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The Past, puzzles, and promise of 6-branes
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Abstract

The fact that both the D6-brane and the orientifold 6-plane have smooth, horizon-free descriptions in M-theory makes them especially useful in understanding certain aspects of brane physics. We briefly review how this connection has been used to understand a number of effects, several of which are associated with the Hanany-Witten transition. One particular outcome is a "confinement mod 2" effect for zero-branes in the background of a single D8-brane. We also discuss an interesting puzzle associated with flux-expulsion from D6-branes in this context. Finally, we discuss the promise of using a similar M-theoretic description of the orientifold 6-plane to understand the consistency of stringy negative energy objects with the 2nd law of black hole thermodynamics.

1 Introduction

This outline for this talk arose in an attempt to find a strong enough unifying theme in my recent work to keep an audience's interest throughout a 50 minute talk. Rather to my surprise, such a theme did exist and, not only did it run through quite a bit of my recent work, but it continues to run through planned future work as well. The theme concerns a certain tool that one can use to uncover certain non-perturbative effects in brane physics by concentrating on the case of six-branes. Thus, the above title was born out of the idea that I would review past work involving six-branes, present some puzzles presently under study involving six-branes, and describe the promising future use of six-branes in addressing what may at first seem like a completely unrelated question.

The feature that makes six-branes unique in string theory is that they admit smooth, horizon-free strong-coupling descriptions in terms of eleven-dimensional supergravity. In the case of the D6-brane, the lift to M-theory is the Kaluza-Klein monopole, while for the orientifold 6-plane it is the Atiyah-Hitchin manifold. These results turn out to provide a handle with which to grasp a variety of non-perturbative effects in brane physics, and we display a selection of such results below.

It turns out that several of the results of interest involve the Hanany-Witten effect. For this reason, we begin with a review of the smooth picture of this effect and then proceed to discuss what one can do with six-branes. Our first application is the construction of supergravity solutions describing the Hanany-Witten effect in which all branes involved are treated as gravitating objects that affect the bulk spacetime fields (and thus the other branes). This construction then leads to a puzzle involving a certain 'flux-expulsion' property of the D6-brane.
We then turn in a rather different direction to discuss how D6-branes may be used to derive and understand a certain ‘confinement mod 2’ effect of D0-branes in the (symmetric) background created by a unit charged D8-brane \[17\]. Finally, we make a further radical change in direction to discuss the issue of the consistency of negative tension string-theoretic constructions with black hole thermodynamics and how the study of orientifold 6-planes promises to provide a resolution.

Despite the wide variety of physical questions that will be discussed, all of these issues will be studied using the same basic fact that six-branes have an easily controlled strong coupling description. While the absence of a gravity/gauge-theory duality \[6\] for D6-branes may sometimes make these branes seem less exciting than their lower dimensional cousins, I hope that the reader is impressed with the variety of issues that can be raised, addressed, and resolved in the context of six-branes.

2 The smooth picture of the Hanany-Witten effect

This section provides a brief review of how the Hanany-Witten brane-creation effect \[5\] is described as a smooth process. While this discussion has nothing to do with six-branes specifically, it will set the context and provide background for two of the sections that follow. The basic picture follows from general principles, but one can also find a one-parameter moduli space of exact BPS solutions describing certain versions of the process in either the worldvolume theory of a test brane in the background generated by another brane \[7, 8, 9\] or in full supergravity \[10\], meaning that both branes are fully coupled to bulk fields and can affect each other. However, in this latter case only so-called ‘near core’ solutions are available. What happens in either the worldvolume theory \[11\] or supergravity \[12\] is that the flux of a gauge field generated by one brane falling on the second brane generates a third kind of charge associated with the new brane.

