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Abstract
Increasing the true progressivity of income taxes can result in greater, not
less, inequality in the distribution of after-tax income.

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I. Introduction

It is commonly argued that progressive tax schedules do not actually promote equality because loopholes and exemptions make actual tax rates no higher for the rich than for the poor. We suggest here that a truly progressive tax system, one without any loopholes or exemptions, can still be ineffective in reducing the inequality of resources available for after-tax consumption. Furthermore, increase in true progressivity can lead to greater after-tax inequality rather than less. We argue that this result is quite possible under circumstances similar to those currently prevailing.

In order to demonstrate the basic idea we employ two different analytic apparatus. The first, in Section II, is a static neo-classical model without explicit income from capital. The second (Section III), is a dynamic Pasinetti-type, Marxian two-class model, with worker and capitalist classes.

In the neoclassical model, the perverse result that increasing progressivity can actually lead to greater inequity arises from a difference across individuals in labor productivity and from the existence of a labor-leisure tradeoff. In this model, the before-tax wage structure depends on the true progressivity of taxes. Indeed, an increase in tax progressivity can produce a more than proportionate increase in relative wages. An increase in tax progressivity can make everyone worse off since aggregate product may decline, but the higher quality labor can be left relatively less worse off even if tax revenues are redistributed per capita. As progressivity increases from zero, the usual effect will be a decrease in after-tax inequality at first, but, eventually, inequality will always begin to increase again when progressivity reaches a sufficiently high level. This happens because skilled workers substitute leisure as the after-tax benefits of their labor decline while firms
are unable to find adequate capital substitutes and therefore bid up wages at a greater rate than the increase in taxation.

In the Marxian dynamic model, the capitalist class receives no labor income, but its members respond to taxation by altering their rates of savings and thereby affect the long-run steady state output. The labor class can also own capital. Provided that the capitalist class does not disappear completely in the steady state, we find that an increase in tax progressivity will inevitably harm the labor class. Whether after-tax equality is increased or decreased depends upon the tax rate. Generally, there is a level of progressivity above which further increases harm laborers more than capitalists. These conclusions are mostly attributable to the (steady-state) result that capital formation occurs until the before-tax return compensates completely for taxes - so the after-tax return is unaffected except by the general equilibrium effect on output.

We are, of course, not the first to suggest the possibility that progressive taxation can have a perverse impact on the income distribution. Feldstein (1973) developed a model similar to the one we use in Sections I and II, but his focus was on the optimal tax under a linear tax schedule and he restricted marginal tax rates to be the same at all levels of income, (whereas we are interested specifically in the impact of differential marginal rates). More recently, Allen (1982) and Stern (1982) assumed linear tax schedules with lump-sum transfers. The dynamic case of Section III was considered by K. Hamada (1967). Our model differs from his in its focus on a tax rate rather than a transfer, and on the relative income distribution as opposed to his concern with finding an optimal transfer pattern. But, it should be noted that he was fully aware of the central phenomenon at work; a shift in income from high-saving capitalists to low-saving workers brings about a lowered equilibrium ratio of capital to labor which makes workers worse off.
Section II below discusses the neoclassical model. Section III develops a dynamic analysis and Section IV considers some generalizations and summarizes the main results.

II. A Neoclassical Model with Two Levels of Labor Quality

A. General Analytics

Imagine an economy producing a single good by employing labor and capital. Assume that there are just two qualities of labor, skilled and unskilled. A more developed model might permit multiple labor qualities and a mechanism whereby individuals endowed with one quality of labor could invest in human capital and change their level of skill. We ignore such refinements for now in order to maintain the simplest possible illustration but we will conjecture about the likely consequences of such extensions in the summary.

The quantities (hours worked per unit of time) of two grades of labor are denoted a and b for skilled and unskilled, respectively. Output is obtained via a neoclassical production function, f(·),

\[ q = f(a, b) = bf(k); \quad k = a/b \quad (1) \]

where \( q \) is the quantity produced of the consumption good. Labor supply is neoclassical, too; we assume that both labor qualities are supplied as functions of their after-tax marginal wage rates and their government subsidies.

