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Pi$^-$—p Elastic Scattering in the Energy Region 500-1500 Mev

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I. INTRODUCTION

The original work of Cool, Piccioni, and Clark$^1$ on the total cross sections for high energy $\pi^-p$ collisions indicated a maximum occurring in an energy region above the well-established $\frac{3}{2}$-$\frac{3}{2}$ state at 200 Mev. It was learned shortly thereafter from work at Berkeley$^2$ that the total cross sections at energies above 2 Bev appeared to be essentially constant at about 28 mb. Subsequent data from the M.I.T. group$^3$ working at Berkeley revealed that the maximum region previously observed at Brookhaven was in fact two peaks. Within the past two years, research at Saclay$^4$ and at Berkeley$^5$ has delineated these peaks with considerable precision and has determined their energies to be at 600-605 Mev for the one at the lower energy, and 890-900 Mev at the higher energy.

In the case of $\pi^-p$ total cross sections,$^4-^6$ a maximum has been observed at 1.35 Bev and the cross sections thereafter decrease to about 28 mb, apparently the same value as that reached by the $\pi^-p$ cross section. There is considerable evidence in the total cross section data that this maximum in the $\pi^-p$ cross section may in fact be due to a complex of states rather than a single state in resonance. There is, in particular, a “shoulder” on the low-energy side of the maximum for pions of about 800-Mev incident energy.

The photoproduction reactions $\gamma p \rightarrow \pi^+p$ and $\pi^-n$ also give rise to pion-nucleon states exhibiting the phenomena manifested in the $\pi^-p$ cross-section data. By utilizing the photoproduction measurements obtained at Cornell and at California Institute of Technology, Peierls$^7$ has attempted to identify the prominent states there seen, and to relate them to the phenomena manifested in the $\pi^-p$ scattering and total cross sections.

The purpose here is to present a survey of elastic scattering data for the $\pi^-p$ system in the energy region of the 600- and 900-Mev peaks. Previous contributions to knowledge of the elastic $\pi^-p$ scattering in the region of the “higher resonances”$^8$ have been made by Crittenden et al.$^8$ at pion energies of 460, 600, and 750 Mev; by Shonle$^9$ at 610, 655, and 750 Mev; by Maglic et al.$^{10}$ at 900 Mev; by Bergia et al.$^{11}$ at 915 Mev; by Erwin and Kopp$^{12}$ at 950 Mev; and by Derado and Schmitz$^{13}$ at 1000 Mev. At somewhat higher energies are the contributions of Chrétien et al.$^{14}$ at 1440 Mev; and Perl$^{15}$ at 1500 and 2000 Mev. The principal emphasis in this presentation, however, is to recent data secured

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$^1$ R. Cool, O. Piccioni, and D. Clark, Phys. Rev. 103, 1082 (1956).
at Berkeley by Wood et al.\textsuperscript{16} This work was done by use of the counter technique, and thus has considerably greater statistical strength than pertains to the previously listed contributions, all of which were obtained by various visual methods. We also make use of the data of Goodwin, Kenney, and Perez-Mendez\textsuperscript{17} which indicate the trend of behavior of the elastic scattering between the 200 new Mev resonance and the region here discussed.

The energies at which the Wood data were obtained are shown in Fig. 1 which displays a smooth curve drawn through the total cross sections as measured at Saclay and Berkeley. They are indicated by the heavy dots at 550, 600, 720, 900, and 1020 Mev, where these numbers refer to the laboratory kinetic energy of the incident pion. The data were secured by the use of an array of scintillation counters shown schematically in Fig. 2. The elastic scattering events were selected by coincidences of pion and proton counters together with the beam monitor counters, the pion and proton counters being connected in pairs which corresponded to the kinematics of elastic scattering events. As Fig. 2 shows, 13 angular positions were secured simultaneously by the array. The unwanted background of coincidences due to inelastic collisions and unassociated radiation was approximated by examining those coincidences which did not satisfy the kinematics of elastic scattering events.

