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Development and Evaluation of Real-Time Volumetric Compton Gamma-Ray Imaging

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Development and Evaluation of Real-Time Volumetric Compton Gamma-Ray Imaging

by

Ross Wegner Barnowski

A dissertation submitted in partial satisfaction of the requirements for the degree of Doctor of Philosophy

in

Engineering - Nuclear Engineering

in the

Graduate Division

of the

University of California, Berkeley

Committee in charge:

Professor Kai Vetter, Chair
Professor Eric Norman
Professor Steven Conolly

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Development and Evaluation of Real-Time Volumetric Compton Gamma-Ray Imaging

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Ross Wegner Barnowski
Abstract

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Doctor of Philosophy in Engineering - Nuclear Engineering

University of California, Berkeley

Professor Kai Vetter, Chair

An approach to gamma-ray imaging has been developed that enables near real-time volumetric (3D) imaging of unknown environments thus improving the utility of gamma-ray imaging for source-search and radiation mapping applications. The approach, herein dubbed scene data fusion (SDF), is based on integrating mobile radiation imagers with real-time tracking and scene reconstruction algorithms to enable a mobile mode of operation and 3D localization of gamma-ray sources. The real-time tracking allows the imager to be moved throughout the environment or around a particular object of interest, obtaining the multiple perspectives necessary for standoff 3D imaging. A 3D model of the scene, provided in real-time by a simultaneous localization and mapping (SLAM) algorithm, can be incorporated into the image reconstruction reducing the reconstruction time and improving imaging performance. The SDF concept is demonstrated in this work with a Microsoft Kinect RGB-D sensor, a real-time SLAM solver, and two different mobile gamma-ray imaging platforms. The first is a cart-based imaging platform known as the Volumetric Compton Imager (VCI), comprising two 3D position-sensitive high purity germanium (HPGe) detectors, exhibiting excellent gamma-ray imaging characteristics, but with limited mobility due to the size and weight of the cart. The second system is the High Efficiency Multimodal Imager (HEMI) a hand-portable gamma-ray imager comprising 96 individual cm$^3$ CdZnTe crystals arranged in a two-plane, active-mask configuration. The HEMI instrument has poorer energy and angular resolution than the VCI, but is truly hand-portable, allowing the SDF concept to be tested in multiple environments and for more challenging imaging scenarios. An iterative algorithm based on Compton kinematics is used to reconstruct the gamma-ray source distribution in all three spatial dimensions. Each of the two mobile imaging systems are used to demonstrate SDF for a variety of scenarios, including general search and mapping scenarios with several point gamma-ray sources over the range of energies relevant for Compton imaging. More specific imaging scenarios are also addressed, including directed search and object interrogation scenarios. Finally, the volumetric image quality is quantitatively investigated with respect to the number of Compton events acquired during a measurement, the list-mode uncertainty of the Compton cone data, and the uncertainty in the pose estimate from the
real-time tracking algorithm. SDF advances the real-world applicability of gamma-ray imaging for many search, mapping, and verification scenarios by improving the tractibility of the gamma-ray image reconstruction and providing context for the 3D localization of gamma-ray sources within the environment in real-time.
To my family and former teachers
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Chapter 1

Overview: Introduction and Motivation

1.1 Applications of Gamma-Ray Imaging

Gamma-ray imaging has many applications in a wide variety of fields, including gamma-ray astronomy, medical imaging, and nuclear security and safeguards. Additionally, the application of gamma-ray imaging to environmental monitoring and nuclear contamination remediation scenarios is being actively researched [1, 2]. There exist several directional imaging methods that characterize the spatial distribution of gamma-ray sources in two-dimensional (2D) angular space. Compton and coded-aperture based methods have been used to provide sensitivity to the spatial distributions of cosmic gamma-ray sources [3, 4], while collimator-based approaches have long been a staple in diagnostic medical imaging [5]. These imaging methods have also been adopted for nuclear security and safeguards applications such as nuclear facility monitoring, nuclear source search (both directed and undirected), and emergency response. Applications of gamma-ray imaging can be divided into two classes: in situ measurements, where the detector system is brought to an environment of interest; or “fixed” scenarios, where samples of the environment (or, in the case of biomedical imaging, patients) are taken to a pre-defined, controlled measurement environment. Volumetric imaging has been implemented for the latter category; for example, 3D imaging is achieved in biomedical applications by leveraging the self-contained nature of the imaging domain. This allows full or partial tomographic image reconstruction methods [6] to be applied for both transmission [7] and emission [8, 9] based imaging modalities. Other approaches that don’t rely on tomographic reconstruction techniques have been studied, though they too rely on the physically constrained nature of the imaging domain [10, 11, 12].

Many of the in situ applications would also benefit from volumetric imaging; i.e., the ability to characterize the distribution of gamma-ray sources in all three spatial dimensions within the original measurement environment. However, there are challenges prevalent in many non-medical imaging scenarios that preclude the direct application of existing volu-
metric methods. For instance, many potential \textit{in situ} applications of gamma-ray imaging are characterized by standoff distances that are large relative to the scale encountered in biomedical imaging. The “standoff” designation in this work represents distances spanning the range of one to tens of meters, commensurate with the scale relevant for many potential \textit{in situ} imaging applications listed above. Thus volumetric imaging approaches for near-field imaging are not generally suitable for \textit{in situ} applications. Additional challenges for the \textit{in situ} case include large variability in the size and complexity of the imaging environment; the inability to acquire orthogonal projections; and comparatively low count rates in many real-world scenarios. Previous work has investigated combining conventional 2D gamma-ray projection imaging techniques with 3D models of the measurement environment [13]. Such methods demonstrate the value of the enhanced context provided by 3D models, but fall short of volumetric imaging as the gamma-ray image itself does not contain any information about the distribution of gamma-ray source in the depth dimension. This work presents the development of a different approach to volumetric imaging to surmount these challenges, enhancing real-time localization capabilities for many potential \textit{in situ} applications of gamma-ray imaging.

1.2 Volumetric Gamma-Ray Imaging

Conventional gamma-ray imaging comprises 2D directional techniques that recover the intensity of gamma-ray sources in angular space, $(\phi, \theta)$. Each pixel in the image represents the intensity of a gamma-ray source along a ray projected from the principal point of the gamma-ray imager to some two-dimensional imaging surface. The geometry of this surface varies depending on the imaging modality, but the recovered image is a measure of gamma-ray intensity along rays emanating from the detector; hence \textit{directional} imaging. A volumetric image reconstruction method by contrast recovers the distribution of gamma-ray sources simultaneously in all three spatial dimensions, $x, y, z \in \mathbb{R}^3$.

1.2.1 Projection Imaging vs. Volumetric Imaging

The combination of directional gamma-ray image reconstruction methods with independently acquired depth information has been investigated as a means of acquiring information about the 3D spatial distribution of gamma-ray sources [13]. Images (either 2D or 3D) acquired independently from multiple modalities can be subsequently co-registered to produce an image overlay. For instance, a directional gamma-ray image can be projected onto a 3D model of the scene in which the gamma-ray image was taken. An example measurement scenario in which gamma-ray imaging may be employed is presented to illustrate the difference between the image overlay technique and full volumetric gamma-ray imaging. Consider the scene shown in Fig. 1.1. For the purposes of illustration, this setup is intended to represent a simplified model of piping in a facility that handles nuclear materials. In this situation, gamma-ray imaging may be employed to localize “holdup”: an unintended aggregation of
Figure 1.1: A simple laboratory mock-up to demonstrate the application of gamma-ray imaging to the localization of radioactive holdup in a nuclear facility. The holdup in this case is represented by a 50 μCi $^{137}$Cs point source in the pipe joint at the indicated location.

nuclear material [14]. The nuclear holdup is represented by a 50 μCi point source of $^{137}$Cs, placed in the front elbow joint of the pipe network as indicated in Fig. 1.1. A 3D model of the scene is acquired independent of the gamma-ray image: in this case, a texturized dense point cloud model, shown in Fig. 1.2. To create the gamma-ray image, a Compton imager with a 4π field-of-view (FOV) is placed directly in front of the pipe setup, with the front elbow joint directly at the center of the FOV. Fig. 1.3 shows a high-resolution gamma-ray image produced by the Compton imager via filtered back projection reconstruction [15]. The 4π directional gamma-ray image is represented as a 2D histogram in angular space, ($\phi, \theta$), with a bin width of 1°. The intensity of the gamma-ray source is encoded in the colormap, blue for relatively low intensity increasing to red for high intensity. To generate the 3D image overlay, the directional gamma-ray image from Fig. 1.3 is aligned with and projected onto the 3D scene model in Fig. 1.2, resulting in Fig. 1.4. The points constituting the 3D model of the scene are colored by the intensity in the gamma-ray image. The direction corresponding to high gamma-ray source intensity in Fig. 1.3 is projected along the entire length of one of the pipes as well as onto the wall behind the piping network. It is not possible to determine the location and extent of the holdup within the piping network from Fig. 1.4. In some cases it may be possible to infer likely locations of point sources given prior knowledge about the scenario. For example, radioactive holdup might be expected to occur at joints within the piping network due to dynamics of flow redirection. An assumption based on this information would limit the probable location of holdup within Fig. 1.4.
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Figure 1.2: A 3D model of the scene containing the pipe network, represented as a collection of colorized 3D points (i.e., a point cloud model). For further detail on how the model is acquired, see section 3.6.1.

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the two elbow joints that exhibit high intensity, but the overlaid image itself contains no information to resolve this ambiguity.

One may note that the placement of the gamma-ray imager in this example, with the source in the center of the field-of-view, represents the worst-case scenario for the 2D reconstruction of this particular scene due to the collinearity of the high-intensity rays with the central pipe. The ambiguity about the location of a point source could be reduced by moving the imager so that the collinearity is eliminated. The process of acquiring multiple perspectives and determining depth via triangulation is at the core of the volumetric imaging approach described in this work, and is discussed in greater detail in section 2.5.

Volumetric imaging is not subject to the ambiguities of overlaying a 2D gamma-ray image on a 3D model as the the gamma-ray source distribution is recovered in all three spatial dimensions. Information about the extent of the source in depth is reconstructed from the gamma-ray data rather than inferred from correlations between the 3D model and the 2D image (though it will be shown the model may be incorporated into the image reconstruction in other ways). Fig. 1.5 shows the result of a full volumetric reconstruction of the holdup scenario described above. The volumetric gamma-ray image is represented by isosurfaces computed from image intensity within a voxelized space. The light blue, green, and gold isosurfaces represent contours at 25%, 50%, and 75% of the maximum reconstructed image intensity respectively, illustrating a point source of gamma-rays at the correct location. The 3D model is also displayed to relate the reconstructed gamma-ray source distribution to
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The image overlay is not without value and provides improved contextual information for the gamma-ray image, in some cases even allowing one to infer the 3D location of a gamma-ray source for simple distributions. Despite this enhancement, overlaying a 2D image onto an independently acquired 3D model is not sufficient to provide depth sensitivity in the general case, especially for the complex source and scene geometries expected in many in situ applications of gamma-ray imaging.

1.2.2 Advantages of Volumetric Gamma-Ray Imaging

The extension of gamma-ray imaging from 2D directional imaging to full 3D spatial imaging has many potential advantages. The gamma-ray source distribution is recovered in $\mathbb{R}^3$ as
opposed to angular space, eliminating ambiguities and artifacts from projection as demonstrated in section 1.2.1. Another advantage is the enhanced context for the gamma-ray image provided by the 3D model. While difficult to quantify, a rendered 3D model of the scene can improve the interpretability of the gamma-ray image compared to conventional overlays on RGB images. However, a 3D model in the absence of a volumetric gamma-ray image information can be misleading. The overlay approach is especially susceptible to faulty inference due to misalignment between the image and the 3D model. The effects of misalignment between the 3D model and the gamma-ray images are illustrated in Fig. 1.6 and 1.7 for the overlay and volumetric approaches respectively. An error of just 3° in the azimuthal angle during the alignment procedure causes the hotspot to be projected onto the horizontal pipe near the back in the image overlay. The image in Fig. 1.6 is readily interpreted as a point source at an incorrect location due to the misalignment. When the same magnitude of misalignment is applied to the volumetric image, the depth information in the gamma-ray
image prevents the incorrect inference of the source location. In fact, the presence of a hot spot at a non-physical location in the volumetric image instead provides evidence of the misalignment.

The ability to determine the 3D spatial distribution of gamma-ray sources from an image is a necessary step towards the ability to determine the quantity of radionuclides in the environment from standoff imaging. Take for example a single point source of a gamma-ray emitting radionuclide. Knowledge of the location of the point source is necessary to account for the geometric $\frac{1}{r^2}$ effect when determining the exposure rate from that source at a given location. For the case of a single point source, simpler methods for determining the source-detector distance are sufficient: employing a laser rangefinder or projecting the 2D image onto a 3D model, for instance. However, quantitative imaging of more complex source distributions such as those encountered in nuclear contamination scenarios requires knowledge of the full spatial distribution of the gamma-ray source(s). In the most general case, a volumetric imaging approach is necessary if one hopes to recover gamma-ray source activities via standoff imaging. The ability to recover the 3D spatial distribution of gamma-ray sources is not sufficient to recover source activity (information about the attenuative environment and a full characterization of the response of the radiation imager are also necessary); but is a necessary step towards general standoff quantitative imaging.
1.3 Previous Work

The approach to volumetric gamma-ray imaging presented herein is based on previous work in which a 3D model of the scene was incorporated into a gamma-ray image reconstruction algorithm. This prior work aimed to demonstrate standoff volumetric gamma-ray imaging and to investigate potential advantages from using the model to constrain the volumetric imaging space. The work in [16] presents the results of a single measurement in which a gamma-ray imager was manually positioned at three separate locations within a laboratory environment. The imager collected data for 24 hours at each location, resulting in the acquisition of thousands of Compton cones. The location and orientation of the gamma-ray imaging system were measured manually at each location. 3D models of the environment were constructed using a non-integrated Light Detection and Ranging (LiDAR) instrument that was also positioned at three independent locations. The image reconstruction algorithm used in [16] is based on the Compton modality; though it relied on thousands of Compton cones collected over 72 hours to reconstruct the gamma-ray image.

This prior investigation demonstrated the feasibility of a volumetric image reconstruction technique based on far-field Compton imaging, but the limitations enumerated above make the procedure impractical for most in situ imaging applications. Some such applications require images be produced on a time scale shorter than the measurement time to provide actionable feedback during the measurement. In many applications, waiting to acquire thousands of Compton cones may not be feasible; especially when trying to localize weak or shielded gamma-ray sources. The need to manually position and measure the location of multiple different sensors makes the procedure in [16] impractical for any measurement scenario where either time or accessibility are restricted. The work presented in this thesis expands on the concepts that were proven feasible in [16] to develop a general approach to real-time volumetric imaging for in situ applications.

1.4 Multi-Sensor Fusion

In general, as the dimensionality of an output space increases, the ability to recover distributions in that space necessitates the probing of each dimension. An increase in the amount of information to be recovered is thus often accompanied by increased complexity in the sensing instrument. Fig. 1.8 illustrates this principle in the context of gamma-ray measurement. The most basic information pertaining to gamma-rays one might be interested in determining about an environment is the presence or absence of gamma-ray sources. This scenario is often termed the “detection” problem. The minimum requirement for instrumentation to address the detection problem is a method for converting gamma-ray radiation into a measurable signal. The Geiger-Müller (GM) counter is a simple instrument designed for this purpose, converting energetic electrons liberated via gamma-ray interactions into a large electrical current impulse via the Townsend avalanche process. GM counters, despite their ubiquity, are among the simplest radiation detection instruments and are incapable of
recovering information beyond simple counting of gamma-ray interactions. One can move around with a GM counter to get a rough idea of the relative strength of radiation fields based on the measured count-rate at various locations, but this approach is very limited for characterizing unknown radiation environments. The next step in characterizing a gamma-ray source is often to identify which specific radionuclides are present. This problem requires spectroscopic instrumentation capable of recovering information about the distribution of the energies of gamma-rays incident upon the instrument. The requirements for gamma-ray instrumentation continue to increase as the applications of gamma-ray measurement become more specific.

Figure 1.8: Increasing the complexity of the inference necessitates the use of instrumentation sensitive to more information in the domain of interest.

One application of gamma-ray measurement is the quantitative determination of the isotopic composition of a given sample. This is an appealing application due to the non-destructive nature of the assay. Other technologies for material quantification, such as mass spectrometry or chemical separation, result in the alteration or destruction of the sample. Precision gamma-ray measurement techniques have been widely employed for quantitative measurement, as in neutron activation analysis \[18\]. For applications of gamma-ray measurement to isotopic quantification, it is important not only to have the gamma-ray sensor(s) completely characterized via extensive calibration measurements \[19\], but the measurement environment must also be accounted for. In practice, this accounting is achieved by tightly
controlling the measurement environment. It is not uncommon to employ gamma-ray spectrometers in a well-geometry and to use a combination of anti-coincidence techniques for background gamma-ray rejection \[20\] and simulation to completely characterize not only the detector response, but the behavior of the entire system including attenuation and scattering of gamma-rays in the materials surrounding the gamma-ray detector. The paradigm of tightly controlling the environment extends beyond quantitative measurements based on spectroscopy to quantitative gamma-ray imaging as well. Quantitative gamma-ray imaging has been demonstrated in the medical field, again relying on extensive characterization of the measurement environment \[21, 22\]. The goal of this work is to develop volumetric gamma-ray imaging capabilities in a different application space, where the environment is not tightly controlled. In the general case of \textit{in situ} measurements, it may be that nothing is known about the environment prior to the measurement. Thus instrumentation must be devised that is sensitive not only to the incident gamma-ray radiation, but can provide additional information about the geometric relationship between the sensor and the environment, as well as information about the environment itself.

Multi-sensor fusion is a technique that relies on integration of data from several sensors for the solution of a specific problem. Auxiliary sensors provide data streams that are complementary, providing information in a domain in which the primary sensor is insensitive. For \textit{in situ} volumetric gamma-ray imaging, a sensor-fusion technique is presented that provides information about the 3D geometry of the imaging environment, as well as enabling mobile detector operation to better sample gamma-ray distributions and provide the multiple perspectives necessary for depth sensitivity in the far-field regime (see section \[2.5\]). This is accomplished by leveraging visual sensors to provide information about the measurement environment and the location of the gamma-ray sensor within it. As the auxiliary sensors provide information about the depth and geometry of the imaging environment, or “scene”, the fusion approach developed in this work has been termed Scene Data Fusion (SDF).

### 1.5 Scene Data Fusion

#### 1.5.1 Introduction

Scene data fusion represents a specific subset of multi-sensor fusion based on integrating simultaneous localization and mapping (SLAM) algorithms and associated sensors with portable gamma-ray imagers. SDF surmounts the limitations of \[10\] by enabling a mobile mode of operation for portable radiation imaging devices. The sensor tracking from real-time SLAM methods provides an estimate of the location and orientation (i.e., pose) of the system as it moves throughout the scene. The pose defining the coordinate frame of the sensor has six degrees of freedom, three translational that specify the displacement of the sensor from a given origin, and three rotational to specify the orientation. A track throughout the scene populated by gamma-ray interactions is produced by synchronizing and spatially registering estimates of the pose of the visual sensor with data from the gamma-ray imager.
This collection of spatially-registered gamma-ray interaction data serves as the input to the volumetric gamma-ray image reconstruction algorithm.

In practice, these data may result in noisy or inaccurate reconstructions due to real-world imaging challenges such as low count rate or limited sampling of the image space. In such cases, the 3D model of the scene can be used to spatially constrain imaging space, improving the image quality in terms of localization accuracy and reduced image noise, as well as decreasing the computation time. SDF is defined by the utilization of both the localization and mapping capability provided by directly integrating SLAM algorithms and auxiliary sensors with instrumentation sensitive to gamma-ray radiation. In principle, there are many ways by which the complementary data can be utilized in the reconstruction due to the multitude of gamma-ray imaging modalities (Compton, collimated, etc.) and the various approaches to the image reconstruction (e.g., filtered backprojection, iterative methods, etc.). This work presents a specific approach based on Compton imaging, which is well-suited to source search and gamma-ray mapping applications due to the inherent wide FOV and sensitivity over a large range in gamma-ray energies corresponding to many radionuclide gamma-ray sources [23].

SDF makes volumetric gamma-ray imaging viable at standoff distances in arbitrary imaging environments. Several advantages of volumetric imaging over directional imaging were discussed in section 1.2 but a mobile mode of operation confers several additional advantages. One of the inherent limitations of static gamma-ray imaging systems is the $\frac{1}{r^2}$ geometric attenuation of the signal from the gamma-ray source. The ability to move around in the measurement environment allows the user to overcome this limitation by moving nearer to the gamma-ray source. This is particularly useful when searching for low activity or shielded gamma-ray sources, and may reduce localization times compared to gamma-ray imagers of comparable sensitivity operated in the conventional, static mode. The mobile mode of operation is also useful for scenarios where a gamma-ray sources are partially shielded: a mobile instrument collects data from multiple perspectives, increasing the likelihood of acquiring data from a viewpoint from which the path through attenuating material is minimized.

1.5.2 Real-Time Imaging

A mobile mode of operation is not sufficient to provide these advantages without real-time feedback from the system. Though its impact is difficult to quantify, real-time feedback is an important component for many measurement scenarios, especially for many of the in situ applications of interest in this work. With results about the location of gamma-ray sources in the scene being updated in real-time, a user can navigate to areas nearer the gamma-ray source to overcome the geometric attenuation of the gamma-ray signal from the inverse-square law. Though this work deals only with manned instruments, this concept could be extended to unmanned auto-navigation systems to allow for route optimization [24] in terms of the gamma-ray signal. Real-time feedback is also required to notify the user when traversal of a particular region results in a change signal intensity, which could indicate a change in the amount of attenuating material between the imager and the gamma-ray source. For these
reasons, as well as operational practicality, the development of a system capable of real-time imaging feedback is a central goal of the work.

1.6 Thesis Overview

This work summarizes the development of a general approach for real-time volumetric gamma-ray imaging, and subsequently evaluates the approach for a variety of gamma-ray imaging scenarios. The main contribution of this work is the SDF concept which, in its current implementation, is suitable for any portable Compton camera. Chapter 2 reviews the Compton imaging modality and discusses its application to standoff volumetric imaging. Chapter 3 introduces the two imaging platforms on which SDF was developed and demonstrated, as well as the software developed for sensor integration and real-time computation. The Volumetric Compton Imager (VCI) served as the initial development platform for SDF and comprises a high-resolution Compton imager based on segmented high-purity germanium (HPGe) detectors. The High Efficiency Multimodal Imager (HEMI) is a hand-held, CdZnTe-based (CZT) instrument that is used to replicate the results achieved with the VCI, as well as to examine more interesting and challenging imaging scenarios that can only be investigated with a hand-portable system. Chapters 4 and 5 summarize qualitative results of real-time volumetric imaging measurements taken with the VCI and HEMI, respectively. Chapter 6 presents a set of measurements and simulations that begin to quantitatively evaluate volumetric Compton image quality and the robustness of the SDF approach. Known sources of uncertainty such as error in the pose estimates and the statistics of gamma-ray acquisition are studied with respect to their impact on the robustness and accuracy of volumetric gamma-ray source localization. Finally, chapter 7 summarizes the study and attempts to provide context for the results as well as motivate further study.

1.6.1 Not Included in this Work

Both HEMI and the VCI were originally developed as conventional (2D) gamma-ray imaging devices. As such, each is capable of directional gamma-ray imaging, but these capabilities are not explored in this work except insofar as to compare relative system performance. Though both HEMI and the VCI can image via both the Compton and coded aperture modalities, only the Compton modality is explored in this work. Volumetric gamma-ray imaging based on the coded aperture modality is possible, but beyond the scope of the present work. While standoff volumetric imaging is an important step towards the determination of source activity for in-situ imaging scenarios, the development of a quantitative imaging approach is also beyond the scope of this work. Quantitative imaging would require a full characterization of the imaging response for specific imaging systems. The development of system-specific response models is not undertaken; instead, the goal is to introduce a general approach suitable for multiple Compton imaging platforms. System characterization is important to
maximize system sensitivity and a necessary step for quantitative imaging, but runs counter to the development of a general approach to real-time volumetric gamma-ray imaging.

It is important to keep in mind that SDF is only a subset of possible data fusion techniques. Other tracking methods such as those relying on global positioning systems (GPS), inertial measurement units (IMU), or some combination thereof including other approaches to visual tracking are not explicitly discussed. Nevertheless, the pose estimate error modes investigated in chapter 6 are relevant for other tracking methods and are considered in a more general context. Similarly, only a single method of fusing the 3D scene model with the gamma-ray data is explored. In principle, there are many ways to utilize the model data acquired from the SLAM algorithm depending on the specific application. For instance, constraining the imaging space to volumes within objects would likely aid in gamma-ray source localization for object interrogation imaging scenarios, but is not explicitly explored in this work. Additional examples are discussed in chapter 7 as possible directions for future research.
Chapter 2

Concepts: Volumetric Compton Imaging

This chapter presents the fundamental principles of volumetric Compton imaging. The first few sections cover the interaction mechanisms of energetic gamma-rays in the context of Compton imaging. Section 2.3 introduces some of the imaging approaches that have been developed based on Compton kinematics, especially methods based on gamma-ray tracking, of which this work is an example. Section 2.3.3 defines a gamma-ray “event”, and describes the type of gamma-ray interactions used in image reconstruction in this work. An understanding of the definitions in this section is essential as the terminology introduced there is used to describe Compton data throughout the remainder of the work. Sections 2.4 and 2.5 explain the basis of the depth sensitivity in the far-field imaging domain. Finally, section 2.6 presents the volumetric image reconstruction algorithm for Compton data developed for this work.

2.1 Gamma-Ray Interactions in Matter

An understanding of the mechanisms by which gamma-rays interact with matter is essential to understanding the design and operation of gamma-ray imagers. The interaction of gamma-rays with materials has been extensively studied in a number of contexts [25, 26]; the discussion here is limited to those mechanisms that are of interest for gamma-ray imaging. The discussion can further be narrowed by limiting it to gamma-rays within a specific energy range. Most of the in situ applications of interest in this work deal with gamma-rays emitted from de-excitation following the decay of radioactive material, which tend to have energies less than several MeV [25]. Furthermore, the imaging approach taken in this work is based on the Compton modality which is generally applicable to gamma-rays in the energy range from several hundred keV to several MeV [27]. Instrumentation has been devised to provide Compton imaging capability for gamma-rays with higher incident energy for astrophysics applications [28, 29, 30], but the instruments used in this work are much smaller
and have limited sensitivity to gamma-rays above several MeV. Of particular interest are
the gamma-ray interaction mechanisms that engender measureable electrical signals in semi-
conductor detectors. Non-ionizing interaction mechanisms such as coherent scattering are
not discussed here. Interaction mechanisms involving gamma-rays with energies outside the
range of interest for Compton imaging are also not discussed here, such as pair production
in the field of an electron. Given the scale of the portable gamma-ray detectors used in this
work (see chapter 3), the effects of diffraction are negligible. The discussion thus focuses
on three photon interaction mechanisms: photoelectric absorption, Compton scattering, and
pair production in a nuclear field.

Due to its central role in Compton imaging, Compton scattering is presented in further
detail in section 2.2 leaving photoelectric absorption and pair production to be discussed
here. Photoelectric absorption is a process whereby an incident gamma-ray interacts with an
atom, resulting in the transfer of all of the incident gamma-ray energy to a single liberated
electron. The total kinetic energy of the liberated electron is equal to the incident gamma-ray
energy, less the atomic binding energy for the shell from which the electron was liberated,
with K-shell electrons the most likely to be liberated. Photoelectric absorption is an
important process in the context of Compton imaging as it is the only interaction mechanism
whereby all of the energy of the incident gamma-ray is transferred to a single electron. The
Compton imaging approach in this work requires that a gamma-ray incident on the detector
must deposit all of its original energy via at least two interactions in the detector. For
any means to directly detect the full energy of the incident gamma-ray, the gamma-ray
event must terminate with a photoelectric absorption in the detector. Pair production on
the other hand is a loss mechanism in the context of Compton imaging. Pair production
becomes the dominant interaction process with increasing gamma-ray energy, especially
in materials with high atomic number, ultimately limiting the effectiveness of Compton
imaging for gamma-rays with energies above several MeV. Imaging techniques based on
determining the momenta of the electron and positron produced via pair production have
been devised, but are not discussed in this work.

