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"The Dilemma of Durable Goods"

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The Dilemma of Durable Goods

1. Under the slogan of the "neo-classical synthesis" the neo-classical general equilibrium theory and Keynesian economics have both been taught in parallel in two class rooms. In one it is taught that the price mechanism works; a general equilibrium exists at which labour is fully employed and the economy is in a state of Pareto-optimum, while in the other, the full employment is taught to be impossible and the market has to be supported and supplemented by conscious and conscientious economic policy activities of the government. These two views should of course be incompatible. As Keynes has pointed out, there must be a hidden hypothesis of "Say's law" behind the neo-classical world; where it is rejected, the theory has to abdicate and to be replaced by a new regime in which unemployment is recognized as a long-standing, unremovable state of affairs that is inevitable. The mechanism of self regulation of the market does not say's law of the market is negated.

where it is deeply disappointing particularly for the author that he has to complete this volume by the final section of the Addendum which establishes a thesis that no general equilibrium of full employment is possible unless the equalization of rates of profits between capital goods is ruled out.

I call this thesis the "dilemma of durable goods" that is very much consistent with Keynes' view of "Say's law". It may be regarded as a microeconomic version of the law which Keynes put in macroeconomic terms. Although Walras seriously but unsuccessfully faced the dilemma (as will be seen in section 2 below), it has been a tradition of the general equilibrium theory since Hicks to avoid the dilemma by ignoring durable goods or treating them unsatisfactorily. I have to say that my DKR is not an exception to this tradition. I now fully acknowledge the weakness of the volume in this respect, but this is because of a development of the theory which has happened during the 45 years after the publication of the volume. Moreover, the "dilemma" has not yet solved unfortunately, or fortunately, so that it remains still to be challenged.

Let us be concerned with an economy with m kinds of durable capital goods, in addition to n kinds of non-durable goods for which the price mechanism works in the sense that the price of each non-durable good changes such that its demand is adjusted to its supply. Each durable capital good has three markets: a market for the new good, another for the second-hand good and the third for capital services. The problem related with the second-hand goods will be discussed in the final section of this article. Until then, we are only concerned with the remaining two markets.
Let us assume that new capital goods which are bought in the market for the new capital goods in the current period will be installed in the buyer's factory in the same period, so that they are available for production from the next period. The total amount of capital good i which is available for production in the current period, $K_i$, is the sum of capital good i installed in the past. We assume all capital goods never deteriorate in their productivity; technological improvements are ignored. These enable us to remain unconcerned with vintages and ages of capital goods. $K_i$ is a simple sum of outputs of i in the past.

Thus, for each capital good there are a pair of markets: one for newly produced capital good i and the other for the capital services which are brought forth by capital good i produced in the past. We then have 2m markets in total. Of course, there are a pair of supply and demand to each of them. The supply of new capital good i is its output which is just produced and made available in the market for sale in the current period, while its demand is the one for investment for the purpose of production in the future. These are designated $X_i$ and $D_i$, respectively. On the other hand, as for capital services, we assume that one unit of the stock of capital good i yields one unit of capital service i for the sake of simplicity. Then the supply of capital service i amounts to $K_i$, which is constant, while its demand $F_i$ is the sum of the firms' demands for the purpose of producing capital goods $X_1, \ldots, X_m$ and non-durable goods $X_{m+1}, \ldots, X_{m+n}$; we thus have

$$F_i = H_{i1} + \ldots + H_{im} + H_{im+1} + \ldots + H_{im+n},$$

where $H_{ij}$, $j = 1, \ldots, m+n$, is the demand for factor i that is needed for producing $X_j$ (durable capital good or non-durable good).

2. Walras is concerned with the case of all production functions being subject to constant returns to scale and the firms choosing production coefficients so as to minimize the unit cost of production. Then the coefficients are obtained as functions of the prices of the factors of production. Let $a_{ij}$ be the amount of factor i which is used for producing one unit of good j. Then:

$$H_{ij} = a_{ij}X_j,$$

where the production coefficient $a_{ij}$ is a function of the factors of production. The price of the product j is determined so as to be equal to its unit cost, that is also a function of factor prices. We have such an equation for each product.

