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IMPACT OF ANISOTROPIC STRESS STATES ON THE THERMAL VOLUME CHANGE OF UNSATURATED SILT

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Abstract: This study is focused on understanding the influence of anisotropic stress states on the thermal volume change of unsaturated, compacted silt specimens. A thermo-hydraulic-mechanical true-triaxial cell was used that permits control of the temperature on all six boundaries of a cubical soil specimen as well as control of the suction within the specimen to provide drained conditions during mechanical loading and temperature changes. Six non-isothermal tests were performed as part of this study, each involving suction application, consolidation to a given isotropic or anisotropic stress state, heating and cooling in stages under drained conditions, and unloading. Specifically, tests having minor to major principal stress ratios of 1.0, 0.7, and 0.5 were performed on specimens having initial degrees of saturation of 0.7 and 0.8, complementing tests on the same soil under similar stress states but saturated conditions published in a previous study. Although compressive thermal axial strains were measured in both the major and minor stress directions, a greater thermal axial strain was observed in the direction of the major principal stress for stress ratios less than 1.0. However, similar thermal volumetric strains were observed in all of the tests regardless of the stress state. A small effect of inherent anisotropy was observed due to the formation of the specimen using compaction. Specimens with a lower initial degree of saturation experienced greater thermal volume changes than specimens closer to saturation, possibly due to thermal collapse of the air-filled voids during heating or thermally accelerated creep after application of a given plastic strain during mechanical loading. An empirical relationship to consider the effects of anisotropic stress states

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and variable saturation was incorporated into an established elasto-plastic model developed for
saturated soils under isotropic conditions, and a good fit was obtained between the measurements
and predictions.

INTRODUCTION

A technique used to improve the energy efficiency of heat pumps for building heating and
cooling systems is to embed closed-loop heat exchangers into drilled shaft foundations to form
energy piles (Brandl 2006; Laloui et al. 2006; Adam and Markiewicz 2009; McCartney 2011;
Murphy and McCartney 2015; Murphy et al. 2015). Heat can be transferred to and from the
ground by circulating fluid through the heat exchangers. When the soil surrounding the energy
pile changes in temperature, potentially irreversible soil volume changes may occur depending
on the stress state, soil mineralogy, and degree of saturation (Campanella and Mitchell 1968;
Demars and Charles 1982; Hueckel and Baldi 1990; Towhata et al. 1993; Burghignoli et al.
2000; Delage et al. 2000; Sultan et al. 2002; Romero et al. 2003; Cekerevac and Laloui 2004;
Salager et al. 2007; Uchaipichat and Khalili 2009; McCartney 2012; Vega and McCartney 2015;
Alsherif and McCartney 2015; Coccia and McCartney 2012, 2016a, 2016b). These thermal
volume changes may affect the lateral stress distribution along the energy foundation, and may
lead to relative movement between the energy pile and surrounding soil (Vega and McCartney
2015). Although constitutive models are available to consider the thermal volume change of soils
(Hueckel and Pellegrino 1989; Hueckel and Borsetto 1990; Cui et al. 2000; Laloui and
Cekeravac 2003; Abuel-Naga et al. 2009), an isotropic stress state is assumed. Accordingly, the
same thermal expansion or contraction is assumed to occur in both the major principal stress
(vertical) and minor principal stress (horizontal) directions when simulating the behavior of soils
surrounding energy piles. Despite the experimental data available on the recoverable and
permanent deformations of unsaturated soil during heating and cooling, the influence of anisotropic stress states on the thermal deformation of unsaturated soils still needs to be better understood before constitutive relationships thermal volume change of soils can be incorporated into design methods for energy piles such as that of Knellwolf et al. (2011).

Coccia and McCartney (2012) studied the effect of anisotropic stress states on the thermal volume change of saturated specimens of compacted Bonny silt using a specially-designed thermo-hydro-mechanical (THM) true-triaxial cell. The results from their testing program showed that the anisotropic stress state does not have a significant influence on the thermally-induced volumetric strain. However, the initial anisotropic stress state does have a significant influence on the magnitude and trend of thermal axial strains in the directions of the major and minor principal stresses. Specifically, during drained heating of anisotropically-consolidated cubical specimens isotropically compressed to normally consolidated conditions before being unloaded in the minor principal stress direction, plastic contraction was observed in the major principal stress direction while less contraction (or even expansion) was observed in the minor principal stress direction. The difference in the thermal axial strains was found to depend on the ratio of the minor to major principal stresses (referred to as the stress ratio K). Coccia and McCartney (2012) proposed that the plastic contraction in the major principal stress direction was because the stress was still at the maximum stress level encountered in that direction, while the lower plastic contraction or expansion in the minor principal stress direction was due to the stress history in that direction that made the behavior similar to an overconsolidated soil.

In addition to the need to perform more in-depth testing to fully understand the influence of anisotropic stress states on the thermal volume change of soils, there are opportunities to improve the experimental approach involving the THM true-triaxial cell used by Coccia and
McCartney (2012). They had intended to perform their tests in plane strain conditions, but found that the thermal expansion of the cell during heating may have led to changes in the intermediate principal stress and strain values. They also only heated the specimen on two opposing faces, requiring a long heating period to reach thermal equilibrium. Accordingly, efforts were made in this study to implement better control of the principal stresses and temperatures applied to each face during testing. Another issue with the approach used by Coccia and McCartney (2012) is that the unloading of the specimen in the minor principal stress direction may have led to a decrease in the mean stress, which may have led to a slight overconsolidation effect in the specimen which could have potentially affected the trends in the thermal volumetric strain. A better approach would be to increase the major principal stress to reach an anisotropic stress state so that the mean stress increases but still remains in normally consolidated conditions. Finally, although Coccia and McCartney (2012) designed their true-triaxial cell with the capability to evaluate unsaturated conditions, they only focused on the behavior of saturated specimens meaning that their work could be complimented by investigations using this feature.

BACKGROUND

Previous studies have observed that the overconsolidation ratio (OCR) has a significant effect on the thermal volume change of a soil specimen when heated under a constant mean stress in drained conditions. Soils with larger OCR values trend to expand during heating while soils with lower OCR values trend to contract (Baldi et al. 1988; Towhata et al. 1993; Cekerevac and Lalouï 2004; Sultan et al. 2002; Abuel-Naga et al. 2007; Vega and McCartney 2015). For example, Cekerevac and Lalouï (2004) observed thermal contraction for saturated kaolinite clay specimens with OCR values between 1 and 2 while thermal expansion was observed for specimens at higher values of OCR of values 6 and 12 (although these specimens were observed
to contract after reaching a certain temperature). Although there have not been many studies on
the thermal volume change of soils under anisotropic stress states except that of Coccia and
McCartney (2012), Hueckel and Pellegrini (1996) found that inherent anisotropy arising from the
orientation of clay particles perpendicular to the direction of consolidation or compaction may
lead to greater lateral thermal axial strains than vertical thermal axial strains in saturated clays
for heating under isotropic stress states.