The muon and electron content of the pion beam was measured by use of a gas Čerenkov counter whose pressure could be varied so as to reveal the response thresholds and contributions of the $\epsilon$'s, $\mu$'s, and $\pi$'s to the total beam intensity. The design of the beam optics and other details of the experiment were given by Wood et al.\textsuperscript{16}.

II. PRESENT STATUS OF ASSIGNMENT OF STATES

Before proceeding with the presentation of the data, it is useful to review the presently existing suggestions for the assignment of states for the 600- and 900-Mev phenomena as given by Peierls\textsuperscript{7} in his analysis of the states appearing in the photoproduction of pions. We consider first the level appearing at 600 Mev (which manifests itself in photopion production as a resonance maximum for incident photon energies of 750 Mev).

From the angular distribution in photoproduction, Peierls, and previously Wilson,\textsuperscript{18} concluded that the angular momentum of this state must be $\frac{3}{2}$, and it is clear from the data that it is purely an isotopic spin $\frac{1}{2}$ state since there is no evidence of a similar phenomenon in the $\pi^+ - p$ scattering at this energy.

Wilson originally considered this to be a $P_1$ state, though Peierls argued from observed energy dependence of the angular distribution that the assignment should be $D_1$.

Sakurai\textsuperscript{19} and Peierls suggested that a measurement of the polarization of the recoiling proton from the photopion reaction $\gamma p - \pi^0 p$, at energies slightly below the second resonance, could indicate the relative parity between this state and that at 200 Mev. This measurement of polarization was subsequently carried out by Stein\textsuperscript{20} who observed the polarization of the protons emitted at 90° in the center-of-mass system and found it to be relatively large. His measurements were carried out at two energies, namely, 550 and 700 Mev for the incident protons; these correspond in the center-of-mass energy produced to a pion-proton scattering experiment with incident pions of 400- and 550-Mev laboratory kinetic energy, respectively. For the lower of these two energies, he obtained a polarization of 0.30±0.12, and for the higher energies 0.59±0.06. If this polarization is interpreted as the result of the interference of the high energy tail of the 200-Mev resonance with the developing amplitude of the 600-Mev resonance, it may be concluded that the states involved in these resonances have opposite parity, and assignment of $D_1$ may consequently be made for the 600-Mev case.

It was subsequently pointed out by Landovitz and Marshall that an alternative interpretation is possible, namely, that interference between states accounting for the polarization may instead be due to the overlapping of the 600-Mev state with the low-energy tail of the 900-Mev maximum; and indeed the fact that Stein's measurement shows a larger polarization at the higher of his two energies might be consistent with this interpretation. However, Peierls further supports the $D_1$ assignment by the energy dependence of the angular


\textsuperscript{17} L. K. Goodwin, R. W. Kenney, and V. Perez-Mendez, Phys. Rev. Letters 3, 522 (1959); also further work submitted for publication.

\textsuperscript{18} R. R. Wilson, Phys. Rev. 110, 1212 (1958).


distribution in the \( \gamma p - \pi^+ N \) reaction. Recent further study of the polarization effect is reported by Peterson and Malloy\(^{21}\) at California Institute of Technology. They calculate expected polarization on the basis of various possible assumptions of interfering states and find that the overlap of the high-energy tail of the \( P_1 \) resonance at 200 MeV with the developing effect of a \( D_1 \) state at 600 MeV gives excellent agreement with their experimental data.

Aside from the evidence based upon angular distribution, the assignment of \( \frac{3}{2} \) for the angular momentum of the 600-Mev level appears to be consistent with the value of the cross section at this energy under certain reasonable assumptions. It has been pointed out previously that if we subtract from the total cross-section curve in this region a nonresonant background of about 30 mb, we are left with a "resonance" contribution of about 25 mb; and if we assume this to be due to the contribution of a single state, the quantitative agreement is reasonable with an angular momentum assignment of \( \frac{3}{2} \) together with an absorption parameter\(^{22}\) value of about 0.1 or 0.2 arising from the complex nature of the phase shifts at these energies.

