower vortex remained, but its impact on the jet cross-section was subdued. The cross-section took on a much more symmetric horseshoe or kidney type of shape, resembling a CVP but with a vertically oriented trail of lower concentration fluid present approximately along the $y = 0$ plane (or slightly positive $y$, less than $y/D = 1$). The lowermost part of this trail was the tertiary vortex, most noticeable at $x/D = 15.5$. These mean cross-sections at $J = 41$ bore a strong resemblance to some of the findings of Kuzo (1995), who studied a broad range of equidensity transverse jets in $(J, Re_j)$ space using PLIF and PIV. As mentioned in Section 4.1, Kuzo observed that the jet cross-section displayed significant asymmetry and multiple vortices apart from the CVP at lower $Re_j$. However, as $Re_j$ was increased at constant $J$, the jet asymmetry increased (as quantified by a parameter related to the spatial skewing of the primary CVP vortices) until suddenly transitioning to symmetry beyond a critical value. This critical value of $Re_j$ was determined to increase with increasing $J$, but no definitive scaling of that transition was obtained (although the crossflow Reynolds number $Re_\infty$ was suggested as a possibility). Based on those observations, it might be expected that as $J$ was lowered at constant $Re_j$ in our experiments, symmetry may have eventually been attained. It would also seem that there was a secondary dependence on $S$, as the lower density jets appeared more symmetric than higher density jets.

As $J$ was lowered to 20 (Figures 4.34-4.37), while the tertiary vortex was no longer present, the asymmetry in the jet cross-section was still apparent and became greater at further distances downstream. In the equidensity case (Figure 4.34), the maximum concen-
tration was found on the negative $y$ side of the centerplane, appearing near the core of a counterclockwise rotating vortex. This stood in contrast to the maximum concentration’s appearance on the positive $y$ side of the centerplane at $J = 41$ (Figures 4.30-4.32). Concentrations in this $J = 20$ vortex core were about twice the concentration found on the opposite side of the jet centerplane. Smith and Mungal (1998) also observed asymmetry in the concentration levels on either side of the CVP structure, and in certain cases it reached this level (their $J = 400$, $Re_j = 16,000$ jet, for example). At a slightly lower density ratio, $S = 0.70$ (Figure 4.35), a similar jet distribution was seen through $x/D = 10.5$, until at $x/D = 15.5$ the jet became skewed severely towards the negative $y$ direction. A slightly more symmetric scalar field was obtained at $S = 0.55$ (Figure 4.36), with evidence of fluid being drawn downwards into wake structures located just on either side of the $y = 0$ centerplane. At $x/D = 15.5$ the negative $y$ side of the jet again attained about twice the concentration level as found on the positive $y$ side. However, at $S = 0.35$ (Figure 4.37), the jet once again became much more symmetric, transitioning from a horseshoe like shape at $x/D = 2.5$ to a kidney shaped turbulent CVP structure at larger $x/D$.

At momentum flux ratios $J < J_{cr}$ for the shear layer’s transition to global instability (regardless of the value of $S$), such as $J = 8$ in Figures 4.38-4.41 and $J = 5$ in Figures 4.42-4.45, much more symmetric cross-sections were observed. Slight differences in the maximum concentration found near each vortex core of the CVP grew larger at further distances downstream, but they were much less severe than in the higher $J$ cases. It seems that in cases in which the jet shear layer was globally unstable, either due to sufficiently low momentum flux or density ratio, the jet was much more symmetric than in cases where the shear layer was convectively unstable everywhere. This phenomenon bears more investigation, perhaps using a ‘symmetry parameter’ related to the scalar field distribution on either side of the jet centerplane in order to describe the jet structural alterations under variation of $J$ and $S$. While the cause of these alterations in the degree of asymmetry in the jet is unclear, the local linear stability analysis of Alves et al. (2007) showed that the growth rates of helical modes of opposite sign in the transverse jet shear layer were slightly different, implying

\footnote{Additional data at $J = 12, 10, \text{and} 2$ are available in Appendix B.}
a tendency towards asymmetry that could be transmitted to the CVP and/or lead to jet structures other than the usual vortex pair. However, in our work, it must be noted that it is of course impossible to create perfectly symmetric injection and crossflow conditions in experiments, and thus the presence and degree of asymmetries may be influenced by very small experimental uncertainties.

