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DIPOLE COUPLING MODEL OF REGGEON EXCHANGE

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ABSTRACT

A complete model for the production mechanism of the double resonance reaction \( \pi N \rightarrow V \Delta \), where \( V \) is a \( J^P = 1^- \) vector meson and \( \Delta \) is a \( J^P = 3/2^+ \) isobar, is presented. The model uses Dipole \( (J = 1) \) coupling at the \( N \)-Regge-\( \Delta \) vertex to predict the quark model results of classes (a) and (b). The remaining class (c) relations are not implied by this model. Only four independent helicity amplitudes are non-vanishing in the Dipole Model, so that quantitative fits of data will be highly overconstrained.
I. INTRODUCTION

The desirability of a model-independent extraction of the production amplitudes from experimental data is well known. However for the double resonance production processes

$$PB \rightarrow V\Delta,$$  \hspace{1cm} (1)

where a pseudoscalar meson $P$ interacts with a baryon $B$ to produce a vector meson $V$ ($J^P = 1^-$) and isobar $\Delta$ ($J^P = 3/2^+$), the set of experimentally observable quantities for unpolarized initial baryons is insufficient to determine the set of 12 independent helicity amplitudes describing the reaction. This situation has led previous investigators to the opposite approach of constructing models for the amplitudes, and then comparing the model predictions with experiment.

In this paper we propose an investigation of processes like reactions (1) by a method which attempts a compromise between these two extremes. We use the marked success of the quark model of Białas and Zalewski in describing relations among decay density matrix elements to infer information about the angular momentum structure of production amplitudes. This analysis leads to a model with only four independent amplitudes, so that observation of the 19 joint decay density matrix elements highly overconstrains a determination of the amplitudes.

Experimental tests of the quark model using data from reactions (1) have established the validity of the additivity assumption of the model. The model predictions of Białas and Zalewski which directly test additivity without further assumption are labeled class (a). More restrictive assumptions about the quark model amplitudes, in particular transformation properties under time reversal and charge conjugation, lead to the predictions of classes (b) and (c). Both of these latter classes rely on the same set
of invariance principles, and thus might be expected to possess joint validity. However it has been noted\(^1\) that a strict interpretation in terms of free quark scattering leads to results at the particle level which depend on the reference frame in which the calculation is performed. Thus Białas and Zalewski were led to consider classes (b) and (c) as independent sets of relations which may hold in only a restricted set of reference frames. In fact the relations of class (c) are expected to be valid in only a single frame.

A relativistic treatment of quark-quark scattering has led Lipkin\(^3\) to conclude that neither class (b) nor (c) should be expected to be valid, so that searches for suitable frames become pointless.

Notwithstanding this objection, we have reinvestigated the derivation of the quark model predictions. Our motivation for this analysis comes directly from experimental data,\(^2\) which, at least for the \(\omega\Delta^{++}\) reaction, unambiguously demonstrates the validity of the class (a) and (b) relations, but also shows class (c) to be badly violated.

We are thus led to an alternative derivation of the class (a) and class (b) results. In particular it has long been realized\(^4\) that a sufficient assumption to derive class (a) for reactions (1) without invoking quarks is that one couples the spins of the initial and final baryons to a resultant spin equal to one. We shall call the model which uses this assumption the Dipole Model.

In Sec. II we present a review of the Dipole Model, mainly to introduce our notation. In Sec. III we analyze the class (b) and class (c) relations from the Dipole Model viewpoint. We find that while class (b) follows with no additional assumptions required beyond those of class (a), the class (c) relations are not naturally motivated by the Dipole Model. A summary and possible extensions of this work are considered in Sec. IV.
II. THE BASIC MODEL

A useful starting point in our discussion of the Dipole Model is the partial wave decomposition\(^5\) of Jacob and Wick t-channel helicity amplitudes:

\[
\mathbf{f}_t^{\lambda_V,\lambda_\pi;\lambda_\Delta,\lambda_N}(\theta_t) = \sum_j (J + \frac{1}{2}) (\lambda_\Delta,\lambda_N | T_j^{(J)}(t) | \lambda_V,\lambda_\pi) d_j^J(\lambda_\Delta - \lambda_N, \lambda_\pi - \lambda_V)(\theta_t)
\]  

(2)

for the s-channel process (1). We now assume

(i) \(J = 1\) dominance,

(ii) factorization,

and (iii) \(\langle \lambda_2 | T_j^{(1)}(t) | \lambda_1 \rangle\) satisfies the spin algebra of \(SU(2)\).

