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The Value of Time in Intercity Transportation: A Study of Thresholds and Discontinuities

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The Value of Time in Intercity Transportation
A Study of Thresholds and Discontinuities

By Rui Wang

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in the
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of the
University of California, Berkeley

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Abstract

The Value of Time in Intercity Transportation
-A Study of Thresholds and Discontinuities

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Previous research studies have provided evidence of the non-uniformity of the value of time, which usually shows a decreasing trend as travel time increases. This work takes an in-depth look at thresholds and discontinuities in the value of time function. A theoretical framework is provided based on microeconomic theory. It is postulated that because of the multiple activities involved in an individual’s activity pattern, and the minimum time requirements associated with these activities, there exist discontinuities in the travel cost function as the travel time encroaches upon the time originally assigned to other activities. The derivation of the indirect utility function of a trip is made in a multi-activity scenario, which shows the existence of discontinuous changes when the lowest time requirements of one or more activities are violated. In such cases, the activity may or may not be canceled. Two models are constructed depending on whether the cancellation is included. In Model 1, which assumes that the activity (of which the lowest time requirement is to be violated) cannot be canceled, the time assigned to the activity cannot be further reduced. As a result, further increase in travel time is at the expense of another activity. Therefore, Model 1 reflects a change of slope in the utility function. In the case of Model 2, the activity can be canceled. The cancellation of the activity results in a quantum change in utility function. Consequently, Model 2 reflects a change in slope together with a quantum decrease in utility.

The impact of the thresholds and discontinuities has long been overlooked, especially in intercity transportation. Using discrete choice modeling, empirical evidence of these discontinuities is found in air travel route choice. The thresholds where the discontinuities occur change with trip characteristics such as direction, and travel purpose. In general, the thresholds of business travelers are lower than those of leisure travelers. Additionally, there is evidence of a second threshold in the data. This is because as travel time keeps increasing after the first threshold is met, travel time starts to encroach on a second activity. The second threshold is hence possible as the binding condition may change again.

Because of the fewer variables involved in the estimation process, Model 1 is generally more stable and requires less computational effort. Based on the estimation results, whether the changes of utility at the thresholds are quantum (model 2) or not (model 1) remains an open question. Due to the limited data available for this study, the comparison...
results between the two models are tentative. More detailed data and in-depth research are needed to ascertain these results.

Numerical examples are used to illustrate the proposed models’ impact on airline network design in the choice of hub location. For future research, suggestions are made to incorporate the notion of thresholds in travel survey design in order to provide better bases for estimating their values and their impact on traveler behavior.
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1 Introduction and Research Objective

Imagine the following scenario: a businessperson working in San Francisco wants to join a three-hour meeting that could occur in any one of three locations: Seattle, Houston, or Boston. Assume that the only consideration in this decision is travel time, everything else being equal. At first glance, Seattle may appear to be an outlier as compared with the other two choices. Although in terms of travel time, the choices follow the sequence of Boston > Houston > Seattle, the perception is likely to be that the difference between Houston and Seattle is considerably larger than that between Houston and Boston. For instance, it is likely that the businessperson can attend the conference by taking an early-morning flight from San Francisco to Seattle and then returning the same day. On the other hand, for a trip to Houston or Boston, a same-day return may not be possible such that the businessperson will be obliged to arrange an overnight stay. These factors, such as the possibility of an overnight stay and the need to miss a day’s work, give rise to significant inconvenience, which is translated into disutility. As a result, the stronger disturbance to activity pattern and the resultant inconvenience once violated contribute to increasing the overall cost of travel in the second scenario. This increase goes significantly beyond the cost reflected in simply comparing the travel time to Seattle with that to either Houston or Boston.

A review of data pertaining to rail systems around the world reveals an interesting phenomenon whereby the travel demand does not vary smoothly with travel time. That is, when rail travel time drops below a certain threshold (around 2.5 hours), there is a drastic increase in travel demand (Vickerman 1997, De Rus 2009, Adler et al. 2010). This phenomenon suggests that there is a need to revise travel demand and mode choice model methodologies by incorporating the time attribute. In current models, the impact of most factors on the utility function is assumed to be constant or to change smoothly per unit change (and hence the level of service and the level of demand are assumed to follow the same pattern). And, in these same models, such relationships are described mathematically in the form of continuous functions. However, observations suggest that there is a threshold value for time such that there is a corresponding discontinuity in the value of the time function, or the utility function.

Failing to account for the discontinuity in the value of time can result in a distortion of the utility function and subsequent errors in demand models derived from it. In trip generation estimation, the overlooked increase in terms of the generalized cost of short- and medium-haul trips tends to lead to an overestimation of demand. Furthermore, in mode choice estimation, ignoring the discontinuity in the value of time and its resultant demand shift between modes leads to an underestimation of the demand for the faster mode (or an overestimation of the slower). Such inaccuracies can be significant enough to bias policy decisions. From the operator’s perspective, it can also distort the estimation of traffic and revenue, and hence can be very critical in system design.

The present research study takes an in-depth look at the impact of travel time on the demand for intercity travel and explores the existence of the threshold and discontinuities in the value of time. The study reconsiders the microeconomic theoretical background of
travel cost. It also justifies the hypothesis of ‘thresholds’ and ‘discontinuities’ both theoretically and empirically. The reformulation of travel time disutility implies ‘economy of travel time difference’ and shows significant impact in analyzing the inputs of transportation system design, which is also discussed by numerical example in this work.

This study is organized as follows: Section 2 reviews related literature covering a range of topics from time allocation theory to scheduling. Section 3 provides an introduction to the methodological framework used herein. Some preliminary models are estimated empirically in Section 5. A more developed model is presented in Section 6, with a cross-comparison between datasets. Section 7 provides the conclusion and a discussion of possible future research directions.
2 Literature Review

It should not be difficult to accept the concept of nonlinear VOT. However, that said, any concept requires theoretical support from multiple aspects in order to test the hypothesis and search for evidence of nonlinearity and discontinuity both in analytical and empirical terms. First, we need to know what the concept encompasses.

It is likely that researchers first encounter the concept in reference to the “activity-based model” and the “travel time budget.” The key aspect of the travel time budget is the belief that people either consciously or unconsciously allocate a certain amount of their everyday time budget (24 hours) to traveling. Rather than minimizing the total travel time, the traveler’s objective is to minimize the departure from that budget (in either direction). This idea originally grew from researchers’ dissatisfaction with the traditional four-step model—i.e., Urban Transportation Planning System (UTPS) modeling—for regional travel demand forecasting. The concept of a stable travel time budget was probably introduced by Tanner (1961) followed by Robinson et al. (1972). Zahavi (1979) developed a Unified Mechanism of Travel (UMOT), which is based on the assumption that travel time and money expenditure are each consistent. Furthermore, Goodwin (1981) discussed how these budgets, if indeed they exist, can be incorporated into the traditional four-step modeling procedure.

Two important facts to note here are that most if not all studies pertaining to the notion of the travel time budget consider only urban travel contexts and that the geographical scale of most of these studies is limited to a metropolitan transportation area. Empirical evidence has shown that a relative constant travel time expense does exist in an aggregated scale. However, when researchers compared the regularity of travel time across space and time (particularly time), they found only limited evidence to support the notion that travel time is regular. There are some studies substantiate the position that travel time is stable over time (Zahavi and Talvitie, 1980; Zahavi and Ryan, 1980; Chumak and Braaksma, 1981; Hupkes, 1982; Barnes and Davis, 2001).

The activity-based model is another attempt to improve UTPS modeling by trying to introduce a series of representations underlying travel behavior. The model is based on “a common philosophical perspective, whereby the conventional approach to the study of travel behavior … is replaced by a richer, more holistic, framework in which travel is analyzed as daily of multi-day patterns of behavior” (Jones et al., 1990). Also, this approach appears to be inherently incompatible with intercity or long-distance transportation, which does not follow a daily or weekly activity pattern in most cases.

I, therefore, decided to consider the fundamental theoretical assumption of the linear value of time functions. In traditional consumer behavior theory, utility was frequently considered a function of the consumption of goods before Becker’s (1965) first attempt to develop a methodology to account for the allocation of time in all non-work activities. In Becker’s postulation, a household’s basic activities such as sleeping and eating are produced through a combination of market goods and time, and thus time should be considered a factor that directly enters the utility function. Therefore, an additional time constraint is introduced into the model. In this theory, by assigning more time to work
rather than to commodity consumption, time is implicitly converted into money. Therefore, according to this concept, non-work time can be valued in the same way as wages are in terms of utility. Johnson (1966) further developed Becker’s model by including work time in the utility function, the subjective value (pleasantness or unpleasantness) of which is then summed with the wage rate to explain the variance of the value of non-work time. Up to this point, time was considered in regard to only two categories: work and non-work. Oort (1969) argued that the value of travel time should be considered an additional category. According to Oort, the value of travel time differs from non-work time because of the additional benefit or travel time in regard to saving time; e.g., travel time can be transformed into additional working time, and the reduction of subjective value of travel discomforts.

De Serpa (1971) provided the first general model that defines the Value of Time (VOT) of all activities. The major difference between De Serpa’s model and previous models is that he considered time and goods to be complements rather than substitutes. Based on this, he introduced another constraint according to which the consumption of certain goods requires a minimum amount of time referred to as the technology constraint. De Serpa also defined three kinds of VOT: the value of time as a resource (VOR), the value of time as a commodity (VOC), and the value of time savings (VTS). Because it can be generalized, this model provides better compatibility than previous models, and it has, therefore, frequently been used as the foundation of later models. Another revolutionary step was taken by Evans (1972), who argued that only time assigned to activities should be considered in regard to determining utility. This position, however, has not been sufficiently recognized. Jara-Diaz (2007) developed a model with a utility function specified in the Cobb-Douglas form with constraints similar to those specified in the De Serpa model.

Some recent research focuses on determining the nonlinearity factor in regard to the value of time functions. In the neo-classical model presented by Train and McFadden (1978), the value of time equals the wage rate. Blayac and Causse (1999) may be the first to provide a theoretical legitimization of some nonlinear representative utility, based on which Kato (2005) developed a model that captures the impact of second- and third-order utility in respect to travel time. In addition, many research studies have discussed variations in the value of time over travel time or distance. According to a number of these studies, the value of travel time decreases as travel distance increases (Hensher 1997, Wardman 1998, 2001, 2004; Hulkrantz and Mortazavi 2001). However, evidence that leads to different or contrary conclusions does exist (De Lapparent et al. 2002, Axhausen et al. 2005) reported in studies predominantly by European and Japanese researchers.