Given the interaction requirements for Compton imaging, the relative probabilities for
the various types of gamma-ray interaction impact the performance of an imaging system.
For instance, Compton imaging requires that the first interaction be a Compton scattering
interaction. The relative probability of Compton scattering in a given detector material then
dictates a fundamental limit on the Compton imaging efficiency of that system. Fig. 2.1
and 2.2 show the relative interaction probability for each of the three interaction mechanisms
over the relevant range of gamma-ray energies for the detector materials used in this work
(see chapter 3). The area of the shaded region over a specific gamma-ray energy represents
the relative probability of interacting via that mechanism (ignoring coherent scattering). To
illustrate the differences between the detector materials, the energy at which a gamma-ray

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1 The HPGe detectors that constitute the VCI system are very similar to the strip detectors used in COSI; though there are only two crystals in the VCI compared to the twelve in COSI.
2 Henceforth “pair production” refers specifically to pair production in a nuclear field.
Figure 2.1: Relative probability of various gamma-ray interaction mechanisms for the first gamma-ray interaction in HPGe vs. incident gamma-ray energy. The shaded area intersected by a vertical line from a specific energy represents the relative probability of that interaction type.

Figure 2.2: Relative probability of various gamma-ray interaction mechanisms for the first gamma-ray interaction in CZT vs. incident gamma-ray energy. The shaded area intersected by a vertical line at a specific energy represents the relative probability of that interaction type.

has an equal probability of interacting via photoelectric absorption or Compton scattering is compared. From Fig. 2.1 and 2.2, this occurs at an energy of 150 keV in HPGe and 260 keV in CZT. In other words, the maximum possible imaging Compton imaging efficiency of 150 keV gamma-rays in HPGe or 260 keV gamma-rays in CZT is 50%. The practical imaging efficiency is reduced by other factors such as detector geometry, differential scattering cross section, and detector granularity; each effecting the ability to detect or resolve individual gamma-ray interactions. A full accounting for these geometric and physical effects is not undertaken for the imagers in this work, though both systems have previously been shown to have Compton imaging efficiencies on the order of a few percent \cite{32, 33} over the energy range of interest.

2.2 Compton Scattering

Compton imaging is based on the kinematics of the Compton scattering process, which occurs when a gamma-ray scatters off of an atomic electron. Fig. 2.3 illustrates the Compton scattering process. $\gamma_0$ represents the incident (un-scattered) gamma-ray which has an initial direction $\vec{\Omega}$ and energy $E_0$. $\vec{\Omega}$ is a unit vector ($|\vec{\Omega}| = 1$) representing the direction of the incident gamma-ray where $\vec{\Omega} \in \mathbb{S}^2$. The scattering center is assumed to be an unbound
electron at rest with no initial momentum. The interaction yields two particles, a liberated scattered electron and the scattered gamma-ray, denoted by $e^-$ and $\gamma$ respectively in Fig. 2.3. The particles resulting from the scattering process each have a post-scatter direction represented by a unit vector in angular space and a scattered energy, as shown in Fig. 2.3. 

Applying conservation of energy to the scattering process yields equation 2.1. Conservation of the relativistic momenta of the particles yields equations 2.2 and 2.3 orthogonal to $\hat{\Omega}$.

$$E_0 + m_e c^2 = \epsilon + E_s \quad (2.1)$$
$$\frac{E_0}{c} = \frac{E_s}{c} \cos \theta + \gamma \beta m_e c \cos \phi \quad (2.2)$$
$$0 = \frac{E_s}{c} \sin \theta - \gamma \beta m_e c \sin \phi \quad (2.3)$$

These equations are sufficient to specify the cosines of the scattering angles in a single dimension for the scattered particles in terms of the incident and scattered energies. Solving for the cosine of the scattering angle for the scattered gamma-ray yields equation 2.4.

$$\cos \theta = \mu_k = 1 + m_e \left( \frac{1}{E_0} + \frac{1}{E_s} \right) \quad (2.4)$$

The cosine of scattered gamma-ray angle is denoted as $\mu_k$; the subscript representing the fact that the value is computed from the kinematics of Compton scattering. A second cosine is given by the dot product of the incident gamma-ray direction and the scattered gamma-ray in equation 2.5.

$$\mu_g = \hat{\Omega} \cdot \hat{\omega} \quad (2.5)$$
$$\mu_k = \mu_g \quad (2.6)$$
The cosine computed from kinematics is equivalent to the cosine computed from the scattering geometry. The equivalency of the geometric and kinematic cosines forms the basis of Compton imaging modalities based on gamma-ray tracking \[34\]. Note also that equation 2.5 defines a cone, giving rise to the conic geometry associated with Compton imaging \[27, 35\]. The conic geometry is consistent with the kinematic equations (2.1, 2.2, 2.3) which, without knowledge of the scattering direction of the liberated electron, only constrain the gamma-ray scattering angle in one dimension. Explicit computation of the geometric cosine requires prior knowledge of the source location and is not included in the procedure for gamma-ray imaging, though it is used to evaluate imaging performance by the Angular Resolution Metric (ARM) \[36\] and other imaging diagnostics \[34\].

The differential cross section that dictates the relative probability of the direction of the scattered photon is given by the Klein-Nishina formula \[37\]. The differential cross section for scattering in germanium (Z=32) for an unpolarized photon source is shown in Fig. 2.4 for several gamma-ray energies in the range relevant for Compton imaging corresponding to common gamma-ray sources. Note the strong bias towards forward-directed scattering over the energy range from 356 keV to 1.333 MeV. The relative likelihood of forward-directed scatter has a strong influence on the design of Compton cameras, which often have multiple planes of detectors. A common Compton camera design is to have multiple detector planes arranged as “scatter planes” and “absorber planes”, often with a lower-Z material in the front plane to maximize the probability of Compton scatter and reduce the effects of doppler broadening \[29, 38\], which fundamentally limits the angular resolution of Compton cameras \[39\].\[3\] The imagers used in this work both follow a similar multi-plane arrangement, though without the specific designation of scatter and absorber planes. Each imager uses the same detector material to provide a more uniform response over 4\(\pi\). The layout of the Compton cameras used in this work is discussed in chapter 3.

\[3\] The derivation of the Compton kinematic formulae is based on the assumption that the scattered electron is initially unbound and at rest. In reality, atomic electrons serve as scattering loci which are atomically bound and have an unknown initial momentum. Gamma-ray sensors are insensitive to the momentum of the bound electron which degrades the angular resolution of the Compton imager, especially with decreasing gamma-ray energy or in detector materials with high atomic number, due to the greater angular momenta of the atomic electrons.
2.3 Compton Imaging

2.3.1 Particle Tracking for Compton Imaging

Both of the particles resulting from the Compton scattering process, the scattered gamma-ray and the liberated Compton electron, bear information about the momentum of the incident gamma-ray. Compton imaging approaches can be categorized based on which of the scattered particles are measured. Methods that rely on the measurement of the momentum of the Compton scattered electron are termed electron-track Compton imaging (ETCI). Approaches have been devised to use this information in conjunction with subsequent gamma-ray interaction positions to break the symmetry of the cone given by equation \( \frac{d\sigma}{d\Omega} \) reducing the imaging...
surface to an arc [40]. Instruments based on this approach have been deployed for 2D imaging applications such as mapping nuclear contamination in Fukushima [41]. It has also been shown that information about the momentum of the Compton scattered electron alone is sufficient to reconstruct the spatio-spectral distribution of the incident gamma-rays [42].

Compton imaging is traditionally based on measuring (or reconstructing) the interaction history of the scattered gamma-ray, otherwise known as gamma-ray track Compton imaging (GTCI). GTCI has dominated Compton imaging for many of the applications discussed in section 1.1 largely due to the preponderance of technology suitable for measuring the position and energy of gamma-ray interactions [43]. The volumetric Compton imaging approach used in this work is also a GTCI method.

2.3.2 Gamma-ray Track Compton Imaging

Given the kinematic and geometric relations in section 2.2 it is possible to generate a probability surface in the incident flux space representing the possible emission locations for each measured Compton event. In the case of volumetric imaging, the incident flux space is $\mathbb{R}^3$, thus the surface representing possible locations of the emission for a particular gamma-ray is given by a cone. Several quantities must be derived from the Compton event to define this cone. The position of the initial Compton scattering interaction and the position of the second interaction by the scattered gamma-ray define the scattering direction $\vec{\omega}$, the symmetry axis of the cone. The opening angle of the cone is given by the kinematics of the scattering in equation 2.4. Spectroscopic instruments are sensitive only to the energy deposited via gamma-ray interactions thus cannot directly measure the energy of the scattered gamma-ray, $E_s$. For the purposes of imaging, it is often assumed that the energy of the initial gamma-ray, $E_0$, is known or directly measured.

$$E_s = E_0 - \epsilon \quad (2.7)$$

Thus $E_s$ can be determined in terms of measured quantities via equation 2.7, where $\epsilon$ represents the energy imparted to the Compton-scattered electron. In this work, the incident gamma-ray energy, $E_0$, is specified by the user setting regions of interest (ROI) in the gamma-ray energy spectrum corresponding to photopeak features or expected gamma-ray energies. Equation 2.4 can thus be rewritten in terms of measured and user-specified quantities (equation 2.8).

$$\mu_k = 1 + m_e c^2 \left( \frac{1}{E_0} - \frac{1}{E_0 - \epsilon} \right) \quad (2.8)$$

Using 3D position-sensitive spectroscopic semiconductor detector systems (see chapter 3), the scatter axis and opening angle of the cone are defined for each Compton event, discussed further in section 2.3.3. The result is a distribution of cone surfaces in $\mathbb{R}^3$, serving as input to the iterative reconstruction method presented in section 2.6.

---

4 Reconstruction of the incident energy and 2D spatial distribution of gamma-rays (3D spatio-spectral reconstruction) without this assumption has been studied [44], but is beyond the scope of this work
2.3.3 Compton Events

The process of reconstructing the interaction history of gamma-rays given the set of signals acquired from the detector is known as gamma-ray event reconstruction. A gamma-ray “event” is the set of all interactions undergone by a gamma-ray within the detector. GTCI requires two parameters to be derived from a gamma-ray event, the cone scattering axis $\vec{\omega}$ and the opening angle of the Compton cone. The scatter axis is computed given the position of the initial Compton scatter interaction and the position of the subsequent interaction by the scattered gamma-ray, and the opening angle is computed given energy deposited in the initial Compton scatter interaction via equation 2.8. Given these requirements, any gamma-ray event that is initiated by a Compton scatter and interacts at least one more time at a distinct location in the detector by any mechanism is a suitable candidate for Compton imaging.

Only a subset of these candidate gamma-ray events potentially suitable for Compton imaging are used in this work. Gamma-ray events that consist of only two interactions are considered, while those of higher multiplicity (from multiple Compton scatters) are ignored. The sum of the energy deposited in the two gamma-ray interactions must within a user-specified ROI, presumably corresponding to the total energy of the incident photon to be imaged. The designation potential Compton event used throughout this work refers specifically to gamma-ray events consisting of two gamma-ray interactions occurring at distinct locations in the detector with a total deposited energy within the user specified ROI. This description is intended to capture true Compton events which have the following interaction history:

1. A gamma-ray emitted from a radionuclide source with initial energy $E_0$ undergoes Compton scattering interaction in the detector.

2. The resulting scattered gamma-ray undergoes a photoelectric absorption interaction within the detector.

There are several mechanisms by which gamma-rays with a different interaction history can be misclassified as Compton events. Gamma-rays with an initial energy greater than $E_0$ can interact via two sequential Compton scattering interactions depositing a total energy within the ROI around $E_0$. These down-scattered events are a source of noise in the Compton image as they are not correlated to the gamma-ray source emitting photons with $E_0$. The event misclassification rate from down-scattering increases with decreasing energy of double-interaction gamma-ray events [45]. For many of the imaging scenarios of interest in this work, misclassified Compton events are not expected to result in prohibitive image noise. It has also been shown that gamma-ray imaging can minimize the effects of gamma-ray background on the image [46] as misclassified events are not expected to map to the imaging space in a coherent fashion.

Much effort has been devoted in the literature to developing event reconstruction techniques that maximize imaging efficiency, i.e., the portion of multi-interaction gamma-ray
events that are used in the image reconstruction \cite{47, 31, 32}. The decision made in this
work to use more simplistic event reconstruction and focus on a subset of imageable events
is driven by several considerations. Advanced event reconstruction techniques are based
on a thorough understanding of the detector response via simulation and characterization
measurements \cite{47}. The characterizations are detector-dependent and incongruous with the
goal of developing an imaging technique that is generally applicable to mobile gamma-ray
 imagers. A stronger emphasis was placed on the portability of the method as opposed to
optimization for a specific system. Additionally, many of the advanced event reconstruction
techniques are computationally intensive at run-time \cite{18}, presenting a challenge from the
perspective of real-time imaging feedback. An event reconstruction algorithm with signifi-
cantly reduced computational load was chosen to guarantee there would be no bottlenecks
in the processing up to the maximum manageable event rate in each detector. Real-time
implementation of advanced event reconstruction techniques and the effect of system-specific
optimization on volumetric gamma-ray imaging are potential topics for further research.

2.4 Far-Field Imaging

Gamma-ray imaging can be categorized as near-field or far-field imaging depending on the
source-detector geometry. In the far-field regime, the distance between the gamma-ray source
and the imager is much greater than the largest dimension of the imager. The far-field
approximation can also be viewed as the point-detector approximation, which assumes the
incident photon flux is sampled at a single point, ignoring solid-angle effects due to finite
detector size. For some gamma-ray imaging applications this assumption is trivially satis-
fied, as in gamma-ray astronomy. When not satisfied, the finite detector area results in a
geometric blur in the gamma-ray image. To visualize the magnitude of this blur, a simple
one-dimensional model is presented where the distance to a point source is reported rela-
tive to the diameter of the detector, \( r_D = \frac{\text{distance to source}}{\text{detector diameter}} \). The magnitude of the geometric
blurring from the finite area of the detector is given by equation \( 2.9 \):

\[
\Delta \theta = \arctan \left( \frac{1}{2r_D} \right)
\]  

(2.9)

This equation can be used to determine a suitable threshold in \( r_D \) beyond which the far-field
approximation is valid. Applying this analysis to the two systems in this work, the distance
at which the geometric blurring is equal to the measured angular resolution of the imagers
(see section 3.5.3) is determined. Given representative values for angular resolutions of the
VCI and HEMI imagers of 3.5\(^\circ\) and 9.5\(^\circ\) respectively, the minimum source distance at which
the geometric blur factor is equivalent to the angular resolution is shown in Fig. 2.5. The
degradation of angular resolution due to the finite size of the imagers used in this work is
negligible if the source is at least 24 cm away from HEMI or 62 cm away from the VCI,
respectively. This condition is generally satisfied in the measurements taken throughout
chapters 4 - 6.
Figure 2.5: Magnitude of the geometric blur due to the finite detector area when the point-detector approximation is used. \( r_D \) is the unitless ratio of the source-detector distance to the diameter of the detector. The distances at which the geometric blur equals the angular resolution of the imager is given for both the VCI (red) and HEMI (green). The degradation of angular resolution from the point-detector assumption is negligible when the source distance is greater than the thresholds specified in this figure.

The far-field designation is also important in terms of depth resolution. The uncertainty in the depth calculated via triangulation scales with the square of the depth normalized by the baseline distance between events: \( \sigma_d \propto r_D^2 \). With a non-mobile detector, the largest distance between measured events is limited by the dimensions of the detector; restricting volumetric imaging for static systems to the near-field. There have been recent approaches to 3D imaging that have leveraged the depth sensitivity of near-field imaging [49], but the near-field requirement is quite limiting for \textit{in situ} applications of gamma-ray imaging. Volumetric imaging in the far-field regime is achieved via a mobile imaging paradigm such that the
triangulation baseline is only limited by the path of the imager through the scene.

2.5 Depth Sensitivity

Volumetric imaging is distinguished from directional imaging in providing sensitivity to the source distribution in the dimension orthogonal to the 2D imaging plane. The coordinate axis that extends outward from the sensor towards the scene being viewed corresponds to the depth dimension. Fig. 2.6 illustrates this convention, which is followed throughout this work. Sensitivity to the distribution of gamma-ray sources in the depth dimension in the volumetric Compton image is based on the principle of triangulation. Triangulation-based techniques are employed in many circumstances with a wide range of sensors. It is the basis of depth sensitivity for binocular and higher-order stereoscopic camera systems [50], as well as other types of depth-sensitive sensors such as the Kinect [51]. There are many approaches to triangulation, especially for cases involving the projective geometry encountered in conventional image formation [52]. Several factors distinguish volumetric Compton imaging from triangulation methods for optical cameras. For the approaches mentioned in [51, 52], a camera calibration matrix (and associated model correcting for lens distortion) is used to project points in the imaging plane into the 3D space for the given coordinate frame. With a

\[\text{Figure 2.6: Illustration of the convention defining the dimension corresponding to depth. The gray rectangles represent a generic two-plane Compton imager while the black trapezoid represents the Kinect. The red arrow emanating from the sensors in the direction of the scene represents the depth dimension.}\]
Compton imager by contrast, there are no optics to account for. The intensity contribution from each Compton cone (convolved with a Gaussian function that broadens the cone) is computed in each voxel. The other major difference is the nature of function that is projected into 3D space. For conventional cameras, each pixel represents a ray defined by the pixel location in the imaging plane, the optical center, and the focal length of the camera (after accounting for any optical distortion) [52]. In Compton imaging, the projected surface is a cone rather than a single ray. The triangulation principle for far-field Compton imaging is illustrated in Fig. 2.7. The three cones illustrated in Fig. 2.7 were measured with the imager positioned at three different locations, represented by the white cubes in the image. The blue arrows emanating from each detector location represent the scatter-axis of each cone. Each cone surface is broadened with a Gaussian kernel with the width given by $\sigma_\theta = 3^\circ$ to account for uncertainty in the scatter axis and opening angle of the cone. The intersection

Figure 2.7: The back-projection of three Compton cones into a voxelized 3D imaging space. The intensity contributions from three individual Compton cones are represented by the three conic surfaces (red, blue, and green). Each cone is convolved with a Gaussian function with $\sigma_\theta = 3^\circ$. The overlap of the cones is illustrated in the orange-yellow colormap, indicating the location of the gamma-ray source at the yellow hot-spot. The scale of the space is intended to illustrate the far-field nature of the imaging.
of the broadened cones is given in the orange-yellow colormap, with increased intensity at the location where all three cones intersect. The scale of the coordinate system shown in Fig. 2.7 is arbitrary, though chosen to emphasize the far-field nature of the imaging.

2.6 Image Reconstruction

Given the distribution of Compton cone surfaces in $\mathbb{R}^3$ that describe from which regions of the imaging space the measured gamma-rays could have originated, the challenge of evaluating the statistics of this distribution to remove the ambiguity inherent in the conic data remains. There are several aspects of the inversion problem for volumetric Compton image reconstruction that distinguish it from existing volumetric gamma-ray imaging approaches that are principally found in the medical imaging field. The far-field nature of the problem presents challenges distinct from those encountered in medical imaging, where the physical domain of the imaging space is pre-defined and limited in physical size. Current far-field Compton image reconstruction methods developed for astronomy [35] and nuclear security [15, 53] based on filtered backprojection are not directly applicable to the 3D problem due to the higher-dimensionality of the imaging space. Another challenge is the inability in general to acquire the perspective necessary for a full tomographic reconstruction. Even truly portable hand-held imagers can suffer from limited spatial sampling due to cluttered or otherwise restrictive environments. Another challenge is the undefined nature of the imaging space, changing in size and scale as the user moves around the environment. These considerations necessitate the development of a reconstruction algorithm for far-field volumetric Compton imaging. The algorithm, based on an iterative maximum likelihood - expectation maximization (MLEM) approach, is briefly discussed here. For further details about the image reconstruction algorithm, the reader is referred to thesis work by Andrew Haefner [34].

2.6.1 MLEM Overview

The imaging domain in $\mathbb{R}^3$ is represented by a regular voxelized grid. The mobile mode of operation results in expansion imaging space as the user moves the instrument through new areas of the scene. It is thus necessary to continually recompute the imaging space based on the extent of the detector path within the scene. The distribution of Compton cones projected into the voxel space determines the voxels from which the measured gamma-rays may have originated. These surfaces are related to the measurement location in $\mathbb{R}^3$ via a Poisson likelihood. The expectation maximization portion of the algorithm maximizes the Poisson likelihood given this description of the imaging space. The solution to this problem is well known [54], and is given by equation 2.10.

$$\lambda_{i}^{n+1} = \lambda_{i}^{(n)} \frac{\sum_{j \cap l} a_{ij} \lambda_{l}^{n}}{s_{i} \sum_{j \cap l} a_{ij} \lambda_{l}^{(n)}}$$ (2.10)

This equation describes the iterative approach to computing the image, where:
• $n$ is the iteration number
• $i$ is the image element (voxel) index
• $j$ is the data (cone) index
• $\lambda_i^{(n)}$ is the image value in voxel $i$ at iteration $n$
• $a_{ij}$ is the system matrix
• $s_i$ is the sensitivity for image element $i$

In this work, list-mode reconstruction is used thus the sensitivities are not computed from $a_{ij}$. In order to facilitate real-time reconstruction, uniform sensitivities are used. This fails to capture some effects and will result in image artifacts for distributed gamma-ray sources. It has been empirically determined (see chapters 4 and 5) that this simplification is not prohibitive for localizing point sources in 3D. Methods for real-time computation of the sensitivities for a mobile imager in an dynamic imaging space require further study.

2.6.2 Scene Data Constraint

SDF not only enables a mobile mode of operation for the radiation imager, but also yields a spatial 3D model of the scene. The impact of incorporating this 3D model into the image reconstruction algorithm is studied in this work. The voxelization of the imaging space lends itself to a simple spatial fusion technique: the model can be used to compute an occupancy grid in the voxelized image space. The solution of equation 2.10 can then be restricted to the occupied voxels. The impact of the application of this technique on the imaging space varies based on the scene geometry, but in many cases greatly reduces the size of the imaging space. In practice, this reduction improves the tractibility of the inversion problem and reduces the computation time. These advantages are explicitly demonstrated in chapters 4 and 5. The simple occupancy-based fusion technique implicitly assumes that gamma-ray sources are on or near the surface of the objects in the scene, and are not present in air or other void space. Equation 2.10 can also be computed without restricting the solution to occupied voxels for scenarios where the scene model is incomplete, unavailable, or the underlying assumption is poor. The voxel occupancy constraint is the only fusion technique investigated in this work, though there are many other ways to utilize the scene data that would be expected to benefit specific imaging scenarios. One example is the object interrogation scenario, i.e attempting to localize a gamma-ray source within a container, which is preliminarily presented in sections 5.3 and 5.4.2. It would be very useful to use the scene model to generate a bounding volume for the container and only voxelize the space within that volume. Utilization of the scene model in other ways such as estimating environmental scattering or attenuation of gamma-ray radiation is also possible. These examples are not explicitly studied in this work, representing potential areas of future study for SDF.
Chapter 3

Implementation: Mobile Gamma-Ray Imaging Platforms

3.1 Overview

This chapter presents details of both the hardware and software used to develop and demonstrate real-time volumetric Compton imaging with SDF. The discussion includes a brief overview of the project timeline to contextualize the development on different systems. A very brief overview of SLAM is presented, along with a discussion of the particular SLAM approach used in this work, an open-source project called RGB-D Slam [55]. SLAM is an expansive and extensively studied problem, and is mentioned here in the broadest terms only to motivate the the decision to use SLAM algorithms for real-time volumetric gamma-ray imaging. A more thorough discussion can be found in numerous places with [56, 57, 58] providing a comprehensive introduction at the time of this writing. Finally, the software framework that was developed for this work is discussed, particularly the design goals and the subsequent implementation to develop applications for multiple gamma-ray imagers.

3.2 Gamma-ray Imaging Systems: VCI and HEMI

Two different gamma-ray imagers were used over the course of the project: the Volumetric Compton Imager (VCI) and the High Efficiency Multimodal Imager (HEMI). At the start of the project, only the VCI was available thus the SDF concept was originally developed using this system. The VCI was originally called the Compact Compton Imager - 2 (CCI-2) and had previously been used to investigate 2D Compton imaging for nuclear security applications [27]. CCI-2 was the system used in [16] discussed in section 1.3, though the system was implemented quite differently in that work and included additional detectors. Much of this previous work with the CCI-2 system was focused on optimizing for angular resolution and imaging efficiency using advanced techniques for gamma-ray event reconstruction [32, 47]. The goals of this project differ significantly, with specific emphasis on real-time computation.
CHAPTER 3. IMPLEMENTATION OF SCENE DATA FUSION

The need for real-time feedback was one of the primary drivers for the development of the software acquisition and analysis framework discussed in section 3.7.

The SDF concept was originally developed and demonstrated on the VCI platform; the HEMI system then became available later in the project. HEMI is appealing as a platform for volumetric Compton imaging for many reasons, the most obvious being its portability. The hand-held mobility of HEMI contrasts with the VCI system which is too large and heavy to move outside of its current laboratory environment. It also allows for the testing of more realistic gamma-ray search and mapping scenarios and provides a greater challenge for visual tracking with SLAM. Despite the limited mobility of the VCI system, it remains a valuable platform for investigating volumetric gamma-ray imaging. The higher position and energy resolutions of the VCI system yield an angular resolution that is a factor of 3 better than what is achievable with HEMI (see section 3.5.3). Implementing SDF on each of these two systems allows for the investigation of a wide range of effects: from controlled laboratory measurements to more realistic search and mapping scenarios. The demonstration of SDF with multiple systems with different properties lends evidence to the claim that SDF is a viable approach for real-time volumetric gamma-ray imaging for existing portable gamma-ray imagers, without requiring significant advances in gamma-ray imaging technology.

Each system is detailed individually in the following sections, along with a comparative evaluation of the system performance in section 3.5. The VCI has undergone significant changes from its prior implementation as CCI-2: for instance the system no longer includes two lithium-drifted silicon [Si(Li)] detectors which constituted the front plane of the CCI-2 imager. The imaging performance of the VCI differs significantly from previously published characterizations of the CCI-2 system [36, 59]. A full characterization of the VCI detectors is not presented here, though the spectroscopic and imaging performance are discussed in section 3.5. The HEMI system by contrast has been thoroughly characterized, with significant changes only in the packaging of the system to improve portability. The reader is referred to [33, 60] for a thorough evaluation of HEMI. Various demonstrations of volumetric Compton imaging with each of the two instruments are presented in chapters 4 and 5.

3.3 Volumetric Compton Imager

The VCI is a cart-based, mobile gamma-ray imaging system consisting of two planar double-sided strip (DSS) HPGe detectors. Each detector is 15.1 mm thick with 37 strip readouts per electrode with 2 mm strip pitch with a maximum sensitive volume of $82.6 \text{ cm}^3$, for a total sensitive HPGe volume of $165.2 \text{ cm}^3$ for the entire instrument. Each detector is square in cross-section with rounded edges from the removal of excess material from the original HPGe crystal. Fig. 3.1 shows one of the two planar HPGe crystals to illustrate the electrode segmentation scheme. The strip electrodes on the opposite face of the detector (facing the table in Fig. 3.1) are oriented orthogonally to the strips on the front face, providing two-

\[1\] In practice, issues with the discrete electronics and electrode connections cause a slight reduction in the number of active strips, reducing the total sensitive volume of HPGe
Figure 3.1: One of the two planar HPGe detectors that constitutes the VCI.

dimensional lateral position sensitivity as a result of the small-pixel effect \[17\]. Each detector is surrounded by a guard ring that is 5 mm along two edges of the detector and 2 mm along the remaining edges. The guard ring is instrumented as it has been shown that guard ring signals can be used to improve the efficiency of strip HPGe detectors \[61\], though these techniques are not applied in this work. Each of the strips is wire bonded to a custom multi-channel connector, which is connected via kapton flexible circuit (flex circuit) to readout pins fed through to the exterior of the cryostat. Fig. 3.2 shows both detectors (though only one is visible in the foreground) mounted within the cryostat. The two detectors are separated by 10 mm to maximize the solid angle subtended by one detector in the other while ensuring there will be no electrical arcing between the detectors from the applied bias. The detector crystals are enclosed in an IR shield to reduce the radiative heat load prior to enclosure in the cryostat (not shown in Fig. 3.2). Each of the 152 channels is instrumented with a high-gain charge-sensitive preamplifier \[62\]. The preamplifier signals from each channel are each sent to a readout module that digitizes and filters the signals, extracting information about the shape and amplitude of the induced signal. The readout module consists of 19 individual 8-channel digitization units from Struck Innovative Systems (SIS GmbH), model number 3302, along with a VME-to-USB2.0 interface (model number 3150) to transfer information to the host computer for subsequent analysis.

The entire system is installed on a large cart to provide the mobility necessary for volumetric imaging with SDF. The detectors and readout instrumentation are accompanied by other equipment necessary for the operation of the system, including:
30L liquid nitrogen dewar.

- NIM bin with preamplifier power supply as well as dual channel high voltage module for biasing the HPGe detectors.
- Uninterruptible Power Supply (UPS) for surge protection and system stability during power outages.
- Desktop computer for readout, analysis, and user I/O.

The cart also includes two adjustable arms that can host various auxiliary sensors. Fig. 3.3 shows the cart fully assembled with all of the equipment used in this work. The cart is
Figure 3.3: The VCI cart.

quite heavy\footnote{The total weight of the cart is roughly 1000 lbs, largely due to a 600 lb stainless-steel plate at the base of the cart to prevent tip-over.} which limits the rate at which it can be moved, with a maximum translational velocity of about 10 cm/s for measurements in this work. This constraint on the mobility influences both the types of measurements of which the system is capable, and specific data processing techniques that can be employed (see section 4.5).

