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Publication Date
1965-10-01
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THE STACKING FAULT ENERGY OF A Ag-In ALLOY
FROM NODE MEASUREMENTS

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October 1965
The Stacking Fault Energy of a Ag-In Alloy
from Node Measurements

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ABSTRACT

Experimental measurements of the inscribed radius, the outer radius and the character of extended nodes have been compared with the theoretical predictions of the recent work by Brown\(^1\) (1964), Siems\(^2\) (1964) and Jøssang et al.\(^3\) (1965). The most comprehensive comparison has been made for the two former theories, which have been found to predict accurately the observed node parameters. Within the range \(\gamma/Gb < 10^{-2}\) the node method appears to be an excellent means of determining the stacking fault energy.
I. INTRODUCTION

The stacking fault energy of a face centered cubic metal is a parameter of considerable importance, in that it determines the equilibrium separation of partial dislocations in metals and is thus a primary factor in governing the dislocation configurations which develop. In addition, the stress corrosion properties of alloys may be closely related to the stacking fault energy (Robertson and Tetelman, 1962) although a detailed interpretation capable of correlating all the experimental results has not yet been obtained.

Although the importance of the stacking fault energy (γ) has been appreciated since Heidenreich and Shockley (1948) showed that dislocations in f.c.c. metals will generally split into two partials separated by a band of faulted crystal, there is still some controversy concerning the absolute magnitude of γ in some materials. There is no shortage of methods of determining γ for the magnitude of the stacking fault energy is of such basic importance in describing the mechanical properties of a metal that a wide range of measurables may be more or less directly related to γ. A review of many of these techniques has recently been made by Christian and Swann (1964). In addition, Gallagher (1964) and Smallman (1965) have examined a number of published results, particularly for copper, gold and silver pointing out that many of the values quoted in the literature are doubtful due to subsequent developments in the theories relating the measured crystal properties to the stacking fault energy.

As far as possible it is desirable that the following conditions should be satisfied when making determinations of the stacking fault energy.
The measurable parameter should be strongly dependent on \( \gamma \).

The dependence of the measurable on \( \gamma \) should be as direct as possible so as to minimize the intervening theory.

The theory relating the measurable to \( \gamma \) should have been rigorously developed.

The measurable should be easily defined and be accurately measured, with a minimum of subjectivity.

The method should preferably be applicable to metals and alloys over a wide range of the stacking fault energy.

Specimen preparation and experimental procedure should if possible be simple and rapid.

It is suggested by the fact that a wide range of techniques have been employed that the above conditions are not completely satisfied by any one of them. In attempting to evaluate the respective worth of the various approaches it is convenient to make a distinction between those which involve macroscopic measurements of crystal properties, and those which depend upon a direct examination of defects within the crystal.

The former category includes determinations from the strain-rate dependence of \( \tau_3 \) (e.g. Peissker, 1965), from an examination of rolling textures (Dillamore et al., 1964), by measuring x-ray peak shifts (Vassamillet and Massalski, 1963), or from the stored energy following cold work (Holt, 1965). All these methods are limited with respect to condition (2) above, and, in addition, condition (3) is not always well satisfied. Because of these basic limitations the present author feels that the most reliable determinations of the stacking fault energy can be made at the present time from observations at the microscopic level.

It is well known that the nature of the dislocation distribution in
metals changes from a cell structure when the stacking fault energy is high, to co-planar groups for low stacking fault energy. This variation is related to the ease with which dislocations may cross-slip, and a qualitative estimate of the magnitude of $\gamma$ may be made simply by inspection of the general dislocation arrangements.

In addition, the nature of vacancy clusters and some specific dislocation interactions can be used to obtain a much more quantitative determination of the stacking fault energy. By comparing the shrinkage rate of faulted and unfaulted loops Kannan and Thomas\textsuperscript{13}(1965), and Edington and Smallman\textsuperscript{14}(1965) have determined the stacking fault energy of Al and Al-Mg alloys. This technique is particularly useful in that it is applicable to metals of high stacking fault energy. Czjzek et al.\textsuperscript{5}(1962) related the stacking fault energy to stability criteria for vacancy tetrahedra and planar configurations, and their theory has been used for a number of determinations of $\gamma$ (e.g. Seeger,\textsuperscript{16}1964; Loretto et al.,\textsuperscript{17}1964). More recently it has been suggested (Humble, Segall and Head, to be published; see Loretto et al.,\textsuperscript{17}1964) that Czjzek et al.'s theory predicts values of $\gamma$ that are too low. Difficulties exist in quenching methods due to the large effect of quenching conditions (plastic deformation during the quench, for instance; see Cotterill and Segall,\textsuperscript{18}1963).