Because the study I designed required intercity travelers’ choice information, and in the U.S. the most widely available intercity transportation information lies in the air travel sector, in the empirical analysis of this work, I focused on the itinerary choice modeling among air travelers. The present study relies on air transportation data. The reasons for adopting data of this type will be discussed in detail in the theoretical framework. There is abundant literature on itinerary choice modeling in the context of air transportation, but this literature seldom overlaps with the kind of studies discussed above. Most of the relevant studies rely on stated preference data, and the behavioral frameworks are mostly some form of logit model: the simple logit model (Abou Zeid et al., 2006), the error
component logit model (Hess et al., 2007), the continuous cross-nested logit model (Lemp et al., 2010), or the mixed logit (with latent variables) model (Brey and Walker, 2011). Estimation of the willingness to pay for delay is usually based on two types of trip purposes: business and leisure, and the numerical results of these estimations vary, from 30.30 $/hour and 4.80 $/hour (Proussaloglou and Koppelman, 1999) to 60 $/hour and 17 $/hour (Adler et al., 2005). It should be noted, too, that business travelers have a greater standard deviation in regard to their willingness to pay than do leisure travelers.

In schedule preference distribution estimation, Mehndiratta and Hansen (1997) adapted time allocation theory by assuming that schedule preference distribution depends on whether the schedule delay occurs during work, leisure, or sleep time.

Although there has been so much discussion on the issues of activity-based modeling, travel time budget, and the non-uniformity of the value of travel time etc., the questions of ‘discontinuity’ and ‘threshold’ are still left to be answered, especially in intercity traveling. I tried to substantiate the proposition of ‘discontinuity’ with a microeconomic theoretical model (Chapter 3) and empirical evidences (Chapter 4 and 5).
3 Theoretical Framework

The choice process as it relates to intercity travel is often based on the travel time of modes that are substantially larger than in urban cases. The travel time values for intercity travel are often on the order of hours rather than minutes as in urban cases. For intercity travel, the travel time often interferes significantly in the daily activities of travelers. It is expected, and has sometimes been observed, that a discontinuous change occurs in demand in response to changes in the travel time of a particular one or when there is a switch between modes. This indicates a change in the value of time. An intuitive explanation for this would be that when the travel time of a certain activity becomes exceedingly long, travel has a significant chance of interfering with other daily activities. For example, if the one-way travel time between two cities is at 2 hours, a traveler can make a round-trip during the day without a significant impact on other activities. With travel time increasing, initially a traveler may be no longer be able to make a one-day round-trip such that the traveler has to arrange for overnight accommodation. This suggests that at a certain level travel time will encroach on other activities in the decision maker’s original plan. Given that the value of time can differ depending on the activity, there will be a discontinuity in the travel time value. This may be perceived as an abrupt increase (or decrease) in the value of time, which is then reflected in the demand for travel. This explanation is in accord with that offered by Time Allocation Theory, introduced by De Serpa (1971).

In this part of the research, I focused on establishing a theoretical framework to connect TAT with the traveler’s decision-making with the purpose of producing a travel behavior model that reflects the discontinuity of the value of time. First, I reviewed the theoretical framework by De Serpa. I also reviewed the derivation of the disutility function of a trip. Based on these theories, I introduced two microeconomic explanations for the discontinuity of the value of time.

Before going into the details of the models, I introduce the following notation system:

- $i$ (from 1 to n) represents a series of activities under consideration during one unit of overall time expansion.
  - $i$ is 1 in the case of the model in section 3.1.2, because only 1 activity is under consideration. As a result, $i$ is omitted in this model for simplification.
- $j$ (from 1 to m) denotes the different transportation modes under consideration in order to perform an activity.
  - $j$ is 1 in the case of the model Section 3.1.1, because only activities, not travel modes are under consideration. Similarly, $j$ is omitted in this model for simplification.
- $T$ is a vector with $i$ entries that indicates the time allocated to the $i$ activities respectively.
  - $T$ is a scalar in section 3.1.2 because $i = 1$.
- $t$ is a matrix with dimension $(i, j)$. The entry $t_{ij}$ is time of travel to conduct activity $i$ if taking mode $j$ (both can be zero if the activity does not involve travel).
o $t$ is a vector with $j$ entries in section 3.1.2, because $i = 1$.

- $d$ is a matrix with dimension $(i, j)$. The entry $d_{ij}$ is time of travel to conduct activity $i$ if taking mode $j$ (both can be zero if the activity does not involve travel). $d_{ij} = 1$ if chosen, $d_{ij} = 0$ otherwise.
  - $d$ is a vector with $j$ entries in section 3.1.2, because $i = 1$.

- $x$ is a vector with $i$ entries that indicates the quantity consumed of activity $i$.
  - $x$ is a scalar in the in section 3.1.2 because $i = 1$.

- $P$ is a vector with $i$ entries that indicates the unit price of activity $i$. $P_i$ is assumed to be exogenous.
  - $P$ is a vector with $j$ entries in the section 3.1.2 because $i = 1$.

- $p$ is a matrix with dimension $(i, j)$. The entry $p_{ij}$ is the price of travel to conduct activity $i$ if taking mode $j$ (both can be zero if the activity does not involve travel). $p_{ij}$ is assumed to be exogenous.
  - $p$ is a vector with $j$ entries in the section 3.1.2, because $i = 1$.

- $a_i$ indicates the minimum time needed to consume a unit of activity $i$.
- $\bar{T}$ is a vector with $i$ entries that indicates the lowest time requirement of activity $i$. $\bar{T}_i$ is assumed to be exogenous.
  - $\bar{T}$ is a vector with $j$ entries in the section 3.1.2, because $i = 1$.

- $\bar{t}$ is a matrix with dimension $(i, j)$. The entry $\bar{t}_{ij}$ is the lowest time requirement of travel to to conduct activity $i$ if taking mode $j$ (both can be zero if the activity does not involve travel). $\bar{t}_{ij}$ is assumed to be exogenous.
  - $\bar{t}$ is a vector with $j$ entries in the section 3.1.2, because $i = 1$.

- $R$ is the overall income of the individual (assumed exogenous).
- $TT$ is the total temporal endowment (assumed exogenous).

### 3.1 Theoretical Background

As mentioned at the beginning of this work, the inspiration of the study is that the thresholds and discontinuities in the value of travel time is a result of that when travel time becomes exceedingly long, its impact on other activities becomes significant. That is to say, the thresholds and discontinuities are suspected to be the impact of a long trip on the multiple objectives of an individual during a certain period of time. In order to construct a theoretical framework that reflects such schemes, the review of two models is provided.

#### 3.1.1 The De Serpa Model

As discussed in Chapter 2, the model by De Serpa provides the foundation of many models. Therefore, the theoretical background review starts with De Serpa’s model. In this model, a series of activities are under consideration in an individual’s activity pattern. The individual tries to maximize his/her overall utility through resource allocation of time and money. The resource allocation problem can be defined as the following utility maximization:
Maximize $U(x, T)$ \hspace{1cm} (3.1)
Subject to
\[ \sum P_i x_i = R \] \hspace{1cm} (3.2)
\[ \sum T_i = TT \] \hspace{1cm} (3.3)
\[ T_i \geq a_i x_i \] \hspace{1cm} (3.4)

Where there are three sets of constraints:

- **Income constraints** that reinforce the overall expenditure equals the individual’s income,
- **Time constraints** that reinforce the total time spent equals the total temporal endowment,
- **Technical constraints** that require the time spent on each unit of activity to satisfy a certain lower bound requirement.

In the following part of the research, the constraints of the utility maximization problems have similar structures.

### 3.1.2 The MVA Model

MVA (1987) developed the first rigorous relationship between the discrete choice modeling functional form and the individual time allocation microeconomic framework. This framework is also based on De Serpa’s time allocation model, in which the exogenous variables are only for the one activity of which mode choice is under consideration. The neo-classical program can be presented as follows:

Max $(x, T, t)$ \hspace{1cm} (3.5)
Subject to
\[ P_x + \sum_{j=1}^{m} d_j p_j = R \] \hspace{1cm} [\lambda] \hspace{1cm} (3.6)
\[ T + \sum_{j=1}^{m} d_j t_j = TT \] \hspace{1cm} [\mu] \hspace{1cm} (3.7)
\[ t_j \geq \bar{t}_j \] \hspace{1cm} [k_j] \hspace{1cm} (3.8)

The model is also designed with three sets of constraints, similar to that of the De Serpa model: the income constraints, the time constraints, the technical constraints.

$\lambda$, $\mu$, and $k_j$ is each the Lagrangian multiplier of the respective constraint. As a result, $\lambda$ indicates the marginal utility of income, whereas $\mu$ is the marginal utility of an additional unit of time. $k_j$ is the marginal utility of decreasing the travel time required of mode $j$. 
The Lagrangian of the problem can be formulated as

\[
L = U(x, T, t_1, \ldots, t_m) + \lambda \left( R - px - \sum_{j=1}^{m} d_j p_j \right) + \mu \left( TT - T - \sum_{j=1}^{m} d_j t_j \right) + \sum_{j=1}^{m} d_j k_j (t_j - \bar{t}_j)
\]

(3.9)

The first-order conditions are

\[
\frac{\partial U}{\partial x} = \lambda p, \quad \lambda > 0
\]

(3.10)

\[
\frac{\partial U}{\partial T} = \mu, \quad \mu > 0
\]

(3.11)

\[
\frac{\partial U}{\partial t_j} = \mu d_j - d_j k_j, \quad k_j \geq 0, \quad j = 1, \ldots, m
\]

(3.12)

The first-order approximation of the utility function is

\[
U \approx c + \frac{\partial U}{\partial x} x + \frac{\partial U}{\partial T} T + \sum_{j=1}^{m} \frac{\partial U}{\partial t_j} t_j
\]

(3.13)

Substituting the first-order conditions gives the indirect utility function. This value is called the ‘indirect utility’ because of the assumption that the utility is maximized while the expenditure is minimized at the same time. The term ‘indirect’ implies the dual problem.

\[
\tilde{V} = c + \lambda px + \mu T + \sum_{j=1}^{m} \left( \mu d_j - d_j k_j \right) t_j
\]

(3.14)

After simplification, and based on the assumption that the mode \( j \) is taken, the indirect utility function of choosing mode \( j \) can be rewritten as

\[
V_j = c - \lambda p_j - k_j t_j
\]

(3.15)

which is a standard form in neo-classical models.

To provide a theoretical framework for the hypothesis, a more generalized theoretical framework is introduced to capture multiple activities within one day. The original four-element structure of each activity is retained: the quantity consumed and the time spent on the activity as a composite good except for traveling are denoted by \( X \) and \( T \), respectively, and the monetary cost and time cost of traveling in order to conduct the activity are denoted by \( p \) and \( t \). An individual is facing multiple activities during a certain time period. One important characteristic of this model is that a fourth set of constraints is introduced, which reflects the lowest time spent requirement on conducting the activity. For instance, if the individual is planning to watch a movie, there would be a lowest time consumption requirement of \( T_{\text{movie}} \), which is the overall show time. Two theoretical models with a similar ideology are proposed and then compared.
3.2 Theoretical Model 1

In the theoretical background review of Section 3.1, two models are presented and discussed. In the De Serpa’s model (Section 3.1.1), multiple activities are involved while no travel mode choice is taken into consideration. In the MVA model (Section 3.1.2), while mode choice is the focus, only one activity is considered. I combined these two models to introduce a theoretical framework that takes into consideration of multiple activities as well as multiple travel modes to conduct each of the activity.

