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LOAN COMMITMENTS AND
CREDIT DEMAND UNCERTAINTY

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LOAN COMMITMENTS AND CREDIT DEMAND UNCERTAINTY

Abstract

We provide an explanation for loan commitments unrelated to borrower credit-worthiness. In our model, banks can use loan commitments to reduce uncertainty regarding their own future funding needs. Given a cost advantage to banks that can acquire such information, there exists an equilibrium demand for commitments by risk-neutral firms. The purchase of the loan commitment and the choice of contract terms reveals the buyer’s private information regarding future credit needs. In order to ensure the sorting of the a priori indistinguishable applicants according to their private information, we show that a usage fee applied to the commitment-holder’s unused credit line is necessary.
I. Introduction

Much of the literature on the banking firm has focused on the randomness of deposits (see Edgeworth (1888), Baltensperger (1980) and Santomero (1984)). Recently, however, Deshmukh et al. (1982), Ricart i Costa and Greenbaum (1983), and Sprinkle (1987) have emphasized the complementary stochastic behavior of bank loan demand. In the two former papers, uncertainty arises from the random exercise of loan commitments. The latter paper provides a more general model in which both deposit and loan uncertainty jointly influence bank behavior. The present paper extends this literature in treating credit demand uncertainty as an object of bank management. Rather than assuming that banks adjust their portfolios to exogenous uncertainty, we have banks managing uncertainty by assembling information in order to improve their predictions of loan demand. Bank funding is then adjusted in light of more informed loan demand estimates.

The model focuses on bank planning; we assume that given advance information regarding loan demand, banks can beneficially prearrange their borrowings. They can therefore be expected to elicit loan demand information from their clientele, and we show that the loan commitment can serve this purpose. In transmitting loan demand information, commitments reduce uncertainty and the costs attendant thereto. However, banks recognize the incentive firms may have to misrepresent, and in equilibrium the policy of the bank must be incentive compatible in the sense that it induces applicant honesty in reporting their information. We show that a loan commitment contract incorporating a usage fee (a fraction of the unused portion of the commitment) and a forward interest rate will be incentive compatible in the
stated sense. Moreover, we show that whereas all (creditworthy) firms will be offered commitments, the bank will prearrange funding only for those that report a sufficiently high probability of borrowing. The usage fee will be higher for firms that report higher expected loan demand whereas the loan rate offered to such firms will be lower. Such a policy will be incentive compatible since firms with a greater likelihood of exercising the commitment will prefer a lower borrowing rate in exchange for a higher usage fee.

Facing uncertain loan demand, banks are assumed to have two distinct sources of funding. They can either borrow after their loan demand is known, or by prearrangement. The latter mode of borrowing is based on anticipated loan demand, but funds are obtainable at a lower interest rate. The assumption of a higher cost for immediate borrowings, following the resolution of loan demand uncertainty, is consonant with traditional inventory-type cash management models (see Orr and Mellon (1961), Kane and Malkiel (1965), and Miller and Orr (1966)). Given an incentive to prearrange borrowings, banks can be expected to expend resources in forecasting loan demand. We view prospective borrowers as knowing more than their banks about their future credit needs and, furthermore, banks recognize their informational disadvantage. Given the value of accurate predictions of loan demand, banks elicit their customers' private information and the loan commitment serves as an instrument of communication.

All loan commitments provide limited guarantees of future credit terms. In the case of fixed-rate commitments, the interest rate on future borrowing is specified whereas for variable-rate commitments, the formula for calculating the future borrowing rate is fixed. Thus, commitments guarantee either a future borrowing rate or a mark-up on the future spot market interest
rate, and the purchase of a commitment is commonly explained in terms of risk abatement or hedging (see Campbell (1978), Thakor et al. (1981), Hawkins (1982), Ho and Saunders (1983), and Melnick and Plaut (1986)). Most of these studies either assume borrower risk aversion or ignore the origins of loan commitment demand. Given the preponderance of corporations in the commitment market, risk aversion is an unappealing assumption. More recent work therefore seeks to establish a role for loan commitments in a risk-neutral setting by focusing on the commitment's possible use in either assessing a prospective borrower's credit risk (Kanatas (1987)), or in controlling this risk (Berkovitch and Greenbaum (1987) and Boot, Thakor, and Udell (1987)). Whereas this paper retains the assumption of universal risk neutrality and a competitive banking environment, it differs from its recent predecessors in focusing on the bank's loan demand uncertainty rather than on the borrowers' creditworthiness. This distinction is the basic contribution of the paper.

