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Author
Dharan, C.K. Hari.

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C. K. Hari Dharan
(Ph.D. Thesis)
October 1968

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THE DYNAMIC BEHAVIOR OF ALUMINUM AT HIGH STRAIN RATES

C. K. Hari Dharan

Inorganic Materials Research Division, Lawrence Radiation Laboratory, Department of Mechanical Engineering, College of Engineering, University of California, Berkeley, California

ABSTRACT

A modified version of the Kolsky thin-wafer technique is used to obtain the dynamic plastic properties of materials at very high strain rates. Data obtained from compression tests on high purity polycrystalline aluminum are presented for strain rates ranging from 4000 to 120,000 sec\(^{-1}\) at room temperature. Specimen size effects and the effect of lateral inertia are taken into account in analyzing the data.

The results plotted as stress vs. strain rate at constant strain, indicate two approximately linear regions followed by a non-linear rise in the stress at the highest strain rates. The slope in the first linear region agrees well with other experimental data and is attributed to dislocation damping by phonon viscosity. The smaller slope in the second region is attributed to a decrease in phonon viscosity effects as a result of the higher dislocation velocities. The damping coefficient in this region agrees well with the phonon scattering and anharmonic radiation theories. The rapid non-linear increase in stress with strain rate in the third region appears to be due to the Lorentz contraction of the core of the dislocation as its velocity approaches that of sound.

The design features of the high velocity impact machine that was constructed for this investigation and the computer program used in analyzing the data are described in separate appendices.
I. INTRODUCTION

The fact that materials behave differently depending upon the rate at which they are deformed, has been known to man since prehistoric times. The early Stone Age man for instance, realized that it was easier to fracture a piece of rock by giving it a sharp blow with another piece of rock rather than by pulling it apart by brute force. This type of material behavior is now known to be caused by stress wave propagation effects. However, it was only after the development of scientific experimental investigation that the basis for a systematic approach to the study of the dynamic behavior of materials began.

Among the earliest experimental work in this field was the work of J. Hopkinson (1872) who measured the strength of steel wires when they were suddenly stretched by a falling weight. He obtained the rather remarkable result that the minimum height from which a weight had to be dropped in order to break the wire was independent of the size of the weight. In other words, under these conditions the effects of two blows were equivalent not when their momenta or energies were the same, but when their velocities were equal. Hopkinson explained this result in terms of the propagation of elastic waves up and down the wire.

Since Hopkinson's experiments, the theory which describes the dynamic behavior of materials when they are subjected to such small impact loads that their deformation is exclusively elastic, is now highly developed and has been well verified by experiments. However, when materials are subjected to such high-impact loads that they suffer permanent as well as elastic deformation, the theory becomes much more
complex, and has only been recently explored. This is because material behavior in the plastic range depends not only on the rate at which the material is strained, but also on its previous strain history.

From an engineering point of view, it is desirable to obtain general constitutive equations describing material behavior at high plastic strain rates. Many experimental techniques have been devised to determine these relations. However, those techniques that depend directly on elastic-plastic wave propagation experiments require a knowledge of the nature of plastic wave propagation beforehand. Since such a knowledge also depends implicitly upon certain assumptions made on the behavior of the material that is to be tested, such experimental techniques are of doubtful value in determining the true nature of plastic behavior at high-impact loads. However, the technique developed by Hauser et al. who used a modified version of the Kolsky thin-wafer method, does not require any such assumptions if certain precautions are taken. 2,3 A modified version of their technique is employed in the present investigation in order to determine material behavior at even higher strain rates (≈ 10^5 sec^-1).

In addition to determining the constitutive equations governing material behavior as a whole, it is essential that the plastic deformation process be understood from a microscopic fundamental point of view. This was not done until 1934, when Taylor, Orowan and Polanyi independently introduced the concept of the dislocation to explain the large discrepancy between the theoretical and observed shear strengths of crystals. Since then, dislocation theory has grown extensively, and has quite successfully dealt with many aspects of plastic deformation phenomena.
The plastic deformation of crystalline materials depends upon the motion of dislocations, and an analysis of the factors that affect the motion of dislocations should yield a true picture of the deformation process. The mechanism of thermal activation which governs the motion of dislocations in aluminum at strain rates ranging from about $10^{-4}$ to $10^3$ sec$^{-1}$ will next be briefly discussed, after which it will be shown that at the higher strain rates ($10^3$ to $10^5$ sec$^{-1}$) obtained in this investigation, dislocations are first viscously damped, and then, at even higher velocities, undergo relativistically limited motion.

A. Thermally Activated Intersection

For strain rates up to about 1000/sec in polycrystalline pure aluminum at room temperature, the strain rate dependence has been well explained by the thermal activation of dislocations across short range barriers. In a perfect crystal, the only intrinsic resistance to a slowly moving dislocation is the Peierl's stress, which is due to the periodic variation of strain energy as a dislocation moves through the crystal lattice. Imperfections such as impurity atoms, precipitates, lattice point defects and other dislocations interact with glide dislocations and offer resistance to their motion. At absolute zero, the mechanical force alone provides the energy to complete the dislocation-imperfection interaction, while at finite temperatures, there is a possibility of successful thermal fluctuations which aid the process.

Figure 1 shows the effect of testing temperature on the resolved shear stress for slip in Al single crystals and is a representative one for slip in fcc metals. Region I is known at the thermally activated region. Under an applied shear stress the mobile dislocations move
rapidly until various segments are arrested at barriers and the dislocations are momentarily held up. Under the random action of sufficiently large thermal vibrations, the dislocations overcome the barriers and move on. At a given shear stress, therefore, a higher temperature will decrease the average arrest periods of the dislocations at barriers because of the larger amplitudes of thermal vibrations, and the strain rate will increase.

Figure 1 shows this type of behavior. \( \tau_A \) is the athermal component of the applied stress and is due to long range barriers that cannot be penetrated by the dislocations by means of thermal activation. \( \tau_T \) (measured at absolute zero) is due to short range barriers that can be overcome by the combination of the applied stress \( (\tau^* + \tau_A) \) plus thermal activation.

Figure 2 shows an example of strain rate phenomena in slightly cold-worked polycrystalline Al. The dynamic behavior over the range where the thermally activated intersection mechanism is the rate-controlling mechanism is indicated by the solid line which shows a linear relationship between \( \ln \dot{\varepsilon} \) and \( \sigma \). This relationship has been predicted extremely well by Seeger's formulation for thermally activated intersection which yields the equations:

\[
\tau = \tau_A + \frac{U_j}{Lb^2} - \frac{kT}{Lb^2} \ln \left( \frac{NAb^2v_0}{L^2Y} \right) \quad T \leq T_c \quad (1.1)
\]

\[
\tau = \tau_A \quad \quad \quad T > T_c \quad (1.2)
\]

where

\( U_j \) is the energy required to form a jog on intersection,
L is the mean spacing between the forest dislocations that are being intersected,

b is the Burgers vector,

kT is Boltzmann's constant times the absolute temperature,

N is the number of points per unit volume at which the dislocations are arrested at barriers,

A is the area swept out per activation,

\( v_0 \) is the Debye frequency, and

\( \dot{\gamma} \) is the shear strain rate.

Although Dorn and Mitchell have shown that a more complicated relationship exists if the dislocations are dissociated into partials, the validity of Eqs. (1.1 and 1.2) has been well established for Al where the constriction energy is negligible. Figure 2 reveals this quite clearly.

B. Viscous Behavior

In Fig. 2, beyond the terminal point shown by \( \nabla \), the stress is higher than \((\tau_A + \tau_T)\) and the velocity of the dislocation is no longer controlled by the thermally activated mechanism. The curve deviates from the linear logarithmic of strain rate versus stress relationship beyond this point, suggesting the operation of a new mechanism. It was thought at first that a limiting strain rate was being approached, but when the data of Fig. 2 are replotted on a linear scale such as in Fig. 3, it indicates that beyond the limit of thermally activated intersection, the strain rate appears to increase almost linearly with some over-stress \((\sigma - \sigma_B)\).

When the applied stress exceeds \((\tau_A + \tau_T)\), the stress is high enough
for the dislocations to be pushed past barriers without assistance from thermal fluctuations. In this range, various energy dissipative mechanisms which serve to prescribe the velocity of dislocations can operate. The linear relationship between \((\sigma - \sigma_B)\) and \(\dot{\varepsilon}\) indicates that viscous damping mechanisms are responsible for material behavior in this region. Various theories have been proposed to explain viscous damping of dislocations. These theories are considered briefly in a later section where a comparison is made between the theories and the results obtained from the present experiments. On comparing the theories, a drag coefficient of damping constant \(B\) is defined where

\[ F = (\tau - \tau_B)b = By \] (1.3)

where \(F\) is the force per unit length on a dislocation moving at a velocity \(v\) with a Burgers vector \(b\).

The drag coefficient \(B\) has been measured experimentally by internal friction tests, torsional stress-pulse tests, and by impact shear and compression tests. In the internal friction measurements, \(B\) is determined usually in the amplitude-independent megacycle frequency region where the dislocations oscillate between pinning points.\(^{10-12}\) The Granato-Lücke theory is used to evaluate \(B\) from the internal friction data.\(^{13}\) In the torsional stress-pulse tests, the dislocation velocity is measured as a function of an input torsional stress pulse.\(^{14,15}\) The impact test, where the stress is measured as a function of the strain rate, is the method used in the present investigation to determine \(B\).
C. **Scope of Present Investigation**

The purpose of the present investigation is to determine the dynamic plastic flow behavior of pure annealed polycrystalline aluminum at very high strain rates (~ $10^5$/sec). While the linear $\dot{\varepsilon}$ vs $\sigma$ region is well established, it is desirable to determine whether this region extends to even higher strain rates, and whether other hitherto negligible effects begin to become more prominent. In the next section, the experimental procedure, the effect of lateral inertia, and the method of data analysis are discussed. In Section III, the results are presented and discussed in relation to specimen size effects, and viscous and relativistic effects. A comparison is then made with other experiments, and with the values of the damping constant obtained from existing theories. Appendix A describes the main features of the high-impact machine that was designed and constructed for the purpose of the present investigation, and Appendix B describes the computer program used in analyzing the data.
II. EXPERIMENTAL TECHNIQUE

A. Specimen Preparation

Polycrystalline aluminum (99.999% pure) was cold drawn into 1/4 in. diam rod involving about 70 percent reduction in cross sectional area. The rod was potted in a resin compound such that it formed the axial center of a cylinder 3/4 in. in diameter. The cylinder was then sliced with an abrasive saw to give three groups of specimen sizes having axial lengths of 1/4 in., 1/8 in. and 1/16 in. Holes of 0.110 in. diam were drilled axially into a few specimens from each group. The two flat surfaces of each specimen were then carefully mechanically polished. Since the specimens were held by the potting compound while being polished, flat surfaces were ensured. Annealing of the entire batch of specimens after removing the potting compound was done in an inert atmosphere at 350°C for one hour. All the specimens were then chemically polished using a solution of 25% sulphuric acid, 70% orthophosphoric acid and 5% nitric acid. A few randomly selected specimens were subsequently etched and the average grain diameter determined to be 0.0068 in. The grains were found to be uniformly distributed both across the cross section, and along the lateral sides of the specimen.