A visual representation of the model structure is illustrated in Figure 3.1.

![Figure 3.1: Model 1 Illustration Example A](image)

As travel time increases, at the beginning, it encroaches upon Activity 1. However, there is a point where the time assigned to Activity 1 can be no longer decreased (which is called the lowest time requirement of Activity 1, and is discussed in more details later). For example, one may need 30 minutes for lunch. He/she may first assign 1 hour for it to ensure some flexibility. If his/her travel time to lunch increases, the person would decrease the time assigned to lunch to account for the extra travel time. However, when 30 minutes are taken away from lunch, which means only 30 minutes are left, his/her time for lunch can be no longer decreased. In such cases, the person may reduce the time assigned to a different activity (grocery shopping for instance), which is represented as Activity 2 in Figure 3.1. The point where the travel time is increase by 30 minutes is hence a threshold.

This is Model 1, the mathematical formulation of which is as follows:
Max $U(x, T, t)$ \hspace{1cm} (3.16)

Subject to

$$\sum_{i=1}^{n} P_i x_i + \sum_{i=1}^{n} \sum_{j=1}^{m} d_{ij} p_{ij} = R \hspace{1cm} [\lambda]$$ \hspace{1cm} (3.17)

$$\sum_{i=1}^{n} T_i + \sum_{i=1}^{n} \sum_{j=1}^{m} d_{ij} t_{ij} = TT \hspace{1cm} [\mu]$$ \hspace{1cm} (3.18)

$$t_{ij} \geq \bar{t}_{ij} \hspace{1cm} [k_{ij}]$$ \hspace{1cm} (3.19)

$$T_i \geq \bar{T}_i \hspace{1cm} [\eta_i]$$ \hspace{1cm} (3.20)

Similarly, the first two sets of constraints are the income and time constraints, respectively, whereas the third and fourth sets of constraints are the technical constraints. Therefore, the Lagrangian of the problem is

$$L = U(X,T,t) + \lambda \left( R - \sum_{i=1}^{n} P_i x_i - \sum_{i=1}^{n} \sum_{j=1}^{m} d_{ij} p_{ij} \right) + \mu \left( TT - \sum_{i=1}^{n} T_i - \sum_{i=1}^{n} \sum_{j=1}^{m} d_{ij} t_{ij} \right) + \sum_{i=1}^{n} \sum_{j=1}^{m} d_{ij} k_{ij} (t_{ij} - \bar{t}_{ij}) + \sum_{i=1}^{n} \eta_i (T_i - \bar{T}_i)$$ \hspace{1cm} (3.21)

At optimality,

$$\frac{\partial U}{\partial x_i} = \lambda P_i, \hspace{0.5cm} \lambda > 0 \hspace{1cm} (3.22)$$

$$\frac{\partial U}{\partial t_{ij}} = \mu d_{ij} - d_{ij} k_{ij}, \hspace{0.5cm} k_{ij} \geq 0, \hspace{0.5cm} i = 1, \ldots, n, \hspace{0.5cm} j = 1, \ldots, n \hspace{1cm} (3.23)$$

$$\frac{\partial U}{\partial T_i} = \mu - \eta_i, \hspace{0.5cm} \mu > 0, \hspace{0.5cm} \eta_i \geq 0 \hspace{1cm} (3.24)$$

The first-order approximation of the utility function would be

$$U = c + \sum_{i=1}^{n} \frac{\partial U}{\partial x_i} x_i + \sum_{i=1}^{n} \frac{\partial U}{\partial T_i} T_i + \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{\partial U}{\partial t_{ij}} t_{ij}$$ \hspace{1cm} (3.25)

with $c$ as an intercept.

The indirect utility of a given set of activity and travel mode choices can be approximated as
The resultant utility function

\[
\bar{V} = c + \sum_{i=1}^{n} \lambda_i p_i x_i + \sum_{i=1}^{n} (\mu - \eta_i) t_i + \sum_{i=1}^{m} \sum_{j=1}^{m} (\mu d_{ij} - d_{ij} k_{ij}) t_{ij}
\]

\[
= c + \lambda \sum_{i=1}^{n} p_i x_i + \mu \sum_{i=1}^{n} T_i - \sum_{i=1}^{n} \eta_i T_i + \mu \sum_{i=1}^{m} \sum_{j=1}^{m} d_{ij} t_{ij} - \sum_{i=1}^{m} \sum_{j=1}^{m} d_{ij} k_{ij} t_{ij}
\]

\[
= c + \lambda \left( R - \sum_{i=1}^{n} \sum_{j=1}^{m} d_{ij} p_j \right) + \mu TT - \sum_{i=1}^{n} \eta_i T_i - \sum_{i=1}^{m} \sum_{j=1}^{m} d_{ij} k_{ij} t_{ij}
\]

(3.26)

where R and TT are constants. To determine the indirect utility of an activity, a given mode b is chosen, and if conditioning on the transportation modes of all the other activities is taken, \( \sum_{i=1}^{n} \sum_{j=1}^{m} d_{ij} p_{ij} \) is constant. The indirect utility function can be rewritten as

\[
V_{ab} = c - \lambda p_{ab} - k_{ab} t_{ab} - \sum_{i=1}^{n} \eta_i T_i - \sum_{i=1}^{m} \sum_{j=1}^{m} d_{ij} k_{ij} t_{ij}
\]

(3.27)

where the vector \( \mathbf{d} \) is assumed to be known and the travel time constraint is binding such that \( k_{ij} \) is continuous. There will be a point of discontinuity when the constraint condition of one of the activities goes from unbinding to binding (\( \eta_j \) changes from zero to non-zero), which is not considered in the neo-classical model. However, in the neo-classical model

\[
V_{ab} = c - \lambda p_{ab} - k_{ab} t_{ab}
\]

(3.28)

It can be observed that the neo-classical model is obtained by assuming that the binding conditions of other constraints do not change. Or, more generally, any continuous utility function is based on this assumption, which is certainly not true in real life. If we consider the point at which the constraint changes from binding to non-binding to be a “threshold,” especially in long-distance travel, such as intercity travel, where the activity-based model of urban travel is no longer valid, the threshold effect cannot be incorporated. However, as the travel time increases, the probability of encountering thresholds will increase. From the empirical framework, it can also be postulated that there can be multiple thresholds in the value of travel time.

Please Recall Figure 3.1, which is a simplified interpretation of model 1 showing how the disutility function (travel cost) changes with travel time. Figure 3.1 represents how the time budget is allocated to different activities when travel time is considered as the only exogenous variable. Again, \( T_{ab} \) denotes the travel time of the trip under discussion. Three activities (1 to 3) are considered in this example. The area underneath represents the time allocated to the trip or the activity. It should be noted that the examples used in this chapter are all simplified and illustrative. They are presented in order to establish the working mechanism underlying the different optimization schemes. The resultant utility function profile is presented in Figure 3.2.
When the travel time is relatively short (at point A), the disutility function increases continuously with travel time before exceeding the first threshold. This is primarily because the travel time takes up only the time allocated to activity 1. At point B, the time allocated to activity 1 reaches its lower-bound requirement, and thus when the travel time requirement increases further, it starts to encroach on other activities, which is activity 2 in this case. As a result, the slope of the disutility function changes after the first threshold. So the value of time changes after point B. A similar process takes place when the time allocated to activity 2 reaches its lower-bound requirement. The travel time disutility function encounters a second threshold, where the value of time changes again. Theoretically, it is possible to assume that even more thresholds exist. However, in empirical research, due to geographical limits, a traveler will encounter only a limited number of thresholds. Model 1 establishes a continuous utility function with a discontinuous value of the time function, which is reflected in slope changes as presented in Figure 3.2.

### 3.3 Theoretical Model 2

The second model proposed is very similar to the first but with stronger conditions. The main difference lies in the cancellation of activities. In model 1, the problem is stated so that each activity requires a minimum time consumption. For example, if the technical constraint of the activity “watching a movie” is 2 hours, the time spent in the theatre will be at least 2 hours. However, in real life, the individual has the choice of missing the movie. That is, if the time assigned to other activities keeps increasing and the time assigned to watching a movie touches the lower bound, the individual can make the choice to miss the movie.

Please recall Figure 3.1 again, similar to which, a illustration of Model 2 is represented in Figure 3.3.
Figure 3.3 shows that when travel time increases to the extent that Activity 1’s lowest time requirement is to be violated, the individual may choose to cancel it. The time assigned to Activity 1 is then taken away by other activities (Activity 2 in the illustrative example). This consideration is in line with the technical constraint in De Serpa’s model where the lower-bound requirement of each activity is proportional to the number of activities consumed. In model 2, the cancellation of the activity means that the unit of consumption drops from 1 to 0. This will only be reflected in terms of the technical constraint. As the result, the most important difference between model 1 and model 2 lies in the last set of constraints for the time allocated to activity i, where $T_i$ can be either greater than a certain lower-bound requirement, or can equal zero. Model 2 can be expressed mathematically as:

$$\text{Max } U(\mathbf{x}, \mathbf{T}, \mathbf{t})$$

Subject to

$$\sum_{i=1}^{n} P_i x_i + \sum_{i=1}^{n} \sum_{j=1}^{m} d_{ij} p_{ij} = R \quad \left[ \lambda \right]$$

$$\sum_{i=1}^{n} T_i + \sum_{i=1}^{n} \sum_{j=1}^{m} d_{ij} t_{ij} = TT \quad \left[ \mu \right]$$

$$t_{ij} \geq \bar{t}_{ij} \quad \left[ k_{ij} \right]$$

$$T_i \geq \bar{T}_i, \text{or } T_i = 0 \quad \left[ \eta_i \right]$$

With the implied assumption that $T_i \geq 0$, the last set of constraints can be rewritten as $(T_i - \bar{T}_i)T_i \geq 0$. 
Therefore, the Lagrangian of the problem is

\[ L = U(X,T,t) + \lambda \left( R - \sum_{i=1}^{n} P_x - \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} p_{ij} \right) + \mu \left( TT - \sum_{i=1}^{n} T_i - \sum_{i=1}^{n} \sum_{j=1}^{m} d_{ij} t_{ij} \right) + \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} k_{ij} (t_{ij} - \bar{t}_{ij}) + \sum_{i=1}^{n} \eta_i (T_i - \bar{T}_i) t_i \]

(3.21)

At optimality,

\[ \frac{\partial U}{\partial x_i} = \lambda P_i, \quad \lambda > 0 \]

(3.22)

\[ \frac{\partial U}{\partial t_{ij}} = \mu d_{ij} - d_{ij} k_{ij}, \quad k_{ij} \geq 0, \quad i = 1, \ldots, n, \quad j = 1, \ldots, n \]

(3.23)

\[ \frac{\partial U}{\partial T_i} = \mu - \eta_i \left( 2T_i - 1 \right), \quad \mu > 0, \quad \eta_i \geq 0 \]

(3.24)

The first-order approximation of the utility function would be

\[ U = c + \sum_{i=1}^{n} \frac{\partial U}{\partial x_i} x_i + \sum_{i=1}^{n} \frac{\partial U}{\partial T_i} T_i + \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{\partial U}{\partial t_{ij}} t_{ij} \]

(3.25)

with \( c \) as an intercept.