3.4 High Efficiency Multimodal Imager

HEMI is a prototype gamma-ray imaging system formatted for hand-held operation in this work. It consists of 96 individual 1 cm$^3$ CZT crystals, each with a co-planar grid electrode configuration \cite{63} and low-power ASIC readout \cite{64}. The detector elements are arranged in a two-plane active mask configuration with the front plane half populated (32 elements) and the back plane fully populated (64 elements) as seen in Fig. 3.4. The active mask
configuration was selected to allow both Compton and coded-aperture modalities, principally providing imaging capabilities for gamma-ray energies from less than 100 keV to several MeV. Only the Compton modality is investigated in this work, with Compton imaging sensitivity demonstrated at gamma-ray energies above 350 keV. For a summary and evaluation of the coded-aperture imaging capabilities of the HEMI system, see [33]. HEMI has a modular design that allows it to be packaged differently for different deployment modes, including hand-held and aerial modes [34]; though only the hand-held mode is investigated in this work. When fully assembled in the hand-held configuration, including on-board battery and all auxiliary sensors (see Fig. 3.5), the system weighs less than 4.5 kg. The battery is sufficient to allow the prototype system to operate for about four hours on a single charge.

Hand-held HEMI and the associated computers required for SDF are shown in Fig. 3.6. HEMI operations, including bias control, data acquisition, and real-time gamma-ray image reconstruction, occur on the tablet which is typically carried by the user along with HEMI while traversing the scene. A second computer runs the SLAM algorithm (see section 3.6.1) to produce pose estimates and the 3D model of the scene using the Kinect data. Due to current limitations of the prototype system, particularly platform compatibility issues, both computers are required to operate HEMI in the volumetric mode. It should be noted that this limitation is not a result of insufficient computational resources on a single platform.

The reconstructed scene model and pose estimates computed via RGB-D Slam are sent at set intervals via a wireless network to the tablet to be incorporated into the volumetric gamma-ray image reconstruction. Fig. 3.7 demonstrates how handheld HEMI is operated.

---

3The entire software stack, including RGB-D Slam and the gamma-ray acquisition and reconstruction framework runs on the same computer for the VCI.
in real-time volumetric mode. The user carries HEMI in one hand, making sure to scan the scene with the Kinect to fill out the 3D scene model. The point cloud model and estimated detector track are rendered in real time on both the tablet and the computer running RGB-D Slam. The gamma-ray image is continuously reconstructed on the tablet as the user moves throughout the scene, giving the operator real-time feedback about the spatial distribution of gamma-ray sources within the 3D scene model.
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Figure 3.6: HEMI along with the Kinect sensor and computers currently required for real-time volumetric Compton imaging in the handheld mode.

Figure 3.7: HEMI operation in the handheld mode using SDF. The system is moved by the operator throughout the scene making sure to capture areas of interest in the Kinect FOV for inclusion in the 3D model. The real-time volumetric reconstruction occurs on the tablet, also carried by the user, providing spectral and imaging feedback.
3.5 Comparison of Imaging Systems

Individual characterizations of the spectroscopic and imaging performance of the two systems can be found in \cite{33, 60} for HEMI and \cite{36, 59} for the CCI-2. It is nonetheless useful to directly compare the systems to emphasize their relative spectroscopic and imaging performance. The comparison of these conventional capabilities informs the expected relative system performance in the volumetric imaging domain.

3.5.1 Experimental Setup

To evaluate the spectroscopic and imaging response of the two systems, a static measurement scenario (i.e., neither the sources nor the detectors are moved during the measurement time) was devised in which both HEMI and the VCI simultaneously participated. Five point sources were placed on a horizontal bar at 39.4 cm intervals as shown in Fig. 3.8. HEMI was placed on the VCI cart next to the VCI detectors separated by 43 cm, as shown in Fig. 3.9. The cart was moved so that the detectors were facing the sources, as shown in Fig. 3.10. The horizontal bar on which the sources were mounted was at a perpendicular distance of 380 cm from the front face of each of the imagers. The true locations of the point sources in angular space are computed from the measured geometry. Table 3.1 lists the activities of

![Figure 3.8: The arrangement of gamma-ray point sources for the measurement described in section 3.5.1.](image)

The nuclide associated with each of the point sources and their activities at the time of the measurement are given in Table 3.1. The point sources are separated from one another by 39.4 cm intervals.
Figure 3.9: The arrangement of the imagers on the VCI cart during the measurement. The detector origins are displaced 43 cm horizontally, and the HEMI origin is 10 cm below the VCI centerline.

Figure 3.10: Depiction of the source-detector geometry for the static multiple-source measurement. The sources are labeled according to their nuclide. The front face of each detector is 380 cm from the horizontal bar on which the point sources are mounted.

<table>
<thead>
<tr>
<th>Source</th>
<th>Activity ($\mu Ci$)</th>
<th>True Source Location - HEMI ($\phi, \theta$)</th>
<th>True Source Location - VCI ($\phi, \theta$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{22}$Na</td>
<td>11.6</td>
<td>($-8.0^\circ, 3.5^\circ$)</td>
<td>($-14.8^\circ, 2.0^\circ$)</td>
</tr>
<tr>
<td>$^{60}$Co</td>
<td>7.2</td>
<td>($-2.1^\circ, 3.5^\circ$)</td>
<td>($-9.1^\circ, 2.0^\circ$)</td>
</tr>
<tr>
<td>$^{54}$Mn</td>
<td>4.3</td>
<td>($3.8^\circ, 3.5^\circ$)</td>
<td>($-3.2^\circ, 2.0^\circ$)</td>
</tr>
<tr>
<td>$^{137}$Cs</td>
<td>9.5</td>
<td>($9.7^\circ, 3.5^\circ$)</td>
<td>($2.7^\circ, 2.0^\circ$)</td>
</tr>
<tr>
<td>$^{133}$Ba</td>
<td>8.1</td>
<td>($15.3^\circ, 3.5^\circ$)</td>
<td>($8.5^\circ, 2.0^\circ$)</td>
</tr>
</tbody>
</table>

Table 3.1: Summary of gamma-ray point sources used for the measurement described in section 3.5.1. The activities of each point source are given, along with the true location of the source in angular space, with $\phi$ representing the azimuthal angle and $\theta$ the polar angle. The source location is given relative to HEMI and VCI origins respectively.

The measurement time was 3 hours and 30 minutes to maximize the number of Compton events in each of the energy ROI’s for imaging and spectral analysis, given the constraints of HEMI’s battery-life.

3.5.2 Spectral Response

Spectral response plays an important role in the detection and identification of specific radionuclides for many of the potential applications considered in this work. Though the tasks of gamma-ray detection and identification are not explicitly studied here, spectral response is
of interest given its role in volumetric Compton imaging. For example, the Compton imaging method in this work assumes knowledge of the incident gamma-ray energy. This assumption is realized by allowing the user to set gates corresponding to ROIs around photopeaks in the gamma-ray energy spectrum. Furthermore, the uncertainty in the determination of gamma-ray energy, of which the detector energy resolution is a measure, has an impact on the uncertainty of the opening angle of the Compton cone and thus directly influences the angular resolution of the Compton imager \[32\]. The efficiency of the detection systems is also important, especially in the context of many of the potential applications of volumetric imaging, where the gamma-ray sources may be weak or shielded and measurement durations limited to the order of minutes as opposed to hours.

The evaluation of the spectral response of a multi-channel system is complicated by the necessary selection and processing to reconstruct gamma-ray events from the list-mode channel data. The spectral response is evaluated for several classes of gamma-ray events: unreconstructed events, single interaction events, and double-interaction (i.e., potential Compton) events. Though the processing steps to derive single and double-interaction events varies significantly between HEMI and the VCI, the comparison between these categories of events is indicative of relative performance. Single interaction events are a good measure of the energy resolution of the system. The energy resolution of each individual channel can be measured, but the overall system resolution will include contributions from each channel as well as folding in the effects of event reconstruction and systematic factors like energy calibration. The uncertainty in the opening angle of the Compton cone is related only to the uncertainty in the energy deposited in the Compton scatter interaction\(^4\). Thus the energy resolution of the single interaction events is correlated to the angular resolution of the imager. The energy resolution from multiple interaction events is not correlated to uncertainty in the opening angle of the Compton cone due to the quadrature summing of the uncertainty associated with measurement each individual gamma-ray interaction. However, the net intensity of photopeaks in the double-interaction spectrum is representative of the number of potential Compton events collected. A comparison of the net area of peaks in the double-interaction spectrum can be used as a proxy for comparing the relative imaging efficiency of the systems.

3.5.2.1 Energy Resolution

First, the energy spectra from each system before the application of event reconstruction are compared. These spectra provide a baseline against which the effects of event reconstruction on the reduction in the number of gamma-ray events can be evaluated. Fig. 3.11 shows the energy spectra produced by the VCI and HEMI during the 3 hour 30 minute measurement. These spectra represent unreconstructed data from the two detectors, reflecting the maximum intrinsic efficiency of the systems, though at the expense of position sensitivity in the VCI. Due to the electrode segmentation in the VCI, the total energy deposited in a gamma-ray

\(^4\) The energy of the incident gamma-ray is explicitly specified by the user as opposed to being computed from the sum of deposited energies
event is determined by summing the energy collected by all channels in the detector. The criterion for determining correlated channels is based on time-coincidence; i.e., all channels that are triggered within some time threshold of one another are considered to have resulted from the same gamma-ray event. The unreconstructed spectrum is produced for the VCI by summing the energy recorded from all strips within a 250 ns coincidence window, reflecting the maximum possible charge drift time in the detectors [32]. No additional processing, such as strip matching for determining gamma-ray interaction position, is applied at this stage.

Several features in the spectra illustrate aspects of the respective responses of each system. Notice the low-energy tailing on the peaks in the VCI spectrum; Fig. 3.12 presents a zoomed view where the phenomenon is clearly illustrated. This tailing is caused by charge loss in the gaps between the strips of the detector [65] [66]. The charge-loss problem has been studied for HPGe strip detectors, and solutions have been proposed based on calibration procedures [65] or signal processing [47] [61]. Additionally, modern HPGe fabrication procedures have
been shown to minimize charge collection in the gap between electrodes [66]. Potential charge-sharing events are ignored in this work, which eliminates these spectral features at the expense of detector efficiency. The HEMI energy spectrum contains very few peaks in the energy region below about 400 keV. The poorer energy resolution of CZT reduces the signal-to-background ratio of HEMI, especially in the spectral regime below several hundred keV. Systematic effects associated with the prototype HEMI system such as increased electronic noise with increasing operation time also contribute to the poor signal-to-background ratio in the energy range below several hundred keV. Gamma-ray event selection and reconstruction has a significant effect on the spectral response, particularly for the VCI.

Fig. 3.13 shows the spectra of the VCI and HEMI from only single interaction gamma-ray events. Note the large drop in the number of events in the VCI spectrum compared to the unreconstructed spectrum in Fig. 3.11 due to the event reconstruction and selection processes. The selection of single interaction events has little impact in the low-energy region of the spectrum, where the majority of gamma-rays interact via a single photoelectric
absorption. The impact of selection becomes more pronounced with increasing gamma-ray energy, with an increase in the number of expected interactions per gamma-ray event. The number of events in the HEMI spectrum by contrast is relatively unaffected by the event selection process. The disparity depends not only on the physics of gamma-ray interactions in the different detector materials, but also the higher degree of complexity of the event reconstruction process for the VCI. Single interaction events in HEMI simply comprise all interactions that are not coincident in time with any other measured signal.

As mentioned, the spectral response for single-interaction events reflects the system energy resolution, which is related to the angular resolution of the Compton imager [32]. The energy resolution is measured at the various photopeaks resulting from gamma-rays emitted from the radionuclide sources given in table 3.1. The measured energy resolution for reconstructed single-interaction events is plotted in Fig. 3.15 and 3.16 for the VCI and HEMI respectively. The recorded resolutions are in agreement with expectations from previous characterizations [33, 32]. The error bars in Fig. 3.15 and 3.16 are given by the uncertainty of the model used to fit the peaks. The trend in Fig. 3.16 illustrates a large contribution from systematic issues from HEMI beyond the effect of statistical uncertainties. Systematic effects such as calibration drift and per-channel noise characteristics that vary with operation time are known issues with the prototype HEMI system; these effects are reflected in the measured system energy resolution over the course of the 3.5 hour long measurement.

The energy spectrum of double-interaction events is important in the context of Compton imaging as the presence and width of peaks in this spectrum can influence the imaging ROIs specified by the user. The energy spectra composed entirely of double-interaction gamma-
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Figure 3.15: Measured energy resolution of the VCI for single interaction events at common gamma-ray energies.

Figure 3.16: Measured energy resolution of HEMI for single interaction events at several gamma-ray energies.

Figure 3.17: Energy spectra of double-interaction events in the VCI and HEMI for the measurement described in section 3.5.1.

Figure 3.18: The same spectra as in Fig. 3.17 zoomed to the energy range from 280 to 780 keV to illustrate spectral features in finer detail.

Both systems is compared in table 3.2 for single and double-interaction gamma-ray events. As mentioned, the energy resolution from double-interaction events is degraded relative to the single interaction events due to the quadrature summing of the uncertainty in the energy
### Table 3.2: Comparison of energy resolution between the VCI and HEMI for both single and double-interaction gamma-ray events. The resolution is measured fitting photopeaks in the single and double-interaction spectra. Note the degradation of the measured energy resolution for single-interaction events above 1 MeV observed in the VCI. These values have a large associated statistical uncertainty as the probability of photoelectric absorption in HPGe above 1 MeV is very low. Thus the photopeaks corresponding to the gamma-rays from $^{60}$Co comprise fewer than 100 events in the spectrum of single-interaction events in the VCI.

<table>
<thead>
<tr>
<th>Photopake Energy</th>
<th>Single Interaction Events</th>
<th>Double Interaction Events</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>VCI Resolution (keV (%))</td>
<td>HEMI Resolution (keV (%))</td>
</tr>
<tr>
<td>356.0 keV</td>
<td>1.90 (0.5 %)</td>
<td>12.40 (3.5 %)</td>
</tr>
<tr>
<td></td>
<td>2.64 (0.7 %)</td>
<td>24.57 (6.9 %)</td>
</tr>
<tr>
<td>511.0 keV</td>
<td>3.21 (0.6 %)</td>
<td>15.25 (3.0 %)</td>
</tr>
<tr>
<td></td>
<td>4.02 (0.8 %)</td>
<td>23.56 (4.6 %)</td>
</tr>
<tr>
<td>662.0 keV</td>
<td>2.30 (0.3 %)</td>
<td>15.36 (2.3 %)</td>
</tr>
<tr>
<td></td>
<td>3.01 (0.5 %)</td>
<td>19.75 (3.0 %)</td>
</tr>
<tr>
<td>834.0 keV</td>
<td>1.78 (0.2 %)</td>
<td>19.10 (2.3 %)</td>
</tr>
<tr>
<td></td>
<td>3.05 (0.4 %)</td>
<td>19.39 (2.3 %)</td>
</tr>
<tr>
<td>1173.0 keV</td>
<td>3.15 (0.3 %)</td>
<td>24.17 (2.1 %)</td>
</tr>
<tr>
<td></td>
<td>3.88 (0.3 %)</td>
<td>22.05 (1.9 %)</td>
</tr>
<tr>
<td>1333.0 keV</td>
<td>4.91 (0.4 %)</td>
<td>23.75 (1.8 %)</td>
</tr>
<tr>
<td></td>
<td>3.16 (0.2 %)</td>
<td>15.51 (1.2 %)</td>
</tr>
</tbody>
</table>

Deposited in the individual interactions that constitute the event. Again, the degraded resolution does not impact the uncertainty of the Compton cone opening angle, as the energy of the incident gamma-ray is specified by the user rather than derived from the sum of energies deposited in the interactions. The degraded energy resolution for double-interaction events can have an impact on the ability to detect peaks in the spectrum in high-background measurement scenarios, as is seen section 5.4.2.

### 3.5.2.2 Imaging Efficiency

The efficiency of the imager for detecting Compton events is an important aspect of detector response, impacting the localization time for gamma-ray sources of a given strength. The limitations to efficiency due to the stringent event selection process has been discussed (see section 2.3.3), thus the efficiency analysis herein represents the lower-bound of the possible imaging efficiency for these systems, especially the VCI. There are many different efficiency metrics [17] depending on the categories of event type and source-detector geometry. The metric most relevant for Compton imaging is the intrinsic Compton imaging efficiency, given by the fraction of photons with energy $E_0$ incident on the detector that result in a Compton event. The imaging efficiency depends on many factors, including the direction and energy of the incident photon. A full characterization of the VCI and HEMI is beyond the scope of the present work; instead, the data from the measurement described in section 3.5.1 are used to coarsely compare the relative imaging efficiency of the two systems. The analysis
Table 3.3: Relative Compton imaging efficiency of the VCI system compared to HEMI. The intrinsic photopeak efficiency of double-interaction gamma-ray events is used as a proxy for the Compton imaging efficiency. The metric is computed by comparing the net area of the photopeaks in the double-interaction spectra from the VCI to those from HEMI, scaled by the expected difference in gamma-ray flux due to the locations of the point sources.

<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>Net Area - VCI</th>
<th>Net Area - HEMI</th>
<th>( \frac{\text{Exposure Rate}<em>{\text{VCI}}}{\text{Exposure Rate}</em>{\text{HEMI}}} )</th>
<th>( \frac{\epsilon_{\text{VCI}}}{\epsilon_{\text{HEMI}}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>356.0 keV</td>
<td>1005</td>
<td>1621</td>
<td>0.95</td>
<td>0.59</td>
</tr>
<tr>
<td>511.0 keV</td>
<td>3373</td>
<td>5665</td>
<td>1.04</td>
<td>0.62</td>
</tr>
<tr>
<td>662.0 keV</td>
<td>734</td>
<td>1390</td>
<td>0.97</td>
<td>0.51</td>
</tr>
<tr>
<td>834.0 keV</td>
<td>247</td>
<td>663</td>
<td>1.00</td>
<td>0.37</td>
</tr>
<tr>
<td>1173.0 keV</td>
<td>207</td>
<td>606</td>
<td>1.02</td>
<td>0.35</td>
</tr>
</tbody>
</table>

is insufficient to specify the true efficiencies of the imaging systems. The issue of event misclassification is not explicitly handled and the measurement provides limited sampling in both photon energy and incident direction. The fact that the sources are all confined to the forward-directed FOV for the systems is vastly insufficient for mapping the 4\( \pi \) response, but is adequate for the sake of relative order-of-magnitude comparison; especially since the imagers are generally pointed towards sources in the scene for many of the measurements in chapters 4 and 5 given the limited Kinect FOV. The net area of the peaks in the spectrum of double-interaction events is used as a proxy for imaging efficiency, which ignores the effects of event misclassification. The differences in the incident directions of the gamma-rays are ignored, and the point detector assumption is applied, further ignoring variability due to finite detector volumes. The differences in expected incident flux due to the location of each of the sources relative to the imagers is taken into account, listed in table 3.3. Scaling by the expected gamma-ray exposure rate at each detector, the ratio of net peak area is compared for several gamma-ray energies. The rough estimate of imaging efficiency presented in table 3.3 suggests HEMI is on the order of two to three times more efficient for potential Compton events over a range of gamma-ray energies as compared to the VCI. The relative efficiency for the VCI system decreases with gamma-ray energy as a result of the event selection limiting to double-interaction events, as higher energy gamma-rays are expected to undergo multiple Compton scatter interactions in the process of depositing their full energy within the detector.

The relative imaging efficiency is expected to be higher at 356 keV for the VCI. The VCI double-interaction spectrum shows a significant photopeak at 352 keV corresponding to \(^{214}\text{Pb}\) present in the background. The energy resolution of HEMI is insufficient to separate these two peaks, thus the contribution from the 352 keV line is included in the estimate of the net area of the 356 keV. This explains why the relative efficiency of the VCI is lower than expected at 356 keV, and would be expected to increase the noise in a Compton image from HEMI from the inclusion of double-interaction events that are not correlated to the
In principle, the effect of the gamma-ray background on spectral response could be explicitly accounted for by subtracting a background spectrum from the energy spectrum attained with the sources present. Unfortunately, the instability of the prototype HEMI system, particularly when it comes to electronic noise, makes it difficult to compare spectra taken from two separate measurements. Taking into account the assumptions and simplifications made for this analysis, the imaging efficiencies of the two systems are expected to be of the same order of magnitude over the relevant energy range, with the disparity in relative efficiency becoming more pronounced with increasing gamma-ray energy.

### 3.5.3 Imaging Response

The measurement described in section 3.5.1 is also used to compare the imaging performance of the two systems. The quality of the Compton image can be assessed in a number of ways (see section 6.1) including image contrast, signal-to-noise ratio (SNR), and image resolution. In this analysis, the impulse response of the two Compton imagers is measured to evaluate the angular resolution of the systems over a range of gamma-ray energies. Image evaluation in terms of other metrics like SNR is left to chapter 6 due to the strong dependencies on the sampling of the gamma-ray source distribution. As with the analysis of imaging efficiency, dependencies on the direction of the incident gamma-ray flux are ignored.

A system-agnostic filtered-backprojection image reconstruction algorithm is used to simultaneously reconstruct each of the point source distributions. The filtered back-projection algorithm includes a regularization parameter which controls the tradeoff between resolution and SNR. The regularization parameter is set to the lowest value that gives a coherent point in each reconstruction. The regularization parameter can range from $0 \leq \lambda \leq 1$ and was set to 0.01 for all of the reconstructions in the VCI, and 0.04 for most of the reconstructions in HEMI. The reconstructions at 356 keV and 834 keV in HEMI used a regularization parameter of 0.08 to compensate for increased noise in these images. Table 3.4 lists the ROI
t selected for the measurement and the resulting number of potential Compton events used in each reconstruction.

Fig. 3.19 and 3.20 give the results of the static image reconstruction for the VCI and HEMI respectively. The $4\pi$ images are visualized as 2D histograms of image intensity in $(\phi, \theta)$ with 1° angular binning. Fig. 3.21 and 3.22 zoom in to the region in angular space where the sources are located to better visualize the point responses. The filtering process makes a direct comparison of the image intensities meaningless, especially given variation in the regularization parameters. Instead, each image is normalized by the maximum image intensity. No additional image processing, such as pixel interpolation, is done prior to display. The true source location measured in 3.1 is plotted as a solid point, indicated by the image legend. Each reconstructed point distribution is accompanied by a contour at 50% of the maximum intensity. The image resolution is then computed taking twice the average distance between the highest-intensity pixel and the contour surface. The result of this computation is represented in the images as well as table 3.4.
Figure 3.19: Results of the filtered backprojection reconstruction over the full $4\pi$ imaging space for the VCI. The contour lines represent the half-maximum intensity for each distribution.

Figure 3.20: Results of the filtered backprojection reconstruction over the full $4\pi$ imaging space for HEMI. The contour lines represent the half-maximum intensity for each distribution.

Figure 3.21: Static reconstruction results from the VCI zoomed to the source region. The colors of the reconstructed distributions correspond to the gamma-ray sources as indicated in the figure legend. The reconstructed point distributions are well-separated. The computed angular resolution is given for each distribution along with the contour line corresponding to 50% image intensity.

The VCI reconstruction of the source distribution given in Fig. 3.21 displays excellent angular resolution and low image noise relative to the image reconstructed from HEMI. The
VCI angular resolution is about $4.5^\circ$ to $6.5^\circ$ over the entire energy range, conforming to expectations of the imaging resolution for the system given the measured energy and position resolution. There is very little image noise above 50% of the maximum image intensity except in the ROI around 834 keV, corresponding to the $^{54}$Mn source, which was a factor two lower in activity compared to the other four sources. The point distributions are accurately reconstructed with the maximum intensity of each reconstructed distribution occurring within $1^\circ$ of the true source location. The computed angular resolution is in agreement with the reconstructed distributions, with a clearly distinguishable point corresponding to each gamma-ray source, which are separated by about $6^\circ$ as noted in table 3.1.

HEMI on the other hand displays poorer angular resolution and increased noise in the reconstructed image. The reconstructed point distributions from HEMI maintain adequate accuracy, with the maximum reconstructed image intensity occurring within $1.5^\circ$ of the true source location for each gamma-ray source, with the exception of the reconstruction at 834 keV, which is accurate to within $2.5^\circ$ degrees. The poorer angular resolution from the HEMI system, about $11^\circ$ FWHM on average over the given energy range, is in agreement

Figure 3.22: Static reconstruction results from HEMI zoomed to the source region. The colors of the reconstructed distributions correspond to the gamma-ray sources as indicated in the figure legend. The reconstructed point distributions cannot be unambiguously resolved, especially at low and high gamma-ray energies. The computed angular resolution is given for each distribution along with the contour line corresponding to 50% image intensity.
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<table>
<thead>
<tr>
<th>ROI Location</th>
<th>ROI Width (keV)</th>
<th>Compton Cones</th>
<th>Angular Resolution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>VCI</td>
<td>HEMI</td>
<td>VCI</td>
</tr>
<tr>
<td>356.0 keV</td>
<td>3.0</td>
<td>10.0</td>
<td>7626</td>
</tr>
<tr>
<td>511.0 keV</td>
<td>3.0</td>
<td>10.0</td>
<td>12330</td>
</tr>
<tr>
<td>662.0 keV</td>
<td>3.0</td>
<td>10.0</td>
<td>3734</td>
</tr>
<tr>
<td>834.0 keV</td>
<td>3.0</td>
<td>12.0</td>
<td>1418</td>
</tr>
<tr>
<td>1173.0 keV</td>
<td>3.0</td>
<td>15.0</td>
<td>1270</td>
</tr>
</tbody>
</table>

Table 3.4: The ROIs used for the static measurement, along with the number of potential Compton events and resulting angular resolution. The angular resolution is given by computing the maximum distance between the pixel with maximum image intensity and the 50% contour line. The measured angular resolution depends on many factors, discussed in further detail in section 6.3, and is consistent with previous characterizations of each instrument [32, 33].

Table 3.4: The ROIs used for the static measurement, along with the number of potential Compton events and resulting angular resolution. The angular resolution is given by computing the maximum distance between the pixel with maximum image intensity and the 50% contour line. The measured angular resolution depends on many factors, discussed in further detail in section 6.3, and is consistent with previous characterizations of each instrument [32, 33].

with expectations from previous characterizations [33] but insufficient to unambiguously resolve all of the point sources in this measurement. Fig. 3.20 also exhibits significant image noise above 50% of the maximum image intensity in the ROI around 356 keV. As mentioned previously, this noise likely results from a combination of factors including the poor signal-to-background ratio in this spectral region due to downscattered gamma-rays, as well as the poorer energy resolution of HEMI. The 352 keV line from $^{214}$Pb is clearly resolved in the VCI spectrum in Fig. 3.18, but not in the HEMI spectrum, thus contributions from this uncorrelated source are included in the image reconstruction. The results from this measurement suggest an angular resolution about 2-3x better for the VCI compared to HEMI over the studied range of gamma-ray energy.

### 3.5.4 Summary

The measurement scenario described in section 3.5.1 provides a quantitative measure of the relative spectral and imaging performance of the VCI and HEMI over a range of gamma-ray energies relevant for Compton imaging. The energy resolution, imaging efficiency, and image resolution of each system are compared. The excellent energy resolution of the VCI is summarized in table 3.2 and is in agreement with previous characterizations of the strip HPGe detectors [32]. The energy resolution impacts the ability of the user to identify peaks by which the imaging ROIs are set, as well as improved imaging performance due to high signal-to-background and lower uncertainty in the Compton cone opening angle. The energy resolution of the HEMI system also agrees with previous characterizations [33] and presents challenges both in setting the imaging ROIs and poorer signal-to-background ratio, especially for gamma-rays with incident energy below several hundred keV. HEMI is estimated to be roughly 2-3x as efficient for acquiring gamma-ray events suitable for Compton imaging when
compared to the VCI in its current configuration. The angular resolution of each system in the Compton mode is conservatively estimated to be about 5° for the VCI and about 11° for HEMI. These results agree with previous characterizations \[32, 33\], and are expected to impact the spatial resolution in the volumetric reconstruction.