Scalar fields obtained in $yz$ planes were analyzed using two common mixing metrics, unmixedness ($U$) and spatial mixing deficiency ($SMD$), in order to quantitatively describe the impact of $J$ and $S$ on mixing and to compliment the trends observed in jet concentration decay and spread in the centerplane. $U$ and $SMD$ are defined as:

\[
U = \frac{1}{L_y L_z} \int \int \frac{(\langle C \rangle - \langle C \rangle_{av})^2}{\langle C \rangle_{av} (1 - \langle C \rangle_{av})} dydz \tag{4.1}
\]

\[
SMD = \frac{1}{L_y L_z} \int \int \left[ \frac{\langle C \rangle - \langle C \rangle_{av}}{\langle C \rangle_{av}} \right]^2 dydz \right]^{1/2} \tag{4.2}
\]

where $\langle C \rangle$ is an ensemble average (temporal mean) at each pixel and av denotes averaging over the chosen spatial domain with dimensions $L_y$ and $L_z$ (Denev et al., 2009; Bothe, 2010). Unmixedness quantifies mixing in terms of the sum of the squared spatial concentration fluctuations, relative to the concentration indicating complete mixing in each temporally averaged domain, thereby resulting in 100% indicating completely unmixed jet and crossflow fluids and 0% complete mixing. It must be noted here that our spatial resolution precluded measurement of complete molecular mixing, and rather the concentration value in each pixel volume (voxel) should be viewed as a spatial mean of the molecular mixing within that volume. Spatial mixing deficiency is simply the root-mean-square of the spatial concentration fluctuation normalized by the spatial average within the domain. Each definition is sensitive to the choice of spatial domain over which to integrate. In our work, we sized the spatial domain to contain the entire $J = 12$ cross-section at its furthest downstream location (and largest size), $x/D = 15.5$, and retained this domain size at each $x/D$ position (though relocating the domain based on the location of each individual image’s concentration centroid).

At lower values of $J$ (as low as 2), the domain size $L_y = L_z = L$ was scaled as $L/\sqrt{J}$ in order to account for reduction in the physical size of the jet cross-section as $J$ is lowered. Values
of $J$ greater than 12 were not considered due to their highly asymmetric shape, which would require reconsideration of the integration domain.