In our analysis, the demand for commitments originates from the incentive banks provide for their acquisition. Recognizing their informational disadvantage, banks offer to share the benefit of their lower funding costs, provided firms disclose their private information. The firm's inducement is an advantageous (partially) predetermined loan rate. Now, unless this loan rate is independent of the information firms provide, there will be an incentive to misrepresent probable credit demand in order to obtain more favorable terms. In equilibrium, however, we should not expect to find contracts that embody incentives to misrepresent. We assume, therefore, that banks adopt a commitment granting policy that sorts applicants according to their private information. In dealing with problems of asymmetric information, we know from the revelation principle (Myerson (1979), Harris and
Townsend (1981)) that a policy in which the commitment contract is contingent on the applicant's reported information cannot be dominated. Therefore, the terms of the loan commitment contract are functions of the applicant's report, and an incentive compatibility restriction is required to ensure applicant candor. We assume that commitment applicants (ranked according to their private information) are arrayed in a continuum, and we adopt Riley's (1979) reactive equilibrium as the appropriate description of bank competition in this environment. Since banks operate in an asymmetrically informed but competitive market, the best they can do is to adopt a policy that provides an honest report of their clients' private information. Competition drives expected profits on each applicant to zero, and maximizes the latter's expected benefit from obtaining a commitment. More detail regarding the reactive equilibrium is provided later.

A paper that complements ours is that by Sealey and Heinkel (1985). We emphasize loan commitments and credit demand uncertainty. Sealey and Heinkel focus on compensating balances and deposit withdrawal uncertainty. In an interesting analysis, they show that compensating balances can be used as a screen by a bank that has depositors (firms) who know their prospective cash needs better than does their bank. Taken together, the two papers explain how the bank can reduce uncertainty on both sides of its balance sheet when its borrowers and depositors are better informed than it is.

The remainder of the paper is organized in three sections. Section II describes the model. Section III discusses the banks' optimal commitment policy, and section IV concludes.
II. The Model

There are three dates, t=0, 1, 2, and universal risk neutrality. Firms are endowed at t=0 with a potential project requiring one unit of funds. Since the firm consists solely of the project, the project alone secures the loan. Based on information obtained at t=1, firms decide whether or not to proceed with the project. If the decision is affirmative, the project is sold at t=2 at a value of Q > 0, provided it succeeds. If the project fails, nothing is realized. At t=0, the firm knows the probability, denoted by q, that the information obtained at t=1 will be favorable, indicating that the project will be undertaken and funds will be needed. Likewise, with probability 1 - q the project will be aborted. The probability q is the firm's private information.

Firms are evaluated at t=0 by the market as either "good" or "bad" credit risks. Let pg be the probability at t=0 that a firm will be viewed as a "good" credit risk at t=1, and denote this state by g. Likewise, 1 - pg is the probability at t=0 that a firm will be viewed as a "bad" credit risk at t=1, and b denotes that state. Since the firm and the project are equivalent, g and b are also project valuations. We now need to describe the probabilities for success and failure of the project. Let \( \gamma_g \) and \( \gamma_b \) denote the probabilities that the project will succeed in states g and b, respectively, and since a firm in the state g has a higher success probability, we have \( \gamma_g > \gamma_b \). The probabilities, pg, \( \gamma_g \), and \( \gamma_b \) are public knowledge and the state of the firm at t=1 is assumed to be costlessly observable. The probabilities, q, pg, \( \gamma_g \), and \( \gamma_b \), are defined in the open intervals (0, 1). The sequencing is depicted in Figure 1.
If a firm proceeds with its project and is in state $g$, it can obtain a spot loan at $t=1$ at the competitive interest factor, $R_g$. However, if the firm is in state $b$ it faces the higher spot interest factor, $R_b$, where the two interest factors equal one plus the corresponding loan interest rates. We will assume that the project’s payoff, $Q > R_b$ so that the firm would be provided with credit at $t=1$ in either of the two states. This assumption is consonant with our focus on loan demand uncertainty rather than credit risk. If the firm obtains spot market credit at $t=1$, the expected value of its equity at $t=0$ is given by

\[
(1) \quad V = \frac{q}{R_f^2} \left\{ Q[p_g \gamma_g + (1-p_g) \gamma_b] - [p_g \gamma_g R_g + (1-p_g) \gamma_b R_b] \right\}
\]

where the first term within ( ) is the expected terminal value of the project, and the second is the expected cost of the spot market credit. The risk-free interest rate factor, $R_f$, is used for discounting.

