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Y\(_1\) (1385 MeV): STUDY OF SPIN AND PARITY BY MOMENT ANALYSIS
FOR \( J = \frac{5}{2}, \frac{3}{2}, \text{ AND } \frac{1}{2} \)

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November 5, 1963
Y_1^* (1385 MeV): Study of Spin and Parity
by Moment Analysis for J=5/2, 3/2, and 1/2*

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ABSTRACT

The possibility that the Y_1^* (1385-MeV) spin is 5/2 has been investigated. Identification of the Y_1^* as an F_5/2 state is excluded; the D_5/2 assignment is found possible, though is not required by the data. Of the states P_3/2, D_3/2, S_4/2, and P_4/2, only P_3/2 is acceptable, the excluded hypotheses having confidence levels of order 10^{-6} or 10^{-7}; this conclusion strengthens slightly the authors' earlier selection of P_3/2, based chiefly on study of the θ_Λ dependence of two components of the decay Λ's polarization. This more definitive report results from the application of the "moment analysis" of Byers and Fenster to a sample of 895 Y_1^* decays obtained from K^-p interactions in the 72-inch bubble chamber.
I. INTRODUCTION

The spin and parity of the $Y_1^*$ (1385 MeV) has been investigated for
$J = 5/2, 3/2, \text{ and } 1/2$. The identification of the $Y_1^*$ as an $F_{5/2}$ state is excluded;
the $D_{5/2}$ assignment is possible, though not required by the data. Confirmatory
evidence has been obtained for the authors' earlier selection of $P_{3/2}$, based
chiefly on the angular dependence of the decay $\Lambda$'s polarization.\(^1\)

A number of reported experiments show considerable anisotropy in the
angular distribution of $Y_1^{*\pm} \rightarrow \Lambda + \pi^\pm$ decay and thus permit the conclusion that the
$Y_1^*$ has a spin $>1/2$.\(^2\) Two experimental groups have found that the average polar-
ization of the $\Lambda$ indicates that the $Y_1^*$ is either $S_{1/2}$ or $P_{3/2}$.\(^3\) A recent study of
$Y_1^{*0} \rightarrow \Lambda + \pi^0$ events concludes from an Adair distribution that spin $3/2$ is probable
but that spin $5/2$ is unlikely.\(^4,5\)

The analysis reported here utilizes the complete angular distribution of all
three components of $\Lambda$ polarization (whereas the earlier study treated only the
$\phi_\Lambda$-averaged distributions of the polarization components in the plane of the $\Lambda$ and
the production normal). The parity conclusion is based on a comparison of the
polarization components transverse and parallel to the $\Lambda$ direction; this comparison
is similar to (but is more general than) that of the normal and "magic-direction"
components of polarization made earlier. In this present study, the formalism of
Byers and Fenster\(^6\) is applied to the distributions of $\Lambda$ direction and $\Lambda$ polarization
resulting from $Y_1^*$ decay.

II. THEORY

As Byers and Fenster have shown, the decay process $Y^* \rightarrow \Lambda + \pi$ can be
conveniently described in terms of expectation values $t_{LM}^M$ of certain spin-space
operators $T_{LM}^M$;\(^7\) these expectation values are designated as "multipole parameters."
Thus the angular distribution of the decay \( \Lambda \) is given by

\[
I(\theta, \phi) = \sum_{L, M} c_{LM}^L Y_{LM}^M(\theta, \phi) \quad \text{with } L \text{ even},
\]

where \( c_{LM}^L \) is a constant determined by \( L \) and \( J \) (the \( Y_1^* \) spin); \( Y_{LM}^M(\theta, \phi) \) is the charge conjugate of the usual spherical harmonic; and the normalization is such that \( \int I(\theta, \phi) \, d\Omega = 1 \). The angular dependence of the \( \Lambda \) polarization is represented by

\[
IP_{1}^m(\theta, \phi) = \sum_{L', M'} C_{L'M'}^{m} Y_{L'}^{M'}(\theta, \phi) \quad \text{with } L' \text{ even},
\]

where \( P_{1}^m \) represents a spherical tensor polarization component

\[
[ P_{1}^0 = P_z, \quad P_{1}^1 = -(P_x + iP_y)/\sqrt{2} \quad \text{and} \quad P_{1}^{-1} = (P_x - iP_y)/\sqrt{2} ]; \quad \text{and } C_{L'M'}^{m} \text{ is a combination of } t_{L'1}^{m+M'} \text{ and } t_{L'-1}^{m+M'} \text{.}
\]

The angles \( \theta \) and \( \phi \) are the polar and azimuthal angles of the \( \Lambda \) momentum as observed in the \( Y_1 \) rest frame (in a coordinate system determined by the production process); the \( z \) axis is most conveniently taken as the normal (n) to the production plane, as all \( t_{L}^{M} \) with \( M \) odd must then vanish (by reason of parity conservation in production).