Whelan\textsuperscript{9}(1959) was the first to show that extended three-fold nodes can provide a direct measure of the stacking fault energy of a crystal. Early measurements on extended nodes observed in transmission electron microscopy by Howie and Swann\textsuperscript{2}(1961) and Siems et al.\textsuperscript{21}(1961) confirmed that $\gamma$ could be readily determined in this way. Subsequent theoretical
treatments have shown that a careful consideration must be made both of the interaction between the partial dislocations in the node, and of the variation of the line energy of the dislocations with character. Contributions to this specific problem have been made by Brown\(^1\)(1964), Siems\(^2\)(1964) and Jøssang et al.\(^3\)(1965). For metals of reasonably low stacking fault energy the node method now appears to be the most favorable, its only major limitation being with respect to condition (5).

It is the purpose of the present paper to compare experimentally measured node parameters (e.g. inscribed radius (width), outer radius, equilibrium partial separation, character) with the predictions of the various theories relating them both to one another and to the stacking fault energy. The present comparison of experiment and theory is undertaken as a prelude to an investigation of the stacking fault energy in a number of alloys, so that the accuracy and limitations of the node method may be clear.

In the next sections the predictions of the published theories relating \(\gamma\) to the node parameters are considered, the experimental results which have been obtained will be presented, and the agreement with experiment of the present theories will be discussed.

II. The Relationships between Node Parameters and the Stacking Fault Energy

Figure 1 shows the notation which is used to describe the conveniently measurable parameters of a node. Whelan's\(^{10}\)(1959) theory considered the equilibrium of the partial dislocations to result from a force \(G b^2/2R\) per unit length due to the line tension (the line tension is taken as
Gb^2/2; b is the Burgers' vector of the partial dislocation, and R its radius of curvature), opposing a force \( \gamma \) per unit length from the stacking fault. On this basis

\[
\gamma = \frac{Gb^2}{2R} \tag{1}
\]

but some of the limitations of Eq. (1) were removed in the formula used by Howie and Swann\(^{20}\)(1961),

\[
\gamma = \frac{Gb^2 \ln (R/\varepsilon)/4\pi KR}{\ln (R/e:\)}/4\pi KR \tag{2}
\]

where \( \varepsilon \) is the cut-off radius (\( \sim b \)) and \( K \) has the value 1 for a screw node, and \( (1 - v) \) for an edge node (in Fig. 1, \( \alpha = 0 \) for a screw node, \( \alpha = 90^\circ \) for an edge node). Brown\(^{1}(1964)\) in a reiterative numerical solution of the problem of node shape made allowance both for the variation of line tension with node character, and for the interaction of the partials, and related both \( w \) and \( R \) to the stacking fault energy. Using the same notation as employed earlier, with \( v \) as Poisson's ratio, the following equations are from Brown and Thölenn\(^{22}\)(1964).

\[
\gamma R/Gb^2 = 0.27 - 0.08 \left( \frac{v}{1-v} \right) \cos 2\alpha + 0.104 \left( \frac{2-v}{1-v} \right) + 0.24 \left( \frac{v}{1-v} \right) \cos 2\alpha \log_{10} (R/\varepsilon) \tag{3}
\]

\[
\gamma w/Gb^2 = 0.055 \left( \frac{2-v}{1-v} \right) - 0.06 \left( \frac{v}{1-v} \right) \cos 2\alpha + 0.018 \left( \frac{2-v}{1-v} \right) + 0.036 \left( \frac{v}{1-v} \right) \cos 2\alpha \log_{10} (\Psi/\varepsilon) \tag{4}
\]

Siems\(^{2}(1964)\) improved on the earlier theory of Siems et al.\(^{21}(1961)\) by allowing for the variation of line energy with dislocation character, and obtained analytical solutions for the width of the node (\( \Psi \)) and for its outer radius (\( R \)). In the present notation Siems\(^{2}(1964)\) equations are,
\begin{align*}
R &= \frac{G b^2}{4\pi Y} \frac{w}{d} \left[ 1 + \frac{v}{(1-v)} \left( \frac{3 \cos 2\alpha + 1}{2} \right) \right] \ln e (R/e) \quad (5) \\
R &= \frac{w \sqrt{3}}{d (\sqrt{3w} - 1)} \left[ 1 + \frac{5v \cos 2\alpha}{(2 - v(1 + \cos 2\alpha))} \right] \ln e (R/e) \quad (6)
\end{align*}

and from Read, 1953,
\[ d = \frac{G b^2}{8\pi Y} \frac{2 - v(1 + 2 \cos 2\alpha)}{1 - v} \quad (7) \]