We assume only labour is exogenous (non-producible) factor, so that we have $m+n$ price-cost equations taken altogether. On the left-hand side of the equations
treatment of the markets of capital services, the real Walras is again very much contrary to the conventional, accepted view of him.

Finally, we have the markets of newly produced capital goods. As their outputs that have been determined above so as to make full employment of the existing stocks of capital goods are supplied at the markets of new goods, there must be the same amounts of demand to buy them in the markets. Walras did not state this fact explicitly, but the existence of such demands is an implicit but necessary assumption for his system to be a complete system of general equilibrium. Thus he has tacitly assumed that for each capital good there is enough demand that is as large as its supply, however large the latter may be. That is to say, the equations,

\[ D_i = X_i, \]

always hold for all \( i = 1, \ldots, m \), regardless of the values of \( X_i \) fixed. Of course, there is no economic justification for them, but once we assume an aggregate identity,

\[ p_1 X_1 + p_2 X_2 + \ldots + p_m X_m = p_1 D_1 + p_2 D_2 + \ldots + p_m D_m, \]

then the equations (2) needed for the equilibrium of the markets of the new capital goods immediately follows from (2'), because investors are indifferent to the markets since the profitability is the same among these capital goods. (2') is referred to as Say's law because it implies that the total supply of capital goods creates exactly the same amount of the total demand for them. We may then conclude that Walras' general equilibrium of capital formation exists only under the unrealistic assumption of Say's law.

3. After Hicks, theorists have been concerned with the economy where each of the firms is provided with a production function which is subject to diminishing returns. With given prices it determines its output so as to maximize its profits. Then the firm's output \( X_i \), inputs \( H_{ji} \), \( j = 1, \ldots, m \), and labour input \( H_{m+n+j} \) are obtained as functions of the prices of factors of production as well as the price of output \( p_i \). We have three kinds of markets: They are for (1) the non-durable products, (2) the factors of production and (3) the durable capital goods. In the market of the first kind, prices of non-durable goods are determined such that their supplies equal their respective demands:

\[ D_i = X_i, \quad i = m+1, \ldots, m+n. \]

The prices of capital services and the price of labour (the wage rate) are determined by:

\[ H_{j1} + \ldots + H_{jm+n} = X_i, \quad j = 1, \ldots, m, \]
handside prices of factors of production including the price of labour, \( P_{m+n+1} \). As we assume no production yielding an excess profit, the price of capital service \( i \) is the income of the capitalist who owns one unit of capital good \( i \). Remembering the depreciations are all ignored and the rates of profits are equalized through all capital goods, we see that prices of products, the rate of profits and the wage rate are ultimate variables on the right-hand side of the price-cost equations. My Theorems 1 and 2 of Chapter III of *Equilibrium, Stability and Growth* (p. 66) together imply that once the value of the real wage rate is specified, then the equilibrium prices of commodities and the equilibrium rate of profits (which I called the "long-run equilibrium" prices and rate of interest, respectively, in the book) are determined. The correspondence between the rate of profits and the real wage rate is, in particular, referred to by Samuelson as the factor price frontier. A similar "frontier" can be traced out between the equilibrium price of commodity \( i \) and the real wage rate \( w \), for all \( i = 1, \ldots, m+n \). The remaining price element, the real wage rate, will be determined such that the full-employment is realized in the labour market. Then the prices of all products and the rate of profits (and therefore the prices of capital services too) are determined, by the frontier for each commodity (or the equation of cost of production).

Thus, contrary to the usual view of Walras, the prices of commodities produced are determined, in his own system, by the price-cost equations, rather than the equations of supply = demand, only the prices of the primary factors of production (in our present case, labour being the single factor) are set so as to make their supplies equal their respective demands. Once the prices are given, the demands for non-capital goods, \( D_i(p, w) \), \( i = m+1, \ldots, m+n \), are determined, then the outputs of these commodities, \( X_i \), are fixed such that they are equal to their demands, \( X_i = D_i(p, w) \), \( i = m+1, \ldots, m+n \). Then the equilibrium conditions for capital services, \( i = 1, \ldots, m \), are written:

\[
\begin{align*}
    a_{11}X_1 + \ldots + a_{1m}X_m + a_{1m+1}X_{m+1} + \ldots + a_{1m+n}X_{m+n} &= K_1, \\
    a_{21}X_1 + \ldots + a_{2m}X_m + a_{2m+1}X_{m+1} + \ldots + a_{2m+n}X_{m+n} &= K_2, \\
    \vdots & \\
    a_{m1}X_1 + \ldots + a_{mm}X_m + a_{m(m+1)}X_{m+1} + \ldots + a_{mm+n}X_{m+n} &= K_m, \\
\end{align*}
\]