An equivalent statement of assumption (iii) is that in the vector space labeled by spin projections along the t-channel axes, the transition operator transforms as a rank one tensor.

While this formulation of the model differs from the original s-channel derivation of Jones\(^6\) it is straightforward to show that the algebraic consequences of the two derivations are identical. In particular we find a set of relations for amplitudes of common vector meson helicity \(\lambda_V\):

\[
\begin{align*}
\mathbf{f}_t^{\lambda_V, 0; -\frac{3}{2}, -\frac{1}{2}} &= 0 = \mathbf{f}_t^{\lambda_V, 0; \frac{3}{2}, -\frac{1}{2}} \\
\mathbf{f}_t^{\lambda_V, 0; \frac{3}{2}, \frac{1}{2}} &= \sqrt{3} \mathbf{f}_t^{\lambda_V, 0; \frac{1}{2}, \frac{1}{2}} \\
\mathbf{f}_t^{\lambda_V, 0; -\frac{1}{2}, \frac{1}{2}} &= \mathbf{f}_t^{\lambda_V, 0; \frac{1}{2}, \frac{1}{2}} \\
\mathbf{f}_t^{\lambda_V, 0; -\frac{3}{2}, \frac{1}{2}} &= \sqrt{3} \mathbf{f}_t^{\lambda_V, 0; \frac{1}{2}, 1} \quad \text{(3c)}
\end{align*}
\]

These relations follow from applying assumptions (iii) to amplitudes with the same value of \(\lambda = \lambda_\Delta - \lambda_N\) and \(\mu = -\lambda_V\), and thus the same rotation
function $d^{(1)}_{\lambda, \mu}(\theta, t)$. An additional restriction on the model amplitudes resulting from overall conservation of parity is that

$$f^{t}_{\lambda V, 0; \lambda' N} = (-1)^{\lambda_A - \lambda_N + \lambda V} f^{t}_{-\lambda V, 0; -\lambda' N}.$$  \hspace{1cm} (4)

We emphasize at this point that both natural and unnatural parity exchange amplitudes are expected to obey the same algebraic rules of Eq. (3).

The constraints on the decay angular distributions of the vector meson and isobar implied by relations (3) and (4) will be developed below. It is useful at this point to evaluate the set of observable density matrix elements in terms of the DM amplitudes. We adopt the following notational scheme for the nonvanishing DM amplitudes:

$$A_0 = f^{t}_{0,0; \frac{1}{2}, \frac{1}{2}}$$  \hspace{1cm} (5a)

$$A_3 = f^{t}_{1,0; \frac{3}{2}, \frac{1}{2}}$$  \hspace{1cm} (5b)

$$B_{1,1} = f^{t}_{1,0; \frac{1}{2}, \frac{1}{2}}$$  \hspace{1cm} (5c)

$$B_{0,3} = f^{t}_{0,0; \frac{3}{2}, \frac{1}{2}}$$  \hspace{1cm} (5d)

and

$$C_{-1,3} = f^{t}_{-1,0; \frac{3}{2}, \frac{1}{2}}$$  \hspace{1cm} (5e)

(so that A is a non-flip amplitude, B is a single flip, and C a double flip).

Following Pilkuhn and Svensson, the general decay angular distribution for $V\Delta$ reactions may be written as

$$W(\Omega_V, \Omega_\Delta) = \sum_{m,m', n,n'} M^{(1)}_{m,m'}(\Omega_V) M^{(3/2)}_{n,n'}(\Omega_\Delta) \rho_{n,n'}^{m,m'}.$$  \hspace{1cm} (6)

Here $M^{(1)}_{m,m'}$ and $M^{(3/2)}_{n,n'}$ are the angular functions describing the decays of the
vector meson and isobar, respectively, and the joint decay matrix elements are defined as

\[ \rho_{n,n'} = \frac{1}{N} \sum_{\lambda} f^{t}_{m,0;n,\lambda} (f^{t}_{m',0;n',\lambda})^* , \]

where

\[ N = 2(|A_0|^2 + \frac{4}{3}(|A_3|^2 + |C_{-1,3}|^2 + |B_{0,3}|^2) + 2|B_{1,1}|^2) \]

so the density matrix has a unit trace.