Similar to the previous model, the indirect utility of a given set of activity and travel mode choices can be approximated as

\[ V_{ab} = c - \lambda P_{ab} - k_{ab} T_{ab} - \sum_{i=1}^{n} \eta_i T_i \left( 2T_i - 1 \right) - \sum_{j=1}^{m} \sum_{i=1}^{n} d_{ij} k_{ij} t_{ij} \]

(3.26)
Figure 3.4: Model 2 Illustration Example B

The utility function profile is presented in Figure 3.4. The beginning of the story is exactly the same: when the travel time is relatively short (at point A), the disutility function increases smoothly with travel time before exceeding the first threshold. The major difference between model 1 and 2 occurs at point B, when the time allocated to activity 1 reaches its lower-bound requirement. In model 2, the traveler may have the choice of canceling activity 1. In this case, there will be a discontinuous change in the disutility function as the travel time exceeds the first threshold. This change is primarily due to the fact that the time originally allocated to activity 1 is now taken up by other activities. The following process is similar: when the travel time requirement continues to increase, it starts to encroach on activity 2. So the slope of the disutility function also changes after the first threshold. Model 2 reflects a change in slope together with a quantum increase in disutility.

3.4 Model Interpretation

How can the thresholds and discontinuities with respect to travel time be found empirically? Ideally, if access to all the information required to represent the model can be obtained, i.e., the overall activity pattern of travelers (Ti, tj, etc.), then we will be able to determine the threshold values and their weights in the disutility function. This approach would be similar to that adopted in the activity-based model for urban settings. Perhaps the most important difference will be the magnitude of the respective time durations. That is, the difference in magnitude may result in completely different problems. This is due to the differences between urban and intercity travel listed as:
Table 3.1: Comparison between urban and intercity traveling

<table>
<thead>
<tr>
<th></th>
<th>Urban</th>
<th>Intercity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Different choice set of travel modes</td>
<td>Bicycle or walking, etc.</td>
<td>Airplane, etc.</td>
</tr>
<tr>
<td>Different collection of activities to be concerned</td>
<td>Picking up kid from school, etc.</td>
<td>An entire day of work, etc.</td>
</tr>
<tr>
<td>Significance in the overall activity pattern</td>
<td>Low</td>
<td>High</td>
</tr>
</tbody>
</table>

It should be emphasized again that the change in magnitude will result in a non-linear change in disutility. In addition to the differences between the models given in Table 3.1, the fundamental cause is that the total time budget must be binding for the magnitude of intercity travel, but not necessarily for the magnitude of urban travel. Due to the smaller magnitude in urban travel models, the activity set is more flexible. That is, as the present study only looks at one piece of the daily activity pattern in reference to the magnitude of urban cases, it is not necessary for the individual to complete all the compulsory activities within the time window, and hence, the overall time budget constraint in the urban setting is not necessarily binding. More importantly, because the time constraints and the activity sets are less flexible in the intercity context than in the urban context, there are more reasons to suspect that the thresholds of certain groups of travelers’ (business, leisure, etc.) in intercity traveling may coincide or at least fall into a certain interval with statistical significance. The threshold(s) can be found with econometric tools. Also, because the thresholds coincide, the group behavior of travelers facing the thresholds would be critical in their respective mode/itinerary choice decisions. From the system operator’s (supplier’s) perspective, the phenomenon also has essential economical implications, as discussed previously. Therefore, the lack of knowledge in intercity traveling leads to a more important question of intercity traveling when travel time becomes exceedingly long.

Due to the change of model settings as well as the lack of information, the empirical work conducted as part of the present study is very exploratory in nature. As a result, rather than incorporating every possible factor into the model, I have adopted simple models in order to capture the discontinuity at a macroscopic level.
4 One-Threshold Model Estimation with Stated Preference (SP) Data

In the previous section, two theoretical explanations are offered to account for the discontinuities of VOT. However, due to the field’s very limited understanding of the issue, the present research offers only an initial-stage inquiry: mere empirical knowledge and no ideal data resource. I decide to take the ambitious step of looking for empirical evidence of the discontinuities with data that was not collected for this particular purpose. My goal in taking this approach was to reach conclusions that would be generalizable.

In this chapter, I start by discussing the choice of mathematical formulation of the utility function. After choosing piecewise linear models, a discussion was made on the properties of the model. Then I introduce a stated preference (SP) data set collected by Boeing, and estimated with one-threshold assumption. I also further stratify the data by travel directions and travel purposes. The results are presented and discussed afterwards.

4.1 Travel Time Utility Function Choice: Piecewise Linear

As suggested in the theoretical framework illustrations and given the limited existing insights into the problem, piecewise linear functions are probably the best representation. It is postulated that the utility function is linear in respect to time between thresholds. The slopes of different segments should also change. For model 2, in particular, a dummy variable should be included at the threshold to simulate the discrete impact.

The estimation began on the basis of observing the change of the likelihood function with respect to different heuristic threshold assumptions. Non-parametric measures were not used for the estimation because it was not feasible to do so given the limitations of the data. With a non-parametric model, the time horizon should be divided into a series of small intervals and the parameter of each interval can be estimated accordingly. Unfortunately, if the interval is defined to be small, there will be too many variables in the utility function, such that both convergence and significance will become problematic. Therefore, the model becomes non-estimable with small time intervals. However, if the interval unit is large, the rough capture of the nonlinearity hinders the model’s intuitive sense. The model becomes non-interpretable, as the parameters may change back and forth drastically and signs can also differ significantly from what is expected.

From another perspective, the piecewise linear model is a simplified version of the non-parametric model. The threshold can be estimated by moving the threshold(s) across the travel time horizon, dividing the travel time into segments accordingly, and then estimating and comparing the log-likelihood values. According to my “simple is best” principle, within my empirical analysis, the research started from the very basic one-threshold assumption. After the estimation comparison results were obtained, the next steps were to determine which model to adopt and whether it made empirical sense to move to a double-threshold estimation. With a confirmation as the answer, the estimation results of the respective datasets were compared in order to determine the relative robustness of the models. The travel time can be entered into the model by cutting it into segments according to the thresholds. In model 2, additional dummy indicators should be added to represent the “jump” at each threshold.
The disutility that results from the travel time is denoted as $-V_{ab}^{T}$. $t_{ab}$ is the total travel time; $I_{ab}$ is a dummy that indicates whether the travel time exceeds the threshold; $k_{ab}^1$ and $k_{ab}^2$ represent the parameter of the travel time before the threshold and after the threshold, respectively, and $k_{ab}^3$ represents the weight of a drastic jump at the threshold. In model 1, where the utility function is continuous and the value of time is discontinuous, the disutility function can be represented as

Figure 4.1: Model 1 Estimation Illustration

Figure 4.2: Model 2 Estimation Illustration
With the mathematical formulation

\[-V_{ab}^T = k_{ab}^1 t_{ab} + I_{ab} k_{ab}^2 (t_{ab} - \tau) = (k_{ab}^1 + I_{ab} k_{ab}^2) t_{ab} - I_{ab} \tau\]

(4.1)

In model 2, where both the utility function and the value of the time function are discontinuous, the disutility function can be represented as

\[-V_{ab}^T = (k_{ab}^1 - I_{ab} k_{ab}^1 + I_{ab} k_{ab}^2) t_{ab} + I_{ab} (k_{ab}^1 \tau - k_{ab}^2 \tau + k_{ab}^3)\]

(4.2)

Secondly, it is necessary to decide on the data resource to use for the empirical estimation. As suggested previously, the contribution of this study pertains to the public transportation sector and the result can be considered as a policy indicator. In intercity settings, the major public modes under consideration are rail and air. Due to the better availability of traveler information data in air transportation in the U.S., the stated preference data (SP) were used from air travel for the empirical estimation. There are several advantages to adopting this type of data. Firstly, as stated at the beginning of the present study, previous research has
shown that a threshold impact does exist in rail travel with revealed preference (RP) data. SP air travel data are an adequate complement for achieving the conclusion of the threshold impact for intercity passenger demand in general. Secondly, as the issue is being discussed within the context of the US, air travel is perhaps the only mode used on a nationwide basis.

The first SP dataset adopted in this section is from the Internet choice survey conducted by Boeing in the fall of 2005. In this survey, the respondents faced three air travel alternatives: alternative 1 is a non-stop flight, alternative 2 is a one-stop flight with no airline change (an in-line connection), and alternative 3 is a one-stop flight with a change of airline (an off-line connection). Other features of the flights are also provided on the survey interface (webpage) as depicted in Figure 3-1.

The dataset records responses from 3,613 respondents (10,839 records), including information such as origin, destination, flying time, trip purpose, round-trip fare, and the direct payer of the trip.

![Boeing Survey Webpage Design](image.png)

Figure 4.5: Boeing Survey Webpage Design
In the preliminary estimation with the SP data, the nested logit model was used as the framework. This model allows a correlation between alternatives within the same nest, whereas in the classical logit model all the alternatives are assumed to be Identically Independently Distributed (IID). That is, the alternatives within the same nest are likely to be of a similar nature. For instance, in travel demand modeling it is frequently assumed that all public transportation modes are in one nest, and all the private modes are assumed to be in another. The nested logit model can be expressed as

\[
P_{ni} = \frac{e^{V_i/\lambda_k}}{\sum_{\ell=1}^{K} \left( \sum_{j \in B_\ell} e^{V_{ij}/\lambda_{\ell}} \right)^{\lambda_{\ell}^{-1}}}
\]

(4.3)

where

- \( n \) denotes the specific individual facing the choice situation.
- \( i (i = 1, 2, 3) \) denotes the mode or itinerary alternative
- \( P_{ni} \) denotes the probability of the individual \( n \) choosing alternative \( i \).
- \( V \) represents the indirect utility function of the individual \( n \) for scenario \( i \).
- \( k \) denotes the nesting structure assumptions. In our estimation, the two connecting flights are in the same nest, whereas the direct flight is in another.
- \( \lambda_k \) is the nest parameter of nest \( k \).

A technical problem occurs once the threshold is introduced: the profile of the log-likelihood value is no longer continuous or derivable. For investigation purposes, the intuitive method is to observe visually. I estimated the maximum log-likelihood value based on different travel time threshold assumptions. In the following figures, the horizontal axis represents the threshold value assumption; the vertical axis indicates the estimated maximum log-likelihood value. The log-likelihood values are then compared with the linear assumption (equivalent to the threshold value set to zero).

