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Diagnosis and Repair for Synthesis from Signal Temporal Logic Specifications

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ABSTRACT

We address the problem of diagnosing and repairing specifications for hybrid systems, formalized in signal temporal logic (STL). Our focus is on automatic synthesis of controllers from specifications using model predictive control. We build on recent approaches that reduce the controller synthesis problem to solving one or more mixed integer linear programs (MILPs), where infeasibility of an MILP usually indicates unrealizability of the controller synthesis problem. Given an infeasible STL synthesis problem, we present algorithms that provide feedback on the reasons for unrealizability, and suggestions for making it realizable. Our algorithms are sound and complete relative to the synthesis algorithm, i.e., they provide a diagnosis that makes the synthesis problem infeasible, and always terminate with a non-trivial specification that is feasible using the chosen synthesis method, when such a solution exists. We demonstrate the effectiveness of our approach on controller synthesis for various cyber-physical systems, including an autonomous driving application and an aircraft electric power system.

1. INTRODUCTION

The automatic synthesis of controllers for hybrid systems from expressive high-level specification languages allows raising the level of abstraction for the designer while ensuring correctness of the resulting controller. In particular, several controller synthesis methods have been proposed for expressive temporal logics and a variety of system dynamics. However, a major challenge to the adoption of these methods in practice is the difficulty of writing formal specifications. Specifications that are poorly stated, incomplete, or inconsistent can produce synthesis problems that are unrealizable (no controller exists for the provided specification), intractable (synthesis is computationally too hard), or lead to solutions that fail to capture the designer’s intent. In this paper, we present an algorithmic approach to reduce the specification burden for controller synthesis from temporal logic specifications, focusing on the case when the original specification is unrealizable.

Logical specifications can be provided in multiple ways. One approach is to provide monolithic specifications, combining within a single formula constraints on the environment with desired properties of the system under control. However, in many cases, a system specification can be more conveniently provided as a set of assumptions about the environment. Controller synthesis from specifications using model predictive control (MPC) framework [23, 24] addresses the problem of categorizing the causes of unrealizability, and how to detect them in high-level robot control specifications. The use of counter-strategies to derive new environment assumptions for synthesis was first proposed by Li et al. [13] and further explored by others [2, 14]. Our approach, based on exploiting information from optimization solvers, has similarities to these techniques as well as to the work of Nuzzo et al. [18] on extracting unsatisfiable cores for satisfiability modulo theories (SMT) solving.

In this paper, we address the problem of diagnosing and repairing specifications formalized in signal temporal logic (STL) [16], a specification language that is well-suited for hybrid systems. Our work is conducted in the setting of automated synthesis from STL via optimization in a model predictive control (MPC) framework [23, 24]. In this approach to synthesis, both the system dynamics and the STL specifications are encoded as mixed integer linear constraints on variables modeling the dynamics of the system and its environment. Controller synthesis is then formulated as an optimization problem to be solved subject to these constraints [23]. In the reactive setting, this approach proceeds by iteratively solving a combination of optimization problems using a counterexample-guided inductive syn-
Given an initial state \( x_0 \), a finite horizon input sequence \( u^H = u_0, u_1, \ldots, u_{H-1} \), and a finite horizon environment sequence \( w^H = w_0, w_1, \ldots, w_{H-1} \), the finite horizon run of the system modeled by the system dynamics in (2) is uniquely expressed as:

\[
\zeta^H(x_0, u^H, w^H) = (x_0, y_0, u_0, w_0), \ldots, (x_{H-1}, y_{H-1}, u_{H-1}, w_{H-1})
\]

where \( x_1, \ldots, x_{H-1}, y_0, \ldots, y_{H-1} \) are computed using (2).

We finally define a finite-horizon cost function \( J(\zeta^H) \), mapping \( H \)-horizon trajectories \( \zeta^H \in \Xi \) to costs in \( \mathbb{R}^+ \).

### 2.2 Signal Temporal Logic

**Signal Temporal Logic (STL)** has been largely applied to specify and monitor real-time properties of hybrid systems [8]. Moreover, it offers a robust, quantitative interpretation for the satisfaction of a formula [7, 6].

An STL formula \( \varphi \) is evaluated on a signal \( \xi \) at time \( t \): \( (\xi, t) \models \varphi \) denotes that \( \varphi \) evaluates to true on \( \xi \) at time \( t \). We instead write \( \xi \models \varphi \); if \( \xi \) satisfies \( \varphi \) at time 0. The atomic predicates of STL are defined by inequalities of the form \( \mu(\xi(t)) > 0 \), where \( \mu \) is a function of signal \( \xi \) at time \( t \). Specifically, \( \mu \) is used to denote both the function of \( \xi(t) \) and the predicate. Any STL formula \( \varphi \) consists of Boolean and temporal operations on such predicates. The syntax of STL formulae is defined recursively as follows:

\[
\varphi := \mu \mid \lnot \varphi \mid \varphi \land \psi \mid \varphi \lor \psi \mid \varphi \Rightarrow \psi \mid \varphi \Rightarrow \psi
\]

where \( \varphi \) and \( \psi \) are STL formulae, \( \mu \) is the globally operator, \( F \) is the finally operator and \( U \) is the until operator. For example, \( \xi \models G_{[a,b]} \psi \) specifies that \( \psi \) must hold for signal \( \xi \) at all times of the given interval, i.e., \( \forall t \in [a, b], (\xi, t) \models \psi \).

Formally, the satisfaction of a formula \( \varphi \) for a signal \( \xi \) at time \( t \) is defined as:

\[
(\xi, t) \models \mu \Leftrightarrow \mu(\xi(t)) > 0
(\xi, t) \models \lnot \varphi \Leftrightarrow \mu(\xi(t)) = 0
(\xi, t) \models \varphi \land \psi \Leftrightarrow (\xi, t) \models \varphi \land (\xi, t) \models \psi
(\xi, t) \models \varphi \lor \psi \Leftrightarrow (\xi, t) \models \varphi \lor (\xi, t) \models \psi
(\xi, t) \models \varphi \Rightarrow \psi \Leftrightarrow \forall t' \in [a, b], (\xi, t') \models \varphi
(\xi, t) \models U_{[a,b]} \psi \Leftrightarrow \exists t' \in [a, b], (\xi, t') \models \psi
\]

A quantitative or robust semantics is defined for an STL formula \( \varphi \) by associating it with a real-valued function \( \rho^\varphi \) of the signal \( \xi \) and time \( t \), which provides a "measure" of the margin by which \( \varphi \) is satisfied [6].

### 2.3 Model Predictive Control

**Model Predictive Control (MPC)**, or **Receding Horizon Control (RHC)**, is a well studied control method for hybrid dynamical systems [7, 10]. In RHC, at any time step, the state of the system is observed and an optimal control problem is solved over a finite time horizon \( H \), for a given set of constraints and a cost function \( J \). When \( f \), as defined in (2), is nonlinear, we assume this optimization is performed at each MPC step after locally linearizing the system dynamics. For example, at time \( t = k \), the linearized dynamics around the current state and time are used to compute an optimal strategy \( u^H \) over the time interval \( [k, k+H-1] \).

Then, only the first component of \( u^H \) is applied, and a similar optimization is solved at \( k+1 \) to compute a new optimal control sequence along the interval \( [k+1, k+H] \) for the model linearized around time step \( k+1 \). While the global optimality of MPC is not guaranteed, the technique is widely used and performs well in practice.