### 3.6 SLAM

Having characterized the performance of the gamma-ray imagers used in this work, attention is now turned to the second essential component of SDF — real-time pose estimation and scene reconstruction. Simultaneous localization and mapping represents a class of problems dealing with the estimation of the location and orientation of a sensor and producing a model of the environment based on measurements of a mobile sensor. The SLAM problem has origins in photogrammetry \[56\], and has been of major interest to the robotics community \[57\] in the context of autonomous navigation in unknown environments. SLAM encompasses a range of problems with varied solutions tailored to specific applications \[58\]. Rather than attempting to provide a comprehensive overview of SLAM problems and potential solutions, particular aspects of the problem at hand serve to narrow the scope of the discussion. One critical element of the SDF approach introduced in section 1.5 is a real-time estimate of the location and orientation of the sensor in the environment. This consideration narrows the scope to a sub-category of SLAM solutions termed online SLAM \[68\]. Online SLAM is characterized by the incremental estimation of the sensor track and 3D model as new measurements become available, as opposed to full or global SLAM, which operates on the complete set of measurements. Another distinguishing factor is that, in order for SDF to be sufficiently general, the SLAM approach must work in unknown, un-labeled environments. This consideration precludes the use of external identifiers such as tags being placed in the environment, and may necessitate the solution of the data association problem; i.e., computing the correspondence between visual features acquired from multiple perspectives. A further distinction is based on the nature of the scene itself: whether the environment is expected to be static or to change over the measurement time. Many of the in situ imaging applications of interest in this work occur in non-dynamic environments, thus a SLAM approach based on static scenes is sufficient. Finally, a central feature of SDF is the incorporation of the 3D model into the gamma-ray image reconstruction. As such, the SLAM approach should provide a dense representation of the environment along with quantitative metric information about the relationship between subregions in the scene (as opposed to topological description). There are many potential SLAM solutions that meet these requirements, which are further differentiated by the sensor (or sensors) that are employed. There exist online SLAM solvers based on LiDAR data \[71\], visual-inertial systems \[72\], stereo-

\[ Not all SLAM approaches rely on the extraction of visual features, relying instead on incremental changes set of 3D points acquired at each frame \[69, 70\].
scopic RGB cameras \cite{73}, and even monocular RGB sensors \cite{74}. Each method has its own set of advantages and limitations; evaluating SDF based on various SLAM approaches is beyond the scope of this work. Instead, a single approach to solving the SLAM problem is selected, with the volumetric imaging results subject to the performance of that specific approach. The SLAM solver chosen for this work is the RGB-D Slam algorithm, discussed in the next section.

### 3.6.1 RGB-D Slam

The SLAM algorithm chosen for this work is called RGB-D Slam, which is presented in full detail in \cite{55}. RGB-D Slam relies on a type of sensor known as an RGB-D camera, where the RGB refers to a standard 2D color camera and the “D” refers to some form of active depth sensor. Over the course of this project, several RGB-D sensors became commercially available, based on different active depth sensing modalities such as structured light \cite{75} or time-of-flight \cite{76}. In this work, a Microsoft Kinect sensor was used to provide RGB images and dense 3D point cloud representations of the environment. The Kinect can provide this information at a frame rate $\leq 30$ Hz and an effective maximum range of 4 – 6 meters depending on the configuration. RGB-D Slam publishes estimates of the 6D pose of the Kinect as well as an aggregate point cloud model of the scene in real time. The coordinate frame of the scene model is set by the first frame acquired from the Kinect, thus subsequent pose estimates represent a transformation from the current Kinect coordinate frame back to the model coordinate frame. The Kinect is kept rigid with respect to the gamma-ray imagers and the relative transformation between the coordinate frame of the Compton camera and the Kinect is determined. Several approaches to determine this transform were explored, including approaches based entirely on remote sensing such as perspective-N-point algorithms \cite{77}. In the end, the Kinect was aligned parallel to the front face of the detectors and the translation between the coordinate frames of the Kinect and Compton imager was physically measured. This registration transform is then applied to the Compton events collected by the detector to correctly orient the Compton cones in the same coordinate frame as the scene model. By synchronizing the tracked pose of the detector with the Compton events, a full Compton event history along the track of the detector is compiled and updated in real time. Both the radiation image as well as the imaging space are continuously recomputed as new Compton events are measured along the detector track, providing real-time feedback to the user.

RGB-D Slam confers several other practical advantages that ultimately led to its selection over other options. The project is entirely open-source \cite{https://github.com/felixendres/rgbdslam_v2}, which enabled the software to be augmented and extended.

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\footnote{The citations here represent only a subset of published approaches to illustrate the breadth of solutions. Even when categorized by sensor type, there are many more approaches than are listed here.}

\footnote{The Kinect that was used in this work is a first-generation model where the depth sensitivity is based on IR structured light technology. A second generation of the sensor has since been released with a different suite of sensors. The information quoted below is only relevant for the first generation Kinect sensor.}
CHAPTER 3. IMPLEMENTATION OF SCENE DATA FUSION

to suit the needs of the SDF development. Furthermore, RGB-D Slam is built upon a flexible open-source framework, the Robot Operating System (ROS) \[78\]. This allowed for straightforward integration with the software framework developed for this work (see section 3.7), which is also based on a distributed process, message-passing paradigm.

No one SLAM algorithm is suitable for every measurement environment. RGB-D Slam is no exception: the Kinect sensor in particular imposes several limitations. The depth sensitivity of the Kinect is based on a projected IR structured light pattern \[79\]. In general, this sensor is not suitable for outdoor use where the ambient IR background washes out the projected structured light pattern. Nor is RGB-D Slam suitable for operation in very low-light environments, such as the inside of an unlit cargo container, due to the reliance on visual features \[80, 81\] extracted from the RGB images. Even in environments with suitable lighting (low ambient IR, adequate lighting for visual imagery), the Kinect has other limitations. Both the accuracy and precision of the sensed depth from the Kinect scale inversely with the depth squared \[51\], resulting in large errors in the pose estimates (and consequently the scene model as well) when significant portions of the visual features are derived from objects that are farther away in the scene. RGB-D Slam includes a graph-optimization backend to minimize the global error in the pose estimates \[55\], yet it is insufficient to entirely overcome this effect. This limitation is especially relevant for measurement scenarios where the dimension of the imaging space is large and has relatively few visual features. The Kinect also has a lower limit on depth sensitivity of about 60 cm \[51\], potentially limiting the distance of closest approach for the imager\[^{8}\]. The Kinect has a field-of-view (FOV) of about 53° horizontal and 47° vertical \[79\]. The limited FOV is very much mitigated by moving around the imaging space, aggregating information as more perspectives are collected. Nevertheless, a sensor with a longer range that could provide depth sensitivity in 4π might be a better match for the volumetric Compton imaging approach, due to the inherently wide FOV associated with Compton imaging \[27\]. The advantages of a real-time tracking and scene reconstruction coupled with software designed for ease of external interfacing far outweigh these disadvantages, allowing for the successful development of SDF with RGB-D Slam as demonstrated in chapters 4 and 5.

There are several additional challenges inherent to many visual tracking approaches that have an impact on volumetric localization of gamma-ray sources. The dependence of the pose estimates on the uncertainty of the visual features has been discussed. The uncertainties from frame-by-frame pose estimation also accumulate as the number of pose estimates increases. The graph-minimization backend RGB-D Slam employs reduces the effect of accumulated error, though it does not eliminate the effect. This impacts not only the pose estimates, but the 3D model as well. Each of these modes of pose error are investigated in chapter 6. There is also the problem of losing tracking during the measurement. This occurs when the feature sets associated with sequential frames fail to meet configured thresholds in the number of corresponding visual features. Lost tracking tends to occur in visually sparse environments,

\[^{8}\] This limitation can be overcome when operating HEMI by turning the Kinect away from an object as the user nears it.
or when the sensor is moving rapidly. This effect is not explicitly explored in this work, but a general consideration for visual tracking systems.

3.7 Software

The development of software to acquire and analyze data for volumetric imaging and display the results in real-time is a central component of this work. Project goals led to the emphasis of specific features in designing the software:

- **Real-Time**: The acquisition, analysis, and display of results must be done in real-time in order to provide actionable feedback during mobile operation.

- **Modularity**: The software developed for real-time volumetric imaging has to be flexible to allow for rapid implementation on multiple systems.

- **Platform Independence**: Related to modularity: some of the existing software for the gamma-ray imagers depends on specific operating systems (OS). It is therefore important that the software framework run on many different operating systems to reduce roadblocks when building applications for different systems.

- **Ease of Interfacing**: It had been clear since the inception of the project that any software developed would need to interface with other software packages. It is therefore important to employ simple communication protocols to allow for rapid integration of information from other software packages (such as RGB-D Slam).

These design goals are admittedly ubiquitous for software development projects (especially when the goal is to develop a flexible framework on which various applications will be built). Nevertheless, adherence to these design considerations facilitated the development of a software framework which has been successfully used to develop real-time volumetric gamma-ray imaging applications for several systems. The framework is built on a distributed memory model with communication based on a message-passing between computational nodes. In this work, the computational nodes are referred to as “threads”. The thread distribution is based on a master-worker model \[82\] where the main (master) thread is responsible for the user interface and controlling a pool of worker threads in which the computation for acquisition and analysis are done. This is a common computational model for Graphical User Interface (GUI) applications to ensure that the UI remains responsive, accepting input from the user and displaying real-time analysis results, while simultaneously handling computationally intensive tasks. Threads are specified by inheriting from a ThreadWrapper base class which standardizes worker-worker and master-worker communication. Several high-level communication protocols are built into the framework including pipes and queues from the Python2.7 multiprocessing library \[83\] as well as network communication based on ZeroMQ \[84\]. The network communication is employed for communication between the gamma-ray imaging framework and external software packages such as RGB-D Slam.
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The multithreaded nature of the framework along with the asynchrony of the inter-thread communication makes efficient use of computational resources. Especially computationally intensive tasks can be forked to several threads responsible for same type of computation to ensure a balanced load and alleviate computational bottlenecks. The multithreaded design also aids in the modularity of the framework, as different threads can be designed for use with specific systems without having to completely redesign a monolithic application for each new imaging system.

Python was chosen as the language in which the framework would be implemented for several reasons. It satisfies the cross-platform requirement as the python interpreter can run on many platforms with minimal complications. The availability of extensive libraries for scientific analysis, such as numpy and scipy, enable rapid development of efficient analysis and imaging algorithms. High-level libraries for multithreading and communication like multiprocessing and python bindings for ZeroMQ made implementing the ThreadWrapper and communications standards straightforward without requiring excessive low-level design. Visualization libraries for the real-time display of data, from single dimensional histograms up to 3D rendered point clouds and isosurfaces, are found in packages like matplotlib, pyqtgraph, and mayavi2. The analysis, communication, and visualization libraries are combined with powerful GUI-design and event-driven programming tools from the Qt framework via the PySide bindings. Finally, the core functionality of the Python interpreter is extended via the CPython API and other packages such as cython. These tools are used to wrap existing, legacy C code and to implement high-performance libraries for computationally intensive tasks like Compton backprojection and gamma-ray event reconstruction. The event reconstruction thread for the VCI was implemented using the CPython API, yielding a greater than 3000x speed-up over a pure python implementation. Thus an application framework, developed in Python and leveraging many pre-existing tools and packages for analysis and display, provides a high-level toolkit with which applications for real-time volumetric imaging are developed for both VCI and HEMI.

Fig. 3.23 and 3.24 represent the thread structure for the VCI and HEMI imagers respectively. Many threads are present in both the HEMI and VCI applications, such as DAQ and event reconstruction threads. These highlight the modularity of the framework: the specific acquisition or reconstruction code is swapped into the appropriate thread depending on the system. Several differences in the design of the applications should be noted. In the VCI, a single thread is used to buffer and synchronize the three data streams (pose estimates, point cloud model, and Compton events) and run the volumetric image reconstruction. With HEMI, the buffering and synchronization are handled in a single dedicated thread, with the synchronized data subsequently passed to a separate thread for volumetric image reconstruction. This structure was determined to be more robust for HEMI as all of the network communication is encapsulated in a single thread, making it easier to isolate network problems when debugging. In all, the python-based gamma-ray imaging framework provides a flexible application development environment that allowed the volumetric imaging concept to be implemented on two different systems without requiring major re-engineering for each application.
Figure 3.23: Threaded software structure for the VCI volumetric imaging application. All software is run on a single computer.
Figure 3.24: Threaded software structure for the HEMI volumetric imaging application. The software is run on two separate computers as described in section 3.4.
Chapter 4

Demonstration: Scene Data Fusion with the VCI

This chapter presents volumetric Compton imaging results from measurements with the VCI. Results from several measurements are presented, including a single point source (sections 4.1 and 4.2), multiple point sources with different energies (section 4.3), and multiple point sources with the same gamma-ray energy (section 4.4). The purpose of these measurements is to demonstrate capabilities; each is accompanied by a qualitative discussion. Evaluation of image quality as well as robustness of SDF with the VCI is discussed in chapter 6.

4.1 Single Point Source

The volumetric reconstruction of a single, monoenergetic point source of gamma-ray radiation demonstrates SDF for the simplest case. A 50 $\mu$Ci $^{137}$Cs point source is placed in the cluttered, indoor laboratory environment shown in Fig. 4.1. The location of a 50 $\mu$Ci $^{137}$Cs point source is indicated by the red arrow. The VCI is pushed along a slightly curved path facing the scene in 4.1. The total length of the cart path is 4.6 m with the source-detector distance reaching a minimum of about 1.5 m near the center of the track. It took 124 seconds to push the cart along this trajectory, collecting a total of 85 full-energy Compton events in the ROI 3 keV wide around 662 keV along the track.

The data collected during the measurement are passed separately to the ML-EM algorithm in both modes: the first without restricting the reconstruction to the voxels occupied by the 3D model, and the second with this constraint. In each case, the 3D model of the scene from RGB-D Slam is plotted in grayscale (in order to contrast with the radiation image), while isosurfaces computed from the reconstructed gamma-ray distribution are displayed in color. The isosurfaces represent the contours at 25%, 50%, and 75% of the maximum image intensity, increasing in shade from light blue to yellow. The red line represents the tracked position of the detector as the cart was pushed through the scene, while the white spheres represent points along the track where full-energy Compton events were recorded. The blue
CHAPTER 4. DEMONSTRATION: SCENE DATA FUSION WITH THE VCI

Figure 4.1: Image of the laboratory environment in which the VCI measurements were taken. The location of the 50 µCi $^{137}$Cs point source is indicated by the red arrow.

arrows emanating from the white spheres represent the scatter-axes of the Compton cones. Only the scattering direction, $\vec{\omega}$, is indicated by the arrows, thus it is possible to have arrows pointed at any orientation relative to the source location (arrows pointing away from the source correspond to Compton back-scatter events). The imaging algorithm runs in near real-time: a non-optimized implementation takes 200 ms to compute the back projection, and another 200 ms to complete 10 EM iterations on a single core of an intel i7 4600U at 2.1 GHz. The number of EM iterations is chosen to balance the real-time requirement with image convergence, with ten being empirically determined to be sufficient for the reconstruction of point sources. The results presented here were computed offline so that the different imaging modes (with model fusion and without) could be directly compared.

Fig. 4.2 shows the resulting image when the constraint is not used, i.e., the entire space along the detector path is voxelized and the reconstruction does not utilize any occupancy constraint. The reconstruction of the source location is comparatively inaccurate and blurred in the direction perpendicular to the track (the depth dimension). Image artifacts are also clearly visible in regions well away from the true source location. In contrast, the artifacts
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Figure 4.2: Volumetric gamma-ray image reconstructed without using the point cloud constraint. The model of the scene is still displayed for context, but was not used in the Compton image reconstruction.

are eliminated and the radiation image more accurately reflects the true source distribution when the model is used to restrict the imaging space, shown in Fig. 4.3. The magnitude of the reduction in the imaging space from the scene model constraint will obviously depend on the scene being viewed. For this measurement, the original image space contained 1.65 million voxels with only 28,000 remaining after applying the constraint; a reduction in the imaging space by roughly a factor of 60.
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Figure 4.3: Volumetric gamma-ray image reconstructed with the point cloud constraint. The location of the $^{137}$Cs point source is accurately reconstructed (cf. Fig. 4.1), and the image noise present in Fig. 4.2 has been eliminated.

4.2 Real-Time Imaging

An attempt is made to illustrate the real-time imaging feedback provided by the software framework, though it is difficult to demonstrate real-time capabilities in print. In this measurement, a single 50 $\mu$Ci $^{137}$Cs point source was placed on a counter in the scene, and the cart is pushed along a horizontal trajectory facing this scene. Fig. 4.4 shows the scene for this measurement, as well as the location of the $^{137}$Cs point source. Fig. 4.5 shows the results of the volumetric reconstruction at 37.5 second intervals. The time interval of 37.5 seconds was chosen to illustrate the imaging results at eight evenly-spaced intervals over the course of the 300 second measurement. In practice, the rate at which the volumetric image is recomputed is configurable in the software. The maximum recomputation rate depends on many factors, the most important being the size of the point cloud and the network bandwidth. If the point cloud is not being used in the reconstruction nor rendered in real-time, then the maximum recomputation rate is governed solely by the speed of the image reconstruction,
CHAPTER 4. DEMONSTRATION: SCENE DATA FUSION WITH THE VCI

Figure 4.4: Point cloud model of the scene for the measurement described in section 4.2. The location of the 50 μCi $^{137}$Cs point source is indicated by the red arrow.

which depends on the number of voxels and number of Compton cones. Despite the limitations of the network and graphical rendering, the recomputation rate is much shorter than the 37.5 seconds shown here, generally around 5-10 seconds for the VCI measurements.

Fig. 4.5 illustrates the evolution of the volumetric image as the cart progresses from right to left at a roughly constant rate throughout the scene. At the beginning of the measurement, the reconstruction is inaccurate and noisy and fail to localize the source due to the low number of Compton cones and short triangulation baseline. By the time the cart has traversed half of the path, having moved a total of 1.57 m in 187 seconds, the volumetric reconstruction has converged on the location of the hotspot, with agreement among the subsequent reconstructions. As the cart continues along the path collecting more Compton data, the reconstructed source location remains constant, with minor fluctuations in the image below 25% of the maximum image intensity. The lack of deviation in the volumetric reconstruction as more and more Compton data is acquired is a strong indication to the user that a point source is present and has been correctly localized.

The panel of images in Fig. 4.5 illustrates another of the operational aspects of the real-time volumetric imaging. The point cloud is not used to constrain the imaging space during the entire measurement to avoid false localization due to an incomplete 3D model, especially early on in the measurement. As the imager progresses through the scene and successive reconstructions become consistent with one another, the user may then wish to use the point cloud to constrain the imaging space to reduce reconstruction time and potentially reduce noise in the gamma-ray image. The rendering of the point cloud is also important to provide context for the volumetric gamma-ray image as the model becomes more complete. Fig. 4.6
shows the results from the final frame in Fig. 4.5 along with the point cloud model. Even though the point cloud is not used to constrain the reconstruction, it still provides important context for interpreting the gamma-ray image.

It should be noted here that the criteria for successful localization based on real-time feedback is entirely heuristic. The agreement among subsequent reconstructions as more Compton data is acquired provides qualitative evidence of the true 3D distribution of gamma-ray sources, though no quantitative measure of the localization certainty is provided. For the simple case of localizing point sources of gamma-rays presented here, a figure-of-merit (FOM) could be devised that might depend on factors like image SNR, number of Compton cones, and the length of the track relative to the hypothesized source location. Issues involving image interpretability and data sufficiency conditions are beyond the scope of this work. In the absence of a quantitative metric of localization performance, the results in Fig. 4.5 illustrate one of the benefits of real-time feedback, as confidence in a correct localization is improved with agreement between subsequent reconstructions.
Figure 4.5: Panel of images showing the progress of the volumetric gamma-ray image as the cart is pushed through the scene from right to left. The images are reproduced here at 37.5 second intervals to limit the number of frames in the image and illustrate the progress over the full course of the 300 second measurement. The reconstructed images in the first four frames are noisy and inconsistent, failing to localize the point source. The last four frames consistently reconstruct a point source at the same location with varying amounts of image noise. The point cloud was not used to constrain the imaging space at any point during the measurement.
Figure 4.6: Result of the volumetric gamma-ray imaging measurement using all of the collected data. The gamma-ray image is the same as the last frame in Fig. 4.5 but the point cloud is also shown here. The model was not used to constrain the imaging space.
4.3 Multiple Point Sources, Different Energies

A third measurement demonstrates the ability to simultaneously reconstruct several gamma-ray point sources in the scene during a single measurement. The measurement also demonstrates volumetric imaging capability over a wider range of incident gamma-ray energies, from 300 keV up to 834 keV. Four gamma-ray point sources were used, each with an activity of 10 μCi. The sources are arranged such that each rests on a different surface, separated from one another by about 25 to 80 cm. Since the gamma-ray sources emit gamma-rays with different energies according to the level structure of the daughter nuclide (following a β decay in the parent nuclide), each of the source distributions can be imaged independently by gating on the spectroscopic data. Fig. 4.7 illustrates the setup for this measurement.

![Image of experimental setup](image)

Figure 4.7: Picture of experimental setup for simultaneous volumetric imaging of multiple point sources with different gamma-ray energies. Each of the sources has an activity of 10μCi.

with the location of each point source indicated in the figure. The color-coding denotes the identity of the point sources in Fig. 4.7 and is consistent throughout this section. Table 4.1 summarizes the spectroscopic ROIs used in the imaging, as well as the color corresponding to the ROI’s and the number of potential Compton events acquired in each ROI during the measurement. The cart was pushed along an arc trajectory with the Kinect facing the scene throughout the measurement. The total measurement time was 4 minutes, 24 seconds with the cart being pushed at an approximately steady pace with a median velocity of $6.5 \text{ cm/s}$. A separate reconstruction process was run for each of the five ROI’s indicated in 4.1.
Table 4.1: Summary of spectroscopic ROI’s specified for volumetric image reconstruction. Five ROI’s are specified, corresponding to five gamma-ray lines from four distinct gamma-ray sources. The color of the corresponding reconstruction and the number of events in each ROI are also given.

<table>
<thead>
<tr>
<th>ROI</th>
<th>Nuclide</th>
<th>Reconstruction Color</th>
<th>Compton Cones</th>
</tr>
</thead>
<tbody>
<tr>
<td>300 – 306keV</td>
<td>$^{133}$Ba</td>
<td>Red</td>
<td>111</td>
</tr>
<tr>
<td>353 – 359keV</td>
<td>$^{133}$Ba</td>
<td>Blue</td>
<td>214</td>
</tr>
<tr>
<td>508 – 514keV</td>
<td>$^{22}$Na</td>
<td>Green</td>
<td>214</td>
</tr>
<tr>
<td>659 – 665keV</td>
<td>$^{137}$Cs</td>
<td>Orange</td>
<td>101</td>
</tr>
<tr>
<td>829 – 839keV</td>
<td>$^{54}$Mn</td>
<td>Purple</td>
<td>69</td>
</tr>
</tbody>
</table>

Fig. 4.8 shows the energy spectrum of two-site, time-coincident gamma-ray interactions collected during the measurement. The gamma-ray events represented in the energy spectrum are not necessarily correlated to the source as discussed in section 2.3.3. Higher energy gamma-rays that down-scatter into the ROI without having deposited their full initial energy in the detector result in event misclassification and increased image noise via the addition of cones that are not correlated to any of the gamma-ray sources. This effect is mitigated somewhat for HPGe due to the excellent energy resolution, which reduces the width of the energy ROI’s. The peaks in Fig. 4.8 are significantly above the background for the coincidence spectrum, indicating the majority of the events in the ROIs are indeed correlated to a specific source. Image noise contributions are expected to be greatest for the reconstruction at 303 keV which has the poorest signal-to-background ratio of the ROIs.

The results of volumetric image reconstruction from each of the five ROI’s are plotted together in the same image. The scatter axes of the Compton cones used in the reconstruction for all five ROI’s are indicated by the colored arrows appearing along the detector track. Fig. 4.9 displays the results from the full reconstruction, without any constraint on the imaging space, whereas Fig. 4.10 gives the results when the solution is constrained to the occupied voxels. Comparing Fig. 4.9 and 4.10 with Fig. 4.7, the reconstructed source location are correct for both the constrained and unconstrained cases. There is increased noise in the volumetric images in the unconstrained case, particularly in the lower energy lines corresponding to $^{133}$Ba, as expected. The unconstrained results also tend to have less precision in the depth dimension; this effect is most pronounced for the reconstruction corresponding to $^{22}$Na, which shows significant spread in the depth dimension in Fig. 4.9 compared to the constrained results in Fig. 4.10. Note the lowest energy ROI’s correspond to the 303keV and 356keV of $^{133}$Ba, therefore we expect that the reconstructed source position be the same for each of these ROI’s. In the constrained case, the maximum image intensity of the two images corresponding to the photopeaks from $^{133}$Ba gamma-rays occur within the same voxel, in addition to reducing the image noise for each reconstruction. The agreement between the two independently reconstructed images corresponding to the same
Figure 4.8: Energy spectrum of time-coincident, double-interaction events from the multiple point source measurement presented in section 4.3. The shaded regions depict the spectroscopic ROIs.

Gamma-ray source provides strong qualitative evidence of an accurate localization. These results demonstrate the ability to simultaneously localize in 3D multiple gamma-ray point sources during a single volumetric measurement by leveraging the spectroscopic signatures from each source.
4.4 Multiple Sources, Same Energy

Point sources of gamma-ray radiation may be expected for some of the in situ applications in this work, such as searching for threat sources or imaging radioactive holdup in nuclear facilities. Measurements of single point sources are also useful for demonstration purposes and to verify system performance. In general however, gamma-ray sources will not always be distributed as a single point in the scene. This is especially true in the nuclear contamination remediation scenario, where the gamma-ray source distribution is potentially spread over vast areas with a complex 3D geometry. Therefore, it is of interest to investigate more complicated gamma-ray source distributions. This is investigated here with a measurement of multiple gamma-ray point sources of the same type, i.e., emitting gamma-rays of the same energy. This case is significantly more complicated than the scenario presented in section 4.3, as distinct gamma-ray sources are no longer spectroscopically separable.

Two $^{137}\text{Cs}$ point sources, each with an activity of 20 $\mu\text{Ci}$ were placed about 30 cm apart, as shown in Fig. 4.11. The unconstrained scenario results in a single hotspot blurred in the lateral direction, while the constrained case clearly distinguishes the two sources. This result demonstrates the conceptual capability of the method to accurately reconstruct more complicated source distributions. It should be noted that the ability to reconstruct distributed gamma-ray sources in general depends on proper weighting of geometric and temporal factors which are not explicitly accounted for in the present treatment (see section 2.6). It is also important to note that the geometry of the scene is conducive to aiding the image reconstruction when the constraint is applied. This issue is addressed in section 4.5 along with several other factors related to the mobility of the VCI that benefit the localization.
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Figure 4.11: Picture of experimental setup for volumetric imaging of two $^{137}$Cs sources on separate posts separated by 30cm.

Figure 4.12: Volumetric image of two $^{137}$Cs sources of equal activity.  
*Left*: Reconstruction without point cloud constraint.  
*Right*: Reconstruction with point cloud constraint.

of point sources with SDF.
4.5 Discussion

The measurements presented in this chapter demonstrate the efficacy of the SDF approach for volumetric gamma-ray imaging, with a variety of different capabilities at least conceptually demonstrated. Many data streams are incorporated into the reconstruction of these images, each with their own biases and variability, many of which are tightly coupled. As discussed in 3.6.1 the accuracy of the pose estimates depend strongly on the visual and geometric makeup of the scene. There are many additional system-specific factors that have an impact on the tractability of the volumetric gamma-ray image reconstruction. For example, the VCI cart is very heavy which limits the mobility of the system. For the measurements presented in this chapter, the cart speed never exceeded 10 cm/s. The rate of rotation of the system was similarly limited due to the mass of the cart. The restrictions on mobility tend to aid the visual tracking as the scene does not change dramatically from frame to frame [55]. Filters that eliminate poor pose estimates are also employed on the basis of the limited cart mobility. For example, a simple rejection filter is implemented that computes the distance between two sequential pose estimates and rejects any new estimate that has a total distance from the previous estimate greater than some configurable threshold. This filter eliminates outliers in the estimates of the detector pose, though at an increased risk of losing the detector track if several sequential estimates are rejected. Another fact that can be leveraged is the detector is at a fixed height. The laboratory space in which the VCI measurements are taken is flat, thus the estimate of the height of the detector should not deviate over the course of the measurement. Any deviation of the estimated detector height beyond a set threshold is an indication that the pose estimates are poor and the detector track (and thus gamma-ray image) may not be reliable. This information can also be applied on a frame-by-frame basis to eliminate outliers in the pose estimate which have poor estimates of detector height.

Fig. 4.13 illustrates the effect of these two filters on the raw pose estimates from RGB-D Slam. The distance filter rejects pose estimates if the computed distance from the previous frame is greater than 20 cm, or the estimate of detector height deviates by more than 5 cm. The unfiltered pose estimates from RGB-D Slam are given in red, whereas the remaining poses after the filter are given in blue in Fig. 4.13. The original, unfiltered track contains several inaccurate pose estimates that are eliminated by these filters. Though this simple filtering method is effective, it increases the likelihood of losing detector tracking during measurements in visually sparse environments.