For simplicity, the tax structure is piecewise linear and progressive with a tax rate \( \tau \) on income below a fixed level \( y^* \) and a higher tax rate \( \tilde{\tau} > \tau \) on marginal income above \( y^* \).
Under these conditions, the labor supply functions will be given by

\[ a = A[(1 - \bar{\tau})w_a, (\bar{\tau} - \bar{\tau})y^* + \alpha G] \cdot n_a \quad (2) \]

\[ b = B[(1 - \bar{\tau})w_b, (1 - \alpha)G] \cdot n_b \quad (3) \]

where \( w_a \) and \( w_b \) are the before-tax wage rates for labor grades \( a \) and \( b \), \( G \) is the total government subsidy distributed to all individuals, \( \alpha \) is the fraction of the subsidy going to skilled laborers, and \( n_a \) and \( n_b \) are the numbers of individuals in each group. In these formulas, the first term is the marginal after-tax wage and the second is the effective intercept, i.e., it is the fixed component of income. The tax cutoff level, \( y^* \), is assumed to be established above the nominal wage income level of the unskilled and below that of the skilled; i.e., \( aw_a/n_a > y^* > bw_b/n_b \).

The after-tax aggregate disposable incomes (consumptions) for members of both labor groups are given by

\[ aw_a - \bar{\tau}(aw_a - n_a y^*) - \bar{\tau}n_a y^* + \alpha G \quad \text{(skilled)} \]

\[ bw_b - \bar{\tau}bw_b + (1 - \alpha)G \quad \text{(unskilled)}. \]

These after-tax formulae show that the skilled group has a more complex situation. The skilled individual's choice problem is illustrated in Figure 1, where the indifference tradeoff of leisure and after-tax consumption is shown for a given nominal wage, \( w_a \). The figure shows also why the term \( (\bar{\tau} - \bar{\tau})y^* \) belongs in the skilled labor supply function. It is simply part of the intercept term for the linear relation between work effort and after-tax consumption.
Figure 1. Work Effort and After Tax Income, Skilled Labor
(Per Capita, n_s=1)

- s = Skilled labor hours worked
- w_s = Skilled wage rate
- y* = Tax bracket break between upper (r) and lower (t) rates
- G = Government revenue
- α = Proportion of government revenue distributed to skilled
The amount collected and redistributed by the government is given by

\[ G = \tilde{e}(aw_a - n_a y^*) + \tilde{e}(bw_b + n_a y^*). \]  

(4)

Finally, firms employ both labor qualities in production (according to \( f(.,.) \)), and since labor is competitively supplied, nominal wages are equated to marginal products,

\[ w_a = f'(k) \]  

(5)

\[ w_b = f(k) - kf'(k) \]  

(6)

where \( k = a/b \). It is easy to see that (6) implies, \( w_b = q/b - w_a a/b \), and thus \( q = w_a a + w_b b \); i.e., output is total before-tax and before-subsidy income.

The system consisting of two labor supply functions, (2) and (3), a government sector, (4), and market-determined wages, (5) and (6) has a total of five endogenous variables, \( a, b, w_a, w_b \) and \( G \). Under non-pathological conditions these are uniquely determined by the system's parameters, \( \tilde{e}, \tilde{e}, y^*, \alpha, n_a, n_b \) and by its exogenous behavioral functions, \( f, A, \) and \( B \). In order to investigate the question of inequality in after-tax incomes, we must solve the system for a given set of parameters and for different levels of tax progressivity.

The simplicity of this model makes the definition of progressivity obvious: it is \( \tilde{e} - \tilde{e} \), the difference between high and low tax rates for a given cutoff income, \( y^* \). For a measure of inequality we will use the simple difference in after-tax consumption between the skilled and unskilled groups,
\[ D = aw_a(1 - \tau) - bw_b(1 - \tau) \]

\[ + (2\alpha - 1) G + n_a y^*(\tau - \bar{\tau}). \]  

(7)

This measures the difference in material well-being, (i.e., not including leisure), of the two labor groups.

Notice that if wages, hours worked, government subsidies, and the lower tax rate were all held constant, then \( \partial D/\partial \tau = n_a y^* - aw_a \), which is unambiguously negative, (since by assumption the nominal wage income of a skilled laborer \( aw_a/n_a \) exceeds the tax cutoff level, \( y^* \)). Thus, the first-order effect of increasing progressivity is to reduce inequality, in concordance with the usual and intuitive reaction of most people when asked about this problem.