For the determination of the \( Y_1^* \) parity, Byers and Fenster transform the above components of \( I(\theta, \phi) \) into components longitudinal and transverse with respect to the \( \Lambda \) direction. Both of these involve only the odd-L \( t_{L}^{M} \) as before, though the dependence on spherical harmonics is very different. The longitudinal component has the form

\[
IP_{\parallel} = IP \cdot \Lambda = \sum_{L, M} c_{LM}^L Y_{LM}^M(\theta, \phi) \quad \text{with } L \text{ odd}.
\]

and the transverse component can be represented as

\[
IP_{\perp} = IP \cdot \Lambda \times (\Lambda \times n) - 11 \overrightarrow{P} \cdot (n \times \Lambda)
\]

\[
= - \sum_{L, M} c_{LM}^L (2L+1)/4\pi \right)^{1/2} \mathcal{A}_{M1}^{L*}(\phi, \theta, 0) \gamma t_{L}^{M*} \quad \text{with } L \text{ odd},
\]
where \( n_{L_1} \) depends on \( L \) and \( J \), \( \mathcal{O}^L_{M_1} \) is the usual rotation matrix, and \( \gamma = +1 \) or \(-1\) in accordance with the \( Y_1^* \) parity \((J = \ell + 1/2 \text{ or } \ell - 1/2)\).

The \( t^M_L \) multipole parameters are proportional to the "moments" \( \langle Y^M_L \rangle \) of the \( I(\theta, \phi) \) and \( I_P(\theta, \phi) \) distributions (and will hereafter be referred to as "moments"). These can be evaluated by multiplying each distribution by the appropriate \( Y^M_L \) (or by \( \mathcal{O}^L_{M_1} \) for the transverse polarization) and integrating. Thus from the angular distribution,

\[
\nu_{L_0} \cdot t^M_L = \int d\omega \, I(\theta, \phi) \, Y^M_L(\theta, \phi) \quad \text{for even } L;
\]

and from the longitudinal polarization,

\[
\nu_{L_0} \cdot t^M_L = \int d\omega \, I_P \cdot \mathcal{A}(\theta, \phi) \, Y^M_L(\theta, \phi) \quad \text{for odd } L.
\]

Finally, from the transverse polarization,

\[
\gamma \, \nu_{L_1} \cdot t^M_{L_1} = (2L+1)^{1/2} \int d\omega \, \mathcal{O}^L_{M_1}(\phi, \theta, 0) \, I_P(\theta, \phi)
\]

for odd \( L \). Since \( P_\perp = -\sqrt{2} \sum_m \mathcal{O}^M_{M_1}(\phi, \theta, 0) \, P^{m*}_1 \), Eq. (7) reduces to

\[
\gamma \, \nu_{L_1} \cdot t^M_{L_1} = (2L+1)^{1/2} \left[ A_L \sum_m \int d\omega \, I_P^m \, Y^{M-m}_{L-1} + B_L \sum_m \int d\omega \, I_P^m \, Y^{M-m}_{L+1} \right],
\]

where \( m \) has values 0, +1, or -1; \( A_L = (L+1)^{1/2} \, C(1, L-1, L; m, M-m) \) and \( B_L = (L)^{1/2} \, C(1, L+1, L; m, M-m) \); and \( P^m_1 \) represents a spherical tensor polarization component. Comparison of the quantities evaluated from expressions (6) and (8), as shown by Byers and Fenster, determines \( \gamma \) and hence the \( Y_1^* \) parity.
III. APPLICATION

In practice, an angular distribution moment is determined by evaluating the appropriate $Y_L^M$ for the $\theta, \phi$ angles of each event (k) and summing over the N events to find the average:

$$\nu_{L0} t_L^M = \left[ \sum_{k=1}^{N} Y_L^M (\theta_k, \phi_k) \right] (1/N).$$  \hspace{1cm} (9)