In this paper, we use STL to express temporal constraints on the environment and system runs during MPC. We then translate an STL specification into a set of mixed integer
linear constraints \[23, 21\]. Given a formula \(\varphi\) to be satisfied over a finite horizon \(H\), the associated optimization is:

\[
\min \ J(\xi^H(x_0, u^H)) \quad \text{s. t.} \quad \xi^H(x_0, u^H) \models \varphi,
\]

which yields a control strategy \(u^H\) that minimizes the cost function \(J(\xi^H)\) over the finite-horizon trajectory \(\xi^H\), while satisfying the STL formula \(\varphi\) at time step 0. In a closed-loop setting, we compute a fresh \(u^H\) at every time step \(i \in \mathbb{N}\), replacing \(x_i\) in \[5\] with \(x_i\) in \[23, 21\].

While \[5\] applies to systems without uncontrolled inputs, a more general formulation can be provided to account for an uncontrolled disturbance input \(w^H\) that acts, in general, adversarially \[21\]. To provide this formulation, we assume the specification is given in the form of an STL assume-guarantee (A/G) contract \[20, 19\] \(C = (V, \varphi_e, \varphi \equiv \varphi_e \Rightarrow \varphi_a)\), where \(V\) is the set of variables, \(\varphi_e\) captures the assumptions (admitted behaviors) over the (uncontrolled) environment inputs \(w\), and \(\varphi_a\) describes the guarantees (promised behaviors) over all the system variables. A game-theoretic formulation of the controller synthesis problem is then represented as a minimax optimization problem:

\[
\begin{align*}
\text{minimize} & \quad \max_{w^H \in \mathcal{W}^e} J(\xi^H(x_0, u^H, w^H)) \\
\text{subject to} & \quad \forall w^H \in \mathcal{W}^e \quad \xi^H(x_0, u^H, w^H) \models \varphi,
\end{align*}
\]

where we aim to find a strategy \(u^H\) that minimizes the worst case cost \(J(\xi^H)\) over the finite horizon trajectory, under the assumption that the disturbance signal \(w^H\) acts adversarially. We use \(\mathcal{W}^e\) in \[6\] to denote the set of disturbances that satisfy the environment specification \(\varphi_e\), i.e., \(\mathcal{W}^e = \{w \in \mathcal{W} | w \models \varphi_e\}\).

**Mixed Integer Linear Program Formulation.**

Following \[23, 21\], we solve the optimization problems in \[5\] and \[6\] by translating the STL formula \(\varphi\) into a set of mixed integer constraints, thus reducing the problem to a Mixed Integer Program (MIP). In this paper, we consider control problems that are encoded as Mixed Integer Linear Programs (MILP).

The MILP constraints are constructed recursively on the structure of the STL specification, and express the robust satisfaction value of the formula. Recall that \((\xi, t) \models \varphi \iff \rho^\varphi(\xi, t) > 0\). The robustness value of a formula with temporal or Boolean operators is expressed recursively as the min or max of the robustness values of the operands over time. These operations can in turn be encoded as mixed integer constraints. For instance, to encode \(\min(\varphi_1, \ldots, \varphi_n)\), we introduce Boolean variables \(z_i\) for \(i \in \{1, \ldots, n\}\) and a continuous variable \(p\). The resulting MILP constraints are:

\[
\begin{align*}
p & \leq \rho^{\varphi_i}, \quad \sum_{i=1}^n z_i & \geq 1, \\
ho^{\varphi_i} - (1 - z_i)M & \leq p \leq \rho^{\varphi_i} + (1 - z_i)M,
\end{align*}
\]

where \(M\) is a constant selected to be much larger than \(|\rho^{\varphi_i}|\) for all \(i\) and \(i \in \{1, \ldots, n\}\). The above constraints ensure that \(p\) takes the value of the minimum robustness and \(z_i = 1\) if \(\rho^{\varphi_i}\) is the minimum. To get the constraints for max, we replace \(\leq\) by \(\geq\) in \[7\].

We solve the MILP with an off-the-shelf solver. If the receding horizon scheme is feasible, then the controller synthesis problem is realizable, i.e., the algorithm returns a controller that satisfies the specification and optimizes the objective. However, if the MILP is infeasible, the synthesis problem is unrealizable. In this case, the failure to synthesize a controller may well be attributed to just a portion of the STL specification. In the rest of the paper we discuss how infeasibility of the MILP constraints can be used to infer the “cause” of failure and, consequently, diagnose and repair the original STL specification.

### 3. A RUNNING EXAMPLE

To illustrate our approach, we introduce a running example from the autonomous driving domain. As shown in Fig. 1, we consider a scenario in which two moving vehicles approach an intersection. The red car, labeled the ego vehicle, is the vehicle under control, while the black car is part of the external environment and may behave, in general, adversarially. The state of the system includes the position and velocity of each vehicle, the control input is the acceleration of the ego vehicle, and the environment input is the acceleration of the other vehicle, i.e.,

\[
\begin{align*}
\ddot{x}_e & = (x_e^{ego}, y_e^{ego}, v_e^{ego}, \dot{x}_e^{adv}, \dot{y}_e^{adv}, \dot{v}_e^{adv}) \\
u_t & = a_t^{ego}, \quad w_t = a_t^{adv}.
\end{align*}
\]

A similar equation holds for the vehicle environment which has, however, constrained to move along the horizontal axis rather than the vertical axis. We assume the ego vehicle is initialized at the coordinates \((0, -1)\) and the other vehicle is initialized at \((-1, 0)\). All units in this example follow the metric system. We would like to design a controller for the ego vehicle to satisfy an STL specification under some assumptions on the external environment, and provide diagnosis and feedback if the specification is infeasible. We discuss the following three scenarios.

**Example 1 (Collision Avoidance).** We want to avoid a collision between the ego and the adversary vehicle. In this example, we assume the environment vehicle’s acceleration is fixed at all times, i.e., \(a_t^{adv} = 2\), while the initial velocities are \(v_0^{adv} = 0\) and \(v_0^{ego} = 0\). We encode our requirements using the formula \(\varphi := \varphi_1 \land \varphi_2\), where \(\varphi_1\) and \(\varphi_2\) are defined as follows:

\[
\begin{align*}
\varphi_1 & = G_{[0, \infty)}((-0.5 \leq \dot{y}_e^{ego} \leq 0.5) \land (-0.5 \leq \dot{x}_e^{adv} \leq 0.5)),
\varphi_2 & = G_{[0, \infty)}(1.5 \leq a_t^{ego} \leq 2.5).
\end{align*}
\]

We prescribe bounds on the system acceleration, and state that both cars should never be confined together within a box of width 1 around the intersection \((0, 0)\), to avoid a collision.

**Example 2 (Non-adversarial Race).** In the race scenario, assuming the adversary’s velocity always exceeds \(0.5\), the ego vehicle must maintain a velocity above \(0.5\). We formalize our requirement as a contract \((\psi_e, \psi_c \Rightarrow \psi_s)\), where \(\psi_e\) are the assumptions made on the environment and \(\psi_c\) are the guarantees of the system provided the environment satisfies the assumptions. Specifically:

\[
\begin{align*}
\psi_e & = G_{[0, \infty)}(v_t^{adv} \geq 0.5), \\
\psi_c & = G_{[0, \infty)}(-1 \leq a_t^{ego} \leq 1) \land (v_t^{ego} \geq 0.5).
\end{align*}
\]
The initial velocities are $v_0^{\text{adv}} = 0.55$ and $v_0^{\text{ego}} = 0$, while the environment vehicle’s acceleration is $a_0^{\text{adv}} = 1$ at all times. We require the acceleration to be bounded by 1.