Thus, measurements of \( R \) and \( w \) lead to a determination of the stacking fault energy, while the character of the node, \( \alpha \), and the appropriate values of \( G \) and \( v \) must be known. The value of \( R \) need only be known approximately as long as \( w \) is measured accurately, since the expressions (3) - (6) are relatively insensitive to the log term. The uncertainty in the magnitude of the cut-off radius, \( e \), (usually taken as of the order of the Burgers' vector of the partial dislocation, \( b \)) makes the ratio \( R/e \) unknown to within a factor \( \sqrt{2} \) even if \( R \) is known with considerable accuracy.

Recently, a third independent comprehensive treatment of the relationship between node parameters and \( \gamma \) has been published by Jóssang et al. (1965). They approximated the true node shape using doubly angular dislocations, and calculated the minimum energy configuration. To determine the stacking fault energy their relationships require a measurement of \( w \), the inscribed radius, together with \( \alpha \), and \( d \) if possible. They do not relate \( R \) to \( \gamma \) which makes it more difficult to establish the accuracy of their results experimentally, since in metals \( d \) cannot usually be measured in the range \( d \sim 50 - 100\AA \), because the dislocation image widths are of the same order. 
Thus, we have the choice of three sophisticated relationships between node parameters and $\gamma$, and an obvious first step is to investigate their consistency, one with another. To do this the following procedure has been followed. A series of imaginary but typical metals have been defined, for each of which $G = 2.5 \times 10^{11}$ dynes/cm$^2$ and the partial Burgers' vector, $b = 1.5\text{Å}$ = the cut-off radius, $c$. In Table I calculations have been made for screw nodes on the theories indicated, for values of Poisson's ratio $v = 0.2$, $0.3$ and $0.4$. In each case two node sizes have been considered, $R = 500\text{Å}$ and $1000\text{Å}$, and to illustrate the theoretical predictions for a wider range of $R$, $R = 2000\text{Å}$ is considered for $v = 0.4$. The comparison has been made for screw nodes since most experimentally observed nodes have characters close to this, the minimum energy configuration.

As can be seen from Eq. (2) the relationship used by Howie and Swann (1961) is independent of $v$ for screw nodes. For the theory of Brown, and that of Siems, $\gamma$ can be calculated both from $R$ and from $w$. Accordingly $\gamma$ is in each case determined from the values of $R$ (Eqs. 3, 5, and 6) and then in the formula relating $\gamma$ to $w$, the value of $\gamma$ just determined is used to discover what $w$ would be for that particular node (using Eqs. 4, 5 and 6). It is interesting to compare the tabulated values of $R$, $\gamma$ and $w$ from the Brown and Siems theories; for all values of $v$, $R/w$ is the same within $3\%$ for $R = 500\text{Å}$, and within $6\%$ for $R = 1000\text{Å}$. The magnitudes of $\gamma$ determined from the identical values of $R$, and almost identical values of $w$, are, however, consistently lower on Siems' theory. With $R = 500\text{Å}$ and $1000\text{Å}$, for $v = 0.2$, $\gamma$ is $14\%$ lower, for $v = 0.3$, $\gamma$ is $10.5\%$ lower, and for $v = 0.4$, $\gamma$ is $7.7\%$ lower, while for $v = 0.4$ and $R = 2000\text{Å}$, $\gamma$ is
5.8% lower.

Since the theory of Jøssang et al. relates only \( w \) to \( \gamma \), the values of \( w \) derived from Brown’s theory (unbracketed in Table I) have been used to calculate values of \( \gamma \). We see that the results thus obtained, are consistently 22% lower than those calculated after Brown. We also see, as Brown (1964) has pointed out, that Howie and Swann's theory in the case of screw nodes leads to lower predicted values of \( \gamma \) than are correct. Contrary to the statement of Jøssang et al. (1965), it is clear that Howie and Swann's published results underestimated the correct values of \( \gamma \).