[Compare with Eq. (5).]  
Polarization moments are more difficult to evaluate because the polarization itself is an average, found by summing direction cosines of decay pion momenta in the $\Lambda$ rest frame; thus

$$\bar{P} \cdot \hat{A} (\theta, \phi) = \left( \frac{3}{\alpha n} \right) \sum_{j=1}^{n} \hat{p}_j \cdot \hat{A} (\theta, \phi), \text{ or } P_1^m (\theta, \phi) = \left( \frac{3}{\alpha n} \right) \sum_{j=1}^{n} \left( \frac{4\pi}{3} \right)^{1/2} Y_1^m (\Theta_j, \phi_j), \hspace{1cm} (10)$$

where $\alpha$ is the $\Lambda$ decay parameter ($\alpha = 0.62$), $j$ is any event with a $\Lambda$ traveling in the $\theta, \phi$ direction in the $Y^*$ rest frame, $\hat{p}$ is a unit vector along the pion direction in the $\Lambda$ rest frame, and $\hat{A}$ is along the direction of transformation into this rest frame. Angles $\Theta_j, \phi_j$ refer to the $\hat{p}$ direction in the $\Lambda$ rest frame.  

Equation (6) for the odd-$L$ moments of longitudinal polarization becomes [with absorption of the sum of Eq. (10) into that of Eq. (6)]:

$$\nu_{L0} t_L^M = \left[ \sum_{k=1}^{N} Y_L^M (\theta_k, \phi_k) \hat{p}_k \cdot \hat{A}_k \right] \left( \frac{3}{\alpha N} \right). \hspace{1cm} (11)$$

Equation (8) for the odd-$L$ moments of transverse polarization becomes

$$\gamma \nu_{L1} t_L^m = (2L+1)^{-1/2} \left[ A_L \sum_{m} \sum_{k=1}^{N} Y_{L-1}^{m-m} (\theta_k, \phi_k) Y_1^m (\Theta_k, \phi_k) \right. \right. \left. \left. + B_L \sum_{m} \sum_{k=1}^{N} Y_{L+1}^{m-m} (\theta_k, \phi_k) Y_1^m (\Theta_k, \phi_k) \right] \left( \frac{4\pi}{3} \right)^{1/2}. \hspace{1cm} (12)$$

The expressions above obviously are sums of complex numbers, so represent separate sums of real and imaginary parts.
Errors are evaluated for the real and imaginary parts of each moment by the use of such expressions as

\[
\delta (\text{Re } t_L^M) = \frac{1}{\sqrt{M_0}} \left\{ \sum_{k=1}^{N} \left[ \text{Re } Y_L^M (\theta_k, \phi_k) \right]^2 - \left[ \sum_{k=1}^{N} \text{Re } Y_L^M (\theta_k', \phi_k') \right]^2 / N \right\}^{1/2}
\]

for the real part of the cross-section moment found from Eq. (9), and

\[
\delta (\text{Re } t_L^M) = \frac{1}{\sqrt{M_0}} \left\{ \sum_{k=1}^{N} (\text{Re } Y_L^M)^2 (\vec{\pi} \cdot \vec{\Lambda})^2 - \left[ \sum_{k=1}^{N} \text{Re } Y_L^M \vec{\pi} \cdot \vec{\Lambda} \right]^2 / N \right\}^{1/2}
\]

for the polarization moment of Eq. (11). (The second term usually is very small in comparison with the first in these equations.)

IV. EXPERIMENT

The formulae developed above were applied to the \( Y_1^* \) decay distributions from 895 specially selected events from a sample of 1650 identified as

\[
K^- + p \rightarrow Y_1^{*\pm} + \pi^\mp
\]

at 1.22 BeV/c incident momentum. The selection criteria were that the \( Y_1^* \) mass be between 1340 and 1430 MeV, and that the production angle be such that \( |\vec{Y} \cdot \vec{K}| \leq 0.8 \). The mass limits gave good separation between \( Y_{1*}^* \) and \( Y_{1*}^- \) bands on the Dalitz plot, shown in Fig. 1; the restriction on production angle enhanced the observed polarization or alignment. (For other details, see the earlier reports.)