(1)

In each of these equations the last \( n \) terms, \( a_{im+1}X_{m+1}, \ldots, a_{im+n}X_{m+n} \), are already fixed, so that these equations contain \( m \) unknowns, \( X_1, \ldots, X_m \). The markets for capital services are cleared by adjusting quantities of outputs rather than the prices of capital services. It must be remembered that in the
(5) \[ H_{m+n+1,1} + \ldots + H_{m+n+1,m+n} = N, \]

respectively, where \( N \) stands for the supply of labour. Finally, the price of capital goods are determined by

(6) \[ D_i = X_i, \quad i = 1, \ldots, m. \]

Equations (3), (4), (5), (6) together give us prices \( p_1, \ldots, p_m, p_{m+1}, \ldots, p_{m+n}, p_{m+n+1} \), and \( q_1, \ldots, q_m \), where \( q_i \)'s are prices of capital services.  

Thus the "price mechanism" works perfectly as is claimed by the mainstream economists. We have full employment of labour (5), together with full utilization of capital stocks (4), in the state of general equilibrium. It is nevertheless true that it is not a state of genuine equilibrium, because the system has no endogenous mechanism which makes the rates of profits equal through all capital goods. In fact, unless some exceptionally favourable circumstances prevail, we do not have

\[ q_1/p_1 = q_2/p_2 = \ldots = q_m/p_m, \]

where \( q_i \) is the price of capital service \( i \).

It is also obvious that there is no mechanism which equates the rates of profits to each other through firms. These mean that this type of equilibrium is established only in a state where the circulation of capital is not perfect but limited. It is a state where the competition in terms of the profitability is obstructed by certain barriers; once they are removed, that equilibrium is evacuated and the economy is trapped in a state with some of the equations, (3) - (6), being violated. Thus it is at least clear that the contemporary general equilibrium theory does not carefully examine the consequences of the inequality in the rate of profits.

4. Garegnani offered another view of Walras in his book published in 1960, which contrasts with my article that appeared in the same year.  

Of course, strictly speaking, only relative prices are determined, because one of the equations (3) - (6) follows from the rest, according to the usual business. Also, in (3) - (6) the equation is replaced by an inequality "\( \leq \)", if we allow for a free good.

1/ In (3)-(6) \( D_i \)'s are given as functions of prices.


of tatonnement algorithm, Garegnani makes up new mixed Walrasian model consisting
of a part determined by equilibrium equations and a part to be adjusted by the
tatonnement procedure. In particular, he views that prices are determined by
the price-cost equations, while the market for capital services are adjusted
according to the rule for tatonnement: the price of capital service \( i \) rises, or
falls, wherever there is an excess demand for, or supply of, the service, respec-
tively. Therefore, roughly speaking, we may say that Garegnani's model is a
mixture of section 2 and section 3 above, that is, the price determination sector
of section 2 and the capital service markets of section 3.

Garegnani then insists that the prices of capital services are doubly
determined, once by the unit costs of production of capital goods assuring the
prices of capital services in proportion to the prices of capital goods and then
twice by the scarcity of the services obtained from historically given capital
stocks. And he concludes that this double-determination implies over-determinacy.
This is, however, a totally wrong conclusion.\(^3\) that

This is seen in the following way. It is true there are double specifications
of a state of general equilibrium in Walras' *Elements*, one in terms of equations
(or more accurately, in terms of inequalities) and the other as a state to be
obtained at the end of the tatonnement process. But it is also true that the
double specifications do neither mean over-specification nor over-determinacy.
They may, in fact, be consistent with each other; this is the essence of my article
in 1960. Starting with a mapping of prices and quantities into themselves which
accords with tatonnement adjustments, I have obtained a fixed point at which
Walras' general equilibrium conditions (equations or inequalities) are all satis-
fied, so that it is a point of general equilibrium. It is now evident that Walras'
tatonnement (i.e. price adjustment according to scarcity and quantity adjustments
according to profitability) leads the economy to a state where the price-cost
equations are all held. By Say's law each \( D_i \) adjust itself to the corresponding \( X_i \)
thus determined.

