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DIRECT EVIDENCE FOR THE MULTIPLE ASSIGNMENTS OF \( \Lambda(1520) \) AND \( \Lambda(1405) \)

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DIRECT EVIDENCE FOR THE MULTIPLETT ASSIGNMENTS OF 
\( \Lambda(1520) \) AND \( \Lambda(1405) \) *

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A measurement has been made of the relative signs of the resonant \( K^-p \to \Sigma\pi \) reaction amplitudes coming from partial waves corresponding to \( \Sigma(1385) \), \( \Lambda(1405) \), and \( \Lambda(1520) \). From this it is shown that \( \Lambda(1405) \) and \( \Lambda(1520) \) are to be described as predominantly unitary singlets.

One of the triumphs of SU(3) has been the consistently correct prediction of the relative signs of resonant reaction amplitudes as derived from formation experiments. In particular, all the better-established \( Y^* \) resonances formed in \( K^-p \) reactions and placed into singlets, octets, and decuplets according to mass formulae have correct relative signs between resonant amplitudes as measured in their \( \Sigma\pi \) and \( \Lambda\pi \) decay modes.\(^1\)\(^2\) Breaking of SU(3) often alters the predicted decay rates considerably, however, unless it is severe the relative signs are unaffected. In this letter we investigate the interferences between the \( J^P = 3/2^+ \) resonance \( \Sigma(1385) \), \( J^P = 1/2^- \) resonance \( \Lambda(1405) \) and \( J^P = 3/2^- \) resonance \( \Lambda(1520) \) as measured in the reaction \( K^-p \to \Sigma\pi \). Taking \( \Sigma(1385) \) to be in a decuplet, we shall find that \( \Lambda(1405) \) and \( \Lambda(1520) \) are consistent with their conventional assignments as unitary singlets.
In this analysis we follow the procedure adopted by Watson et al. of parameterizing the low-momentum $K^-p$ coupled-channel amplitudes by constant reaction complex scattering lengths and constant/phases, apart from the $D_{03}$ amplitudes corresponding to $\Lambda(1520)$ which are written as Breit-Wigner resonances. All partial waves through $J = 3/2$ are included. The old experimental data spanning the momentum region 250 to 513 MeV/c (1470 to 1570 MeV c.m. energy) have been greatly augmented (about 100-fold at 390 MeV/c) by our more recent bubble chamber experiment in the region 300 to 450 MeV/c. This partially completed experiment has so far yielded new angular distributions in the $K^0_n$, $\Sigma^0\pi^0$, and $\Lambda\pi$ channels, and polarizations in the channels containing $\Sigma^+\pi^0$, $\Sigma^0\pi^0$, and $\Lambda$ hyperons. Although the question of relative phase of the resonant amplitudes enters for this particular study only in the $K^-p \rightarrow \Sigma\pi$ reactions, we have included other channels in the analysis in order to simultaneously establish the best values for the amplitudes.

The reaction amplitudes for $K^-p \rightarrow \Sigma\pi$ may be written in terms of the $I = 0$ and $I = 1$ $\Sigma\pi$ reaction amplitudes as

\begin{align}
T_{\Sigma^\pm \pi^\pm} &= \frac{1}{\sqrt{6}} T_0 \pm \frac{1}{2} T_4', \\
T_{\Sigma^0 \pi^0} &= -\frac{1}{\sqrt{6}} T_0.
\end{align}

If the amplitude is resonant it is proportional to

\begin{equation}
g_{N\Sigma Y^*} g_{\Sigma\pi Y^*} / (E_R - E - \frac{i\Gamma}{2}),
\end{equation}

where the coefficient of the coupling of each channel to the $Y^*$ may be obtained from the Clebsch-Gordan coefficient of SU(3). Table I shows each of these couplings for a decuplet $Y^*_4$, i.e., $\Sigma(1385)$, and for a $Y^*_0$ belonging to a singlet, octet, or 27-plet. The subscripts on these real coupling constants indicate to which multiplet each belongs. We see that for the reaction
The numerator in the amplitude in (2) is negative: \(-g_{10}^2/6\).