The various \( t_L^M \) moments determined are presented in Table I. (The negative-\( M \) moments are omitted, as they give no additional information, \( t_L^{-M} \) being equal to \((-)^M t_L^{M*} \). There are two evaluations for each odd-\( L \) moment, one from longitudinal and one from transverse polarization. These are compared below. The power series in sines and cosines necessary to describe the data are obtained by using the experimental moments of Table I in the expressions (1) through (4); e.g., the longitudinal polarization for the assumption of the \( Y_1^* \) state \( P_{3/2} \) is given by
\[ I \mathcal{P} \cdot \hat{\Lambda} = n_{10} t_1^0 Y_1^0 + n_{30} t_3^2 Y_3^{2*} + n_{10} t_2^0 Y_2^{0*} + n_{30} t_2^2 Y_2^{-2*} \]

\[ = (3/4\pi)^{1/2} [0.126 t_1^0 \cos \theta - 0.379 \langle(3/2)^{1/2}\rangle^2 + (\text{Re} t_3^2 \sin^2 \theta \cos \phi \cos 2\phi) \]

\[ + \text{Im} t_3^2 \sin^2 \theta \cos \phi \sin 2\phi] - 0.379 t_3^0 (\sqrt{7}/2\sqrt{3})(5 \cos^3 \theta - 3\cos \theta) \].

To decide the spin of the \( Y_1^* \), the maximum complexity of non-zero moments was determined. A chi-squared was constructed, of the form

\[ \chi^2 = \sum_{i,j} (t_i - \langle t_i \rangle) G^{-1}_{ij} (t_j - \langle t_j \rangle) \]

where \( t_i \) or \( t_j \) represents a real or imaginary part of any \( t_i^M \) moment (e.g., \( \langle \text{Re} Y_2^{2,i} \rangle \)) and \( \langle t_i \rangle \) or \( \langle t_j \rangle \) designates the expected value; the moments included were those needed to describe the decay of a spin 5/2 system (17 real numbers). The \( G^{-1} \) is the inverse error matrix; each term of the error matrix is given by

\[ G_{ij} = \delta(t_i - \langle t_i \rangle) \delta(t_j - \langle t_j \rangle) = (1/N^2) \sum_{k=1}^{N} \left( \frac{Y_i - \langle Y_i \rangle}{\langle Y_j \rangle} \left( \frac{Y_j - \langle Y_j \rangle}{\langle Y_j \rangle} \right) \right). \]

[For diagonal terms Eq. (18) is the same as Eq. (13) or Eq. (14)]. Data from the angular distribution and from all polarization distribution were used.

The \( \chi^2 \) for \( Y_1^* \) spin equal to 1/2 was found by using for the expected \( \langle t_i \rangle \) the experimental values for \( t_{10} \), \( \text{Re} t_{11} \), and \( \text{Im} t_{11} \), but taking all higher-L \( \langle t \rangle \)'s to be zero (as required for spin 1/2). The \( \chi^2 \) for spin 3/2 was obtained in a similar way. The results are stated, with the number of degrees of freedom, in Table II. A.

To determine the parity of the \( Y_1^* \) for each spin hypothesis, it was necessary to test equality of the corresponding moments for longitudinal and transverse components of polarization with the \( \gamma \) of Eq. (12) assumed equal to +1 or -1.

The \( \chi^2 \) formed was

\[ \chi^2 = \sum_{i,j} (t_i^L - \gamma t_i^T / \gamma') G^{-1}_{ij} (t_j^L - \gamma t_j^T / \gamma'). \]
where the indices $i$ and $j$ again designate any $\text{Re} t_{L}^{M}$ or $\text{Im} t_{L}^{M}$ (including only those permitted to be non-zero for a given spin), and where superscripts $L$ and $T$ denote longitudinal or transverse moments. Each element of the error matrix has the form

$$G_{ij} = \left< \delta \left( t_{i}^{L} - \gamma t_{j}^{T} / \gamma' \right) \delta \left( t_{j}^{L} - \gamma t_{j}^{T} / \gamma' \right) \right>.$$  \hspace{1cm} (20)

The constant $\gamma'$ was given a value of $+1$ to test for $\ell = J - 1/2$ and $-1$ for $\ell = J + 1/2$. Results of the parity test are given in Table II, B.