B. Theory of the Experimental Technique

It is well known from one-dimensional wave propagation theory that the relation between the particle velocity \( v \) and the longitudinal stress \( \sigma \) in a long elastic rod is given by

\[
v = \frac{\sigma}{\rho c}
\]  

(2.1)
where $\rho$ is the density of the elastic medium and $c$ the longitudinal wave velocity. If such an elastic rod is impacted at velocities greater than

$$v_y = \frac{\sigma_y}{\rho_c} \quad (2.2)$$

where $\sigma_y$ is the yield strength of the material, then yielding would occur, and Eq. (2.1) would no longer be valid. Hence the Kolsky thin-wafer technique, where the specimen is sandwiched between an input elastic rod and an output elastic rod, is limited to impact velocities less than $v_y$ for the rod material. The largest values of $v_y$ for presently known high strength elastic materials is probably less than 3000 in. per sec.

To overcome this difficulty in this investigation, since higher velocities were involved, the input rod was discarded and (see Fig. 4) the projectile was allowed to impact the specimen directly. The specimen was backed up by an elastic output rod as usual.

Upon impact by the projectile (see Fig. 5), the particles in the specimen at interface 1 are given a velocity $v_1$ equal to the projectile velocity with a finite rise time. The stress transmitted by the specimen, $\sigma_x$, travels down the elastic output bar where the magnitude of the strain as a function of time is recorded by the strain gages on the bar. Since the stress in the specimen is well below the yield strength of the bar material, the bar behavior is purely elastic and the stress can be calculated using Young's modulus. The particle velocity in the specimen at interface 2 is given by

$$v_2 = \frac{\sigma_E}{\rho_c E} \quad (2.3)$$

The subscript $E$ refers to the elastic output bar. Then, if $a$ is the gage
length of the specimen, the average strain rate in the specimen at any time $t$ is

$$
\dot{\varepsilon} = \frac{v_1 - v_2}{a} \tag{2.4}
$$

The average strain is then

$$
\varepsilon = \int_0^t \frac{(v_1 - v_2)}{a} \, dt \tag{2.5}
$$

and the average stress is $\sigma_x = \frac{A_E}{A_s} \sigma_E$ at time $t$. $A_E$ and $A_s$ are the cross sectional areas of the elastic output bar and specimen, respectively.

Figure 5 shows the procedure for solving Eqs. (2.4) and (2.5) from the properly phased stress-time relationship measured by the gages on the output bar. Since the work done to strain the small specimen is negligible compared to the kinetic energy of the massive projectile, constant velocity impact may be assumed. In addition, no permanent plastic deformation was noticed on the polished impact-end of the projectile which was inspected at the end of each test.

C. Correction for Lateral Inertia

During a high speed compression test, in addition to the axial particle velocity, the radial and tangential particle velocities of the specimen material may achieve high values. The resulting necessary accelerations require high compressional stresses in all three directions, and therefore, the hydrostatic part of the stress tensor is high. As a consequence, the specimen remains to some extent constrained showing an apparently highly resistance to compression than would be anticipated for the plastic behavior of a metal investigated in ideal one-dimensional compression.
In a recent paper, Klepaczko and Hauser\textsuperscript{17} investigated the effects of lateral inertia in impact experiments. Their analysis pertinent to the situation involved in the present investigation is given below.

Consider a cylindrical specimen with initial dimensions of radius $a_0$ and length $l_0$. At any time during deformation the actual dimensions are radius $a$ and length $l$.

Let the face perpendicular to the $x$-axis at $x = l_0$ be given an independent arbitrary velocity $v_x(t)$. Assuming that there is no volume change (i.e., incompressibility), the lateral velocity at a specimen point $r = r$ is

$$v_r(t) = \frac{r}{2l} v_x(t) \quad 0 \leq r \leq a \quad (2.6)$$

The above equation implies that even for constant axial velocity, the lateral velocity $v_r(t)$ increases with increasing axial strain (decrease in $l$). Because of the acceleration of the specimen material in the $r$ direction a component of lateral stress $\sigma_r$ is necessary. If the deformation process is treated as quasi-static and it is assumed that there is no friction at the faces of the specimen, the stress $\sigma_r$ does not depend on $x$. Thus from conservation of momentum in the radial direction
\[
\frac{d\sigma_r}{dr} = \rho \frac{dv_r}{dt} \tag{2.7}
\]

where \( \rho \) denotes the density of the material. Integrating Eq. (2.7),

\[
\sigma_r = \rho \int \frac{dv_r}{dt} \, dr + c \tag{2.8}
\]

To obtain \( \sigma_r \), the term \( \frac{dv_r}{dt} \) must be known. For this purpose, Eq. (2.6) can be used, and after differentiation with respect to time

\[
\frac{dv_r}{dt} = \frac{r}{2l} \left( \frac{3x^2}{2} + \frac{dv_x}{dt} \right) \quad 0 \leq r \leq a \tag{2.9}
\]

In this case,

\[
l = l(1-\epsilon_x), \quad r = \frac{r_0}{\sqrt{1-\epsilon_x}} \tag{2.10}
\]

Equation (2.9) then becomes

\[
\frac{dv_r}{dt} = \frac{1}{2(1-\epsilon_x)^{3/2}} \frac{r_0}{l_0} \left[ \frac{3x^2}{2l_0(1-\epsilon_x)} + \frac{dv_x}{dt} \right] \quad 0 \leq r_0 \leq a_0 \tag{2.11}
\]

Substituting Eq. (2.11) into Eq. (2.8) and integrating

\[
\sigma_r = \frac{\rho}{4l_0(1-\epsilon_x)} \left[ \frac{3x^2}{2l_0(1-\epsilon_x)} + \frac{dv_x}{dt} \right] \left[ \frac{a_0^2}{(1-\epsilon_x)} - r^2 \right] \tag{2.12}
\]

The boundary condition used to evaluate \( c \) in Eq. (2.8) is \( \sigma_r = 0 \) for \( r = a \).

From Eq. (2.12) it can be seen that the lateral component of the stress \( \sigma_r \) is not constant along the \( r \) direction. The maximum value of \( \sigma_r \)
occurs at $r = 0$, i.e., at the center of the specimen. For convenience, $\sigma_r$ may be assumed to be independent of $r$ and equal to $K(\sigma_r)_{\text{max}}$. The exact value of $K$ is unknown, and since lateral friction has been neglected, $K = 1$ is a satisfactory assumption. For this case,

$$\sigma_r = \frac{\rho a_o^2}{4 I_o (1 - \varepsilon_x)} \left[ \frac{3 v_x^2}{2 I_o (1 - \varepsilon_x)} + \frac{d v_x}{dt} \right]$$  \hspace{1cm} (2.13)

For a constant velocity impact, which is the case in the present investigation,

$$\sigma_r = \frac{3}{8} \rho \left( \frac{a_o}{I_o} \right)^2 \frac{v_x^2}{(1 - \varepsilon_x)^3}$$  \hspace{1cm} (2.14)

The strain record $\varepsilon_x(t)$ is known since it is determined from experimental data by the method described in the previous section; the velocity of impact $v_x$ is also known, and therefore, $\sigma_r(t)$ can be calculated.

Next, the Von Mises condition of plasticity is assumed. This gives

$$\sigma = \sigma_x - \sigma_r, \quad \sigma_r = \sigma_o, \quad \nu = 1/2$$

where $\sigma$ denotes the flow stress in an ideal compression test, and $\sigma_x$ is the measured stress in the $x$-direction. Hence

$$\sigma(t) = \sigma_x(t) - \frac{3}{8} \rho \left( \frac{a_o}{I_o} \right)^2 \frac{v_x^3}{(1 - \varepsilon_x)^3}$$  \hspace{1cm} (2.15)

The computer program evaluates $\varepsilon_x(t)$ for each time increment by using Eq. (2.5). Then $\sigma_r(t)$ is calculated for each time increment from Eq. (2.14) and subtracted from the experimental record $\sigma_x(t)$ to give the corrected stress $\sigma(t)$. 
The corrected stress, the experimental stress, and the inertia stress for a typical test are shown as functions of strain in Fig. 6. The impact velocity in this test was \(4336\) in/sec. It can be seen that the lateral inertia stress constitutes about \(17.5\) percent of the axial measured stress, and so at these high strain rates the effect cannot be neglected. The average strain rate for this test was about \(2.5 \times 10^4\) sec\(^{-1}\) for strains higher than \(0.12\).

D. Apparatus

1. General Arrangement

Figure 7 shows the general arrangement of the apparatus. A high pressure air gun is used to accelerate a \(2\) in. diam., \(6\) in. long high-strength, hardened, steel projectile weighing \(5.3\) lbs. to velocities ranging from \(100\) to \(1000\) ft. per sec. The projectile impacts the relatively soft specimen which is held in alignment and contact with the output bar by means of a small section of heat collapsible plastic tubing. The end of the output bar is held in an aligning fixture mounted on the end of the gun barrel by means of a plastic sleeve. A close-up of the specimen holder is shown in Fig. 8, and Fig. 9 shows the projectile, specimen and output bar. Before each shot the specimen is aligned with the projectile which sits in the end of the barrel. When aligning is completed, the projectile is sucked into the breech by means of a vacuum pump.

The specimen chamber encloses the specimen holder and is connected to a vacuum pump so that impact occurs in a vacuum. This was done to avoid the effect of air pressure being built up between the specimen and the projectile before actual impact. The magnitude of this effect for a
1/4 in. diam specimen at atmospheric pressure and for a velocity of impact of 1000 ft/sec was estimated to be about 5500 psi on the specimen. (Appendix A)

After impact, the projectile, specimen, and output bar with the shattered fragments of the plastic sleeve are hurled through a Mylar diaphragm at the end of the specimen chamber into an 8 foot long sand box to be slowed down and recovered. A detailed description of the gun and associated equipment is given in Appendix A.

2. Measurement of Impact Velocity

The velocity of the projectile just before impact is measured by the arrangement shown in Fig. 10. The velocity measuring device consists of two sets of light source, 0.010 in. wide slits and photodiodes, 2 in. apart. The time taken by the projectile to cut the two narrow beams of light is recorded by a time interval counter. With this arrangement, the error in velocity measurement is estimated to be less than one percent.

The rise time of the velocity of interface I of the specimen is a function of the uniformity of contact between projectile and specimen at the instant of impact. A number of tests were carried out involving direct impact of the projectile on the output bar, but without the specimen, and the average rise time was found to be 4 microseconds, independent of the velocity of impact. Figure 11 shows one such result. The velocity of impact was 6110 in./sec.

3. Strain Gage Circuits

Two active 350 ohm strain gages are placed diametrically opposite each other on the output bar and connected in series across an Altec 103 Constant Current Source and a Tektronix 547 oscilloscope. Figure 4 shows
Figure 13 shows a photograph of the actual instrumentation set up, and Fig. 14 is a photograph of the stress-time history in a typical test (Test No. 12).
III. RESULTS AND DISCUSSION

A. Specimen Size Effects

The geometry of the specimen should effect (a) the lateral inertia correction term, (b) the time taken for the stress gradients across the specimen to diminish, and (c) the correction for the radial frictional restraints at the ends of the specimen. The lateral inertia correction has been described in detail in the preceding section and therefore, only the last two effects will be discussed here.