**Example 3 (Adversarial Race).** We discuss another race scenario, in which the environment vehicle’s acceleration $a_0^{\text{adv}}$ is no longer fixed, but varies up to a value of 2. Initially, $v_0^{\text{adv}} = 0$ and $v_0^{\text{ego}} = 0$. Under these assumptions, we would like to guarantee that the velocity of the ego vehicle exceeds 0.5 if the speed of the adversary vehicle exceeds 0.5, while maintaining an acceleration in the $[−1, 1]$ range. Altogether, we capture the requirements above via a contract $(\phi_w, \phi_s \rightarrow \phi_\gamma)$, where:

$$
\phi_w = G_{[0,\infty)} (0 \leq a_0^{\text{adv}} \leq 2),
\phi_s = G_{[0,\infty)} ((v_0^{\text{adv}} > 0.5) \rightarrow (v_0^{\text{ego}} > 0.5)) \land (|a_0^{\text{ego}}| \leq 1).
$$

4. **PROBLEM STATEMENT**

In this section, we define the problems of specification diagnosis and repair in the context of controller synthesis from STL. We assume the discretized system dynamics $f_d$ and $g_d$, the initial state $x_0$, the STL specification $\varphi$, and a cost function $J$ are given. The controller synthesis problem, denoted $\mathcal{P} = (f_d, g_d, x_0, \varphi, J)$, is to solve (1) (when $\varphi$ is a monolithic specification of the desired system behaviors) or (2) (when $\varphi$ represents a contract between the system and the environment).

If diagnosis fails, the diagnosis problem is, intuitively, to return an explanation in the form of a “subset” of the original problem that is already infeasible when taken alone. The repair problem is to return a “minimal” set of changes to the specification that would render the resulting controller synthesis problem feasible. To diagnose and repair an STL formula, we focus on its atomic predicates and the time intervals of its temporal operators. We start by providing a definition of the support of a formula’s atomic predicates, i.e., the set of times at which the value of a predicate affects satisfiability of the formula. We build on this definition to formalize the set of repairs that we allow.

**Definition 1 (Support).** The support of a predicate $\mu$ in an STL formula $\varphi$ is the set of times $t$ such that $\mu(t)$ appears in $\varphi$.

For example, given $\varphi = G_{[6,10]}(x_i > 0.2)$, the support of predicate $\mu = (x_i > 0.2)$ is the time interval $[6, 10]$.

**Definition 2 (Allowed Repairs).** Let $\Phi$ denote the set of all possible STL formulae. A repair action is a relation $\gamma : \Phi \rightarrow \Phi$ consisting of the union of the following:

- A predicate repair returns the original formula after modifying one of its atomic predicates $\mu$ to $\mu^*$. We denote this sort of repair by $\varphi[\mu \mapsto \mu^*] \in \gamma(\varphi)$;  
- A time interval repair returns the original formula after replacing the interval of a temporal operator. This is denoted $\varphi[\Delta_{a,b}] \mapsto \Delta_{a^*,b^*}] \in \gamma(\varphi)$ where $\Delta \in \{G, F, U\}$.

We can compose repair actions to get a sequence of repairs $\Gamma = \gamma_0(\gamma_1(\ldots(\gamma_n(\varphi) \ldots))$. Given an STL formula $\varphi$, we denote as $\text{REPAIR}_\Gamma(\varphi)$ the set of all possible formulae obtained through allowed sequences of repairs on $\varphi$. Further, given a set of atomic predicates $\mathcal{D}$ and a set of time intervals $\mathcal{T}$, let $\text{REPAIR}_{\mathcal{D}, \mathcal{T}}(\varphi) \subseteq \text{REPAIR}(\varphi)$ denote the set of repair actions that act only on predicates in $\mathcal{D}$ or time intervals in $\mathcal{T}$. We are now ready to formulate the problems addressed in this paper, namely that of diagnosis and repair of a monolithic specification $\varphi$ (general diagnosis and repair), and of an $\Lambda/G$ contract $(\varphi_e, \varphi_e \rightarrow \varphi_s)$ (contract diagnosis and repair).

![Figure 2: Diagnosis and repair flow diagram.](image)

**PROBLEM 1 (GENERAL DIAGNOSIS AND REPAIR).**

Given a controller synthesis problem $\mathcal{P} = (f_d, g_d, x_0, \varphi, J)$ such that $\mathcal{P}$ is infeasible, find:

- A set of atomic predicates $D = \{\mu_1, \ldots, \mu_d\}$ or time intervals $T = \{\tau_1, \ldots, \tau_d\}$ of the original formula $\varphi$,
- $\varphi' \in \text{REPAIR}_{\mathcal{D}, \mathcal{T}}(\varphi)$, such that $\mathcal{P}' = (f_d, g_d, x_0, \varphi', J)$ is feasible, and the following minimality conditions hold:
  1. (time interval minimality) if $\varphi'$ is obtained by predicate repair $\varphi'[\tau_i] = (\tau_i^1, \tau_i^2)$ are the non-empty repaired intervals, and $|\tau_i^1|$ is the length of interval $\tau_i$,
  2. (atomic predicate minimality) if $\varphi'$ is obtained by predicate repair $\varphi'[\mu_i^e] = (\mu_i^e, \mu_i^e')$ for $i \in \{1, \ldots, m\}$, and $|\mu_i^e| = |\mu_i^e'| = |\mu_i^e| = |\mu_i^e'|$.

**Problem 2 (Contract Diagnosis and Repair).**

Given a controller synthesis problem $\mathcal{P} = (f_d, g_d, x_0, \varphi \equiv \varphi_e \rightarrow \varphi_s, J)$ such that $\mathcal{P}$ is infeasible, find:

- Sets of atomic predicates $\mathcal{D}_e = \{\mu_1^e, \ldots, \mu_n^e\}$, $\mathcal{D}_s = \{\mu_1^s, \ldots, \mu_n^s\}$ or sets of time intervals $\mathcal{T}_e = \{\tau_1^e, \ldots, \tau_r^e\}$, $\mathcal{T}_s = \{\tau_1^s, \ldots, \tau_r^s\}$, respectively, of the original formula $\varphi_e$ and $\varphi_s$,
- $\varphi'_e \in \text{REPAIR}_{\mathcal{D}_e, \mathcal{T}_e}(\varphi_e)$, $\varphi'_s \in \text{REPAIR}_{\mathcal{D}_s, \mathcal{T}_s}(\varphi_s)$,
- $\mathcal{P}' = (f_d, g_d, x_0, \varphi', J)$ is feasible, and $\mathcal{D} = \mathcal{D}_e \cup \mathcal{D}_s$, $\mathcal{T} = \mathcal{T}_e \cup \mathcal{T}_s$, and $\varphi'$ satisfy the minimality conditions of Problem 1.