In the supergravity description, this effect follows from the fact that the ‘brane-source’ charge of the D4-brane (see \[12\]) is not conserved \[10, 12\]. This in turn is a straightforward consequence of the modified Bianchi identity satisfied by the gauge invariant Ramond-Ramond four-form field strength \(\tilde{F}_4 = dC_3 + A_1 \wedge H_3\) of which the D4-brane is a magnetic source. We have the relation

\[
d\tilde{F}_4 + F_2 \wedge H_3 = *j_{D4}^{bs},
\]

where the right hand side is the brane-source current (which vanishes in the absence of an explicit D4-brane source). Here, \(F_2\) is the usual IIA Ramond-Ramond two-form field strength and \(H_3 = dB_2\) is the Neveu-Schwarz field strength. Taking an exterior derivative of (1) shows that \(d * j_{D4}^{bs}\) does not vanish. Instead, a flux of \(F_2\) falling on an NS5 brane (where \(*j_{NS5}^{bs} = dH_3 \neq 0\) or a flux of \(H_3\) falling on a D6-brane (where \(*j_{D6}^{bs} = dF_2 \neq 0\) acts as a source or
sink of D4-brane charge. Some of the subtleties of defining charge and working with brane-source currents are discussed in [12], but it is enough for us that this result leads to the Hanany-Witten effect and the associated creation of a D4-brane as described below.

The diagram below shows various stages in this process for the case of an NS5-brane moving past a D6-brane to make a D4-brane [10]. Similar results also follow for Dp and Dp' branes whenever $p + p' = 8$, see e.g. [8, 9] for a worldvolume description of the D3/D5 case. At stage (i) when the NS5-brane is far from the D6-brane, the center of the NS5-brane subtends a small angle at the D6-brane and captures only a small amount of flux from the D6. As a result, essentially no D4 charge is induced in the region shown and one has only a flat NS5-brane. Then, as the NS5-brane approaches the D6-brane (ii), it subtends a larger angle and begins to capture some flux, generating some D4 charge. This charge corresponds to D4-branes lying inside the NS5-brane and running outward along this brane to infinity.

When the NS5-brane is dragged past the D6-brane (iii), all of the flux from the D6-brane is captured in the part of the NS5-brane close to the D6-brane. Capturing one quantum of flux corresponds to the creation of one quantum of fundamental string charge, so that the thin neck of NS5-brane approximates a single D4-brane. However, the NS5-brane captures flux of the opposite sign in the region where the neck joins the asymptotically flat part of the NS5-brane. The flux captured in this region is half of that generated by the D6-brane, so that a net one-half quantum of D4 charge reaches infinity along the NS5-brane. This last statement is true in each of the stages (i, ii, iii, iv), though only in stage (iii) are all of the relevant parts of the NS5-brane visible in figure 1. In stage (iv), the neck has narrowed so as to become difficult to resolve and all that remains is a D4-brane string stretching between an NS5-brane and a D6-brane.

The above picture seems to follow from general properties of the supergravity field equations, but it is important to check them by studying exact solutions in detail. This will be particularly clear in a moment when we discuss the 6-brane ‘puzzle,’ which is an apparent exception to the above story. The known exact solutions come in several forms, the first of which [7] considers a test D2-brane in the background created by a six-brane. This case is particularly tractable using the M-theory description in which we have an M2-brane in a Kaluza-Klein monopole background. In this case, any holomorphic curve represents a BPS...
configuration of the M2-brane. By moving the M2-brane past the monopole, one can watch the formation of a string that connects the monopole to the two-brane. Test brane solutions were also studied in [8, 9] for the case of a fundamental string stretching between a D5-brane and a D3-brane, but in this case one must work much harder to solve the differential equations for the BPS configuration as one does not have the shortcut of simply looking for holomorphic curves.

3 Supergravity Solutions and a Puzzle

It is interesting, however, to go one step further and to solve for the full supergravity solutions beyond the test brane approximation; i.e., to go to the stage in which both branes are actively coupled to the bulk field. One would expect that this would show a ‘back-reaction’ of the D2-brane on the D6-brane. It turns out that such solutions can in fact be constructed in what is known as the ‘near-core limit’ using a simple trick introduced by [13, 14].