Figure 4.46(a,b) shows the result obtained with $U$ and $SMD$ for the equidensity jet between $12 \geq J \geq 2$ as a function of $x/D$. While both indices dropped at further distances from the point of injection, indicating increased mixing, there were differences between the trends in $U$ and $SMD$. At $x/D < 10$, $U$ showed a non-monotonic increase with $J$, while $SMD$ showed clearly that jets of higher $J$ also had higher $SMD$ at all downstream locations. In Figure 4.46(c,d), the spatial coordinate was scaled as $x/\sqrt{JD}$, yielding a collapse of the $SMD$ data to a logarithmic trend, while $U$ at varying values of $J$ actually became more divergent. When the density ratio $S$ was varied at a constant value of $J$, as shown for the case of $J = 8$ in Figure 4.47, we found that lower density jets attained higher $U$ and $SMD$, while the attempted $x/\sqrt{JD}$ scaling did not by itself fully capture the alterations in mixing. These trends are consistent with the measurements of scalar centerline concentration decay in the $y = 0$ centerplane discussed in Section 4.2.1, which indicated that jets reached lower centerline concentrations (implying improved mixing) as either $J$ was decreased or as $S$ was increased. In Figure 4.48, all $(J, S)$ combinations are shown, revealing that a logarithmic scaling of $SMD \sim \ln(\sqrt{JD}/x)$ was obtained for each density ratio $S$, with lower density jets remaining slightly less mixed (higher SMD) at equivalent scaled downstream locations. Trends in $U$ appeared similar, although the attempted scaling with $\sqrt{JD}$ did not yield complete collapse of the data.
Figure 4.30: Mean jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (41, 1.00)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure 4.31: Mean jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (41, 0.70)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure 4.32: Mean jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (41, 0.55)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure 4.33: Mean jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (41, 0.35)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure 4.34: Mean jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (20, 1.00)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure 4.35: Mean jet fluid concentration fields in four \( yz \) cross-sections of the \( (J, S) = (20, 0.70) \) transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure 4.36: Mean jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (20, 0.55)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure 4.37: Mean jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (20, 0.35)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure 4.38: Mean jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (8, 1.00)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure 4.39: Mean jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (8, 0.70)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure 4.40: Mean jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (8, 0.55)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure 4.41: Mean jet fluid concentration fields in four $yz$ cross-sections of the $(J,S) = (8,0.35)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure 4.42: Mean jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (5, 1.00)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure 4.43: Mean jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (5, 0.70)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure 4.44: Mean jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (5, 0.55)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure 4.45: Mean jet fluid concentration fields in four $y z$ cross-sections of the $(J, S) = (5, 0.35)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure 4.46: Unmixedness ($U$, left) and Spatial Mixing Deficiency ($SMD$, right) as computed from the mean scalar fields at four $x/D$ positions in $S = 1.00$ transverse jets at several momentum flux ratios $J$. Scaling of the $x/D$ spatial coordinate by $\sqrt{J}$ is shown in (c,d).
Figure 4.47: Unmixedness ($U$, left) and Spatial Mixing Deficiency ($SMD$, right) as computed from the mean scalar fields at four $x/D$ positions in $J = 8$ transverse jets at several density ratios $S$. Scaling of the $x/D$ spatial coordinate by $\sqrt{J}$ is shown in (c,d).
Figure 4.48: Unmixedness ($U$, left) and Spatial Mixing Deficiency ($SMD$, right) as computed from the mean scalar fields at four $x/D$ positions at several $(J, S)$ conditions. Here, each symbol represents a different value of $J$: $\bigcirc$, $J = 12$; $\times$, $J = 10$; $\bigtriangledown$, $J = 8$; $\square$, $J = 5$; $\bigtriangleup$, $J = 2$. Scaling of the $x/D$ spatial coordinate by $\sqrt{J}$ is shown in (c,d).
4.2.3 Comparison Amongst Waveform Types for Acoustic Excitation of a Globally Unstable Transverse Jet

As an extension of the studies of Davitian et al. (2010b), who used smoke visualization to examine the impact of acoustic forcing on jet penetration and spread, PLIF measurements were made in a \((J,S) = (5,1.00)\) transverse jet under a range of forcing waveforms. Davitian et al. (2010b) found that square wave forcing at particular duty cycles and at low frequencies relative to the jet shear layer’s natural frequency yielded improved jet penetration and spread in globally unstable transverse jets, while sinusoidal forcing at equivalent amplitudes of forcing yielded little impact. The optimal duty cycles agreed well with the insights of Gharib et al. (1998) and Johari (2006) on the ideal forcing conditions to create minimal interaction between adjacent vortex rings. The differences in the impact of the forcing waveform is particularly large in globally unstable jets, where sinusoidal forcing may be required to be exceedingly large in order to have an impact on jet behavior, unless the forcing frequency is selected to be very close to the natural shear layer roll-up frequency. A comparison between sinusoidal forcing and square wave forcing was undertaken here, with the forcing frequency held constant at 1/20\(^{th}\) of the shear layer’s fundamental mode frequency (as measured by a hotwire prior to imaging). This frequency was chosen simply to be sufficiently low for purposes related to the creation of square waves within the frequency response of the actuation system, since forcing at the underlying forcing frequency as well as numerous higher harmonics (Fourier modes) is required, as outlined in Hendrickson (2012).