The effect of temperature on the volume change behavior of unsaturated soils has been
evaluated in several studies (Romero et al. 2003; Francois et al. 2007; Salager et al. 2008; Tang
Uchipichat and Khalili (2009) studied the effect of the thermal volume change of normally
consolidated and overconsolidated soils having constant suction conditions. Similar to saturated
soils, they observed thermal expansion in overconsolidated specimens during drained heating-
cooling tests between 25 to 60 °C, but observed contraction under lower overconsolidation
ratios. They did not observe a significant effect of the initial degree of saturation on the thermal
volume change, but the fact that several different stress paths were applied to the specimens
before heating may have affected the influence of this variable. Alsherif and McCartney (2015)
performed heating tests on compacted Bonny silt at low suction (0.04 MPa) and high suction
(300 MPa) magnitudes, and found that soils heated under low suction contract but those at high
suctions expand. Their results were reinterpreted by Alsherif and McCartney (2016) in terms of
the mean effective stress and OCR, and it was observed that the trends are similar to those for
saturated specimens of compacted Bonny silt tested by Vega and McCartney (2015).
MATERIAL

Bonny silt was used in this study as its thermal volume change has been investigated under different stress states and degrees of saturation in previous studies (Coccia and McCartney 2012; Vega and McCartney 2015; Alsherif and McCartney 2015). Bonny silt has a fines fraction of 83%, a liquid limit of 25, and plastic limit of 21, so it is classified as ML (inorganic low plasticity silt) according to the Unified Soil Classification Scheme (USCS). The specific gravity of the particles is 2.65. The specimens tested in this study were prepared using compaction in order to reach initial degrees of saturation of 0.7 and 0.8. All cubical soil specimens were prepared using static compaction to reach the same initial dry unit weight of 16.2 kN/m$^3$, which corresponds to an initial void ratio of 0.59. This dry unit weight corresponds with about 98% of the maximum dry unit weight from the standard Proctor compaction curve (16.6 kN/m$^3$). The target gravimetric water content values investigated in this study were 15.5 and 17.5%, which correspond to initial degrees of saturation of 70% and 80%, respectively, and are both wet of the optimal gravimetric water content for the standard Proctor compaction effort (14%).

Before compaction, the soil was mixed with water until the target gravimetric water content was reached. It was then sealed within a five-gallon bucket for 24 hours to allow the water content to homogenize within the soil. A mechanical press was used to compress the specimen in six lifts of equal height within a cubical aluminum mold having a side length of 178 mm. The under-compaction technique was used to ensure uniform compaction of the specimen. The top of each compacted lift was scarified to minimize any potentially weak planes within the cubical specimen. To remove the specimen from the mold, the specimen was first pushed from the mold using the press before removing the side walls so that this process does not pull on the surfaces of the specimen. The compacted specimen was covered immediately with plastic wrap to avoid
changes in the degree of saturation while the rest of the components of the cubical cell were being assembled. Pictures of the mold and specimen during and after preparation are presented by Shanina (2015).

**EXPERIMENTAL APPROACH**

**Experimental Setup**

The true-triaxial cell used in this study was originally developed by Mould (1983) and updated by Takata (2000). In its typical configuration, this cell uses six flexible latex bladders reacting against aluminum mechanical loading plates to apply principal stresses to the faces of a cubical soil specimen, following the approach outlined by Ko and Scott (1967). The deformations of each face of the true-triaxial cell are measured using spring-loaded linearly variable differential transformers (LVDTs). Coccia and McCartney (2012) adapted the cell for thermo-hydro-mechanical testing by replacing the bottom and top faces of the cell with rigid face plates. This study used replaced only the bottom face of the cell with a rigid face plate, similar to the approach of Hoyos and Macari (2001), Hoyos et al. (2008), and Hoyos et al. (2012). As shown in the cross-section schematics of the true-triaxial cell in Figures 1(a) and 1(b), the bottom face in the z-direction is a rigid face plate that contains porous disks that independently apply pore air pressure ($u_a$) or water pressures ($u_w$), while the other faces in the x- and y-directions are flexible bladders that are used to apply total principal stresses. The bottom rigid face plate incorporates heating elements to control temperature on this face of the specimen and hydraulic ports to control the pore water and air pressures within the specimen (or to provide drainage during mechanical loading or changes in temperature). This rigid face plate is referred to as a hydro-thermal face plate.
This study involved additional modifications to the THM true-triaxial cell to address some of the issues encountered by Coccia and McCartney (2012). First, the THM true-triaxial cell was configured to apply principal stresses in all three orthogonal directions by maintaining a mechanical loading plate on the $z_1$ face of the cell, as shown in Figure 1(a). This permits application of anisotropic stress states to a cubical soil specimen that correspond with the bedding planes associated with compaction of the soil specimen (i.e., major principal stress in the vertical direction orthogonal to the bedding planes and minor principal stresses applied equally in the two horizontal directions parallel to the bedding planes). This approach also permits independent measurement of the principal strains in all three orthogonal directions. Further, the modified system permits incorporation of temperature control on each face of the cell by circulating pressurized, heated water through five of the bladders and circulating water through the heating elements in the hydro-thermal face plates. This uniform heating permitted thermal equilibrium to be reached in a shorter period of time than the approach of Coccia and McCartney (2012).

Although the particular anisotropic stress state investigated in this study (stress in the vertical direction equal to the major principal stress, and stresses in the two horizontal directions equal to each other and corresponding to the minor and intermediate principal stresses) correspond to a triaxial compression stress state that could be investigated in an advanced thermal triaxial cell, there are several advantages for using this advanced testing device. First, the cubical cell permits tests to be performed in stress-control conditions. This is desired when measuring thermal volume changes so that the device does not provide displacement constraints on the specimen that could lead to a change in stress state in the specimen. This can only be achieved in a triaxial frame with feedback control or incorporating a pneumatic piston to control the axial stress such
as that used by Alsherif and McCartney (2015). The cell also permits evaluation of much larger specimens than in a conventional triaxial cell which better permits better assessment of inherent anisotropy effects in compacted specimens due to the larger sample size. Even though the thermal axial strains may be similar in experiments performed in the true-triaxial cell and in a conventional triaxial cell, a larger specimen will have larger deformations that are easier to measure within the sensitivity of available measurement systems.