The key point is that the charge N IIA D6-brane solution lifts to the charge N Kaluza-Klein monopole solution in M-theory. In particular, for the unit charge case the M-theory solution is completely smooth and so is well approximated by flat space at the center. As a result, there is a Kaluza-Klein reduction of flat space that yields the leading approximation to the D6-brane geometry near the singularity. This is the ‘near-core’ D6-brane solution. The observation is that it is straightforward to add another brane to this flat space and thereby obtain the ‘near-D6 brane’ part of a solution in which the D6-brane intersects an F1- or D4-brane. These solutions follow by simply applying the same Kaluza-Klein reduction to the M-theory solution describing ‘an M2- or M5-brane in flat space;’ i.e., to the usual M2- or M5-brane solution. This process was begun in [14] and completed in [10], where it was shown that an appropriate family of such reductions in fact describes the near-D6 brane versions of stages (i-iv) in the Hanany-Witten process for a D2- or NS5-brane being pulled past a D6-brane. Similarly, the multiply charged case can be obtain by first taking an orbifold quotient and then reducing the result.

We refer to the reader to [10] for the details of these solutions, but we mention here an interesting puzzle that one finds after a bit of study. As already mentioned, one would expect that constructing such full supergravity solutions would show the ‘back-reaction’ of the D2- or NS5-brane on the D6-brane. Certainly, a brane-charge argument indicates that, for example, any $F_4$ or $H_3$ flux falling on the D6-brane must result in the creation of F1- or D4-branes. However, one does not see this in the solutions of [10]. Instead, there seems to be a ‘flux-expulsion’ effect associated with D6-branes which is reminiscent of the ‘superconducting branes’ phenomenon [13].

To begin to understand this effect, consider any massless type IIA solution containing D6-branes. This of course provides a solution to 11-dimensional supergravity in which the D6-branes are replaced by the cores of Kaluza-Klein monopoles. This solution has a Killing field $\lambda^{11}$ which vanishes at the core of each monopole. The natural boundary condition to impose on the D6-branes
is that the corresponding 11-dimensional solutions (or an appropriate multiple cover in the multiply charged case) be smooth at these cores. But now consider the 11-dimensional four-form field strength $F_{4}^{(11)}$. If it is smooth then $F_{4}^{(11)} \cdot \lambda^{11}$ must vanish when $\lambda^{11}$ does and in particular at any core. Since $H_{3} = F_{4}^{(11)} \cdot \lambda^{11}$, it follows that $H_{3}$ will vanish at any D6-brane. Note that since the lowest Fourier mode around the circle will again give some smooth field, this conclusion also holds in cases where the 11-dimensional solution does not have an exact translation symmetry along $\lambda^{11}$ but which can be treated perturbatively. The same argument also applies to the dual field, so that $*F_{4}^{(11)}$ should also vanish at a D6-brane. Here $*_{11}$ is the eleven-dimensional Hodge dual. This is the flux that causes D6-branes to produce fundamental strings, so no fundamental string charge should be induced on a D6-brane when a D2-brane is dragged past it. This is also related to a surprising property of a T-dual type IIB solution involving D5-branes and Kaluza-Klein monopoles.

A similar sort of flux-excluding property was studied in [15]. For the 'superconducting' branes considered in that work, the normal component of some field strength was forced to vanish on the horizon. The situation here is somewhat different, however, as now the entire field strength $H_{3}$ or $*F_{4}$ must vanish at the brane.

Although the above 11-dimensional argument for flux-expulsion meshes nicely with the unexpected results of [10] and [16], certain aspects of this story remain quite puzzling. For example, flux is clearly not expelled from the NS5-brane or from a corresponding D2-brane crossing a D6-brane. Yet, the D6-brane is connected to these other branes by dualities. Thus, at least naively it appears that application of supergravity dualities can transform the solutions of [10] into ones in which D6-branes do in fact admit flux from other branes. Nevertheless, finding the mechanism through which this works, or what alternative resolution string theory provides remains a puzzle to be solved by further study of D6-branes.