It is satisfactory that the independent calculations of Siems and Brown, in the one case analytical and the other numerical, should give predicted values of \( \gamma \) that differ only within the limits mentioned above. The results calculated from all three theories show a consistent variation of \( \gamma \) with node parameters, although the absolute values are somewhat different. Rather than attempt an appraisal of the theoretical techniques employed by the above authors, an experimental survey has been made of a relatively low stacking fault energy silver-indium alloy \((e/a = 1.23)\). The principal measurements made have been of the inscribed radius, \( w \), the character of the node, \( \alpha \), and wherever possible, of \( R \), the outer radius. Attempts have also been made to measure \( d \), the equilibrium separation of the partial dislocations, and the difficulties involved in these measurements will be mentioned below. By choosing an alloy of low stacking fault energy the nodes were sufficiently large to enable the measurements to be made with small percentage error.
III. Experimental Procedure and Results, and Comparison with the Theory of Brown (1964)

The silver-indium alloy was prepared from high purity oxygen free elements. Subsequent analysis by Johnson and Matthey showed that the indium content was 12.5 wt.%, and that 61 p.p.m. of Cu, Fe, Mg and Th were present. Gas analysis indicated an oxygen content of 3 p.p.m. The alloy, sealed in an evacuated Vycor capsule was quenched into water, and after cold-rolling to a thickness \( \approx 7 \) mils was homogenized at 500\(^\circ\)C for 48 hrs. After light deformation in torsion the strips were thinned for electron microscopic examination. The rolling texture led to specimen normals close to [110], and nodes were photographed in bright field using either 002 or \( \{111\} \) type reflections. A [\( \{111\} \)] specimen normal is more convenient in that the nodes lie in the plane of the foil, thus being less susceptible to disturbance by the foil surfaces, and also making a projection correction unnecessary. However, by taking pictures with the foil normal always close to [110] the same correction factor (\( \approx 10\% \)) applied to all the nodes.

In order to determine the node parameters with the best possible accuracy particular attention was devoted to photographic procedure and improvements in measuring devices. It was found that the optimum microscope magnification was x20,000 and so that the node character, \( a \), could be determined diffraction patterns were always taken. For the example shown in Fig. 2 the specimen normal is close to [110], and the operative reflection is 002, so that \( a \) may be unambiguously obtained from one photograph. Due to the reflecting conditions \( g.b = 0 \) for one node arm, and from the contrast in the node network it is clear that arm B is the one in question. Since the traces of [0\( \{1\} \)], [101] and of [\( \{101\} \)], [011]
coincide the node character may now be determined irrespective of whether
the node lies on the (111) or (111) plane. In this case, $\alpha_A = 14^\circ$, 
$\alpha_B = 19^\circ$ and $\alpha_C = 23^\circ$, which is typical of the accuracy of the character
determinations which have been made.

The effect of variations in the imaging conditions has also been
examined. Using the 002 reflection, six pictures were taken of the same
nodes considerably varying $s$, the deviation from the Bragg position, and,
although the quality of the pictures was markedly affected, in the two
cases investigated the inscribed radius was constant to within $\pm 7\%$. A
more detailed investigation utilizing dark field is planned, since the
effect of image width is of importance particularly when the inscribed
radius of small nodes is being measured.

For the large nodes found in the present alloy, the node radii
(inscribed and outer) were measured by projecting an image of the original
plate on to the screen of a Vanguard Motion Analyzer, at a further
magnification of from x2.5 to x16. This instrument is equipped with
cross wires enabling translations to be measured to 0.001" in two directions
at right angles, and is particularly useful for measurements of this kind.

For the smaller nodes found in alloys of higher stacking fault energy an
intermediate negative is prepared from the original plate at a further
magnification of x17.5, so that employing the Vanguard the nodes may be
examined at a total magnification of up to 5 million. In this way,
inscribed radii as small as 70Å, and outer radii of 300Å have been
measured with a reproducibility of better than 10%. It is worth pointing
out that one of the few advantages of measuring $R$ rather than $w$ is that
the image width of the partials does not affect the accuracy of the determination, whereas for small nodes particularly it is important to correct the measured value of $v$. Measurement of a small node's inscribed radius to ±10% does not, therefore, imply that $\gamma$ is at once known to the same accuracy.

It has been found that using the geometrical theorem illustrated in Fig. 1, namely,

$$z^2 = (2R - x)x$$

leads to a much less subjective determination of the outer radius, $R$. The lengths $2z$ and $x$ can be measured in the present apparatus with considerable accuracy and reproducibility, and the method has been found to be much superior to attempting to fit a circle to the curved part of the node. Care must be taken in choosing the chord length, $2z$, so as to include only the curved section of the partials which approximates closely to a circle. Clearly, a compromise must be reached, since for a short chord length the measurement of $x$ particularly is subject to large errors.