The pore water pressures are applied through a high air-entry (HAE) porous ceramic disk that only allows water to pass until reaching a suction of 300 kPa. The pore air pressures are applied through coarse porous disks that have a very low air entry suction (less than 0.1 kPa). Matric suction in the specimens can either be measured or controlled using the ports at the base of the cell. To measure the matric suction, the air pressure is maintained at atmospheric conditions and the pressure in the water reservoir behind the HAE ceramic disk is measured in the same manner as a tensiometer. Specifically, the water pressure within the reservoir beneath the HAE ceramic disk was monitored using a pressure sensor. If a compacted soil is placed atop the ceramic disk, there will be a tendency to draw water through the disk into the specimen due to the gradient formed by the initial suction in the specimen. Negative water pressures up to approximately -80 kPa can be measured using this approach. At equilibrium, the negative water pressure measured within the reservoir is expected to be approximately equal to the suction within the unsaturated specimen as the pore air pressure is assumed to be zero.

Alternatively, the suction can be controlled using the axis translation technique (Hilf 1956). In this case, positive pore air and water pressures can be applied independently to the base of the specimen through the low air entry and high air entry disks, respectively, with a difference (u_a – u_w) being equal to the matric suction in the specimen. Although it is straightforward to control
the suction in the specimen to provide drained conditions, the particular configuration evaluated in this study shown in Figure 1(a) is not optimal for changing the suction in the specimen because water must flow from the bottom face of the specimen to the upper corners of the specimen through capillarity, which can be time consuming and may lead to air entrapment. Accordingly, the approach used in this study is to measure the initial suction in the compacted specimen using the tensiometer approach, and then subsequently apply this same suction using the axis translation technique to permit drained mechanical compression and drained heating experiments to be performed. This approach avoids the need to wait for complicated water flow processes before applying thermo-mechanical loads to the specimen. Although this approach may be affected by the different soil structures induced by compaction, both initial degrees of saturation investigated in this study correspond to compaction wet of optimum so the soil structure soil not be a major factor.

It should be noted that during transient heating of the specimen, the results from this test cannot be considered to be representative of an element test because the temperature distribution may not be uniform through the volume of the cubical specimen. To investigate the transient heating response, the cubical specimen should be treated as a boundary value problem, as noted by Zhang and Kurimoto (2016). However, at equilibrium it is assumed that the temperature, suction, and degree of saturation will be uniform throughout the cubical specimen and that the results will be representative of an element test. Accordingly, the heating approach used in this study is to apply changes in temperature in stages to obtain equilibrium volume changes that correspond to a uniform change in temperature across the volume of the cubical specimen.
Machine Deflection Evaluation

Before performing tests on the soil, it is necessary to characterize the deflections of the THM true-triaxial cell during changes in stress or temperature. Although the true-triaxial cell was designed to be as rigid as possible, application of stresses to the specimen will cause the space frame to expand outward which may affect the displacement measurements of the LVDTs. The approach used to evaluate the mechanical and thermal machine deflections was to first perform tests on an aluminum cube having known mechanical and thermal properties. The mechanical machine deflections were calculated by subtracting the elastic deflection of the aluminum cube from the average deflections of the LVDTs mounted on the face plates. The deflection of the aluminum cube was calculated as follows:

\[ \delta = \frac{\sigma \times L}{E} \]  

(1)

where \( \delta \) is the elastic deflection of the aluminum cube, \( \sigma \) is the axial stress in kPa, \( L \) is the length of the aluminum cube, which is equal to 178 mm and \( E \) is the Young’s modulus of the aluminum cube, which is equal to 69 GPa. A positive machine deflection is defined as compression. The slopes of the machine deflections in each principal stress direction \( M_x \), \( M_y \) and \( M_z \) were calculated as follows:

\[ M = \frac{d_m}{\sigma} \]  

(2)

where \( M \) is the slope of machine deflection (mm/kPa), \( d_m \) is the mechanical machine deflection (mm) and \( \sigma \) is the axial stress (kPa). The average mechanical machine deflections and the values of \( M \) in each direction during loading are presented in Figure 2(a). The measurements from the three LVDTs used to define these average mechanical machine deflections were nearly identical, indicating negligible bending of the aluminum face plates mounted to the space frame. The
values of mechanical machine deflections can be subtracted from the measured deformation
results from isothermal compression tests on soils during changes in stress. The $z_1$ face showed a
softer response than the other directions, possibly because the bottom face in the $z$ direction is a
rigid plate. The application of anisotropic stresses in the $z$ direction still resulted in linear elastic
behavior, reflected in the same slope of the mechanical machine deflection curve.

It is critical to consider the deformation of the true-triaxial cell during heating and cooling in
order to consider the effects of thermal expansion of the space frame when inferring the
thermally induced volume change of soil specimens. After applying an isotropic stress state of
350 kPa to the aluminum cube (the same stress state used in the tests on the soils that will be
discussed later), the system was heated from an ambient room temperature of 20 °C to a
temperature of approximately 50 °C in three 10 °C intervals, then cooled back to ambient
temperature in one stage. The change in temperature of the cell during heating is shown in Figure
2(b). The thermal machine deflections were calculated by subtracting the expected thermo-elastic
deflection of the aluminum cube from the measured deflections. The LVDTs were used to
measure the deflections of the five faces in the $x$, $y$ and $z$ directions during thermal cycling. The
theoretical displacement of the aluminum cube in each direction was calculated as follows:

$$d_{T,al} = \alpha_{al} \times L_{al} \times \Delta T$$

(3)

where $d_{T,al}$ is the thermal displacement of the aluminum cube, $\alpha_{al}$ is the linear coefficient of
thermal expansion of the aluminum (equal to $23 \times 10^{-6}$ m/m°C), $L_{al}$ is the initial length of
aluminum cube and $\Delta T$ is the change in temperature applied during the test. The thermal
machine deflections were calculated as the difference between the measured deflections from the
LVDTs and the theoretical value of $d_{T,al}$ for a given change in temperature. Because the thermal
machine deflections were found to be thermo-elastic, the slopes of the thermal machine
deflection curves during heating $H_x$, $H_y$ and $H_z$ were calculated as follows:

$$H = d_T / \Delta T$$

(4)

where $H$ is the slope of thermal machine deflection (mm/$^\circ$C), $d_T$ is the thermal machine
deflection (mm) and $\Delta T$ is the change in temperature ($^\circ$C). Although not shown here, Shanina
(2015) found that the slopes of the thermal machine deflection curves during cooling were also
relatively linear and similar to those during heating, so the slopes during heating were used to
correct the thermal machine deflections through the entire test for simplicity.

