Title
HIGH-FIELD MAGNET DEVELOPMENT ANALYSIS. CONDUCTOR POSITION ERRORS DUE TO FRICTION, PART I

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An often used magnet design and construction technique involves winding the conductors into an annular form, then pushing them into place by applying a force to one edge, and finally baking the coil to set the resin. Ideally the conductors end up with equal compressive forces and equal spacing. But friction between the inside of the annular form and the coil makes the compressive force less at the fixed end than at the pushed end, and results in unequal conductor spacings, and in turn distortion of the magnetic field.

The effect of friction on conductor position aberrations is easily calculated, and an analysis is presented here. The effect of conductor position aberrations on magnetic field aberrations is a bit more difficult to calculate, and is deferred.
The dimensions and the forces per unit dimension perpendicular to the paper are as illustrated; \( \mu \) is the coefficient of friction.

Consideration of equilibrium of the element in radial and circumferential directions yield, respectively:

\[
\sigma_0 h (\frac{1}{2} d\theta) - (\sigma_0 + d\sigma_0) h \frac{1}{2} d\theta - \mu \sigma_0 a d\theta = 0
\]

or \( \sigma_0 h = \sigma_r a \) (1)

\[
\sigma_0 h - (\sigma_0 + d\sigma_0) h + \mu \sigma_0 a d\theta = 0
\]

or \( h d\sigma_0 = \mu \sigma_0 a d\theta \) (2)

Upon eliminating \( \sigma_r \) between Eqs. 1 and 2 we get

\[
\frac{d\sigma_0}{\sigma_0} = \mu d\theta
\]

Upon integration, and subject to the condition that \( \sigma = \sigma_0 \) at \( \theta = \alpha \), we get

\[
\sigma_0 = \sigma_0 e^{-\mu (\alpha - \theta)}
\]
A few values might be amusing:

\[ \sigma / \sigma_0 \]

\[ \mu \rightarrow 0.01 \quad 0.02 \quad 0.05 \quad 1 \quad 2 \quad 5 \quad 1 \]

\[ \theta \downarrow \]

\( 30^\circ \) | 0.995 | 0.990 | 0.974 | 0.949 | 0.901 | 0.770 | 0.592 \\
\( 60^\circ \) | 0.990 | 0.979 | 0.949 | 0.900 | 0.811 | 0.592 | 0.351 \\
\( 90^\circ \) | 0.984 | 0.969 | 0.924 | 0.854 | 0.730 | 0.456 | 0.205

... Then again, they might not.

The position aberrations resulting from the non-uniformity of the compressive stress depend on the shape of the stress-strain curve for the conductor. A solution for a nice decile linear stress-strain curve would be pretty useless since the curve for typical conductors is pretty curvy. So we will represent the stress-strain curve by a power series:

\[ \varepsilon = C_1 \sigma + C_2 \sigma^2 + C_3 \sigma^3 + \ldots + C_n \sigma^n \quad (5) \]

(\( \varepsilon \) is the strain, \( + \) is compression, and \( \sigma \) is the compressive hoop stress formerly called \( \sigma_0 \)).

The circumferential displacement at any position \( \theta \) from its unstressed position is:

\[ \delta = \delta_0 \int_0^{\theta} \varepsilon \, d\theta \quad (\text{+ is clockwise}) \quad (6) \]

Upon eliminating \( \varepsilon \) and \( \sigma \) between (3), (4), and (5) and integrating we obtain

\[ \delta = \frac{\delta_0}{\mu} \sum_{n=1}^{\infty} C_n \sigma_0^i \left( e^{-i\mu(x-\theta)} - e^{-i\mu x} \right) / i \quad (7) \]
A + $\theta = \alpha$, $s$ is

$$s_0 = \frac{a}{\mu} \sum_{i=1}^{\infty} c_i \alpha (1 - e^{-i\mu \alpha})/i$$  \hspace{1cm} (8)

If the friction $\mu$ were zero the displacement would be

$$s' = s_0 \frac{\theta}{\alpha}$$  \hspace{1cm} (9)

so the position aberration, $\Delta$, is simply

$$\Delta = s' - s$$  \hspace{1cm} (10)

($\Delta$ is the linear position error of a conductor in the circumferential direction, $+$ is counter-clockwise.)

In a typical application, we know the displacement we need at the pushed end where $\theta = \alpha$. So using Eq. 8, letting $\theta = \alpha$ and knowing $s_0$, we solve for $s_0$ by iteration or other sneaky means. We then plug this into Eq. 7 to evaluate $s$ at any $\theta$, get the corresponding $s'$ from Eq. 9, and then get $\Delta$ from Eq. 10.

If there are no more than two terms in the stress-strain equation, Eq. (5) — and two should be enough for Government work — then we can get a closed-form solution, as follows:

$$\sigma_0 = \left( \sqrt{B^2 + 4AC} - B \right) / 2A$$

where

$$A = \frac{1}{2} c_2 (1 - e^{-2\mu \alpha})$$

$$B = c_1 (1 - e^{-\mu \alpha})$$

$$C = \mu s_0 / \alpha$$
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