It is straightforward to evaluate the DM expressions for each of the 19 terms which enter the orthogonal expansion of Eq. (6). The results are summarized in Table I. For convenience the relations between the joint decay density matrix elements and the equivalent statistical tensors, which are directly related to angular averages of spherical harmonics, are also presented. The DM is seen to fully describe the \( V\Delta \) joint decay density matrix in terms of five (complex) amplitudes, and thus in terms of five magnitudes and four relative phases. Thus data which can measure well all 19 correlation terms will severely overconstrain the model, and provide many stringent tests.

We observe from our method of derivation that the relations of Eqs. (3a-d) follow from the recoupling coefficients for the addition of a \( J = 1 \) state to a spin-1/2 nucleon to form a spin-3/2 isobar. If we adopt this rule as our definition of dipole coupling, then the Dipole Model embraces a large class of models, including elementary particle as well as Reggeized exchange models. For example, the M1 coupling rules of Stodolsky and Sakurai \(^{10}\) for \( \pi^0\Delta^{++} \) production emerge naturally from Eqs. (3) and (4) when one applies the restriction \( \lambda_V = 0 \). Another model contained within the scope of the DM is that of Maor, \(^{11}\) who considers M1 coupling for natural parity exchanges.
in the t-channel, and a vanishing of the coupling of unnatural parity amplitudes to amplitudes other than the nonflip, nonflip term $A_0$ [Eq. (5a)]. We shall reconsider Maor's model below (see Sec. III).

We next consider the relationship of the quark model to the Dipole Model. The class (a) relations of Ref. 1 follow directly from the assumption of additivity: scattering amplitudes are a coherent sum of single quark-quark scattering terms with the nonscattered quark(s) acting as spectators. This assumption, plus the conventional quark structure of the mesons and baryons, is sufficient to derive the full set of class (a) predictions. If the quark model is interpreted in terms of quark-quark scattering amplitudes, five independent complex amplitudes must be considered, just as in the DM. Then nonlinear relations among density matrix elements also emerge, again in a completely equivalent fashion to the DM.

To investigate the angular momentum content of the nonrelativistic quark model we note that quark-quark scattering in both the $S = 0$ and $S = 1$ states are allowed. However to couple at the $N-\Delta$ (or $\pi-V$) vertex with $L = 0$ requires that $S = 1$. Furthermore at the meson vertex each quark-quark scattering state corresponds to a single value of the vector meson helicity, hence leading to relations between amplitudes with the same value of $\lambda_V$. We have noted above that the rules of Eq. (3) above follow directly from the Clebsch-Gordan coefficients of the coupling of the spin of the nucleon with a $J = 1$ state to form the $\Delta$ spin state. Thus the nonrelativistic quark model and the Dipole Model correspond to the coupling of a $J = 1$ system at the $N-\Delta$ vertex. It is equally straightforward to verify that relations (3a-d) above are sufficient to establish the validity of the class (a) predictions. Therefore the validity of the class (a) relations may be interpreted as supporting the hypothesis of additivity.
(of single scattering amplitudes), or, equivalently, of $J = 1$ coupling.

III. FURTHER CONSEQUENCES OF THE DIPOLE MODEL

In Sec. II we discussed the well-known result that coupling of a $J = 1$ system reproduces the class (a) quark model predictions of Białas and Zalewski for $\Delta\Lambda$ production without invoking the existence of quarks. A set of necessary and sufficient conditions to obtain the class (a) results are given above in Eqs. (3,4).

To pursue the further consequences of the DM it is necessary to relate amplitudes with the vector meson in different helicity states. Using the symmetry property of the $d^{J}_{\lambda \mu}$

$$d^{J}_{\lambda, \mu}(\beta) = d^{J}_{-\mu, -\lambda}(\beta),$$

and noting that the two nonvanishing single-flip amplitudes $B_{1,1}$ and $B_{0,3}$ are proportional to the same angular function, we find

$$B_{1,1} = \sqrt{2/3} B_{0,3}. \tag{8}$$

Further relations among the DM amplitudes only follow from a separation of the full amplitudes into natural and unnatural parity exchange contributions. These other relations will be sensitive to the detailed specification of the $s$ and $t$ dependence of the $d^{(1)}_{\lambda \mu}$, and in particular to the form of the nonasymptotic function assumed, since different $d^{(1)}_{\lambda \mu}$ must be related. Equation (8) is derived independently of assumptions about the nonasymptotic forms, and so should enjoy an energy independent validity.