5. **MONOLITHIC SPECIFICATIONS**

Fig. 2 represents the workflow adopted to diagnose inconsistencies in the specification and provide constructive feedback to the designer. In this section, we describe our solution to Prob. 1 as summarized in Alg. [1]. Given a problem $\mathcal{P}$, defined as in Sec. 4, the method $\text{GenMILP}$ reformulates (6) in terms of the following MILP:

$$
\begin{align*}
\text{minimize} & \quad J(\xi^H) \\
\text{subject to} & \quad f_i^{\text{dyn}} \leq 0 \quad i \in \{1, \ldots, m_d\} \\
& \quad f_k^{\text{est}} \leq 0 \quad k \in \{1, \ldots, m_e\},
\end{align*}
$$

where $f^{\text{dyn}}$ and $f^{\text{est}}$ are mixed integer linear constraint functions over the states, outputs, and inputs of the finite horizon trajectory $\xi^H$ associated, respectively, with the system $1$For technical reasons, our minimality conditions are predicated on a single type of repair being applied to obtain $\varphi'$.
IIS) \[1, 5\] of constraints irreducibly inconsistent
the-art MILP solvers to provide an infeasible subset of constraints if any single constraint is removed. For
infeasible; (ii) the optimization problem \( f \), defined in Problem 1, as follows. The slack vector
variables \( s \) denote both categories of constraints \( f^{\text{syn}} \) and \( f^{\text{adv}} \) in the feasibility problem \( M \). We reformulate \( M \) as the following feasibility problem with slack:

\[
\min \sum_{s \in \mathbb{R}^{\mathcal{J}}} ||s||
\text{subject to } f_i - s_i \leq 0 \quad i \in \{1, \ldots, |\mathcal{J}|\}
\]

\[
f_i \leq 0 \quad i \in \{\mathcal{I} + 1, \ldots, m\}
\]

\[
s_i \geq 0 \quad i \in \{1, \ldots, |\mathcal{I}|\},
\]

where \( s = [s_1, \ldots, s_{|\mathcal{I}|}] \) is a vector of slack variables added to the set \( \mathcal{I} \) obtained after the latest call of Diagnosis. Note that not all the constraints in the original optimization Eq. (13) can be modified. For instance, the designer will not be able to arbitrarily modify constraints that can directly affect the dynamics of the system, i.e., constraints encoded in \( \mathcal{P}^{\text{syn}} \). Solving Eq. (14) is equivalent to looking for a set of slack variables that make the original control problem feasible while minimizing a suitable norm \( ||\cdot|| \) of the slack vector. In most of our applications, we choose the l1-norm, which tends to provide sparser solutions for \( s \), i.e., nonzero slacks for a smaller number of constraints. However, other norms can be used, including weighted norms based on the set of weights \( \lambda \). If Problem (14) is feasible, \( \text{ExtractFeedback} \) uses the solution \( s^* \) to repair the original infeasible specification \( \varphi \). Otherwise, an infeasible problem is returned for another round of diagnosis to retrieve further constraints to relax. Next, we provide details on the implementation of \( \text{ExtractFeedback} \).

Given a minimum norm solution \( s^* \) to Eq. (14), the slack variables \( s^* \) are mapped to a set of predicate repairs \( s_\mathcal{P} \), as defined in Problem 2 as follows. The slack vector \( s^* \) in Alg. 3 includes the set of slack variables \( \{s^*_i, t\} \), where \( s^*_i, t \) is the variable added to the optimization constraint associated with an atomic predicate \( \mu_i \in \mathcal{D} \) at time \( t \), \( i \in \{1, \ldots, d\} \). We then set \( \forall i \in \{1, \ldots, d\} \),

\[
s_i = \mu_i - \mu_i = \max_{t \in \{1, \ldots, H\}} s^*_i, t, \quad i \in \{1, \ldots, d\}, \quad H \text{ being the time horizon for \( \mathcal{D} \), and \( s_\mathcal{P} = \{s_1, \ldots, s_d\} \).}
\]

To find a set of time-interval repairs instead, we proceed as follows:

\[
s_i = \mu_i - \mu_i = \max_{t \in \{1, \ldots, H\}} s^*_i, t, \quad i \in \{1, \ldots, d\}, \quad H \text{ being the time horizon for \( \mathcal{D} \), and \( s_\mathcal{P} = \{s_1, \ldots, s_d\} \).}
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\[
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\]

To find a set of time-interval repairs instead, we proceed as follows:
1. The slack vector $s^*$ in Alg. 3 includes the set of slack variables $\{s^*_{\nu,t}\}$, where $s^*_{\nu,t}$ is added to the optimization constraint associated with atomic predicate $\mu_t \in \mathcal{D}$ at time $t$.

2. We convert every temporal operator in $\varphi$ into a combination of $\mathcal{G}$ (timed or untimed) and untimed $\mathcal{U}$ by using the following transformations:

$$F_{[a,b]}\psi = \neg G_{[a,b]}\neg\psi,$$

$$\psi_1 U_{[a,b]}\psi_2 = G_{[a,b]}(\psi_1 U \psi_2) \land F_{[a,b]}\psi_2,$$

where $U$ is the untimed (unbounded) until operator. Let $\hat{\varphi}$ be the formula obtained from $\varphi$ after these transformations.

3. We construct the syntactic parse tree of $\hat{\varphi}$ based on $P$. Each node is an operator, and the leaves are atomic predicates. The nodes of the parse tree of $\hat{\varphi}$ can be partitioned into three subsets, $\nu$, $\kappa$, and $\delta$, respectively associated with the atomic predicates, Boolean operators, and temporal operators ($\mathcal{G}$, $\mathcal{U}$) in $\hat{\varphi}$. We traverse this parse tree from the leaves (atomic predicates) to the root and recursively define for each node $i$ a new support interval $\sigma^*_i$ as follows:

$$\sigma^*_i = \begin{cases} 
\sigma'_i \cap \sigma_j & \text{if } i \in \nu \cup \kappa \cup \delta \\
\sigma'_i \cap C(i) & \text{if } i \in \delta \mathcal{G}
\end{cases} \quad \text{(16)}$$

where $C(i)$ denotes the children of node $i$, while $\delta \mathcal{G}$ and $\delta \mathcal{U}$ are, respectively, the subsets of nodes associated with the $\mathcal{G}$ and $\mathcal{U}$ operators. We observe that a $\mathcal{G}$ node has a single child. Therefore, with some abuse of notation, we use $C(i)$ in (16) to denote a single node in the parse tree.

4. We define the interval repair $\tilde{\tau}_j$ for each (timed) temporal operator node $j$ in the parse tree of $\hat{\varphi}$ as $\tilde{\tau}_j = \sigma^*_j$. If $\tilde{\tau}_j$ is empty for any $j$, no time-interval repair is possible. Otherwise, we map the set of intervals $\{\tilde{\tau}_j\}$ to a set of interval repairs $\mathcal{T}'$ for the original formula $\varphi$ according to the transformations in step 2 and return $\mathcal{T}'$. We provide an example of predicate repair below, while time interval repair is demonstrated in Sec. 4.

**Example 4 (Collision Avoidance).** We diagnose the specifications introduced in Example 7. To formulate the synthesis problem, we assume a horizon $H = 10$ and a discretization step $\Delta t = 0.2$. The system is found infeasible at time $t = 5$. Intuitively, given the limits on acceleration, the ego vehicle should be the set of slack values that allow $\varphi_1$ and $\varphi_2$ at time $t = 6$. To ensure that the ego vehicle enters the forbidden box at the same time, we calculate the limits on the acceleration of the ego vehicle, both the cars end up entering the forbidden box at the same time.

To avoid collision, we use $\varphi_1$ by adding slacks to all of its predicates, such that $\varphi_1 = (0.5 - s_{11} \leq y_{i,5} \leq 0.5 + s_{11}) \land (0.5 - s_{12} \leq y_{i,6} \leq 0.5 + s_{12})$.

**Table 1:** Slack variables for horizon, with $\Delta t = 0.2$, and $H = 10$. We then obtain $[s_{11}, s_{12}] = [0.82, 0]$ which ultimately gives $\varphi_2 = G_{[0, \infty)}(0.68 \leq a_{i,5}^\text{ego} \leq 2.5)$. The ego vehicle should then slow down to prevent entering the forbidden box at the same time as the other car.

Our algorithm offers the following guarantees, for which a proof sketch is given below. The complete proofs can be found in the extended version of this paper [14].