RGB-D Slam includes an optimization routine that attempts to minimize the global error of edges in a pose graph which consists of the individual pose estimates [55]. This routine is more robust than the simple frame-by-frame filter approach and provides additional capabilities like loop closure. These globally optimized poses are not used for volumetric imaging in this work due to complications with time synchronization between the optimized poses and the radiation data. Integration with the optimized poses is a much more robust

1 The point cloud model however is constructed using the optimized poses, which explains the model consistency despite the poor pose estimates.
Figure 4.13: Illustration of the effect of applying a filter on the pose estimates by thresholding on detector height and frame-to-frame distance. A track consisting of the unfiltered pose estimates is shown in red, whereas the track from the filtered pose estimates is given in blue. Several erroneous pose estimates, indicated by the portions of the red track that deviate from the blue track, are eliminated by these simple filters.

solution than the simple filter described here and merits further investigation. Yet even this approach does not completely eliminate error in the pose estimates; the effect of uncertainty in the pose estimates on volumetric localization is investigated in chapter 6.

Another aspect to consider is the tradeoff between the maximum allowable uncertainty in the pose estimate and the robustness of the tracking algorithm. The visual tracking can be configured in a number of ways in an attempt to improve the frame-by-frame pose estimates. For example, the number of visual feature correspondences required between frames can be increased, along with many other parameters associated with the visual tracking. Increasing data association requirements or tightening thresholds in the tracking make it more likely that the estimation of the relative pose between frames will fail, resulting in lost tracking. The configuration of RGB-D Slam used in this work was chosen empirically such that tracking is rarely lost in the laboratory environments in which the presented measurements take
place. For a further discussion of tracking robustness and an evaluation of RGB-D Slam in particular, see [55].

The scene geometry also directly impacts the gamma-ray imaging when the model is used to constrain the imaging space in the reconstruction. In many of the measurements presented in this chapter, the gamma-ray point sources were located on a post, stand, or similar object. When the point cloud constraint is applied, these structures behave like geometric delta functions in the imaging space, depending on their dimension relative to the voxel size. Portions of the image intensity concentrated in voxels that neighbor those occupied by the post that themselves do not contain any points will likely contribute their signal to the nearest occupied voxels when the constraint on the image space is applied. Sources positioned on such structures would be expected to have reconstructed distributions with lower spread as a result of the lack of continuity and smoothness in the image space. These considerations make it very difficult to draw general conclusions about the impact of the model constraint on volumetric gamma-ray imaging. Many of these issues are addressed in chapter 5, where SDF is demonstrated on a much more mobile system with a wider variety of scene and source geometries.
Chapter 5

Demonstration: Scene Data Fusion with HEMI

The major limitation of the VCI is the inability to assess SDF on a system without any restrictions in mobility (see section 4.5). The HEMI system is not subject to these limitations, and is used to study the efficacy of SDF given a truly mobile system. HEMI is not only truly mobile, but also portable, and can be taken to new measurement locations outside of the lab in which the VCI measurements were taken. This provides access to scenes with varying geometries and visual makeups. Demonstrating SDF in different environments mitigates concerns that the successful demonstrations in chapter 4 are dependent on the structure and visual makeup of that particular environment. The increased mobility also allows for testing applications that were impractical with the VCI, such as remote interrogation of sealed objects for gamma-ray sources. The increased mobility presents new challenges as well, especially for the visual tracking component of SDF. HEMI is much lighter than the VCI and is limited only by the walking speed of the operator (vehicular mounting is not investigated in this work). There is a similar concern with rotational velocity, as HEMI is much easier to turn than the VCI. The increased velocities may increase the likelihood of failed tracking, potentially limiting the usefulness of SDF for some real-time imaging scenarios. Another result of HEMI’s mobility is the ability to traverse the scene in a circular manner with certain parts of the scene being viewed at different times. This situation results in a loop-closure scenario [92] and has increased potential for model inconsistencies. The additional capabilities and challenges of true mobility and portability are investigated in this chapter.

5.1 Real-Time Reconstruction

Though the arrangement of computers required to operate HEMI with SDF is more complex than the VCI (see section 3.4); the detector tracking, scene model reconstruction, and the volumetric gamma-ray image reconstruction are still all performed in real time. The feedback provided to the user from real-time reconstruction is especially advantageous with HEMI,
enabling the user to augment their path throughout the scene. The user can spend more time in regions of high signal to mitigate the $\frac{1}{r^2}$ factor that limits the static imagers; or, in the case of very little detected signal, move to unexplored regions in the scene. This was true of the VCI system as well; however, acting on real-time feedback was much less realistic due to the mass of the cart. The real-time feedback from the reconstruction is illustrated in Fig. 5.1. The six frames in Fig. 5.1 are extracted from a demonstration video in which the results of the reconstruction at 8-second intervals are merged with a conventional video of the HEMI operation. The total path taken over the course of the 40 second measurement is indicated by the red track that accumulates over successive frames. At the beginning of
the measurement, the track is limited and not much of the scene model is captured. With so few data, the point cloud constraint is toggled off. Using the constraint will result in a faulty reconstruction due to the incomplete model and lack of imageable gamma-ray events. As the model becomes more complete and the gamma-ray source distribution more tightly localized, the constraint can be toggled on to decrease the computation time of the image reconstruction. The size of the system matrix in equation 2.10 is given by the product of the number of Compton cones with the number of voxels. The computation time and the physical memory requirements thus increase throughout the measurement as both the imaging space and number of Compton events increase. Employing the occupancy constraint can greatly reduce the number of voxels and help with these limitations, but will cause imaging errors when the assumption\(^1\) is not suitable. The computational resources of modern desktop or laptop computers are adequate for real-time reconstruction for measurements of the scale presented in this work. Computational challenges must be addressed for SDF to scale to larger environments \(O(1000m^3)\) and environments with relatively high gamma-ray fluxes.

5.2 Multiple Point Source Reconstruction

Several additional measurements are presented to provide qualitative evidence of the gener-ality of the SDF approach. The goal is to demonstrate the same capabilities that were shown with the VCI as evidence of the efficacy of the SDF approach independent of the gamma-ray imaging system used. The reconstruction of multiple point sources with different gamma-ray energies is chosen for this purpose, analogous to the VCI measurement presented in section 4.3. Two measurement scenarios are considered in which multiple point sources with different characteristic gamma-ray energies are placed throughout a scene. One of the measurements takes place in the same laboratory environment as the VCI measurements, to focus on the differences between operation of HEMI and the VCI while holding the environmental factors relatively constant. A second measurement takes place in a new environment to demonstrate volumetric imaging capabilities outside of the laboratory environment.

5.2.1 Known Measurement Environment

The first measurement takes place in the laboratory space in which all of the VCI measurements were conducted. Though the physical location is the same, the scene traversal is more complex due to the enhanced mobility of HEMI. The VCI was generally restricted to linear or slowly arcing paths; in this measurement, HEMI is carried in a roughly circular path around the entire lab space. The 3D model of the scene acquired from the measurement is shown in Fig. 5.2 with the true location of three gamma-ray point sources indicated in the figure. Note that the VCI cart has become part of the scene for this measurement. The identity and activity of the point sources used in this measurement are given in table 5.1

\(^1\) Again, the simple point cloud constraint assumes gamma-ray sources are distributed on or near the surfaces of objects in the scene
Table 5.1: Summary of gamma-ray point sources used in the measurement described in section 5.2.1, as well as the user-selected ROI’s and the number of coincident events in each ROI.

<table>
<thead>
<tr>
<th>Nuclide</th>
<th>Activity ($\mu$Ci)</th>
<th>ROI (keV)</th>
<th>Compton Events</th>
<th>Reconstruction Color</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{22}$Na</td>
<td>10</td>
<td>501 – 521</td>
<td>188</td>
<td>Green</td>
</tr>
<tr>
<td>$^{137}$Cs</td>
<td>10</td>
<td>647 – 677</td>
<td>122</td>
<td>Orange</td>
</tr>
<tr>
<td>$^{54}$Mn</td>
<td>5</td>
<td>814 – 854</td>
<td>61</td>
<td>Purple</td>
</tr>
</tbody>
</table>

along with the corresponding color that is used throughout the analysis in this section.

The total measurement time was 108 seconds, resulting in an energy spectrum of potential Compton events given in Fig. 5.3 with the, user-selected ROIs highlighted. Table 5.1 lists the ROI’s as well as the number of potential Compton events collected in each of the regions. As in the VCI measurements, the imaging space is partitioned into $10 \text{ cm}^3$ voxels, and 10 EM iterations are used in each independent reconstruction. The 3D model constraint is applied to each of the reconstructions.

The result of the volumetric image reconstruction along with a rendering of the track taken through the environment is given in Fig. 5.4. A zoomed view from the same perspective is shown in Fig. 5.5 to more clearly illustrate the reconstructed source distributions. There
are several interesting features captured in Fig. 5.4 and 5.5. By comparison with the setup in Fig. 5.2, the locations of the $^{22}\text{Na}$ and $^{137}\text{Cs}$ point sources are accurately reconstructed. The distribution corresponding to the $^{54}\text{Mn}$ source however is offset from the true source location by a few voxels. Several factors may have contributed to the inaccuracy, one of the primary being the relatively few number of Compton cones collected in the ROI. An evaluation of volumetric reconstruction accuracy versus the number of Compton cones is presented in chapter 6. This measurement also illustrates several of the issues associated with the detector tracking with a more mobile system. The measurement began with the user walking forward and to the left, towards the VCI cart, and ended with the user walking back towards the starting point at a greater distance from the visual features of the scene. The effects of this increased distance are readily visible in the track as the pose estimates near the end of the track are much noisier than those at the beginning. Though there is increased jitter in the pose estimates near the end of the track, the computed track does not exhibit large accumulated error even though the globally optimized pose estimates are not
Figure 5.4: Volumetric reconstruction of the source distribution for the measurement described in Fig. 5.2. The reconstructed distributions corresponding to each of the ROI’s are shown according to the color scheme in table 5.1. The scatter axes of the Compton events are shown in their corresponding colors along the track, which is rendered in white.

used (see section 4.5). This result further motivates the study of the impact frame-to-frame pose error on volumetric imaging in section 6.4.2.

The track also illustrates the close proximity with which HEMI was brought to each of the sources during the measurement. This is significant for several reasons, some beneficial and some detrimental to the localization. On the one hand, HEMI was brought so near several of the sources that the assumption of far-field imaging may have been violated at several locations where the signal from the point sources is highest. This may have led to loss of precision in the reconstruction, as could potentially be the case with the $^{22}$Na reconstruction. On the other hand, note the density of arrows indicating Compton scatter axes when HEMI is near the point sources along the track. The increased countrate at certain locations along the track coarsely localizes the sources. The application of proximity-based localization techniques are beyond the scope of the present work, though this measurement clearly indicates the potential for the application of such techniques with a hand-portable
system. Proximity-based localization would have several advantages including improved efficiency (eliminating the coincidence requirement for Compton events) and the ability to employ non-imaging radiation detectors. A discussion of different localization approaches would necessarily be extensive and is not included in the present work; though the application of proximity-based localization with SDF systems is a compelling subject for further research.

5.2.2 New Measurement Environment

Volumetric reconstruction with HEMI was demonstrated in the same laboratory environment as the VCI measurements in section 5.2.1. Though there were significant differences with the VCI measurements, most notably the longer, circular path through the environment and the proximity to the gamma-ray sources, volumetric imaging has yet to be demonstrated outside of this specific laboratory environment. RGB-D Slam has been shown to work in several different environments [55], so it is expected that the tracking will be relatively robust any indoor scene with adequate visual features. A new environment would be expected to have a not only a different visual and geometric makeup, but a different gamma-ray background as well. Thus the demonstration of SDF in multiple environments qualitatively supports the assertion that the approach is robust to differences in the scene and gamma-ray background.

HEMI was moved to a different indoor laboratory, and another measurement with multiple point sources with different gamma-ray energies was conducted. Fig. 5.6 shows a 3D model of the environment, as well as the location of three gamma-ray point sources placed in the scene. The most significant feature of the new environment in terms of gamma-ray back-
Table 5.2: Summary of gamma-ray point sources used in the measurement described in section 5.2.2, as well as the user-selected ROIs and the number of potential Compton events in each ROI.

<table>
<thead>
<tr>
<th>Nuclide</th>
<th>Activity (µCi)</th>
<th>ROI (keV)</th>
<th>Compton Events</th>
<th>Reconstruction Color</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{133}\text{Ba}$</td>
<td>10</td>
<td>346 – 366</td>
<td>222</td>
<td>Red</td>
</tr>
<tr>
<td>$^{22}\text{Na}$</td>
<td>10</td>
<td>501 – 521</td>
<td>222</td>
<td>Green</td>
</tr>
<tr>
<td>$^{137}\text{Cs}$</td>
<td>10</td>
<td>647 – 677</td>
<td>121</td>
<td>Orange</td>
</tr>
</tbody>
</table>

The total time for the measurement was about 57 seconds, yielding an energy spectrum of coincident, potential Compton events given in Fig. 5.7. The 3D model is used to constrain the reconstruction to the occupied voxels. The result of the volumetric image reconstruction along with a rendering of the track taken through the environment is given in Fig. 5.8. Note that the $^{133}\text{Ba}$ source is taped behind the cabinet door. In order to see the reconstructed...
Figure 5.7: Spectrum of coincident, two-site events acquired over the course of the 57 second measurement.

Energy Spectrum - Coincident Events
Measurement Time = 56.5 s

The locations of both the $^{133}$Ba and the $^{22}$Na point sources are accurately reconstructed. The reconstructed location of the $^{137}$Cs point source is offset in the z-direction from the true source location, similar to the offset seen in the $^{54}$Mn reconstruction from the measurement in section 5.2.1. The cause of the spatial offset is unknown, though the reconstructed location is still within two voxel-widths of the true source location. The reconstruction at 356 keV corresponding to the $^{133}$Ba source is interesting for several reasons. There is increased noise in the image relative to the other two distributions, likely due to the poor signal-to-background in this energy region as evidenced by the spectrum in Fig. 5.7. Despite the increased image noise, the reconstructed source location is accurate, though this is not immediately obvious in Fig. 5.8 due to the density of the point cloud. It is important to note that the real-time software includes the ability to manipulate the 3D images by panning, rotating, zooming, etc. Thus a user would be able to see the true source location by interacting with the 3D...
The results from these measurements demonstrate volumetric point source localization ability commensurate with the VCI system. The first measurement emphasizes the differences in instrumentation and system mobility. The second measurement demonstrates results from a completely different environment with a different visual and geometric constitution, as well as a different gamma-ray background. The results, along with those given in the remainder of the chapter, provide evidence of the generality of the SDF method for volumetric imaging with SDF to the 3D localization of sources within objects for which there is no visual line-of-sight. This application is explored further for more challenging scenarios in sections 5.3 and 5.4.2.
imaging with portable gamma-ray imagers. Extensions of SDF-based volumetric imaging to even more challenging scenarios including outdoor environments and distributed gamma-ray sources can be found in [34].

5.3 Object Interrogation

Object interrogation is an example of a directed-search scenario that would benefit from real-time volumetric gamma-ray imaging. In this scenario, there is an a priori indication of the presence of a gamma-ray source somewhere within a specific container or set of containers. One may then be interested in the 3D location of the gamma-ray source within the container without having to open and manually inspect it. An example of a container that is of interest to the nuclear security community is a ship-borne cargo container [93]. Unfortunately, a measurement setup that included an indoor cargo container was not available for this work.

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2 The indication may be given by spectroscopic or event rate information from the mobile imager itself.
The principle can be demonstrated nonetheless, albeit at a smaller scale, with more readily available containers. One aspect that makes the object interrogation scenario particularly challenging is the unsuitability of the point cloud constraint used in many of the other volumetric measurements. Recall that the simple occupancy constraint presented thus far is based on the assumption that gamma-ray sources are distributed on or near the surface of the objects in the 3D model. Thus the imaging space is not constrained by the 3D model even after all the data has been acquired, presenting a more challenging reconstruction scenario. Other constraints can be applied that are well-suited to object interrogation: the selective voxelization of the volume within the container of interest, for instance. The development of this capability in real-time is not investigated in this work, though it is a compelling research topic for this application of volumetric imaging.

As a demonstration of the object interrogation scenario, a cardboard box approximately 2 ft × 3 ft × 3 ft in dimension is used to house a radioactive source. A 10 µCi $^{22}$Na source is placed at the center of the box, as shown in Fig. 5.10. The source is then covered with a foam cutout (similar to the base of the box seen in Fig. 5.10). The box is closed and placed with one face backed up against a table, as shown in Fig. 5.11. This is done intentionally to limit the area from which the box can be viewed, a more challenging scenario than if a full 360° track around the box were possible.

The measurement begins with HEMI on one side of the box, and a roughly semi-elliptical path is taken around it, ending up 180° from the starting location. The path around the box and the resulting model of the scene are illustrated in Fig. 5.12. The total measurement time was 67 seconds, resulting in the collection of 95 Compton cones in an ROI 10 keV wide around 511 keV. The location of the point source is correctly recovered in Fig. 5.12, though it is difficult to verify from this perspective, which is intended to highlight the track and model. Fig. 5.13 gives the volumetric reconstruction with the view zoomed into the
Figure 5.12: Illustration of the track taken and resulting 3D model acquired during the 67 second object interrogation measurement during which 95 Compton events in the ROI around 511 keV were acquired. The volumetric reconstruction is included, though the portion that extends into the box is not visible from this view due to the density of the point cloud model from this perspective.

box. The location of the $^{22}$Na is properly reconstructed by both the green isosurfaces, and the small green cross which indicates the maximum of the reconstructed image intensity in Fig. 5.13. This measurement demonstrates the feasibility of applying hand-held gamma-ray imaging systems to object interrogation via SDF, even without any constraint on the imaging space. The development of real-time 3D object identification and dynamic voxellization would be expected to further improve localization capabilities for this application. The object interrogation scenario is further investigated in section 5.4.2, where SDF is applied to detect and localize hidden SNM.
5.4 IAEA Gamma-ray Imaging Workshop

In October 2015, HEMI participated in a technology demonstration workshop hosted by the International Atomic Energy Agency (IAEA). The purpose of the workshop was to evaluate gamma-ray imaging technologies in the context of user-scenarios of interest to the IAEA. The scheduled measurements were primarily designed to evaluate conventional static gamma-ray imagers, though some time was set aside for mobile measurements to demonstrate the new technology. The results of these mobile measurements are presented here, and can be divided into two categories: general radiation search and mapping applications, and object interrogation. The mobile measurements taken with HEMI at the demonstration workshop are interesting for several reasons. First, they demonstrate the deployment of the prototype HEMI system in an entirely unknown environment, with a completely different geometric and visual makeup than those previously studied. Second, the demonstration workshop included measurements with gamma-ray sources that are not available in our laboratory, including
$^{239}$Pu, allowing the demonstration of volumetric imaging in a context more closely aligned with nuclear security and safeguards.

The measurement area was an indoor laboratory space, approximately 4m wide, 3m high, and 10m long. At one end of the room, a gantry was erected to house the static imaging systems, shown in Fig. 5.14. The gantry faced a gridded poster board used to position the gamma-ray sources for many of the measurements. The poster board as viewed from the upper-right portion of the gantry is shown in Fig. 5.15. The mobile measurements were all taken in the area behind the poster board; 3D models of this area of the lab are included in the volumetric imaging results.

Figure 5.14: Gantry that occupied roughly one third of the available measurement space in the laboratory. Several static imagers (not shown) were mounted here during the workshop.

Figure 5.15: The poster board used to position the gamma-ray sources for many of the static measurements. In this image, the poster board is viewed from a location in the upper-right corner of the gantry, looking towards the back of the room. The volumetric measurements took place in the area of the room behind this poster board.
5.4.1 General Search and Mapping Scenarios

This section presents results from measurements generally describes as applications of volumetric gamma-ray imaging to gamma-ray search and mapping scenarios, similar to many of the measurements in chapter 4 and the first sections of this chapter. These include applications where it is of interest to localize gamma-ray sources in 3D in real-time, a potential fit for some inspection tasks relevant to the IAEA. The results from three different measurements are presented, each distinguished from the types of measurement scenarios previously investigated. The localization of a shielded point source is investigated in the measurement in section 5.4.1.1. Section 5.4.1.2 investigates SDF in the presence of a gamma-ray point source several orders of magnitude more active than previously measured sources. The results from two final measurements of a complex gamma-ray source distribution including over a dozen $^{137}$Cs and $^{60}$Co point sources is presented in section 5.4.1.3.

5.4.1.1 Shielded Point Source Measurement

![Figure 5.16](image1)

**Figure 5.16:** HEMI in operation during the first mobile measurement of the IAEA workshop. The location of a shielded $^{137}$Cs source is indicated by the red arrow. Several SNM sources containing $^{235}$U were also present, but not imaged in the Compton mode (see footnote).

![Figure 5.17](image2)

**Figure 5.17:** The 3D model of the scene acquired during the mobile measurement.

Fig. 5.16 shows HEMI in operation during the first mobile measurement of the workshop. The corresponding 3D model of the scene acquired during the measurement is shown in Fig. 5.17. In this measurement, a shielded 12 µCi $^{137}$Cs source was placed on the second shelf of a cart in the middle of the room, with several SNM sources containing $^{235}$U on the top level.
Starting from the front of the poster board, HEMI is carried along an arcing path towards the area of the room behind the poster board. The $^{137}$Cs source container is the small red box which is visible in the corner of the cart in Fig. 5.16. The source is covered by about 0.25 in of lead on the top of the container. The measurement time for this run was 52 seconds, yielding a total of 57 Compton cones in the ROI from 652 - 672 keV. The image space is constrained by the point cloud model during the reconstruction. Note that this is the fewest number of potential Compton events acquired in any measurement presented thus far due to the lead shielding the source.

Despite the limited number of Compton events, the reconstructed image has relatively high intensity in the region corresponding to the true source location in Fig. 5.18. The number of contour surfaces used to represent the volumetric image in Fig. 5.18 is increased by a factor of 10 compared to other images to better capture spatial nuances in the distribution. There is significant noise in the volumetric gamma-ray image, largely concentrated near the edge of the scene model (the hotspot in the upper-right corner of the image). Features of this type, i.e., high-intensity regions concentrated near the edge of the scene are not uncommon, resulting from cones that do not significantly intersect the constrained imaging space. The 3 keV gamma-ray from the decay of $^{235}$U is too low in energy to be efficiently imaged with HEMI via Compton imaging. A volumetric coded-aperture modality which would be capable of imaging in this energy regime is beyond the scope of this work.

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3 The 186 keV gamma-ray from the decay of $^{235}$U is too low in energy to be efficiently imaged with HEMI via Compton imaging. A volumetric coded-aperture modality which would be capable of imaging in this energy regime is beyond the scope of this work.
noisy image further motivates an investigation of the accuracy of the volumetric localization as a function of the number of Compton events, undertaken in chapter 6. Nevertheless, this measurement demonstrates some of the advantages of SDF even in the relatively challenging scenario of a shielded source: real-time 3D localization within very short measurement times; potentially relevant for search and inspection tasks of interest to the IAEA and other similarly-tasked organizations.

5.4.1.2 Volumetric Imaging of High Activity Point Source

The second mobile measurement from the gamma-ray imaging workshop demonstrates the efficacy of volumetric imaging in the presence of more intense gamma-ray fluxes. All of the previous measurements in this work tend toward the acquisition of very few Compton events. This measurement explores the opposite end of the intensity range, evaluating SDF with very high Compton event rates in the presence of a strong gamma-ray point source. For this measurement, a 1.6 mCi $^{60}$Co source is positioned near the center of the room. The poster board seen in Fig. 5.15 is moved aside to provide more space for movement in the lab. The source is partially withdrawn from the lead cask in which it is transported, with the top portion of the source fully exposed. During a 50 second long measurement, HEMI is carried along a roughly linear path past the source. The energy spectrum of coincident, double-interaction gamma-ray events is given in Fig. 5.19. Peaks corresponding to the the 1173 keV and 1332.5 keV lines emitted from the decay of $^{60}$Co are clearly visible. The reconstructed gamma-ray source distributions for each of the ROI's indicated in Fig. 5.19 are shown in Fig. 5.20 and 5.21. The volumetric gamma-ray distribution is computed for each of the two characteristic gamma-rays simultaneously and independently. The agreement between the reconstructed source distributions provides strong evidence of the robustness of the localization for this measurement. Note the density of arrows corresponding to the scatter axes of Compton events along the track of the measurement. This $^{60}$Co source is orders-of-magnitude more active than the sources used in all the measurements presented thus far, with 1650 coincident events in the ROI around 1173 keV, and 1128 such events in the ROI around 1333 keV. This measurement scenario may not have many practical analogs in nuclear security, though operating in fields of high gamma-ray intensity is certainly relevant for some applications, such as recovering lost industrial or medical gamma-ray sources or operating in a Fukushima-like environment. More importantly, the results demonstrate that feasibility of SDF both for environments with high gamma-ray intensity, as well as at gamma-ray energies above 1 MeV.

5.4.1.3 Complex Gamma-ray Source Distribution

During the second day of the workshop, a measurement scenario was set up where systems with mobile capability were given the opportunity to investigate a more complicated source distribution. This measurement included both point gamma-ray sources as well as fresh plates of LEU fuel. The 186 keV gamma-ray from the decay of $^{235}$U in the fuel plates is not
Figure 5.19: Spectrum of coincident, potential Compton events collected during the 50 second volumetric measurement of a 1.6 mCi $^{60}$Co source. The user-selected ROI’s for Compton imaging are given in blue and red.

Imaged in this measurement due to the poor Compton imaging efficiency in HEMI in this energy range. Instead, it is the multiple point sources emitting higher energy gamma-rays that are investigated with via volumetric Compton imaging. These consisted of 11 $^{137}$Cs point sources and 6 $^{60}$Co point sources of varying, unspecified activity; though the total activity of these sources is much less than the strong $^{60}$Co source from the previous measurement. Fig. 5.22 and 5.23 show the true distribution of the sources, highlighting the locations of the individual point sources on the poster board. The 11 point sources of $^{137}$Cs are distributed on the top half of the poster board, and the six $^{60}$Co sources distributed along the bottom half of the poster board, including the lower left and right corners. There are also four point sources above the poster board in the upper-right corner, which include a $^{137}$Cs point source. Two individual mobile measurements were taken with this source distribution. Given the number of sources and their complex spatial distribution, a single longer measurement would
have been preferable in an attempt to spatially separate the sources, but was not possible given the constraints of the workshop.

The first measurement lasted for one minute, resulting in the energy spectrum displayed in Fig. 5.24. 243 potential Compton events were collected in the ROI around 662 keV and 186 potential Compton events in the ROI around 1173 keV during the 60 second measurement.

The results of the volumetric gamma-ray image reconstruction are shown in Fig. 5.25. The ROI around 662 keV corresponding to $^{137}$Cs yields an image spread horizontally along the top of the poster board, commensurate with the location of the majority of the $^{137}$Cs sources. The $^{60}$Co image shows concentrated intensity along the bottom half of the poster board, also in agreement with the true location of the $^{60}$Co point sources. The individual point sources are not resolved, though this is expected given the limitations on measurement time and the detector track coupled with the angular resolution of HEMI. The measurement emphasizes the advantage of a mobile imager able to overcome the $\frac{1}{r^2}$ effect to increase sensitivity relative to a static imager, resulting in the accurate reconstruction of coarse features of the distribution in a very short measurement.

The results from the second measurement with this source distribution are shown in Fig. 5.26 and 5.27. A different path is taken in this measurement, which was slightly longer at 82 seconds. Again, the coarse features of the gamma-ray source distribution are generally captured in the reconstruction. The orange isosurfaces corresponding to the reconstruction in the ROI around 662 keV is again distributed along the top half of the poster board, as expected. The result is in good agreement with the actual distribution of $^{137}$Cs point sources, especially given the point sources above the poster board (see Fig. 5.22). The image corresponding to the $^{60}$Co distribution is again concentrated on the lower half of the
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Figure 5.22: Illustration of gamma-ray source distribution for the measurement described in section 5.4.1.3. The location of the $^{137}$Cs point sources are indicated on the poster board. There is an additional $^{137}$Cs point source above the gridded part of the poster board.

Figure 5.23: The same source setup described in Fig. 5.22 but with the locations of the $^{60}$Co point sources highlighted instead.

poster board. One of the individual $^{60}$Co is successfully resolved in the lower right corner, though the remaining 5 sources are not individually resolved.

The results of this second measurement are presented here for several reasons. For one, the rough agreement between the reconstruction from independent measurements provides qualitative evidence of the robustness of the reconstruction. More importantly, the results from these particular measurements illustrate one of the common error modes of the SLAM tracking: a discontinuity in the scene model is clearly visible in Fig. 5.27. This is an example of failed loop-closure [92], a manifestation of accumulated error in the pose estimate over the course of a measurement. Accumulated error is common to many tracking algorithms as a result of sequential pose estimation. Overcoming this particular tracking error mode is critical to improving the robustness of SDF and is being actively researched [93, 94]. Longer measurements with this source setup would have been preferable in order to evaluate the limits of HEMI’s volumetric imaging resolution in a distributed gamma-ray source scenario. Nevertheless, the results demonstrate the ability to reconstruct the coarse features of a complex gamma-ray source distribution in 3D with very short measurement times.
Figure 5.24: Spectrum of coincident, potential Compton events collected during the 60 second volumetric measurement around the source distribution for this measurement. The user-selected ROI’s for Compton imaging are given in orange and blue.