Garegnani did not see this connection between the equation approach
through Say's law and the adjustment approach in Walras. It is more unfortunate that the so-called
Neo-Ricardians' (such as Eatwell's) attacks on Walras are based on this short-
sighted view of him by Garegnani. This seems also to imply that the latter did

\(^3\) Unfortunately, on the basis of this false statement, Lord Eatwell magnified
the scale of falsification in his PhD thesis by saying that Walras can avoid the
Garegnani over-determinacy only when only one good is produced, others being not
produced because they are less profitable than the one produced. Of course, this
statement is entirely wrong. Whereas Garegnani too gave some consideration to
the inequality approach, it did not lead him to the correct understanding of Walras.

not properly appreciate the fact that outputs of new capital goods are regarded by Walras as perfectly flexible. Thus he missed to point out Walras' weak point that the demand functions of new capital goods are absent in Walras' economy, so that if they are explicitly introduced to avoid Say's law, the system has to suffer from another kind of over-determinacy as I have discussed in section 2 above, that is totally different from the one alleged by Garegnani.

5. For those neo-classical economists who see the general equilibrium theory from the point of view of its interpretation as is presented in section above, the von Neumann theory of economic equilibrium is very different from their own theory. But for those who correctly take Walras as we have seen him in section 2, von Neumann is no one different from him. Of course there are obvious differences between them: (1) Von Neumann was concerned with a state of balanced growth, while Walras did not make such a restriction upon capital accumulation. (2) The former ruled out consumers' choice, while it was a main concern of the latter. (3) The former allowed for joint production, while the latter ruled out it. (a) As for labour, as discussed before and will be repeated later, the former made some peculiar assumptions, but they may be replaced by usual assumptions on demand for and supply of labour which are acceptable for the latter. Removing these von Neumann assumptions from his model, I tried to reduce it to a Walrasian model in my Theory of Economic Growth, Chapter VI - VIII. In the following I deal with this version, but the same argument mutatis mutandis holds for the original von Neumann model too.

Such a von Neumann-Walras model consists two sets of conditions (or inequalities): (1) Price-cost inequalities and supply-demand inequalities. The former implies that the total prices obtained from a unit operation of an activity does not exceed its unit cost including the normal profits. The latter implies that the demand for a good does not exceed its supply. This condition holds not only for products but also for the goods used for production, including labour and land, though I neglect land throughout the following.

These sets of conditions are not independent from, but coupled with, each other. That is to say, if price-cost condition holds for an activity with strict inequality, then it is not utilized for production at all. Secondly, if supply-demand condition holds for a commodity with strict inequality, it cannot have a positive price and should be a free good. These two relations, which I call the rule of profitability (stating that an unprofitable activity should not be employed) and the rule of free goods (stating that the commodity supplied in excess should not have a positive price), hold together only in the state of general equilibrium. In other disequilibrium states, as well as the equilibrium state, however, the following powerful identity holds:
\[ \sum X_i E_i + \sum F_j p_j + Gw + (I - A) + (S - I) = 0, \]

where \( E_i \) = excess profits of an activity \( i \), \( X_i \) = its level of activity, \( F_j \) = excess demand for commodity \( j \), \( p_j \) = its price, \( G \) = excess demand for labour, \( w \) = the wage rate, \( A \) = the increase in the stocks of goods from the current period to the next, \( I \) = the total amount of investment demand, and \( S \) = savings. I call this identity the (extended) Walras' law.\(^4\)

Identity (7) is derived in the following way. Let \( T \) be the total purchasing power available in the society which consists of the total supply of labour plus the profits obtained from the production activities in the previous period. The former equals the total amount of the excess supply of labour plus the total wages for labour to be paid by the firms. While the latter equals the income earned by selling outputs that have been produced by the activities in the past and just become available in the market at the beginning of the current period (this part being designated by (a)) minus the costs spent for production in the past (designated by (b)). Then (a) is equal to the excess supply of commodities plus the total consumption and the total amount of commodities demanded for production. This last, together with the wage payment, gives the costs spent for production in the present period, that is equal to the value of output left over to the next period minus the excess profits. The former may exceed the value of output left over to the present from the past. This last is shown to be equal to the part (b) above defined in terms of the cost prices, as we assume that the rule of profitability prevailed in the previous period.