4 Confinement and Charge quantization

We now turn to a related puzzle [17] which, though we will use 6-branes in its study, is most easily stated in terms of D0- and D8-branes. Consider for example a system with a single D0-brane and a single D8-brane and suppose that the boundary conditions are such that the ten-form Ramond-Ramond gauge field takes the symmetric values $\pm \frac{1}{2}$ of the fundamental quantum on either side of the D8-brane domain wall. Then, the brane-source charge arguments above (or, equivalently the arguments of [18, 19] in type IIA supergravity and the arguments of [20, 22, 23, 24, 25, 26, 27] in related contexts) lead to the conclusion that exactly 1/2 of a fundamental string must end on the D0-brane. While this seems to be at odds with charge quantization, several possible resolutions immediately present themselves. One possibility is that the half-string is a mere artifact of some accounting scheme (see, e.g. [28, 29, 30, 31]) and that it is not in fact in conflict with charge quantization. Another possibility is that
such symmetric boundary conditions for D8-branes are not actually allowed, and that the Ramond-Ramond gauge ten-form field strength $F_{10}$ must take integer values. A final possibility is that $F_{10}$ is allowed to take half-integer values but that, in such backgrounds, D0-brane charge is allowed to occur only in multiples of 2. We will conclude that this final scenario is in fact correct by considering the T-dual D2/D6 system and again using the description of D6-branes as Kaluza-Klein monopoles in M-theory.

The same question of course arises for the D2/D6 case. It is useful to first establish notation and we recall that, supposing the D6-brane is oriented along the $x_0, x_1, x_2...x_6$, the unit charged D6-brane solution takes the form

$$ds_{\text{string}}^2 = V^{-1/2} dx_0^2 + V^{1/2} dx_\perp^2,$$
$$e^{2\phi} = V^{-3/2},$$
$$A_1 = \frac{1}{2} (1 - \cos \theta) d\psi,$$
$$F_2 = \frac{1}{2} \sin \theta d\theta \wedge d\psi$$

(2)

where we have introduced $dx_\perp^2 = -dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2 + dx_5^2 + dx_6^2$ and $dx_\perp^2 = dx_7^2 + dx_8^2 + dx_9^2$, along with $V = 1 + \frac{1}{r^2}$, $r = x_7^2 + x_8^2 + x_9^2$, $\theta = \cos^{-1} \left( \frac{-x_9}{r} \right)$, and $\psi = \tan^{-1} \left( \frac{x_7}{x_8} \right)$. Here, to simplify the formulas we have set the radius $R_{10}$ of the M-theory circle to one.

If the D2-brane is extended in two directions (say, $x_7, x_8$) orthogonal to the D6-brane, then it will capture half of the flux from the D6-brane and must therefore have half of a fundamental string ending on the D2. In [17], this question was studied using the method of [7]; i.e., by considering test M2-branes in the Kaluza-Klein monopole background. There certainly do exist configurations in which the D2-brane is extended orthogonally to the D6-brane, and for these cases the issue is merely one of proper accounting. In particular, while there is indeed 1/2 unit of fundamental string 'brane source' charge in this system (in particular, this charge can be shown to flow along the D2-brane world-volume to infinity), brane-source charge is not in general quantized (see [12]). Instead, the measure of charge that is quantized is known as the 'Page charge.' While the value of this charge is not gauge invariant, its value for this configuration is an integer in any gauge [17]. In particular, in simple gauges one finds either zero or one units of fundamental string charge.

However, one can show [17] that the distinction between brane-source and Page charge is important only for the case that the fundamental string charge runs to infinity along the worldvolume of the D2-brane.[4] Note, however, that the flux of fundamental string charge or world-volume gauge field to infinity will obstruct any attempts to compactify this solution in the directions along the D2-brane. Because the flux is only outward, no consistent identifications can be imposed on solutions with such a flux. From the worldvolume perspective,
this is just the familiar statement that the total charge coupled to the gauge
field must vanish on a compact worldvolume. As a result, such configurations
cannot be compactified and therefore are not in fact related by T-duality to the
D0/D8 case.