A criticism often brought up in the discussion of the thin-wafer technique is that the measured average stress, strain and strain rate do not approximate the actual stress, strain and strain rate. (See Discussion of Ref. [9], p. 114). This of course is a danger, and depends critically on the specimen length and the velocity of impact.

A basic requirement in the Hopkinson-type compression test is that the sandwiched specimen must be thin enough such that the following condition is satisfied:

\[ t_1 = \frac{a}{C_0} \ll t_2 \text{ at } \epsilon \]  

where \( t_1 \) is the time taken for a longitudinal elastic wave to transverse a specimen of gage length \( a \) at a longitudinal wave velocity of \( C_0 \); and \( t_2 \) is the time taken to attain a strain of \( \epsilon \) at which point the stress, strain, and strain rate are measured. This is so because the time \( t_2 \) at which the variables are measured must be large enough for several internal reflections to occur in the specimen if a true average state is to be reached. This is essential for the stress gradients existing between the two faces of the specimen to diminish. As the velocity of impact is increased in an effort to obtain higher strain rates, the time \( t_2 \) taken
to produce a given strain, decreases until a point is reached when condition (3.1) is no longer true. Beyond this, the values measured will correspond to transient conditions in the specimen, and hence are invalid.

Figure 15 shows the data points obtained for 10% strain plotted as true compressive stress versus true strain rate. For the sake of simplicity, the points shown are those that have been corrected for lateral inertia. Figure 16 is a plot of the ratio of $t_2/t_1$ versus the percentage deviation in stress $\sigma'/\sigma$ (defined in Fig. 15). From Fig. 16, it can be seen that the percentage deviation in the stress because of the aforementioned effect drops rapidly as $t_2/t_1$ increases and becomes zero beyond $t_2/t_1 = 8.5$. Thus, there must be time allowed for about eight elastic transits to occur in the specimen. Fortunately, this effect is easily identifiable since its presence involves sharp drops in the stress at set distances along the strain rate axis in Fig. 15 corresponding to certain specimen lengths.

It is justifiable therefore, in studying the mechanical behavior of the material to discard those tests for which condition (3.1) is not satisfied. This has been done for data obtained at 5, 10, 15 and 20 percent strains plotted in Figs. 17-20, where both the uncorrected points and the points corrected for lateral inertia are shown. The trend of the curves in these plots will be discussed in the next section.

In addition to setting a limit to the maximum strain rate that can be measured, the geometry of the specimen also affects the relative significance of the frictional restraints at the ends of the specimen. This effect should be of greater importance in the shorter specimens resulting in a higher measured stress. In an effort to determine the
significance of this effect, tubular specimens of the same gage lengths as the solid specimens were also tested. Data from these specimens are included in Figs. 17-20, and show no significant deviation from data obtained from the solid specimens. (No lateral inertia correction was applied to the tubular specimen. Any such correction would, in any case, be quite small.) The importance of frictional restrain therefore appears to be small. Lindholm has also found no appreciable deviation in stress in a plot of true stress versus the specimen $l/d$ ratio at constant strain and strain rate for strains above 4 percent. Below 4 percent strain, a slight deviation was observed, although the trend is opposite from what one would expect, the stress increasing slightly with the $l/d$ ratio.

B. Dislocation Damping and Relativistic Effects

Figures 17-20 show the experimental data obtained from this investigation plotted in terms of true compressive stress versus true compressive strain rate for 5, 10, 15 and 20 percent strains. Included also are the corresponding data points for which the lateral inertial correction was not applied.

In Regions I and II, the stress is linearly related to the strain rate with the slope changing at a strain rate $\dot{\varepsilon}_c$; these regions may be termed the linear-viscous regions. Dorn, Mitchell and Hauser noticed a linear relationship in tests performed on polycrystalline aluminum at strain rates above the thermally activated region suggesting that the rate controlling mechanisms governing the motion of dislocations changes to viscous damping type mechanisms. Similar behavior was also found to be the case at high strain rates for $\text{AgMg}$. This linear region (corresponding to Region I in Figs. 17-20) was investigated in detail by Kumar,
Hauser and Dorn, who performed impact shear tests on oriented single crystals of aluminum and zinc. The maximum shear strain rate obtained was about $2.6 \times 10^4$ sec$^{-1}$.

The dislocation damping constant $B$ in the linear viscous region can be evaluated from the data obtained from impact tests as follows:

From dislocation theory, the macroscopic plastic shear strain rate $\dot{\gamma}$ is related to the motion of dislocations by the equation

$$\dot{\gamma} = \rho_m b v$$  \hspace{1cm} (3.2)

where $\rho_m$ is the density of mobile dislocations cm$^{-2}$, $b$ is the Burgers vector, and $v$ the average velocity of the dislocations. The force per unit length on the moving dislocations as the result of a net applied shear stress $(\tau - \tau_B)$ is given by

$$F = (\tau - \tau_B)b$$  \hspace{1cm} (3.3)

Then,

$$B = \frac{F}{v} = \frac{(\tau - \tau_B)b^2 \rho_m}{\dot{\gamma}} = \alpha \rho_m$$  \hspace{1cm} (3.4)

where

$$\alpha = \frac{\tau - \tau_B}{\dot{\gamma}}$$  \hspace{1cm} (3.5)

$\alpha$ is the slope of the stress-strain rate curve in the linear regions.

In Region III in Figs. 17-20, the stress increases rapidly with strain rate in a non-linear fashion. This rapid increase in stress (more evident in Figs. 19 and 20) may be attributed to the fact that relativistic effects become more significant in the motion of dislocations at these
The theory of the motion of screw dislocations at velocities approaching the velocity of sound has been worked out by Frank, Eshelby, and Leibfried and Dietz. Weertman has considered the more complicated behavior of edge dislocations. The main result of all these theories is that dislocation motion at high velocities is quite analogous to the motion of particles in special relativity theory. The energy of the screw dislocation becomes infinite as the velocity of transverse sound waves is approached. This velocity therefore sets an upper limit to the speed of a dislocation.

For the case of a dislocation that experiences a viscous-type drag due to the phonon viscosity effect proposed by Mason, the use of the Lorentz transformation

\[ x' = x - vt, \quad y' = y, \quad z' = z \]

results in the well known "Lorentz contraction" of the width of the dislocation:

\[ a = a_0 \left(1 - \frac{v^2}{c^2}\right)^{1/2} \]  \hspace{1cm} (3.6)

where \( a_0 \) is the "rest" width of the dislocation, \( v \) is the velocity of the dislocation, and \( c \) is the velocity of sound.

Mason's expression for the damping constant due to phonon viscosity drag is

\[ B = \frac{b^2 \eta}{8ma^2} \]  \hspace{1cm} (3.7)

This expression is derived (following the treatment by Mason) in a later
section. \( \eta \) is the phonon viscosity and \( b \) the Burgers vector. Substituting Eq. (3.6) in Eq. (3.7), the force per unit length on the dislocation may be written as

\[
F = B'v = \frac{Bv}{\left(1 - \frac{\nu^2}{c^2}\right)}
\]  

(3.8)

Using Eqs. (3.2), (3.3), (3.4) and (3.5) in Eq. (3.8),

\[
\tau - \tau_B = \frac{\alpha'v}{\left(1 - \frac{\nu^2}{c^2}\right)}
\]  

(3.9)

In terms of compressive stress \( \sigma \) and compressive strain rate \( \dot{\varepsilon} \), one may make use of the von Mises relations for isotropic materials

\[
\tau = \frac{\sigma}{\sqrt{3}}, \quad \dot{\gamma} = \sqrt{3} \dot{\varepsilon}
\]

Then Eq. (3.9) becomes

\[
\sigma - \sigma_B = \frac{\alpha_c \dot{\varepsilon}}{\left(1 - \frac{3 \dot{\varepsilon}^2}{\rho_m b^2 c^2}\right)}
\]  

(3.10)

where \( \alpha_c = \frac{\sigma - \sigma_B}{\dot{\varepsilon}} \)  

(3.11)

From Eq. (3.5) and the von Mises relations

\[
\alpha = \alpha_c / 3
\]  

(3.12)

It is interesting to note that an expression similar to Eq. (3.8) is
obtained for the retarding force on a charged particle moving in an isotropic flux of electromagnetic waves. For this case, the retarding force

\[ F = \frac{4}{3} \left( \frac{W \sigma}{1 - \frac{v^2}{c^2}} \right) \frac{v}{c} \]

\( W \) = energy density
\( \sigma \) = the Thomson scattering cross section
\( u \) = velocity of the particle
\( c \) = velocity of light

For low velocities \( (v \ll c) \) the above expression becomes

\[ F = \left[ \frac{4}{3} \frac{W \sigma}{c} \right] v \quad v \ll c \]

This expression is similar to Leibfried's original expression for the drag force per unit length experienced by a dislocation as a result of phonon scattering.

\[ F = \left[ \frac{1}{10} \frac{E_0 \sigma}{\bar{v}} \right] v \quad v \ll \bar{V} \]

\( \bar{V} \) = average Debye velocity for phonons
\( E_0 \) = thermal energy density

Hence, for high dislocation velocities one can expect a relation similar to Eq. (3.8).

Equation (3.10) was used in the present investigation to estimate a value for \( \rho_m \) which was assumed to remain nearly constant for a given strain. This was done by obtaining the best fit of Eq. (3.10) for a particular value of \( \rho_m \) to the data obtained at high strain rates (dotted curves in
assumed by Mason and Rosenberg. B then, at 300°K from the internal friction data, reduces to $2.53 \times 10^{-4}$ dyne-sec/cm$^2$.

The results of the present investigation and those of the above mentioned experiments are compared in Table II. It must be noted that in the torsional stress-pulse experiments and in the internal friction measurements, the velocities attained by the dislocations are small. For example, in the torsional stress-pulse tests, the maximum velocity that was measured was 2800 cm/sec$^{15}$

In the impact shear tests of Ferguson et al. if $\rho_m$ is taken to be $2.4 \times 10^7$ cm$^{-2}$, the maximum average velocity of the dislocation (corresponding to the maximum measured shear strain rate of $\dot{\gamma} = 2.6 \times 10^4$ sec$^{-1}$) becomes $v = \dot{\gamma}/\rho_m = 3.8 \times 10^4$ cm/sec. However, the average velocity of the dislocations corresponding to an equivalent critical shear strain rate of $\dot{\gamma}_c = \sqrt{3} \dot{\varepsilon}_c$ in Figs. 18-20, range in value (depending upon the strain) from $3.8 \times 10^4$ to $5.1 \times 10^4$ cm/sec. Hence in comparing the results of the present investigation with the above mentioned experimental results, only Region I is applicable. It can be seen from Table II that the value of the damping constant $B_1$ determined in the present investigation, is in very good agreement with the other experimental results. The lower value for the damping constant ($B_{II}$) obtained in Region II in Figs. 17-20 will be discussed in the next section.

In the case of Region III, at the present time there has been no other experimental evidence to show the presence of relativistic effects. However, the close fit of the relativistically corrected equation of motion with the experimental data obtained in the present investigation strongly suggests that relativistic effects are responsible for the rapid increase in stress
at very high strain rates. It must be noted that in the region of the highest strain rates that were measured, the degree of fit of Eq. (3.10) is extremely sensitive to the choice of $\rho_m$. For example, a 10% change in $\rho_m$ causes the dotted curve in Figs. 17-20 to deviate sharply away from the experimental data points.