5.4.2 Object Interrogation Scenario

On the last day of the technology demonstration workshop, there was a brief open period where the opportunity arose to take measurements that were not explicitly specified in the workshop agenda. The chosen scenario is an extension of that introduced in section 5.3 with a more complex container geometry housing special nuclear material (SNM). This measurement is of particular interest for nuclear security and safeguards applications due to the 3D localization based on the gamma-ray emissions from $^{239}\text{Pu}$.

Fig. 5.28 shows the basic setup for this measurement. Several containers of various size were stacked together to simulate a pile of cargo. Half of the room was cleared of obstructions so a partial path was possible around the stack of containers. Two gamma-ray point sources of unknown activity are placed in the scene: an unshielded, high-burnup $^{239}\text{Pu}$ and a $^{137}\text{Cs}$ source. The $^{137}\text{Cs}$ was placed in the upper-right corner of the box furthest to the rear of
the stack. The $^{239}$Pu source was placed in the upper left corner (approximately 4 inches from each edge) of the left-most box in the stack. Fig. 5.28 shows the approximate location of both of the sources. This particular $^{239}$Pu sample contained a large amount of $^{241}$Am, providing a very high background flux of 60 keV gamma-rays. Fig. 5.29 shows the spectrum of single gamma-ray interactions (i.e., no time coincidence window applied) in HEMI with the source configuration given in Fig. 5.28. This spectrum was measured over the course of a measurement lasting 42 minutes and 25 seconds, of which HEMI was employed in the mobile mode for about two minutes with the $^{239}$Pu source present. The remainder of the time was spent in static deployment no nearer than 1.5 m to the $^{239}$Pu source. The 60 keV peak from $^{241}$Am is the most prominent feature of the spectrum, with an average count rate of nearly 2400 counts/second in the ROI from 50 to 70 keV. The high flux of 60 keV gamma-rays gives rise to other spectral features, such as a visible peak near 720 keV due to random coincidence between a 662 keV and a 60 keV gamma-ray within a single detector element. Despite the strong background component from the 60 keV gamma-ray flux and the very small intensities of the 375 keV and 414 keV gamma-rays from $^{239}$Pu decay [97], the presence of $^{239}$Pu can
Figure 5.26: Energy spectrum of coincident, two-site interaction gamma-ray events resulting from the 82 second measurement. There are 163 potential events in the ROI around 662 keV, and 105 cones in the ROI around 1173 keV.

still be identified using spectroscopic analysis techniques [98], the results of which are shown in Fig. 5.30.

The mobile portion of the measurement consisted of a single volumetric imaging run. The measurement time was 130 seconds netting 260 Compton events in the ROI around 414 keV corresponding to $^{239}$Pu and 292 Compton events in the ROI around 662 keV. The data from each ROI are passed to two independent volumetric imaging threads which reconstruct the two source distributions. The results from each reconstruction are plotted with a single scene model in Fig. 5.31, with the distribution corresponding to the 662 keV ROI in red and the distribution corresponding to the 414 keV ROI in green. The locations of the $^{137}$Cs and $^{239}$Pu point sources are correctly recovered in the volumetric reconstruction. The reconstruction is quite noisy as the 3D model is not used to constrain the imaging space so that the region inside the containers will be voxelized. The the contours of the volumetric source distribution are represented by 20 isosurfaces evenly scaling up with image intensity to capture low-amplitude features in the reconstructed image.

Note that the spectrum shown in Fig. 5.29 and 5.30 is for non-coincident gamma-ray interactions, and taken over the course of a much longer measurement. This information is sufficient for the spectral identification of $^{239}$Pu, but additional event reconstruction is required for Compton imaging. The high flux of 60 keV gamma-rays manifests as an even more significant background once the time-coincidence requirement is applied. Fig. 5.32 gives the spectrum of double-interaction gamma-ray events. The peak at 662 keV is clearly visible,
but as in the case of single interaction events, the most prominent features of the spectrum are related to the high background of 60 keV gamma-rays. There is again a peak at 720 keV, though it is even more pronounced in this spectrum due to the coincidence requirement. The most prominent feature is a wide peak centered around 120 keV corresponding to random coincidence between two 60 keV gamma-rays. These features illustrate the magnitude of the background signal for double-interaction events due to false coincidence with the 60 keV gamma-rays. A closer look at the ROI around 414 keV in Fig. 5.34 displays no discernible peak. Yet the volumetric imaging was successful in correctly localizing the location of the $^{239}\text{Pu}$ point source by imaging events in this ROI. Conventional 2D gamma-ray imaging has been shown to improve sensitivity to gamma-ray sources due to the incoherence of non-source photons in the imaging space [99]. Thus the volumetric localization is still successful even in
To test the hypothesis that the photopeak is suppressed due to the high false-coincidence rate, an energy filter is applied to discard all events where the energy deposited in one of the interactions is in the range from 50 - 70 keV. The effect of this filter can be clearly seen in the energy spectrum in Fig. 5.35 compared to the unfiltered data. The net count rate in the 414 keV ROI increases by a factor of three, despite an net decrease in double-interaction events due to the filter, illustrating the rejection of the background signal from false coincidence. The effect of the energy filter on the volumetric imaging is shown in Fig. 5.33. The parameters used for the volumetric reconstruction are identical to those used earlier. The false-coincidence filter results in a reduction of image noise and a more tightly concentrated high-intensity region corresponding to the point source. These results confirm the hypothesis that the false-coincidence background does significantly hinder the
ability to identify the photopeak in the energy spectrum of double-interaction gamma-ray events. The application of volumetric imaging improves sensitivity to the presence of the $^{239}\text{Pu}$ source, even when the spectroscopic signal is suppressed. It would be incorrect to claim that the volumetric imaging offers improved detection sensitivity over spectroscopic analysis as a full-fledged comparison to dedicated spectral detection methods is not undertaken here. The results do however illustrate the power of volumetric imaging as the $^{239}\text{Pu}$ point source was accurately localized in 3D within minutes even in the presence of a strong, uncorrelated background.

Figure 5.30: The region of interest of the spectrum in Fig. 5.29 corresponding to the expected spectral lines from $^{239}\text{Pu}$ at 375 keV and 414 keV.
Figure 5.31: Volumetric image reconstruction results from a 130 second mobile measurement with both a $^{137}$Cs and high-burnup $^{239}$Pu source present, as shown in Fig. 5.28. The Compton cone axes corresponding to both the ROIs around 414 keV and 662 keV are given by the blue arrows, while the detector path is indicated in red. The volumetric images corresponding to the 414 keV and 662 keV ROIs are represented by the green and red contour surfaces, respectively. 15 contour surfaces have been used in this image to represent the distributions as opposed to the fewer surfaces which have been used in the preceding volumetric image representations. The contours dispersed in space away from the container stack illustrate low-intensity image noise as the reconstruction is not constrained to the 3D model.
Figure 5.32: Spectrum of double-interaction events resulting from the measurement described in section 5.4.2.
Figure 5.33: Reconstructed volumetric gamma-ray image from double-interaction events in the ROI around 414 keV. The left image shows the volumetric reconstruction with the original data (a replication of the result in Fig. 5.31 excluding the 662 keV reconstruction). The image on the right was produced using filtered data as described in the text. The number of overall cones is reduced from 260 to 195 by the filter, but the image noise is greatly reduced.

Figure 5.34: The coincidence spectrum zoomed to show the ROI around 414 keV. The expected peak does not deviate significantly from the background in this region.

Figure 5.35: The coincidence spectrum in the same ROI after the event filter has been applied. The expected peak at 414 keV is now visible as a significant deviation from background.
5.5 Discussion

The results presented in this chapter illustrate the efficacy of the SDF approach for real-time volumetric imaging with HEMI. SDF is successfully demonstrated for a variety of imaging scenarios with a more mobile gamma-ray imager. The measurements in sections 5.1, 5.2.1 and 5.2.2 replicate capabilities previously demonstrated with the VCI; namely the ability to accurately localize multiple gamma-ray point sources emitting gamma-rays of various different energies in real-time. Though these measurement scenarios mirror those of chapter 4, the results are subject to several key differences. Different imaging environments were investigated to demonstrate the independence of SDF from the visual and geometric constitution of any specific scene. The hand-held nature of HEMI allows the user to get much nearer to gamma-ray point sources, a demonstration of one of the major advantages of a mobile imaging approach. The real-time data analysis provides feedback not only in the imaging domain, but also spectroscopic information, with coarse localization information encoded in the count rate in spectroscopic ROIs along the track. This feedback can be a powerful tool for identifying high-signal regions during the measurement. The extension of this information to a proximity-based localization method warrants further research.

The improved mobility confers new challenges in addition to these benefits. The hand-held system achieves much higher rotational and translation velocities compared to the VCI, and the simple filters for poor pose estimates used with the VCI can no longer be applied. This manifests in greater uncertainty in the pose estimates via several mechanisms. In section 5.2.1, greater uncertainty in the poses is visible in the portion of the track at a greater distance from the visual features in the environment. A second pose uncertainty mechanism is visible in section 5.4.1.3, where accumulated pose error results in a failed loop-closure in the model. The results of these measurements motivate the study of the two classes of pose uncertainty on volumetric imaging in chapter 6.

Finally, a set of measurements from a workshop hosted by the IAEA demonstrate the deployment of the prototype HEMI system in a completely new and unknown environment. These measurements further demonstrate the ability to localize gamma-ray sources in 3D, for both strong and weak or shielded sources with very limited measurement times, on the order of 30-120 seconds. It should be noted that the volumetric mode doesn’t require short measurement times — but measurement times allocated for mobile operation were quite limited, as the workshop was primarily dedicated to static gamma-ray imagers. The application of volumetric gamma-ray imaging to object interrogation has been discussed several times throughout the chapter. The final object interrogation measurement is of special significance due to the demonstration of 3D localization in the presence of gamma-ray background, and is particularly relevant for nuclear security and safeguards given the 3D localization of SNM given only the gamma-ray signature from that material.
Chapter 6

Quantitative Evaluation of Scene Data Fusion

The previous chapters have demonstrated volumetric gamma-ray image reconstruction via the SDF method for a variety of scenarios. A host of inputs have been qualitatively investigated, spanning ranges in incident gamma-ray energies and activities; different numbers and distributions of gamma-ray point sources; scenes with varying geometries and visual makeup; and different gamma-ray imaging systems. The results of the demonstration measurements convey a convincing argument that SDF is viable for potential applications involving 3D mapping of gamma-ray sources. Several advantages and limitations of the SDF method have been identified in the measurements presented in chapters 4 and 5. For example, one demonstrated benefit is the ability to localize gamma-ray point sources in 3D having collected relatively few Compton events; fewer than 100 in many cases. The ability to accurately localize gamma-ray sources given very low signal is advantageous when searching for weak or shielded gamma-ray sources. Several results from HEMI measurements further suggest the advantages of a truly mobile system; overcoming the effect of geometric attenuation and the additional information from varying proximity to the source (see section 5.2.1).

This chapter presents a more quantitative evaluation of the volumetric Compton image reconstruction against variations in the input data. A series of measurements is taken with each instrument to quantify the reliability of the gamma-ray localization as a function of the number of acquired Compton events, both in terms of accuracy and repeatability. One limitation that has been identified in the presented measurements, particularly in section 5.4.1.3, is the impact of the uncertainty in the pose estimate on the gamma-ray localization. The tracking method is subject to several different modes of uncertainty, and it is important to understand their impact on the 3D localization. The effects of just one component of the pose uncertainty, the translational estimation, are studied in section 6.4. Another source of uncertainty has been identified in the context of 2D Compton imaging: the uncertainty associated with the Compton cone opening angle. The impact of the “Compton data quality”, i.e., the relative uncertainty in the opening angle of the Compton cones used in the image reconstruction, is also investigated in the volumetric domain.
The SDF approach relies on many more data streams than static imaging approaches, and the dependencies are quite complicated. The pose estimates and scene model are both derived from the properties of the scene being viewed, and it is difficult to draw general conclusions about SDF for all imaging scenarios based only on properties of the scene. An extensive evaluation of this sort would require a vast number of measurements, and could still not be generalized beyond the specific SLAM algorithm used in this work. It is also very difficult to isolate specific variations in the input data input, such as count rate or pose uncertainty, without also constraining or biasing the data. Ground-truth measurements with external validation of system pose, as well as geometric simulation, are used in an effort to isolate the effects of variation in each of these dependencies. The tracking, model, and gamma-ray image reconstruction also depend on many configurable parameters; mapping out this parameter space is beyond the scope of this work as the results would be specific to the individual measurement as opposed to SDF in general. Instead, the goal here is to conduct a preliminary investigation of effects that are expected to impact mobile Compton imagers, and illustrate methods for evaluating their impact on volumetric localization.

6.1 Quantifying Volumetric Imaging Performance

Quantitative evaluation necessitates selecting appropriate metrics by which the volumetric images can be evaluated. There are several approaches on how the quantification problem can be framed. One is to abstract the localization to a binary problem; classifying a reconstruction as “successful” based on a comparison of the location of the maximum or mean intensity of the reconstructed image to the known true source location. This analysis provides a straightforward way to analyze the 3D localization but has several drawbacks. Image interpretation is complicated and simply extracting low-order moments in the reconstructed image intensity is insufficient to capture the information conveyed by the image. The dependence on a single parameter extracted from the full volumetric image is susceptible to bias. Take Fig. 6.1 for instance, repeated here from section 5.4.1.1 for convenience. The voxel containing the highest image intensity in the foreground of the image does not correspond to the true source location. According to the definition of “localization success” given above, this reconstruction would be classified as a failure, despite the aggregation of image intensity in the region corresponding to the true source location. Artifacts of the type seen in Fig. 6.1 are not uncommon when the image space constraint is used, with voxels near the edge of a scene model showing high intensity due to Compton cones that minimally intersect the imaging space. Thus the treatment of localization as a binary problem is susceptible to bias due to image artifacts and noise resulting from operational considerations such as the completeness of the imaging space from the track taken or limitations of the Kinect sensor. Nevertheless, such an approach is used to evaluate the robustness of the 3D localization with respect to variation in the number of Compton cones acquired during a measurement.

Volumetric localization can also be evaluated in terms of accuracy and precision rather than in the context of robustness for which the binary approach is suitable. In the absense
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Figure 6.1: Volumetric gamma-ray image reconstruction of the measurement described in section 5.4.1.1. Only 57 potential Compton events were acquired during the measurement. The volumetric reconstruction is noisy, with a single voxel containing the maximum image intensity in the foreground of the image near the edge of the model. Despite this image artifact, there is still significant intensity from the reconstructed image concentrated in the region corresponding to the true source location.

of imaging noise, the accuracy and precision of the localization can be described by evaluating reconstructed distributions corresponding to point sources of gamma-rays. The distance between the reconstructed source location and the true, known location of the source measures the accuracy of the reconstruction. The precision is quantified in terms of the width of the reconstructed point distribution along each spatial dimension. These metrics are used in conjunction with ground-truth measurements and volumetric simulation to evaluate the effects of Compton event rate, uncertainty in the cone opening angle, and translational pose uncertainty on volumetric gamma-ray source localization.

Volumetric images can also be assessed in terms of more conventional metrics of image quality: image resolution, signal-to-noise ratio (SNR), and image contrast [100]. The angular resolution for the imaging systems has been previously reported in section 3.5.2. Traditionally, image resolution is an intrinsic property of the imager, but dependencies on the measurement environment through the pose estimates make it difficult to decouple the image resolution from the imaging environment. Another method for quantifying the resolution of an imaging system is to measure the minimum distance by which two sources can be distinctly separated in the image space. There are several issues associated with the current volumetric reconstruction algorithm that complicate this procedure. If the scene
model is being incorporated into the image reconstruction, the reconstruction will depend on the geometry of the scene itself, as is clearly demonstrated in Fig. 4.12. Even without the scene model, the reconstruction of the source distribution in the depth dimension depends on the detector track. These dependencies make it very difficult to measure the system resolution via this method as well. Given these considerations, the spatial resolution is instead characterized in terms of the width of the reconstruction distribution in $\mathbb{R}^3$, analogous to the methods used to assess angular resolution for conventional 2D imagers. Though the reconstructed distribution itself depends on more than the intrinsic properties of the imager, this evaluation at least provides a quantitative measure of spatial resolution.

Image contrast quantifies the difference between image characteristics in terms of intensity or luminance, and is another metric by which images are conventionally evaluated. The measurements in this work are taken with point sources; there are no measurements in the presence of gradients of source activity or coherent gamma-ray background to provide non point-like features in the image. Thus the only features to be distinguished in the image are the reconstructed point sources and the only additional source of variation is from image noise. In this regime, the image contrast is determined by the ratio of the feature intensity to the variation in the image background; which is also described by the image signal-to-noise ratio (SNR). The volumetric images in this work are therefore evaluated in terms of SNR, which also describes the contrast of the images of point sources. Image contrast is an important metric for applications with distributed gamma-ray sources, such as nuclear contamination remediation. Reconstructed volumetric images of gamma-ray point sources are evaluated in terms of SNR by comparing the reconstructed image intensity in a region around the known location of a gamma-ray source to the variability of the image intensity outside of the source region, as given in equation 6.1:

$$SNR = \frac{\mu_{source}}{\sigma_{background}}$$  \hspace{1cm} (6.1)

The $\mu_{source}$ term represents the mean signal from the known source region, where the $\sigma_{background}$ represents the root-mean-square (RMS) variability of the reconstructed intensity in voxels outside this region. The definition of SNR in equation 6.1 is an extension of the metric often applied to evaluate biomedical imaging modalities [100]. The SNR metric can also be coupled to the concept of image interpretability by such thresholds as the Rose criterion [101]. Note that the computation of the SNR requires the source region to be known, necessitating either ground-truth measurements or simulations studies where the true source location is externally measured within some global coordinate frame. Evaluating the volumetric images in this way reduces the effects of systematic biases and image artifacts inherent in the binary quantification approach.

The metrics introduced here are employed throughout this chapter to quantify the volumetric imaging performance with reference to uncertainties in various inputs. The binary approach evaluating the “success” of volumetric image reconstruction is used to evaluate the robustness of the localization as a function of number of Compton events. The volumetric reconstructions produced with varying number of Compton events are also analyzed in terms
of image SNR. The accuracy and spatial resolution of the 3D localization are evaluated using simulated Compton data in an attempt to isolate the effect of various forms of translational pose uncertainty. Finally, the dependence of the accuracy and precision of the 3D localization on uncertainty in the Compton cone is also studied here to inform further development of volumetric Compton imaging.

6.2 Volumetric Imaging Vs. Number of Compton Events

One of the primary results from the demonstrations in chapters 4 and 5 is the capability to accurately localize gamma-ray sources with relatively few measured Compton events. This makes the approach viable when searching for weak or shielded sources, and allows the SDF approach to be implemented with existing mobile gamma-ray imagers without requiring significant improvements in imaging efficiency over existing systems. Yet each of the presented reconstructions resulted from a single measurement, with no quantitative indication of the accuracy of the measurement, nor any information about the repeatability of the result. The reconstructions from measurements in chapters 4 and 5 more often than not successfully localized point sources; in practice, this is not always the case, especially when few Compton events are collected. In this section, a high-activity point source is measured and a downsampling procedure is applied to simulate the effect of taking many measurements under identical environmental conditions with the same detector track in the presence of a gamma-ray incident flux of varying intensity. The data are used to evaluate the accuracy and robustness of the 3D localization, where robustness refers to the probability of correctly localizing the point source in 3D.

The location of the strong point source is measured via external (i.e non-imaging) methods, along with the pose of the imager (i.e., the pose estimates from SLAM are not used) to eliminate the effects of pose uncertainty on the image reconstruction. Similarly, the model of the scene is not incorporated into the reconstruction to acquire an unbiased quantitative measure for imaging accuracy that does not explicitly depend on the scene model. A $^{137}\text{Cs}$ source with an activity of 832 $\mu\text{Ci}$ and is measured from various locations along a linear track to ensure ample Compton signal is acquired during the measurement for the subsequent downsampling procedure. The measured Compton events are then downscaled along the track while preserving the statistics of the acquisition to simulate an identical measurement of a $^{137}\text{Cs}$ point source of weaker activity. A ground-truth measurement is taken with each detector system to evaluate the performance of each imager.

6.2.1 Measurement Setup and Validation

Each imager (both HEMI and VCI in separate measurements) is moved along a linear path in front of a 832 $\mu\text{Ci}$ $^{137}\text{Cs}$ source, measuring for 30 seconds at eleven locations at roughly even intervals along the track. HEMI is placed on a cart for this measurement to limit motion
in all but the horizontal direction and aid in the external measurement of the pose of the imager (the VCI is already on a cart). The orientation, height, and depth (i.e., perpendicular distance to the gamma-ray source) of the imager is held constant during the measurement within the limits of uncertainty in the ground-truth location determination. The location of the cart along the trajectory and the location of the point source were physically measured using tape measures and a floor-grid. The uncertainty in the measured orientation and location of the cart is estimated to be less than $1^\circ$ and 5 cm respectively.

Figure 6.2: Illustration of the measurement setup for the experiment described in section 6.2. The externally-measured detector poses are indicated by the white cubes, with the red track corresponding to the VCI and the green track corresponding to HEMI (two separate measurements with the same source configuration). The location of the 832 $\mu$Ci $^{137}$Cs source on top of the blue cart is indicated by the white cross-hair.

Fig. 6.2 illustrates the setup for the measurements. The measurement locations of each of the imagers are indicated by the white cubes, with the green track corresponding to the HEMI measurement and the red track corresponding to the VCI measurement. Though both tracks are plotted with the same model in Fig. 6.2, the measurements were taken at different times. The carts were placed at discrete positions along the same linear trajectory (albeit
at different points) in order to externally measure the pose of the instruments using the floor-grid. The cart on which HEMI was mounted on for this experiment was much smaller than the VCI cart, so a slightly longer trajectory was taken: the total track length was 3.35 m for the HEMI measurement and 2.97 m for the VCI measurement. The measurement points are evenly spaced at roughly 30 cm intervals, though the intervals for the VCI are less regular due to the difficulty of precision maneuvers with the heavy cart. The measurement time at each location was 30 seconds, resulting in a total live time of 5 minutes and 30 seconds evenly distributed along the track. Neither the positions of the detectors on their respective carts, nor the starting location of the two measurements could be matched. As a result, HEMI is about 25 cm closer to the source than the VCI. The source and scene geometry remained unchanged between the two measurements. The coordinate frames for each of the two measurements are defined by the locations of the first measurements, thus the expected source location is different for the two measurements. The source is located at $(2.08\text{m}, 1.66\text{m}, -0.36\text{m})$ in the VCI coordinate frame, and $(1.75\text{m}, 1.46\text{m}, 0.135\text{m})$ in the HEMI frame, where $x$ corresponds to depth, $y$ to the direction of cart motion, and $z$ the height of the detector (opposite gravity). Though the origins of the coordinate frames differ, the orientation of the two detectors is the same between the measurements.

2952 Compton cones were collected during the HEMI measurement, and 3257 Compton cones by the VCI. HEMI was previously shown (see section 3.5.2.2) to have an imaging efficiency roughly a factor of two greater than that of the VCI. In addition, the perpendicular distance between HEMI and the point source is about 25 cm less than that of the VCI. Given these facts, many more Compton events would be expected to be measured by HEMI than by the VCI. The disparity is explained by a cut that is applied to the HEMI data to remove Compton events with a lever-arm of less than 2.1 cm. This eliminates events comprising adjacent or diagonally-adjacent interactions, for which the uncertainty in the position of the interactions results in a very large uncertainty in the scatter axis of the Compton cone. This cut on the data has been empirically determined to vastly improve the robustness of the localization with HEMI, and is therefore applied here. The lever-arm filter can reduce the Compton event rate by nearly 50%, explaining much of the disparity in the number of collected events. No external cuts (beyond those implicit in the event reconstruction) are applied to the VCI data. The remainder of the disparity is explained by the width of the energy ROIs that are applied in selecting Compton events. Double-interaction events with total energy within $662 \pm 3 \text{ keV}$ were used in the VCI measurement, while those with total energy within $662 \pm 10 \text{ keV}$ were used in the HEMI measurement. The width of the energy gates represent different fractions of the total energy resolution of the respective detectors, thus a smaller fraction of potential Compton Events are considered from HEMI than from the VCI. The width of the energy gates is chosen to be consistent with the measurements in chapters 4 and 5.

The full set of Compton events collected from each measurement are reconstructed to validate the external measurements of the detector poses and source location. Fig. 6.3 shows the reconstructed source distributions using all of the Compton events from each of the two measurements. Note the good agreement between the externally measured source location
Figure 6.3: Volumetric image reconstruction using all 3257 Compton events collected during the VCI measurement (left) and 2952 Compton events collected during the HEMI measurement (right). The voxel size is 10 cm$^3$, consistent with previous measurements (and mandated by computational memory). The isosurfaces represent contours at 25%, 50%, and 75% of the total image intensity. The expected location of the point source is indicated by the red cross-hair, illustrating the good agreement between the expected and reconstructed location in each case. Numerical values for the reconstructed source location are given in Table 6.1.

(represented by the red cross-hair in the images) and the reconstructed source location. The reconstructed source location is measured using two different methods: taking the center of the voxel corresponding to the maximum image intensity, and computing the centroid of a 3D Gaussian model fit to the reconstructed distribution. The expected (i.e., externally measured) and observed (i.e., reconstructed) source locations resulting from this procedure are tabulated in Table 6.1. For each measurement, the source location derived from the image reconstruction is within one voxel-width of the true source location, providing confidence in the correctness of the externally measured pose estimates and source location.

Having validated the measured detector and source locations, the Compton data are subjected to the downsampling procedure. A list-mode approach to downsampling is used to preserve the original sampling of Compton data along the detector track. Each Compton event is paired with a random variable drawn from a uniform distribution. The Compton event is retained for a given trial only if the random variable meets a threshold determined by a scaling on the activity of the original source. For example, the original source activity is 832 $\mu$Ci, so to simulate a source with an activity of 200 $\mu$Ci, the threshold for the random variable is set at $\frac{200}{832} = 0.24038$. Representing the downsampled set of Compton cones as having resulted from a point source with lower activity requires several conditions to be met. The geometry within a single measurement is self-consistent, though since the detector locations are not exactly the same between the HEMI and the VCI measurements, the scaled activities from the down-sampling procedure cannot be compared between the
Table 6.1: Comparison of expected source location (externally measured) with the source location computed from the volumetric reconstruction (shown in Fig. 6.3). The reconstructed source location is presented several ways: by recording the center of the voxel with the maximum image intensity, in addition to a fitting procedure. The results from both of these methods are presented. The distance between the expected and observed source locations is given as a measure of the accuracy of the reconstruction for these measurements.

measurements. In other words, the relative localization success rate of HEMI at 80 $\mu$Ci versus 20 $\mu$Ci is a meaningful comparison, whereas the comparison of localization success rate between HEMI and the VCI at 20 $\mu$Ci is not. The notion of scaled activity is only meaningful if the gamma-ray source is of the same type, as differences in the intensities of gamma-rays emitted from different sources are not accounted for. Since the same source was used for each of these measurements, this limitation is acceptable, though attempting to map the “activities” here back to expected event rates from some of the other gamma-ray sources used in chapters 4 and 5 is incorrect. It is also assumed that contributions of Compton events in the ROI around 662 keV from background are negligible due to the high intensity of the point source.

The downsampled data are presented in terms of scaled activity to reflect that the resulting number of downsampled events is still subject to the original sampling along the track. Yet it is useful to think about the results in terms of the number of Compton events acquired during a measurement. This is a better metric for comparison to the measurements in chapters 4 and 5 as it reflects the total amount of Compton signal collected during the measurement independent of the strength of the source. The number of Compton events acquired during any given trial from the downsampling procedure cannot be fixed as it is subject to the sampling of the original distribution. Fig. 6.4 shows the distribution of Compton events resulting from the downsampling procedure applied to HEMI and the VCI. The mean number of Compton events expected at each activity is given by the white line, with the error bars corresponding to 2x the standard deviation from the Poisson model. The data

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$^1$ Though the number of Compton events may be in the same range, the results from measurements here are still not directly comparable to those in previous chapters due to the long list of uncontrolled variables (detector path, pose uncertainties, signal-to-background, etc.).
Figure 6.4: Distribution of the number of Compton events at each of the subsampled activities. The white line represents the average number expected given the scaling factor, and the error bars are 2x the standard deviation expected from a Poisson model. The distributions are normalized so the color scale represents the fraction of total events in each bin. There is good agreement between the Poisson model and the subsampled distributions. The distributions are generated from 3000 independent runs at each scaling factor (i.e., each column represents a histogram of 3000 data points).

are normalized such that the colorbar indicates the fraction of total events sampled at each activity (3000 trials at each scaled activity for both instruments). There is good agreement between the observed distribution of the number of subsampled events with expectation given by the Poisson model. Parameters from the expected and observed distributions of Compton events are tabulated in 6.2, confirming agreement between the expectation and the results of the downsampling. It is worth noting the significant overlap in the distributions of the number of Compton events with increasing activity. As a result, the evaluated robustness and accuracy are expected to be correlated at higher values of scaled activity.

