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A Secular Quark Gluon Plasma Phase Preceding Black Hole Formation

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General Relativity (GR) predicts that trajectories of radiation quanta in passing by a star are deflected by twice the amount predicted in Newtonian gravity. Eddington’s historic test of GR in 1919 hinged on this prediction. Here we consider instead emission from a very massive star. The trajectories of emitted quanta bend away from the normal to the surface of such a star. However, as long as z < \( \sqrt{3} - 1 \) where z is the gravitational redshift,

\[ z = (1 - R_s/R)^{1/2} - 1 \]  

the emitted quanta nevertheless manage to evade entrapment and move away to infinity.

But if the body should be so compact as to lie within its “photon sphere”, i.e., \( R < (3/2)R_s \) or z > \( \sqrt{3} - 1 \), then only the radiation emitted within a cone defined by semi-angle \( \theta_c \):

\[ \sin \theta_c = \frac{\sqrt{27}}{2} (1 - R_s/R)^{1/2} (R_s/R) \]  

will be able to escape eventually [2, 3]. Radiation emitted in the rest of the hemisphere would eventually return within the compact object. This effect of gravitational radiation trapping has been considered for static compact objects having z < 2 [3, 4].

Another fundamental result of GR is the existence of the vacuum Schwarzschild solution which apparently predicts the existence of non-charged black holes (BH) characterized by an Event Horizon (EH) having \( z = \infty \). In fact the formation of an EH might be seen as the ultimate result of the previous phenomenon of bending/trapping of light in a strong gravitational field.

It is believed that when stars of a few solar masses collapse, a hot proto-neutron-star (PNS) is formed first. Because of the compactness of a PNS, collapse generated radiation is trapped by matter-radiation interaction in the PNS. Since the PNS has a modest value of z ~ 0.1 - 0.2, there is no gravitational trapping of radiation and neutrinos diffuse out of the star in about \( \sim 10^6 \) to form a static NS [5]. For static objects, z < 2 and the effect of gravitational trapping, if it occurs, would not be dramatic. In general it is found that, the effect of neutrino trapping may further delay the formation of a NS and the mass of the NS could be higher than the canonical value of \( 1.4M_\odot \), where \( M_\odot \) is the solar mass.

The collapse of very massive stars would not result in the formation of a static and cold NS; on the other contrary, collapse in such a case would fall inexorably towards the \( z = \infty \) black hole (BH) stage. This seems reasonable both because (i) there is an upper limit on the mass of cold objects and (ii) there cannot be any cold/static spherical configuration for z > 2.

But by definition, the \( z = \infty \) state must be preceded by intermediate states having arbitrarily large but finite z. The external spacetime associated with any contracting and radiating object is represented by the radiating Vaidya metric [6] which at the boundary (\( r = R \)) of the
body has the form,

$$ds^2 = (1 - 2GM/Rc^2)du^2 + 2duR - R^2(d\theta^2 + \sin^2 \theta d\phi^2)$$

(3)

where $u$ is the retarded time, $\theta$ is the polar and $\phi$ is the azimuth angle. As the collapse/contraction proceeds both $M(R)$ and $R$ decrease. Simultaneously, $g_{00} = (1 - 2GM/Rc^2) \to 0$ as an event horizon forms. By definition, there is no upper limit on $z$ in this case because $(1 + z) = g_{00} \to \infty$ in the same limit.

The effect of radiation trapping during these intermediate high-$z$ states has never been considered, though previously Kembhavi and Vishveshvara [4] observed that “If neutrinos are trapped, they will not be able to transport energy to the outside, and this can have serious consequences on the thermal evolution of the star. These considerations might become especially interesting in the case of a collapsing phase which leads to the formation of a compact, dense object.”

In this Letter, we want to point out precisely this effect of radiation trapping during continued gravitational collapse in a qualitative manner. At high $z$, $R \approx R_c$ and from Eq.(2), one can see that, $\sin \theta_c \to \theta_c \approx (27/2)(1 + z)^{1/2} [2, 3]$ Therefore the solid angle of escaping radiation is

$$\Omega_c \approx \pi \theta^2_c \approx 27\pi / 4(1 + z)^2$$

(4)

The chance of escape of radiation therefore decreases as $\Omega_c/2\pi \approx (27/8)(1 + z)^2$. Consequently, as the collapse generates internal heat/radiation, the energy density and pressure of trapped radiation at least as fast as

$$\rho_r \sim R^{-3}(1 + z)^2.$$

(5)

This means that if without trapping, $10^{10}$, neutrino/photons would escape a particular spot on the surface, with gravitational trapping, only 1 of every $10^{10}$ quanta would escape for $z = 10^3$. Even without any gravitational trapping, in the regime of high density and temperature of the collapsing matter, radiation is trapped in the collapsing matter because of radiation-matter interaction. This is the reason that though the free fall time of a PNS could be $< 1$ ms, its actual collapse time (even without any gravitational trapping of radiation) is much larger $\sim 10$s. Thus actually $\rho_r$ would start rising much faster than $R^{-3}(1 + z)^2$ during the $z \to \infty$ process because of the joint effect of matter-radiation interaction and gravitational trapping.