**Theorem 1 (Soundness).** Given a controller synthesis problem $P = (f_\mathcal{A}, g_\mathcal{A}, x_\mathcal{A}, \varphi, J)$, such that $P'$ is inflexible at time $t$, let $\varphi' \in \mathcal{REPAIR}_{T}(\varphi)$ be the repaired formula returned from Alg. 4 for a given set of predicates $\mathcal{D}$ or time-intervals $T$. Then, $P'' = (f_\mathcal{A}, g_\mathcal{A}, x_\mathcal{A}, \varphi', J)$ is feasible at time $t$ and $(\varphi', T, \mathcal{D})$ satisfy the minimality conditions in Prob. 7.

**Theorem 2 (Completeness).** Assume the controller synthesis problem $P = (f_\mathcal{A}, g_\mathcal{A}, x_\mathcal{A}, \varphi, J)$ results in $P''$ being infeasible at time $t$. If there exist a set of predicates $\mathcal{D}$ or time-intervals $T$ and $\Phi \in \mathcal{REPAIR}_{T}(\varphi)$ for which $\forall \Phi \in \Phi$, $P'' = (f_\mathcal{A}, g_\mathcal{A}, x_\mathcal{A}, \varphi, J)$ is feasible at time $t$ and $(\varphi', T, \mathcal{D})$ is minimal in the sense of Theorem, then Alg. 4 returns a repaired formula $\varphi'$ in $\Phi$.

**Proof Sketch.** We start by discussing the case of soundness for predicate repair. Let $\mathcal{M}$ be the MILP encoding of $P$ as defined in [13]. $\mathcal{M}'$ is the encoding of $P''$, and $\mathcal{M}''$ the feasible MILP obtained from Alg. 4 together with the optimal slack set $\{s_{\nu,t} \in \mathcal{D}, t \in \{1, \ldots, H\}\}$. We note that $\mathcal{M}'$ and $\mathcal{M}''$ are both relaxed versions of $\mathcal{M}$. Moreover, each constraint with a nonzero slack variable in $\mathcal{M}''$ is relaxed in $\mathcal{M}'$, and offset by the largest slack value over the horizon $H$. Since $\mathcal{M}'$ is feasible, $\mathcal{M}'$, and subsequently $P''$, are feasible. To prove that $(\varphi', \mathcal{D})$ satisfy the predicate minimality condition, by Definition 4.1 at least one predicate in $\mathcal{D}$ generates a conflicting constraint and must be repaired. Moreover, because Alg. 4 finds all the IISs in the original optimization problem and allows relaxing any constraints in the union of the IISs, repairing any predicate outside of $\mathcal{D}$ is redundant. Therefore, if a formula $\varphi'$ is obtained from $\varphi$ after repairing a set of predicates $\mathcal{D}$, then the associated repair set $s_{\mathcal{D}}$ is seen as a repair set on the same predicate set as $s_{\mathcal{D}}$. Finally, by the norm minimization in (14), we conclude $\|s_{\mathcal{D}}\| \leq \|s_{\mathcal{D}}\|$. We now consider the MILP formulation $\mathcal{M}'$ associated with $\varphi'$ in the case of time-interval repairs. For each atomic predicate $\mu_t \in \mathcal{D}$, $\mathcal{M}'$ includes only the associated constraints evaluated over time intervals $s^*_t$ for which the slack variables $\{s^*_{\nu,t}\}$ are zero. Such a subset of constraints is trivially feasible. Moreover, because of the structure and the manner in which slacks are added, if the constraints corresponding to the atomic predicates in $\mathcal{D}$ have slack zero, so will any constraints enforcing Boolean or temporal combinations of these predicates. Thus, $\mathcal{M}'$ is feasible. To show the satisfaction of the minimality condition, we observe that Alg. 4 selects, for each $\mu_t \in \mathcal{D}$, the largest interval $s^*_t$ such that the associated constraints are feasible, i.e., their slack variables are zero after norm minimization. Because feasible intervals for Boolean combinations of atomic predicates are obtained by intersecting these maximal intervals, and then propagated to the temporal operators, the length of the intervals of each $G$ operator in $\varphi$, and finally of the temporal operators in $\varphi$, will be maximal.

To prove completeness, we first observe that Alg. 4 always terminates with a feasible solution since the set of MILP constraints to diagnose and repair is finite. Let $\mathcal{D}$ be the
set of predicates modified to obtain \( \phi \in \Phi \) and \( D' \) the set of diagnosed predicates returned by Alg. 1. Then, because \( D' \) includes all the predicates responsible for inconsistencies, as argued above, we conclude \( D \subseteq D' \). By Eq. (14), \( |s_p| \leq |s_p'||\), hence \( \varphi' \in \Phi \). Further, if \( \phi \in \Phi \) repairs a set of intervals \( T = \{ \tau_1, \ldots, \tau_i \} \), then there exists a set of constraints associated with atomic predicates in \( \varphi \) which are consistent in the MILP associated with \( \phi \) and make the overall problem feasible. Then, the relaxed MILP associated with \( \phi \) after slack norm minimization will include a set of constraints admitting zero slacks over the same set of time intervals, thus terminating with a set of non-empty intervals \( T' = \{ \tau'_1, \ldots, \tau'_i \} \). Finally, because Alg. 1 finds the longest such intervals, we are guaranteed that \( |\tau'_i| \geq |\tau_i| \) for all \( i \in \{1, \ldots, I\} \), hence \( \varphi' \in \Phi \) holds.

In the worst case, Alg. 1 solves a number of MILP problem instances equal to the number of atomic predicates in the STL formula. While the complexity of solving a MILP is NP-hard, the actual runtime depends on the size of the MILP, which is quadratic in the size (number of predicates and operators) of the STL specification.

6. **CONTRACTIONS**

In this section, we consider specifications provided in the form of a contract \( (\varphi, \varphi_\circ \rightarrow \varphi_z) \), where \( \varphi_\circ \) expresses the assumptions and \( \varphi_z \) captures the guarantees. To repair contracts, we capture tradeoffs between assumptions and guarantees in terms of minimization of a weighted norm of slacks. We now describe our results for both non-adversarial and adversarial environments.

6.1 **Non-Adversarial Environment**

For a contract, we distinguish between controlled inputs \( u_c \) and uncontrolled (environment) inputs \( u_e \) of the dynamical system. In this section we assume that the environment signal \( w^H \) can be predicted over a finite horizon and set to a known value for which the controller must be synthesized. With \( \varphi \equiv \varphi_\circ \rightarrow \varphi_z \), equation (17) reduces to:

\[
\begin{align*}
\min_{u^H} & \quad J(\xi^H(x_0, u^H, w^H)) \\
\text{subject to} & \quad \xi^H(x_0, u^H, w^H) \models \varphi.
\end{align*}
\]

Because of the similarity of Eq. (17) and Eq. 6, we diagnose and repair a contract using the same methodology illustrated in Sec. 4. However, to reflect the different structure of the specification, i.e., its partition into assumption and guarantees, we adopt a weighted sum of the slack variables in Alg. 1 allocating different weights to predicates in the assumption and guarantee formulae. We provide the same guarantees as in Thms. 3 and 2 where \( \varphi \equiv \varphi_\circ \rightarrow \varphi_z \) and the minimality conditions are stated with respect to the weighted norm.