Another method investigated for treating polarization data was a ratio technique also advanced by Byers and Fenster. \(^{13}\) A ratio of the experimental evaluations of the two parts of Eq. (12) (the $A_{L}$ sum and the $B_{L}$ sum) was compared with theoretically predicted ratios for various spin-parity hypotheses. The evaluations of a $\chi^2$ [which tested $(\text{part A}) = \text{part B}) \times R$, with $R$ the predicted ratio] were almost identical with those of the parity $\chi^2$ discussed above and presented in Table II; however, the results permitted no discrimination between spin 1/2 and spin 3/2.

Additional attempts were made to discriminate between spin-3/2 and spin-5/2 hypotheses. The experimental moments of Table I were used to evaluate

$$\sum_{L, M} (2L+1) \left| t_{L}^{M} \right|^2 \leq 2J + 1$$  \hspace{1cm} (21)

and

$$2 \sum_{L, M} (2L+1) \left| t_{L}^{M} \right|^2 \geq 2J + 1,$$  \hspace{1cm} (22)

both of these deriving from general properties of the density matrix. \(^{14}\) Both inequalities were found well satisfied, within errors, for $J = 3/2$ and $J = 5/2$.

Another study of spin hypotheses involved the explicit solution for spin $J$ from the Byers-Fenster relation between longitudinal and transverse polarization moments. \(^{6}\) The equality

$$t_{L}^{M} (\text{longitudinal}) = t_{L}^{M} (\text{transverse})$$  \hspace{1cm} (23)
demands that
\[ \left( \frac{1}{\gamma_{L0}} \right) \mathcal{P}^{(LM)}_\parallel = \left( \frac{1}{\gamma_{L1}} \right) \mathcal{P}^{(LM)}_\perp, \]  
(24)

where \( \mathcal{P}^{(LM)}_\parallel \) and \( \mathcal{P}^{(LM)}_\perp \) are the \( L, M \) moments of the distributions in Eqs. (3) and (4), and are evaluated according to Eqs. (11) and (12). As shown by Byers and Fenster, Eq. (24) is equivalent to
\[ \gamma (2J+1) = \left[ L(L+1) \right]^{1/2} \frac{\mathcal{P}^{(LM)}_\perp}{\mathcal{P}^{(LM)}_\parallel}. \]  
(25)

This equation should hold for every value of \( L \) and \( M \) for which a polarization moment can be defined. The use of the four lowest moments (proportional to \( t^0_1, \Re t^2_3, \Im t^2_3, \) and \( t^0_3 \)) to evaluate \( J \) from Eq. (25) indicates that the spin is likely to be \( 3/2 \) rather than \( 5/2 \). (See Table III.) A simple \( \chi^2 \) of the form
\[ \chi^2 = \sum_i (J_i - J_i^0)^2 / (\delta J_i)^2 \]  
(26)
yields confidence levels of 0.005 for \( J^0 = 5/2 \) and 0.22 for \( J^0 = 3/2 \). However, these \( \chi^2 \) values cannot be considered reliable. [They differ substantially from those of the parity \( \chi^2 \) (Eq. 19), though based on the same \( \mathcal{P}^{(LM)}_\parallel - \mathcal{P}^{(LM)}_\perp \) relation, Eq. (24); the differences are much too great to be accounted for by the neglect of correlated errors in Eq. (26)]. The experimental values \( J_i \) of Eq. (26) are not Gaussianly distributed, as they depend on the ratio of \( \mathcal{P}^{(i)}_\perp \) to \( \mathcal{P}^{(i)}_\parallel \). The "parity \( \chi^2 \)" however, does test a Gaussianly-distributed quantity,
\[ \left( \frac{1}{\gamma_{L0}} \right) \mathcal{P}^{(i)}_\parallel - \left( \frac{1}{\gamma_{L1}} \right) \mathcal{P}^{(i)}_\perp. \]

The value of \( J \) was varied in small increments in the "parity \( \chi^2 \)" with first the four moments appropriate to spin \( 3/2 \) and then the nine moments appropriate to spin \( 5/2 \). Only a slow rise in each \( \chi^2 \) is observed as \( J \) is increased from \( 3/2 \) to \( 5/2 \); the confidence levels for the four-moment \( \chi^2 \) are 0.45 and 0.21, respectively. (See Fig. 2.) It is evident from the form of the nine-moment \( \chi^2 \)
as a function of \( J \), that the inclusion of the \( L = 5 \) moments has no significant effect on the \( \chi^2 \) (as might be expected from the fact that these had near-zero values); the \( \chi^2 \) minimum is still near \( J = 3/2 \). These results represent the best reliable discrimination between \( J = 3/2 \) and \( J = 5/2 \) obtained from the data.