The importance of Eq. (8) in the context of the quark model is that it is a sufficient (and for nonzero $B_{0,3}$ and $B_{1,1}$ also a necessary) condition for the validity of the relations of class (b). This result follows from an inspection of the class (b) relations in terms of DM amplitudes. Thus
the DM prediction of the class (b) relations does not require additional assumptions beyond those of class (a). By contrast, the quark model leads to these relations only if an additional assumption is made.

The use of rotational invariance to derive Eq. (8) suggests that, in a sense, this relation is more kinematic than dynamic. However the $B_{0,3}$ amplitude receives contributions only from unnatural parity exchange amplitudes, while for the $m = 1$ state of the vector meson both parity exchange amplitudes may contribute. Thus models which predict Eq. (8) (non-trivially) either relate exchanges of opposite parity or forbid a natural parity contribution to $B_{1,1}$.

The DM model elects the latter method, as a simple derivation shows. From the DM Clebsch-Gordan coefficients

$$f^t_{1/2,1/2} = f^t_{1,1/2,1/2},$$

while from overall conservation of parity

$$f^t_{1,1/2,1/2} = - f^t_{-1,1/2,1/2}.$$  

Hence

$$f^t_{1/2,1/2} = - f^t_{-1,1/2,1/2},$$

so that Eq. (2) implies that for $J = 1$,

$$\langle \frac{1}{2}, \frac{1}{2} | T^{(1)}(t) | 1,0 \rangle d^{(1)}_{0,-1}(\theta_t) = - \langle \frac{1}{2}, \frac{1}{2} | T^{(1)}(t) | -1,0 \rangle d^{(1)}_{0,1}(\theta_t).$$

We thus conclude that the DM with parity conservation in the production process predicts

$$\langle \frac{1}{2}, \frac{1}{2} | T^{(1)} | 1,0 \rangle = \langle \frac{1}{2}, \frac{1}{2} | T^{(1)} | -1,0 \rangle.$$
However the exchange of a trajectory with either natural \( P = (-1)^J \) or unnatural \( [P = (-1)^{J+1}] \) parity implies that\(^5\)

\[
\langle \frac{1}{2}, \frac{1}{2} | T^{(1)} | 1, 0 \rangle = P(-1)^{J+1} \langle \frac{1}{2}, \frac{1}{2} | T^{(1)} | -1, 0 \rangle .
\]

Therefore

\[ P(-1)^{J+1} = +1 , \]

demonstrating that only unnatural parity amplitudes can contribute to \( B_{1,1} \) in the DM. This method of derivation does not rely on a Regge-type leading power of \( s \) approximation and therefore is not restricted to asymptotic energies.

We have thus shown that the successes of the quark model predictions of classes (a) and (b) can be understood in terms of Dipole coupling in the t-channel. Our derivation suggests that relations between observable density matrix elements are expected to be valid in the t-channel coordinate system, but may be only approximately satisfied in the s-channel. Hence our prediction for the quark model additivity frame (i.e., that frame for which the use of spectator quarks is a good approximation) agrees with the t-channel conjecture of Białas, Kotanski, and Zalewski.\(^{14}\)

It is worth noting, however, that the approximate equality of the helicity crossing angles for the vector meson and isobar leads\(^2\) to the validity of the class (b) quark model relations in the s-channel as well as the t-channel coordinate system. In the static limit\(^{15}\) all crossing angles become equal to the s-channel center-of-mass scattering angle \( \theta_s \). Hence static models also predict the validity of the class (b) relations in both frames, since amplitudes in one frame may be obtained from those in the other by a rotation of angle \( \theta_s \) about the production normal.