**Example 5 (Non-Adversarial Race).** We consider Example 3 with the same discretization step \( \Delta t = 0.2 \) and horizon \( H = 10 \). The MPC scheme results infeasible at time 0. In fact, we observe that \( \psi \), as true as \( \hat{v}_{adv}^x > 0.5 \) and \( \hat{v}_{adv}^x = 1 \) at all times. Since \( \hat{v}_{adv}^x = 0 \), the predicate \( \psi' = \Box_{10s} (v_{s0} \geq 0.5) \) in \( \psi' \) is found to be false. As in Sec. 3, we modify the conflicting predicates in the specification by using slack variables as follows: \( v_{adv}^x + s(t) \geq 0.5 \) and \( v_{s0}^x + s(t) \geq 0.5 \). Moreover, we assign weights to the assumptions (\( \lambda_0 \)) and guarantee (\( \lambda_z \)) predicates, our objective being \( \lambda_0 + \lambda_z \). By setting \( \lambda_0 > \lambda_z \), we encourage modifications in the assumptions by falsifying it, which would provide a trivial solution. We instead prefer setting \( \lambda_0 < \lambda_z \), obtaining the slack values in Table 2, which leads to the following predicate repair:

\[
\psi' = \Box_{10s} (v_{s0} \geq -0.01).
\]

<table>
<thead>
<tr>
<th>time</th>
<th>0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1</th>
<th>1.2</th>
<th>1.4</th>
<th>1.6</th>
<th>1.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_{\Delta} )</td>
<td>0.31</td>
<td>0.31</td>
<td>0.11</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2: Slack variables used in Example 2 and 5

![Figure 3: Parse tree of \( \psi \equiv \psi \circ \rightarrow \psi_z \) used in Example 2 and 5](image)

We can also modify the time interval of the temporal operators associated with \( \psi \) to repair the overall specification. Based on the slack values in Table 2, we conclude \( \sigma_1 = \sigma_2 = [0, 9] \) (the optimal slack values for these predicates are always zero), while \( \sigma_3 = [3, 9] \). For the syntax tree in Fig. 3, we have \( \sigma_1 = \sigma_1, \sigma_2 = \sigma_2, \) and \( \sigma_3 = \sigma_3 \) for the temporal operator nodes that are parent nodes of \( \mu_3, \mu_2, \) and \( \mu_3 \).

Since none of the above intervals are empty, a time interval repair is indeed possible by modifying the time interval of the parent node of \( \mu_3 \), thus achieving \( \tau_1 = \sigma_2 \). This leads to the following proposed sub-formula \( \psi_\circ = \Box_{[0, 6, \infty]} (v_{s0} \geq 0.5) \).

In this example, repairing the specification over the first horizon is enough to guarantee controller realizability in the future, and we can keep the upper bound of the \( G \) operator at infinity.

6.2 **Adversarial Environment**

When the environment behaves adversarially, the control synthesis problem assumes the structure in (9). In this paper, we allow \( w_t \) to lie in an interval \( [w_{min}, w_{max}] \) at all times; this corresponds to the STL formula \( \Phi = G_{[0, \infty]} (w_{min} \leq w_t \leq w_{max}) \). We decompose a specification \( \varphi \) of the form \( \varphi_\circ \land \varphi_\circ \rightarrow \varphi_\circ \), representing the contract, as \( \varphi \equiv \varphi_\circ \rightarrow \psi \), where \( \psi \equiv (\varphi_\circ \rightarrow \varphi) \). Our diagnosis and repair method is summarized in Alg. 1.

We first check the satisfiability of the control synthesis problem by examining whether there exists a pair of \( u^H \) and \( w^H \) for which Prob. 6 is feasible (CheckSAT routine):

\[
\begin{align*}
\min_{u^H, w^H} & \quad J(\xi^H(x_0, u^H, w^H)) \\
\text{subject to} & \quad \xi^H(x_0, u^H, w^H) \models \varphi, \\
& \quad w^H \models \varphi_\circ \rightarrow \varphi_\circ.
\end{align*}
\]

If (18) is unsatisfiable, we use the techniques introduced in Sec. 5.2 and 6.1 to diagnose and repair the infeasibility. Therefore, we assume that (18) is satisfiable, hence there exist \( u^H \) and \( w^H \) that solve (18). To check realizability, we use the following CEGIS loop (SolveCEGIS routine).

By first fixing the control trajectory to \( u^H_0 \), we find the worst case disturbance trajectory \( w^H_1 \) that minimizes the robustness value of \( \varphi \) by solving the following problem:

\[
\begin{align*}
\min_{w^H} & \quad \rho^x(\xi^H(x_0, u^H, w^H), 0) \\
\text{subject to} & \quad w^H \models \varphi_\circ \land \varphi_\circ.
\end{align*}
\]

with \( u^H = u^H_0 \). The optimal \( w^H \) from (19) will falsify the specification if the resulting robustness value is below zero.

If this is the case, we look for a \( u^H_2 \) which solves (17) with the additional restriction of \( w^H \in W_{adv} = \{w^H\} \). If this

3A tolerance \( \rho_{\text{min}} \) is selected to accommodate approximation errors, i.e., \( \rho^x(\xi^H(x_0, u^H_0, w^H), 0) < \rho_{\text{min}} \).
step is feasible, we once again attempt to find a worst-case disturbance sequence \( w^H_u \) that solves \((10)\) with \( u^H = u^L \): this is the counterexample-guided inductive step. At each iteration \( i \) of this CEGIS loop, the set of candidate disturbance sequences \( W_{\text{cand}} \) expands to include \( w^H_u \). If the loop terminates at iteration \( i \) with a successful \( u^H \) (one for which the worst case disturbance \( w^H_u \) in \((10)\) has positive robustness), we conclude that the formula \( \varphi \) is realizable.

The CEGIS loop may not terminate if the set \( W_{\text{cand}} \) is infinite. We, therefore, run it for a maximum number of iterations. If SolveCEGIS fails to find a controller sequence prior to the timeout, then \((17)\) is infeasible for the current \( W_{\text{cand}} \), i.e., there is no control input that can satisfy \( \varphi \) for all disturbances in \( W_{\text{cand}} \). We conclude the specification is not realizable (or, equivalently, the contract is inconsistent). While this infeasibility can be repaired by modifying \( \varphi \) based on the techniques in Sec. 12 and 13, an alternative solution is to repair \( \varphi \) by minimally pruning the bounds on \( w_i \) (RepairAdversarial routine). To do so, a basic linear search procedure is implemented as follows. Let:

\[
\begin{align*}
    w_u &= \max_{w_i} w_{i,t} & w_i &= \min_{t \in \{1, \ldots, H-1\}} w_{i,t},
\end{align*}
\]

and define \( s_u = w_{\text{max}} - w_u \) and \( s_l = w_l - w_{\text{min}} \). The differences \( s_u \) and \( s_l \) are used to update the range for \( w_i \) in \( \varphi \) to a maximal interval \([w_{\text{min}}, w_{\text{max}}] \subseteq [w_{\text{min}}, w_{\text{max}}] \) and such that at least one \( w^H_u \in W_{\text{cand}} \) is excluded. Specifically, if \( s_u \leq s_l \), \([w_{\text{min}}, w_{\text{max}}] \) is set to \([w_{\text{min}}, w_{\text{max}}] \), otherwise \([w_{\text{min}}, w_{\text{max}}] \) is set \([w_{\text{min}}, \epsilon] \) being a suitable (small) constant; otherwise \([w_{\text{min}}, w_{\text{max}}] \) is set \([w_{\text{min}}, w_{\text{max}}] \). We implement an improved version of the above procedure, which allows optimizations over subsets of the time sets in \( \{1, \ldots, H-1\} \) based on the time instants at which an infeasibility occurs. Moreover, we use binary search over the range of \( w_i \) for faster convergence. Finally, we use the updated formula \( \varphi' \) to run SolveCEGIS again until a realizable control sequence \( u^H \) is found. In Alg. 4, for a predicate repair procedure, FindMin provides the solution with minimum slack norm over all solutions repairing \( \psi \) and \( \varphi \).