For sinusoidal forcing, the peak-to-peak amplitude was approximately 20\% of the mean jet velocity. A sawtooth waveform was also utilized, along with square waves with duty cycles (the ratio of the temporal pulse width to the pulsing period) ranging from 10\%-50\%. The root-mean square of the jet velocity perturbation was maintained at a constant level between all cases, equal to that of the sinusoidal case. In this manner, an equivalent forcing level was applied to the jet with each different waveform. Waveform shaping was performed prior to the PLIF measurements via a hotwire placed in the jet potential core and a microphone in the jet pipe. A closed-loop controller was designed to account for the
nonlinear response of the shaker/piston/nozzle actuation system, details of which can be found in Hendrickson (2012). A primary control loop was established using feedback from the microphone and subsequently adjusted to account for nonlinear harmonic coupling as measured by the hotwire. After the system converged to the periodic input required to yield the desired square waveforms, the hotwire loop was opened and the hotwire removed for closed-loop control during PLIF data acquisition.

Sample instantaneous $y = 0$ centerplane concentration field images are shown in Figure 4.49, for the $J = 5$ globally unstable unforced jet as well as different forced conditions. Particularly under square wave forcing at 20% and 30% duty cycles, deeply penetrating structures were created, resulting in an improved jet penetration. This impact on penetra-
Figure 4.50: Mean jet fluid concentration fields in the centerplane \((y = 0)\) of \(J = 5\) transverse jets, both unforced (a) and under acoustic excitation at \(f_f = f_0/20\) (b)-(i), with amplitude of forcing matched between each waveform at \(u'_{\text{rms}}/U_j = 0.2\).

The visible alterations in jet structure were relatively minor in the \(yz\) fields, except in two cases: 15\% duty cycle square wave forcing, which caused the jet to become asymmetric (higher concentrations at negative \(y\)), and 20\% duty cycle, where the deep penetration of individual vortex structures occurring at \(f_f\) resulted in a mean vertical stretching of the jet cross-section.

Using the same mixing metrics \(U\) and \(SMD\) presented and utilized for the unforced jet measurements under variation of \(J\) and \(S\), the impact of acoustic forcing on mixing is shown in Figure 4.53. The integration domain was the same for each forcing case and \(x/D\) location. All forcing conditions, including sinusoidal, yielded mixing enhancement
as defined by either metric. The resultant improvement may be viewed more easily in (c,d), where $U$ and $SMD$ are plotted as a percentage difference relative the unforced case at each $x/D$ position. In general, the mixing enhancement relative to the unforced case was reduced at further distances downstream. Indeed, the case of 20% duty cycle forcing yielded the greatest reduction in $U$ and $SMD$, and was therefore the most well mixed (although the quantitative reduction level differed between metrics). Somewhat surprisingly, sinusoidal forcing, which had little alteration in visible centerplane penetration and spread compared with the unforced jet, was not far behind this optimal case in terms of its impact on mixing. As found in our comparisons between unforced jets of varying $J$ and $S$, alterations in centerplane jet penetration and spread do not necessarily correlate with trends in mixing. This sample of results indicates that mixing enhancement may be optimized in the same manner as penetration and spread at certain duty cycles of square wave forcing, but that the improvement over sine wave forcing in globally unstable jets may not be as significant. A more complete examination of mixing that includes forced convectively unstable transverse jets could shed more light on the impact of different types of forcing waveforms.
Figure 4.51: Mean jet fluid concentration fields three $yz$ cross-sections of $J = 5$ transverse jets, both unforced (a)-(c) and under acoustic excitation at $f_f = f_0/20$ (d)-(o), with amplitude of forcing matched between each waveform at $u_{rms} / U_j = 0.2$. 
Figure 4.52: Mean jet fluid concentration fields three $yz$ cross-sections of $J = 5$ transverse jets under acoustic excitation at $f_f = f_0/20$, with amplitude of forcing matched between each waveform at $u'_{rms}/U_j = 0.2$. 
Figure 4.53: Unmixedness ($U$, left) and Spatial Mixing Deficiency ($SMD$, right) as computed from the mean scalar fields at three $x/D$ positions in the $J = 5$ transverse jet under acoustic excitation. In (c) and (d), the percentage reduction in $U$ and $SMD$ from the unforced case, indicating mixing enhancement, is shown for each different type of forcing waveform.
CHAPTER 5