Banks are assumed to have access to two kinds of borrowing contracts. If funds are needed immediately, they are obtainable at a unit cost, $r + c$, where $r \geq R_f$ and $c > 0$. Alternatively, if borrowing is prearranged, funds are available at $r$. Hence, the bank must pay a premium for immediacy. This is a stylized way of capturing the benefit of knowing future loan demand. Such cost structures are well documented in a variety of contexts; for example,
Goldfeld and Kane (1966) view the Federal Reserve discount window as a facility providing immediate funds at a "penalty" rate, whereas Battacharya (1980), and John and Williams (1985) assume that firms incur costs if borrowings are needed unexpectedly. The premium paid for immediacy provides an incentive for banks to learn about future loan demand. However, without a corresponding disincentive for early acquisition of funds, the bank would prearrange funds without limit. Thus, if there is a cost saving obtained by planning, there must also be a penalty if prearranged borrowings are not employed according to plan. We therefore assume that banks face a "disposal cost" of \( d \) per unit of prearranged borrowings that are not employed to finance customer projects. This cost is the difference between the bank's borrowing rate, \( r \), and the (lower) rate it can expect to obtain from investing in liquid assets. In light of reserve and capital requirements and deposit insurance premia, it is not surprising that banks do not prearrange borrowings to finance their securities portfolios or to place the funds into the Federal Funds market. Since the "disposal" cost is an important element of our model, it should be emphasized that this cost must exist if banks face immediacy costs; i.e., costs of acquiring funds on demand.

Given that banks can reduce their costs by correctly anticipating loan demand, they have an incentive to learn \( q \) as early as possible. To induce firms to disclose their private information at \( t=0 \), banks offer to share their cost savings in the form of favorable credit terms. More specifically, banks commit at \( t=0 \) to provide credit at \( t=1 \) at a loan rate, \( R \), that is lower than the firm's appropriate spot loan rate in state \( b \). However, if the firm realizes state \( g \), it can borrow in the spot market at a more favorable rate than under the commitment and will therefore not exercise its commitment.
Thus, firms obtaining commitments are assured of being able to borrow at \( t=1 \) at a rate not exceeding \( R \).

If \( R \) depends on the firm's reported information, there will be an incentive to misrepresent in order to obtain a favorable loan rate. Recognizing this, banks incorporate a usage fee, \( \alpha \), as part of the commitment contract, and \( \alpha \) and \( R \) jointly induce accurate reporting. The usage fee is a fraction of the unused amount of the loan commitment, and is paid to the bank at \( t=1 \). Thus, if the commitment is for one dollar, then \( \alpha \) is paid to the bank at \( t=1 \) if the firm chooses not to proceed with the project, which occurs with probability \( 1-q \). Likewise, the firm will pay \( \alpha \) if it realizes the state \( g \) at \( t=1 \) and undertakes the project, in which case it will borrow in the spot market at a rate more favorable than that provided by the commitment.

We assume that the firm borrows the usage fee in the spot market when the commitment is not exercised. Consequently, in the state \( i=g \), the firm will borrow in the spot market the one unit of funds needed for the project, if it is undertaken, as well as the commitment fee to pay the bank. If the project is aborted, then only the fee is borrowed. In state \( i=b \), the firm will borrow the commitment fee only if the project is not undertaken.

For a firm with a commitment, the \( t=0 \) (discounted) expected value of the project's equity, is given by

\[
V_c = \frac{q}{R_f^2} \left\{ q (p_g \gamma_g + (1-p_g) \gamma_b) - p_g \gamma_g R_g (1+\alpha) - (1-p_g) \gamma_b R \right\} \\
- \frac{\alpha}{R_f^2} \left[ p_g \gamma_g R_g + (1-p_g) \gamma_b R_b \right].
\]
The first term in (1) is the expected terminal value of the project whereas the second and third are the expected costs of debt in states g and b, respectively. Note that if \( i = g \) and if the project is undertaken, then the firm borrows \( 1 + \alpha \) in order to pay the usage fee to the bank. In state \( g \), one unit is borrowed at the rate \( \gamma_g \) and is repaid with probability \( \gamma_g \). The last term in equation (2) is the discounted value of the commitment fee, \( \alpha/R_f \), which is borrowed by the firm at \( t = 1 \) if the project is aborted.

An applicant who reveals her probable credit demand, described by \( q \), by applying for a commitment may or may not receive one, depending on the applicant’s characteristics and the bank’s lending policy. If rejected, or equivalently here, if it chooses not to apply for a commitment at \( t = 0 \), the firm will obtain spot credit at \( t = 1 \), if the project is to be undertaken.

The firm must expect to benefit from a commitment if it is to apply for one. Hence, \( V_c \), the firm’s equity value if it receives a commitment, must be at least as great as \( V \), the firm’s equity value if it waits until \( t = 1 \) to finance the project in the spot market. Defining \( U \) as

\[
U = V_c - V,
\]

we have the individual rationality condition,

\[
U \geq 0.
\]

Thus, (4) must constrain the bank’s commitment granting policy.

Substituting (1) and (2) into (3) gives the expected benefit to a firm from receiving a commitment,

\[
U = \left[ q(1-p_g)\gamma_b(R_b-R)/R_f^2 \right] - \left( \alpha/R_f \right)^2 \left[ p_g \gamma_g R_g + (1-q)(1-p_g)\gamma_b R_b \right]
\]
The first term is the firm's benefit in state $b$ when it exercises its commitment and therefore incurs the debt $R$ rather than borrow in the spot market at $R_b$. If constraint (4) is to be satisfied, the value of this benefit realized at $t=2$ if the project succeeds, with probability $q(1-p_g)\gamma_b$, must exceed the expected value of the usage fee, depicted by the second term in (5).