V. COMPARISON WITH EARLIER ANALYSIS

The coefficients or moments obtained in the study presented here were checked by comparing distributions obtained with the functions of lesser complexity that the authors previously derived and fitted to the data. The distributions given in Eqs. (1) through (4), with experimental \( t^M_L \) substituted, were averaged over the azimuthal angle \( \phi \) and then compared with the angular distribution and polarization distributions of the earlier \( Y^* \) report. \(^1\) \([\text{IP}_\parallel] \) was compared with 
\[
(N \mathbf{P} \cdot \mathbf{\hat{n}} + N \mathbf{P} \cdot \mathbf{\hat{m}})/2 \cos \theta, \text{ and } \text{Re}[\text{IP}_\perp] \text{ was compared with } (N \mathbf{P} \cdot \mathbf{\hat{n}} - N \mathbf{P} \cdot \mathbf{\hat{m}})/2 \sin \theta].
\]
The relative magnitudes of coefficients and their errors compared very well.

VI. CONCLUSIONS

The assignment of spin \( 1/2 \) to the \( Y^*_4 \) is excluded by the existence of angular distribution and polarization moments of higher order \((L = 2, 3)\) than permitted. Spin \( 3/2 \) seems quite acceptable, since moments expected for spin \( 5/2 \) are consistent with zero. Spin \( 5/2 \) is not required by the data. (See Table II for \( \chi^2 \) values).

The parity assignment demanded by longitudinal and transverse polarization moments is \( \gamma = +1 \) or \( P_{3/2} \) for spin \( 3/2 \). This confirms the earlier report. \(^1\) Although the data do not suggest spin \( 5/2 \), an assignment of \( D_{5/2} \) would be very much preferred to \( F_{5/2} \). (See Table II.)

A further study now in progress treats the \( Y^*_4 \) data with a maximum likelihood approach to find moments. Preliminary results on data ranging from 1.15 to 1.30 BeV/c are similar to the results given above.
A qualifying remark should be made with respect to confidence levels stated in Table II. As is no doubt obvious to the informed reader, quantitative values can be changed by a small amount of background. Thus the $10^{-6}$ or $10^{-7}$ confidence levels should not be taken too literally; but conclusions should be weighed appropriately with other pieces of evidence (which also have statistical and background uncertainties). It is reassuring that several experiments showing strong effects give consistent results for the $Y_4^*$. Perhaps the application of such techniques as the moment analysis to $Y_4^*$ events in new $K^*$ and $\pi$ experiments will permit discrimination between $P_{3/2}$ and $D_{5/2}$ hypotheses.

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FOOTNOTE AND REFERENCES

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5. Robert K. Adair (Yale University) has recently attempted to reproduce the results of several experiments by assuming special forms of backgrounds with the production of spin-1/2 \(Y_1^{*}\)'s. ("Background Amplitudes and the Spin of the \(Y_1^{*}\)" preprint). We believe that the small background in our experiment (probably < 5%, certainly < 10%) requires such special assumptions to account for observed high-order moments as to make the hypothesis unlikely. The three experiments cited by Adair to support spin 1/2 are somewhat limited by statistics and background; two of these have been simply interpreted by the
experimenters to support spin 3/2 (references 3, 4).


7. These \( T^M_L \) operators are the irreducible tensors defined in M. E. Rose, Elementary Theory of Angular Momentum (John Wiley and Sons, Inc., New York, 1957) or A. R. Edmonds, Angular Momentum in Quantum Mechanics (Princeton University Press, Princeton, New Jersey, 1957). The normalization of each \( T^M_L \), as in Byers and Fenster, is such that the density matrix for the \( Y^*_1 \) spin states is

\[
\rho = (1/2J+1) \sum_{L,M} (2L+1) T^{M*}_L T^M_L.
\]

8. These expressions result from the requirement that the angular distribution be a scalar quantity (as it is specified by one number for each \( \theta, \phi \)); and that the polarization be a vector quantity, i.e., a tensor of rank one. The \( T^M_L \)'s have the same form in spin space that the \( Y^M_L \)'s have in coordinate space; e.g.,

\[
T^4_4 = -(S_x + i S_y)/[2J(J+1)]^{1/2}.
\]

For use of the \( T^M_L \)'s in collision problems, see the theoretical study of W. Lakin, Phys. Rev. 98, 139 (1955), and the experimental work of J. Button and R. Mermod, Phys. Rev. 118, 1333 (1960), on vector and tensor polarizations of deuterons.