Conclusions and Future Work

Consistent observation of the transitional nature of the upstream shear layer instabilities in the variable density transverse jet has been linked with the onset of global instability, in line with the work of experimentalists in several other transitional shear flows, including the circular cylinder wake (Provansal et al., 1987), the low density axisymmetric free jet (Monkewitz et al., 1990), and the countercurrent jet shear layer (Strykowski and Niccum, 1991). In an identical fashion to the findings of Megerian et al. (2007), the shear layer of the variable density transverse jet exhibited strengthened oscillations at higher $St$ that initiated closer to the jet exit than the preferred modes of free jets of corresponding density ratios and Reynolds number. For jets in the range $1.00 \geq S \geq 0.55$ and the realm of high momentum flux ratio ($J \geq 10$), the characteristics of the power spectra obtained in the shear layer indicated that the jet near-field was likely convectively unstable. The fundamental shear layer mode oscillations underwent jet edge tone-induced frequency shifting in the spatial domain, evidencing a variable response to localized forcing that would not occur in a self-excited shear flow. The instabilities were also rather weak, and after saturating in amplitude at a particular downstream location, transferred energy to a subharmonic mode via the mechanism of vortex pairing.

As $J$ was lowered beyond 10 and/or as $S$ is lowered below 0.45-0.40, the growth of a region of absolute instability in the flowfield became evident. Global oscillations with distinct peaks dominated the power spectra, and the Strouhal number of oscillation underwent a shift in trend, from slightly increasing with decreasing $J$ above the transition, to sharply decreasing with decreasing $J$ below the transition. At low enough momentum flux ratios, the vortex pairing process was completely inhibited in the near-field, up to $s/D = 5.0$. However, this
process of energy transfer from the fundamental mode to its subharmonic appeared to be enhanced at lower density ratios, a result which was not expected in consideration of the known transition to global instability in the low density free jet (which occurs below a critical jet-to-surroundings density ratio). This behavior, along with the apparent lack of Strouhal number scaling of the global mode as the jet density was lowered, may be indicative of a different type of global mode occurring in low density transverse jets than in higher density ones, similar to the mode competition between shear layer and ‘jet column’ type modes in variable density axisymmetric jets under external flow (Jendoubi and Strykowski, 1994).

The nonlinear stability theory of Landau was utilized in an attempt to estimate the critical value of the control parameter, which appears to be $J$, at which a global mode arises. This effort was successful in evincing the strong likelihood that a supercritical Hopf bifurcation to a global mode occurs in the jet shear layer. However, the effects of localized forcing were enough to skew the measurements such that a consistent estimation of $J_{cr}$ throughout the jet near-field was not attained. Consideration of the spatial evolution of the unforced shear layer spectra, as summarized above, is thought to be the best method for determination of the critical control parameter.

The response of the jet to low level single-tone acoustic forcing provided further evidence of a transition to global instability. At large $J$ and $S$, the shear layer acted as an effective amplifier of small perturbations, locking in to the applied forcing. Lowering $J$ and/or $S$ sufficiently resulted in the shear layer becoming resistant to the same (low) level of external forcing. However, the globally unstable transverse jet may be impacted by higher amplitude acoustic forcing in a manner that is linearly dependent on $|f_f - f_0|$, as would be expected for a flow having undergone a Hopf bifurcation to a global mode.

Stereo PIV measurements in the jet near-field provided a non-intrusive, quantitative confirmation of the shear layer instability character in the equidensity transverse jet under variation of $J$. Instantaneous velocity fields showed increased magnitude of out-of-plane motion in the jet wake at lower values of $J$, likely related to the strengthening of upright wake vortices.
Concentration field measurements acquired using acetone PLIF in the unforced transverse jet centerplane, as well as in several $yz$ planes, revealed a high degree of asymmetry in the mean about the $y = 0$ centerplane at larger values of $J$ and $S$. The asymmetry was significantly reduced as $J$ and $S$ were brought below their respective critical values for the transition to global instability, in general agreement with the symmetry trends discussed by Kuzo (1995). The apparent tendency towards asymmetry in the high momentum flux ratio jets may indicate the predominance of helical instability modes in these convectively unstable transverse jet flows.