Accurate measurements of $R$ are essential if the theoretical predictions of Brown, and of Siems are to be investigated. That this is the case is illustrated in Fig. 3, which shows the variation of $\gamma R/Gb^2$, $\gamma w/Gb^2$ and $R/w$ as a function of node character, $x$, calculated from Brown's theoretical relationships. As can be seen from Eqs. (3) and (4), $v$ and $R/e$ must be known to make these plots. The relationships between $v$, $G$, and the elastic constants are those of Aerts et al.,$^{24}$ 1962, and the elastic constants for Ag-In alloys have been taken from Bacon and Smith,$^{25}$ 1956. The value of $R$ used in the term $\log R/e$ is the mean determined from all the measurements.
on this alloy, $\bar{R} = 2200 \pm 150\text{Å}$.

Figure 3 illustrates well why the inscribed radius, $v$, is the best node parameter to measure in that it is practically independent of node character. On the other hand, $R$ varies by a factor $\sim 2$ from screw to edge character, and the character of each node which is used must be determined with accuracy, due to this dependence of $R$ on $\alpha$. A limitation on the comparison of theory with experiment is that in the present work nodes have only been found in the range of character, $\alpha = 0$ to $\alpha = 40^\circ$ (marked in Fig. 3). In order to test the theory measurements of $\gamma R/Gb^2$ and/or $R/w$ are required as a function of character. Brown's theory predicts a decrease of $\sim 2\%$ in $\gamma R/Gb^2$ and $R/w$ from $\alpha = 0$ to $\alpha = 40^\circ$. Similar predictions are made by Siems theory so that our main concern is to establish how accurately the Brown-Siemens theories fit the experimental results, and good absolute agreement will indicate that their theories are preferable to that of Jössang et al., on account of the systematic discrepancies to be found in Table I.

Figure 4 illustrates the distribution of node character determined experimentally for this alloy. Forty two nodes have been used to determine $\bar{v}$, the mean value of the inscribed radius, and the character has been unambiguously obtained for 38 of these as shown in the figure. Only the ten best nodes have been used for measurements of the outer radius, $R$. In taking measurements of the three outer radii it has been found that, having allowed for projection effects, very few of them are completely symmetrical. Due to this, they are, of course, not completely described by the theories, in which the calculations are for minimum energy configuration.
symmetrical nodes. It is clear from Fig. 4 that the nodes which have been investigated do not behave ideally as described by the theories since the peak in the distribution as a function of $\alpha$ does not occur at $\alpha = 0$, the screw orientation. Screw nodes are, however, the minimum energy configuration, and the results seem to imply that the nodes are prevented from reaching their most favorable equilibrium configuration by internal stresses, more particularly due to pinning of the dislocation lines which have joined to form them, or, as proposed by McLean (1963), and discussed further in the light of evidence from heating experiments in Cu-7.3% Al by Christian and Swann (1964), as a result of a solute impedance force. That the screw node is not the most frequently observed has also been found by Ericsson (1965) in 67% Co/33% Ni for which at 310°C the median value of $\alpha$ was $\sim 30^\circ$, while none of the nodes were within 10° of pure screw orientation. In work to be published by the present author, a variation of $\alpha$ was observed while a specimen of Ag-12.5% In was heated in the electron microscope, with some node arms rotating 15 - 20°. In addition, irreversible collapse of extended dislocations occurred. It is proposed to examine such changes in detail in order to separate the possible contributions to the non-symmetrical shape due to dislocation pinning, from internal stresses, and image forces near the foil surface, from solute impedance and in some cases from anisotropy.

However, despite the indications that the nodes generally observed cannot be fully described on the assumption that they are in the equilibrium lowest energy configuration the comparison with theory which is to be made should enable the probable magnitude of the deviation to be estimated.
Figure 5 illustrates the measurements made of $w$ as a function of $a$. Less than half a dozen of the values lie more than $\pm 120^\circ$ from $\bar{w}$, and the standard deviation about the mean is only $15^\circ$. As one expects from the theory, $w$ has no obvious dependence on $a$, although the spread is too large to claim this with complete confidence.

Figure 6 shows the determinations of $R$ as a function of $a$. Once again the spread is such that one can claim no more than that the experimental points are consistent with the theories of Brown, and Siems, and that the agreement is good considering the difficulty attached to measuring $R$. Apart from two rather high values, the points lie close to the theoretically predicted curves, and show the required systematic decrease in $R$ with increasing $a$.