Now we may summarize the above, rather tedious description of accounting relations into the following equation:

\[ T = \text{the excess supply of commodities and labour} - \text{the excess profits} + \text{consumption} + \text{the increase in the stocks of goods from the current period to the next}. \]

Hence

\[ \text{The excess demand for commodities and labour} + \text{the excess profits} + T - \text{consumption} - \text{the increase in the stocks of goods} = 0 \]

The term, \( T - \text{consumption} \), represents savings. Therefore, the above identity can be put, in symbols, in the form (7); that is the extended Walras law. In the usual case which assumes the excess profits to be included in \( T \), that term constitutes a part of \( S \), and (7) reduces to the usual form as is given by (19) on page 92 of my Walras' Economics.

\(^4\) See Morishima, 1969, p. 139.
The state of general equilibrium is defined as:

(i) no process yielding excess profits, \( E_i < 0 \),
(ii) no excess demand for any commodity, \( F_j < 0 \),
(iii) no excess demand for labour, \( w < 0 \),
(iv) the stocks of goods increased in the current period being equal to investment demands for use in the future, \( A = I \).

By the rule of profitability, \( X_i = 0 \) if \( E_i < 0 \) in (i), while by the rule of free goods \( p_j = 0 \) if \( F_j < 0 \) in (ii) and \( w = 0 \) if \( G < 0 \) in (iii). Therefore we have \( X_i E_i = 0, F_j p_j = 0 \) and \( Gw = 0 \) in the state of general equilibrium, so that it follows from (7) that \( S = I \) (savings = the total amount of investment demands) holds in equilibrium. We can show that the von Neumann equilibrium is a special case of the general equilibrium analysis of this type. In fact, we obtain his model of balanced growth where \( X_j = (1 + g)X_j,-1 \) (\( X_i,-1 \) is the level of activity in the previous period and \( g \) is the common rate of growth; \( X_i = p_j = p_j,-1 \) and \( w = w,-1 \) are the price and the wage rate in the previous period), there is no consumer choice for the workers and the capitalists, and finally there is neither excess supply nor excess demand for labour as he assumes that labour is exported or imported as soon as we find an excess supply of excess demand in the labour market.\(^5\)

The conditions (i), (ii), (iii), (iv) above correspond to the equations in section 2 above; that is, (i) to Walras' price-cost equations and (ii) to (1). Consequently, a state of general equilibrium is obtained in the same way as Walras found solutions to his system of equilibrium of capital formation.\(^6\) First, we find a price-wage system which satisfies (i) and (iii) above. As the normal profits may be included in the cost of production in the form of the prices paid for capital services, the conditions for equal rate of profits are equivalent to Walras' price-cost equations. Secondly, \( X_i \)'s are adjusted so as to fulfil (ii), i.e. (1) as in section 2. We note that the present system of von Neumann-Walras type allows for joint production, the number of activities is not necessarily equal to the number of goods. Consequently, all the conditions (i), (ii), (iii) are put in inequality forms, rather than the equation forms in section 2; therefore, the activity levels \( X_i \), the prices \( p_j \) and the wage rate \( w \) have to be adjusted.

\(^5\) For more detailed argument, see Morishima, 1969, Chapter VIII.

\(^6\) We may therefore say that Walras is a legitimate precursor of von Neumann. I do not understand that Joan Robinson was hostile to Walras, whereas she was rather sympathetic to von Neumann.
so as to fulfil the rule of profitability and the rule of free goods. Finally, (iv) corresponds to \( (2') \). Where all these conditions are realized, it follows from Walras' law that savings equal investment demand, \( S = I \). On this point we need rather careful comments.

The most fundamental assumption of the von Neumann model is that it takes one period to complete any production activity. This means that the outputs of current activities appear only in the beginning of the next period. Their prices are therefore determined in the markets of the next period, so that we can only make expectations of them in the current period. Of course the people buy the same kinds of commodities in the current period, but they are the products of the activities carried out in the previous period. To avoid expected prices, von Neumann assumes that the prices prevailing in the current markets will continue to hold in the next period. We have, however, no economic rationale for neglect of expected prices.