Throughout the analysis, $\rho_m$ is assumed to be independent of the stress, and hence constant in the stress versus strain rate curve drawn for a given strain. The experimental data of Ferguson et al. and of the present investigation show this to be true, since if $\rho_m$ did change with the stress, the slope would not be constant. One may argue, however, that since $\alpha$ is a measure of $B/\rho_m$, $B$ could also be a function of $\rho_m$ and change proportionately with $\rho_m$, thus keeping $\alpha$ constant. Kumar, for example, has suggested that dislocation-imperfection interaction may result in a viscous damping $B_D$ that could depend on the total dislocation density and hence on $\rho_m$.22 However, theoretical calculations performed by this author and also by Gorman et al.16 indicate that energy losses for a dislocation intersecting forest dislocations is very small compared to the energy of the moving dislocation so that this effect on the total damping is negligible. Recently, Victoria et al.31 performed impact shear tests at different temperatures on aluminum single crystals prestrained in such a way as to introduce a large number of forest dislocations. The objective was to determine if dislocation-dislocation interaction would cause a greater amount of damping, i.e., an increase in $\alpha$. However, their results indicate that within the limits of experimental scatter, the damping obtained is about the same as the damping obtained in the annealed aluminum tests by Ferguson et al. Only the total stress level is raised as one would expect,
since the introduction of more defects raises the long range back stresses, which the applied stress has to overcome in order to move the dislocations in a viscous manner.

D. Comparison With Theoretical Viscous Damping Mechanisms

1. Phonon Interactions

Phonons are quantized lattice vibrations that possess momentum and energy-like real particles and can therefore exchange energy and momentum with the moving dislocation. Two theories have been proposed to account for dislocation-phonon interactions. The first theory that will be discussed was proposed by Mason who considered the phonons in the lattice to comprise a "gas." The viscosity of such a gas can then be calculated from gas kinetic theory.

a. The phonon viscosity theory. This theory may be summarized as follows: A shear stress can be resolved into a compressive stress and an orthogonal tension stress. If a step change in shear stress is impressed on a region of a crystal, the radiation pressure of the phonon gas in the compression direction is increased with corresponding increase in phonon temperature, while the radiation pressure and temperature are decreased in the tension direction. These changes in the radiation pressure cause an instantaneous increase in the modulus of rigidity of \( \Delta \mu \). The increase in modulus relaxes with a characteristic time \( t_{\text{rp}} \) determined by the rate at which the phonons approach equilibrium. One may write

\[
\Delta \mu \cdot t_{\text{rp}} = \eta \quad (3.16)
\]

where \( \eta \) is a viscosity.
Treating phonons as particles possessing an average momentum \( \bar{p} \), then from gas kinetic theory, the viscosity of the gas is:

\[
\eta = \frac{1}{3} N \bar{p} \bar{l}
\]  

(3.17)

where \( N \) is the number of particles per unit volume and the mean free path \( \bar{l} \) is given by:

\[
\bar{l} = \bar{V} t_{rp}
\]  

(3.18)

where \( \bar{V} \) is the average phonon velocity (Debye velocity). Substituting Eq. (3.18) into Eq. (3.17)

\[
\eta = \frac{1}{3} N \bar{p} \bar{V} t_{rp}
\]  

(3.19)

\( N \bar{p} \bar{V} \) is the thermal energy density of the gas. Since the gas in question consists of phonons that possess different modes of vibration, the thermal energy density of the gas may be written as \( DE \), where \( D \) is an elastic non-linearity constant which depends on an evaluation of elastic third-order moduli and \( E_0 \) is the thermal energy density of the solid. Hence

\[
\eta = \frac{DE}{3} \cdot t_{rp}
\]  

(3.20)

from (3.16) and (3.20)

\[
\Delta \mu = \frac{DE}{3}
\]  

(3.21)

The thermal relaxation time \( t_{rp} \) for phonons is given by:

\[
t_{rp} = \frac{3K}{CV^2}
\]  

(3.22)

where \( K \) is the thermal conductivity and \( C \) is the lattice specific heat per unit volume, hence
A dislocation moving through the lattice has a strain field associated with it, and hence at any point in the medium the strain is a function of time. The energy loss associated with the phonon viscosity $\eta$ is determined from the product of the viscosity times the appropriate strain rate squared. The simplest dislocation to consider is the screw dislocation since it is associated with the simple shearing strain

$$
\gamma = \frac{b}{2\pi r}
$$

(3.24)

where $r$ is the distance from the center of the dislocation. If the dislocation is moving with a velocity $v$, we have,

$$
r_2 = r_1 - vdt \cos \theta
$$

(3.25)

at a point $P$.

Then the rate of change of shearing strain at $P$ is

$$
\frac{(\gamma)_2 - (\gamma)_1}{dt} = \dot{\gamma} = \frac{b}{2\pi} \left( \frac{1}{r_2} - \frac{1}{r_1} \right) = \frac{b}{2\pi \Delta t} \left( \frac{r_1 - r_2}{r_1 \cdot r_2} \right) \approx \frac{b}{2\pi} \frac{v \cos \theta}{r_1^2}
$$

(3.26)

The energy dissipated (per unit volume) is the stress $\tau$ times the
strain rate, or
\[ \tau \dot{\gamma} = \eta (\dot{\gamma})^2 = \frac{\dot{\omega}}{\dot{V}} \]  

(3.27)

\( \dot{\omega} \) is the energy dissipated per second per unit length of the dislocation.

Considering a cylinder of unit length surrounding the dislocation, then

\[ \dot{\omega} = \int_V \eta (\dot{\gamma})^2 \, dV \]

\[ = \int_a^\infty \int_0^{2\pi} \frac{b v^2 \cos^2 \theta}{4\pi^2 r^4} \eta \, r \, d\theta \, dr \]

(3.28)

\[ = \frac{b v^2 \eta}{4\pi^2} \int_a^\infty \frac{dr}{r^3} \int_0^{2\pi} \cos^2 \theta \, d\theta \]

\[ = \frac{b v^2 \eta}{8\pi a_0^2} \]

(3.29)

To determine the drag coefficient \( B_{pv} \) as a result of phonon viscosity, it may be noted that the velocity attained by the dislocation is determined by the equation

\[ F = \tau b = B_{pv} \nu \]

(3.30)

The energy dissipated (per sec per unit length) is equal to the force per unit length times the velocity

\[ \dot{\omega} = F\nu = B_{pv} \nu^2 = \frac{b v^2 \eta}{8\pi a_0^2} \]

(3.31)

\[ \therefore B_{pv} = \frac{b^2 \eta}{8\pi a_0^2} \]

(3.32)
For edge dislocations

\[ B_{pv} = \frac{3}{4} \left[ \frac{b^2 \eta}{8\pi(1-\nu)^2 a_o^2} \right] \]  

(3.33)

which is close to Eq. (3.32). In Eq. (3.33), \( \nu \) = Poisson's ratio. From Eq. (3.32), Eq. (3.23) becomes

\[ B_{pv} = \frac{b^2}{8\pi a_o^2} \left[ \frac{DE_0K}{CV^2} \right] \]  

(3.34)

The non-linearity constant \( D \) can be calculated from third order elastic moduli. Mason and Rosenberg in a later paper, determined \( D \) for aluminum to be 3.95.28

The drag coefficient in Eq. (3.34) depends critically on the radius \( a_o \) which is the "cut off" radius used in Mason's analysis. There are two limitations on the choice of \( a_o \). The first limitation is that the concept of the phonon as an acoustic wave transmitted through the medium may breakdown sufficiently close to the dislocation because of the non-linear terms in the elastic energy. Mason used \( a_o = 3/4 b \). However, if \( a_o = b \) is used, one obtains good agreement with the results of the present investigation.

The other limitation has to do with the size of the region around the dislocation which can exchange energy with phonons. In the neighborhood of the dislocation, the strain changes discontinuously when the extra plane moves over by the Burgers vector \( b \). To determine whether the material in the neighborhood of the dislocation should be included in the calculation, the criterion is whether the phonon modes can be equilibrated at a time \( t_{rp} \) less than that between jumps. Consider a point at a distance
a from the dislocation and near its core. If the strain at this point changes discontinuously in a time $t$, then energy is lost to the phonons only if

$$t \geq t_{rp}$$

or

$$a/v \geq t_{rp}$$  \hspace{1cm} (3.35)$$

where $v$ is the velocity of the dislocation. Hence the excluded radius

$$a_o = t_{rp} v$$  \hspace{1cm} (3.36)$$

Suzuki et al. first suggested that this should be the cutoff radius. For points at distances $a$ less than $a_o$, the time during which the strain changes is less than the phonon relaxation time and hence no energy is lost to the phonons. However, from the first limitation, the minimum value of $(a_o)_{min} \sim b$. With this value of $a_o$, the velocity $v$ beyond which the above equation should be used for $a_o$ is

$$v_c = \frac{(a_o)_{min}}{t_{rp}} = \frac{b}{t_{rp}}$$  \hspace{1cm} (3.37)$$

At $300^\circ K$, $t_{rp} = 0.762 \times 10^{-12}$ sec which is obtained from Eq. (3.22), using the values $K = 0.08 \times 10^8$ erg/sec-cm$^{-2}$K, $C = 2.3 \times 10^7$ erg/cm$^3$-K, $v = 3.7 \times 10^5$ cm/sec. Then $v_c = 3.76 \times 10^4$ cm/sec. Mason justifies his use of a constant value of $a_o$ on the basis that the velocities attained by dislocations in internal friction measurements is small compared to $v_c$. For a dislocation density of $\rho_m = 2.4 \times 10^7$ cm$^{-2}$, $v_c$ corresponds to a shear strain rate of $\dot{\gamma}_c = 25800$ sec$^{-1}$ which is just the maximum strain rate reached by
Ferguson et al.\textsuperscript{23} Hence their choice of a constant value of $a_0$ is also justifiable.

However, for strain rates greater than $\dot{\gamma}_c$, Eq. (3.36) must be used for the excluded radius. The above mentioned value of $\dot{\gamma}_c$ corresponds to an equivalent critical compressive strain rate of

$$
\dot{\varepsilon}_c = \frac{\dot{\gamma}_c}{\sqrt{3}} = \frac{\rho_m b v c}{\sqrt{3}}
$$

$$
= 6.24 \times 10^{-4} \rho_m
$$

The values of $\rho_m$ obtained in the present investigation, range from $1.1 \times 10^7$ at $5\%$ strain, to $2.01 \times 10^7$ at $20\%$ strain. This results in a range of compressive strain rates from $6850$ sec\(^{-1}\) to $12500$ sec\(^{-1}\). This is seen in Figs. 18-20, where the experimental curve changes its slope near these critical strain rates.