Our analysis of the bank's commitment granting policy and its decision to prearrange funding proceeds backwards as in the standard dynamic programming problem. Given that it has sold a commitment, the bank must determine its prearranged borrowings. At this time, the bank has the same information as its commitment holders since the customer has disclosed its private information regarding $q$ through its purchase of a loan commitment. The bank chooses the amount of funding to prearrange for a commitment holder, characterized by $(q,p_g,\gamma_g,\gamma_b)$. Recall that at $t=1$, the commitment holder will either sacrifice the usage fee, $\alpha$, and borrow $1+\alpha$ in the spot market, abort the project and again give up the usage fee, or exercise the commitment. If we let $I$ denote the amount of funds the bank prearranges for a commitment holder, then the bank's expected discounted profit at $t=0$, after it learns its customer's $q$, is given by

$$
(6) \quad \pi = q(1-p_g)(\gamma_b R - \tau I - (r+c)(1-I))/R_f^2 + (\alpha - \phi dI)(q p_g + (1-q))/R_f
$$

where the terms in the first $[ ]$ describe the expected loan repayment, if the project is undertaken in state $i=b$, with probability $q(1-p_g)$, less the bank's funding cost which is $r$ for the pre-arranged amount, $I$, and $r+c$ for the remaining $1-I$. The terms in the second $[ ]$ describe the probability that the usage fee will be paid at $t=1$ and the bank will incur the expected cost $\phi dI$. 
The latter is the expected cost to the bank of disposing of funds it has prearranged but cannot use for financing customer projects at t=1. While the customer with a commitment will not borrow in state i-g or if the project aborts, the bank may still be able to lend the prearranged funds to another credit applicant. If we define $\phi$ as the probability that the bank has no other spot market alternative at t=1, then its expected cost at t=0 of having to later dispose of funds is \([(1-q) + qp_g]\phi d/R_f\).

Once the commitment has been sold and $q$ is disclosed, the bank chooses $l$ to maximize its expected profits. Given the linearity in $l$ of the bank’s profits, equation (6), the bank’s funding decision becomes

\[
(7) \quad l = \begin{cases} 
1 & \text{if } q \geq q_c \\
0 & \text{if } q < q_c 
\end{cases}
\]

where the critical value of $q$ is given by,

\[
q_c = R_f\phi d/[(R_f\phi d + c)(1-p_g)].
\]

Thus, the bank prearranges funding to satisfy the prospective needs of its commitment holder, provided the customer satisfies some predetermined minimum probability of borrowing. For commitment holders with $q < q_c$, the bank will not prearrange funding. Note that if the expected disposal cost, $\phi d$, is zero, then $q_c=0$ and the bank prearranges funding for all commitment holders; if the "cost of immediacy," $c$, is zero, then $q_c = 1$, and the bank prearranges funding for none of the commitment holders.
We have assumed that commitment holders in state $g$ at $t=1$ who undertake their projects will prefer to borrow at $R_g$ rather than at $R$. Thus, if we denote by $V_1$ the firm's expected discounted equity value at $t=1$ if it is in state $g$ and consequently borrows at the spot rate, $R_g$, and by $V_{1c}$ if it exercises its commitment at $t=1$ and borrows at $R$, we have

$$V_1 = \left(\frac{\gamma_g}{R_g}\right)[Q-R_g(1+\alpha)].$$

and

$$V_{1c} = \left(\frac{\gamma_g}{R_g}\right)[Q-R].$$

If the spot loan is to be chosen over the commitment loan in state $g$, we must have $V_1 > V_{1c}$ or equivalently $R_g(1 + \alpha) < R$. Similarly, in state $b$, the commitment holder is assumed to exercise the commitment and borrow at $R$ rather than at the spot rate, $R_b$. Comparing $V_1$ and $V_{1c}$, but now for state $b$, we find that $R_b(1 + \alpha) > R$.

The bank may incur a cost if a commitment holder for whom the bank has arranged funds does not undertake its project; thus, those applicants more likely to exercise their commitments would be expected to obtain more favorable terms. Consequently, applicants would be expected to exaggerate the likelihood of exercising their commitments and banks can be expected to recognize this proclivity. In equilibrium, loan commitment contracts cannot provide incentives for misrepresentation. Thus, we constrain the bank's commitment policy to be incentive compatible, and from (5), we have

$$q(1-p_g)\gamma_b [R_b - R(q)] - \alpha(q) [p_g \gamma_g R_g + (1-q) p_b R_b] \geq$$

$$q(1-p_g)\gamma_b [R_b - R(\hat{q})] - \alpha(\hat{q}) [p_g \gamma_g R_g + (1-q)(1-p_g) p_b R_b]$$
for all \( q, \hat{q} \) in \((0,1)\), where \( q \) is the true value and \( \hat{q} \) is the reported value. This constraint requires that the firm's expected benefit, \( U \), from applying for a commitment and misrepresenting its \( q \) will not exceed that obtained by being truthful.