**Example 6 (Adversarial Race).** We consider the specification in Example 3. For the same horizon as in the previous examples, after solving the satisfiability problem, for the fixed \( u^L_1 \), the CEGIS loop returns \( a^{\text{adv}} = 2 \) for all \( t \in \{0, \ldots, H - 1\} \) as the single element in \( W_{\text{cand}} \) for which no controller sequence is found. We then choose to tighten the environment assumptions to make the controller realizable and shrink the bounds on \( a^{\text{adv}} \) by using Alg. 4 (with \( \epsilon = 0.01 \)). After a few iterations, we finally obtain \( w_{\text{min}}' = 0 \) and \( w_{\text{max}}' = 1.24 \), and therefore \( \phi_w = G_{[0,\infty)}(0 \leq a^{\text{adv}} \leq 1.24) \).

To account for the error introduced by \( \epsilon \), given \( \varphi' \in \text{REPAIR}_{\tau}(\varphi) \), we say that \( (\varphi', D, \tau) \) are \( \epsilon \)-minimal if the magnitudes of the predicate repairs (predicate slacks) or time-interval repairs differ by at most \( \epsilon \) from a minimal repair in the sense of Problem 2. Assuming that SolveCEGIS terminates before reaching the maximum number of iterations, the following theorems state the properties of Alg. 4.

**Theorem 3 (Soundness).** Given a controller synthesis problem \( P = (f_a, g_a, x_0, \varphi, J) \), such that \((6)\) is infeasible at time \( t \), let \( \varphi' \in \text{REPAIR}_{\tau}(\varphi) \) be the repaired formula returned from Alg. 4 for a given set of predicates \( D \) or time intervals \( T \). Then, \( \varphi' = (f_a, g_a, x_0, \varphi, J) \) is feasible at time \( t \) and \( (\varphi', D, \tau) \) is \( \epsilon \)-minimal.

**Theorem 4 (Completeness).** Assume the controller synthesis problem \( P = (f_a, g_a, x_0, \varphi, J) \) results in \((6)\) being infeasible at time \( t \). If there exist a set of predicates \( D \) and time intervals \( T \) such that there exists \( \Phi \subseteq \text{REPAIR}_{\tau}(\varphi) \) for which \( \forall \psi \in \Phi, P' = (f_d, g_d, x_0, \psi, J) \) is feasible at time \( t \) and \( (\varphi, D, \tau) \) is \( \epsilon \)-minimal, then Alg. 4 returns a repaired formula \( \varphi' \).

**Proof Sketch.** When \( \psi = \varphi \rightarrow \psi \) is modified using Alg. 4, soundness and completeness are guaranteed by Thm. 6 and the termination of the CEGIS loop. Assuming Alg. 4 modifies the atomic predicates in \( \varphi_w \), the RepairAdversarial routine and \((20)\), together with the termination of the CEGIS loop, assure that \( \varphi_w \) is repaired in such a way that the controller is realizable and \( \epsilon \)-optimal. This gives us soundness. For completeness, we assume there exists a minimal norm repair for the atomic predicates of \( \varphi_w \), which returns a maximal interval \([w_{\text{min}}, w_{\text{max}}] \subseteq [w_{\text{min}}, w_{\text{max}}] \). Then, given the termination of the CEGIS loop, repeated application of \((20)\) and RepairAdversarial will produce a predicate repair such that the corresponding interval \([w_{\text{min}}, w_{\text{max}}] \) makes the control synthesis realizable and is maximal within an error bounded by \( \epsilon \). Hence, \( \varphi' \in \Phi \) holds.

### 7. CASE STUDIES

We developed the toolbox DIARY (Diagnosis and Repair for sYnthesis) implementing our algorithms. DIARY uses YALMIP 15 to formulate the optimization problems and GUROBI 16 to solve them. It interfaces to different synthesis tools, e.g., BluSTL17 and CnSPrSTL18. Here, we summarize some of the results of DiaRY for diagnosis and repair.

#### 7.1 Autonomous Driving

We consider the problem of synthesizing a controller for an autonomous vehicle in a city driving scenario. We analyze the following two tasks: (i) changing lanes on a busy road; (ii) performing an unprotected left turn at a signalized intersection. We use a simple point-mass model for the vehicles on the road. For each vehicle, we define the state as \( x = [x, y, \theta, v]^\top \), where \( x \) and \( y \) denote the coordinates, and \( \theta \) and \( v \) represent the direction and speed, respectively. Let \( u = [u_1, u_2]^\top \) be the control input for each vehicle, where \( u_1 \) is the steering input and \( u_2 \) is the acceleration. Then, the vehicle's state evolves according to the following dynamics:

\[
\begin{align*}
x &= v \cos \theta \\
y &= v \sin \theta \\
\dot{\theta} &= v \cdot u_1 / m \\
v &= u_2,
\end{align*}
\]

where \( m \) is the vehicle mass. To determine the control strategy, we linearize the overall system dynamics around the initial state at each run of the MPC, which is completed in less than 0.1 seconds. For each simulation run, we compute the cost-to-go of the MPC for each vehicle, and we use the solution with the lowest cost-to-go as the controller. We then apply the controller input to the vehicles, and we repeat the process until the system converges to a stable state. We use the following performance metrics to evaluate the controller:

- **Safety:** The vehicles should avoid collisions.
- **Efficiency:** The vehicles should minimize energy consumption.
- **Comfort:** The vehicles should minimize jolts and vibrations.
- **Scalability:** The controller should work for different vehicle speeds and road conditions.

We implemented the controller in C++, and we used the YALMIP 15 and GUROBI 16 optimization libraries to solve the optimization problems. The controller is tested on a set of scenarios, including changing lanes and performing unprotected left turns. The results show that the controller is able to achieve the desired performance metrics and that it is scalable to different vehicle speeds and road conditions.
than 2 s on a 2.3-GHz Intel Core i7 processor with 16-GB memory. We further impose the following constraints on the ego vehicle (i.e., the vehicle under control): (i) a minimum distance must be established between the ego vehicle and other cars on the road to avoid collisions; (ii) the ego vehicle must obey the traffic lights; (iii) the ego vehicle must stay within its road boundaries.

7.1.1 Lane Change
We consider a lane change scenario on a busy road as shown in Fig. 4a. The ego vehicle is in red. Car 1 is at the back of the left lane, Car 2 is in the front of the left lane, while Car 3 is on the right lane. The states of the vehicles are initialized as follows: \( x_{0\text{Car } 1} = [-0.2, -1.5, \frac{5}{2}, 0.5]^\top \), \( x_{0\text{Car } 2} = [-0.2, 1.5, \frac{5}{2}, 0.5]^\top \), \( x_{0\text{Car } 3} = [0.2, 1.5, \frac{5}{2}, 0]^\top \), and \( x_{0\text{ego}} = [0.2, -0.7, \frac{5}{2}, 0]^\top \). The control inputs are initialized as follows: \( u_{0\text{Car } 1} = [0, 1]^\top \), \( u_{0\text{Car } 2} = [0, -0.25]^\top \), \( u_{0\text{Car } 3} = [0, 0]^\top \), and \( u_{0\text{ego}} = [0, 0]^\top \). The objective of ego is to safely change lane, while satisfying the following requirements:

\[
\begin{align*}
\varphi_{\text{acc}} &= G_{[0,\infty)} ([u_2] \leq 1) \quad \text{Acceleration Bounds (22)} \\
\varphi_{\text{vel}} &= G_{[0,\infty)} ([v] \leq 1) \quad \text{Velocity Bounds}
\end{align*}
\]