If $a_0$ from Eq. (3.36) is used as the cut-off radius, the damping due to phonon viscosity becomes inversely proportional to the velocity of the dislocation squared. Thus it should drop down very quickly with increased strain rates. The reduced slope in Region II in Figs. 17-20 may be attributed to this effect. The predominant damping in this region then ceases to be of the phonon viscosity type, and approximates damping due to phonon scattering, which will be discussed next.

b. The phonon scattering theory. The various mechanisms by which dislocation damping can be caused by phonon scattering have been reviewed by Lothe.\textsuperscript{34} At temperatures below the Debye temperature for materials with a negligible Peierl's barrier, such as aluminum, there are two significant phonon scattering mechanisms. The first mechanism is the scattering of
phonons by the dislocation strain field in a manner analogous to the refraction of light. The second mechanism is the absorption of energy from phonons by the dislocation and subsequent vibration of the dislocations with re-radiation of phonons. Lothe finds that the drag coefficient from both types of scattering is about

\[
B_{ps} \sim \frac{\varepsilon b}{5\nu}
\]

At room temperature, this works out to be \(0.724 \times 10^{-4}\) dyne-sec/cm², which is close to the low damping obtained in the present investigation in Region II. (See Table II.)

2. **Electron Interactions**

The free electron gas in a metal can also cause viscous-type damping of dislocation. An electronic viscosity can be calculated and the drag coefficient determined from Eq. (3.32). The value of the drag coefficient is again uncertain because of the uncertainty in the proper value of the core radius. Mason and Rosenberg computed the drag coefficient due to electron viscosity as a function of temperature, and assumed a constant core radius of \(10^{-7}\) cm. Their results indicate that the effect due to electron viscosity is negligibly small at room temperature.

**E. Other Effects**

1. **Anharmonic Radiation from the Dislocation**

The vibrational energy of the atoms in the highly distorted region of the dislocation core is increased when the dislocation moves due to the anharmonic coupling forces between atoms. The radiation of this
energy away from the dislocation contributes to dislocation damping. Lothe\(^3\) has estimated the effect of anharmonic radiation for an edge dislocation and obtained

\[ B = \frac{E_o b}{12\gamma} \] (3.39)

This equation is uncertain because of the difficulty in estimating the vibrational frequency and energy changes that occur in the dislocation core. Equation (3.39) gives a value for the drag coefficient at room temperature \(= 0.3 \times 10^{-4} \) dyne-sec/cm\(^2\) and is lower than the value obtained in Region II in the present investigation (see Table II).

2. **Thermoelastic Effect**

Eshelby\(^3\) demonstrated that the compressive and tensile stresses around a moving edge dislocation will cause irreversible heat flow to take place and thus will dissipate energy. Calculations using the equation of Weiner\(^7\) and Lothe\(^4\) show that this effect results in a drag coefficient for aluminum three or more orders of magnitude less than that determined in this experiment. The low rate of energy dissipation in aluminum due to this effect is a result of the high thermal conductivity of aluminum which makes the compression and tension around the edge dislocation occur in a nearly isothermal manner.

3. **Glide Plane Viscosity**

Gilman\(^3\) has proposed that the dislocation drag coefficient is primarily due to viscous effects in the dislocation core which are neglected by Mason and Rosenberg. According to his assumption the slip plane can be considered to contain a sheet of a Newtonian viscous fluid that is an atomic dimension in thickness. When the dislocation moves across
the slip plane this fluid is deformed in simple shear. The stress required to shear the fluid gives rise to a damping force on the dislocation. He shows that the viscous losses in the core region exceed those in the bulk of the crystal by a factor of about eight. Gilman suggests that the value of viscosity computed by Mason using the phonon viscosity theory is the appropriate value to employ, for metals, in his expression for the drag coefficient, B.

A quantitative analysis of glide plane viscosity cannot be performed since there is as yet no way of estimating an appropriate value of the viscosity. Phonon viscosity probably is not applicable to the dislocation core since phonon viscosity depends on phonon-phonon interactions, and, is therefore not applicable to regions smaller than the phonon mean free path.

The comparison of the theory with the results of the present experiment indicates that below a certain critical strain rate, the phonon viscosity appears to be the predominant damping mechanism governing the mobility of dislocations at room temperature. Above the critical strain rate (corresponding to a critical velocity of the dislocations) the damping reduces, indicating that phonon viscosity becomes less effective. This may be attributed to a larger cut-off radius associated with the fast-moving dislocation within which no phonon interaction is possible. At these higher velocities, phonon scattering and anharmonic radiation appear to be the important mechanisms. These give lower values for the drag coefficient which is in agreement with the results obtained in the present investigation in Region II. However, the uncertainties involved in the phonon scattering and anharmonic radiation theories, permit only
an order of magnitude comparison. Since no experiments were performed at different temperatures, the temperature dependence of the drag coefficient in Region II could not be determined. This may have thrown more light on the type of mechanism that is prevalent in Region II since it is known that phonon scattering and anharmonic radiation damping drop more rapidly with decreasing temperature than phonon viscosity damping. The values of the drag coefficient obtained from the various theories are compared with the results obtained in the present investigation in Table II.
IV. SUMMARY AND CONCLUSIONS

1. A method is described by means of which the dynamic plastic properties of materials at very high strain rates can be obtained.

2. Data obtained from compression tests on high purity annealed polycrystalline aluminum are presented for strain rates ranging from 4000 to 120,000 sec\(^{-1}\) at room temperature.

3. The effect of lateral inertia in a high impact compression test is taken into account in analyzing the data.\(^{17}\)

4. Time must be allowed for at least eight transits across the specimen thickness at the elastic wave velocity for reliable data. This ensures the measurement of average values of stress, strain and strain rate.

5. The results are plotted as stress vs. strain rate at constant strains of 5, 10, 15 and 20 percent.

6. The results indicate two approximately linear regions followed by a non-linear rise in the stress at the highest strain rates.
   a. The slope in the first linear region agrees well with other experimental results\(^{15,22,28}\) and is attributed to dislocation damping by phonon viscosity.\(^{10,11,28}\) The dislocation drag coefficient in this region ranges from 1.75\times10^{-4} \text{ dyne-sec/cm}^2 to 2.66\times10^{-4} \text{ dyne-sec/cm}^2.
   b. The smaller slope in the second linear region indicates a decrease in the dislocation damping and appears to be due primarily to phonon scattering and anharmonic radiation effects.\(^{34}\) The dislocation drag coefficient in this region ranges from 0.45\times10^{-4} \text{ dyne-sec/cm}^2 to 0.75\times10^{-4} \text{ dyne-sec/cm}^2.
   c. The rapid non-linear increase in stress with strain rate in the third region appears to be due to the Lorentz contraction of the core.
of the dislocation as its velocity approaches that of sound. This is a relativistic effect.

7. The design features of the high velocity impact machine that was constructed for this investigation and the computer program used in analyzing the data are described in separate appendices.
Table I  Experimentally obtained values of $\rho_m$, $\alpha_I$, and $\alpha_{II}$. The corresponding drag coefficients $B_I$ and $B_{II}$ were calculated from Eq. (3.4).

<table>
<thead>
<tr>
<th>True strain (%)</th>
<th>$\rho_m \times 10^{-7}$ (a)</th>
<th>$\alpha_I$ (dyne-sec/cm²)</th>
<th>$\alpha_{II}$ (dyne-sec/cm²)</th>
<th>$B_I \times 10^4$ (dyne-sec/cm²)</th>
<th>$B_{II} \times 10^4$ (dyne-sec/cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1.1</td>
<td>-</td>
<td>8270</td>
<td>-</td>
<td>0.745</td>
</tr>
<tr>
<td>10</td>
<td>1.55</td>
<td>13800</td>
<td>4820</td>
<td>1.75</td>
<td>0.598</td>
</tr>
<tr>
<td>15</td>
<td>1.77</td>
<td>15400</td>
<td>3220</td>
<td>2.23</td>
<td>0.465</td>
</tr>
<tr>
<td>20</td>
<td>2.01</td>
<td>16100</td>
<td>2760</td>
<td>2.66</td>
<td>0.454</td>
</tr>
</tbody>
</table>

(a) obtained by fitting Eq. (3.10).
Table II  Dislocation damping coefficients at 300°K

<table>
<thead>
<tr>
<th>EXPERIMENTS</th>
<th>DAMPING COEFFICIENT B (dyne-sec/cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Impact shear tests°²² (with ( p_m = 2.4 \times 10^7 ) cm⁻²)</td>
<td>( 2.42 \times 10^{-4} )</td>
</tr>
<tr>
<td>2. Internal friction measurements°²⁸</td>
<td>( 2.53 \times 10^{-4} )</td>
</tr>
<tr>
<td>(with ( N = 1.25 \times 10^6 ) cm⁻²)</td>
<td></td>
</tr>
<tr>
<td>3. Torsional stress-pulse tests°¹⁵</td>
<td>( 2.47 \times 10^{-4} )</td>
</tr>
<tr>
<td>4. Present investigation (( p_m = 1.1 \times 10^7 ) to ( 2.01 \times 10^7 ) cm⁻²)</td>
<td>( B_I ) ( 1.75 \times 10^{-4} ) to ( B_{II} ) ( 0.45 \times 10^{-4} ) to ( 0.74 \times 10^{-4} )</td>
</tr>
</tbody>
</table>

| THEORIES                                  |                                      |
|-------------------------------------------|                                      |
| 1. Phonon viscosity theory°¹⁰,¹¹,²⁸ (with \( a_0 - b, D = 3.95 \)) | \( 2.0 \times 10^{-4} \)          |
| 2. Phonon scattering theory°³⁴            | \( 0.724 \times 10^{-4} \)          |
| 3. Anharmonic radiation°³⁴                | \( 0.30 \times 10^{-4} \)          |
APPENDIX A: DESIGN FEATURES OF THE HIGH VELOCITY IMPACT MACHINE

The apparatus is shown in Fig. 7. It is essentially a large air gun capable of accelerating a 2 in. diameter, 6 in. long cylindrical steel projectile to a maximum velocity of 100 ft/sec. The primary restrictions in the design of such a gun are a) the size of the gun, b) cost and c) safety of operation. Size is restricted by the space available for housing this equipment (not more than 36 ft. by 10 ft.). This is small when compared to the sizes of many of the high velocity devices now in existence, especially when one considers the mass of the projectile that has to be accelerated. Cost, an equally important consideration for obvious reasons, is estimated to be in the region of $18,000. This is again low. Safety regulations for on-campus experimental work obviated the use of explosives for accelerating the projectile, allowing only the use of high pressure gas at 2000 psi. All of these restrictions narrowed the choice of a suitable design considerably.

Many excellent hypervelocity machines have been built but the principles involved in their operation could not be utilized because almost all of them used an explosive charge to fire the projectile\(^ {39-42}\). Besides, the equipment was almost always large, complex and expensive.

The most common of the devices used is the two stage gas gun. A heavy piston is accelerated in the first stage by burning a powder charge. The piston compresses the light gas ahead of it (usually helium) to a high temperature and pressure. This pressure keeps rising until a small tapered projectile plugged into a smaller barrel (the second stage) breaks loose and is fired. The projectiles used are usually small (a few grams),
and extremely high velocities can therefore be obtained (up to 20,000 ft/sec).