Once expectations are allowed for, the profitability is calculated on the basis of outputs evaluated in terms of expected prices and inputs in terms of current prices. It is evident that the choice of techniques is affected by more or less precarious elements of price expectations. Also the levels of current production activities depend on the demands which we may expect in the next period when outputs become available. The outputs of the next period are evaluated at their expected prices in this period. Their total sum is discounted by \( 1 + \) the rate of normal profits (because it is equal to the rate of interest). This is compared with the same sum for the previous period, and the difference between them gives the stocks of goods increased in the current period, denoted by \( A \). In this difference, the first term refers to the total stocks available in the next period, a part of which is put for replacement of the existing stocks and the rest provides a net increase in the stocks.

Let \( D_j \) be the investment demand for commodity \( j \) and \( \phi_j \) its expected price that is formed on the basis of the current prices. The total of investment demands amounts to

\[
I = \sum \phi_j D_j.
\]

In order for \( A \) to be accepted and useful in the next period, \( A \) should be equal to \( I \) as is required by (iv). If this holds, together with other conditions (i), (ii), (iii) satisfying the rules of profitability and free goods, we obtain

\( S = I \) from \( (7) \). This means that investment demands \( D_j \) are sufficiently flexible; \( I \) does not equal \( A \) and, therefore, otherwise \( I \) does not reach \( S \), a gap remaining between them. Then, even though techniques are chosen in an efficient way, that is to say, condition (i) satisfies
the rule of profitability, (ii) and (iii) violate the rule of free goods wherever $S = A > I$. Thus, there must be either unemployment of labour at a positive wage rate, or a commodity which is in excess supply at a positive price.

This is the conclusion obtained by Keynes; it occurs when investment demands $D_j$ cease flexible at some levels, so that I does not reach S. When Say's law prevails, $D_j$'s are created flexibly, so that there is no barrier for the aggregate demand to reach S. This Keynesian case is obtained only where Say's law dose not hold. We obtain this conclusion because we deal with an economy with capital goods and are confronted with the "dilemma of durable goods". We have first to take account of the equalization of rates of profits of all capital goods; then a capital good and the capital service from it have no two prices which can change independently. We have only one price to clear two markets, the market for a new capital good $k$ and the market for the capital service offered by the existing stock of the capital good $k$. If the latter is cleared by adjusting the price of capital service, then the price of the corresponding capital good is also fixed, so that the market for a new capital good is cleared only by changing the quantity of the capital good produced. If there is enough demand for them, both markets are cleared. In order to have this we must assume Say's law. Where we negate it because of its implausibleness, we have markets of capital goods left uncleared. In such circumstances it is highly likely to have unemployment of labour too.

It is ironic to see that this conclusion has similar effects as Garegnani's rejective view of Walras' capital theory which he derived by reasoning wrongly. Rejecting his argument, I have instead found that the equal rate of profits hinders the economy from settling at a state of general equilibrium with full employment of labour and full use of stocks of goods, unless Say's law whose unrealistic character is obvious is accepted.\(^7\) Keynes accepted the equal rate of profits\(^8\) and rejected Say's law. It is seen from the above that I have also made the same choice. Then the Walrasian full-employment and full-use equilibrium is impossible and should accordingly be replaced by a weaker one that allows for unemployment and other disequilibrium elements. This might perhaps be a change that Garegnani wanted to have, when he wrongly declared an internal inconsistency or "overdeterminacy" of Walras' theory of capital.

\(^7\) M. Morishima, 1977, p. 95; 1989. With regard to Hicks' stance towards this problem see Morishima, 1994.

\(^8\) Keynes accepts that marginal efficiencies of capital goods are equalized throughout. This is his version of the equal rate of profits.
there has so far been no general equilibrium analysis concerning long-run establishment of the equal rate of profits.