Thus, the issue remains. However, we have learned that we must focus on
the case in which no fundamental string brane-source charge flows along the
D2-brane to infinity. Nonetheless, the D2-brane will necessarily intercept some
flux from the D6-brane and it is clear that some fundamental string charge must
somehow flow off of the D2-brane. The only remaining possibility is that this
charge will in fact flow to the D6-brane itself. Since the fundamental string
in this context is nothing but a deformation of the D2-brane worldsheet, this
means that we must consider solutions where the D2-brane actually intersects
the D6-brane. It is here that the M-theory context is particularly useful, as what
appears to be a singular intersection in the IIA description becomes merely the
smooth passage of an M2-brane through the core of a Kaluza-Klein monopole. In
particular, while the D6-brane singularity would prevent one from determining
the true structure of the intersection from the IIA perspective, from the 11-
dimensional perspective it is clear that valid D2/D6 intersections are exactly
those for which the M2-branes remain smooth at the Kaluza-Klein monopole
core.

The general holomorphic such intersection is analyzed in [17]. However,
for simplicity we examine here only the special case in which the D2-brane is
placed on the surface \(x_9 = 0\). To describe the 11-dimensional description of this
surface, we must first establish our conventions for the Kaluza-Klein monopole
in 11-dimensions. A useful form of this metric is [7, 32, 33]

\[
\mathrm{ds}^2 = -dx_9^2 + V dv d\bar{v} + V^{-1} \left( \frac{dw}{w} - f dv \right)^2,
\]

where

\[
f = \frac{x_9 + r}{2v^p},
\]

correcting a small typographic error in [7, 33]. The complex coordinates \(v\) and
\(w\) define one of the complex structures on the Euclidean Taub-Nut space. They
are related to the ten-dimensional coordinates through

\[
v = x_7 + ix_8
\]

\[
w = e^{-(x_9 + ix_10)} \left( -x_9 + \sqrt{x_9^2 + |v|^2} \right)^{1/2},
\]

and Kaluza-Klein reduction takes place along the Killing field \(\partial_{x_{10}}\). Such coordi-
nates are smooth so long as \(v \neq 0\) or \(x_9 < 0\). Note that \(x_{10}\) ranges over [0, 2\(\pi\)]
consistent with our setting \(R_{10} = 1\). In addition, a careful check will show that
this space has a \(Z_2\) symmetry of the form \((w, v) \rightarrow (\frac{w}{w^p}, v)\).

A holomorphic curve that reduces to the surface \(x_9 = 0\) surface can be found by noticing that the symmetry \((w, v) \rightarrow (\frac{w}{w^p}, v)\) changes the sign of \(x_9\),
so that any surface which is invariant under this symmetry must lie at \(x_9 =\)
0. The surface \( w^2 = v \) is invariant in this way, and careful investigation \(^{17}\) shows that it remains smooth at the origin \( (v = w = 0) \). However, because it contains \( w^2 \), upon dimensional reduction we find \( \text{two} \) D2-branes lying at \( x_9 = 0 \). The corresponding sheets of the M2-brane lie at \( x_{10} = \psi \) and \( x_{10} = \psi + \pi \). Deforming this surface to move the asymptotic parts of both D2 branes to large \( |x_9| \) therefore produces a string-like piece of D2-brane connected to the D6-brane and carrying a full unit of fundamental string charge and tension.

Note that the two D2-branes cannot be separated from one another as, when the angle \( \psi \) increases by \( 2\pi \), we must move from one brane to the other. It turns out (see \(^{17}\)) that in fact all D2-brane configurations that can be compactified have a similar structure, with a pair of D2-branes forming a Riemann surface whose a branch point lies at the location of the D6-brane. As a result, the same behavior should be found in the T-dual D0/D8 system. That is, in the background of a unit charged D8-brane, D0-branes should only occur in pairs and these pairs must be connected to the D8-brane by (integer) fundamental strings. We refer to this effect as “confinement of the D0-branes in pairs.”

It is interesting to remark that a \( \mathbb{Z}_2 \) quotient of the above solution leads to the charge 2 Kaluza-Klein monopole and the charge 2 D6-brane. Such a \( \mathbb{Z}_2 \) quotient identifies the two sheets of the D2-brane, leading to a single D2-brane in the charge 2 D6-brane background. Thus, the confinement effect disappears in the charge 2 D8-brane background.