Figure 7, in which $R/w$ is plotted against $a$, affords the best test of the theoretical predictions, since the ratio of the two radii $R$ and $w$ shows fewer of the fluctuations caused by abnormally large or small nodes in Figs. 5 and 6. It is satisfactory that both the absolute magnitudes of the experimentally determined ratio and its variation with $a$ are in good agreement with theory. The values of $R/w$ (screw) from experiment and theory are in agreement to better than 2.5%. Once again the theoretical curve of Siems lies close to that of Brown, indicating as one might expect from Table I that, especially for alloys with a large Poisson's ratio, both the theories describe the node configuration accurately. The present results, in conjunction with the comparisons made previously by Brown and Thölén,1964, and Loretto,1964, provide strong evidence for the validity of the theories of Brown and Siems in determining stacking fault
energies.

It does appear, however, from the systematic discrepancies in Table I, and the good absolute agreement of the experimental results with the theories of Brown, and Siems, that Jøssang et al.'s theory provides a slightly less accurate relationship between node parameters and the stacking fault energy. This appears to stem from the fact that Jøssang et al. related $\gamma$ to a configuration of straight dislocation segments which differed sufficiently from the true node shape to effect the contributions from the interactions between partials, and from the variation of line tension with character.

It is particularly pleasing in Fig. 7 that the experimental ratios $R/w$ are in such close absolute agreement with those predicted by theory. Although $R/w$ is not excessively dependent on node configuration, it could nevertheless be affected by, for instance, a solute impedance force. An estimate of the magnitude by which $R/w$ could be changed by such a force is readily obtained, and we shall make a calculation for a node typical of those in the present work. Using the notation of Fig. 1, the effect of solute impedance is to alter the relative magnitudes of $w$ and $x$ in the sum $(w + x)$ through a variation in the position of the partial dislocation, while we may consider the position of the node arms to be unaltered, although $z$ may change due to the altered curvature of the partials. Using Eq. (8) we have

$$8R = 1/2\left(\frac{2z}{x^2}\right) 8z + 1/2\left(-\frac{z}{x^2} + 1\right) 8x$$

(9)

In a typical case, for $R/w = 5.2$ the chord measured will have $2z - \frac{4}{5}w$ and $x \approx 0.4w$. Thus, $8R = 58z - 128x$, and we find,
\[ \delta(R/w) = \delta x/w (R/w - 12) + 5\delta z/w. \]  

(10)

where \( \delta x \) and \( \delta z \) are both positive. The extent to which \( R/w \) will be altered from its value in the absence of a solute impedance force clearly depends on the relative magnitudes of \( \delta x \) and \( \delta z \). If, as one limiting case, we take \( \delta z = 0 \), i.e. the change in node shape is only marked near the center of the partials, then even for a small change \( \delta x = 0.1w \), we find from Eq. (10) that \( \delta(R/w) = -0.7 \). However it is also clear from Eq. (10) that \( \delta(R/w) = 0 \), if for \( \delta x = 0.1w \) we also have \( \delta z = 0.14w \). It is by no means obvious, therefore, that distortion of the node will result from a solute impedance force, but a series of pictures under identical diffracting conditions on the same node after various annealing treatments should reveal more information.

Although it is clear that forces other than those allowed for in the theory are operative (evidence from lack of complete symmetry of nodes, and their distribution as a function of character) it appears that their effect is not sufficient to prevent the major predictions of the theory (Figs. 6 and 7) from being met experimentally.

It is possible, of course, that forces are present which change the scale of the node without influencing the absolute magnitude of \( R/w \), or the variation with character of \( R \), \( w \) and \( R/w \), and in such a case the value of \( \gamma \) calculated on the present theories will be incorrect. It is not immediately obvious what the effective magnitude of the stacking fault energy will be as far as the mechanical properties of a crystal are concerned in the presence of a solute impedance force. The fact that only a small temperature dependence of \( \gamma \) has been observed in Cu-Ge, Cu-Ga,
Cu-Zn and Ni-Co alloys (Haasen and King, 1962; Thornton, Mitchell and Hirsch, 1962; Pfaff, 1962) suggests, however, that the short-range solute impedance force is relatively ineffective. For a gliding dislocation the force will be in the same direction on both partials. The most meaningful value of \( \gamma \) will therefore be determined on nodes which have been annealed following formation in order to eliminate the solute impedance force.

Before going on to calculate \( \gamma \) from \( R \) and \( w \), a comment is necessary on the measurements of \( d \), the equilibrium separation of the partial dislocations in the node arms. No previous measurements of this parameter appear to have been published for metals. Using Eq. (7), the expected variation of \( d \) with node character, \( \alpha \), can be plotted for various values of \( \gamma \). A microdensitometer was used to determine \( d \), a number of traverses being made on each node arm. The measured values of \( d \) were much higher than expected for the known value of \( \gamma \). To within the rather large experimental error they did, however, show the expected variation with \( \alpha \), and have the expected absolute magnitude if a constant amount, \( 75\text{Å} \) was subtracted from each value. In Fig. 7, \((d_{\text{measured}} - t_0/\pi)\) is plotted, with \( t_0 \) the extinction distance for the operative reflection, and \( t_0/\pi \) is the approximate amount by which the dislocation image exceeds the separation of the partials. Since \( t_0/\pi \) is the maximum image width, Whelan, 1959.