Another scheme that appeared promising employed diaphragms in the firing section with pressure differentials across each diaphragm. A solenoid-operated plungerburst the first diaphragm, thereby increasing the pressure differential across the second causing that to fail and so on—so that a pressure wave hits the projectile plugged into the barrel and accelerates it rapidly. This seemed suitable for the present purpose but it still has the undesirable features of requiring a complicated valving system and requiring the changing of diaphragms after every firing. In another device, the projectile is held in place by a break-pin which snaps when the pressure in the pressure chamber is raised to a certain valve. The disadvantage here is that the exact instant of firing is uncertain.

The ideal solution to the problem would be a solenoid-operated valve which has an extremely small time of operating so that when opened, the pressure would act over the cross section of the projectile nearly instantaneously and thus accelerate it to the required velocity in a minimum distance. However, high pressure solenoid-operated gas valves having a low time of operation and also a diameter large enough to minimize throttling (about 2 in. dia.) are presently unavailable.

A. Present Scheme - The Firing Mechanism

The firing mechanism is shown schematically in Fig. 24. The principle employed here is that a net force can act on the projectile only if the cross section of one end is exposed to the high pressure gas. In this scheme, one end of the projectile is inserted into the evacuated gun barrel while the other end is held flat against the surface of an electromagnet and does not come into contact with the high pressure gas. Any gas leaking
into the interface between the magnet and the projectile is exhausted through the vent. In this position the projectile is in static equilibrium. To fire the projectile, the two solenoid valves are energized simultaneously. This results in isolating the vacuum pump and allows the high pressure air from the pressure vessel to come into contact with the back end of the projectile pushing it forward into the barrel against the attractive force of the electromagnet. The high pressure air surrounding the projectile is now able to act over its cross section and thus accelerate it down the barrel. Firing is therefore accomplished almost instantaneously with minimum throttling losses.

The entire firing mechanism is contained in a standard flanged "cross" which is readily available. Two opposite ends of the cross hold the magnet assembly and the gun barrel while one of the ends perpendicular to these is connected to the pressure vessel. The fourth end is sealed by a blind flange which may be opened for cleaning and inspection, (See Fig. 25). The projectile is loaded from the muzzle end of the barrel and the specimen is aligned with respect to its polished impacting face. After this is done, the projectile is sucked into the breech by means of the vacuum pump shown in Fig. 25. Thus the breech need not be opened after every firing. Figure 26 shows the opened breech revealing the electromagnet and the projectile.

B. The Gun Barrel and Pressure Vessel

Let \( p \) be the instantaneous pressure acting on the cross sectional area \( A \) of the projectile. The effect of the Mach number of the fluid stream will be neglected for the present. Also neglected are the frictional losses of the gas flowing through the pipe and losses due to sliding
friction between the sides of the projectile and the gun barrel.

Let \( p_1 \) be the initial pressure and \( V_1 \) be the initial volume (i.e., the volume of the pressure chamber and the cross). If the weight of the projectile is \( W \), then the force acting on the projectile at any instant is

\[
F = \frac{W}{g} v \frac{dv}{dx} = pA \tag{A.1}
\]

where \( v \) is the instantaneous velocity of the projectile.

Assuming that expansion is adiabatic,

\[
p = p_1 \left( \frac{V_1}{V} \right)^k \tag{A.2}
\]

where \( k \) is the ratio of specific heats (\( =1.4 \) for air). Substituting Eq. (A.2) in Eq. (A.1),

\[
\frac{W}{g} v \frac{dv}{dx} = p_1 \left( \frac{V_1}{V} \right)^k A \tag{A.3}
\]

The instantaneous volume

\[
V = (V_1 + Ax) \tag{A.4}
\]

where \( x \) is the distance traveled by the projectile. Equation (A.3) then becomes

\[
\frac{W}{g} v \frac{dv}{dx} = \frac{p_1 V_1 \left( V_1 / V \right)^k}{(V_1 + Ax)^k} \tag{A.5}
\]

Integrating Eq. (A.5) from \( x = 0 \) to \( x = 1 \) and from \( v = 0 \) to \( v = v_f \)

\[
\frac{W}{2g} v_f^2 = \frac{p_1 V_1}{(k-1)} \left( \frac{1}{V_1} - \frac{1}{(V_1 + A)^{k-1}} \right) \tag{A.6}
\]
For the desired maximum velocity of 1000 ft/sec and assuming the initial maximum pressure to be 2000 psi and the initial volume to be 2000 in.³, the length \( l \) of the barrel calculated from Eq. (A.6), is equal to 18 ft. The final length of the barrel was made equal to 20 ft. to take into account frictional losses of the gas and frictional losses due to sliding friction between the projectile and the gun barrel.

C. The Support Structure

The cross is mounted on two 8×9 in. welded steel angles \( \frac{5}{8} \) in. thick which was adequate to withstand the maximum thrust of an estimated 3000 lbs. The gun barrel is mounted along its length on seven smaller supports which can be adjusted to align the barrel. Alignment is done by reflecting a laser beam off the polished surface of a test projectile as it is pulled through the barrel by a wire. The deviation of the reflected beam from the incident beam at each support station then gives the degree of adjustment required. The rear supports and the adjustable supports are mounted on a heavy (200 lb/ft) wide flange I-beam 26 ft. long, which is bolted to fixtures embedded in concrete in the floor.

D. The Vacuum System

The volume ahead of the projectile should be maintained at a vacuum so as to minimize the effect of compressed gas when the projectile and the target impact. To get an idea of the degree of vacuum that will be needed a study is made below of the behavior of gas trapped between impacting cylindrical objects. An exact treatment of the problem will not be attempted, the object here being to get a rough order-of-magnitude idea of what will occur.
The behavior of a gas which is being "squeezed" between two surfaces is known in two limiting cases:

(a) If the velocity of approach is very small, so that the pressure within the film is no more than a few per cent larger than the ambient pressure then the gas can be regarded as incompressible. The volume flow rate of gas leaving the gap is equal to the rate at which the volume of the gap is decreasing.

(b) If the velocity of approach is very high, in effect the gas does not have time to flow and compresses instead. In the limiting case the gas acts as if it were completely contained as in a frictionless piston and cylinder arrangement and behaves like an air spring.

The parameter which is a measure of whether the compression is slow or rapid involves an inverse cube of the gap height $h$, so that there will be a continuous variation from case (a) to case (b) as the cylinders approach at essentially constant velocity.

Since for the present case appreciable pressures are generated only in the last instant before impact, then limiting case (b) is of interest. This can be written as

$$PH = \text{constant} \quad (A.7)$$

where

$$p = \frac{p(r,t)}{p_a}$$

$p(r,t)$ = film pressure at a radius $r$ of the impacted cylinder at time $t$

$p_a$ = ambient pressure

$H = \frac{h}{h_0}$

$h_0$ = reference distance between the impacting cylinders

$h$ = an arbitrary distance between the impacting cylinders.
In order to estimate the constant, the continuous variation between limits (a) and (b) will be divided into two discontinuous regions. Arbitrarily, the gas film will be regarded as incompressible until the average pressure in the film is twice that of surroundings. Then this quantity of gas will be regarded as "trapped", obeying Eq. (A.7). The juncture between the two regions will allow evaluation of the constant:

For an incompressible film, the average pressure in the film is given by:

$$p_{\text{avg}} = p_a + \frac{3}{2} \frac{\mu V r_0^2}{h_0^3}$$

where
- $\mu = $ absolute viscosity of the gas
- $V = $ velocity of impact
- $r_0 = $ radius of target

setting $p_{\text{avg}} = 2p_a$ and solving for the required value of $h_0$ gives

$$h_0 = \frac{3}{2} \left( \frac{\mu V r_0^2}{p_a} \right)^{1/3}$$

(A.8)

From this point on the gas is assumed to be trapped and to obey Eq. (A.7) written as

$$p_{\text{max}} = \frac{p_a h_o}{h_{\text{min}}}$$

(A.9)

Clearly this equation predicts an infinite pressure if $h_{\text{min}} = 0$. The limiting value $h_{\text{min}}$, rather than being zero, should be the value at which contact of the high points on the opposing surfaces begins i.e., equal to the surface roughness and waviness of the impacting surfaces.

As an example, for room conditions
\[ p_a = 15 \text{ psi} \]
\[ \mu = 2.8 \times 10^{-9} \text{ lb} \cdot \text{sec}/\text{in}^2 \]
\[ r_o = \frac{1}{8} \text{ in. (radius of specimen)} \]
\[ V = 12,000 \text{ in/sec.} \]

then from Eq. (A.8)
\[ h_o = 0.36 \times 10^{-2} \text{ in.} \]

substituting this in Eq. (A.9)
\[ p_{\text{max}} = \frac{15 \times 0.36 \times 10^{-2}}{h_{\text{min}}} \]

Taking \( h_{\text{min}} \) as \( 10^{-5} \text{ in.} \) gives
\[ p_{\text{max}} = 5500 \text{ psi} \text{ which is very large.} \]

However, if \( p_a \) is lowered by a factor of \( 10^{-2} \), i.e. \( p_a = 0.15 \text{ psia} \), a pressure readily attained by ordinary vacuum pumps,
\[ p_{\text{max}} = 55 \text{ psi} \text{ which may be neglected.} \]

E. The Silencer

The pressure of the air exhausting from the muzzle may be as high as 1200 psi. It is therefore essential that the air be expanded gradually if noise levels are to be maintained at a bearable level. For this purpose, a silencer was constructed. It consists of a 36 in. diameter, 52 in. long steel drum installed between the specimen chamber and the sand drum so that the air exhausting from the muzzle first expands in the vacuum chamber and
and then into the silencer. One side of the drum is perforated to allow
the air to then flow into another chamber where after passing over a
series of baffles, it exhausts into the atmosphere. The inside surfaces
of the silencer are coated with a 3/4 in. layer of heavy sound deadening
paint to muffle the sound. In actual tests this design proved to be re-
markably efficient and no further modifications were necessary.

F. The Projectile Recovery System

The kinetic energy of the projectile travelling at 1000 ft/sec is
about 82,000 ft-lbs. This energy has to be absorbed quickly, in order to
stop the projectile within a reasonable distance. Sand was chosen as the
energy absorbing medium and was found to be quite suitable in subsequent
testing.

Few theoretical investigations have dealt with the penetration of
projectiles in sand and earthwork; however numerous empirical formulas
that describe penetration of military projectiles into various materials
have been developed over the years. Some of the earliest and best ex-
perimentation in penetration was done by the French between 1835 and 1845.
These were the famous Metz experiments and are described in Helie's
"Traite de Ballistighe" published in 1884 (from Ref. (44)). During this
period, the French were concerned with spherical cannon balls whose velo-
cities were about 1800 ft/sec, whose weights were about 24 lbs. and whose
diameters were about 7 or 8 inches. They fired into masonry, earth, sand,
coal and oak and developed the following penetration formula

\[ a = \frac{m}{2BA} \log_e \left( 1 + \frac{b}{a} V_o^2 \right) \]  \hspace{1cm} (A.10)
where \( S \) = distance penetrated
\( m \) = mass of projectile
\( A \) = cross sectional area of projectile
\( V_0 \) = velocity of impact
\( a, b \) = constants depending upon the material that is penetrated.