9. "Polarization" will always refer to \( \bar{P}(\theta, \phi) \), the fractional number of decay \( \Lambda \)'s times the \( \Lambda \) polarization for some \( \theta, \phi \) direction.

10. Particle four-momenta are Lorentz-transformed from the laboratory system to the production c.m., then to the \( Y^*_4 \) rest frame, and finally (\( \Lambda \) decay products) to the \( \Lambda \) rest frame. The \( \theta, \phi \) angles for the \( \Lambda \) and the \( \Theta, \Phi \) angles for the \( \pi \) are determined by taking appropriate direction cosines with respect to axes obtained by "direct Lorentz transformations" of \( x, y, \) and \( z \) axes (incident \( \bar{R} \times \hat{R}, \bar{R} \) and \( \hat{R} \) directions) to each successive system. The direct Lorentz "transformation" is simply the orienting of \( x, y, \) and \( z \) axes the same in the new system as in the old, with respect to the \( \beta \) direction of the
conventional Lorentz transformation. (See H. P. Stapp, Relativistic Transformations of Spin Directions, Lawrence Radiation Laboratory Report, UCRL-3096, December 30, 1957, unpublished.)


12. Moments higher than \( L = 5 \) have not been examined; and a \( \chi^2 \) for spin 5/2 is not presented. From the fact that all 10 independent parameters for the \( L = 4 \) and \( L = 5 \) moments were consistent with zero and from the lack of any evidence for \( (\hat{A} \cdot \hat{n})^6 \) polarization terms in the earlier study, these are expected to be zero.

13. N. Byers and S. Fenster (University of California, Los Angeles, Department of Physics), department report, June 1963, and private communication.

Table I. \( t^M_L \) moments.

### A. from \( I(\theta, \phi) \)

<table>
<thead>
<tr>
<th>J</th>
<th>L</th>
<th>M</th>
<th>Re ( t^M_L )</th>
<th>Im ( t^M_L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2, 3/2, 5/2</td>
<td>0</td>
<td>0</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td>3/2</td>
<td>2</td>
<td>+2</td>
<td>-0.018±0.022</td>
<td>-0.028±0.023</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0</td>
<td>-0.118±0.034</td>
<td></td>
</tr>
<tr>
<td>5/2</td>
<td>2</td>
<td>+2</td>
<td>-0.017±0.021</td>
<td>-0.026±0.021</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0</td>
<td>-0.110±0.032</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>+4</td>
<td>-0.025±0.024</td>
<td>-0.011±0.024</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>+2</td>
<td>0.014±0.025</td>
<td>0.042±0.025</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0</td>
<td>0.027±0.037</td>
<td></td>
</tr>
</tbody>
</table>

### B. from \( I \tilde{P}(\theta, \phi) \) - Longitudinal component

<table>
<thead>
<tr>
<th>J</th>
<th>L</th>
<th>M</th>
<th>Re ( t^M_L )</th>
<th>Im ( t^M_L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2</td>
<td>1</td>
<td>0</td>
<td>-0.019±0.100</td>
<td></td>
</tr>
<tr>
<td>3/2</td>
<td>1</td>
<td>0</td>
<td>-0.043±0.225</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>+2</td>
<td>-0.134±0.048</td>
<td>-0.066±0.051</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0</td>
<td>0.247±0.073</td>
<td></td>
</tr>
<tr>
<td>5/2</td>
<td>1</td>
<td>0</td>
<td>-0.066±0.343</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>+2</td>
<td>-0.246±0.088</td>
<td>-0.121±0.094</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0</td>
<td>0.454±0.134</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>+4</td>
<td>0.046±0.042</td>
<td>0.025±0.043</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>+2</td>
<td>0.045±0.044</td>
<td>0.008±0.046</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0</td>
<td>0.030±0.065</td>
<td></td>
</tr>
</tbody>
</table>