Another important gap that the present study is designed to address is that of using clock travel time to analyze travelers’ responses. Elapsed travel time has always been the measure adopted in model estimations. However, following the logical implication in every time allocation model, including the two in the present study, when travelers make travel-related decisions, the factor of time should be considered in the form of clock time. That is, for a flight from SFO (San Francisco) to JFK (New York), the elapsed travel time in the airplane is 5 hours; however, the actual time subtracted from an individual’s overall time budget is the clock travel time, which is 9 hours (taking into account the time zone difference). To be consistent with the theoretical setup, I used the clock travel time for the estimation. In traditional linear models, this factor would not be an issue because as long as the same OD pair is concerned; the time zone difference will always be the same. In a linear model, only the difference in the utility function matters. The time zone impact for the same OD pair will not result in any utility difference between different options. However, in the new models proposed herein, the time zone difference will have an additional impact. Additionally, 1.5 hours is added to the clock travel time to simulate the access/egress time. Of course, more detailed access/egress information could be considered.
should such information be available in future studies. In linear VOT models, neither of the two revisions would have any impact because the relative utility cancels out.

![Utility Comparison](image)

**Figure 4.6: Utility Comparison**

Figure 4.4 depicts the estimation mechanism. The horizontal axis indicates different assumed threshold values, whereas the vertical axis refers to the negative log-likelihood values. Each point on each line represents a different threshold assumption and its corresponding log-likelihood estimation. For maximum likelihood estimation, the lowest point is optimal. The dashed line indicates the estimation results from model 1, and the continuous line represents the estimation results from model 2. Obviously, the estimation results from model 1 are smoother than those from model 2, which means the parameter of the threshold can be estimated analytically with model 1, but not for model 2. Tables 4.1 and 4.2 provide the optimal estimation results (points circled in black) of some of the variables. The estimation results of linear model can be observed when the threshold is assumed to be zero.
### Table 4.1: Optimal Estimation Result of Model 1

<table>
<thead>
<tr>
<th>Threshold</th>
<th>3.67</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log-likelihood</td>
<td>-2429.6378</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Estimation</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time Segment 1</td>
<td>-1.1001</td>
<td>0.0809</td>
</tr>
<tr>
<td>Time Segment 2</td>
<td>-0.8433</td>
<td>0.0327</td>
</tr>
</tbody>
</table>

### Table 4.2: Optimal Estimation Result of Model 2

<table>
<thead>
<tr>
<th>Threshold</th>
<th>3.95</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log-likelihood</td>
<td>-2425.0305</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Estimation</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time Segment 1</td>
<td>-1.2174</td>
<td>0.0884</td>
</tr>
<tr>
<td>Time Segment 2</td>
<td>-0.8739</td>
<td>0.0355</td>
</tr>
<tr>
<td>Jump</td>
<td>0.2890</td>
<td>0.0874</td>
</tr>
</tbody>
</table>

### 4.2 Comparison of Models

I have set out the previous estimation example in order to show the respective advantages and disadvantages of the two models proposed herein. For the specific estimation I conducted with this dataset, model 1 shows a strong advantage over model 2. As illustrated in Figure 4.1, model 1 can be estimated without linear search, which is not possible in the case of model 2. This difference between the estimation algorithms of the two models implies that more computational effort is necessary for model 2 than for model 1. Additionally, for the same reason that it requires this additional computational effort, model 2 is also less robust than model 1. The high computational effort and low robustness of model 2 is further complicated by another major problem with the current model: the lack of intuitive sense associated with the estimation results. For instance, in the estimation results just listed, the estimation parameter of the “jump” turned out to be positive, which indicates that indirect utility increases once travel time exceeds the threshold. That is, at the threshold boundary, passengers prefer a longer trip. This is not always the case and can even be a killing factor in regard to choosing between models, as the results are obviously anti-intuitive according to classical models.
Table 4.3: Comparison between models 1 and 2

<table>
<thead>
<tr>
<th></th>
<th>Pros</th>
<th>Cons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>• The threshold can be estimated analytically as an exogenous variable</td>
<td>• Relatively weak conditions</td>
</tr>
<tr>
<td></td>
<td>• Less computational effort</td>
<td>• Less sensible implications</td>
</tr>
<tr>
<td></td>
<td>• Robust for almost all of the cases</td>
<td></td>
</tr>
<tr>
<td>Model 2</td>
<td>• Implications are closer to real travelers’ behavior</td>
<td>• Requires abundant computational effort</td>
</tr>
<tr>
<td></td>
<td>• Stronger condition with more significant economic and political implications</td>
<td>• Lacks robustness</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Estimation results sometimes do not make intuitive sense</td>
</tr>
</tbody>
</table>

Although it is certainly possible to list the reasons to criticize the estimation results of model 2, the question under consideration is whether this anti-intuitive result might actually make sense. The answer is yes. Redmond and Mokhtarian (2001) found that people may assign positive utility to the travel time of commuting due to several reasons: ‘the benefits associated with a typical work destination (such as opportunities for socializing, shopping and other activities), the benefits associated with activities that can be conducted while traveling (listening to music, making phone calls, reading, transitioning between work and home roles), and an intrinsic enjoyment of travel itself’ (Redmond and Mokhtarian, 2001).

The explanation lies in the bundling effect of the activities. That is, as travel time increases, the possibility that the travel time duration can accommodate another activity also increases. For example, when the travel time is less than 2.5 hours, the major portion of the trip is spent on access, egress, acceleration, and deceleration. This means that it is difficult for a traveler to perform any work productively during the trip. However, when the travel time increases, the passenger has a disproportional increase in the travel time during which he/she can conduct some work efficiently. The same principle can be applied to overnight flights. It is only when the travel time is exceedingly long (transcontinental) that overnight flights are possible. Although the airfare for overnight flights may not be as low as that of non-overnight flights, the former bring savings on accommodation, which is not taken into account when only the travel activity is considered. As a result, it is certainly possible to consider the at first glance anti-intuitive results to be reasonable. To understand the underlying working mechanism and consequences, it is necessary to improve the models and survey methods, which will be discussed later. For the purpose of this
dissertation and given the limitations of the data, I conclude that model 1 is more sensible and robust than model 2 is. Therefore, I consider only model 1 in the following sections.

4.3 Variance Across Travelers
The value of travel time changes depending on the travel group because different travelers willingness differ in regard to the amount they are willing to pay depending on the purpose of their travel. The value of travel time for business travelers is always higher than that of leisure travelers. Similarly, I also suspect that the thresholds change between traveler types. Travel purpose is one of the variance factors taken under consideration, and thus I compared the difference in thresholds between business and leisure travelers in the same way as I did for the value of travel time. Additionally, as I am concerned with clock time, travel directions should also affect the threshold values. This is because for instance, for east–west bound travelers, the travel clock time is shorter than the elapsed time (longer in the case of west–east bound, equal in the case of north–south or south–north bound). As a result, the binding conditions, and hence the travel time threshold values, change among the three types of travelers. Therefore, I also stratified the samples according to the three types of travel directions.

Table 4.4: Comparison of Results

<table>
<thead>
<tr>
<th></th>
<th>Threshold (±Standard Error)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall</td>
<td>3.7 (± 0.2)</td>
</tr>
<tr>
<td>North–South</td>
<td>3.4 (± 0.4)</td>
</tr>
<tr>
<td>East–West</td>
<td>4.1 (± 0.7)</td>
</tr>
<tr>
<td>West–East</td>
<td>5.1 (± 1)</td>
</tr>
<tr>
<td>Business</td>
<td>2.8 (± 0.4)</td>
</tr>
<tr>
<td>Leisure</td>
<td>3.8 (± 0.4)</td>
</tr>
</tbody>
</table>

Table 4.4 lists the estimation results with the stratified data. The result is still far from conclusive. As indicated in the previous log-likelihood plot, Figure 4.4 is to a certain extent bi-modal, which implies that one-threshold estimation is not sufficient. Although the second threshold may not be significant, its impact on the distortion of the overall estimation result should be understood. As a result, it is still critical to determine whether and to what extent it has a bias on the results. Therefore, I continue with the two-threshold estimation in the next chapter, in which only model 1 was adopted for estimation.
5 Estimation of the Two-Threshold Model

In the single threshold estimation, our goal was to determine only the global maximum of the likelihood values (the lowest point in the negative log-likelihood plot). But if I take another look at Figure 4.4, it is evident that there always appears to be a local maximum at the far tail of the time span (6–8 hours). The evidence presented in the previous chapter leads us to suspect that a second threshold does exist, especially on a national basis, where the longest direct flight travel time is on the scale of 5–10 hours. It has already been shown in our theoretical framework that as travel time increases, it may encroach on the time originally allocated to multiple activities, and hence changes the binding conditions of multiple constraints. Consequently, it is already evident in the theoretical framework that there can be multiple thresholds in the value of travel time.

I tested the model with the hypothesis of two thresholds in this chapter. Based on a comparison between the one-threshold estimations of the two proposed models, I concluded that model 1 fits better for our current data and knowledge condition. Due to this reason, although I used both model 1 and 2 to test the two-threshold hypothesis, only the results from model 1 is presented here.

This chapter starts with the two-threshold estimation results of the Boeing survey data set, which is adopted in Chapter 4 (single threshold estimation). Then I introduced a second data set, collected by Resource Systems Group (RSG) in 2012. I estimated used model 1 and two-threshold assumptions to calibrate the utility function. The results from the two data sets are compared afterwards.

5.1 Two-Threshold Estimation with Boeing Survey Data

I used the same data source as the one adopted in the previous chapter. All the other variables under consideration are the same except for the additional piece of segmentation of travel time. In this model, two travel time thresholds are considered and the time disutility function is therefore a 3-segment piecewise linear function.
Figure 5.1: Estimation Result of two thresholds with overall Boeing Data

I used methods similar to one-threshold estimation. Figure 5.1 is also a plot of the negative log-likelihood against the threshold values. This time, I allowed two thresholds to change at the same time. In this figure, the first threshold changes from 2 to 5 hours, whereas the second changes from 5 to 8 hours. The optimal combination will again be the lowest point in the figure (approximately 3.7 vs. 7).

Figure 5.2: Estimation Result of two thresholds with overall Boeing Data by axes
5.2 Two-Threshold Estimation with RSG Survey Data

In order to obtain more conclusive evidence, I included an additional survey dataset. The Stated Preference dataset was collected by the Resource Systems Group (RSG) in 2012. In this survey, the respondents were asked to recall a recent trip they had made. Choice scenarios were simulated based on these trips. The survey comprised 3,123 respondents, each facing 8 choice scenarios with 2 choices within each. Flight information included in this dataset is similar to that of the Boeing data (e.g., origin, destination, travel time, fare, number of connections, and socio-demographic information about the travelers). One point to note here is that because the travel scenarios are simulated the differences between alternatives are exaggerated and the travel time is usually much longer than travel time in real life. I used a neo-classical model to estimate this model (without nests). Figure 5.1 presents a case of the estimation results using the RSG dataset.

![Figure 5.3: Estimation Result of two thresholds with overall RSG Data](image-url)
5.3 Analysis of the Results

I also checked the results against those of model 2 in this case. In the two-thresholds estimation, model 1 remains robust for most of the cases. Based on the comparison between and the integration of models 1 and 2, the following is a list of the thresholds estimated with each of the two datasets. The threshold values left blank are the ones insignificant. As can be observed from the log-likelihood surface, the confidence intervals of the first threshold do not change much comparing the single threshold estimation. However, the confidence interval of the second threshold is usually 1 or larger. There are a lot more details about the second threshold left to be explored.