We have derived the simple quark model relations by treating both natural and unnatural parity exchanges on an equal footing, assuming that both interact
at the N-Δ vertex with total $J = 1$. Thus in $\rho^0\Delta^{++}$ production $\pi$ exchange acts like a $J^P = 1^+$ amplitude, while $A_2$ exchange acts like a $J^P = 1^-$ amplitude. The DM thus suggests exchange degeneracy in the sense that the $B$ coupling in $\omega\Lambda^{++}$ production is expected to have the same rotational properties as the $\pi$ coupling in $\rho^0\Delta^{++}$, as well as the $\rho$ in $\omega\Lambda^{++}$ compared to the $A_2$ in $\rho^0\Delta^{++}$. The dynamics specific to a given trajectory persists, of course, in the DM, where it may be viewed as residing in the reduced matrix element

$$\langle \lambda_2 | T^{(1)}(t) | \lambda_1 \rangle = C(j_1,1,j_2;\lambda_1,\lambda_2,\lambda_1) \langle j_2 | T^{(1)}(t) | j_1 \rangle .$$

The Regge signature factor, the Regge residues, as well as other possible $t$-dependent terms, will appear in an explicit model in the reduced matrix elements.

Turning to the class (c) quark model predictions, the analog of Eq. (8) is

$$B_{1,1} = 0 = B_{0,3} .$$

That is, the class (c) relations are valid in the coordinate system for which the single helicity-flip amplitudes vanish. The DM does not suggest the existence of such a frame except at $\theta^0$ production angle, when conservation of $J_z$ leads to Eq. (9). Hence from the DM viewpoint the observed violation of the class (c) relations for $\omega\Lambda^{++}$ is not unexpected.

Returning briefly to Maor's model, we note that this model reproduces at large $s$ all of the quark model predictions. Insofar as the quark model has more parameters than Maor's model (in particular as shown above $B_{0,3}$ and $B_{1,1}$ need not be zero in the quark model, while in Maor's model $B_{0,3} = 0$ and only the natural parity contribution to $B_{1,1}$ is allowed to be nonvanishing), more constraints on the joint decays are predicted by the latter model.
One of the constraints most sensitive to the decoupling of the unnatural
parity exchange amplitude from spin-flip terms is
\[ \rho_{1,1} - \rho_{1,-1} = 0. \]
At 3.7 GeV/c the observed value for \( \rho^0\Delta^+ \) production was\(^{16}\)
\[ \rho_{1,1} - \rho_{1,-1} = 0.106 \pm 0.014 \]
compared to a value of \( \rho^{0,0} \) in the momentum transfer region \( t' < 0.1 \,(\text{GeV}/c)^2 \)
of \( 0.81 \pm 0.02 \). Thus one may view Maor's model as explaining the dominant t-
channel exchange amplitudes, but not a model which can be used to probe sec-
ondary contributions such as those which lead to a nonvanishing \( \rho_{1,1} - \rho_{1,-1} \).

IV. SUMMARY AND POSSIBLE EXTENSIONS

The Dipole Model is here defined by the recoupling coefficients of a
rank one tensor in the t-channel. For Vector Meson-Isobar (V\(\Delta\)) double reso-
nance production we have shown that the Dipole Model fully duplicates the
experimentally valid class (a) and (b) constraints of the nonrelativistic quark model. Further, the class (b) relations are obtained in the DM with
no additional assumptions required beyond those already necessary to derive
class (a). In contrast, the quark model does require further assumptions.
The class (c) relations of the quark model, which in general do not appear substantiated by experiment, rely on further assumptions in both models.

The Dipole Model is readily extended to other reactions
\[
\begin{align*}
P + B & \rightarrow P + \Delta \\
P + B & \rightarrow V + B \\
B + B & \rightarrow B + \Delta \\
B + B & \rightarrow \Delta + \Delta.
\end{align*}
\]
For these reactions the quark model predictions are found carried over to the Dipole Model intact.