In equilibrium, though, the supply of labor is a function of the tax rates. Notice from the skilled labor supply function, (2), that an increase in progressivity can reduce the total hours worked by skilled labor. This tends to increase the nominal wage, provided the marginal product of skilled labor is decreasing, (5); and this effect is transmitted throughout the system. Unskilled labor also experiences a change in its nominal wage (via (6)) which, of course, alters the hours worked by the unskilled. Government revenue responds to the new nominal incomes of both labor qualities and this feeds back via government subsidies to cause still further changes in wages and hours worked. The final effect on income inequality is far from apparent. It depends upon the system's parameters and behavioral functions. In the next section, we illustrate that the effect of progressivity on inequality can be either positive or negative.
II. B. Solutions for the Response of After-Tax Inequality to Tax Progressivity.

The general solution procedure involves differentiating the system of equations, (2) (3), (4) and (5) and (6) with respect to $\tilde{\tau}$ and using (7) to determine the response of the inequality measure D. Sensible qualitative restrictions on the sign patterns produce an indeterminate result in this exercise, so we have chosen instead to examine a specific parameter model.

The response of after-tax income inequality to tax progressivity was studied with the following specification:

\begin{align*}
\text{number of individuals} & \quad n_a = n_b = 1 \\
\text{unskilled} & \quad b = 1 \\
\text{Cobb-Douglas} & \quad q = a^\gamma, \quad 1 > \gamma > 0 \\
\text{Production} & \\
\text{skilled} & \quad a = \frac{[(1-\tilde{\tau})w_a]^\delta}{[(\tilde{\tau}-1)y^\alpha + \alpha G]^\beta}, \quad 1 > \delta > 0 \\
\text{labor supply} & \\
\end{align*}

By (9), we set equal the number of individuals in the two labor qualities. By (10), we make a further simplification that unskilled labor is perfectly inelastic; the same hours are worked regardless of wage rates. By virtue of the Cobb-Douglas production function, (11), the two equilibrium wage rates are respectively,

\begin{align*}
w_a &= \gamma a^{\gamma-1} \\
w_b &= (1 - \gamma)a^\gamma
\end{align*}

for skilled and unskilled labor, respectively.
Finally, the skilled labor supply function (12) specifies a positive response to after-tax marginal wages, with elasticity of response $\delta$, and a response to non-marginal income $(\bar{\tau} - \tau)y^{*} + \alpha G$ ruled by a second elasticity parameter, $\beta$, which we will take as positive in the numerical illustrations.

Even with these added assumptions, the system is rather complex and we found it necessary to proceed sequentially and, at the final step, to enlist the aid of a computer. We begin by substituting the equilibrium wages from (13) into the government resource function, (14), the labor supply function (12), and the inequality measure, $D$, in (7). Differentiating the resulting system, we obtain

$$
\begin{bmatrix}
1 & -D_a & -D_G & dD & D_{\tau} \\
0 & -G_a & 1 & da & G_{\tau} d\bar{\tau} \\
0 & (1 - A_a) & -A_G & dG & A_{\tau}
\end{bmatrix}
$$

(14)

The resulting partials are given in the appendix.

The determinant of the $3 \times 3$ matrix $\Gamma$ on the left on (14) is

$$
\Gamma = G_a A_G - (1 - A_a) < 0 ,
$$

(15)

(Since $G_a > 0$, $A_G < 0$ and $(1 - A_a) > 0$, $\Gamma < 0$; see the appendix), hence the solution is given by

$$
dD/d\bar{\tau} = D_{\tau} - \left[ G_{\tau} A_G + D_G (1 - A_a) \right] + A_{\tau} [G_a D_G + D_a] / \Gamma
$$

(16)

$$
da/d\bar{\tau} = -\left[ A_G G_{\tau} + A_{\tau} \right] / \Gamma
$$

(17)

$$
dG/d\bar{\tau} = -\left[ G_{\tau} (1 - A_a) + G_a A_{\tau} \right] / \Gamma .
$$

(18)
Only the sign of (17) is uniquely determined for all parameter values. The appendix shows that \( A_{G \tau} + A_{\tau} < 0 \) and since \( \tau < 0 \) also, \( da/d\tau < 0 \); i.e., increasing tax progressivity results in fewer hours worked by the now more heavily-taxed skilled individuals.