Table 5.1: Comparison of Results

<table>
<thead>
<tr>
<th></th>
<th>Boeing 2005</th>
<th>RSG 2012</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Threshold 1</td>
<td>Threshold 2</td>
</tr>
<tr>
<td>Overall</td>
<td>3.8 (± 0.3)</td>
<td>7 (± 0.9)</td>
</tr>
<tr>
<td>Direction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>North–South</td>
<td>5.7 (± 0.4)</td>
<td>6 (± 0.8)</td>
</tr>
<tr>
<td>East–West</td>
<td>3.8 (± 0.7)</td>
<td>7.2 (± 1.1)</td>
</tr>
<tr>
<td>West–East</td>
<td>3.6 (± 0.8)</td>
<td></td>
</tr>
<tr>
<td>Purpose</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Business</td>
<td>2.9 (± 0.5)</td>
<td>6.7 (± 1.0)</td>
</tr>
<tr>
<td>Conference</td>
<td>3.5 (± 1.1)</td>
<td></td>
</tr>
<tr>
<td>Leisure</td>
<td>3.9 (± 0.4)</td>
<td>6 (± 0.8)</td>
</tr>
</tbody>
</table>
It can be observed that

- For the Boeing survey dataset, the first threshold generally lies between 3 to 4 hours, whereas the second threshold (once observed) is around 7 hours.
- As for the RSG dataset, probably due to survey design differences, the first threshold observed is usually around 6 hours, which is significantly higher than the results from the Boeing survey. The second threshold in the RSG data is not significant for most of the cases. As a matter of fact, the first threshold in the RSG data is more consistent with the second threshold shown in the Boeing data, which may be because the first threshold in the RSG data is not significant, again due to survey design differences.
- Business travelers show a stronger inclination to shorter threshold values than to higher threshold values.
6 Numerical Example and Illustration

In this chapter, I am using some numerical examples to illustrate the contribution of the proposed model. Model 1 is used for the demonstration because of its stronger empirical evidence and robustness in utilization as compared to model 2. However, the impact of model 2 can be expected to be greater than that of model 1—a point that can be considered in future work when more established results become available.

The chapter starts with a simple illustration of the change of value of travel time difference between models (the traditional one and the proposed one). The illustration shows that around 10% overestimation or underestimation may occur in the traditional model. A second implication of ‘asymmetrical system design’ can also be obtained from the piecewise linear model, which shed light on hub location in airline decisions. From there I used two different hub choice scenarios to discuss the optimal strategy with the proposed model.

6.1 Value of Travel Time Difference: Comparison with Linear Model

To start with, I use the parameters estimated in Chapter 5 to set up the illustrative example. Imagine the following case: there is a direct flight connection between airports A and B. The airline is considering introducing a connecting flight through airport C. For the connecting flight to be considered competitive, the generalized cost of this flight for travelers cannot exceed that of the direct flight. The generalized cost includes only travel expense and travel time in this case. In this experiment, I try to investigate the decrease in airfare of the connecting flight as compared with the direct flight fare—a decrease that makes the connecting flight more “competitive.” More importantly, I want to determine how the difference in airfare changes with the travel time of the direct flight, which is defined as the base time and the increase in travel time from base time to the connecting flight travel time, i.e., the value of the travel time difference between the connecting flight and the direct flight. I introduce the term “value of travel time difference,” which is the total monetary value of the travel time difference between two equivalent flights. In the following section, I explore some of the properties of the “value of travel time.”
As discussed before, the travel time cost function can be presented as:

\[ V_{ab} = c - \lambda p_{ab} - V_{ab}^T \] (6.1)

where \( V_{ab}^T \) is the disutility function of travel time.

Therefore, for the two itineraries to maintain the same level of utility, we need

\[ V_{ab1} = V_{ab2} \] (6.2)

and hence

\[ V_{ab1} = c - \lambda p_{ab1} - V_{ab1}^T = V_{ab2} = c - \lambda p_{ab2} - V_{ab2}^T \] (6.3)

Therefore we need the fare difference \( p_{ab2} - p_{ab1} \) to have the property of

\[ p_{ab2} - p_{ab1} = (V_{ab1}^T - V_{ab2}^T) / \lambda \] (6.4)

So we can calculate the fare difference accordingly

![Figure 6.2: Monetary value of the travel time difference](image)

Figure 6.2 depicts how the airfare difference changes with the travel times and travel time differences. The horizontal axis indicates the base time. Each line represents a different assumption of travel time increase between the direct and connecting flights, ranging from 0 to 2 hours. The vertical axis is the fare decrease of the connecting flight compared with that of the direct flight (in dollars), in order for the connecting flights to maintain the same
level of generalized cost with the direct ones, or the value of the travel time difference. For example, the line at the top of the figure represents the case in which there is a 2-hour travel time difference. As a result, the line indicates the monetary value equivalence of a 2-hour difference based on the travel time cases. As the base time increases from 1.5 hours to approximately 1.75 hours, the value of the travel time difference does not change. From 1.75 hours, however, the travel time difference starts to decrease until 3.7 hours, after which the difference is constant. It can also be observed that the slope or the rate of decrease is the same for all the cases.

Based on the assumptions I made in model 1, the value of the travel time difference decreases with the magnitude of the difference. Because of the decreasing value, the lines do not intersect with each other. Instead, they present in an ascending order as the time difference increases. Therefore, the lines become “denser” toward the right-hand side of the axis. An intuitive interpretation of the figure is that the line space “shrinks” as the base time increases, which causes the decreasing slope in between of the two turning points in every value of time function plot. Although this is only a simplified numerical example we are discussing now, similar results and changes are expected to appear in general.

Figure 6.2 is a comparison of the fare difference (or the value of the travel time difference) predicted by the proposed model and the traditional model for the 2-hour travel time increase case. Because of the constant value of time adopted in traditional models, the value of the travel time difference does not change throughout the base time horizon. In the proposed model, the line retains the shape described in the previous paragraph. In the 2-hour difference case, and compared with the prediction produced by our proposed model, in low travel time cases, the fare difference may be as high as 10%. In long travel time cases, the overestimation in the fare difference may reach as high as 20%.

![Figure 6.3: Comparison of the monetary value of the travel time difference](image-url)
We can observe from Figures 6.1 and 6.2 that depending on the particular case the fare difference can be positive or negative. This error will be carried into the demand estimation later. In the traditional model, the fare difference is underestimated when travel time is short, which implies that the travel demand of the connected flight will be underestimated for the same travel expense assumption. However, when the travel time is long, the fare difference tends to be overestimated, which results in an overestimation of demand.

Although the elaboration is relatively tedious and manipulative, the actual misinterpretation in terms of monetary value can be quite significant, as discussed. As a matter of fact, the assumption of a 2-hour travel time increase from a direct flight to a connecting flight is probably a very reasonable one. Given the rerouting, the acceleration and deceleration processes, together with the layover time at the airport, the extra time a connection takes usually exceeds 2 hours. That is, in reality, the estimation error has a pretty good chance of exceeding 10 or even 20%.

The even smaller difference in travel time (0–1.9 hours) in the example is a representation of a comparison between different connected cases, which allows a closer travel time difference. These cases still happen very often in reality, and also have implications for the hub choice problem in reality.

The previous numerical simulation illustrated the contribution of the new model in a quantitative way. In returning to the fundamental interpretation of the calibrated model and summarize its implications, I find that the central contribution of this model can be captured in the following statement: *the value of saved time in a shorter connection or network should be given more weight than the same amount of travel time saved in a larger connection or network*. This point also has suggests the importance of the hub-and-spoke system for a second time: it implies that as travel time increases passengers become more indifferent to additional travel time increases. Especially in the traditional model, other factors—monetary cost, operation convenience etc.—start to reveal their significance at an increasing speed.

It is intuitive to think that in considering the hub (or connecting airport) that a traveler would choose the one closest to the mid-point of the connection (in case 1), or as close as possible to the mid-point, if such a location exists. Here, I want to consider the operation factor, which constitutes another level of complexity in regard to airline network design. This factor is parallel to the attracted demand issue that we are discussing. One obvious issue associated with operation complexity is that of average speed. The high maximum speed of which airplanes are capable coupled with the limited acceleration that passengers can withstand means that airplanes take a long time to reach full speed. For instance, in the case of a 1-hour flight, the plane may only be at full-speed for 10 minutes. The same problem arises when it comes to other high maximum-speed modes, such as high-speed rail. However, I will leave the issue of operation to future research. At this point, I want to note that travel distance is seldom a good representation of travel time for air travelers (and it is likely that the same holds true for high-speed rail travelers as well).

However, what our findings imply is that an *economy of travel time does exist* such that there is a diminishing marginal cost when travel time exceeds the threshold. This is in addition to the well-known *economy of distance* in air and rail travel. When the travel time
is below the threshold, the economy of travel time does not prevail, which is probably the reason this fact was not discovered. This is also the reason why the symmetrical intuition will not work in long-distance travel modeling.

6.2 Asymmetrical Connecting Condition

In this section, I begin with the simplest connecting conditions with certain assumptions to analyze the change of generalized cost with respect to different connecting points. In traditional models, the generalized cost does not change with connecting locations as long as the overall travel time does not change. However, based on the proposed models, the utility function does have an advantage when the hub is located relatively close to the origin and/or the destination.

The model implies that the asymmetrical design in Case 2 (Figure 6.5) only has an advantage over the symmetrical design in Case 1 (Figure 6.4) (for the same connection situation, such as layover time, online or offline, etc.) in terms of the value of travel time. This will always be true for long flights, but needs to be at the same time under the assumption of one threshold.

By providing services for the three airports, the airline is facing three markets: A-B, A-C, C-B (where the location of C may change as implied in Figure 6.4 and Figure 6.5). If I assume that the market size of A-C₁ is equal to A-C₂, and C₁-B is equal to C₂-B, I want to decide the optimal hub location. For the purpose of system optimization, and with all the other conditions assumed to be the same, I want to minimize the overall travel time disutility. Because the travel time disutility between A and B is the same, it does not have a impact on the final result. So I only need to sum up the travel time Disutility of A-C and C–B.

To provide a numerical illustration, I also assumed that:

a) For different connection points on the same route, the travel times are also different, due to the significant impact of the economy of distance. In this example, I simplified the problem by assuming that the fly times are always the same for different connecting cases.
b) More importantly, when a connection point is introduced, it usually involves a detour from the shortest path between the origin and the destination. In the simplified example, I only assumed that the connection points exist continuously on the shortest path between the origin and the destination.

c) I also assumed that different connecting situations all have the same layover time, which is already included in the overall flying time.