6.2.2 Robustness of Volumetric Localization

As a first assessment, the probability of a successful reconstruction is investigated as a function of the number of Compton events. The distance between the true and reconstructed source location is taken as a measure of localization accuracy, which is subsequently thresholded to define a successful reconstruction.

Fig. 6.5 shows the distributions of the deviation from the expected source location at 5 $\mu Ci$ intervals up to 100 $\mu Ci$ for the measurements taken with the VCI and HEMI respectively.
Table 6.2: Table correlating the scaled activity to the distribution of the number of Compton events. The expected number of Compton events is computed from the measurement data: $\frac{A_{scaled} \times N_{comes}}{A_{total}}$ and the uncertainty is given the standard deviation expected from the Poisson model. The observed number of Compton events is given by the mean of the distribution at each scaled activity (the columns in Fig. 6.4) and the uncertainty is the sample standard deviation of each distribution.
Fig. 6.5 conveys a lot of information which is discussed in several contexts; first in terms of robustness, with a discussion of specific features of the distributions and apparent biases to follow in section 6.2.3.1. Each column in these figures represents the distribution in localization accuracy resulting from 3000 independent trials at each of the scaled activities. The distributions are binned in 5 $\mu$Ci bins along the x-axis and with a bin width of 10 cm along the y-axis to match the voxel size. The color scale in these Fig. indicates the fraction of the 3000 independent trials contained in each bin. The median localization accuracy, given by the solid white line, improves with increasing source activity, as expected. The variability in the distributions also decreases with increasing source activity; the dashed line representing twice the sample standard deviation of the accuracy distribution at each scaled activity.

The information contained in Fig. 6.5 can also be viewed in terms of success rate, as described in section 6.1. Fig. 6.6 shows the fraction of successful reconstructions for various thresholds in localization accuracy. The success rate at the 10 cm accuracy threshold is subject to systematic bias that is discussed more thoroughly in section 6.2.3.1. The trend of increasing localization robustness with increasing number of Compton events is even more clearly demonstrated in Fig. 6.6. The results from each instrument are relatively consistent,

Figure 6.5: Distribution of the distance between the expected and measured source locations vs. scaled activity for the ground truth measurements with the VCI (left) and HEMI (right). The distributions result from 3000 independent samples at each downsampled activity. The measured source location is determined by the maximum image value in this case. The color scale represents the fraction of total trials (along each column) for which the reconstructed source location falls within a certain distance from the true source location.
Figure 6.6: The fraction of total downsampled trials that resulted in a successful localization of the point source. A successful reconstruction is one in which the reconstructed source location is within some distance of the expected source location. Success rates are given for each scaled activity with distance thresholds of 10, 20, and 30 cm. The errorbars are determined by the number of trials at each scaled activity (3000), not the variance in the measured accuracy.

with roughly a 60% success rate for localizing a 40 $\mu Ci$ point source within 20 cm of the true point source location, increasing to roughly 80% for a 70 $\mu Ci$ source.

It is important to re-emphasize here some of the limitations of this procedure for quantifying localization robustness. As these results are based off of a single measurement from each instrument, the conclusions about the localization success rate are subject to the specifics of the measurement, most notably the shape and extent of the detector track. Only a single track geometry is measured, and with discrete measurement locations as opposed to the continuous measurement during the mobile operation modes shown in chapters 4 and 5. Each measurement represents only a single sample in $\frac{\text{distance to source}}{\text{detector track length}}$, which is expected to have a large impact on the ability to reconstruct the point source location in the depth dimension. Furthermore, the detector trajectory was tightly controlled, not subject to the pose uncertainties inherent in online pose tracking. These represent just a few of the dependencies that must be considered for a general evaluation of localization robustness of the SDF method. Nevertheless, the results from the downsampling procedure quantify the repeatability of the volumetric localization, providing an order-of-magnitude estimate for the success rate and accuracy of volumetric localization for similar measurements vs. the number of recorded Compton events.
6.2.3 Volumetric Image Quality Analysis

The data generated from the downsampling procedure are also analyzed in the context of volumetric image quality, particularly localization accuracy and image SNR. The set of volumetric data from the procedure described in section 6.2.1 are used for this purpose.

6.2.3.1 Volumetric Localization Accuracy

Fig. 6.7 re-visualizes the distributions in localization accuracy, i.e., the calculated distance between the known and reconstruction point source location. In this case, the reconstructed location of the point source is determined by applying three independent Gaussian fits, one along each dimension, through the maximum of each reconstructed image. This yields an estimate of the reconstructed source location at finer resolution than the voxel size. The resulting distributions in Fig. 6.7 and 6.9 converge around a non-zero value, representing a systematic inaccuracy in the reconstruction. The results from the VCI data, summarized in Fig. 6.7, are investigated first. A perfect alignment between the true and reconstructed source locations would result in distributions with a baseline at 0 m; the distributions here, however, exhibit a baseline offset of around 5 cm. The distance between the expected and measured source location is evaluated along each dimension independently to investigate whether the offset is concentrated in a single dimension. Fig. 6.8 shows the distance distribution along the dimensions corresponding to depth, horizontal location, and height for the 3000 trials at 100 μCi, which had the lowest variability in the aggregate distance distribution. The distributions in the dimensions corresponding to the track traversal and height of the detector are in good agreement with the expected source location, indicated by the vertical black line. The offset is seen in the depth dimension, with the centroid of the distribution of the measured depths about 5 cm beyond the expected depth, revealing the source of the bias seen in Fig. 6.7. Unfortunately, the current measurement is not sufficient to determine whether this bias results from the volumetric reconstruction, as the uncertainty in the “true” source location from the ground-truth measurement is on the order of 5 cm.

The same analysis is also applied to the data from the HEMI measurement. Fig. 6.9 is analogous to Fig. 6.7 illustrating the distribution of distances between the known and reconstructed source location determined by applying Gaussian fits to the reconstructed images. The HEMI distributions tend to converge to a baseline offset of 10-15 cm with increasing activity; a larger offset than that seen in the VCI case, beyond the limits of ground-truth measurement uncertainty. Fig. 6.10 gives the distribution of the reconstructed source location in the dimensions corresponding to depth, horizontal motion, and height for the 3000 reconstructed images at 100 μCi. As with the VCI data, horizontal location and height of the source agree well with the externally measured values. In this case, the depth too is in good agreement with the expected value, with an offset of only 4 cm from the expected source location, within the uncertainty of the ground-truth measurement of the true source location. Given these results, the inaccuracy of the HEMI reconstruction cannot be explained by the uncertainties in the measurement of the true source location. It is possible that the leverarm
cut described in section 6.2.1 is not stringent enough, resulting in a distribution of Compton cones with high uncertainty in the opening angle. It is also possible that the computation of the system matrix does not properly account for the overlap of the Compton cone with the voxelized space. The interplay between the cone opening angle and the discretization of the imaging space is not fully understood at this time. Further investigation is necessary to explain the inaccuracy in the HEMI reconstruction.
Figure 6.8: The distribution of the reconstructed source location along the depth, horizontal, and height dimensions. The reconstructed values are given by the distributions in blue, with a bin width of 2 cm, and the expected values from the ground-truth measurement are given by the black lines. Note the good agreement in the y and z dimensions and the 5 cm offset in the depth dimension.

### 6.2.3.2 Volumetric Signal-to-Noise

As discussed in section 6.1, the extraction of the image maximum or centroid is subject to incorrect classification of the source location due to image artifacts and noise. The likelihood of incorrect localization increases as the number of Compton events used to generate the image decreases. Images formed from fewer Compton events are expected to have a lower signal-to-noise ratio, increasing the chance that the maximum point in the image corresponds to image noise rather than convergence at the true source location. To visualize this effect, the reconstructed images from the downsampled data are analyzed in terms of SNR via equation (6.1). Since the true source location is known for these measurements, the regions of the volumetric image corresponding to the “source” and “non-source” voxels are defined a priori. The 2 x 2 x 2 voxel cube around the expected source location is taken as the source region for the images, while the remaining voxels are labelled as not corresponding to the point source. Fig. 6.11 shows the evolution of the distributions in volumetric SNR with increasing gamma-ray signal from the point source. The image SNR generally increases with the number of Compton events used to generate the image, as expected. The SNR distributions tend to converge around an average SNR value of 40 in the case of the VCI and 20 in the case of HEMI as the number of Compton events increase. An SNR of 20-40 is significantly above the Rose criterion [101], suggesting that point-source distributions can
Figure 6.9: Distribution of the distance between the true and reconstructed source locations vs. scaled activity for the HEMI ground-truth measurement. The reconstructed source location is determined by fitting a Gaussian model along each dimension through the maximum of the reconstructed image. The distributions converge around a distance of about 12 cm between the true and measured source location, indicating a systematic inaccuracy in the reconstruction.

be clearly distinguished in the reconstructed image. The SNR distributions from images formed with fewer numbers of cones (less than 100 or so, corresponding to a scaled activity of around 30 $\mu$Ci) have large variability, including a significant portion of reconstructions with SNR > 40. This is explained by the very low variability in the background when there are very few cones, as the majority of the image space may not be intersected by any cones,

2 The Rose criterion specifically deals with contrast-to-noise ratio (CNR), though for the case of a point source distribution in the presence of negligible background, the SNR is assumed to be equivalent to the CNR.
Figure 6.10: The distribution of the reconstructed source location along the depth, horizontal, and height dimensions for the 3000 trials corresponding to a scaled activity of 100 µCi. The reconstructed values are given by the distributions in blue, with a bin width of 2 cm, and the expected values from the ground-truth measurement are given by the black lines.

thus exhibiting zero signal. In such cases, the variability in non-source voxels will be very low, but if even a small percentage of the cones intersect the source region, the mean signal in these voxels will be non-zero, resulting in a high SNR. Thus the highly variable SNR distributions below 30 µCi suggest insufficient sampling of the image space with the given Compton data. These results demonstrate that volumetric SNR is not a sufficient metric for determining image convergence or interpretability, though suggest that the coverage of the imaging space is an important factor in evaluating image quality.
Figure 6.11: Volumetric SNR computed using equation [6.1] with the downsampled Compton data from the VCI (left) and HEMI (right) measurements. A general trend of increasing SNR with source activity is clearly visible in the VCI case, with the distributions in SNR converging to an average value of about 40 at higher activities. The SNR distributions for images with fewer cones (below 20 $\mu Ci$) display large variability in the SNR.

6.3 Localization vs. Compton Cone Uncertainty

In Compton imaging, not every Compton cone contributes equally to the imaging space. The parameters describing the Compton cone, the scattering axis and cone opening angle, are derived from measured quantities in the detector: the interaction positions and deposited (and total) energy of the event, respectively. The uncertainty in these measured quantities manifests as an uncertainty attributed to the Compton cone. Unlike conventional image formation, where each photon maps from the image space to real space in a uniform way, each cone from every individual Compton event will have a different uncertainty when mapping to the image space. The gamma-ray image comprises an aggregation of these cones, thus the precision of the image, i.e., the Compton image resolution, directly depends on the distribution of Compton cone uncertainties. This dependence is qualitatively apparent in the comparison of the Compton image resolution of the VCI versus that of HEMI. The VCI has both finer position and energy resolution, resulting in finer angular resolution in the Compton image compared to HEMI (see section 3.5). This also causes the observed variation in Compton image resolution with incident gamma-ray energy, as gamma-rays with different energy will result in different uncertainties in deposited energy as well as different

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3 A ray from the pixel location in the image space to a point in $\mathbb{R}^3$ determined by the principal point, focal point, and subject to any additional optics
probabilities for interaction histories\textsuperscript{4} affecting the distribution of uncertainties associated with the cone scatter axes. The effect of the uncertainty of Compton cones on conventional 2D Compton imaging has been previously studied \textsuperscript{32}. The goal here is to extend this analysis to the volumetric domain. It is important to note that the total image reconstruction, even in the static case, depends on many other factors aside from the distribution of Compton cone uncertainties, such as the angular sampling of the imaging space \textsuperscript{34}. Perfect knowledge of the distribution of uncertainty in the Compton cones is not sufficient to completely pre-determine the all the characteristics of the resulting image. The Compton image also depends on the reconstruction method, whether by analytic methods like filtered back-projection \textsuperscript{15} or the computation of a system matrix for iterative methods (as is done here). These considerations serve to limit the scope of the investigation here: the goal being to undergo a primary assessment (again, from only a single measurement) so that the impact on the imaging can be compared with other effects on an order-of-magnitude basis. Identifying the impact of these individual effects on the volumetric imaging informs the direction of future research to maximize improvements in volumetric imaging.

A value for the uncertainty of the opening angle of the cone is computed for each of the Compton events in list-mode. A model for relating the measured position and energy resolution to the uncertainty of opening angle of the Compton cone is presented in \textsuperscript{32}.

\[
\delta \mu = \left( \frac{1}{A_0^2} \sigma_{A_0}^2 + \frac{1}{A_d^2} \sigma_{A_d}^2 + 2 \left( 1 - \mu^2 \right) \frac{\sigma_r^2}{r_{12}^2} \right) \frac{1}{2} \tag{6.2}
\]

In equation \textsuperscript{6.2} \( \mu \) is the cosine of the cone opening angle, \( \mu = \cos \theta \); \( A_0 \) is the normalized total gamma-ray energy, \( A_0 = \frac{E_0}{m_e c^2} \); and \( A_d \) is the normalized energy of the scattered gamma-ray, \( A_d = \frac{(E_0 - \epsilon)}{m_e c^2} \). As \( E_0 \) is pre-determined using spectral gates in this work, the uncertainty in energy of the incident gamma-ray is assumed negligible\textsuperscript{5} \( \sigma_{A_0} = 0 \), so the uncertainty in the cone opening angle depends only on the last two terms in equation \textsuperscript{6.2} \( \sigma_{A_d} \) is given by the energy resolution of the imager at the energy deposited in the first Compton scattering interaction, which in principle can range from 0 keV up to the backscatter energy, \( \epsilon_{\text{max}} = E_0 \left( \frac{2E_0}{m_e^2} \right) \). The value for \( \sigma_{A_d} \) can be determined by interpolating the data in table \textsuperscript{3.2} for single-interaction events. For the final term in equation \textsuperscript{6.2} the lever-arm distance \( (r_{12}) \) is computed by taking the distance between the two 3D interaction locations that constitute the Compton event. The position resolution, \( \sigma_r \), is assumed to be 2 mm in all dimensions, as no sub-strip interpolation methods are used in this work. This is a

\textsuperscript{4} Dictated by physics (Klein-Nishina differential cross-section) coupled with source-detector geometry; other detector properties like position resolution and granularity; and additional biases from event selection and reconstruction.

\textsuperscript{5} This would not be the case for gamma-rays originating from sources other than nuclear de-excitation, such as 511 keV photons from annihilation, which are impacted by the uncertainty of the momentum of the positronium.
very conservative estimate of the position resolution, as even without sub-strip interpolation methods, the position resolution from a Gaussian model is less than the strip pitch. Thus equation 6.2 can be used to compute the uncertainty in the cone opening angle for all Compton events.

Several assumptions are made in the derivation of the model described by equation 6.2. For instance, the model assumes that the uncertainty in cone scatter axis represented by the last term in equation 6.2 can be directly included in uncertainty of the cone opening angle. In reality there is a complex geometrical relationship between these two terms, and further study is necessary to fully map out the relationship between \( \sigma_{\phi} \), the uncertainty in the scatter axis, and \( \sigma_{\theta} \), the uncertainty in the cone opening angle. There are also complications with the incorporation of position uncertainty in equation 6.2. The model assumes that the interaction position is modelled by a normal distribution, and only allows for a single degree of freedom to describe the position uncertainty. In practice, the x, y, and z dimensions of the location may be sensed by entirely different means, and be described by different distributions. The event reconstruction in the VCI is one such example, where the location in x is computed from signal analysis on the vertical strips, the y position from signals on the horizontal strips, and the depth of interaction from the difference in signal arrival time. The 2 mm value used for the position resolution is a conservative simplification which will result in the over-prediction of the Compton cone uncertainty.

6.3.1 Model validation

A static measurement is taken with the VCI to investigate the validity of the uncertainty model given by equation 6.2 before it is applied to the volumetric Compton data. The 832 \( \mu Ci \) point source is placed directly in front of the VCI imager (i.e., at \((0^\circ, 0^\circ)\) in angular space) at a distance of 4.57 meters, ensuring geometric blurring from the point detector approximation is negligible. Equation 6.2 is applied to all potential Compton events in a 3 keV window around 661.62 keV in the energy spectrum, a total of over 100,000 events. The uncertainty in the cone opening angle, \( \delta_{\theta} \), is computed from the \( \delta_{\mu} \) resulting from equation 6.2. Fig. 6.12 shows the distribution of the uncertainty in the Compton cone opening angle for a subset of events collected during the measurement. The uncertainty in the opening angle is largely governed by the distribution of leverarms for the Compton events at this energy, given the superior energy resolution of the HPGe detectors. Discrete features can be seen in the high angular uncertainty regions due to the strip pitch of the HPGe detectors (sub-strip interpolation is not used in this work). Note the wide range of cone opening angle uncertainties from this set of events, including a large number of events with uncertainties in the cone opening angle greater than 40°. To evaluate the validity of these calculated uncertainties, an event selection procedure is applied to the data where subsets of the total number of Compton events are selected according to thresholds in the computed \( \delta_{\theta} \).

An equal number of Compton events is randomly drawn from within six different regions in the computed cone uncertainty distribution. The angular uncertainty of these cones is evaluated using the known energy and source location in angular space by several different
Figure 6.12: Distribution of the uncertainty of the Cone opening angle computed by applying equation 6.2 to the potential Compton events collected during the static measurement. The uncertainty regions used throughout the remainder of the validation analysis are indicated by the colorized regions.

metrics. The ARM metric is given by the smallest distance between a Compton cone and the true source location in angular space. Fig. 6.13 shows distributions of the ARM metric for the 10,000 events corresponding to each cone uncertainty region. Each distribution is fitted with a Lorentzian model, the width of which provides a quantitative measure of the measured angular uncertainty from that set of cones. As a second measure of angular uncertainty, a directional filtered back projection algorithm is used to generate an image from each set of 10,000 cones. The resulting images are fit with a 2D Gaussian model to measure the FWHM of the reconstructed distributions.

Fig. 6.14 shows the angular resolution, measured both by the width of the ARM distribution and the FWHM of the Compton image, versus the expected angular uncertainty for the events given by equation 6.2. The magnitude of the measured angular uncertainty does not agree with the predictions from the uncertainty model. The measured angular uncertainty is generally lower than what is predicted by the uncertainty model, especially for events with
Figure 6.13: ARM distributions computed from 10000 events in each of the 6 uncertainty regions shown in the legend. The ARM distributions are each fit with a 1D Lorentzian model which gives an estimate of the width of the distribution. These widths constitute the blue line in Fig. 6.14.

a higher predicted cone uncertainty. This is likely due to the model assumptions discussed above, especially with regards to position resolution and the correspondence between the uncertainty in the cone scatter axis and the cone opening angle. Note however that measured angular uncertainty does decrease with decreasing predicted uncertainty, verifying that the trend in predicted uncertainty is correct to first order. The general overprediction suggests that the model is conservative over the cone opening angles of interest. These results suggest that the model can be applied to the volumetric data to get an order-of-magnitude estimate of the impact of Compton cone uncertainty of volumetric imaging.

6.3.2 Volumetric Localization vs. Compton Cone Uncertainty

Having verified that the uncertainty model in equation 6.2 correlates to the measured angular uncertainty to first order, the analysis is extended to the volumetric domain to evaluate the impact of cone uncertainty on 3D localization. Data from the ground-truth measurement described in section 6.2.1 are used for this analysis. As in the validation section, only data
Figure 6.14: The measured angular resolution of 10,000 Compton events drawn from six different regions in the $\delta_\theta$ parameter, given on the x-axis. The angular resolution is quantified by both the ARM metric (blue) and the FWHM of a 2D Gaussian fit to a gamma-ray image produced by filtered back projection (red). The error bars in each case represent the goodness of fit for the 1D lorentzian and 2D gaussian models, respectively. Though the magnitude of the measured angular resolution does not agree with that predicted by the model, the general trend of improving angular resolution as events with lower predicted angular uncertainty are used is verified.

from the VCI measurement are due to the unexplained biases observed in the HEMI ground-truth measurement. The list-mode cone uncertainty is computed for all 3257 Compton events collected during the ground truth measurement. Fig. 6.15 shows the distribution, which agrees well with the corresponding distribution of uncertainties observed from the static measurement. The colorized regions in Fig. 6.15 correspond to event selection boundaries for Compton events. As in the static case, there are insufficient events within a single bin of expected cone uncertainty to generate Compton images free of statistical effects. Furthermore, the predicted $\delta_\theta$ strongly depends on the lever-arm; thus selections based on $\delta_\theta$...
Figure 6.15: Distribution of the uncertainty of the Cone opening angle computed by applying
equation 6.2 to the 3257 potential Compton events collected during the ground-truth mea-
surement. The colorized regions correspond to event selections made based on expected cone
uncertainty. There are 750, 867, and 560 Compton events in the $\delta \theta < 10^\circ$, $10 \leq \delta \theta < 20^\circ$,
and $20 \leq \delta \theta < 30^\circ$ regions, respectively.

corresponds to a lever-arm cut, especially with increasing predicted cone uncertainty. This
can cause biasing in the angular sampling of the imager, ultimately impacting the gamma-ray imaging [34]. In an effort to lessen the effect of these biases and statistical effects, relatively large bins in the $\delta \theta$ parameter are chosen.

Fig. 6.16 displays the results of volumetric reconstruction on the three categories of
Compton event designated in Fig. 6.15. The event selection resulted in 750 cones with
$\delta \theta < 10^\circ$, 867 cones with $10 \leq \delta \theta < 20^\circ$, and 560 cones with $20 \leq \delta \theta < 30^\circ$. This number of
cones is well above the threshold identified in section 6.2.3.1 for an accurate reconstruction,
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Figure 6.16: Volumetric gamma-ray reconstruction for Compton events with different predicted angular uncertainties. The three images resulted from 750, 867, and 560 Compton events in the $\delta \theta < 10^\circ$, $10^\circ \leq \delta \theta < 20^\circ$, and $20^\circ \leq \delta \theta < 30^\circ$ regions, respectively. The images are shown from the same perspective, and each gamma-ray distribution is represented by 8 isosurfaces, representing increasing image signal in 10% increments from 10% up to 90%. The point cloud was not used to constrain the imaging space, and 10 EM iterations were used in reconstructing each image.

thus statistical effects are not expected to dominate the imaging results. Variation in the reconstructed distributions can reasonably be attributed to the event selection based on the $\delta \theta$ parameter. The increasing variance of the reconstructed distribution with increasing Compton event uncertainty is immediately clear in Fig. 6.16. Unfortunately, there are not enough events in each of the $\delta \theta$ regions to repeat the downsampling procedure described in 6.2.1. Instead, the spatial resolution of each image is quantified by assessing each of the reconstructed volumetric images in 6.16. Fig. 6.17 illustrates the distributions in Fig. 6.16 by taking a 1D slice along each dimension through the maximum signal in each of the volumetric images. Each axis in 6.17 represents a slice through one dimension, while the colors correspond to the Compton events used in the reconstruction, blue for events with $\delta \theta < 10^\circ$, green for events with $10^\circ \leq \delta \theta < 20^\circ$, and red for events with $20^\circ \leq \delta \theta < 30^\circ$. The resolution in each spatial dimension is given by the FWHMs of the Gaussian models fit to the data in each dimension, which are tabulated in 6.3. The accuracy in each dimension is also tabulated in 6.3, determined by the distance between the known source location and the centroid of the reconstructed distribution. Recall that the systematic uncertainty in the true source location is about 5 cm in each dimension. The accuracy of the reconstruction is within these bounds in all dimensions except depth. This suggests that the uncertainty in the cone opening angle degrades the accuracy of the reconstructed depth of a gamma-ray point source. Though this conclusion is derived from a single measurement with a linear track with only a single sample in $\frac{\text{Source Distance}}{\text{Track Length}}$, it is consistent with expectations from other triangulation-based methods [51]. The spatial resolution in every dimension degrades as the uncertainty in cone opening angle for the Compton events increases, as is expected. Thus the uncertainty of the opening angle of the Compton cones tends to limit the precision of
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Figure 6.17: Representation of the volumetric distribution along each dimension through the maximum of the reconstructed image. The dots represent the volumetric image data along the 1D slice, while the dotted lines correspond to a 1D Gaussian model applied to the data to assess the angular resolution in each dimension. Each image is normalized by the maximum image signal so the width of the distributions may be visually compared. The colors correspond to those in the volumetric reconstructions in Fig. 6.16. The true source location along each dimension is represented by the black line.

<table>
<thead>
<tr>
<th>Event Selection</th>
<th>Number of Events</th>
<th>Accuracy (cm)</th>
<th>Resolution (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>X</td>
<td>Y</td>
</tr>
<tr>
<td>$\delta \theta &lt; 10^\circ$</td>
<td>750</td>
<td>1.7</td>
<td>1.9</td>
</tr>
<tr>
<td>$10 \leq \delta \theta &lt; 20^\circ$</td>
<td>867</td>
<td>5.6</td>
<td>0.1</td>
</tr>
<tr>
<td>$20 \leq \delta \theta &lt; 30^\circ$</td>
<td>560</td>
<td>10.6</td>
<td>2.6</td>
</tr>
</tbody>
</table>

Table 6.3: Measured spatial accuracy and resolution from the volumetric images created from Compton events with different uncertainties in cone opening angle. $X$ represents the depth, $Y$ the horizontal displacement, and $Z$ the height. The accuracies are given by the distance between the reconstructed source location and the externally measured source location in each dimension. The resolutions are given by the FWHM of the Gaussian model fit to the volumetric image along each of the three dimensions, shown in Fig. 6.17.

the localization in all three dimensions, while also degrading the accuracy of the computed depth.

6.4 Pose Uncertainty

In RGB-D Slam, the pose of the sensor is estimated based on visual and depth information derived from views of the scene. As with any other calculated quantity, each pose estimate has an associated uncertainty given the uncertainties in the sensed values. This is significant in the context of SDF as the pose estimates are used to position and orient the measured
Compton data in the environment, so any disparity between the true and estimated translational or rotational component of the pose has an effect on the gamma-ray image. The pose estimation of RGB-D Slam in particular is based on the extraction of salient visual features from the RGB images\cite{55}. This has the advantage that it reduces the feature space from 640 x 480 pixels\footnote{The resolution of the first-generation Kinect sensor.} down to a much smaller set for comparison, which benefits the real time computation\cite{55}. The features from the RGB image are then transformed into the coordinate frame of the depth image from the Kinect to extract the 3D location of the feature. Thus each frame is decomposed into a set of 3D features which are then correlated to the feature sets from other frames. The number of frames between which feature sets are compared is configurable in the RGB-D Slam software, though in this work the comparison is made with the features from the three previous frames, as well as 17 frames uniformly selected along the entire trajectory.

Some aspects of the uncertainty in pose estimation are specific to RGB-D Slam. One such instance is the magnitude of the degradation of pose estimates as the distance between the Kinect and the scene increases. This is a consequence of the depth accuracy and resolution of the Kinect, which scale inversely with the square of the distance to objects\cite{51}. If a large fraction of the visual features in a given frame correspond to objects that are greater distances away from the sensor, the lack of certainty in the 3D location of these features swiftly degrades the pose estimate. This behavior can be seen in Fig.\ref{fig:6.18} where the same scene is viewed by the Kinect along an identical track at 3 different distances. Note the degradation of the pose estimates for measurements where the Kinect is further away.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{kinect_tracks}
\caption{Example of three horizontal detector tracks of identical length taken with the Kinect at different perpendicular distances from the scene. The green, blue, and red tracks were taken with the Kinect at a distance of roughly 1.2, 2.3, and 3.6 m from the nearest visual features in the FOV, respectively. The two images represent two different views of the tracks. Error from poor individual pose estimates are clearly visible from both perspectives, and the effect of accumulated error is visible from the perspective in the right frame. These tracks qualitatively illustrate the degradation of the pose estimates as the majority of the visual features are at a further distance from the sensor.}
\end{figure}
CHAPTER 6. QUANTITATIVE EVALUATION OF SCENE DATA FUSION

from the back wall (where the majority of the visual features are concentrated). Another potential disadvantage of this approach is that salient visual features tend to be located in high-contrast regions of the RGB image, many of which are corners or edges of objects. As such, there are often depth discontinuities very near the visual features, which can cause large errors in the 3D position of the feature if the RGB and depth images are misaligned by even a few pixels. Methods for dealing with these outliers such as RANSAC estimation [102] mitigate this issue. These mechanisms introduce errors in the pose estimates, explaining some sources of single-frame pose errors (i.e., non-cumulative errors).