For sand, \( a = 620 \text{ psi} \) and \( b = 0.0115 \frac{\text{lb sec}^2}{\text{ft}^2 \text{in}^2} \)

Using these values, the depth of penetration for a 2 in. diameter, 6 in. long steel projectile travelling at 1000 ft/sec, is equal to 7 ft. from Eq. (A.10). In the actual tests, Eq. (A.10) predicted the depth of penetration quite accurately at the high velocities but failed at low velocities, the penetration distance being larger than predicted. However in no case did the penetration depth exceed 7 feet.

The sand drum is shown in Fig. 7. It is 8 ft long, 30 in. in diameter and is mounted horizontally on rollers. A 1/4 in. perforated steel sheet is fitted along its diameter. Sand is filled only up to the level of the perforated sheet. After firing, the drum is rotated through 180° allowing the sand to sieve through the perforated sheet leaving the projectile, output rod and specimen which can then be collected through access doors mounted on the side of the drum.
APPENDIX B - DATA ANALYSIS AND COMPUTER PROGRAM

The digitalizer has an output consisting of two five digit integer numbers representing \( Y \), the stress axis and \( X \), the time axis, respectively. The axes on the oscilloscope trace photograph are first aligned with the axes of the digitalizer. The first card of the data deck punched by the digitalizer is the fiducial card which represents the scaling factor to be used when converting from the digitalizer output to the photograph. This is obtained by punching the coordinates of the diagonally opposite nodes of a 1 cm square grid on the photograph. If these two coordinates are \( \text{FIDY}(1), \text{FIDX}(1) \), and \( \text{FIDY}(2), \text{FIDX}(2) \), then

\[
\text{SCALEX} = \text{Absolute value of } [\text{FIDX}(2) - \text{FIDX}(1)] \quad \text{(B.1)}
\]

and

\[
\text{SCALEY} = \text{Absolute value of } [\text{FIDY}(2) - \text{FIDY}(1)] \quad \text{(B.2)}
\]

where \( \text{SCALEX} \) and \( \text{SCALEY} \) represent the scale factors relating the five digit coordinates of the digitalizer to one centimeter on the photograph. Beginning from the next card, coordinates of the oscilloscope trace are punched starting from the origin \( Y(1), X(1) \) which is the point on the trace where the stress just begins to rise from the trace base line. The number of points taken is \( N \) which may be from 80 to 140. Then the time and experimental stress at any point \( I \) is

\[
\text{TIME}(I) = \text{Abs. value of } [X(I) - X(1)] \frac{\text{SFT}}{\text{SCALEX}} \quad \text{(B.3)}
\]

\[
\text{SIGEXP}(I) = \text{Abs. value of } [Y(I) - Y(1)] \frac{\text{SPS}}{\text{SCALEY}} \frac{(\text{OUCD})}{(\text{OD}^2 - \text{DI}^2)} \quad \text{(B.4)}
\]
where

SPT is the time in sec per cm,

and SFS is the psi per cm on the oscilloscope,

OUTOD is the diameter of the output rod,

OD is the outside diameter of the specimen,

and DI is the inside diameter of the specimen (in the case of solid specimens DI = 0.)

Thus Eq. (B.4) is the experimental stress in the specimen in terms of the stress measured in the output rod. Then from Fig. 5,

\[ V_2(I) = \frac{\text{SIG}_{\text{EXP}}(I)}{\text{RHOC}} \quad (B.5) \]

where RHOC is the product of the density times the longitudinal wave velocity in the Ti alloy output bar (= 88.0 lb sec^2/\text{in}^2). From the point of view of the analysis, the origin is the point at which the projectile makes contact with interface I of the specimen (see Fig. 5). This point is $\frac{GL}{CO}$ sec before the origin used in the digitaliser; GL is the specimen gage length and CO is the velocity of the longitudinal wave in the specimen.

Then,

\[ V_1(I) = \frac{VX}{4 \times 10^{-6}} \text{ TIME}(I) + \frac{GL}{CO} \quad (B.6) \]

If

\[ \text{TIME}(I) + \frac{GL}{CO} < 4 \text{ microsecs} \]

then

\[ V_1(I) = VX \quad (B.7) \]

where VX is the velocity of the projectile. The strain rate is

\[ \text{STRARAT}(I) = \frac{V_1(I) - V_2(I)}{GL} \quad (B.8) \]

The strain is determined from Eq. (2.5) which in this analysis works out to be
\[ \text{STRAIN}(I) = \text{STRAIN}(I-1) + \frac{(\text{TIME}(I) - \text{TIME}(I-1))}{2} \]

\[
(\text{STRAIT}(I) + \text{STRAIT}(I-1)) + \frac{\text{GL}^2}{\text{CO}} \frac{\text{VX}}{8 \times 10^{-6}}.
\]

(B.9)

Integration is performed incrementally by using the trapezoidal rule which gives sufficient accuracy since a large number of increments \( N \) are used in the analysis. The last term in the above expression is the small area of the triangle OAB in Fig. 5. The true compressive strain is then

\[ \text{TRUSTR}(I) = -\ln [1 - \text{STRAIN}(I)] \]

(B.10)

The true experimental compressive stress is

\[ \text{TRUSIG}(I) = \text{SIGEXP}(I) [(1 - \text{STRAIN}(I)] \]

(B.11)

and the true strain rate is

\[ \text{TRUSTRAT}(I) = \frac{\text{STRAIT}(I)}{1 - \text{STRAIN}(I)} \]

(B.12)

The lateral inertia correction stress \( \text{SIGIN}(I) \) is a true stress and is determined from Eq. (2.14). The corrected true stress is then

\[ \text{SIGCOR}(I) = \text{TRUSIG}(I) - \text{SIGIN}(I) \]

The correction for lateral inertia is employed only for the solid specimens and not for the hollow specimens.

The computer program following the above line of analysis is given on the next page, followed by the data input and the corresponding output for test No. 12, the photographic record of which is shown in Fig. 14.
COMPUTER PROGRAM

ANALYSIS OF HIGH STRAIN RATE DATA

PROGRAM HIVE1(INPUT,OUTPUT,TAPE1=INPUT)
DIMENSION SIGFXP(140), SIGIN(140), SIGCOR(140), TRUSIG(140),
   STRARAT(140), STRAIN(140), TRUSTR(140), TIME(140), V2(140), V1(140)
   X(140), Y(140), FIDX(140), FIDY(140), TRSTRAT(140)
INTEGER X, Y, FIDX, FIDY
NAMELIST/DATA/RHOC,DFNS,CO,OUTOD,NTEST,VX,Gl,OD,DI,SFS,SFT,N
   STRAIN(1)=0
1 READ(1,DATA)
   READ 100,(FIDX(I),FIDY(I),I=1,2)
100 FORMAT(10X,4I5)
   READ 100,(Y(I),X(I),I=1,N)
110 FORMAT(10X,4I5)
   SCALEX = IABS(FIDX(2)-FIDX(1))
   SCALEY = IABS(FIDY(2)-FIDY(1))
   DO 10 I=1,N
   TIME(I) = (IABS(X(I)-X(1)))/SCALEX
   SIGFXP(I) = (IABS(Y(I)-Y(1)))/SCALEY*(OD**2-DI**2))
   V2(I) = SIGFXP(I)/(RHO)
   IF (TIME(I)-4.0E-06-GL/CO)) 15,16,16
5 V1(I)=VX/4.0E-06*(TIME(I)+GL/CO)
   GO TO 17
16 V1(I)=VX
17 STRARAT(I)=(V1(I)-V2(I))/GL
10 CONTINUE
(Computer Program - Continued)

DO 11 1=1:N
   STRAIN(1)=STRAIN(I-1)+{(TIME(I)-TIME(I-1))/2}*(STRAP(1)+STRAP(I-1))
   +((GL/CO)**2)*VX/8.06
   TRISTR(1)=ALOG(1.-STRAIN(I))
11 CONTINUE
DO 12 I=1:N
   IF(D1.GT.0.) GO TO 30
   SIGIN(I)=3.*DENS/32.*(STRAP(I)*OD)**2/(1.-TRISTP(I))*3
   GO TO 31
30 SIGIN(I)=0
31 TRUSIG(I)=SIGEXP(I)*(1.-STRAIN(I))
   SIGCOR(I)=TRUSIG(I)-SIGIN(I)
   TRSTRAT(I)=STRAT(1)/(1.-STRAIN(I))
12 CONTINUE
PRINT 41,TST,ROH,C,DENS,OD,DI,GL,SV,ST,FS,N,CO,OUT
41 FORMAT(*EXPERIMENTAL RESULTS*//,**TEST NUMBER*13,/* PRODUCT OF
   DENSITY AND LONGITUDINAL SOUND SPEED OF OUTPUT BAR (IN SF/2 IN-3)
   *=G15.6/* SPECIMEN DENSITY (LP SF/2 IN-4) *=G15.6/* SPECIMEN
   OD (IN) *=G15.6/* SPECIMEN ID (IN) *=G15.6/* SPECIMEN GAGE
   LENGTH (IN) *=G15.6/* PROJECTILE VELOCITY (IN SEC-1) *=G15.6/
   ** TIME SCALE FACTOR (SEC PER CM) *=G15.6/* STRESS SCALE FACTO-
   R (PSI PER CM) *=G15.6/* NUMBER OF STRESS VALUES *=G13/*
   ** LONGITUDINAL SOUND SPEED IN SPECIMEN (IN PER SEC) *=G15.6/
   ** OD OF OUTPUT BAR (IN) *=G15.6/* //2X
   ** TIME *.6X* EXPTL. STRESS*.5X* TRUE EXPTL. STRESS*.7X* INERTIA
   ** STRESS*.6X* TRUE STRESS*.8X* TRUE STRAIN*.3X* TRUE STRAIN RATE
   **/)
DO 44 I=1:N
PRINT 42*(TIME(I),SIGEXP(I),TRUSIG(I),SIGIN(I),SIGCOR(I),TRISTR(I)
   TRSTRAT(I))
44 CONTINUE
42 FORMAT(7(13H,G15.6))
GO TO 1
END
DATA INPUT FOR A TYPICAL TEST

(Oscillograph shown in Fig. 14)

\[ \text{DATA RHOC=88.0, DENS=0.200251, CO=303700, OUTOD=0.319,} \]
\[ \text{MTEST=12, VX=1436.0, GL=0.7510, ON=0.248, OL=0.0000, FS=6188.0, FT=0.000010,} \]
\[ M=900.} \]

17 50153701345701277025
12 36793122317051354781777318882385088200232362324324667
12 398843260240333327604801902976414793338941890434514232533942
12 4269834210430233448643473472843751349674413535264451335596
12 44791358674507836142646236560457403684946941272014640437558
12 466693794449611383424716238824441440395614766839609478840421
12 4799140812481644120463104149748633420944838472074871543251
12 4889943067490454400049125452894028145836494314624462946769
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12 5181554069519955530062151560863395856781652658372704309057893
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12 543756236054593631795490063905552136142526379686505647566524
12 559106718756110670565634868652566436945556917701927277270966
12 5748476145773077335541333730285838973311589694745556896075239
12 59195758995946376624597857738360217814660567701040608279567
### COMPUTER OUTPUT FOR A TYPICAL TEST
(Oscillograph shown in Fig. 14)