Finally, some comments on the second-hand markets on durable capital goods. First, let k be a capital good of t years old and k+1 be the same capital good of t+1 years old. Von Neumann distinguishes k+1 from k and treat them as different commodities. But k+1 is produced only by k. A process, which produces commodity j by using capital good k by one unit and other factors of production by appropriate units, makes not only one unit of commodity j but also one unit of the one year older capital good, k+1, left over, available in the markets of the next period. Thus he regards k+1 as a joint output of j. According to this treatment the profitability of the process is calculated by

$$\phi_j + \phi_{k+1} - \beta(\Sigma^* a_{ji}p_i + p_k + w_l)$$

where $\phi_j$ and $\phi_{k+1}$ are expected prices of the respective goods, $\Sigma^*$ is the summation over all goods used for production of j and k+1, $\beta = 1 +$ the rate of normat profits (or the rate of interest). Where the above expression takes on a positive value, the process brings forth excess profits; if it is zero, it yields only normal profits. We assume this in the following.

Let us write:

$$\beta p_k - \phi_{k+1} = \beta p_k - p_k + p_k - \phi_{k+1} = rp_k + \delta p_k,$$

where $r$ = the rate of normal profits and $\delta = (p_k - \phi_{k+1})/p_k =$ the rate of depreciation. Then the profitability equation,

$$\phi_j + \phi_{k+1} - \beta(\Sigma^* a_{ji}p_i + p_k + w_l) = 0,$$

may be put in the form:

$$\phi_j = \Sigma^* a_{ji}p_i + \delta p_k + w_l + r[the \ total \ unit \ cost \ including \ the \ cost \ of \ using \ capital \ good \ k]$$

As far as this price-cost equation is concerned, von Neumann's accounting that treats the older capital good k+1 as joint output is not different from the conventional one. It distinguishes various second-hand capital goods according with their ages as well as their kinds; and hence we have many second-hand markets. Then we have the expected prices $\phi_{k+1}, \phi_{k+2}, \ldots$ of old capital goods. If enough amount of demand is not expected for k+1 old capital good, it would become free and would therefore be discarded. We arrive at the conclusion that investment demands concerning old capital goods are crucial in deciding their economic lifetime. Indeed, to discard or not to discard old capital goods is a greatly important part of the problem of investment. This important problem is not dealt with properly by von Neumann, since he is only concerned with the state of balanced growth equilibrium.
Secondly, let $X_k$ be the output of capital good $k$. If there is enough demand for $k$, however large $X_k$ may be, we say that Say's law holds in the market of $k$. Since it can be shown that the total value of $X_k$, $\sum p_k X_k$, for all capital goods equals the aggregate savings, the total value of demand for capital goods, i.e. the aggregate investment, equals the aggregate savings, where Say's law prevails in every market. Thus we see that the law holds macroeconomically in exactly the same sense as Keynes means, wherever it prevails microeconomically, i.e. in every capital good market.

Next we show that the converse is also true. To see this that the total amount of investment, that is equal to the aggregate savings because of the macroeconomic Say's law, is allocated among capital goods according to the order of their rates of profits. Then, where these rates are equalized, investors are indifferent among capital goods, so that $S = \sum p_k X_k$ is distributed such that $X_k = D_k$ for every $k$ because $I = \sum p_k D_k$ and $S = I$ by Say's law. We thus obtain microeconomic Say's law.

Thirdly, we have so far been concerned with the problem of the "dilemma of durable goods", on the assumption that these are all real (non-monetary) commodities. However, it is evident that money is also a durable commodity, so that it has two markets: rental and stock markets. What I have called the securities market in this volume is the lending and borrowing market, or the buying and selling market of money services for one week. That is to say, it is the rental market of money. On the other hand, the money market, where the demand for cash balances is equated to the existing amount of money is the stock market of money.

As we have seen in this volume that Hicks wanted to show the equivalence of the loanable fund theory of interest based on the rental market of money and the liquidity preference theory on the stock market of money. I have reviewed his argument and reached the conclusion that although they are not equivalent from the genetic-causal point of view, they obtain the same level of the rate of interest in the state of temporary general equilibrium. Like
Hicks, I have, however, neglected in this analysis, the problem of the dilemma of durable goods entirely.