With the stated simplified assumptions, I also assumed that the total flying time is always 6 hours. In other words, I defined that

\[ T_{AB} = T_{AC} + T_{CB} + T_{Layover} = 6 \text{ hours} \]  

(6.5)

Figure 6.5 is a simplified illustration of how the cost or disutility changes with the location of the connection point for the same route.

\begin{figure}
\centering
\includegraphics[width=0.8\textwidth]{disp_2.png}
\caption{Disutility vs. Connecting airport location with long travel time}
\end{figure}

If I recall cases 1 and 2, the horizontal axis represents the travel time between A and C. Therefore, the connecting location approaches B as the point moving toward the right-hand side of the axis. This is a very simplified example in that

The location of the connection is represented by \( T_{AC} \). The disutility decreases as the \( T_{AC} \) approaches either end of the axis. The disutility, or generalized cost, is highest when the connecting point is located around the mid-point of the connecting path. Therefore, when the connection is located relatively close to the origin or the destination, it incurs lower disutility on the users.

Traditional models assume that the introduction of a connection has an impact on the generalized cost of a flight. Our model actually implies a smoother shift from the direct flight to connecting flights. As Figure 6.6 shows, as the connection point approaches the origin or the destination, the disutility decreases. Additionally, the disutility of the travel time is highest when the connection point is around the mid-point, which is counter-intuitive according to traditional models. Further, as predicted before, the disutility may not change with changing connection points, as depicted in Figure 6.7.
Figure 6.7: Disutility vs. Connecting airport location with short travel time

When the total travel time is relatively short compared with the threshold time and the two segments do not incur the issue of the non-uniform value of time, then the location of the connection point (based on the pre-mentioned simplification conditions) will not affect the overall disutility caused by travel time.

Now I recall the three simplifying assumptions pertaining to acceleration/deceleration, detouring, and layover times. What will happen if these assumed conditions are relaxed? The non-linearity would become more significant because of the detouring and acceleration/deceleration processes. That is, the nonlinearity impact I discovered from our model is complementary, and hence adds to the original embedded non-linear impact.

6.3 Hub-and-Spoke System

The insight of asymmetrical optimality also coincides with our current hub-and-spoke system, which is asymmetrical as most of the cases show. This is due to the geographical situation of the nation to a very large extent. Figure 6.8 depicts the locations of major airports, which have the potential of becoming hubs, in the US, which clearly indicates non-uniform density across the nation. The density appears to be particularly low in the Midwest region. There are clusters of major airports along the East and West Coasts at the same time. Therefore, for long-distance travel especially transcontinental travel, airlines face the choice of placing the hub in the less-populated area in the middle of the country or somewhere close to either the origin or the destination.
I start by asking what the resulting network change will be once the new model is introduced. And, I expect to answer that the airline will prioritize shorter connections over longer connections for the same travel time saving. This expectation can be illustrated by comparing case 3 with case 4.

Figure 6.9: Case 3
Cases 3 and 4, as shown in Figure 6.9 and Figure 6.10, is a qualitative representation of the connection between cities A, B, C, and D. Assume that an airline company is considering choosing to locate a hub city between cities E and F in order to serve markets in A, B, C, and D, with major connections between A and C and between B and D. It should be emphasized that this is a qualitative example; our intention is not to calculate the travel time from the link lengths.

The major difference between the two networks is that there is a longer detour in the B–D connection in case 3, while the longer detouring occurs in A–C connection in case 4. Therefore, case 3 favors the markets in A–C whereas case 4 favors B–D. In traditional models, case 4 may be valued as equivalent to case 3, or in some extreme cases case 4 may even be valued more highly than case 3 for the same (or in some cases a longer) detour. However, based on the conclusions I just provided above, the new models proposed herein would value case 4 more highly than case 3. This is because the penalty it introduced via the detour only affects the longer connection, and hence the penalty is less valued in the utility function.

Of course, if the market of A–C is significantly larger than that of B–D, the implications will be quite different than the current ‘same size’ assumption, because the airline would certainly prioritize the A-C marker. I will only look at the generalized cost without the impact of demand. The demand issue can be introduced later without interrupting the original analysis.

6.3.1 Example I
In this extreme case example, I focus on three cities: San Francisco, Los Angeles, and San Antonio. And, I only consider two of the possible markets: San Francisco–Los Angeles
and San Francisco–San Antonio. The market of Los Angeles–San Antonio is not considered.

Figure 6.11: Example I

The airline is considering introducing a connection between San Francisco and San Antonio to serve as its southwestern hub. Two options are under consideration: Las Vegas and Los Angeles. The connecting cases and their respective travel times are given in Table 6.1.
In the case of connecting through Las Vegas, the connecting airport is almost exactly on the shortest path between San Francisco and San Antonio; therefore, there is no detour for the flights between San Francisco and San Antonio, and hence a limited penalty besides that of the connection itself. However, for the market between San Francisco and Los Angeles, the detour is significant, and so, therefore, is the penalty it introduces.

**Table 6.1: Travel Time (from the Internet)**

<table>
<thead>
<tr>
<th>Origin–Destination</th>
<th>Travel Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>San Francisco–Las Vegas</td>
<td>1 hr 30 min</td>
</tr>
<tr>
<td>Las Vegas–Los Angeles</td>
<td>1 hr 5 min</td>
</tr>
<tr>
<td>Las Vegas–San Antonio</td>
<td>2 hr 40 min</td>
</tr>
<tr>
<td>Overall Travel Time</td>
<td>5 hr 15 min</td>
</tr>
</tbody>
</table>

In the case of connecting through Los Angeles, although the detour between San Francisco and San Antonio is much more significant than the detour in the Las Vegas case, its advantages in the San Francisco–Los Angeles market is way more significant. The advantages include the direct flight itself and the reduced penalty in regard to travel time. However, in comparison with the San Francisco–Los Angeles market, the Los Angeles case has an advantage that overrides the disadvantage just stated. Yet, the total travel time
difference is not very significant, especially if I consider the fact that the layover times at the connecting airports are not included.

![Figure 6.13: San Francisco, Los Angeles, San Antonio - Scenario 2 with the hub located in Los Angeles](image)

It could be argued that the overall travel time in the Las Vegas case (5 hours 15 minutes) is longer than that in the Los Angeles case (4 hours and 30 minutes). Therefore, merely attributing the disadvantage to the economy of travel time is far-fetched. However, even though the layover time at Los Angeles airport is 45 minutes longer than that at Las Vegas, would the airline choose Las Vegas as the hub for the case I constructed? The answer is still probably not. The reality, which is entirely expected, is that travelers are seldom willing to transfer for a connection that is as short as SF–LA. Consequently, the idea of using Las Vegas as a hub airport serving SF–LA is unrealistic.

<table>
<thead>
<tr>
<th>Origin–Destination</th>
<th>Travel Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>San Francisco–Los Angeles</td>
<td>1 hr 25 min</td>
</tr>
<tr>
<td>Los Angeles–San Antonio</td>
<td>3 hr 5 min</td>
</tr>
<tr>
<td>Overall Travel Time</td>
<td>4 hr 30 min</td>
</tr>
</tbody>
</table>

Table 6.2: Travel Time (from internet)
Travelers’ unwillingness to make such a transfer is due at least in part to the competition between air travel and other modes of transportation, such as rail and highway. However, the highway (8 hours) and rail (12+ hours) each entail more than double the travel time of flying. Therefore, the impact in terms of modal competition is very limited. There is still a disadvantage in regard to connecting through Las Vegas because there it is not possible to control travel time below the threshold and because of the economy of travel time.

6.3.2 Example II

In this section, I use a more realistic example to discuss some more realistic and broader problems than I have considered so far. I focus on four cities: San Francisco; Washington, DC; Minneapolis; and San Antonio. These markets are similar to those of cases 3 and 4, as discussed above: San Francisco–Washington, DC, and Minneapolis–San Antonio. The hub airports through which the flights connect could be Pittsburgh, Dallas, or Washington, DC.

![Map](image)

Figure 6.14: Example II

As noted, given that this example involves a broader geographic region, more complicated airline operational and competition issues may arise. I do not want to introduce more assumptions to make this example into an “ideal case”; instead, our focus is on the overall operations. Instead of coming up with a conclusion whereby a certain case is posited as ideal, I analyzed the three cases in terms of their respective pros and cons.
Figure 6.15: San Francisco; Washington, DC; Minneapolis; and San Antonio – Scenario 1 with the hub located in Pittsburgh

Table 6.3: Travel Time (from the Internet)

<table>
<thead>
<tr>
<th>Origin–Destination</th>
<th>Travel Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>San Francisco–Pittsburgh</td>
<td>5 hr 10 min</td>
</tr>
<tr>
<td>Pittsburgh–Washington, DC</td>
<td>1 hr 5 min</td>
</tr>
<tr>
<td>Minneapolis–Pittsburgh</td>
<td>2 hr 5 min</td>
</tr>
<tr>
<td>Pittsburgh–San Antonio</td>
<td>2 hr 10 min</td>
</tr>
</tbody>
</table>

Table 6.4: Travel Time by Direction

<table>
<thead>
<tr>
<th>Direction</th>
<th>Overall Travel Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minneapolis–San Antonio</td>
<td>6 hr 15 min</td>
</tr>
<tr>
<td>San Francisco–Washington, DC</td>
<td>4 hr 15 min</td>
</tr>
<tr>
<td>Overall Flying Time</td>
<td>10 hr 25 min</td>
</tr>
</tbody>
</table>
Figure 6.16: San Francisco; Washington, DC; Minneapolis; and San Antonio, – Scenario 2 with the hub located in Dallas

Table 6.5: Travel Time (from the Internet)

<table>
<thead>
<tr>
<th>Origin–Destination</th>
<th>Travel Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>San Francisco–Dallas</td>
<td>3 hr 30 min</td>
</tr>
<tr>
<td>Dallas–Washington, DC</td>
<td>2 hr 55 min</td>
</tr>
<tr>
<td>Minneapolis–Dallas</td>
<td>2 hr 20 min</td>
</tr>
<tr>
<td>Dallas–San Antonio</td>
<td>1 hr 5 min</td>
</tr>
<tr>
<td>Overall Flying Time</td>
<td>9 hr 55 min</td>
</tr>
</tbody>
</table>

Table 6.6: Travel Time by Direction

<table>
<thead>
<tr>
<th>Direction</th>
<th>Overall Travel Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minneapolis–San Antonio</td>
<td>6 hr 25 min</td>
</tr>
<tr>
<td>San Francisco–Washington, DC</td>
<td>3 hr 25 min</td>
</tr>
<tr>
<td>Overall Flying Time</td>
<td>9 hr 50 min</td>
</tr>
</tbody>
</table>
Figure 6.17: San Francisco; Washington, DC; Minneapolis; and San Antonio – Scenario 3 with the hub located in Washington, DC

Table 6.7: Travel Time (from the Internet)

<table>
<thead>
<tr>
<th>Origin–Destination</th>
<th>Travel Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>San Francisco–Washington, DC</td>
<td>5 hr 10 min</td>
</tr>
<tr>
<td>Minneapolis–Washington, DC</td>
<td>2 hr 40 min</td>
</tr>
<tr>
<td>Washington, DC–San Antonio</td>
<td>2 hr 20 min</td>
</tr>
<tr>
<td>Overall Flying Time</td>
<td>10 hr 10 min</td>
</tr>
</tbody>
</table>

Table 6.8: Travel Time by Direction

<table>
<thead>
<tr>
<th>Direction</th>
<th>Overall Travel Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minneapolis–San Antonio</td>
<td>5 hr 10 min</td>
</tr>
<tr>
<td>San Francisco–Washington, DC</td>
<td>5 hr</td>
</tr>
<tr>
<td>Overall Flying Time</td>
<td>10 hr 10 min</td>
</tr>
</tbody>
</table>

In the case of Pittsburgh, there is a long detour in the North–South bound (Minneapolis–San Antonio connection), whereas the East–West bound (San Francisco–Washington, DC) is close to the direct flight path. The situation is the opposite in the case of Dallas. The case of Washington, DC, is close to that of Minneapolis, except that the conditions are even stronger for Washington, DC: the East–West bound is exaggerated into a direct flight,
whereas the detour in the North–South bound is even longer. It should be noted that only Washington, DC, includes a direct flight, and its overall travel time is the lowest. It should also be noted that, as discussed in Example I, the travel time could be leveraged by layover time. Therefore, although the overall flying time is different, the actual travel time difference could be a lot shorter than at first appears. Also due to the layover time, the East–West bound flights have a good chance of exceeding the second threshold, which is around 7 hours. As estimated in Chapter 5, the value of the travel time decreases even more after the second threshold has been passed. Consequently, even though both the North–South and the East–West bounds exceed the first threshold, only the East–West bound exceeds the second one, and hence the economy of travel time still prevails.