Neither after-tax income inequality (16) nor government revenue (18) have unambiguous signs. Concerning the latter, however, government collections are maximized at a positive tax rate. The components of (18) are signed but \( G_{\tau}(1 - A_{\tau}) > 0 \) while \( G_{a} A_{\tau} < 0 \), so the result is indeterminate; but for zero tax rates, \( G_{a} = 0 \), and so the first imposition of a small tax will indeed increase government revenue. However, at confiscatory levels of taxation, as the upper tax approaches 100\%, \( A_{\tau} \) approaches \(-\infty\) and certainly for marginal taxes somewhat below 100\%, increases in progressivity lower revenue, \( dG/d\tau < 0 \).

For different parameter values, the solutions are presented numerically in a series of tables below. Table 1A, for instance, shows the response of after-tax inequality \( dD/d\tau \) for various levels of the upper and lower tax rates and for \( \gamma = .65, \delta = .70, \beta = .70, y^{*} = .60, \) and \( \alpha = .10 \). Companion Tables 1B, 1C, 1D, give, respectively, total output, government revenue and relative wages of unskilled labor. Table 1E gives \( dG/d\tau \), the response of government revenue to changes in the higher tax rate.

The results are constrained by two infeasible regions which are indicated by blank spaces. For example, given the parameters of Table 1, it is not possible to have \( \bar{\tau} \) less than 6 percent and \( \bar{\tau} \) greater than 60 percent. This upper left-hand infeasible region is caused by the tax bracket cutoff \( y^{*} \) being constrained to lie below the pre-tax income of the skilled. From (13), their pre-tax income is \( ya^Y \) or \( yq \), where \( q \) is total output. Thus for \( y^{*} = .60 \) and \( \gamma = .65 \), \( q \) must exceed 60/65 = .923 and which it does not in the top left of the table; (cf. Table 1B).
The lower right infeasible region occurs because the skilled wage rate must exceed the unskilled wage rate. As Table 10 shows, the two wage rates are very close for combinations of high \( \tau \) and low \( \bar{\tau} \).

These tables are for illustration and we have experimented to determine the effects of different parameter values on these results. Experimentation has shown that \( y^* \) has the more substantial effect on \( dQ/d\bar{\tau} \). For sufficiently high levels of \( y^* \), the entire feasible region displays the perverse effect, \( dQ/d\bar{\tau} > 0 \). In other words, tax progressivity increases inequality even for the lowest degrees of progressivity. Furthermore, increasing the subsidy mitigates the perverse effect of tax progressivity on inequality. Evidently, as the skilled subsidy declines, hours worked by the skilled (and total output) declines as well. This effect, however, may be reversed for other values of the labor supply elasticities. The response of government revenue to progressivity, \( dQ/d\bar{\tau} \), is uniformly negative in the feasible region. Finally, experimentation has indicated that the maximum government revenue usually occurs for very low levels of taxation (at least for the parameter values in the ranges considered here).

III. Progressive Taxation in a Dynamic Capital/Labor Framework

In this section, we examine the impact of progressive income taxation in a dynamic setting. Our primary interest is in the long run dynamic impact of taxation, and to obtain definitive results we will simplify the model as much as possible. The basic framework is the Pasinetti-type Marxian two-class model. In such models a capitalist class is identified as a group whose sole source of income is from its holdings of capital. The worker class may also hold capital in our model, but it also derives income from labor services. A distinguishing feature of these models is that the long run equilibrium rate
of profit is determined solely by the savings propensity of the capitalist class.

Subscript $c$ denotes the capitalist class and $L$ the workers. The long run savings propensities are $S_c$ and $S_L$ respectively. Production is by an ordinary neoclassical (per capita) production function $f(.)$. Labor is inelastically supplied at $L$, and total capital,

$$K = K_c + K_L,$$

the sum of the capitalist and labor holding. Let $\delta$ denote the rate of capital depreciation plus the rate of growth of the labor force; then, without taxes, the dynamic growth equations are

$$\dot{K}_c = S_c Y_c - \delta K_c,$$
$$\dot{K}_L = S_L Y_L - \delta K_L,$$

where

$$k_L = \frac{K_L}{L},$$
$$k_c = \frac{K_c}{L},$$

$$k = k_L + k_c,$$

$Y_c$ = per capital income of capitalists,

and

$Y_L$ = per capita income of workers.