**Experimental Procedures**

The first step in preparing the THM true-triaxial cell for a test is to saturate the HAE ceramic
disk. As mentioned, this HAE ceramic disk is used to facilitate measurement of the negative
water pressure in unsaturated soils using the tensiometer approach, to apply water pressures
using the axis translation technique, and to measure potential outflow or inflow of water into the
specimen during drained compression or heating. Accordingly, it is critical for the HAE ceramic
disk to be saturated with water before beginning a test. The HAE ceramic disk was first placed
into the recess in the rigid bottom platen of the THM true-triaxial cell in air-dry conditions. Then
a bead of RTV silicon sealant was placed around the edge of the ceramic to provide a hydraulic
seal that prevents short-circuiting of air around the edges of the HAE ceramic disk. After the
silicon sealant cured, water was flushed through the channel beneath the ceramic disk. Next, a
special pressure-saturation device was placed on top of the ceramic disk, which is described in
detail by Shanina (2015). The device consists of a steel chamber with an O-ring seal at the base
that is tightened onto the face of the rigid platen using three screws. After placement of the
chamber atop the HAE ceramic disk, a vacuum of -70 kPa was applied to the base of the disk for
292 24 hours to de-air the ceramic. This chamber was then filled with de-aired water under a pressure
293 of 70 kPa, and the same vacuum on the bottom side of the HAE ceramic disk was maintained.
294 De-aired water was then permitted to flush downward through the HAE ceramic disk overnight.
295 Water flow was oriented downward to avoid putting upward stresses on the seal between the
296 hydro-thermal plate and the HAE disk. After this, the HAE ceramic was assumed to be water-
297 saturated, and air breakthrough was not observed in any of the experiments.

298 Next, the compacted soil specimen was placed carefully on top of one of the flexible latex
299 bladders outside of the THM cell so that the compaction lifts are perpendicular to the bladder
300 face. The THM cell incorporates a tilting apparatus that can be used to facilitate placement of the
301 soil specimen within the frame, which is described in detail by Mould (1983). After attaching the
302 hydro-thermal face plate to the bottom of the frame (which is aligned with the z-axis), the frame
303 was tilted 90° around the hinge-point. The specimen and the flexible bladder were then inserted
304 into the THM cell from the bottom upward so that the z-face of the specimen, which is
305 perpendicular to the compaction lifts, would be in contact with the hydro-thermal face plate.
306 Then the frame was tilted back into the normal configuration.

307 Next, four thermocouples were installed into the system to measure the spatial distribution in
308 temperature during the test. Two thermocouples were inserted into the compacted soil specimen,
309 a third thermocouple was placed between the $y_1$ face of the compacted soil specimen and the
310 bladder, and the fourth thermocouple was left outside to measure potential changes in the
311 ambient room temperature. The ambient room temperatures were found to be relatively stable
312 during testing and are reported by Shanina (2015). The thermocouples in the soil specimen were
313 inserted using a needle so that they would measure the temperature near the mid-plane of the
specimen and in between two lifts at similar distances from the lower hydro-thermal face plate and the upper $z_1$ bladder.

The bladders and mechanical loading plates were then assembled onto the true-triaxial cell frame. In the next step, fifteen LVDTs were placed on the top of the mechanical loading plates of the true triaxial cell. Each loading face contains three LVDTs, as three points are needed to define the orientation of a plane. Once the true-triaxial cell is assembled, the initial suction in the compacted, unsaturated silt was measured using the tensiometer approach described above.

A seating normal stress of 10 kPa was then applied to all of the bladders on the $x$- $y$- $z$-faces of the cubical soil specimen. This initial total stress was applied to ensure initial contact between the flexible latex bladders and the cubical specimen without causing significant deformations to the compacted specimen. The LVDT readings measured after equilibration under this initial seating stress were then zeroed to serve as a baseline reading from which to base further deformations of the specimen. After application of the seating stress, the initial value of suction measured within the specimen was applied using the axis translation technique. This involved increasing the total stress, pore air pressure, and pore water pressure in stages to maintain a constant suction within the specimen equal to the initial suction. The axis translation approach permits the pore water pressure applied to the bottom of the specimen to be positive, which minimizes the likelihood that the water will cavitate. The constant matric suction pressure was achieved by directly measuring the difference between the pore water and air pressures of the specimen ($u_a-u_w = 20$ kPa for $S_r = 0.7$ and $u_a-u_w = 10$ kPa for $S_r = 0.8$). The value of pore air pressure should be maintained greater than the value of pore water pressure, and both less than the value of total net stress. The isotropic and anisotropic loading-unloading tests were
performed by increasing or decreasing the pressurized water through the bladders in increments and allowing the excess pore water pressure to dissipate.

Stresses were applied to the compacted soil specimens in order to reach three different anisotropic stress states represented by stress ratio $K$ of 1.0, 0.7, and 0.5. In this study, it is assumed that the horizontal principal stresses are equal to the minor principal stress $\sigma_h = \sigma_x = \sigma_y = \sigma_3 = \sigma_2$, and that the vertical stress is the major principal stress ($\sigma_v = \sigma_z = \sigma_1$). As mentioned, the stress ratio is equal to the ratio of the minor principal stress to the major principal stress ($K = \sigma_3/\sigma_1 = \sigma_y/\sigma_z = \sigma_x/\sigma_z$). During mechanical loading, room-temperature water was used to pressurize the flexible bladders around the soil specimen. The specimen was first loaded isotropically up to 350 kPa in all cases, then the stress in the $z$ direction was increased to reach the different target $K$ values. This approach is different than that used by Coccia and McCartney (2012) and ensures that the mean stress never decreases during the tests and that the specimens remain normally consolidated during application of the anisotropic stress states.

After reaching the desired value of the anisotropy coefficient $K$, the soil specimens were heated in three stages then cooled in a single stage. This study focuses on the results from the heating stages of the experiments to facilitate comparison between the different conditions investigated, but the full set of test results can be found in Shanina (2015). The target rate of increasing water temperature was 0.5°C/hr to follow the approach of Uchaipichat and Khalili (2009). During heating, the water within the bladders was circulated through a pressurized reservoir that contains a heating coil. When the heating coil is activated, the water used to pressurize the flexible bladders heats up, and applies the same temperature to all sides of the cubical specimen. Because only one heating reservoir was available, and because the temperature at all six faces of the specimen should be the same, a copper circulation coil was placed inside...
the pressurized fluid within the bladder on the upper z-face. This approach permitted independent application of pressures to the z face bladder from the x and y face bladders, but uniform temperatures on all of the faces.
EXPERIMENTAL RESULTS

Six non-isothermal tests were conducted using the THM true-triaxial cell under constant suction conditions to characterize the effect of anisotropic stress states on the thermal volume changes of unsaturated soils. These tests were performed on compacted, cubical specimens of Bonny silt having different initial degrees of saturation and under different stress ratios $K$. The compaction conditions for these specimens are summarized in Table 1 along with the initial suction $\psi_0$, degree of saturation $S_0$, and temperature $T_0$. The tests performed on specimens with initial degrees of saturation of 0.7 and 0.8 complement tests on saturated specimens of compacted Bonny silt evaluated by Coccia and McCartney (2012). The name designations for each test along with the axial stresses applied to the specimens in the different tests are summarized in Table 2. The void ratios at the beginning of heating are also presented in this table, which are different from the initial values given in Table 1 due to the application of the isotropic stress state followed by the anisotropic stress state in some tests. The compression curves for these different tests are compared in Shanina (2015), who also presented time series of axial stress, axial displacement and temperature for the different tests. This study focuses on the synthesized average thermal axial strains measured in the different tests. It should be noted that because the heating tests were performed in drained conditions, the applied matric suction was constant during the tests. The outflow from the specimens were monitored during mechanical loading and heating, but negligible changes in degree of saturation were observed. This observation is consistent with the trends in degree of saturation during heating experiments on the same soil performed in a different experimental setup by Coccia and McCartney (2016a).