An assignment for the 900-Mev maximum has not been given with as much confidence as the 600-Mev case allows. Peierls suggests that it has \( J = \frac{3}{2} \) upon the basis of the photopion angular distribution associated with it. The development of interference terms in the energy dependence of the angular distribution suggests even parity for the state (if it is a single state); so the tentative assignment is \( F_1 \). Our own belief, on the basis of the \( \pi^--p \) scattering angular distribution to be presented, is that the 900-Mev phenomenon cannot be adequately explained by a single state in resonance, but that it is rather a superposition of at least a strong \( D \)-state interaction together with \( F_1 \).

### III. SURVEY OF ELASTIC SCATTERING DATA

We now turn to the examination of the elastic scattering data and seek to identify some of the trends which may be observed by considerations of the shapes of the curves at various energies.

Figure 3 shows a series of nine angular distributions consisting of the data of Goodwin et al.\(^{17}\) at 290, 370, and 427 Mev, the data of Wood et al.\(^{18}\) at 550, 600, 720, 900, and 1020 Mev, and the data of Chrétien et al.\(^{14}\) at 1300 Mev. To allow comparison with some of the previously existing information, the results of Bergia et al.\(^{11}\) at 915 Mev are shown in Fig. 4, and those of Erwin and Kopp\(^{13}\) at 950 Mev appear in Fig. 5.

The normalization of the new data by Wood et al. is unfortunately not independently achieved with accuracy comparable to that with which the shapes of the curves are determined, owing to some degree of failure of the absolute monitor of their pion beam to follow faithfully the beam intensity variations. However, it was ascertained that this difficulty did not distort the shapes of the angular distributions.

The normalization of the curves shown was adjusted to provide best agreement conjointly with the \( 0^\circ \) differential cross sections calculated by dispersion theory from total cross-section data,\(^{23}\) and with the total elastic-scattering cross sections inferred from the Rochester Conference report of Falk-Vairant and Valladas\(^{24}\) and from the work of Shonle.\(^{9}\)

The curves drawn through the experimental points in the Wood data (Fig. 3) are those determined by a best-fit expansion in powers of \( \cos \theta \) (or alternatively in Legendre polynomials). A discussion of highest powers required is given in Sec. IV.

In surveying Fig. 3 we observe certain trends in the shapes of the angular distributions as the energy is increased. The analysis of the data of Goodwin\(^{17}\) indicated the presence of \( D \) waves in the elastic scattering by the time the energy had reached 427 Mev. However, the first of the curves of the Wood data obtained at 550 Mev suggests no pronounced growth of \( D \)-wave characteristics up to this point.

Going from 550 to 600 Mev, up to the peak of the 600-Mev resonance curve, the major change in the scattering phenomenon is that of an enhanced forward

![Fig. 2. Array of scintillation counters in Wood experiment. An elastic event required a coincidence of counts in \( M_1, M_2, S_0, \pi_1, \) and \( p_1 \) where \( \pi_1 \) and \( p_1 \) satisfied elastic kinematics.](image)
peak, as if the additional magnitude of the cross section might largely be provided by diffraction scattering caused by absorptive interactions. There is also evidence of the beginning of a turning over of the differential cross section curve near 180°, which trend is seen to be developing strongly near 720 Mev, in the valley between the 600- and 900-Mev maxima.

The backward hump which appears strongly at 720 Mev is built up in a pronounced manner at the 900-Mev maximum, accompanied by a rather complete suppression of the yield precisely at 180°, and then substantially dies away as seen in Wood's curve at 1020 Mev. The data of Chrétien at 1300 Mev indicates that the cross section at 180° has again reached a value other than

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**Fig. 3.** Display of sequence of angular distributions in elastic scattering at increasing energies. (See text for identification of origins of data.)
zero and the backward hump has essentially disappeared. The data of Perl\textsuperscript{14} obtained at 1.5 and 2 Bev indicate a continuance of the state of affairs shown by the Ch\'retien data. The backward hump which appeared so dramatically between 720 and 1020 Mev and which reached its maximum at the 900-Mev energy is clearly due to a contribution of the spin-flip scattering amplitudes.

IV. ANALYSIS OF THE ANGULAR DISTRIBUTION DATA

In fitting the data by expansions of the type \(\sigma(\theta) = \Sigma a_n \cos^n \theta\), the highest power of \(n\) to be employed for a given set of points was determined in the usual manner by reference to the dependence of \((X^2/d)^4\) upon \(n\). \((X^2\) is the usual deviation parameter, and \(d\) is the number of "degrees of freedom" available in the fit.)