5 The future promise of the six-brane

In this final section, we give a preview of other results one may hope to obtain from a study of six-branes. This latter study is of quite a different nature, and involves the relationship between negative energy objects and the generalized second law of thermodynamics. Let us first state the issues involved, and we will then fore-shadow briefly why a study of six-branes, this time of the orientifold variety, should allow us to understand the situation.

The following discussion is motived by the recent appearances of negative energy objects in various large extra dimension scenarios (see, e.g. \(^{34-46}\)). It is, of course, important that any extra-dimensional scenario be stable, and negative energies can be problematic. Placing a negative tension brane at a orbifold fixed plane has been shown to remove perturbative dynamical instabilities. Nevertheless, one still worries about the second law of thermodynamics, particularly as generalized to include the entropy of black holes. To see the point, consider a process in which a bit of a negative tension brane is lowered into a black hole (or, equivalently, in which a black hole is thrown at a negative tension brane). Heuristically, one expects that as negative energy matter flows into the black hole, the black hole will shrink and reduce in entropy. In the semi-classical limit, the entropy of a black hole should dominate over all other forms of entropy and one expects a reduction in black hole area to lead to a violation of the generalized second law of thermodynamics.

In \(^{47}\), exact constructions were given of spacetimes representing collisions in
2+1 dimensional gravity between BTZ black holes and 1+1 dimensional negative tension branes at orbifolds. The results are interesting in that one can prove that if the system were to settle down to some equilibrium state representing a black hole attached to the brane, then a violation of the second law would necessarily result. However, in this case such clear violations are avoided by the onset of a catastrophe. Instead of settling down to some equilibrium state, a new spacelike singularity forms that extends far outside of what one would naively have called the black hole, reaching out to engulf the entire brane and, in a certain sense, the entire universe.

One is led to wonder whether this behavior is somehow typical or whether other sorts of branes are better behaved. In particular, a priori this might be an artifact of our low-dimensional setting, and a higher dimensional study is currently in progress in collaboration with Joel Rozowsky, Pedro Silva, and Mark Trodden. However, one suspects that the above catastrophe is somehow related to the more basic issue involving black hole thermodynamics. It seems likely that negative tension objects should be allowed only in the case that they have some property that guarantees compatibility with black hole thermodynamics.

This is where we come to the connection with six-branes. We begin by recalling that certain negative tension orientifolds do in fact arise in string theory (see, e.g., [48, 49] for recent reviews). String theoretic calculations do not indicate any instabilities of such objects, and given the history of string theory one might well suspect that such objects could teach us something new about gravitational physics. Thus, one expects that such orientifolds are the best candidates for negative tension objects that may be compatible with black hole thermodynamics.

As with the puzzles discussed above, it appears that the six-plane case can shed further light on the issues. It turns out that (at strong coupling) there is an M-theoretic understanding of the orientifold six-plane [1, 2, 3] in terms of the Atiyah-Hitchin manifold [4]. The resulting 11-dimensional spacetime is in fact a smooth Ricci-flat (hyperkähler) manifold whose structure near infinity resembles a $Z_2$ quotient of a negative tension Kaluza-Klein monopole. As a result, one may describe the collision of a black hole with such an orientifold as a problem in pure vacuum (i.e., source-free) Einstein-Hilbert gravity in eleven dimensions. Under such conditions, the Raychaudhuri equation leads in the usual way [5] to the conclusion that the total horizon area must increase during the collision. Our violations of the generalized second law of thermodynamics will not arise in this context. As the various stringy negative tension orientifolds are related by T-duality, one expects that the other orientifolds of string theory also have properties such that the second law of thermodynamics is upheld in collisions with black holes.

Since we have seen that the orbifold boundary condition itself is not sufficient to make negative tension compatible with black hole thermodynamics, it would be interesting to understand in more detail just what properties of these orientifolds enforce the second law. One suspects that one need only probe the details of the Atiyah-Hitchin manifold for the answers. A study of this form is currently in progress in collaboration with Simon Ross. We hope to report the
results of this work soon.

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