Table 6.9: Overall Travel Time Comparison

<table>
<thead>
<tr>
<th>Hub Airport</th>
<th>Overall Flying Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minneapolis</td>
<td>10 hr 25 min</td>
</tr>
<tr>
<td>Dallas</td>
<td>9 hr 55 min</td>
</tr>
<tr>
<td>Washington, DC</td>
<td>10 hr 10 min</td>
</tr>
</tbody>
</table>

Although the overall flying time for Dallas is shorter than that for Washington, DC, the disutility of the case of the latter is probably less. If I consider the optimality of the utility function as the primary reason, without considering other operation issues, the case of Washington, DC, is probably the most optimal even though the flying time in the case of Dallas is shorter. Example II, in this sense, has implications very similar to those of Example I.

The insights that can be obtained from the comparison are as follows:

A. Travel Time Penalty: Minneapolis > Dallas > Washington, DC

B. North–South Bound Advantage: Pittsburgh > Dallas > Washington, DC

C. East–West Bound Advantage: Washington, DC > Pittsburgh > Dallas

D. Northeastern Market: Pittsburgh ≥ Washington, DC > Dallas

E. Southwestern Market: Dallas > Pittsburgh = Washington, DC

Once the emphasis of the markets is clarified, there may also be other issues to consider, such as the issue of demand. For instance, the East–West (San Francisco–Washington, DC) direction would probably be the one with the higher travel demand. However, the actual demand of the market that the airline is targeting depends on how it positions itself. For instance, the airline may decide to enter the heated competition of the East–West bound (connecting through Washington, DC), or it may consider itself to be more competitive in the North–South market (connecting through Dallas). It may already be primarily serving the Northeastern region, and hence prefer to connect its flights with all the major airports in the region (connecting through Pittsburgh). The airline may not have a hub in certain
airports. American Airlines, for example, does not have a hub in Washington, DC; therefore, to prioritize its East–West bound connection, it would have to connect flights through Pittsburgh.

The steps from A to E can serve as metrics for an airline system evaluation based on different markets or issues that an airline is considering. After all, disutility is still the first factor to be analyzed.
7 Summary and Conclusion

7.1 Findings

It can be imagined that when travel time increases from 1 hour to 2 hours, the impact on travelers’ decisions is significantly greater than the impact of an increase from 5 to 6 hours. Travelers become less sensitive to travel time as it increases. The decreasing marginal cost of travel time has long been recognized. In the present research, I hypothesized that there are thresholds in travel time beyond which a decrease in the value of travel time becomes significant. There exist discontinuities of value of travel time at this threshold.

To test our hypothesis, I proposed 2 models for explanation, both of which are piecewise linear, which may (model 2) or may not (model 1) be associated with quantum changes at the thresholds. I used two air travel survey datasets: one conducted by Boeing in 2005 and one conducted by RSG in 2012. In order to take into account the access and egress process, I added 1.5 hours to the travel time to simulate the impact. The models are grounded in a theoretical framework derived from De Serpa’s time allocation theory. To estimate the models, clock travel time, meaning the difference between local arrival time and local departure time, was used for estimation.

The estimation results suggest the following findings:

1. There is clear evidence of a threshold at a travel time of around 3.7 hours (including the 1.5-hour access and egress time). This result could be considered consistent with the travel time threshold (3 to 3.5 hours) found in rail travel demand analysis.

2. The value of time changes with descriptive factors of the trip such as travel purpose and travel direction. The results showed that the threshold of business travelers (2.8 hours) is lower than that of leisure travelers (3.8 hours). The threshold should also change with travel directions because of the time zone difference. Travelers “gain” time by traveling from east to west, and “lose” time traveling in the opposite direction. From the theoretical point of view, the change is due to the overall time budget change associated with different time zones. With limited data, I was only able to conclude that the threshold does change with travel directions with a certain level of significance.

3. Again, based on limited data, I found model 1 to be more stable and robust than model 2. As stated in Chapter 3, model 2 has stronger conditions, but also requires more data. Our results also show some evidence of the quantum change suggested in model 2, although this is far from conclusive.

4. There is evidence suggesting the existence of a second threshold, which is also consistent with the theories in both models 1 and 2. I concluded that within the travel time scope of air travel within the US, there are two travel time thresholds.

The quantitative results of the thresholds from the two datasets should be considered tentative, especially as the datasets are very limited in regard to details. However, I can conclude that thresholds and discontinuities do exist in regard to the value of travel time. Whether the changes at the thresholds are quantum (model 2) or not (model 1) remains an open question.
It should be noted that the decreasing marginal cost of travel time, which has been observed for decades, is usually captured by multinomial functions. However, our findings suggest that smooth functions may not be an adequate form of travel time disutility. The change of the value of time is only the most significant at the threshold. The decrease of marginal cost, or the economy of travel time, only prevails when travel time exceeds the threshold.

Therefore, for the same amount of increase in travel time, travelers suffer a greater penalty when the travel time is below the threshold. In air travel, where the travel time cannot decrease below the direct flight time, the implication can be rephrased as follows: for the same amount of travel time increase (usually caused by a connection and/or a detour), the penalty is higher for a shorter connection.

In practice, the findings imply the need for a network of hubs predicated on an asymmetrical design, when transfers do not take place centrally. It is intuitive to think that when a connection is introduced, the midpoint or somewhere close to the middle is ideal. However, via a simplified numerical simulation of the change of travel time disutility with respect to different hub locations, the disutility, or the cost, is highest when the hub is located close to the middle. The disutility decreases as it approaches the origin or the destination. Similar implications can be drawn from the demand for rail travel.

For system designs that involve more than three airports, the implication is still that the system designer should favor the shortest possible connections. However, our results show that the detours and/or connections should be directed toward the longer connections if possible, sometimes even at the cost of longer overall travel time. Additionally, the hub should be located close to major airports instead of at the geographical mid-point.

Airlines face differ in regard to the respective reality constraints they face and in regard to the marketing strategies they use, and hence they also differ in terms of market preference. Travel time is not the only criterion based on which airlines choose their hub locations. Still, with the proposed model estimation, I am able to provide better and clearer evaluation metrics than have been presented to date.

Although in our study, I considered air travel in regard to route choice, the theories and the findings can be applied to all intercity transportation modes, in both mode and route choice situations. Some revision should be considered for other modes, however. For instance, the working mechanism of rail transportation systems is quite different from that of air transportation systems. For instance, whereas the speed of air travel cannot be increased, rail systems still have great potential to increase speed, which leads us to the topic of high-speed rail. When travel speed becomes a controllable variable, there are more available options for the network design. And, thus, the threshold and potential causal optimality in operations become even more critical. The travelers’ behavior theory itself is completely transferable to other transportation modes.

Similarly, the results can contribute to demand analysis and to policy-making, especially for multimodal transportation system design. The threshold affects the evaluation of the level of service significantly.
7.2 Future Work

As discussed, the present research is limited by the limited nature of the available datasets. Specifically, the limitations are as follows:

- Insufficient travel time information in regard to access and egress time.
- Insufficient number of travel respondents to achieve close stratification.
- Insufficient details to test the model exactly in line with the theoretical framework.

Most of these limitations could be resolved by better data support.

In future studies, improvements could be made both to the theoretical framework and to the empirical study. In regard to the theoretical framework, it is necessary to explain the decrease in disutility at thresholds in the previous estimations. In the present study, I have tended to explain the positive jump as arising from bundling activities.

Consequently, the model could be improved by accommodating the bundling effect between activities. For instance, a traveler might bundle traveling and sleeping together during an overnight flight, or she/he might bundle working and traveling together. In these situations, the other activities that travelers might conduct simultaneously with traveling provide extra utility. However, the question remains unanswered as to whether the positive parameter estimated for the dummy variable at the threshold is a result of the bundling effect or of insufficient relevant data. Future studies should be able to answer this question.

If the answer is activity bundling, the thresholds of bundling would need to be understood. For instance, at what threshold would a traveler start to consider bundling a trip with a work activity? What should be the threshold of bundling with sleeping, or even other activities that I have not yet taken into consideration? Furthermore, it is necessary to keep in mind the thresholds for “utility-consuming” incidents such as canceling other activities in the activity pattern, such as I have included in the present research. Under what conditions do positive quantum utility changes prevail, and what are the conditions of the negative cases? Or do quantum utility changes and negative cases occur simultaneously? All these questions suggest that there are many more factors and questions to explore before we can be said to have reached a full understanding of the discontinuity of the value of travel time.

In terms of data collection, the problem can be tackled in two directions: survey that asks about traveler’s perception of thresholds directly and indirectly. For the direct survey methods, questions about travelers’ perception of different travel times can be asked and aggregated. For the indirect survey methods, questions that relates to every single detail about the traveler’s activity pattern, itinerary details should be included.

A study of the overall daily activity patterns of travelers should be considered with a designed survey, and investigated thoroughly. However, even before such research is conducted, more work is needed to understand the exact time magnitude and level of detail required for the data collection—the survey design. New intercity transportation survey design methods should be considered. The survey should emphasize the correlation between the trip and other activities in which the traveler engages. Better links should be provided between intercity travel analysis and daily activity patterns, in much the same way as is done in urban travel analysis. The driven utility is the foundational aspect in
efforts to understand the value of time and even in terms of understanding transportation economics in general. To ignore the links results is to disregard the non-linearity of the value of time.

One stratification factor that should be taken into consideration is the change of threshold with the departure and arrival times. For instance, if a traveler is considering making a business trip from San Francisco to New York on an overnight flight because otherwise he/she would miss a day of working time, especially when time zone difference is considered. There are also many other activities that are closely connected with some particular clock time, such as lunch, dinner, and exercise. Due to the nature of this topic, the role of clock time should also be emphasized. The emphasis should be seen both in the data collection process and the analyzing process.
8 References


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