A different class of reactions which may be considered is tensor meson \((J^P = 2^+)\), \(T\), production:

\[ P + B \rightarrow T + \Delta . \]

From factorization we are led to conclude that \(J = 1\) is still dominant at the baryon vertex, so that Eqs. (3-4) carry over to \(\Delta\) production [as does Eq. (8)]. Furthermore the matrix element

\[ \langle \lambda_T | \tau^{(J)} | \lambda_\pi \rangle \]

transforms as a scalar under rotations since \(J_\pi = 0\) (as above for vector meson production). Hence the recoupling coefficients for \(\Delta\) production are identical to those for \(V\Delta\) production, and the quark model results of classes (a) and (b) are immediately recovered. In addition, the vanishing of amplitudes with two units of helicity flip at the baryon vertex implies (using the recoupling coefficients) a similar vanishing at the meson vertex, so that

\[ \rho_{2,m} = 0 \]

where \(\rho_{2,m}\) is the density matrix element of the tensor meson in the t-channel coordinate system, and \(m\) is arbitrary. Our general result is thus that natural parity states of spin \(> 1\) produced opposite a \(J^P = 3/2^+\) isobar have an amplitude structure no more complicated than that of \(V\Delta\) production. We therefore expect all joint decay density matrix elements to vanish except those that are allowed for \(V\Delta\) production, and further, that the nonvanishing joint decay density matrix elements obey constraints identical to those of \(V\Delta\) production.

It is difficult to assess the importance of amplitudes not described by the Dipole Model. In reactions where pole dominance appears a reasonable
assumption our description should lead to an elucidation of the secondary production amplitudes. We note that the presence of absorption leads to a violation of our factorization hypothesis. However Eqs. (3,4) and (8) relate amplitudes which involve the same number of helicity flips. We expect these relations to persevere, then, in the presence of absorptive corrections which depend only on the net helicity flip.

Finally, we note that the general validity of the quark model predictions of classes (a) and (b) also establishes the quantitative validity of the Dipole Model. Thus experimenters should be able to perform least squares fits to their $V\Delta$ data, and extract the basic amplitudes of Eq. (5). Such a fit is highly overconstrained, and so provides a stringent test of the model. Also, of course, the amplitudes themselves have intrinsic interest for their ability to distinguish among competing theoretical models.

ACKNOWLEDGMENTS

The authors wish to acknowledge stimulating conversations with Gerson Goldhaber, Uri Maor, Alan Martin, and George Trilling.
Table I. Dipole model expressions for the measurable parameters in VΔ production. The amplitudes are defined in Eq. (5) of the text, and the density matrix elements in Ref. 8.