Besides setting the usage fee, \( \alpha \), and the borrowing rate, \( R \), as functions of the firm's reported information, we allow banks to establish a standard, consisting of a range of acceptable reported values of \( q \), for providing commitments. We don't explicitly allow banks to randomize their decision on granting commitments; however, we can show that randomization would not be optimal.

The market for commitments is assumed to be competitive. Riley (1979) has shown that in an asymmetrically informed competitive market, a Nash equilibrium will not generally exist with a continuum of unobservable quality types. Riley's alternative is a reactive equilibrium which he proves is unique and Pareto optimal among the set of informationally consistent policies. In our context, the latter set is defined by:

(a) the commitment applicant prefers the commitment contract constructed for his type over any contract offered to any other applicant, and

(b) the bank expects to break even on each commitment applicant.

The characterization in (a) is our incentive compatibility constraint, (9), whereas (b) requires that the bank's expected profits, given in (6), be zero. Thus, with a continuum of unobservable firm types, there cannot be a pooling
equilibrium in which commitment applicants remain unidentified and the bank breaks even across applicant types. We adopt the reactive equilibrium concept and, following Riley, we note that if a bank offers a commitment contract that provides it with positive expected profits, it will evoke a reaction by another bank that will lead to expected losses for the first bank. Furthermore, any other departures from the equilibrium set of commitment contracts by other banks do not impose expected losses on the reacting bank. Therefore, no bank would wish to offer any contract outside the equilibrium set. Banks will compete by offering commitments consistent with the revelation principle, i.e., requiring a candid report of each applicant's private information, and in equilibrium, the expected profits earned on each applicant will be driven to zero.

Given the competitive market, the reactive equilibrium set of commitment contracts will be those that maximize banks' expectation of the benefit to firms of applying for a commitment, and that simultaneously satisfy the appropriate constraints. Letting the banks' prior expectations of firms' private information be described by the cumulative probability distribution, \( F(q) \), and strictly positive density, \( f(q) \), with support \((0,1)\), the equilibrium set of contracts is the solution to:

\[
\text{(10) } \quad \begin{align*}
\text{Max} & \quad \int_0^1 U(q) \, dF(q) \\
\{ & \begin{align*}
\alpha(q), \ R(q) \end{align*} \}
\end{align*}
\]

subject to  \( \begin{align*}
(a) \quad & U \geq 0 \\
(b) \quad & U(q, q) \geq U(q, q) \quad \forall \ q, \ q' \\
(c) \quad & \pi = 0,
\end{align*} \)
where $U(q) = U(q, q)$ and is given by (5). Constraint (a) is the individual rationality condition given by (4), (b) is a restatement of the incentive compatibility restraint, (9), and (c) is the banks' zero profit condition with $\pi$ given by (8). We will later verify that the solution to (10) is indeed a separating equilibrium; i.e., it is the reactive equilibrium set of commitment contracts.

III. The Optimal Loan Commitment Policy

Since the banks' commitment policy, in equilibrium, cannot embody incentives for applicants to misrepresent, condition (10) must be satisfied. In the appendix, we show that a more useful form of this incentive compatibility condition is given by:

Lemma 1: The global incentive-compatibility condition can be expressed in terms of the local representation

\begin{equation}
U'(q) = \left[ (1-p_g) \gamma_b / R_e \right] \left\{ R_b \left[ 1 + \alpha(q) \right] - R(q) \right\} ,
\end{equation}

and

\begin{equation}
U''(q) \geq 0 ,
\end{equation}

where primes indicate first and second derivatives with respect to $q$.

Proof: Given in the Appendix.
The right hand side of (11) is the discounted cost saving to the firm from having a commitment, conditional on undertaking the project and realizing the "bad" state, \( b \), at \( t-1 \). Since we know that \( R_b(1 + \alpha) \geq R \), the expected benefit of the commitment is non-decreasing in the applicant's probability of undertaking the project, or \( U'(q) \geq 0 \). If the latter inequality were reversed, the debt under the commitment would exceed that obtainable in the spot market and therefore commitments would never be exercised, and therefore never be purchased. Furthermore, it seems plausible that commitment applicants who are more likely to proceed with their projects and borrow from the bank should be provided with a greater expected benefit \((U'(q)) \geq 0\) since the bank then would be less likely to incur the cost of disposing of prearranged funds. The procedure we use to solve (10) separates the commitment applicant pool into those with \( q \) above the critical level, \( q_c \), for whom the bank will prearrange funding provided they are granted commitments, and those with \( q \) below \( q_c \) for whom the bank will not prearrange funding, even if they are given commitments. The bank's maximization problem is solved separately in these two regions. Global incentive compatibility will be maintained by ensuring that \( U(q) \) is continuous at \( q = q_c \). In the appendix, we prove the following result:

**Lemma 2:** The bank grants commitments only to applicants for whom it will subsequently prearrange funding.

Consequently, applicants with values \( q < q_c \) for whom the bank would not prearrange financing if they held commitments are indeed not granted commitments. The bank cannot achieve non-negative expected profits for these applicants and still provide \( U \geq 0 \). Therefore, they are offered contracts having \( \alpha = 0 \) and \( R - R_b \), providing no apparent expected benefit to spot credit;
i.e., \( U=0 \). Consequently, such firms do not apply for commitments.

We now want to examine the dependence of the usage fee on the reported \( q \). Our results are:

**Lemma 3**: The usage fee, \( \alpha(q) \), is increasing in \( q \) for all \( q \geq q_c \); the maximum usage fee is the bank's expected cost, \((1 - p_g)\phi d\), of disposing of unused funds.

**Proof**: Given in the Appendix.

We turn now to the borrowing rate:

**Lemma 4**: The commitment loan rate is decreasing in \( q \); that is, applicants more likely to undertake their projects, all else the same, receive lower commitment-loan interest rates.

**Proof**: Given in the Appendix.

The increase of the usage fee and decrease in the loan rate with \( q \) is obviously incentive compatible. Those firms knowing that they are more (less) likely to undertake their projects will choose a higher (lower) usage fee in exchange for a lower (higher) interest rate.

We have assumed that the solution to (10) is the set of separating reactive equilibrium commitment contracts; we now want to verify that this is indeed correct. In \( R\alpha \) space, consider the utilities and corresponding bank profits for applicants with \( q \geq q_c \). The applicant's indifference curve from
(5) can be shown to be negatively sloped,

\[
(dR/da)_{dU=0} = [p_g \gamma_R R_g + (1-q)(1-p_g) \gamma_b R_b] / [q(1-p_g) \gamma_b] < 0,
\]

and linear, with higher \( q \) applicants having indifference curves that are less steep. Similarly, the bank's zero profit locus has negative slope,

\[
(dR/da)_{\pi=0} = -[q p_g + (1-q) R_f] / (1-p_g) \gamma_b < 0,
\]

and is also linear. The zero profit locus is likewise less steep for higher \( q \) applicants, but steeper for any given applicant than the corresponding indifference curve. Now, consider Figure 2 which depicts the indifference curves, \( U_1 \) and \( U_2 \), and corresponding zero profit loci for two applicants described by \( q_1 \) and \( q_2 \), with \( q_2 > q_1 \).

---

**Figure 2**

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Points \( C_1 \) and \( C_2 \) represent the candidate contracts for applicants \( q_1 \) and \( q_2 \) for a reactive equilibrium. Now, consider an offer of a pooling contract, \( \hat{C} \), by another bank; this new contract would improve the utility of both \( q_1 \) and \( q_2 \) applicants and consequently attract them. Assume further that the bank's zero profit locus corresponding to contract \( \hat{C} \) lies below point \( \hat{C} \) so that \( \hat{C} \) is indeed profitable to the bank. If \( C_1 \) and \( C_2 \) are to be reactive equilibrium contracts, another bank must be able to attract the better \( (q_2) \) applicants.
away from $C$ and thereby impose expected losses on the bank offering $\hat{C}$. In
Figure 3, $C^*$ is a contract that will indeed attract the $q_2$ applicants but
leave the $q_1$ applicants at $\hat{C}$.

The bank offering $\hat{C}$ will now be left with only the lower quality applicants,
thereby incurring expected losses, while the worst outcome for the bank
offering $\hat{C}$ is the loss of all its customers, which is no worse than if it did
not offer $\hat{C}$. Thus, $\hat{C}$ will not be offered in the first instance and
consequently neither will $C^*$; the equilibrium contracts will be $C_1$ and $C_2$.
The argument can be extended for any two types, and therefore for the
continuum of types having $q \geq q_c$ establishing that the solution to (10) is
indeed the reactive equilibrium set of commitment contracts.