#### EXPERIMENTAL RESULTS

<table>
<thead>
<tr>
<th>TEST NUMBER</th>
<th>PRODUCT OF DENSITY AND LONGITUDINAL SOUND SPEED OF OUTPUT BAR 1B SEC 1-3</th>
<th>88.000000</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPECIMEN CD (IN.)</td>
<td>1.40000</td>
<td></td>
</tr>
<tr>
<td>SPECIMEN IN (IN.)</td>
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<td></td>
</tr>
<tr>
<td>SPECIMEN CASE LENGTH (IN.)</td>
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</tr>
<tr>
<td>PROJECTILE VELOCITY (IN SEC-1)</td>
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<tr>
<td>TIME SCALE FACTOR ESE PER CM</td>
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</tr>
<tr>
<td>STRESS SCALE FACTOR (PSI)</td>
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<td>NUMBER OF STRESS VALUES</td>
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<td></td>
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<tr>
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<td></td>
</tr>
<tr>
<td>OD OF OUTPUT BAR (IN.)</td>
<td>0.314000</td>
<td></td>
</tr>
</tbody>
</table>

#### TIME | EXPL. STRESS | TRUE EXPL. STRESS | INERTIA STRESS | TRUE STRESS | TRUE STRAIN | TRUE STRAIN RATE |
<table>
<thead>
<tr>
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</tr>
</tbody>
</table>

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The table above contains experimental results for a typical test as shown in the oscillograph in Fig. 14. The test details include the product of density and longitudinal sound speed of the output bar, the specimen dimensions, and various experimental conditions. The table lists the time in seconds, the experimental stress, the true experimental stress, the inertia stress, the true stress, the true strain, and the true strain rate for each data point. The values are presented in scientific notation and are accurate to the specified number of significant figures.
ACKNOWLEDGEMENTS

I am deeply indebted to Professor F. E. Hauser for his patient guidance and many valuable suggestions during the course of this project. I also wish to express my gratitude to Professor J. E. Dorn for his continued interest and support.

In addition, thanks are due to Dr. Maximo P. Victoria, post graduate fellow, for many interesting and useful discussions; and to Dr. Janusz Klepaczko, visiting professor, and Mr. Dale Klahn, graduate student, for their assistance in conducting the experiments. Lastly, I am indebted to Mr. Les Seaborn for his indispensable help in setting up the equipment.

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REFERENCES


FIGURE CAPTIONS

Fig. 1. Effect of temperature and strain rate on the flow stress of aluminum single crystals.  

Fig. 2. Prediction of the dynamic plastic behavior in the intersection region for pure Al at $T = 77K$.  

Fig. 3. Effect of stress on the strain rate in dynamic tests in pure Al at 295K.  

Fig. 4. Schematic arrangement of instrumentation.  

Fig. 5. Determination of stress, strain and strain rate from strain gage output $\sigma_e(t)$.  

Fig. 6. Stress vs. strain diagram for a typical test. Velocity of impact = 4336 in/sec.  

Fig. 7. The apparatus. 1 - cross, 2 - pressure vessel, 3 - gun barrel, 4 - vacuum chamber, 5 = silencer, 6 - sand drum.  

Fig. 8. Specimen holder. 1 - light sources, 2 - photo diodes and amplifiers, 3 - vacuum chamber ( can be drawn across and bolted to flange ); 4 - specimen aligning screws 5 - gun barrel.  

Fig. 9. 1 - projectile, 2 - specimen (held by a small section of heat collapsible plastic tubing), 3 - output rod, 4 - ground wire (to trigger scope), 5 - plastic sleeve, 6 - strain gage leads.  

Fig. 10. Arrangement for the measurement of projectile velocity just before impact.  

Fig. 11. Oscillograph from direct impact on Ti-6Al-4V output rod. Velocity of impact = 6110 in/sec. Vertical axis (stress in output rod) = 123,800 psi/div. Horizontal axis (time) = 20 micro seconds/div.
Fig.12. Variation of stress level with impact velocity for direct impacts on Ti-6Al-4V output rods.

Fig.13. Instrumentation. 1 - oscilloscope, 2 - trigger, 3 - time interval meter, 4 - constant voltage source for photodiode lights.

Fig.14. Oscillograph of a typical test. Velocity of impact = 1436 in/sec. Vertical axis (stress in output rod $\sigma_b$) = 6,199 psi/div. Horizontal axis (time) = 10 micro seconds/div. The data input and computer output for this test is shown in Appendix B.

Fig.15. All obtained data points for 10 percent strain showing deviations because of stress gradients across the specimen.

Fig.16. Percentage deviation in stress vs. number of transits at longitudinal elastic wave velocity across specimen. At least 8 to 9 transits are required for equilibrium to be reached.

Fig.17. True compressive stress vs. true strain rate at 5 percent strain.

Fig.18. True compressive stress vs. true strain rate at 10 percent strain.

Fig.19. True compressive stress vs. true strain rate at 15 percent strain.

Fig.20. True compressive stress vs. true strain rate at 20 percent strain.

Fig.21. Shear stress vs. shear strain-rate for dynamic shear in Al single crystals, (after Ferguson et al.22).

Fig.22. Velocity of dislocations vs. applied resolved shear stress (after Gorman et al.15).

Fig.23 Dislocation drag coefficient vs. temperature (after Gorman et al.25).

Fig.24. Schematic of firing mechanism.

Fig.25. Breech end of gun. 1 - normally closed solenoid valve, 2 - normally open solenoid valve, 3 - vacuum hose.

Fig.26. Opened breech to show 1 - electromagnet and 2 - projectile.
Fig. 1. Effect of temperature and strain rate on the flow stress of aluminum single crystals. See Ref. 5.
Fig. 3

\( \dot{\varepsilon}_{\text{AVE}}, \) AVERAGE STRAIN RATE/SECOND

\( \sigma_{\text{AVE}}, \) AVERAGE STRESS IN KSI

- SLOW TENSION
- DYNAMIC
- PREDICTED LIMIT FOR INTERSECTION (\( \dot{\varepsilon} = 120/\text{SEC.} \))

I ELASTIC
II THERMALLY ACTIVATED INTERSECTION
III LINEAR VISCOUS
FIG. 4 SCHEMATIC ARRANGEMENT OF INSTRUMENTATION.

SPECIMEN, 0.250 IN. DIA.

OUTPUT ROD, 0.3195 IN. DIA.

PROJECTILE, 2 IN. DIA.

STRAIN GAGES

CONSTANT CURRENT SOURCE

OSCILLOSCOPE
AS: Cross sectional area of specimen.

$A_E$ = Cross sectional area of output rod.

$C_0$ = Longitudinal elastic wave velocity in specimen.

FIG. 5 DETERMINATION OF STRESS, STRAIN AND STRAIN RATE FROM STRAIN GAGE OUTPUT $\sigma_E(t)$. 

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FIG. 6 STRESS vs STRAIN DIAGRAM FOR A TYPICAL TEST. VELOCITY OF IMPACT = 4336 IN/SEC.

$\dot{\varepsilon}_{AV} = 2.5 \times 10^4$ SEC$^{-1}$

FOR $\varepsilon \geq 0.12$
Fig. 8
FIG. 10 ARRANGEMENT FOR THE MEASUREMENT OF PROJECTILE VELOCITY JUST BEFORE IMPACT.
Fig. 11
FIG. 12 VARIATION OF STRESS LEVEL WITH IMPACT VELOCITY FOR DIRECT IMPACTS ON Ti6Al4V OUTPUT RODS.

SLOPE = 88.0 lb·sec²/in.⁴

VELOCITY OF IMPACT, \( v \times 10^{-3} \) IN/SEC

COMPRRESSIVE STRESS, \( \sigma \times 10^4 \) PSI

STATIC YIELD STRENGTH, PSI
174,500

STATIC ULTIMATE STRENGTH, PSI
188,000

O.3195 in. Dia. Ti6Al4V Output Rod
FIG. 17

TRUE COMPRESSIVE STRESS, $\sigma \times 10^3$ PSI

TRUE STRAIN RATE, $\dot{\varepsilon} \times 10^{-3}$ SEC$^{-1}$

- △ UNCORRECTED
- △ CORRECTED FOR LATERAL INERTIA
- ○ ANNULAR SPECIMENS

EQ. 3.10 WITH $\rho_m = 1.1 \times 10^7$ CM$^{-2}$

$\langle \alpha \rangle_\Pi = 3\langle \alpha \rangle_\Pi = 24,810$ DYNE-SEC/CM$^2$

$\varepsilon_c = 6800$ SEC$^{-1}$

REGION I
REGION II
REGION III
\( \Delta \) CORRECTED FOR LATERAL INERTIA

\( \circ \) ANNULAR SPECIMENS

EQ. 3.10 WITH
\( \rho_m = 1.55 \times 10^7 \text{ CM}^{-2} \)

\( (\alpha_c)_\Pi = 3 (\alpha)_\Pi \)

\( = 14,460 \text{ DYNE-SEC/CM}^2 \)

\( (\alpha_c)_I = 3 (\alpha)_I = 41,400 \text{ DYNE-SEC/CM}^2 \)

\( \dot{\varepsilon}_c = 10,450 \text{ SEC}^{-1} \)

FIG. 18
FIG. 19

TRUE COMPRESSIVE STRESS, $\sigma \times 10^{-3}$ PSI

- UNCORRECTED
- CORRECTED FOR LATERAL INERTIA
- ANNULAR SPECIMENS

EQ. 3.10 WITH $\rho_m = 1.77 \times 10^{-7}$ CM$^{-2}$

$(\alpha_c)_{II} = 3(\alpha)_I$

$= 9660$ DYNE-SEC/CM$^2$

$(\alpha_c)_I = 3(\alpha)_I = 46,200$ DYNE-SEC/CM$^2$

REGION I
REGION II
REGION III

$\dot{\varepsilon}_c = 10,950$ SEC$^{-1}$

TRUE STRAIN RATE, $\dot{\varepsilon} \times 10^{-3}$ SEC$^{-1}$

0 10 20 30 40 50 60 70 80 90 100
FIG. 20

TRUE COMPRESSIVE STRESS, $\sigma \times 10^{-3}$ PSI

TRUE STRAIN RATE, $\dot{\varepsilon} \times 10^{-3}$ SEC$^{-1}$

- UNCORRECTED
- CORRECTED FOR LATERAL INERTIA
- ANNULAR SPECIMENS

EQ. 3.10 WITH $\rho_m = 2.01 \times 10^7$ CM$^{-2}$

$[\alpha_c]_I$ = 3 $[\alpha_1]_I$ = 48,300 DYNE-SEC/CM$^2$

$[\alpha_c]_I$ = 3 $[\alpha_1]_I$ = 8280 DYNE-SEC/CM$^2$

REGION I

REGION II

REGION III

$\dot{\varepsilon}_c \times 12,400$ SEC$^{-1}$
Fig. 21
RESOLVED SHEAR STRESS, $10^6$ dynes/cm$^2$

c. ROOM TEMPERATURE (23°C)

Fig. 22

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-92-

Fig. 23
TO VACUUM PUMP

NORMALLY OPEN SOLENOID VALVE

FROM PRESSURE CHAMBER

NORMALLY CLOSED SOLENOID VALVE

HIGH PRESSURE

PROJECTILE

BARREL (EVACUATED)

ELECTROMAGNET

CROSS PIPE FITTING

Fig. 24
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