### C. from \( I \tilde{P}(\theta, \phi) \) - Transverse component

<table>
<thead>
<tr>
<th>J</th>
<th>L</th>
<th>M</th>
<th>( \gamma ) Re ( t^M_L )</th>
<th>( \gamma ) Im ( t^M_L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2</td>
<td>1</td>
<td>0</td>
<td>-0.051±0.061</td>
<td></td>
</tr>
<tr>
<td>3/2</td>
<td>1</td>
<td>0</td>
<td>-0.056±0.068</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>+2</td>
<td>-0.077±0.041</td>
<td>0.040±0.041</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0</td>
<td>0.272±0.058</td>
<td></td>
</tr>
<tr>
<td>5/2</td>
<td>1</td>
<td>0</td>
<td>-0.057±0.070</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>+2</td>
<td>-0.095±0.050</td>
<td>0.048±0.050</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0</td>
<td>0.332±0.071</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>+4</td>
<td>0.026±0.037</td>
<td>-0.029±0.038</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>+2</td>
<td>0.012±0.038</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0</td>
<td>0.015±0.055</td>
<td></td>
</tr>
</tbody>
</table>
Table II. $\chi^2$ values.

### A. $Y_1^*$ spin

<table>
<thead>
<tr>
<th>$Y_1^*$ spin</th>
<th>$\chi^2$</th>
<th>Degrees of freedom</th>
<th>Confidence level</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/2$</td>
<td>77.0</td>
<td>24</td>
<td>$2 \times 10^{-7}$</td>
</tr>
<tr>
<td>$3/2$</td>
<td>10.8</td>
<td>15</td>
<td>0.79</td>
</tr>
</tbody>
</table>

### B. $Y_1^*$ parity

<table>
<thead>
<tr>
<th>$Y_1^*$ state</th>
<th>Parity</th>
<th>$\chi^2$</th>
<th>Degrees of freedom</th>
<th>Confidence level</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{1/2}$</td>
<td>(-)</td>
<td>0.07</td>
<td>1</td>
<td>0.80</td>
</tr>
<tr>
<td>$P_{1/2}$</td>
<td>(+)</td>
<td>0.34</td>
<td>1</td>
<td>0.56</td>
</tr>
<tr>
<td>$P_{3/2}$</td>
<td>(+)</td>
<td>3.7</td>
<td>4</td>
<td>0.45</td>
</tr>
<tr>
<td>$D_{3/2}$</td>
<td>(-)</td>
<td>44.9</td>
<td>4</td>
<td>$&lt;10^{-7}$</td>
</tr>
<tr>
<td>$D_{5/2}$</td>
<td>(-)</td>
<td>7.6</td>
<td>9</td>
<td>0.57</td>
</tr>
<tr>
<td>$F_{5/2}$</td>
<td>(+)</td>
<td>45.3</td>
<td>9</td>
<td>$8 \times 10^{-7}$</td>
</tr>
</tbody>
</table>

Table III. $J$ values from Eq. (25).

<table>
<thead>
<tr>
<th>Moment: $t_1^0$, $Re t_3^2$, $Im t_3^2$, $t_3^0$</th>
<th>$J$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$2.1 \pm 14.1$</td>
</tr>
</tbody>
</table>
FIGURE LEGENDS

Fig. 1. Dalitz plot of $\Lambda\pi^+\pi^-$ events from $K^-p$ interactions at 1.22 BeV/c.
Projection of the events onto the $\Lambda\pi^+$ mass axis is displayed to the right of the figure; the curve represents the fitting of Breit-Wigner resonance expressions to the $\Lambda\pi^+$ and $\Lambda\pi^-$ systems.

Fig. 2. Dependence of the "parity $\chi^2$" (Eq. 19) on spin $J$. The solid line represents the $\chi^2$ obtained from the four moments ($L = 1, 3$) appropriate to spin $3/2$; the dotted line represents the $\chi^2$ evaluated for the nine moments ($L = 1, 3, 5$) appropriate to spin $5/2$. The number of degrees of freedom $f$ is 4 in the former case and 9 in the latter.
Mass squared of $\Delta\pi^+$ pair (BeV$^2$)

Mass of $\Delta\pi^+$ pair (MeV)

Number per 10 MeV
\[ \chi^2 \]

\[ \text{Spin } J \]

\[ f = 9 \]

\[ f = 4 \]
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