IV. Conclusions

We have provided a new explanation for the existence and the pricing of
bank loan commitments in an environment of competition among banks and
universal risk neutrality. Whereas earlier explanations either assume risk
aversion, or alternatively focus on the role of commitments in controlling
credit risk, the present model focuses on the commitment's ability to inform
the bank about its uncertain loan demand. In a setting where loan demand
uncertainty affects bank costs, the commitment becomes an instrument for
mitigating bank funding costs by facilitating borrowing plans.
We assume that given advance information regarding loan demand, banks can benefit by prearranging their borrowing. They can therefore be expected to elicit that information from their clientele, and we show that the loan commitment can serve this purpose. In communicating loan demand information, commitments reduce uncertainty and the costs associated thereto. However, banks recognize firms’ possible incentive to misrepresent, and in equilibrium the banks’ policy must induce all applicants to report truthfully. We show that a commitment contract with a usage fee and a forward interest rate will be incentive compatible in the stated sense. Moreover, we show that the bank will offer commitments and prearrange funding only for those that report a probability of borrowing that exceeds some threshold. The usage fee will be higher for firms that report higher expected loan demand, whereas the loan rate offered to such firms will be lower. Such a policy will be incentive compatible since firms with a greater likelihood of exercising the commitment will prefer a lower interest rate in exchange for a higher usage fee.

Explanations of loan commitments based on borrower risk aversion provide no rationale for the observed price structure of commitments. Models that explain both the existence and design of commitments focus on the borrowers' unobservable default risk. The model offered here provides an empirically distinguishable explanation. In particular, it is the bank’s desire to learn the likely credit needs of its customers that accounts for the existence and characteristic design of loan commitments. Our model should be viewed as one among an expanding set of competing, but not mutually exclusive hypotheses. Although the credit risk and loan demand hypotheses apply in a risk neutral setting and thereby avoid the awkward preference assumptions of the risk-sharing explanation, none of the competing hypotheses has faced the rigor of
empirical testing. But this is not accidental, since the latter two explanations tie back to private information necessitating indirect testing and therefore considerable ingenuity. Given the necessary data, there is an obvious test that can reject both the credit risk and loan demand models. We have shown how the usage fee and borrowing rate vary with the prospective borrower's privately known probability of borrowing. Interpreted more generally, q in our model can be viewed as the fraction of the maximum borrowing that the commitment holder intends to activate. In that case, our model can be rejected by comparing commitment-holders' borrowing rates and usage fees with borrowings done under commitments. Similarly, the credit risk models predict a relationship between the borrower's unobservable default risk and the commitment terms. These models can be tested by examining commitment rates and fees for commitment holders grouped in observable risk classes, e.g. as described by a rating service. At the minimum, there should be variation in commitment terms within observable risk classes if these models are not to be rejected. It is not implausible, however, to expect that both credit risk and loan demand models are consistent with the data. In that case, a joint test would be necessary to determine which has greater explanatory power.
References


Miller, M., and D. Orr, "A Model of the Demand for Money by Firms," *Quarterly
Journal of Economics 80 (August 1966), 413-35.


Proof of Lemma 1:

To show that (9) implies (11) and (12):
Rewrite (9) for a firm having a probability of undertaking the project of $q$ who reports $\tilde{q}$,

\begin{align*}
(A1) \quad U(\tilde{q}, q) &= (q/R_f^2)(1 - p_g)\gamma_b[R_b - R(\tilde{q})] \\
&\quad - [\alpha(\tilde{q})/R_f^2][p_g\gamma_b R_g + (1 - \tilde{q})(1 - p_g)\gamma_b R_b] \leq U(q, q)
\end{align*}

where $R(\tilde{q})$ and $\alpha(\tilde{q})$ indicate that the commitment terms are functions of the firm's reported value of $q$.

After rearranging, we can express (A1) as

\begin{align*}
(A2) \quad U(\tilde{q}, q) &= U(\tilde{q}, \tilde{q}) + (q - \tilde{q})(1 - p_g)(\gamma_b/R_f^2)\left\{ R_b[1 + \alpha(\tilde{q})] - R(\tilde{q}) \right\}.
\end{align*}

If we reverse the roles of $q$ and $\tilde{q}$, we find

\begin{align*}
(A3) \quad U(q, \tilde{q}) &= (\tilde{q}/R_f^2)(1 - p_g)\gamma_b[R_b - R(q)] \\
&\quad - [\alpha(q)/R_f^2][p_g\gamma_b R_g + (1 - \tilde{q})(1 - p_g)\gamma_b R_b] \leq U(q, q)
\end{align*}

or,

\begin{align*}
(A4) \quad U(q, \tilde{q}) &= U(q, q) + (\tilde{q} - q)(1 - p_g)(\gamma_b/R_f^2)\left\{ R_b[1 + \alpha(q)] - R(q) \right\}.
\end{align*}
Therefore,
\[ (q - \hat{q})(1 - p_g)(\gamma_b/R_f^2) \left\{ R_b[1 + \alpha(\hat{q})] - R(\hat{q}) \right\} \]
\[ \leq U(q, q) - U(\hat{q}, \hat{q}) \leq \]
\[ (q - \hat{q})(1 - p_g)(\gamma_b/R_f^2) \left\{ R_b[1 + \alpha(q)] - R(q) \right\}. \]