We will ignore depreciation offsets and assume, for simplicity alone, that capitalists pay taxes at the rate $\tau$ and that total taxes are returned as a subsidy to workers. After tax income for capitalists and workers is therefore
\[ Y_c = (1 - \tau)k_c f'(k), \]

and

\[ Y_L = f(k) - \tau k f'(k) + f'(k)k_L + \tau k f'(k). \]

Labor income subsidy

In a long run stationary state, the capitalists' accumulation equation is given by

\[ \dot{k}_c = \{S_c (1 - \tau) f'(k^*) - \delta\} k_c^* = 0. \]

If the capitalist class does not become insignificant, then

\[ f'(k^*) = \frac{\delta}{S_c (1 - \tau)} \]

(Another equilibrium with \( k_c^* = 0 \) is also possible, see Samuelson and Modigliani [1966], but we will ignore this regime.)

Notice that a rise in taxes, \( \tau \), will raise the equilibrium marginal product and lower the equilibrium capital stock and per capital output. This, in turn, lowers the wage to workers. In the extreme Marxian-Malthusian model, workers own no capital stock and only consume, \( S_L = 0 \). It is easy to see that in this case, despite the subsidy, workers are unequivocally harmed by the tax increase. A worker's income is now

\[ Y_L = f(k^*) - k^* f'(k^*) + \tau k^* f'(k^*) \]

\[ = f(k^*) - (1 - \tau) k^* f'(k^*), \]

and

\[ \frac{dY_L}{d\tau} = [ f'(k^*) - (1 - \tau) f'(k^*)] \frac{dk^*}{d\tau} \]

\[ = \tau f'(k^*) \frac{dk^*}{d\tau} < 0, \]
unambiguously, since
\[(1-\tau)f'(k^*) = \delta S_c \]
and does not change with \(\tau\).

Despite the fact that worker income falls, it is possible that the gap between the capitalists and the workers actually narrows. The income gap
\[Y_c - Y_L = 2(1-\tau)k^*f' - f\]
and
\[\frac{d}{d\tau} (Y_c - Y_L) = [2(1-\tau)f' - f'] \frac{dk^*}{d\tau}\]
\[= (1-2\tau)f' \frac{dk^*}{d\tau}\]
\[\geq 0 \text{ as } \tau \geq 1/2.\]

In other words, at tax rates above 50%, not only do workers suffer absolutely, they also lose relative (in an absolute sense) to capitalists as taxes are further increased. Figures 2 and 3 illustrate the two possibilities. Since at \(\tau = 1\), \(Y_L = Y_C = 0\), at \(\tau = 1/2\) we must have the total income of the working class, \(Y_L > Y_C\), capitalist's income. If, as in Figure 2, at \(\tau = 0\), \(Y_L > Y_C\), then the gap widens as \(\tau\) increases to 50%. But, if at \(\tau = 0\), \(Y_L < Y_C\), then, as illustrated in Figure 3, the gap narrows as \(\tau\) rises and at \(\tau = 1/2\) it has been reversed.

With workers saving as well as consuming they can also gain from the increase in rents, and the welfare effect, while no longer obvious, is, in fact, still ambiguous. Now, \(k_L^*\) is determined by
\[\dot{k}_L = S_L \{f(k^*) - f'(k^*) [k^* - k_L^* - \tau k_c^*]\} - \delta k_L^* = 0.\]
Laborers and Capitalists' Income

$Y_L$

$Y_C$

---

0 1/2 1 $\tau$ (tax rate)

Figure 2. After-Tax Incomes versus tax rate on capitalists, higher labor income.

---

$Y_L$

$Y_C$

---

0 1/2 1 $\tau$

Figure 3. After Tax Incomes versus tax rate on capitalists, reversal case.
Figure 2. After-Tax Incomes versus Tax Rate on Capitalists, Higher Income Labor Case

Figure 3. After-Tax Incomes versus Tax Rate on Capitalists, Reversal Case
Differentiating with respect to \( \tau \) yields the steady state change in worker's per capita income (and, therefore, welfare in this one good world),

\[
\frac{dY_L}{d\tau} = \left[ -\frac{S_C \tau}{S_C - S_L} \right] f'(k^*) \frac{dk^*}{d\tau} < 0,
\]

if the savings rate of the capitalist class is in excess of worker's propensity to save.

In the alternative case where \( S_L \geq S_c \), there is, in fact, a different regime where the steady state solution, \( k_L^* = k^* \) and \( k_c^* = 0 \) prevails, i.e., capitalists become asymptotically negligible relative to workers. This is easy to see, since,

\[
\dot{k}_L = S_L Y_L - \delta k_L
= S_L \{f(k) - (1-\tau)f'(k)k + (1-\tau)f'(k)k_L\} - \delta k_L
= S_L \{f(k) - (1-\tau)f'(k)k + \frac{\delta}{S_c} k_L\} - \delta k_L
\geq S_L \{f(k) - (1-\tau)f'(k)k\}
> S_L \tau f'(k^*) k^*
> 0
\]

if \( S_L \geq S_c \) and \( k_c^* > 0 \). As before, we will assume that this regime is not the relevant one.