For the 002 and 111 reflections used, \( t_0/\pi \approx 75\text{Å} \), and thus, having made some allowance for the breadth of the image, reasonable agreement with theory is obtained. It appears, however, that \( d \) cannot be a good parameter in metals, due to the difficulty in obtaining accurate measurements, and
the large fractional size of corrections which must be made.

IV. Calculation of \( \gamma \) from the Present Results

The inscribed radius, \( w \), is the parameter from which \( \gamma \) is most readily calculated, and \( w = 412 \pm 15 \text{Å} \). In addition, on the theories of Brown and Siems, \( \gamma \) may be obtained from the individual values of \( R \) and \( \alpha \). No correction has been made for the image width pending a more complete investigation, since good agreement has been obtained between \( \gamma \) determined from \( w \) and \( R \), indicating that the relatively small correction needed in \( w \) may be compensated for when measuring the node. Table I indicates that \( \delta \) need only be known to \( \pm 0.05 \) in order to determine \( \gamma \) to within 10%—it is usually known with considerably more precision than this. The value of the appropriate shear modulus cannot always be obtained for alloys from data in the literature, but an approximate value can be obtained from measurements of the Young's modulus, together with a comparison of the moduli of the alloying elements.

The magnitude of the cut-off radius, \( \varepsilon \), can only be estimated, and is of the order of the partial Burgers' vector \( b \). A variation in \( \varepsilon \) by a factor of 4 varies the calculated value of \( \gamma \) by \( \pm 20\% \). In Table II the stacking fault energy is tabulated as determined from the three recent theories, for three values of the cut-off radius.

Both Siems' and Brown's theories give results in good agreement from calculations using the outer radius, \( R \). For instance, on Brown's theory for \( \varepsilon = b \), \( \gamma = 5.67 \text{ ergs/cm}^2 \). It appears, therefore, from the excellent agreement in Table II that both Brown's and Siems' theories can be used with confidence in evaluating \( \gamma \).
Table II

The Stacking Fault Energy Determined from Radius Measurements

<table>
<thead>
<tr>
<th></th>
<th>$\gamma$ (Brown) $\text{ergs/cm}^2$</th>
<th>$\gamma$ (Siems) $\text{ergs/cm}^2$</th>
<th>$\gamma$ (Jøssang et al.) $\text{ergs/cm}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon = b$</td>
<td>5.55</td>
<td>5.47</td>
<td>4.6</td>
</tr>
<tr>
<td>$\varepsilon = b/2$</td>
<td>5.98</td>
<td>6.00</td>
<td>5.1</td>
</tr>
<tr>
<td>$\varepsilon = b/4$</td>
<td>6.41</td>
<td>6.52</td>
<td>5.5</td>
</tr>
</tbody>
</table>

As far as the random errors in the present work are concerned, $\gamma$ has been determined with some accuracy by making measurements on a large number of nodes. In addition good data have been used for $G$ and $v$. Possibilities of further error arise from the magnitude of $\varepsilon$, and from a systematic error of some sort in the node measurements, e.g. effect of diffraction contrast on node image size, uncertainty in original plate magnification, film shrinkage, inapplicability of theory due to further effects as discussed earlier.

Taking $\varepsilon = b$ as the most reasonable value of the cut-off radius, the results indicate that $\gamma = 5.6 \pm 0.3 \text{ ergs/cm}^2$ is not unduly optimistic. If allowance is made for a rather smaller value of the cut-off radius being possible, then it is more reasonable to quote $\gamma \sim 6.0 \pm 0.7 \text{ ergs/cm}^2$. 
In conclusion it is appropriate to comment on the extent to which the node method satisfies the conditions suggested earlier.

The strength of the dependence of $\gamma$ on $R$ and $w$ is clear from Table I, and is satisfactory.

The dependence of the node parameters on $\gamma$ is certainly direct in that the area and the shape of the fault depends only on the equilibrium between the repulsive and attractive forces. An annealing treatment may be necessary in some alloys to eliminate the solute impedance force so that the most meaningful value of $\gamma$ is determined.