For \( q > \hat{q}, \)
\[ (1 - p_g)(\gamma_b/R_f^2) \left\{ R_b[1 + \alpha(\hat{q})] - R(\hat{q}) \right\} \leq (1 - p_g)(\gamma_b/R_f^2) \left\{ R_b[1 + \alpha(q)] - R(q) \right\}, \]
and therefore \( (1 - p_g)(\gamma_b/R_f^2) \left\{ R_b[1 + \alpha(q)] - R(q) \right\} \) is non-decreasing in \( q. \)

Dividing (A5) by \( q - \hat{q} \) and taking the limit as \( \hat{q} \to q \), we have

\[ (A6) \quad U'(q, q) = (1 - p_g)(\gamma_b/R_f^2)[R_b(1 + \alpha) - R] \]
and since the right-hand side is non-decreasing in \( q \), we also have

\[ (A7) \quad U''(q, q) \geq 0. \]

Now, to show that (11) and (12) imply (9):

\[ U(q, q) - U(\hat{q}, q) \]
\[ = [U(q, q) - U(\hat{q}, \hat{q})] - (q - \hat{q})(1 - p_g)(\gamma_b/R_f^2) \left\{ R_b[1 + \alpha(\hat{q})] - R(\hat{q}) \right\} \]
\[ - \int_{\hat{q}}^{q} U'(x, x)dx - (q - \hat{q})U'(\hat{q}, \hat{q}) \geq 0 \]

since \( U'(q, q) \) is non-decreasing in \( q. \)

Therefore, \( U(q, q) \geq U(\hat{q}, q) \) Q. E. D.
Proof of Lemma 2:

First, we consider \( q < q_c \) and consequently, \( \ell = 0 \). Setting the bank's expected profits, (6), to zero and combining it with the expression for \( U \), (5), we have,

\[
(A8) \quad U = - [q p_g + (1-q)](\alpha/R_f)^2(r + c - R_f) \leq 0
\]

where the competitive spot market rates, \( R_g = (r + c)/\gamma_g \) and \( R_b = (r + c)/\gamma_b \), have been used. Since \( r + c - R_f \geq 0 \), only \( \alpha = 0 \) will provide firms having \( q < q_c \) with a non-negative benefit from purchasing a commitment. Thus, \( \alpha = 0 \) and consequently \( R = R_b \) and \( U = 0 \) for such firms.

Now, consider \( q \geq q_c \) and therefore, \( \ell = 1 \). Setting (6) to zero and using it with (5) and (11), we have,

\[
(A9) \quad U = \left\{ (1-p_g)(c + \phi d R_f)(q - q_c) - \alpha (r + c - R_f)[q p_g + (1-q)] \right\} + R_f^2
\]

and

\[
(A10) \quad U' = \left\{ (1-p_g)(c + \phi d R_f)(q - q_c) + \alpha [R_f + q(1-p_g)(r + c - R_f)] \right\} + q R_f^2.
\]

Combining (A9) and (A10) results in

\[
(A11) \quad U'(q) + [a(q)/b(q)]U(q) = h(q)/R_f^2 b(q)
\]

with \( a(q) = R_f + q(1-p_g)(r + c - R_f) > 0 \)
\[ b(q) = q[q_p \phi_g + (1-q)(r + c - R_f)] \geq 0 \]
\[ h(q) = (q - q_c)(1-p_g)(c + \phi_d R_f)(r + c) \geq 0 \]

The solution to (A11) is

\[ (A12) \quad U(q) = e^{-Y(q)} \int_{q_c}^{q} e^{Y(x)} \frac{h(x)}{R_f} b(x) dx \geq 0 \]

where the lower limit on the integral is taken to be \( q_c \) since \( h(q) < 0 \) for \( q < q_c \). Thus, commitments are granted and \( U(q) > 0 \) for firms having \( q > q_c \), with \( U(q_c) = 0 \).

Q. E. D.

Proof of Lemma 3:

Differentiating (A9) and equating it to (A10), we have

\[ (A13) \quad \alpha'(q) = R_f [(1-p_g)\phi d - \alpha] / [(r+c-R_f)[q_p \phi_g + (1-q)]q] \]

\[ > 0 \text{ if } \alpha < (1-p_g)\phi d \]

Now differentiating (A10) and using (A7), we find

\[ \alpha \leq (1-p_g)\phi d \]

and consequently, the usage fee is non-decreasing in \( q \); i.e.,

\[ \alpha'(q) \geq 0. \]

Q. E. D.
Proof of Lemma 4:

To show that $R'(q) \leq 0$, differentiate (5) and equate to the incentive-compatibility condition (11); the result is

$$R'(q) = -\alpha'(q)[q p_g + (1-q)(r + c)/q(1-p_g)\gamma_b \leq 0$$

Q. E. D.
SEQUENCE OF EVENTS

FIGURE 1
Figure 2

Figure 3