<table>
<thead>
<tr>
<th>Term</th>
<th>Element</th>
<th>Stat. tensor</th>
<th>Expressiona</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\rho_{0,0}^1$</td>
<td>$\frac{1}{2}[1 - 2\sqrt{6} J_{10}^{20}]$</td>
<td>$2[</td>
</tr>
<tr>
<td>2</td>
<td>$\text{Re} \rho_{1,0}^1$</td>
<td>$- \sqrt{2} \text{Re} J_{10}^{20}$</td>
<td>$\text{Re}(\frac{4}{3}(A_3 - C_{-1,3})B_{0,3}^* + 2B_{1,1}A_0^*)$</td>
</tr>
<tr>
<td>3</td>
<td>$\rho_{1,-1}^1$</td>
<td>$2 J_{20}^{20}$</td>
<td>$2 \text{Re}(\frac{4}{3}A_3 C_{-1,3}^* -</td>
</tr>
<tr>
<td>4</td>
<td>$\rho_{3,3}^3$</td>
<td>$\frac{1}{2}[1 + 2\sqrt{3} J_{02}^{02}]$</td>
<td>$</td>
</tr>
<tr>
<td>5</td>
<td>$\text{Re} \rho_{3,1}^3$</td>
<td>$- \sqrt{3/2} \text{Re} J_{01}^{02}$</td>
<td>$\text{Re}((A_3 - C_{-1,3})B_{1,1}^* + B_{0,3}A_0^*)$</td>
</tr>
<tr>
<td>6</td>
<td>$\text{Re} \rho_{3,-1}^3$</td>
<td>$\sqrt{3/2} \text{Re} J_{02}^{02}$</td>
<td>$1/\sqrt{3} \text{Re}(2A_3 C_{-1,3}^* -</td>
</tr>
<tr>
<td>7</td>
<td>R.7</td>
<td>$\sqrt{6} J_{00}^{22}$</td>
<td>$\frac{2}{3}(</td>
</tr>
<tr>
<td>8</td>
<td>R.8</td>
<td>$2 \text{Re} J_{20}^{22}$</td>
<td>$\frac{4}{3}(\text{Re} A_3 C_{-1,3}^* + \frac{3}{2}</td>
</tr>
<tr>
<td>9</td>
<td>R.9</td>
<td>$- \sqrt{2} \text{Re} J_{10}^{22}$</td>
<td>$\frac{2}{3} \text{Re}((A_3 - C_{-1,3})B_{0,3}^* - 3B_{1,1}A_0^*)$</td>
</tr>
<tr>
<td>10</td>
<td>R.10</td>
<td>$\sqrt{3} \text{Re} J_{02}^{22}$</td>
<td>$\frac{2}{\sqrt{3}} \text{Re}(A_3 C_{-1,3}^* +</td>
</tr>
<tr>
<td>11</td>
<td>R.11</td>
<td>$- \sqrt{3} \text{Re} J_{01}^{22}$</td>
<td>$\text{Re}((A_3 - C_{-1,3})B_{1,1}^* - 2B_{0,3}A_0^*)$</td>
</tr>
<tr>
<td>12</td>
<td>R.12</td>
<td>$\frac{1}{\sqrt{2}} \text{Re} J_{22}^{22}$</td>
<td>$\frac{1}{\sqrt{3}}</td>
</tr>
<tr>
<td>13</td>
<td>R.13</td>
<td>$\frac{1}{\sqrt{2}} \text{Re} J_{22}^{2-2}$</td>
<td>$\frac{1}{\sqrt{3}}</td>
</tr>
<tr>
<td>14</td>
<td>R.14</td>
<td>$- \frac{1}{\sqrt{2}} \text{Re} J_{21}^{22}$</td>
<td>$- \text{Re}(A_3 B_{1,1}^*)$</td>
</tr>
<tr>
<td>15</td>
<td>R.15</td>
<td>$\frac{1}{\sqrt{2}} \text{Re} J_{22}^{2-1}$</td>
<td>$\text{Re}(B_{1,1} C_{-1,3}^*)$</td>
</tr>
<tr>
<td>16</td>
<td>R.16</td>
<td>$- \text{Re} J_{12}^{22}$</td>
<td>$- \frac{2}{\sqrt{3}} \text{Re}(A_3 B_{0,3}^*)$</td>
</tr>
<tr>
<td>17</td>
<td>R.17</td>
<td>$- \text{Re} J_{1-2}^{22}$</td>
<td>$\frac{2}{\sqrt{3}} \text{Re}(C_{-1,3} B_{0,3}^*)$</td>
</tr>
<tr>
<td>18</td>
<td>R.18</td>
<td>$\text{Re} J_{11}^{22}$</td>
<td>$\text{Re}(A_3 A_0^* + B_{1,1} B_{0,3}^*)$</td>
</tr>
<tr>
<td>19</td>
<td>R.19</td>
<td>$- \text{Re} J_{22}^{11}$</td>
<td>$\text{Re}(- C_{-1,3} A_0^* + B_{1,1} B_{0,3}^*)$</td>
</tr>
</tbody>
</table>

a. Each term below should be multiplied by a factor of $(1/N)$ to agree with the normalization of Eq. (7).
REFERENCES AND FOOTNOTES

*Work supported by the U. S. Atomic Energy Commission.
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2. See, for example, G. S. Abrams and K. W. J. Barnham, Phys. Rev. D7, (1973), as well as references cited therein.
17. The necessity of setting \( J = 2 \) at the meson vertex for tensor meson production is at odds with our analysis of Eq. (2). This prior analysis need only be generalized to contain a \( J \) value at the meson vertex \( J_v \), and one at the baryon vertex \( J_\Delta \). \( J_v \) is then chosen to make \( \langle \lambda_v | T^{(J_v)} | \lambda_r \rangle \) a scalar, while \( J_\Delta = 1 \).
18. Since the general decay angular distribution of \( J = 2 \) states into two
spinless bosons is quite different from that of the similar decay of
\( J = 1 \) states, the quark model predictions that are applicable to both
\( V \) and \( T \) production must be written in terms of density matrix elements
rather than statistical tensors.

analysis.


21. This conclusion requires the approximate validity of the DM in the
helicity coordinate system.
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