That the theory relating the measurables to $\gamma$ is rigorous has been demonstrated in the present work. Further work is, however, necessary to understand the reasons for the deviations from complete agreement with theory.

The inner radius of a node can be defined and measured with ease, and although measurement of $R$ is more difficult it is nevertheless a reasonably good parameter.

A limitation of the method is that nodes are visibly extended only at relatively low stacking fault energies. The upper limit may be defined as the value of $\gamma$ at which $w$ equals the dislocation image width, $\gamma t_0 / \pi$.

Thus, from Brown's theory, in general

$$\frac{\gamma w}{G b^2} \sim 0.3$$

and for $w \sim t_0 / \pi$, and $b \sim 1.5 \times 10^{-8}$ cm, when low order reflections are used the node method is useful up to $\gamma_{\text{max}}$, where

$$\frac{\gamma_{\text{max}}}{G b} \sim 10 \times 10^{-3}$$
ACKNOWLEDGMENTS

Many thanks are due to J. Washburn and G. Thomas for helpful comments.
This work was supported by the United States Atomic Energy Commission.

REFERENCES

Table I

$\gamma$ As Determined after Howie and Swann (1961), Brown (1964), Siems (1964) and Jössang et al. (1965)

All calculations are for:

- $G = 2.5 \times 10^{11}$ dynes/cm$^2$
- partial Burgers' vector, $b = 1.5\,\AA$
- screw nodes ($\alpha = 0$)
- cut-off radius $\varepsilon = b = 1.5\,\AA$

After Howie and Swann  After Brown and Siems*  After Jössang et al.

<table>
<thead>
<tr>
<th>$\nu$</th>
<th>$\gamma$ (ergs/cm$^2$)</th>
<th>outer radius (Å)</th>
<th>$\gamma$ (ergs/cm$^2$)</th>
<th>inscribed radius (Å)</th>
<th>$\gamma$ (ergs/cm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>6.2</td>
<td>500</td>
<td>(9.7)</td>
<td>(119)</td>
<td>116</td>
</tr>
<tr>
<td></td>
<td>3.5</td>
<td>1000</td>
<td>(5.3)</td>
<td>(239)</td>
<td>226</td>
</tr>
<tr>
<td>0.3</td>
<td>6.2</td>
<td>500</td>
<td>(11.5)</td>
<td>(108)</td>
<td>108</td>
</tr>
<tr>
<td></td>
<td>3.5</td>
<td>1000</td>
<td>(6.3)</td>
<td>(216)</td>
<td>212</td>
</tr>
<tr>
<td>0.4</td>
<td>6.2</td>
<td>500</td>
<td>(13.8)</td>
<td>(102)</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>3.5</td>
<td>1000</td>
<td>(7.6)</td>
<td>(203)</td>
<td>193</td>
</tr>
<tr>
<td></td>
<td>1.9</td>
<td>2000</td>
<td>(4.2)</td>
<td>(403)</td>
<td>386</td>
</tr>
</tbody>
</table>

*The values of $\gamma$ and $\omega$ in parentheses are those calculated after Siems.
FIGURE CAPTIONS

Figure 1. The parameters of an extended node.

Figure 2. Character determination of an extended node.

Figure 3. The variation of $\gamma R/Gb^2$, $R/w$ and $\gamma w/Gb^2$ with character for the present alloy, calculated after Brown.

Figure 4. Node distribution as a function of character.

Figure 5. The measurements of $w$, the inscribed radius.

Figure 6. The theoretically predicted variation of $R$ with $\alpha$, and the experimental points.

Figure 7. $R/w$ vs. $\alpha$, experimental and theoretical (after Brown).

Figure 8. The variation of $(\bar{\gamma}_{\text{meas}} - \bar{\gamma}_{/\gamma})$ with node character.
Fig. 2
Calculated for
\[ \bar{R} = 2200 \text{ Å} \]
\[ \epsilon = b_p = 1.67 \text{ Å} \]
\[ \nu = 0.48 \]
Fig. 4
Inscribed radius (w) vs node character (a)

\[ \bar{w} = 412 \text{ Å} \]

Fig. 5
Theoretical curves:
- Brown (1964)
- Siems (1964)

Fig. 6
Experimental \( \frac{R}{w} \)\text{_{screw}} = 6.22

Theoretical \( \frac{R}{w} \)\text{_{screw}} = 6.07

--- Best fit (least squares) line through experimental points, \( \frac{R}{w} = 6.22 - 0.0356 \alpha \)

--- Theoretical curve (after Brown, 1964)

Fig. 7
Theoretical curve
(after Read, 1953)
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