We can also use the steady state equation to examine the relationship between taxation and government revenues. On a per-capita basis

\[
G(\tau) = \text{government revenues} = \tau k_c^* f'(k^*)
= \frac{\delta \tau}{S_c (1-\tau)} k_c^*.
\]
Clearly
\[ G(0) = 0, \]
and as \( \tau \to 1 \), if this is feasible, then
\[ k_c f'(k) < kf'(k) < f(k) \to 0, \]
hence
\[ G(1) = 0. \]

While \( G(\tau) \) is obviously positive between \( \tau = 0 \) and the maximum feasible tax rate, the exact shape of \( G(\tau) \) depends upon the particular production function. We can, however, determine the maximum feasible tax rate. From the steady-state equation we find
\[ k^*_c = \left[ \frac{S_L - (1-\tau)S_c \alpha}{(S_L - S_c) \alpha (1-\tau)} \right] k^*, \]
where
\[ \alpha = \frac{k^* f'(k^*)}{f(k^*)} = \text{capital's share of output}. \]

For \( k_c > 0 \), we require that
\[ \alpha (1-\tau) S_c > S_L, \]
which further restricts the tax rate so that
\[ \tau \leq 1 - \frac{S_L}{\alpha S_c}, \]
and as \( \tau \to 1 - \frac{S_L}{\alpha S_c}, \) \( G(\tau) \to 0 \)

The simplest case to examine in detail is the one where workers only consume, and
\[ G(\tau) = \tau k^* f'(k^*). \]

Differentiating, we obtain
\[ G'(\tau) = \frac{f'}{(1-\tau) f''} [kf'' + \tau f'] \]
\[ \tau > 0 \text{ as } \tau > \tau^*, \]

where

\[ \tau^* = - \frac{k_f''}{f'} = 1-\alpha = \text{labor's share of output}, \]

for a Cobb-Douglas technology. In other words, in this case, government revenue is maximized at a tax rate exactly equal to labor's share of total output and rises monotonically as \( \tau \) approaches \( 1-\alpha \) and falls as \( \tau \) rises further.

These results might at first seem suspicious; but the intuition is that progressive taxation, by reducing capitalists' incomes, reduces their savings and lowers the equilibrium capital stock. This lowers the wage income of workers, and, even if workers also hold capital, their increased return and their tax subsidy will not be sufficient to raise their real income. In addition, the gap between capitalist and worker incomes can also increase with tax increases.

A somewhat richer two-class model would sacrifice simplicity by having capitalists also work. In a model with capitalists also working, the possibility of taxing labor and capital income differentially arises, and by offering lower taxes on capital it might be possible to offset the reduced incentive to invest in the presence of a progressive tax structure. This would be an interesting route to pursue.

In sum, then, we have shown in the context of a two-class model, that progressive taxation will worsen the lot of the workers (and of the capitalists). These are stationary state comparisons, however, and we have not examined the transition to the steady state. The immediate impact of an (unanticipated) increase in taxes, as expected, raises the worker's income by the subsidy at the original capital stock and lowers capitalist's income. A full analysis would explore whether this transient effect is sufficient to
counter the steady state result in a world where future utility is discounted and is related to the choice of a savings rate.

IV. Summary and Conclusion

In the above models, under a wide class of circumstances increasing tax progressivity causes greater after-tax inequality of consumption. How robust is this result? The following extensions serve as a basis for summarizing the models' contents while at the same time offering speculations on their generalizations - beyond the obvious ones of adding more labor types and a more complex tax schedule.

A. The Functional Forms

The numerical results presented in section II depend on Cobb-Douglas type production and labor supply functions and an inelastic supply of unskilled labor. An important improvement would be a model with more general functional forms. In particular, it is instructive to consider the effect of an elastic unskilled labor supply and to allow for the marginal product of labor to be bounded from below as the number of hours worked increases.