In event of the rate of profits being equalized, we have to conclude differently. Let us assume, for the simplicity sake, that there is no non-durable commodity. Let $E_o$ be the excess demand for money, $E_i$ the excess demand for securities, $E_i, i = 2, ..., n$, the excess demand for $i$ in its rental market, $E_j, j = i + n - 1$, where $i = 2, ..., n$, the excess demand for $j$ in the stock market $j$. We may then put Walras' law in the form:

$$E_o + E_i + \sum_p^i E_i + \sum_j^i E_j = 0.$$

If we assume the equal rate of profits for good $i = 2, ..., n$, that is, if

$$\frac{p_2}{p_{n+1}} = \frac{p_3}{p_{n+2}} = ... = \frac{p_n}{p_{2n-1}},$$

then the price mechanism cannot work perfectly because of the dilemma of durable goods; so we cannot have

$$E_i = 0, \ i = 2, ..., 2n-1$$

simultaneously. In case of price mechanism working in the rental markets and the investment demand being insufficient for new capital goods, we would have

$$E_i = 0, \ i = 2, ..., n; \ E_j < 0, \ j = n+1, ..., 2n-1,$$

so that

$$E_o + E_i > 0.$$

Thus rental and stock markets for money are not cleared simultaneously. This means that if one accepts the liquidity preference theory as Keynes did, he should reject the loanable fund theory held by neoclassical economists such as D. H. Robertson. Their equivalence is obtained where (8) is not imposed. This is because the dilemma of durable goods gives rise to a dilemma between the two money markets. The two theory are not consistent with each other because equations $E_o = 0$ and $E_i = 0$ do not hold simultaneously unless, in addition to the rate of interest, some other element such as the quantity of money may be taken as an instrument for adjusting the stock market of money. This market
Now we assume, according to the liquidity preference theory, that the rate of interest \( r \) is determined such that \( E_0 = 0 \). Then the rates of profits listed in (8) all adjust themselves so as to equal the \( r \) thus determined. The gap in the effective demand, \( \sum_{n+1}^{2n-1} P_j \beta_j \), is also determined correspondingly. The government will then try and fill up the gap at least partly. Let \( \beta_j, j = n+1, \ldots, 2n-1 \), be the demand for good \( j \) of the government. To make the expenditure of the amount \( \sum P_j \beta_j \), the government must raise the same amount of money; to do so it must issue the securities of the same amount. Let the government's supply of the securities be \( S^G_2 \) which is

\[
S^G_2 = \sum P_j \beta_j
\]

In this equation it is assumed that the amount of money the government acquires by issuing the securities is immediately spent for buying commodities \( j = n+1, \ldots, 2n-1 \), so that there is no change in the government's balance of money.

Adding (9) to Walras' identity, we obtain

\[
E_0 + (E_1 - S^G_2) + \sum P_i E_i + \sum P_j (E_j + \beta_j) = 0.
\]

In this we have \( E_0 = 0, E_i = 0, i = 2, \ldots, n \) because the interest rate and the rental prices are adjusted such that the corresponding markets are cleared. However, \( E_j + \beta_j \) may be still negative wherever the government expenditure is insufficient. The remaining negative excess demands if exist are "rationed" according to some method of quantity adjustments. These may disturb \( E_0 \) and \( E_1 - S^G_2 \). If excess demand or supply appears in these markets, the 'rationing' is carried out in the monetary sectors too. Thus, where Say's law does not hold, both price-interest rate mechanism and quantity adjustment mechanism must work in order to establish an equilibrium under insufficient effective demand. I regard this as the synthesis of the general equilibrium theory and Keynesian economics that would perhaps very
profits listed in (8) all adjust themselves so as to equal the \( r \) thus determined. The gap in the effective demand, \( 2^m - 1 \beta_j E_j \), is also determined correspondingly. The government will then try and fill up the gap at least partly. Let \( \beta_j \), \( j = n+1, \ldots, 2n-1 \), be the demand for good \( j \) of the government. To make the expenditure of the amount \( \Sigma \beta_j \), the government must raise the same amount of money; to do so it must issue the securities of the same amount. Let the government's supply of the securities be \( S^G_1 \) which is
\[
(9) \quad S^G_1 = \Sigma \beta_j \beta_j
\]
In this equation it is assumed that the amount of money the government acquires by issuing the securities is immediately spent for buying commodities \( \beta_j \), \( j = n+1, \ldots, 2n-1 \), so that there is no change in the government's balance of money.

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91 Alternatively, we may take the loanable fund theory; then the interest rate takes on the value making \( E_1 = 0 \). This value is different from the value making \( E_0 = 0 \) because of the inequality above. Thus the two interest theories are not equivalent under Anti-Say's law.