Fig. 6 shows the behavior of this goodness-of-fit parameter, as \(n\) is increased, for the various energies of the Wood data. The two lowest energies do not require terms beyond \(n=3\). The three highest energies clearly require terms through \(n=5\), but the curves are not improved by including terms beyond \(n=5\).

The variation of the coefficients \(a_n\) as functions of pion energy form the basis for such conclusions as we can extract from the present data. Figure 7 shows this behavior over the energy range 200-1300 Mev. The data involved here are from the work of Goodwin et al.,\textsuperscript{17} Shonle,\textsuperscript{9} Bergia et al.,\textsuperscript{11} Ch\'retien et al.,\textsuperscript{14} and Wood et al.,\textsuperscript{15} with minor renormalization in one case.

It is at first thought surprising that both \(a_3\) and \(a_4\) should be so near to zero at 600 Mev, where we have been led to believe that a strong interaction in the \(D_1\) state exists. In fact, only \(a_3\) and \(a_4\) indicate maxima at this energy. Between 700 and 900 Mev, \(a_4\) and \(a_5\) increase dramatically to maxima at 900 Mev, where \(a_5\) also maximizes in a negative sense.

The utility of a decomposition into partial waves in the analysis of such data at these energies is somewhat questionable, since one must deal with a very large array of parameters. It is clear that we must include orbital states as high as the \(F\) state, each with its possibilities for \(J\) values, and that the phase shifts are complex; also, we have a mixture of the \(T=rac{3}{2}\) and \(T=rac{1}{2}\) isotopic spin states. Thus, we are confronted with 28
variables to be adjusted in fitting the experimental data. Furthermore, the strongly inelastic character of the collisions causes the cross sections to be relatively insensitive to variations in the real parts of the phase shifts. Consequently, it is typically possible to fit a given experimental angular distribution curve with various sets of parameter values. In fact, the problem is not uniquely determined in a mathematical sense without further types of data, such as measurements of polarization of the recoil proton as a function of scattering angle.

Nevertheless, a computer program of sufficient complexity to calculate phase shift parameters through $G$ waves has been prepared by R. Cence, P. McManigal, and D. Hagge in anticipation of further data, including polarization measurements and $\pi^\pm$ studies. When applied to this presently existing data, the program delivers many possible sets of parameters giving satisfactory agreement with the experimental points. At 600 Mev, for example, several of the solutions cluster near the values for $\alpha$ (real part of phase shift) and $b$ (absorption parameter) shown in Table I. These values have not been decomposed into the parameters for the isotopic spin states separately, since an extensive array
of $\pi^+ - p$ differential cross sections does not yet exist. But this set is compatible with the results of Walker, Davis, and Shephard$^{26}$ in their earlier analysis of the data of Goodwin$^{17}$ and of Crittenden$^{18}$ et al.$^{8}$ In this set we do not see any clear evidence of $D$-state resonance; the most outstanding aspect being strong absorption in $P$ states as was implicit in our earlier examination of the coefficients of powers of $\cos \theta$ (see Fig. 7). Even in the decomposition into parameters for the separate $s$-spin states attempted by Walker, data at 550 Mev indicates that terms beyond $\cos 3 \theta$ are clearly not required even at 1020 Mev indicates that $P_3$ is the highest order amplitude present. The $\cos \theta$ term requires $D$ and $F$ interference.

The strong forward-direction scattering is well described by a diffraction model in which the radius of the absorbing volume is about 1.2 $f$, which indicates strong interaction of the incident pion with the pion cloud of the proton.

The pronounced bump in the backward region, together with the essentially complete suppression of the scattering at $180^\circ$ (at 900 Mev), calls for a superposition of spin-flip amplitudes through $F$ waves. A faithful reproduction of the 900-Mev curve can be achieved by empirical superposition of amplitudes involving Legendre polynomials through $P_3$ and associated Legendre functions through $P_4$ which, by the coefficients required, implies that $P$, $D$, and $F$ waves are all prominent at 900 Mev.