B. Human Capital Investments and Dynamics

Given the obvious fact that multiple skills are extant in every economy and that the distribution of skilled hours supplied can be altered over time, might we anticipate that dynamic investment in human capital would eventually offset our simple progressivity effect? Perhaps the easiest way to answer this question is by comparing human capital investment under zero progressivity
with the same activity under a progressive tax structure. In the zero progressivity case, one should observe low-skilled laborers investing in education or training until the risk-adjusted present value of the expected lifetime after-tax cash flow stream from the investment just equals the current cost of training. Those individuals who undertake the investment and move to a higher skill level will enjoy higher wages and, to the extent that a greater number of skilled workers results in a reduction in the skilled wage rate relative to the unskilled rate, further equality will be produced indirectly.

This process would be impeded by the imposition of a higher marginal tax rate on skilled wages. A higher tax reduces the benefits of investing in training. Since the expected lifetime after-tax consumption stream is reduced, fewer unskilled workers will find a human capital investment worthwhile and thus greater inequality will remain in every period. Claude Montmarquette (1974) has investigated the impact of human capital in a model that includes some of these effects.

C. Ownership of Taxed Physical Capital in the Short-Run

Our simple static model (Section II) assumes that all material consumption results from work effort during the same period. No savings nor consumption from previous stored-up labor is allowed. Most readers will probably regard this as the most likely candidate for insuring the absence of a perverse result in the short-run. Obviously, if the capitalist has no choice but to employ his resources to produce current income and if capitalists are in high tax brackets, total after-tax income equality might be more likely with a sudden imposition of greater tax progressivity. While the skilled laborer can consume leisure if he is too heavily taxed, no such obvious alternative exists for the capitalist. 2
In the long run, to the contrary, as we showed in section III, the existence of capital may be consistent with even greater inequality as a result of tax progressivity. The reasoning can proceed by analogy to investment in human capital (see above). Any investment is a means by which labor can be stored to produce higher incomes in later periods. Thus, physical capital, as embodied labor, is a means by which the poor can eventually augment their direct work effort. In addition, physical capital is superior to human capital when the latter is not marketable. Thus, physical capital can serve as a buffer against unexpected fluctuations in labor income, thereby decreasing the variability of income inequality as well as its average level.

If a progressive tax impinges more heavily on the income from capital, less new capital will be formed. In fact, in an efficient capital market, the expected after-tax return from new capital includes an adjustment for present and for all anticipated future taxes. An anticipated increase in the tax rate will simply cause an equal and offsetting increase in expected pre-tax returns so that the after-tax return will be unchanged. This implies a reduction in new capital formation but it does not necessarily imply a reduction in the value of old capital that cannot be withdrawn from market production. Since the pre-tax rental rates will be increased in all subsequent periods as less new capital is formed to compete with that already in place, there may be no long-run change in total after-tax inequality. Inequality could even be increased, just as with labor considered in isolation.

D. Conclusion

In summary, after-tax labor income can become less equal in response to increased tax progressivity because pre-tax wage progressivity is affected by
the tax structure. Similarly, increased progressivity of taxes on capital may leave the inequality of after-tax capital income unchanged or exacerbated depending on the response of pre-tax expected returns to the tax structure.
Appendix

Partial Derivatives of the Cobb-Douglas System

The system specified by equations (9) - (13) of the text result in the following formulae for after-tax inequality, $D$, government revenue, $G$, and skilled labor supply, $a$:

$$D = a^\gamma [\gamma (1 - \bar{\tau}) - (1 - \gamma)(1 - \bar{\tau})] + (2\alpha - 1) G + y^*(\bar{\tau} - \bar{\tau}) \quad (A-1)$$

$$G = \bar{\tau}(\gamma a^\gamma - y^*) + \bar{\tau}[(1 - \gamma)a^\gamma + y^*] \quad (A-2)$$

$$a = [(1 - \bar{\tau})\gamma a^{\gamma-1}]^\delta [(\bar{\tau} - \bar{\tau})y^* + \alpha G]^\beta \quad (A-3)$$