The thermal axial strains for the specimens with an initial degree of saturation of 0.7 having different initial stress ratios are shown in Figures 3(a), 3(b), and 3(c). The first observation from
the data is that the thermal axial strains are contractile for all of the different stress ratios. This is consistent with the behavior of normally consolidated unsaturated silts observed by Uchaipichat and Khalili (2009) and Coccia and McCartney (2016a). The second observation is that with decreasing stress ratio, the thermal axial strain in the major stress direction (z) is observed to increase, while the thermal axial strains in the minor stress directions (x and y) are observed to decrease. For the isotropic test with K=1.0, the thermal axial strains in the x, y and z directions are similar, while for the anisotropic test with K=0.5, the thermal axial strain in the z direction is significant while it is negligible in the x and y directions. This observation is consistent with that of Coccia and McCartney (2012), who tested saturated specimens of Bonny silt. A comparison of the magnitude of thermal axial strains with those of Coccia and McCartney (2012) will be discussed later. Although not the focus of this study, Shanina (2015) reported the data from during cooling in these tests and observed irrecoverable, plastic deformations at the end of the test that would be expected when heating and cooling a normally-consolidated soil. A comparison between the thermal axial strains for the tests with an initial degree of saturation of 0.8 are shown in Figures 3(d), 3(e), and 3(f). The observations are consistent with those for the specimens with an initial degree of saturation of 0.7, although the magnitude of thermal axial strains are slightly smaller for the specimens with an initial degree of saturation of 0.8. Shanina (2015) also observed permanent deformations after cooling in these experiments.

The thermal volumetric strains for the three tests with an initial degree of saturation of 0.7 are shown in Figure 4(a). The thermal volumetric strain is equal to the sum of the three thermal axial strains measured in Figures 3(a), 3(b), and 3(c). The first observation is that the thermal volumetric strain for the three tests is relatively consistent despite the different stress ratios. This is consistent with the observation from Coccia and McCartney (2012) for saturated specimens of
Bonny silt. The thermal volumetric strains for the specimens with an initial degree of saturation of 0.8 are shown in Figure 4(b). The thermal volumetric strains are smaller than those for the specimens with an initial degree of saturation of 0.7, but a similar observation can be drawn that the thermal volumetric strains are relatively independent of the stress ratio.

**ANALYSIS**

**Influence of Stress Induced Anisotropy**

A synthesis of the thermal axial strains for the thermal axial strains at a change in temperature of 27 °C for all cases in the major and minor principal stress directions versus different K values are presented in Figure 5(a). Some of the tests were performed to higher temperatures, in which case, the results were linearly interpolated from the trend at the nearest measurement to estimate the thermal axial strain at 27 °C. Although the same conclusions drawn from Figure 3 can be drawn, an interesting observation is that the soils with an initial degree of saturation of 0.7 show greater thermal axial strains, with a greater difference between the thermal axial strains in the major and minor direction. The results are similar to those of Coccia and McCartney (2012), even though their anisotropic stress states were reached by unloading the specimen in the minor stress direction and thus incorporating an overconsolidation effect. Instead, the results in this study may indicate that greater thermal strains occur in the direction that has accumulated more plastic strains due to mechanical loading. This hypothesis is consistent with the thermally accelerated secondary compression model proposed by Coccia and McCartney (2016b).

A synthesis of the volumetric strain versus the stress ratio for the specimens having initial degrees of saturation of 0.7 and 0.8 is shown in Figure 5(b) for a change in temperature of 27 °C. Although there is some slight fluctuation in the curves with K, the thermal volumetric strain is
not as sensitive to the value of K as the thermal axial strain. Further, it is clear that the specimens
with an initial degree of saturation of 0.8 have a consistently lower thermal volume change that
those with an initial degree of saturation of 0.7.

As Coccia and McCartney (2012) evaluated Bonny silt under saturated conditions, their
results were compared to those of this study in terms of the thermal volumetric strain in Figure 6.
It is not possible to fairly compare the thermal axial strains because Coccia and McCartney
(2012) applied the major and minor principal stresses in the x- and y-directions, applied the
anisotropic stress state by unloading, and did not control the principal stress in the z-direction.
Nonetheless, the comparison in Figure 6 permits an approximate assessment of the influence of
the initial degree of saturation on the thermal volumetric strain. Although there is some scatter,
the thermal volumetric strains for the specimens with an initial degree of saturation of 1.0 are
slightly lower on average than those with an initial degree of saturation of 0.8, reflecting that a
greater amount of thermal collapse may occur in the dryer soils under normally consolidated
conditions. Coccia and McCartney (2016a) observed a greater amount of contraction with
decomposing degree of saturation up to a certain point (S0 of 0.56), but then observed a lower
amount of contraction.

Model for Anisotropy Effects

The thermal volumetric strain for the cubical soil specimen can be calculated from the axial
thermal strains as follows:

\[ \varepsilon_{vT} = \varepsilon_{xT} + \varepsilon_{yT} + \varepsilon_{zT} + (\varepsilon_{xT} \times \varepsilon_{yT} + \varepsilon_{xT} \times \varepsilon_{zT} + \varepsilon_{yT} \times \varepsilon_{zT}) + \varepsilon_{xT} \times \varepsilon_{yT} \times \varepsilon_{zT} \]  

(5)

where \( \varepsilon_{vT} \) is the total thermal volumetric strain and \( \varepsilon_{xT}, \varepsilon_{yT}, \varepsilon_{zT} \) are the thermal axial strains in the
x, y and z directions. This equation can be simplified by assuming that the higher order terms are
negligible because the thermal axial strains are very small. Accordingly, the thermal volumetric
strain can be expressed as follows:

$$\varepsilon_{vt} = \varepsilon_{xt} + \varepsilon_{yt} + \varepsilon_{zt}$$  \hspace{1cm} (6)

In the true-triaxial experiment, the three orthogonal thermal axial strains are assumed to be equal
to the principal strains. Most well-established thermo-elasto-plastic models use an isotropic
thermal yield surface (Cui et al. 2000; Abuel-Naga et al. 2009), and focus on prediction of the
thermal volumetric strain rather than the individual axial strains. Although a thermo-elasto-
plastic model could be developed that considers thermal yielding individually in each direction, a
simpler approach was followed in this study. Specifically, using the fact that the volumetric
thermal strain is not sensitive to the stress ratio $K$ [i.e., Figure 6(b)], the volumetric strains
predicted from the established isotropic thermo-elasto-plastic models can be partitioned using an
empirical relationship based on the results presented in this study. Further, an empirical
relationship can also be incorporated to account for the influence of the initial degree of
saturation on the parameters of the established thermo-elasto-plastic model.