The solid blue line in Fig. 4 is the trajectory of ego as obtained from our MPC scheme, while the dotted green line is the future trajectory pre-computed for a given horizon at a given time. MPC becomes infeasible at time \( t = 1.2 \) s when the no-collision requirement is violated, and a possible collision is detected between the ego vehicle and Car 1 before the lane change is completed (Fig. 4b). Our solver takes 2 s, out of which 1.4 s are needed to generate all the IISs, consisting of 39 constraints. The run time is negligible with respect to the time needed to encode the original optimization problem, which is typically higher by an order of magnitude. To make the system feasible, the proposed repair increases both the acceleration bounds and the velocity bounds on the ego vehicle as follows:

\[
\begin{align*}
\varphi_{\text{acc}} &= G_{[0,\infty)} ([u_2] \leq 3.5), \quad \varphi_{\text{vel}} = G_{[0,\infty)} ([v] \leq 1.54).
\end{align*}
\]

When replacing the initial requirements \( \varphi_{\text{acc}} \) and \( \varphi_{\text{vel}} \) with the modified ones, the revised MPC scheme allows the vehicle to travel faster and safely complete a lane change maneuver, without risks of collision, as shown in Fig. 4b.

7.1.2 Unprotected Left Turn
In the second scenario, we would like the ego vehicle to perform an unprotected left turn at a signalized intersection, where the ego vehicle has a green light and is supposed to yield to oncoming traffic, represented by the yellow cars crossing the intersection in Fig. 5. The environment vehicles are initialized at the states:

\[
\begin{align*}
\begin{array}{c}
x_{0\text{Car } 1} = [-0.2, 0.7, -\frac{5}{2}, 0.5]^\top \\
x_{0\text{Car } 2} = [-0.2, 1.5, -\frac{5}{2}, 0.5]^\top
\end{array}
\end{align*}
\]

while the ego vehicle is initialized at \( x_{0\text{ego}} = [0, 0, -0.7, 0]^\top \). The control input for each vehicle is initialized at \([0, 0]^\top \). Moreover, we use the same bounds as in (22).

The MPC scheme becomes infeasible at \( t = 2.1 \) s. The solver takes 5 s, out of which 2.2 s are used to generate the IISs, including 56 constraints. As shown in Fig. 5a, the ego vehicle yields in the middle of intersection for the oncoming traffic to pass. However, the traffic signal turns red in the meanwhile, and there is no feasible control input for the ego vehicle without breaking the traffic light rules. Since we do not allow modifications to the traffic light rules, the original specification is repaired again by increasing the bounds on acceleration and velocity, thus obtaining:

\[
\begin{align*}
\varphi_{\text{acc}} &= G_{[0,\infty)} ([u_2] \leq 11.903), \quad \varphi_{\text{vel}} = G_{[0,\infty)} ([v] \leq 2.42).
\end{align*}
\]

As shown by the trajectory in Fig. 5b, under the assumptions and initial conditions of our scenario, higher allowed velocity and acceleration make the ego vehicle turn before the oncoming cars get close or cross the intersection.

7.2 Aircraft Electric Power System
Fig. 6 shows a simplified architecture for the primary power distribution system in a passenger aircraft [20]. Two power sources, the left and right generators \( G_0 \) and \( G_1 \), deliver power to a set of high-voltage AC and DC buses \( B_0, B_1, DB_0, \) and \( DB_1 \) and their loads. AC power from the generators is converted to DC power by rectifier units \( R_1 \) and \( R_2 \). A bus power control unit (controller) monitors the availability of power sources and configures a set of electromechanical switches, denoted as contactors \( (C_0, \ldots, C_4) \), such that essential buses remain powered even in the presence of failures, while satisfying a set of safety, reliability, and real-time performance requirements [20]. Specifically, we assume that only the right DC bus \( DB_1 \) is essential, and use our algorithms to check the feasibility of a controller that accommodates a failure in the right generator \( G_1 \), by rerout-
ing power from the left generator to the right DC bus in a time interval which is less than or equal to $t_{\text{max}} = 100$ ms. In addition, the controller must satisfy the following set of requirements, all captured by an STL contract.

**Assumptions.** When a contactor receives an open (close) signal, it shall become open (closed) in $80$ ms or less. Let the time discretization step be $20$ ms, $i \in \{0, \ldots , 4\}$ be a set of Boolean variables describing the controller signal (where 1 (0) stands for “closed” (“open”)), $c_i$ be a set of Boolean variables denoting the state of the contactors. The system assumptions are a conjunction of formulas of the form: $G_{[0, \infty)}(c_i \rightarrow F_{[0,4]}c_i)$, providing a model for the discrete-time binary-valued contactor states. The actual delay of each contactor is modeled using an integer (environment) variable $k_i$ for which we require: $G_{[0, \infty)}(0 \leq k_i \leq 4)$.

**Guarantees.** If a generator becomes unavailable (fails), the controller shall disconnect it from the power network in $20$ ms or less. Let $g_0$ and $g_1$ be Boolean environment variables representing the state of the generators, where 1 (0) stands for “available” (“failure”). We encode the above guarantees as $G_{[0, \infty)}(g_0 \rightarrow F_{[0,1]}c_i)$. A DC bus shall never be disconnected from an AC generator for $100$ ms or more, i.e., $G_{[0, \infty)}(\neg b_i \rightarrow F_{[0,5]}b_i)$, where $b_i$ $i \in \{0, \ldots, 3\}$ is a set of Boolean environment variables denoting the status of a bus, where 1 (0) stands for “powered” ("unpowered"). Additional guarantees expressed as STL formulas, include: (i) If both AC generators are available, the left (right) AC generator shall power the left (right) AC bus. $c_0$ and $c_1$ shall be closed. (ii) If only one generator is available, all buses shall be connected to it. (iii) Two generators must never be directly connected.

We apply the diagnosis and repair procedure in Sec. 2 to investigate if there exists a control strategy that satisfies the specification above over all possible values of contactor delays. Fig. 3 shows the controller is unrealizable; a trace of contactor delays equal to 4 at all times provides a counterexample, which leaves DB2 unpowered for $160$ ms, exceeding the maximum allowed delay of $100$ ms. In fact, the controller cannot close $C_2$ until $C_1$ is tested as being open, to ensure that $G_1$ is safely isolated from $G_2$. To guarantee realizability, Alg. 1 suggests to modify our assumptions to $G_{[0, \infty)}(0 \leq k_i \leq 2)$ for $i \in \{0, \ldots, 4\}$. Alternatively, by interpreting the provided counterexamples, it is possible to relax the guarantee on $DB_1$ to $G_{[0, \infty)}(\neg b_0 \rightarrow F_{[0,8]}b_0)$. The execution time is $3926$ s, which includes formulating and executing $3$ CEGIS loops, requiring $6$ optimization problems.

### 8. CONCLUSION

We presented a set of algorithms for diagnosis and repair of STL specifications in the setting of controller synthesis for hybrid systems using a mixed integer programming approach. Given an unrealizable specification, our algorithms detect possible reasons for infeasibility and suggest repairs to make it realizable. We showed the effectiveness of our approach on the synthesis of controllers for several applications. As future work, we plan to investigate techniques that better leverage the structure of the STL formulas, handle a broader range of environment assumptions, and apply to the control of human-in-the-loop systems as explored in [14].

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### 10. REFERENCES