Quantification of jet mixing established that particularly in the case of high cross-sectional asymmetry, trends in the jet centerline concentration and spread may not directly correlate with mixedness. Lowering the momentum flux ratio $J$ at a constant density ratio $S$ resulted in more rapid initiation of centerline concentration decay and lower jet spread in the far-field (as defined by a 5% concentration threshold). The spatial mixing deficiency $SMD$ was found to scale approximately as $SMD \sim \ln(\sqrt{JD/x})$. Lowering $S$ while maintaining constant $J$ produced slower concentration decay, higher far-field spread, and higher $SMD$ (indicating a reduction in mixing).

The characterization of the variable density jet shear layer in terms of its stability properties has strong implications for the effectiveness of varying methods of controlled excitation. Previous studies by Shapiro et al. (2006) and Davitian et al. (2010b) showed that a multi-pronged approach could be taken for control of global jet penetration and spread, dependent on whether the shear layer was globally unstable. In the convectively unstable case, control was achieved by low or moderate sinusoidal forcing, with little improvement under variation of the forcing waveform. In the absolutely or globally unstable case, sinusoidal forcing at very large amplitudes (dependent upon the forcing frequency) was required to impact jet penetration and spread, with significant benefit seen by forcing with square waveforms of particular frequencies and duty cycles. In this regime, the generation of discrete pulses of vorticity was critical to overtaking the self-excited oscillations. In the current work, PLIF measurements in a $(J,S) = (5, 1.00)$, acoustically forced and globally unstable transverse jet found that while an optimal duty cycle of square wave forcing produced the highest degree
of mixing, its improvement over sinusoidal forcing at the same level of excitation (quantified by $u'_{rms}$) was not as significant as the improvements in penetration and spread found in the aforementioned studies. At further distances downstream of injection, the mixing enhancement relative to the unforced jet was reduced.

While this work has established several new features of the variable density transverse jet and its forced response, there are many useful new directions in which these studies may be continued. These areas include:

- Analysis of the variation of acoustic forcing waveform to a wider range of flow conditions, including the convectively unstable regime. Such a study will have important implications for the development of strategies for control of jet penetration, spread, and/or mixing.

- The use of proper orthogonal decomposition on PIV data in the jet near field, both in the centerplane and in $yz$ cross-sections. Extracting the shapes and magnitudes of the most energetic modes may provide insight as to the interactions between vortex systems in the jet shear layer and wake under variation of $J$ and $S$.

- Exploration of the alterations in jet structural asymmetry under $(J, S)$ variation, also including the $Re_j$ and $Re_\infty$ effects. An understanding of the predominance of different types of instability modes (i.e. helical or axisymmetric dominance) may be established.

- Variation of the injector type, including pipe flow injection as well as injectors elevated from the crossflow wall. The work of Hallberg and Strykowski (2006) in low density free jets of varying injector pipe length and Megerian et al. (2007) in transverse jets of different injection distance from the wall showed that the onset of global instability may be strongly influenced by both factors.

- Introduction of chemical reaction. As discussed by Juniper et al. (2009), the coupling of heat release with acoustic forcing may be significantly reduced in a globally unstable diffusion flame, thereby impacting the onset of high-amplitude thermoacoustic oscillations that have been so problematic in industrial combustors.
APPENDIX A