In order to do this, an approach similar to that proposed by Coccia (2011) was used to define
an empirical relationship between the thermal axial strains in the major and minor principal
stress directions and the stress ratio. The thermal axial strain ratio ($\Omega$) was defined to relate the
thermal axial strains in the minor principal stress directions ($x$ and $y$) and the major principal
stress direction ($z$), as follows:

$$\Omega = \varepsilon_{yt} / \varepsilon_{zt}$$  \hspace{1cm} (7)

The trend in $\Omega$ observed in the current study for Bonny silt under different stress ratios ($K = \sigma_y / \sigma_z$) and different degrees of saturation is shown in Figure 7. This figure also includes the data
from Coccia (2011). The first observation is that the trend between $W$ and $K$ does not appear to
be sensitive to the degree of saturation, except for a strongly negative value of $\Omega$ for the
saturated specimen under the lowest K value of 0.5. This point was assumed to be an outlier in
the development of a best fit power law relationship, which is shown in Figure 7. Although
Coccia (2011) used a different form of equation to represent the nonlinear decreasing trend in $\Omega$,
the additional results presented in this study better establish the trend between $\Omega$ and K. The
relationship for $\Omega$ is only extended to a value of $K = 0.325$, as this is the minimum value of K
corresponding to shear failure for this soil (corresponding to a friction angle of 33°). The
experimental and best-fit values of $\Omega$ for the specimens with $K=1$ are not equal to 1.0 even
though this corresponds to an isotropic stress state because of inherent anisotropy effects
associated with how the specimen was compacted with lifts perpendicular to the z direction.

Combining Equations 6 and 7 leads to the following relationship:

$$\varepsilon_{zT} = \frac{\varepsilon_{vT}}{2\Omega + 1}$$

which can be written in differential form as follows:

$$d\varepsilon_{zT} = \frac{d\varepsilon_{vT}}{2\Omega + 1}$$

The differential form of the thermal axial strain in the y or x directions can be calculated
similarly, as follows:

$$d\varepsilon_{yT} = \frac{d\varepsilon_{vT} \Omega}{2\Omega + 1}$$

By incorporating combined incremental forms of the elastic and plastic volumetric strains from
the constitutive model of Cui et al. (2000), Equation 9 can be rewritten as follows:

$$d\varepsilon_{zT} = \frac{1}{2\Omega + 1} [\alpha_a dT + \alpha_p (\exp(\alpha_p \Delta T) - 1) dT]$$

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while Equation 10 can be rewritten as follows:

$$d\varepsilon_{sT} = \frac{\Omega}{(2\Omega+1)} \left[ \alpha_2 dT + \alpha_p \left( \exp\left(\alpha_p \Delta T\right) - a \right) dT \right]$$

(12)

where $\alpha_2$ is a constant parameter representing the drained coefficient of thermo-elastic expansion of a soil obtained from a cooling test performed at a slow rate, $\alpha_p$ is a constant parameter that depends on the overconsolidation ratio, $a$ is a constant shape parameter representing the evolution of the plastic thermal volumetric strain with temperature, $\Delta T$ is the total change in temperature ($^\circ$C) from a reference temperature, and $dT$ is the increment in temperature from one step to another ($^\circ$C). The value of $\alpha_2$ was calculated to be approximately $-0.0007/^\circ$C from cooling tests on Bonny silt reported by Shanina (2015). Cui et al. (2000) assumed that $\alpha_p = -\alpha_2/(1-a)$ for normally consolidated soils. In this case, Equation 11 can be rewritten as follows:

$$d\varepsilon_{sT} = \frac{1}{(2\Omega+1)} \left[ \alpha_2 dT - \frac{\alpha_2}{1-a} \left( \exp\left( -\frac{\alpha_2}{1-a} \Delta T \right) - a \right) dT \right]$$

(13)

and Equation 12 can be rewritten as follows:

$$d\varepsilon_{sT} = \frac{\Omega}{(2\Omega+1)} \left[ \alpha_2 dT - \frac{\alpha_2}{1-a} \left( \exp\left( -\frac{\alpha_2}{1-a} \Delta T \right) - a \right) dT \right]$$

(14)

The incremental form of the model of Cui et al. (2000) was used in this study so the combined effects of elastic thermal expansion and plastic thermal contraction expected during heating of normally consolidated soils could be superimposed atop each other. This assumes that some elastic thermal expansion will always occur during heating. The advantage of this approach is that the elastic thermal expansion can be separated from the overall observed thermal contraction in the experiments, which permits a more accurate definition of the model parameter $a$. 

The text continues with further explanations and equations related to the model.
In addition to the influence of the anisotropic stress state, the influence of unsaturated conditions observed in Figure 6(a) needs to be incorporated into the prediction of the thermal volumetric strain. The thermal axial strain for unsaturated soils was estimated by assuming that the $\alpha_p$ parameter in the model of Cui et al. (2000) is also a function of the degree of saturation. Specifically, Equation 14 was updated by modifying the definition of $\alpha_p$ assumed by Cui et al. (2000) to include the product of a constant parameter $b$ and the effective saturation at the start of heating ($S_{e,0} = S_0 - \frac{S_{res}}{1 - S_{res}}$, where $S_0$ is the initial degree of saturation, and $S_{res}$ is the residual degree of saturation) in the denominator, as follows:

$$d\varepsilon_T = \frac{1}{(2\Omega + 1)} \left[ \alpha_2 dT - \frac{\alpha_2}{bS_{e,0}(1-a)} \left( \exp \left( -\frac{\alpha_2}{bS_{e,0}(1-a)} \Delta T \right) - a \right) dT \right]$$

(16)

Similarly Equation 15 can be modified as follows:

$$d\varepsilon_T = \frac{\Omega}{(2\Omega + 1)} \left[ \alpha_2 dT - \frac{\alpha_2}{bS_{e,0}(1-a)} \left( \exp \left( -\frac{\alpha_2}{bS_{e,0}(1-a)} \Delta T \right) - a \right) dT \right]$$

(17)

This modified form of the Cui et al. (2000) model assumes that the initial effective saturation affects the thermal axial strain, and that any potential changes in degree of saturation during heating do not have a major effect on the value of the thermal axial strain. As no change in degree of saturation was observed for this soil during heating by Shanina (2015) and Coccia and McCartney (2016a), this assumption is reasonable. However, this is a topic that may need further study for other soils.