Another behavior is common not only to many SLAM approaches, but many other tracking algorithms as well, including those based on GPS or inertial navigation. Tracking methods are often susceptible to accumulated error owing to the fact that subsequent pose estimates are based on the set of previous pose estimates. This is the “dead-reckoning” problem common to odometry and other forms of online, incremental navigation. One common manifestation of accumulated error is seen in navigational loops, where the sensor returns to a location it has previously been but the estimate of the pose significantly deviates from the previous estimate at that location. This is referred to as the loop-closure problem, and is actively studied for vision and tracking applications [95, 96]. There are several approaches taken in an attempt to mitigate the effects of accumulated error in the real-time SLAM setting. RGB-D Slam relies on a graph-based non-linear optimization approach, implemented with the libg2o package [103]. In this approach, each pose estimate (and corresponding point cloud) is added as a node to a pose graph. For every frame with a successful match to a previous frame, a node is added to the pose graph along with an edge connecting the new node to any other nodes for which the feature matching was successful. After a configurable number of nodes have been added to the graph (for this work, RGB-D Slam is configured to run the graph optimization after the addition of every two new nodes), the software attempts to minimize the error along the edges of the pose graph. This routine, referred to as the SLAM “back-end” in [55], aims to introduce global consistency to the pose estimates, reduce the effects of accumulated error from online odometry, and resolve loop-closures. Accumulated error is prevalent in many tracking algorithms, thus it is worthwhile to study the impact it has on volumetric gamma-ray imaging, as the results would still be relevant for other mobile imaging systems based on different tracking methods.

6.4.1 Evaluation Methodology

In sections 6.2 and 6.3, a ground-truth measurement was used to eliminate the effects of pose uncertainty so that the impact of event statistics and quality on volumetric localization could be studied. The goal of this evaluation is to do the opposite: control variables such as counting statistics and Compton cone uncertainty to isolate the effect that pose uncertainty has on volumetric localization. The two error modes detailed in this section are studied via simulation to isolate the geometric effects of pose uncertainty on the volumetric localization. The effect of uncertainty in the translational components of the pose are examined by applying errors to known, simulated positions according to both an independent
(non-cumulative) and accumulated error model. Fig. 6.19 gives a visual representation of

the types of pose uncertainty generated along a simulated detector track. A simulated point
source is positioned at \((0, 0, 2)\) meters in the coordinate frame while the detector takes a
semicircular track around the source with a radius of two meters. The detector position
is sampled 100 times along the track and various types of uncertainty are applied to those
samples. The detector positions with non-cumulative error are given by the red points, while
the cumulative error is given by the green points. In each case, the error is applied to the
sample points using a Gaussian model with a variance specified by the user. In Fig. 6.19,
the non-cumulative instance of pose error is given a standard deviation of \(\sigma = 3\) cm in each
spatial dimension. The cumulative error mode is given a smaller standard deviation of only
\(\sigma = 1\) cm to illustrate how even small deviations result in large pose errors over long tracks.
Only the translational component of the pose estimation is investigated in this work.

6.4.2 Simulation Results

A linear detector track 4 m long with a single point source positioned two meters away from
the center of the track is chosen for the simulation study (this geometry closely matches many
of the source-detector geometries from the measurements in chapter 4 and the ground-truth measurement in section 6.2.1. The simulation assumes a point-detector model commensurate with the far-field assumption that is implicit in the volumetric imaging measurements. The position of the detector is sampled along the track and Compton events are generated via Poisson sampling at each location. The scatter-axes of the Compton cones are computed by sampling three independent univariate Gaussian distributions with equivalent variance, one for each coordinate direction, and appropriately normalizing the result. This procedure accounts for neither the differential probability for gamma-ray scattering direction governed by the Klein-Nishina formula, nor the effects of detector geometry, eliminating any biases that result from these effects. The gamma-ray point source is monoenergetic with $E_0 = 662$ keV. The energy deposited in the Compton scatter interaction is determined by equating the kinematic cosine to the geometric cosine (equation 2.6) and solving equation 2.4 for $E_s$. This procedure results in a set of Compton events generated along a detector track with no pose error and no uncertainty in the cone opening angle, subject only to geometric signal attenuation ($\frac{1}{r^2}$) and Poisson sampling. Thus the effects of source-detector geometry are sufficiently isolated that any impact on the volumetric reconstruction can be attributed to translational pose uncertainty. As so many physical effects are eliminated in the simulation procedure, the results of the analysis are only to be compared with other simulation results to quantify the relative effect of translational pose uncertainty. A comparison between the simulation results and measurements with estimates of the pose uncertainty are less meaningful as the measurements will obviously be subject to many of the physical effects that are ignored in this analysis.

Once the Compton event history for a single simulated trial is computed using this procedure, the two error modes are applied. Applying the translational pose error after the events have been generated along their true location on the path accurately captures the effect of pose uncertainty in the context of detector tracking: the measured Compton events are displaced from their true location according to the error in the pose estimate. The upper-bound of the magnitude of the translational uncertainty is set based on an evaluation of RGB-D Slam [55], which reported a translational RMS error of 22 cm for a measurement mapping an indoor room. Magnitudes of the standard deviation for the independent and accumulated pose errors were chosen to be $\sigma_{\text{ind}} = 4$ cm and $\sigma_{\text{accum}} = 2$ cm respectively.

The results of a single simulation trial are given in Fig. 6.20. The true source location is indicated by the cross-hair, and the results of volumetric reconstruction of all three event histories are shown. The blue arrows and contour surfaces represent the original Compton data free of pose error, while the data in green and red are from the applications of independent and accumulated translational error models, respectively. The reconstructed results are evaluated in terms of the accuracy and precision of the reconstruction as described in section 6.4. The reconstructed images resulting from each independent trial are fit with Gaussian models in each of the three dimensions that pass through the mean of the image intensity distribution. The centroids of these three fits represent the location of the reconstructed point source in $\mathbb{R}^3$, which is then compared to the true source location specified for the simulation. Consistent with previous evaluations in this chapter, the localization accuracy is given by
Figure 6.20: A single trial of the geometric simulation with a linear detector track with $\sigma_{ind} = 4\, \text{cm}$ and $\sigma_{accum} = 2\, \text{cm}$. The blue reconstruction corresponds to the original Compton data before the application of any translational error. The green data give the events with the independent error model applied, and the red data with the accumulated error model. The true source location is specified by the cross-hair.

the distance in $\mathbb{R}^3$ between these two points. The precision of the localization is given by the FWHM of the fitting functions in each spatial dimension. The impact of pose error on the accuracy and precision of volumetric localization is summarized in Fig. 6.21 and 6.22 for the given simulation geometry. These distributions result from 1000 independent simulation trials using $\sigma_{ind} = 4\, \text{cm}$ and $\sigma_{accum} = 2\, \text{cm}$ for the independent and accumulated error models respectively. The precision presented in Fig. 6.22 reflects the distribution of the precisions measured for each individual reconstruction. The precision in this case is not simply given by the width of the distribution of measured distances, but by the sum of the variances in each dimension for each individual reconstruction. The imaging space is represented by 10 cm$^3$ voxels, consistent with previous evaluations.
Figure 6.21: Distribution of localization accuracy: distance between true and reconstructed source locations in $\mathbb{R}^3$ for 1000 trials.

The images generated from Compton events in which the independent pose uncertainty model has been applied show degraded precision of the reconstructed point source location. In other words, the spatial imaging resolution decreases with larger uncertainty in the independent error model, limiting the ability to spatially differentiate multiple gamma-ray sources with the same energy, or capture distinct features in a distributed gamma-ray source.\(^7\) The accuracy of the 3D localization is not significantly impacted however, at least at the current ratio of the pose uncertainty relative to the voxel size. The accumulated pose error however, although smaller in magnitude, severely degrades both the accuracy and precision of the 3D localization. The effects of accumulated error will obviously scale with track length, worsening the accuracy and precision of the gamma-ray source localization as the track length increases. Thus in the selection of a tracking modality for a mobile gamma-ray imager, priority should be placed on systems that minimize accumulated error.

\(^7\) This limitation is a result of the translational pose uncertainty only. The additional complications involved with reconstructing distributed gamma-ray sources are not addressed.
Figure 6.22: Distribution of the localization precision: FWHM of Gaussian models fit to reconstructed volumetric images for 1000 trials.

6.5 Discussion

The goal of the analyses presented in this chapter is to quantitatively estimate the impact of several known phenomena on various aspects of volumetric imaging. Important considerations such as the accuracy of the volumetric localization, spatial imaging resolution, and the robustness of the SDF approach for localizing point sources have all been investigated, supporting limited conclusions about the application of SDF to gamma-ray point source localization. One of the main results from the downsampling procedure is to provide strong evidence that the volumetric localization of gamma-ray point sources is repeatable, even when the sources are relatively weak, and the robustness of the localization increases with the number of Compton events. This is a non-trivial result, indicating that limitations in the repeatability of the localization are dependent on external factors like imaging statistics and sampling of the imaging space, not inherent to the method itself. Another important consideration is the fact that these results are based on a single measurement that did not leverage one of the primary advantages of mobile detector operation: moving within close
proximity of the source. For the downsample measurements, the detector never approached nearer than 1.75 m (2 m in the VCI case) to the source. The results then are conservative from the perspective of evaluating the volumetric localization, failing to capture the benefits of mobile operation with real-time feedback. With these considerations, the robustness results represent a lower bound for these instruments, with expected improvements when operational benefits are accounted for.

The accuracy of volumetric localization evaluated in terms of quantity of Compton data also follows the same important trend of improving with increasing quantities of Compton data. The results from this preliminary evaluation do however suggest the requirement for further study. Both the VCI measurement and the HEMI measurement exhibited a systematic spatial inaccuracy, biased to varying degrees away from the externally measured source location. In the case of the VCI, it was found that this systematic offset was concentrated in a single dimension (depth), and was within the limits of precision of the ground truth measurement. Thus the VCI results cannot conclusively attribute any of the unexpected behavior to the image reconstruction. The accuracy results from the HEMI measurement on the other hand do not abide a similar explanation. The magnitude of the observed inaccuracy, about 12 cm, is greater than what would be expected from uncertainty in the ground-truth measurement of the source location\textsuperscript{8}. The bias is not found to be concentrated in any single dimension, providing further evidence that the result is not simply due to measurement error. It is still possible that the observed offset is largely explained by error in the ground-truth measurement of the source location: An error of 5 cm in each dimension would yield a total offset on the order of 10 cm. A more precise ground-truth measurement is required, perhaps involving a motorized track and laser range-finding to replace the measurement grid. In spite of the unexplained offset in the determination of volumetric accuracy with HEMI, the general trend of improving localization accuracy with increasing quantities of Compton data provides evidence in support of the efficacy of SDF for accurate localization of gamma-ray point sources in 3D.

The quantity of Compton data was shown to impact the robustness and accuracy of volumetric localization; similarly, the quality of the data has also proven to be an important consideration. Using equation 6.2 as a first-order estimate of the uncertainty in the opening angle of the Compton cone, the effect of Compton event quality on volumetric localization was evaluated. The results showed decreasing image resolution in every spatial dimension with increasing uncertainty of the opening angles of the Compton cones used to reconstruct the image. It is important to note that for each of the three event selection regions; cones with $\delta \theta < 10^\circ$, $10^\circ \leq \delta \theta < 20^\circ$, and $20^\circ \leq \delta \theta < 30^\circ$, a sufficient number of Compton events were acquired for an accurate reconstruction according to the results in Fig. 6.6\textsuperscript{9}. The degradation of 3D spatial resolution with Compton events with greater cone uncertainty emphasizes the link between 2D angular resolution and the full spatial resolution. The results of the cone

\textsuperscript{8} The same physical measurement grid was used for both the VCI and HEMI measurements, so the expected uncertainty between each would be of the same order.

\textsuperscript{9} The same data set was used for both of these analyses, so the direct comparison with the number of cones used here is valid.
quality analysis coupled with the downsampling analysis results illustrate the existence of an important tradeoff between the quantity and quality of Compton data. This tradeoff exists in many imaging domains, often described as the compromise between sensitivity and specificity, or efficiency versus resolution. Investigation of how best to balance this tradeoff would be expected to improve the robustness and accuracy of volumetric imaging. In principle, one can determine this balance via event selection, selecting only Compton cones with smaller uncertainty at the expense of imaging efficiency. In practice, an approach to governing this tradeoff based on event selection is not desirable. Selecting only a subset of events has readily apparent drawbacks; eliminating the spatial information conveyed by these uncertain events as well as potentially biasing the imaging in other ways, such as restricting the angular sampling of the imager. The events with high cone uncertainty are entirely comprised of events with short leverarms (at least in the case of the VCI, where the contributions of energy resolution to cone uncertainty are very small). Eliminating these events is akin to a leverarm filter, which may alter the angular sampling capabilities of the imager. Limited angular sampling is known to have an effect for static Compton imaging [34], though it may be mitigated in the case of a mobile imager through a 3D environment. In any case, further study is required to determine an appropriate method for incorporating the cone uncertainty in the image reconstruction. The uncertainty in the cone opening angle could be accounted for in the computation of the system matrix in equation 2.10, for example. A model that more accurately predicts the expected cone uncertainty is also required to appropriately capture these effects in the image reconstruction. In principal, evaluation of the cone uncertainties, Compton event statistics, and related degree of sampling of the imaging space could be combined to derive a figure-of-merit to give a quantitative measure of the confidence associated with the volumetric reconstruction.

Having examined several properties intrinsic to volumetric Compton imaging, a preliminary analysis of one of the external dependencies associated with SDF was undertaken in evaluating the effect of of pose uncertainty on volumetric localization. Though the implementations of SDF described here rely on a specific SLAM approach, i.e., RGB-D Slam, the uncertainty in pose estimates is decomposed into two different classes that are relevant for many tracking algorithms: uncertainty in sequential estimates and the accumulation of those uncertainties over the entire track. Accumulated error has been shown to be manifest in track estimates from SLAM even after the application of back-ends for loop closure and global track consistency [55]. The effects of these two types of positional error were evaluated via a geometric simulation procedure, which eliminates all effects from the physics of gamma-ray interaction and detector geometry. Thus the results from the simulation study are subject only to the effects of the track geometry coupled with the source location. The simulation results demonstrate the effects of these two types of uncertainty in the translational pose estimates. The accuracy of the volumetric reconstruction is significantly degraded by the effects of accumulated pose error, even when the magnitude of the individual errors are small relative to the length of the track and source-detector distance. The independent pose error by contrast only degrades the spatial resolution in accordance with the magnitude of the uncertainty relative to the track and source geometry. These results influence what
is to be expected from SDF systems relying on different sensors and/or SLAM approaches. For an accurate reconstruction, the selection of a tracking method that effectively reduces accumulated error in the tracking is essential.

It is also important here to keep in mind the many limitations of the evaluations conducted in this chapter. One very important limitation is the fact that the evaluations are all carried out in terms of the spatial localization of point sources. This allows for straightforward evaluation of imaging properties, but provides limited insight for more complicated imaging scenarios, particularly distributed gamma-ray sources. A distributed gamma-ray source would be expected in many compelling real-world applications, particularly nuclear contamination monitoring and remediation. The procedures here fall in line with impulse response measurements for system evaluation; the more complex scenario of imaging response to distributed gamma-ray sources is not addressed. There are also significant limitations in the image reconstruction approach itself that prevent a more thorough quantitative evaluation. As has been previously mentioned, the sensitivity term in equation 2.10 is simply set to unity in this work, ignoring finer details about the coupling of the imaging space to the detector response. This is a reasonable simplification to allow the reconstruction of gamma-ray point sources in a system-agnostic setting, though a more realistic evaluation of system performance requires an accounting for the sensitivity. Other limitations in the volumetric reconstruction result from computational constraints. The computation memory requirements scale inversely with the cube of the voxel size. For example, representing an imaging space with a total volume of 1 m$^3$ with a voxel size of 1 m$^3$ requires a single voxel; a voxel size of 0.5 m$^3$ requires 8 voxels and 0.25 m$^3$ 64 voxels, etc. This scaling is the primary factor that limits the voxel size to 10 cm$^3$ in the demonstration measurements of chapters 4 and 5 and the evaluations in this chapter. A more efficient representation of and computation within the imaging space is necessary to be able to explore the effect of variation in the discretization. Voxel size is not the only parameter of the volumetric imaging that remains unexplored. Other parameters such as the number of EM iterations or the fixed cone opening angle[10] will have an impact on the imaging, though they are not explored in this work.

One element that is required for future development, evaluation, and testing of volumetric imaging is a validation dataset. This is a common paradigm in other fields such as computer vision, where a known, curated dataset is used to benchmark and evaluate different algorithms or proposed improvements. The ground-truth measurements introduced in section 6.2.1 represent the first two entries of such a dataset, but far more measurements covering a wider range of tracks and source-detector geometries are required to provide a more comprehensive validation set. Within the broader scope of system evaluation, the measurements and simulation studies presented here represent only single samples within a large parameter space with many complex dependencies. Nevertheless, the measurements

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[10] Rather than incorporate the list-mode uncertainties, the current version computes the system matrix assuming a cone opening angle equivalent to the angular resolution of each system: 3° for the VCI and 10° for HEMI.
and simulation study represent a necessary first step towards a quantitative understanding of the efficacy and performance of standoff volumetric imaging with SDF.
Chapter 7

Closing Remarks

7.1 Summary

Standoff gamma-ray imaging can be applied in many scenarios relevant to several different fields, including nuclear security, safeguards, emergency response, and nuclear contamination remediation. More generally, gamma-ray imaging is a suitable tool for any application where the ultimate goal is to localize gamma-ray sources, or to characterize the distribution of gamma-ray sources in an extended environment; generally described as gamma-ray search or gamma-ray mapping scenarios. Conventional 2D gamma-ray imaging techniques have been applied to a number of these scenarios, including mapping contamination after a nuclear accident [1, 2] and other environmental monitoring [104], nuclear security [105, 106, 107, 108], and facility monitoring [13, 23]. In fact, many applications for gamma-ray imaging have become so compelling that some 2D imaging technologies have been recently made commercially available [109, 110].

Many of these demonstrated applications would greatly benefit from the extension of 2D imaging into full volumetric imaging; expanding the source localization and characterization capabilities to full 3D. Volumetric imaging has been an important tool in medical diagnostic imaging for decades, yet the approaches used in this regime depend on the limited physical extent of the imaging space. Thus the volumetric imaging methods used for medical imaging are not directly applicable to in situ applications of gamma-ray imaging, indicating the need for a different approach for far-field volumetric gamma-ray imaging. The potential for the extension to full 3D spatial gamma-ray image reconstruction was recognized and shown to be feasible with the development of a 3D imaging algorithm based on the Compton modality [16].

The work in [16] demonstrated that 3D reconstruction was possible by sampling the gamma-ray distribution in a given environment from multiple perspectives. However, the measurement approach had several drawbacks that make it infeasible for general imaging applications: the manual positioning of the detector at several locations is time-consuming and subject to error; the measurement times at each

\[1\] The above references reflect only a small portion of active research in applications of gamma-ray imaging.
CHAPTER 7. CLOSING REMARKS

location are long (on the order of 24 hours); and many thousands of Compton cones were acquired for the reconstruction. The work presented in this thesis overcomes these limitations by integrating auxiliary sensors and online SLAM solvers with mobile gamma-ray imagers to provide detector tracking capability and a 3D model of the environment in real-time. This approach, scene data fusion, engenders multiple advantages due to the mobile mode of imaging operation, including:

1. The ability to acquire gamma-ray data from the multiple perspectives necessary for a full 3D reconstruction of the spatial distribution of gamma-ray sources.

2. Detector tracking combined with real-time imaging and spectral feedback allow the user to overcome the geometric attenuation of the signal; a fundamental limit to static gamma-ray imaging modalities.

3. The sampling of gamma-ray distributions from multiple perspectives can help in scenarios where gamma-ray sources are partially shielded.

The 3D model of the environment provides additional advantages. Though difficult to quantify, the robustness and interpretability of volumetric gamma-ray images are arguably greater than directional gamma-ray images overlayed on conventional 2D RGB images. The 3D model can also be directly incorporated in the gamma-ray reconstruction, constraining the imaging space thus improving the tractability of the inversion problem while reducing computation time.

Given the potential benefits of SDF for a wide variety of potential applications, the development was pursued in this and other work in several ways. A real-time iterative 3D gamma-ray image reconstruction algorithm was developed based on the Compton modality. Compton imaging is particularly well suited for many source search and mapping applications due to the wide field-of-view and sensitivity over a wide range of gamma-ray energies. The SLAM approach chosen for this work was RGB-D Slam, a specific real-time SLAM algorithm based on RGB-D cameras such as the Microsoft Kinect. Both RGB-D Slam and the 3D gamma-ray imaging algorithm were integrated by means of a flexible software framework implemented in Python and C. The software framework and auxiliary Kinect sensor were then combined with two separate gamma-ray imagers, the VCI and HEMI. The VCI is a cart-based mobile gamma-ray imager comprising two planar HPGe detectors with segmented readouts providing sensitivity to the 3D position of gamma-ray interactions, as well as the excellent energy resolution characteristic of HPGe. HEMI by contrast is a CdZnTe-based system comprising 96 individual 1 cm$^3$ CZT crystals each with a co-planar grid readout arranged in a two-plane, active mask configuration. HEMI is packaged in a light-weight, hand-portable format for this work. The two gamma-ray imagers have very different characteristics. For example, the angular resolution of the VCI is nearly 3x better at around 4° at 662 keV compared to 11° in HEMI. Yet HEMI is much more mobile that the VCI, allowing the exploration of new environments, investigation of new applications, as well as testing the robustness of SDF for more challenging and realistic scenarios. Each
of these systems is used to demonstrate real-time volumetric gamma-ray imaging using the SDF method. The ability to simultaneously localize multiple gamma-ray point sources of different energy with measurement times on the order of 10’s of seconds is demonstrated with each system, as well as demonstrating the impact of incorporating the scene model into the reconstruction. The spatial reconstruction of gamma-ray source distributions spanning a wide range of gamma-ray energies, measurement environments, and gamma-ray source activities is successfully demonstrated using both imagers, providing qualitative evidence of the feasibility of SDF. More challenging scenarios, including imaging multiple gamma-ray point sources of the same energy, localizing shielded gamma-ray sources, and localizing gamma-ray sources within objects demonstrate the potential of SDF for many compelling applications.

Having demonstrated the efficacy of SDF for many different in situ gamma-ray imaging scenarios, a more quantitative evaluation of SDF-based imaging was undertaken. The volumetric imaging is evaluated from a number of perspectives: the accuracy of 3D localization and spatial image resolution, the signal-to-noise ratio of the volumetric images, and the repeatability of the localization. Many of these metrics are evaluated in terms of variation in the data input into the image reconstruction algorithm. For instance, the robustness of the volumetric localization of a point gamma-ray source is evaluated against the number of Compton events acquired during the measurement via a downsampling procedure. The results from this study provide quantitative evidence of the feasibility of SDF for localizing gamma-ray point sources in 3D, with a high rate of successful recovery of the source distribution with as few as 100 Compton cones, corresponding to measurement times on the order of 10’s of seconds even for gamma-ray sources with activities $\leq 10 \mu Ci$. The accuracy of the reconstruction is likewise studied vs. Compton event rate using both the VCI and HEMI. Reconstructed images are coarsely evaluated in terms of the uncertainty of the cone opening angle, extending an analysis of the impact of Compton data quality [32] to three dimensions. As SDF depends on auxiliary data streams, potential sources of uncertainty in these data are identified and one of them, uncertainty in the translational pose estimate, is investigated to measure its impact on volumetric localization. The results provide quantitative evidence of the efficacy of SDF and highlight considerations and requirements for SDF imaging systems beyond the specific instruments studied in this work. For instance, the study of Compton event rate and Compton cone uncertainty identifies the existence of a trade-space between quantity and quality of Compton data. Though the detector tracking relies on the specific RGB-D Slam approach, many of the uncertainties in the pose estimates are of a form prevalent with other tracking modalities, both visual and non-visual. The importance of minimizing the effect of accumulated error in the selection of a tracking method has been shown to be paramount to the accuracy of a mobile gamma-ray imaging system. The quantitative evaluations are limited in scope, probing only a small region of a high-dimensional parameter space that depends in complicated ways on the visual and geometric make-up of the environment, and the path of the detector through that environment. Nevertheless, these preliminary evaluations highlight the importance of some dependencies in terms of improving SDF for future applications.
7.2 Conclusion

Standoff volumetric gamma-ray imaging has many potential applications in nuclear security, safeguards, and other source-search or gamma-ray source mapping scenarios. The additional information in depth over existing directional (2D) gamma-ray imaging methods is valuable for many in situ applications of gamma-ray imaging, and is a necessary step towards the ability to quantify gamma-ray source activities from standoff imaging measurements. While volumetric imaging has been a staple of medical diagnostics for decades, the challenges faced in standoff imaging scenarios, particularly limited sampling capabilities; low count rates; and undefined, large-scale imaging spaces necessitate the development of new methods. This work demonstrates that these challenges can be overcome by a mobile mode of operation for gamma-ray imagers, with additional benefits from incorporating complementary information about the imaging environment into the gamma-ray image reconstruction. The ability to track the location and orientation of the gamma-ray imager in real-time, as well as a 3D model of the scene, is provided by integrating a real-time SLAM algorithm and auxiliary sensors with a mobile gamma-ray imager. This integration is referred to as scene data fusion, as the pose estimation and scene reconstruction from the SLAM solver are derived from direct observation of the scene while it is being traversed. Operating in the mobile mode, the imager can acquire gamma-ray data from the multiple perspectives necessary for deriving the full spatial distribution of gamma-ray sources based on the principle of triangulation. Scene data fusion also allows additional information about the scene, e.g., a 3D model encapsulating the geometry of the environment, to be directly incorporated to the gamma-ray image reconstruction. The additional constraint on the imaging space from this auxiliary information can reduce image noise and greatly reduce computation time an more efficient use of computational resources. The ability to localize gamma-ray point sources in real-time was demonstrated with multiple different imaging platforms via SDF and a real-time 3D Compton image reconstruction algorithm. The gamma-ray source localization capability is further improved by the mobile mode of operation, using the real-time imaging feedback to move to regions with high gamma-ray signal, overcoming the $\frac{1}{r^2}$ factor that limits static gamma-ray imagers. Subsequent evaluation has shown that the 3D localization capability is accurate and repeatable for gamma-ray sources with activities on the order of 10 $\mu$Ci with measurement times less than a minute.

This work presents a concept, implemented on two different mobile gamma-ray imaging systems, and demonstrates results with each instrument for a wide variety of imaging scenarios, in addition to a preliminary evaluation of the performance and dependencies of the presented imaging approach. It is important to keep in mind that the implementation, measurements, and evaluation presented here represent only a single, specific instance of the more general concept of scene data fusion. This work has presented a specific implementation of volumetric imaging based on the Compton modality, but the SDF approach does not exclude other imaging modalities such as coded-aperture or other collimation schema. In fact, the tracking and 3D model provided by SLAM can even be coupled with non-imaging gamma-ray sensors and still provide localization capability via proximity, i.e., proximity
imaging, where variations in gamma-ray intensity as a mobile sensor moves throughout the scene are used to reconstruct the spatial distribution of gamma-ray sources. Even within the realm of volumetric Compton imaging, the current work is narrow in scope relative to the potential areas for exploration. The imaging systems here are hand-portable and require manual operation. Many compelling applications motivate the study of other platforms beside human operators; nuclear contamination mapping via aerial sensors for instance. SDF is quite relevant in this domain as well, and one could envision the extension of the gamma-ray sensor fusion concept to autonomous systems. In demonstrating the feasibility of and applications for a particular instance of SDF, the author hopes to have provided a strong case for future research in the integration of gamma-ray sensors with mobile, scene-sensing systems.

In all, this work is intended to motivate the potential advantages of volumetric gamma-ray imaging, and detail the conception, implementation, demonstration, and a preliminary evaluation of the approach used to achieve it. Real-time tracking from SLAM not only enables real-time volumetric imaging, but allows for the exploration of applications based on detector tracking without the need for highly-specialized instruments. There is much to be explored and evaluated in the mobile imaging paradigm, but it is clear that SDF provides new capabilities and can address issues that are fundamental limitations in static scenarios like geometric $\frac{1}{S^2}$ attenuation. Though not explicitly investigated in this work, the ability to reconstruct the full 3D distribution of gamma-ray sources in the general case is a necessary step towards the development of standoff quantitative gamma-ray imaging. Neither the VCI nor the HEMI imagers were optimized for imaging efficiency or precision, yet accurate volumetric images are consistently reconstructed with a demonstrated robustness for very few Compton events. The applicability of gamma-ray imaging to a wide variety of radiation search and mapping scenarios is greatly enhanced by SDF, providing 3D localization capability with improved environmental context in real time without requiring significant improvements over modern portable gamma-ray imaging technology.
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