Thus, it is not possible to satisfy the 900-Mev data by a single state ($F_3$) in resonance; a superposition of at least $D$ and $F$ in strong interaction is required, and absorptive processes may be predominant in view of the forward "diffraction."

With reference to the $D-F$ interference in this region, it is of interest to recall that in the $\pi^+ - p$ total cross section as measured by both the Saclay$^2$ and Berkeley$^3$ groups, there appears a "shoulder" on the low-energy side of the 1350-Mev peak, suggesting a "resonance" in the $T=\frac{3}{2}$ system for incident pions of about 800 Mev. Since the $T=\frac{3}{2}$ state contributes to the $\pi^+ - p$ system in the proportion $\frac{1}{2}(T_1)+\frac{3}{2}(T_1)$, we may expect such a phenomenon to manifest itself in some manner in the $\pi^+ - p$ scattering even though its effect upon the $\pi^+ - p$ total cross section is not evident in the region of the 900-Mev maximum. But if this 800 Mev, $T=\frac{3}{2}$ phenomenon were a $D$-state interaction, then this could help to account for the prominent $D-F$ interference which builds up so strongly in the 700- to 900-Mev region.

Carruthers and Bethe$^{26}$ have also considered the possibility of such effects from a $D_4$ state in the $T=\frac{3}{2}$ system at this energy.

The general picture emerging from these data is compatible with the models proposed by Peierls$^7$ and by Carruthers and Bethe,$^{26}$ utilizing the final-state isobar interaction and initial $\pi-\pi$ interactions to infer the importance of $D_4$ and $F_4$ for the 600- and 900-Mev phenomena in $\pi-p$ collisions. The nature of the scattering is such as to imply absorption processes to be prominent, as these models require. The evidence does not point to single states in resonance, in the usual sense of elastic-scattering resonance, as an explanation of the phenomena.

### Table I. Typical set of phase-shift parameters for $\pi^- - p$ scattering at 600 Mev.

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_4$</td>
<td>46.6°</td>
<td>0.65</td>
</tr>
<tr>
<td>$P_4$</td>
<td>$-13^\circ$</td>
<td>0.30</td>
</tr>
<tr>
<td>$D_4$</td>
<td>$-19^\circ$</td>
<td>0.46</td>
</tr>
<tr>
<td>$D_4$</td>
<td>$3.9^\circ$</td>
<td>1.0</td>
</tr>
</tbody>
</table>

is evidence for strong $D$ and $F$ waves in superposition; and the fact that terms beyond $\cos \theta$ are clearly not required even at 1020 Mev indicates that $P_1$ is the highest order amplitude present. The $\cos \theta$ term requires $D$ and $F$ interference.

### Possible Interpretations

#### A. Energy Region near 600 Mev

The fact that terms in $\cos \theta$ and $\cos^3 \theta$ are small in the scattering at 600 Mev does not necessarily argue against interaction in a $D$ state; for if $D_4$ were the highest order partial wave present there would certainly be no $\cos \theta$ term, and it is possible to select phase-shift parameters for the $P_4$ and $D_4$ interactions which also eliminate the $\cos \theta$ term. There is, in fact, clear evidence for $D$ and $P$ waves superimposed given by the existence of the $\cos \theta$ term at energies below 600 Mev as shown in the behavior of $\alpha_4$ in Fig. 7. Since, as shown in Fig. 6, terms beyond $\cos \theta$ are not required to fit the Wood data at 550 and 600 Mev, we may infer that the $D$ state interaction is $D_4$ rather than $D_4$.

The most reasonable conclusion is that amplitudes through $D_4$ are present as we increase the incident pion energy toward 600 Mev, and that the behavior of $\alpha_3$ results from superposition of the $P_4$ amplitude from the high-energy tail of the $(3-3)$ state at 200 Mev with a developing $D_4$ amplitude; but the phases progress in such a manner as to bring $\alpha_3$ near zero at 600 Mev.

#### B. 900-Mev Region

The rapid increase of $\cos \theta$ and $\cos^3 \theta$ terms between 700 and 900 Mev, where they reach maximum values,