Comparisons of the measured and simulated thermal axial strains for the specimens with an initial degree of saturation of 0.7 are shown in Figures 8(a), 8(b), and 8(c) for K values of 1.0, 0.7, and 0.5, respectively. The differential thermal axial strains from Equations 16 and 17 were
integrated to define the total thermal axial strains. A good fit is observed between the model and experimental results for a value of $a$ equal to 0.323. Similarly, comparisons of the measured and simulated thermal axial strains for the specimens with an initial degree of saturation of 0.8 are shown in Figures 8(d), 8(e), and 8(f) for $K$ values of 1.0, 0.7, and 0.5, respectively. A value of $b$ equal to 1.95 was observed to provide a good fit to this data, as well as to the saturated data of Coccia and McCartney (2012) that is not shown here for brevity but is shown in Shanina (2015).

A plot of the thermal volumetric strains for the same specimens with degrees of saturation of 0.7 and 0.8 along with the model simulations are shown in Figures 9(a) and 9(b), respectively, and a good fit is also observed to the data. Although this model is simple and empirical, it shows how an established isotropic thermo-elasto-plastic model such as that of Cui et al. (2000) can be extended to predict the thermal axial strains in principal stress directions for different anisotropic stress states. Nonetheless, the role of unsaturated conditions through the $b$ parameter needs to be further verified through testing of other soils over a wider range of suction.

**DISCUSSION**

The results from this study confirm the importance of considering the influence of anisotropic stress states on the thermal volume change of soils. Natural soils typically have an inherent anisotropy, corresponding to at-rest or $K_0$ conditions. As $K_0$ is typically less than 1 in natural soil deposits, this means that the vertical stress is greater than the horizontal stress. If the soil under this stress state is heated, then different magnitudes of thermal axial strains are expected. The value of $K$ may change more significantly in the case of the installation of an energy pile, which typically involves excavation and a lower $K$ value that is closer to active earth pressure conditions.
When the soil surrounding an energy pile changes in temperature, potentially irreversible soil volume changes in the soil may occur. If the soil is overconsolidated, it is likely that the soil will expand in both the vertical and horizontal directions elastically. In this case, the thermal volume changes are not expected to be significant. More significant changes are expected to occur if the soil is closer to normally consolidated conditions in the vertical direction, in which case contraction would be expected in both the vertical and horizontal directions as observed in this study. The contractile thermal axial strain in the horizontal direction may lead to a reduction in the lateral stress distribution along the energy foundation, and may lead to relative movement due to the existing mechanical stress being transferred from the foundation to the surrounding soil. Further, the thermal axial strain the vertical direction will be greater than that in the horizontal direction, and may lead to dragdown forces on the energy pile that are superimposed atop any mechanical or thermo-mechanical strains that are predicted to occur using a load-transfer analysis such as that of Knellwolf et al. (2011). Nonetheless, for the soil that was evaluated in this study, the magnitudes of thermal axial strain are all less than 1%, which is relatively small. The only likelihood that these thermal axial strains will affect the performance of the foundation would be if it were heavily loaded close to its ultimate capacity.

The isotropic thermo-elasto-plastic model of Cui et al. (2000) was adapted in this study to empirically consider the effects of anisotropic stress states as well as to include the influence of unsaturated conditions. This model is simple to use in the fact that it can predict the thermal axial strains for a given change in soil temperature from the value of predicted thermal volumetric strain from an established isotropic thermo-elasto-plastic model. In this case, a finite element model may be used to predict the change in temperature of the soil as a function of space and time using a transient conduction analysis. After this, the model can be used to estimate the
thermal axial strains in a de-coupled manner (assuming the thermal axial strains are just due to
changes in temperature and are independent of the mechanical stresses). These strains would
have to be superimposed on top of a mechanical stress-strain analysis to see if mechanical
changes in pile behavior would occur.

Regarding the role of unsaturated conditions, this study indicates that this is an important
variable to consider. Different trends in the magnitude of thermal volume change were observed
for the soil tested in this study than in previous studies such as Uchaipichat and Khalili (2009).
Nonetheless, the tests in this previous study underwent undrained heating and cooling cycles and
several different loading and unloading cycles before heating, which may have had a cumulative
effect on the results. A mechanism for the increasing trend in thermal volume change with
decreasing thermal volume change was not explicitly proposed in this study due to the relatively
limited number of specimens with different degrees of saturation, although this behavior may be
due to the collapse of air voids during drained heating, or to thermally accelerated creep after
application of plastic strains during mechanical loading (Coccia and McCartney 2016b).

CONCLUSIONS

This study involved the development of a thermo-hydro-mechanical true triaxial cell which is
capable of measuring the thermal deformations of unsaturated soils under various anisotropic
stress states. In addition to calibrating and characterizing the response of the true-triaxial cell,
several experiments were performed on specimens having different minor to major stress ratios
and to different initial degrees of saturation. The specimens were all loaded to normally
consolidated conditions isotropically before application of the different anisotropic stress states.
The major conclusions that can be drawn from the evaluation of the results from these test
include:
• The thermally-induced axial and volumetric strains during heating for all tests on the normally consolidated, compacted Bonny silt specimens showed contractile behavior, regardless of the stress state and the initial degree of saturation.

• The plastic thermal contraction trends in the volumetric strains observed in this study are consistent with those published in the literature for normally consolidated soils.

• With decreasing values of stress ratio K, the thermal axial strains in the major principal stress direction (z) were observed to increase, while the thermal axial strains in the minor principal stress directions (x, y) are observed to decrease. However, for isotropic conditions (K = 1), the thermal axial strains were slightly greater in the x and y directions than in the z directions even though the stresses were the same. This was attributed to the effects of inherent anisotropy associated with how the specimens were formed by compacted.

• Consistent with observations from tests on saturated Bonny silt specimens reported by Coccia and McCartney (2012), the thermal volumetric strains were relatively similar regardless of the stress ratio K. This indicates that anisotropic stress states may lead to different thermal deformations in different directions but the same overall volumetric response.