On the other hand, the locally measured Eddington luminosity i.e., the luminosity for which the outward radiation pressure on the plasma counterbalances the inward pull of gravity is [2]

$$L_{ed} = \frac{4\pi G M c}{\kappa}(1 + z)$$

(6)

where $\kappa$ is the appropriate opacity. Essentially, $L_{ed}$ corresponds to a critical radiation pressure of

$$p_{ed} = \frac{GM}{\kappa R^2}(1 + z)$$

(7)

Since the trapped radiation pressure $p_r \sim R^{-3}(1 + z)^2$, while $p_{ed} \sim R^{-2}(1 + z)$, the former must catch up with the latter at some appropriate finite value of $z$ and $R$. Then the trapped radiation pressure would exactly counterbalance the inward pull of gravity and the catastrophic collapse would be dynamically halted by it! Once this quasi-stable stage is reached, $R$ and $z$ would become practically constant on short time scales and there would be no further rise in the value of $p_r$. In a very strict sense, however, an eternally collapsing object (ECO) would still be contracting on extremely long time-scales! This is so because as long as an horizon is not formed, i.e., $z < \infty$, the body would radiate and $M$ would continue to decrease. Consequently the metric would remain non-static and, in response, $R$ too, would decrease. Actually $z$ would continue increasing $z \to \infty$ while $R$ would hardly change, i.e., the evolution would primarily take place in $z$ space. It is this infinitesimal decrease in the value of $R$ and attendant much higher secular increase in the value of $z$ and $\rho_r$ which would generate just enough energy (at the expense of $M_c^2$) to maintain the Eddington luminosity seen by a distant observer is [2]

$$L_{ed}^\infty = \frac{4\pi G M c}{\kappa(1 + z)} \approx 1.3 \left(\frac{M}{1M_\odot}\right) 10^{38}(1 + z)^{-1} \text{erg/s}$$

(8)

where $M_\odot$ is the solar mass. Since $L^\infty = -c^2dM/du$, the time scale associated with this phase is

$$u = \frac{\dot{M}/c^2}{L^\infty} = \frac{M(c^2/c^2dM/du)}{L^\infty} = \frac{\kappa(c(1 + z))}{4\pi G}$$

(9)

Obviously, $u \to \infty$ irrespective of the value of $\kappa$ as the BH stage ($z = \infty$) would be reached. Thus the Eddington-limited contracting phase actually becomes eternal and the object in this phase may be called an Eternally Collapsing Object (ECO). Since for photons, $\kappa_r \sim 0.4 \text{cm}^2/\text{g}$, but, for neutrinos $\kappa_\nu$ is smaller by an extremely large factor of $\sim 10^{14}$, we will have $u_\nu < u_c$. Consequently, initial transition to the ECO phase may be dominated by huge $\nu$-emission with a time scale $u_\nu$. But as far as eventual secular ECO phase is concerned, it should be governed by photonic time scale $u_c$ because it is much easier to maintain a $L_{ed}$ caused by photons than by neutrinos. Somewhat similar thing happens for the formation of a hot NS in from a PNS: initial time scale of $\sim 10$s is dictated by $\nu$ emission, while the hot NS cools for thousands of years by photon emission. However, as mentioned earlier, while trapped photons can escape by diffusion from a NS with $z \sim 0.1$, they remain practically trapped for ever in an ECO with $z \gg 1$. It
is because of the same difference in the value of $z$, that there is no significant trapped radiation pressure effect in a NS.

For this era of quasi-stability by trapped photons, one can verify that $u_c \gg Hubble\ time$, and therefore the observed black hole candidates (in present epoch) must be in this ECO phase ($z = finite \gg 1$) rather than in the BH phase ($z = \infty$). In fact Robertson & Leiter [7] have shown that the stellar mass black-hole candidates do not have any EH ($z = \infty$) in the present epoch; on-the-other-hand they have an extremely large but finite $z \sim 10^7$. Since in the high $z$ regime, $R \approx R_s$, the proper energy density, $\rho = \rho_\pm + \rho_r$, of the ECO is

$$\rho = \frac{M}{(4\pi/3) R_s^3} = \frac{3c^6}{32\pi G^3 M^2} \tag{10}$$

It has been shown elsewhere that when a self-luminous object is radiating at its Eddington luminosity, $\rho_\pm / \rho_c \sim (1+z)^8$. And since for the ECO, $z \gg 1$, its energy density is dominated by radiation rather than by baryonic rest mass, $\rho_r \gg \rho_c$. Although Ref. (8) has shown this in a transparent way, one might try to appreciate this result in the following qualitative way:

The formation of a PNS corresponds to collapse to a gravitational potential well of depth $z \sim GM/Rc^2 \sim 0.1$ and release of gravitational potential energy of about $-zMc^2$. But the formation of an ECO corresponds to collapse to much deeper gravitational potential well ($z \gg 1$) with attendant gravitational binding energy $\sim -zMc^2$. The positive internal energy of the ECO or any self-gravitating quasistatic object must be comparable to the negative gravitational potential energy for hydrostatic balance. Thus when a high $z$ ECO is in quasistatic state equilibrium, its trapped radiation energy $\sim zMc^2$ must overwhelm its baryonic energy, i.e., one must necessarily have $\rho_r \gg \rho_c$ for $z \gg 1$ ECO.

Thus an *Eternally Collapsing Object* (ECO) is an ultrarelativistic fireball of radiation, with particle pairs interpersed with baryons much like the plasma in the very early universe. The average temperature of the ECO is obtained by noting $\rho \approx \rho_c = aT^4$, where $a$ is the radiation constant in Eq. (9):

$$T = \left(\frac{3c^4}{8\pi\sigma G}\right)^{1/4} R_s^{1/2} \approx 600 \left(\frac{M}{M_\odot}\right)^{1/2} \text{MeV} \tag{11}$$

Thus for a $M = 6M_\odot$ BHC/ECO, the local temperature is $\approx 250$ MeV. Hence such stellar mass BHCS/ECOs could be in a Quark Gluon Plasma (QGP) phase. As of now, it is believed that a bulk QGP phase existed only in the very early universe and in the present epoch perhaps in the cores of neutron stars. Possibly a QGP phase may at best be momentarily created in high-energy-accelerator experiments. But now we arrive, through a detailed analysis, at the possibility that a bulk and ever-lasting QGP phase exists within the so-called stellar mass BHCs.

Although this discussion specifically considered formation of ECOs of a few stellar masses it is valid for all mass scales provided the progenitor density and gravity are suitably high to cause continued collapse towards BH formation, i.e., $z \to \infty$. And although enhanced matter-radiation interaction would accelerate the formation of ECOs, ECOs would nevertheless occur even in the absence of any matter-radiation interaction simply because $p_{\text{trap}} / \rho_c$ grows as $\sim (1+z)/MR$. However, for a supermassive ECO, the local internal temperature would be lower; for instance, for $M \sim 10^6 M_\odot$, one would have $T \sim 60$ keV.

Most of the numerical and analytical studies of GR radiative collapse implicitly or explicitly assume $\rho_r \ll \rho_c$ and hence they cannot obtain this ECO phase of $p_{\text{trap}} \gg \rho_c$. Note, if the transport of heat/radiation would be naively treated as *radially outward flow of a (null) fluid, one may not at all reproduce the effect of bending of radiation*. In principle, one needs to do ray tracking for each emitted photon/neutrino in extremely strong gravity. But in practice, this may be impossible, and one can probably write a program to approximately incorporate this effect by hand. Further while $R = R_s[1-(1+z)^2]^{-1}$ changes only modestly from $(3/2)R_s$ to $R_s$, $z$ would change infinitely faster from $(\sqrt{3}-1)$ to $\infty$ and appropriate commensurate binning of $R$ and $z$ would be quite difficult. Thus, numerically, it would be extremely difficult to capture this dramatic effect of radiation trapping and probably no study has ever attempted to do the same even though it is inevitable.

Interestingly, Cuesta, Salim and Santos [7] have attempted to see whether collapse of Supermassive stars can produce an ECO. And they have found that collapse of (Newtonian) Supermassive Stars first produces an ECO, radiating at its Eddington limit, rather a static BH. However they have not incorporated this dramatic effect of $I_{\text{trap}} \sim (1+z)^2$ (actually much faster) as $z \to \infty$. Neither have they considered the general possibility that, in GR, it is very much possible to have a state with $\rho \approx \rho_c$ instead of $\rho \ll \rho_c$ because of the dramatic rise of trapped radiation density.

In conclusion, since $p_{\text{trap}} \sim R^{3}(1+z)^2$ while the local Eddington pressure $p_e \sim R^{2}(1+z)$, as continued collapse would proceed to form a black hole with $z = \infty$, the former must equal the latter at appropriate finite value of $z$ and $R$. Then radiation pressure would make the evolution quasistatic. And irrespective of the value of opacity and any other details of the initial collapsing phase, this phase becomes eternal (as $z \to \infty$) and is called the *eternally collapsing (ECO) phase.*

Mathematically, however, the ECO is approaching the static configuration of a $z = \infty$ BH, *asymptotically*, to honor the exact vacuum Schwarzschild solution. Finally, since low mass ECOs could have a local temperature of
~ 250 GeV, they could comprise QGP floating in a sea of extremely hot and dense neutrino pairs.

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