Additional Centerplane Scalar Field Data

Figure A.1: Sample instantaneous jet fluid concentration fields in the centerplane \((y = 0)\) of \(J = 12\) transverse jets at four different values of the jet-to-crossflow density ratio, \(S\), with \(Re_j\) fixed at 1800.
Figure A.2: Mean jet fluid concentration fields in the centerplane ($y = 0$) of $J = 12$ transverse jets at four different values of the jet-to-crossflow density ratio, $S$, with $Re_j$ fixed at 1800.
Figure A.3: Sample instantaneous jet fluid concentration fields in the centerplane ($y = 0$) of $J = 10$ transverse jets at four different values of the jet-to-crossflow density ratio, $S$, with $Re_j$ fixed at 1800.
Figure A.4: Mean jet fluid concentration fields in the centerplane ($y = 0$) of $J = 10$ transverse jets at four different values of the jet-to-crossflow density ratio, $S$, with $Re_j$ fixed at 1800.
Figure A.5: Sample instantaneous jet fluid concentration fields in the centerplane \((y = 0)\) of \(J = 2\) transverse jets at three different values of the jet-to-crossflow density ratio, \(S\), with \(Re_j\) fixed at 1800.
Figure A.6: Mean jet fluid concentration fields in the centerplane ($y = 0$) of $J = 2$ transverse jets at three different values of the jet-to-crossflow density ratio, $S$, with $Re_j$ fixed at 1800.
Figure A.7: Transverse jet scalar distribution metrics at $J = 12$, $S = 1.00$, 0.70, 0.55 and 0.35. (a) Scalar penetration, 5% concentration threshold (b) Jet spread in the trajectory-normal direction along the scalar centerline, 5% concentration threshold. (c) Maximum concentration decay along the scalar centerline.
Figure A.8: Transverse jet scalar distribution metrics at $J = 10$, $S = 1.00$, 0.70, 0.55 and 0.35. (a) Scalar penetration, 5% concentration threshold (b) Jet spread in the trajectory-normal direction along the scalar centerline, 5% concentration threshold. (c) Maximum concentration decay along the scalar centerline.
APPENDIX B

Additional Mean $yz$ Plane Scalar Fields
Figure B.1: Mean jet fluid concentration fields in four $yz$ cross-sections of the $(J,S) = (12, 1.00)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure B.2: Mean jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (12, 0.70)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure B.3: Mean jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (12, 0.55)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure B.4: Mean jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (12, 0.35)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure B.5: Mean jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (10, 1.00)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure B.6: Mean jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (10, 0.70)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure B.7: Mean jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (10, 0.55)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure B.8: Mean jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (10, 0.35)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure B.9: Mean jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (2, 1.00)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure B.10: Mean jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (2, 0.70)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure B.11: Mean jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (2, 0.55)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
APPENDIX C

Sample Instantaneous $yz$ Plane Scalar Fields

Figure C.1: Sample instantaneous jet fluid concentration fields in four $yz$ cross-sections of the $(J,S) = (41, 1.00)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure C.2: Sample instantaneous jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (41, 0.70)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure C.3: Sample instantaneous jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (41, 0.55)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure C.4: Sample instantaneous jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (41, 0.35)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure C.5: Sample instantaneous jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (20, 1.00)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure C.6: Sample instantaneous jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (20, 0.70)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure C.7: Sample instantaneous jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (20, 0.55)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure C.8: Sample instantaneous jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (20, 0.35)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure C.9: Sample instantaneous jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (12, 1.00)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure C.10: Sample instantaneous jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (12, 0.70)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure C.11: Sample instantaneous jet fluid concentration fields in four $yz$ cross-sections of the $(J,S) = (12,0.55)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure C.12: Sample instantaneous jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (12, 0.35)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure C.13: Sample instantaneous jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (10, 1.00)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure C.14: Sample instantaneous jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (10, 0.70)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure C.15: Sample instantaneous jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (10, 0.55)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure C.16: Sample instantaneous jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (10, 0.35)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure C.17: Sample instantaneous jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (8, 1.00)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure C.18: Sample instantaneous jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (8, 0.70)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure C.19: Sample instantaneous jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (8, 0.55)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure C.20: Sample instantaneous jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (8, 0.35)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure C.21: Sample instantaneous jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (5, 1.00)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure C.22: Sample instantaneous jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (5, 0.70)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure C.23: Sample instantaneous jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (5, 0.55)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure C.24: Sample instantaneous jet fluid concentration fields in four $yz$ cross-sections of the $(J,S) = (5, 0.35)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure C.25: Sample instantaneous jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (2, 1.00)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure C.26: Sample instantaneous jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (2, 0.70)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Figure C.27: Sample instantaneous jet fluid concentration fields in four $yz$ cross-sections of the $(J, S) = (2, 0.55)$ transverse jet. Plotted using a linear colormap, where a lack of indicated concentration values is due to poor quantitative agreement between the centerplane and cross-sectional data.
Bibliography