• Specimens with a lower initial degree of saturation were observed to show greater thermal axial strains. The data reported by Coccia and McCartney (2012) for saturated specimens of the same soil are consistent with these trends in that greater thermal axial strains are measured for the unsaturated specimens than the saturated specimens. The trends with degree of saturation differed from that of Uchaipichat and Khalili (2009), who observed a very slight decrease in thermal volume strain with decreasing degree of saturation. However, their tests
involved application of multiple loading stages and undrained heating and cooling cycles before the drained heating tests.

- The isotropic elasto-plastic model of Cui et al. (2000) for saturated soils was combined with empirical relationships to account for the effects of anisotropic stress states and the initial degree of saturation. An empirical relationship between the ratio of thermal axial strains in the major and minor principal stress directions and the stress ratio was developed that appears to be insensitive to the degree of saturation. This relationship also incorporated the effects of inherent anisotropy in the compacted specimens. The adapted thermo-elasto-plastic model is capable of considering the effects of anisotropic stress states as well as the influence of unsaturated conditions for Bonny silt, although further confirmation is required for other soils over a wider range of unsaturated conditions.

ACKNOWLEDGMENTS

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Table 2: Applied stresses in the tests having different stress ratios K

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Fig. 2: Machine deflections for the true-triaxial cell: (a) Mechanical; (b) Thermal

Fig. 3: Thermal axial strains for the specimens with different initial degrees of saturation and stress ratios: (a) S = 0.7, K = 1.0; (b) S = 0.7, K = 0.7; (c) S = 0.7, K = 0.5; (d) S = 0.8, K = 1.0; (e) S = 0.8, K = 0.7; (f) S = 0.8, K = 0.5

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Fig. 9: Comparison between predicted and observed thermal volumetric strains for specimens with: (a) $S = 0.7$ and different stress ratios; (b) $S = 0.8$ and different stress ratios

Table 1: Initial conditions of the soil specimens evaluated in this study

<table>
<thead>
<tr>
<th>Test</th>
<th>Stress ratio, $K = \sigma_y/\sigma_z$</th>
<th>Gravimetric water content, $w$ (%)</th>
<th>Dry density, $\rho_d$ (kg/m$^3$)</th>
<th>Initial void ratio</th>
<th>Initial degree of saturation, $S_0$</th>
<th>Measured initial suction, $\psi_0$ (kPa)</th>
<th>Initial temperature*, $T_0$ (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>K1.0-S0.7</td>
<td>1.0</td>
<td>16.08</td>
<td>1650</td>
<td>0.594</td>
<td>0.712</td>
<td>20.0</td>
<td>19.70</td>
</tr>
<tr>
<td>K0.7-S0.7</td>
<td>0.7</td>
<td>15.32</td>
<td>1661</td>
<td>0.584</td>
<td>0.690</td>
<td>20.0</td>
<td>19.21</td>
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<tr>
<td>K0.5-S0.7</td>
<td>0.5</td>
<td>15.99</td>
<td>1651</td>
<td>0.593</td>
<td>0.709</td>
<td>20.0</td>
<td>19.25</td>
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<td>K1.0-S0.8</td>
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<td>K0.7-S0.8</td>
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<td>18.39</td>
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<td>0.808</td>
<td>10.0</td>
<td>19.10</td>
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<td>K0.5-S0.8</td>
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<td>17.70</td>
<td>1655</td>
<td>0.589</td>
<td>0.790</td>
<td>10.0</td>
<td>19.20</td>
</tr>
</tbody>
</table>

*Note: The initial temperature is the highest temperature previously experienced by the soil

Table 2: Applied stresses in the tests having different stress ratios $K$

<table>
<thead>
<tr>
<th>Test</th>
<th>Stress ratio, $K = \sigma_y/\sigma_z$</th>
<th>Stresses at the start of heating</th>
<th>Void ratio at the start of heating</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\sigma_x'$ (kPa)</td>
<td>$\sigma_y'$ (kPa)</td>
</tr>
<tr>
<td>K1.0-S0.7</td>
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<td>274</td>
<td>274</td>
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<tr>
<td>K0.7-S0.7</td>
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<td>274</td>
<td>274</td>
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<tr>
<td>K0.5-S0.7</td>
<td>0.5</td>
<td>274</td>
<td>274</td>
</tr>
<tr>
<td>K1.0-S0.8</td>
<td>1.0</td>
<td>268</td>
<td>268</td>
</tr>
<tr>
<td>K0.7-S0.8</td>
<td>0.7</td>
<td>268</td>
<td>268</td>
</tr>
<tr>
<td>K0.5-S0.8</td>
<td>0.5</td>
<td>268</td>
<td>268</td>
</tr>
</tbody>
</table>
Figure 2

(a) Change in axial stress, $\Delta \sigma_a$ (kPa)

Mech. machine defl., $d_M$ (mm)

- $M_x = 0.000030$ mm/kPa
- $M_y = 0.000028$ mm/kPa
- $M_z = 0.000039$ mm/kPa

Graph showing data for X-face, Y-face, and Z-face.

(b) Change in temperature, $\Delta T$ (°C)

Therm. machine defl., $d_T$ (mm)

- $H_x = 0.0038$ mm/°C
- $H_y = 0.0040$ mm/°C
- $H_z = 0.0033$ mm/°C

Graph showing data for X-face, Y-face, and Z-face.
Figure 3

Change in temperature, $\Delta T$ (°C)

- X-face
- Y-face
- Z-face

(a) Thermal axial strain, $\varepsilon_{at}$ (%) $S = 0.7$ $K = 1.0$

(b) Thermal axial strain, $\varepsilon_{at}$ (%) $S = 0.5$ $K = 0.5$

(c) Thermal axial strain, $\varepsilon_{at}$ (%) $S = 0.7$ $K = 0.7$

(d) Thermal axial strain, $\varepsilon_{at}$ (%) $S = 0.8$ $K = 1.0$

(e) Thermal axial strain, $\varepsilon_{at}$ (%) $S = 0.8$ $K = 0.7$

(f) Thermal axial strain, $\varepsilon_{at}$ (%) $S = 0.8$ $K = 0.5$
Figure 5

(a) Thermal axial strain, $\varepsilon_{at}$ (%)

- X (S=0.70)
- Y (S=0.70)
- Z (S=0.70)

$\Delta T = 27 ^\circ C$

Stress ratio, K

(b) Thermal vol. strain, $\varepsilon_{vt}$ (%)

- S = 0.70
- S = 0.80

$\Delta T_{ave} = 27 ^\circ C$

Stress ratio, K
Figure 7

The diagram shows the relationship between the ratio of minor to major thermal axial strains, $\Omega$, and the stress ratio, $K$. The equation $\Omega = 1.13K^{3.79}$ is given, and the degree of saturation is indicated with different markers for $K = 0.7$, $0.8$, and $1.0$. The graph visualizes how $\Omega$ decreases as $K$ decreases, with distinct points for each degree of saturation.