The partial derivatives required to solve the system (14) are therefore,

$$D_a = w_a[\gamma (1 - \bar{\tau}) - (1 - \gamma)(1 - \bar{\tau})] \quad (A-4)$$

$$D_G = 2\alpha - 1 \quad (A-5)$$

$$D_\tau = y^* - \gamma a^\gamma < 0 \quad (A-6)$$

$$G_a = [\gamma \bar{\tau} + (1 - \gamma)\bar{\tau}]w_a > 0 \quad (A-7)$$

$$G_\tau = \gamma a^\gamma - y^* > 0 \quad (A-8)$$

$$A_a = \delta(\gamma - 1) < 0 \quad (A-9)$$

$$A_G = -\beta\alpha/[(\bar{\tau} - \bar{\tau})y^* + \alpha G] \quad (A-10)$$

$$A_\tau = -a \left[ -\frac{\delta}{1 - \bar{\tau}} + \frac{\beta y^*}{(\bar{\tau} - \bar{\tau})y^* + \alpha G} \right] < 0 \quad (A-11)$$
The signs of most of these partial derivatives are unambiguous. In the case of $A_G$ and $A_{\tau}$, the sign depends on the elasticity, $\beta$, which we assume is positive. The signs of $D_{\tau}$ and $G_{y}$ are determined because the total per capita wages of skilled labor, $ya^y$, exceeds the tax bracket cutoff level of income $y^*$. 

Only the signs of $D_a$ and $D_G$ are ambiguous; $D_a$ will be negative if only a small fraction of government revenue is reallocated to the wealthier citizens and it will be positive if the wealthy succeed in obtaining a reallocation more proportional to their tax payments.

Concerning $D_a$, at a confiscatory level of marginal tax, $\bar{\tau} = 1$, it is negative whereas for zero progressivity, $\bar{\tau} = 0$, it is positive for $y > \frac{1}{2}$. 
Table 1A

Response of Income Inequality, D, to a Change in the Upper Tax Rate, τ, dD/dτ

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<tr>
<th>Upper Tax Rate, τ</th>
<th>LOWER TAX RATE, τ</th>
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PARAMETERS:
Production elasticity, γ = 0.65, equation (11)
Skilled labor supply elasticities, δ = 0.7, β = 0.7, equation (12)
Income for higher tax, γ* = 0.60.
Proportion redistributed to the skilled, α = .10
Table 1B
Total Output, q

<table>
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<th>Upper Tax Rate, ( \bar{\tau} )</th>
<th>LOWER TAX RATE, ( \tau )</th>
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PARAMETERS:
- Production elasticity, \( \gamma = 0.65 \), equation (11)
- Skilled labor supply elasticities, \( \delta = 0.7, \beta = 0.7 \), equation (12)
- Income for higher tax, \( \gamma^* = 0.60 \)
- Proportion redistributed to the skilled, \( \alpha = 0.10 \)
Table 1C

Government Revenue, G

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PARAMETERS:
- Production elasticity, $\gamma = 0.65$, equation (11)
- Skilled labor supply elasticities, $\delta = 0.7$, $\beta = 0.7$, equation (12)
- Income for higher tax, $\gamma^* = 0.60$
- Proportion redistributed to the skilled, $\alpha = .10$
Table 1D
Relative Wages, \( w_b / w_a \) (%)

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<tr>
<th>Upper Tax Rate, ( \bar{\tau} )</th>
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<th>0.06</th>
<th>0.08</th>
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<th>0.12</th>
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PARAMETERS:
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Table 1E
Response of Government Revenue, G, to a Change in the Upper Tax Rate, \( \tau \)

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\frac{dG}{d\tau}
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Notes

1. See Pasinetti [1962].

2. There is, however, a more subtle mechanism for capital analogous to the short-run leisure/labor substitution. Leisure is simply a euphemism for utilizing a certain fraction of each day in activities which are not taxed. The individual engaged in leisure is not necessarily relaxing beside his pool. He is often producing something material for himself, his family, or for the underground (i.e., non-taxed) market. The distinguishing feature of leisure's product is its non-sale. It is used directly or exchanged outside the normal marketplace.

   A similar possibility is available to owners of some types of capital goods. Machinery and land can be adapted to direct-use production, at least to a certain extent. We would expect the imposition of a tax on capital (or a progressive income tax on presumably richer capitalists), to prompt the withdrawal of some capital goods from their previous use. This would, of course, raise the rental returns on the remaining capital if the marginal product of capital is declining -- just as a substitution of leisure raises the wage rate.

   Whether the immediate withdrawal of capital from market production can lead to a more than proportional and immediate offsetting increase in rental-rate progressivity may seem doubtful. The mechanism exists for this possibility, but it is probably weaker than in labor's case and we expect, therefore, that tax progressivity is less likely to produce an immediate increase in inequality if capital income is a significant and increasing fraction of total income.
References