Similarly for \(\bar{K}N\)→\(\Lambda(1520)\)→\(\Sigma\pi\), if \(\Lambda(1520)\) is a unitary singlet the numerator is \((3/32)^{1/2} g^2_4\), and if \(\Lambda(1520)\) is a 27-plet it is \((-3/800)^{1/2} g^2_{27}\). Thus in the interference between a decuplet and a singlet the numerators are of opposite sign; between a decuplet and a 27-plet they are of the same sign. For an octet \(Y_0\) the sign of the numerator depends on the relative strengths of the D and F couplings. For \(\Lambda(1520)\) the only known \(J^P = 3/2^-\) octet into which it could possibly fit would make the sign the same as for the decuplet. For the possibility of an octet \(\Lambda(1405)\), the partially completed \(J^P = 1/2^-\) octet has an insufficient number of known decay rates to fix the D and F couplings and therefore to predict the relative sign.

Figure 1 shows the Argand diagrams for the \(S_{01}\), \(P_{13}\), and \(D_{03}\) amplitudes (the subscript notation is \(L_{1,2J}\)) in the \(\bar{K}N\) and \(\Sigma\pi\) channels. The trajectory of the \(D_{03}\) amplitude in energy is shown by the dashed circle with a vector indicating its value at resonance; the vectors labeled \(S_{01}\) and \(P_{13}\) are shown at 395 MeV/c (corresponding to the energy of the \(D_{03}\) resonance). The \(\bar{K}N\) amplitudes in Fig. 1a are not relevant to this discussion since the relative signs are always positive. However, they do show that the elastic channel amplitudes do have the correct orientations to be described by the behavior beyond resonance of \(\Lambda(1405)\) and \(\Sigma(1385)\). The \(\Sigma\pi\) amplitudes in Fig. 1b display the signs with which we are concerned in this letter, namely that relative to the \(D_{03}\) amplitude of \(\Lambda(1520)\), the \(S_{01}\) amplitude representing the high-energy tail of \(\Lambda(1405)\) has the same sign. The \(P_{13}\) amplitude given by the high-energy tail of \(\Sigma(1385)\), however, shows that at its resonant energy this amplitude is negative imaginary, i.e., opposite in sign to \(\Lambda(1520)\), which demonstrates that the signs of the \(\Sigma\pi\) amplitudes for \(\Lambda(1520)\) and \(\Lambda(1405)\) relative to \(\Sigma(1395)\) are as expected.
from unitary singlets. It also excludes the assignment of \( \Lambda(1520) \) in the \( 3/2^- \) octet or the assignments of \( \Lambda(1520) \) or \( \Lambda(1405) \) into 27-plets.\(^8\)

Having seen that the \( S_{01} \) and \( P_{13} \) amplitudes at our energy have phases in both \( \bar{K}N \) and \( \Sigma\pi \) which are quite consistent with an interpretation that they are the high-energy tails of their respective resonances, we have altered the parameterization of these states from the previous constant scattering-length description to that of simple Breit-Wigner resonances. Threshold dependences were given by appropriate phase-space and centrifugal barrier factors. Re-minimization of all amplitudes yielded fits with higher \( \chi^2 \) than the previous parameterization,\(^9\) but nonetheless gratifyingly good considering the simplicity and highly constrained nature of this model, in which there are no free phases for the resonant amplitudes. In these fits we have fixed the masses and partial widths at resonance of \( \Sigma(1385) \) and \( \Lambda(1405) \) at their experimental values,\(^10\) whereas the \( \bar{K}N \) partial widths (of course closed below 1432 MeV) were left free to be determined by this experiment. For \( \Sigma(1385) \) the ratio of the reduced \( \bar{K}N \) width thus obtained to that expected by SU(3) was 1.5 , and for \( \Lambda(1405) \) it was 8.0 . This indicates that deviations from SU(3) symmetry are small for \( \Sigma(1385) \) but are quite appreciable for \( \Lambda(1405) \), as has been noted previously.\(^11\)

It is perhaps worthwhile to exhibit the data that are most relevant to the sign determination. The interference between \( \Sigma(1385) \) and \( \Lambda(1520) \) appears most clearly in the \( A_3 \) coefficient of the Legendre polynomial expansion of the angular distribution, since this coefficient arises only from \( P_3 - D_3 \) interference. From Eq. (4), if we take the difference in the \( A_3 \) coefficients between \( \Sigma^+\pi^- \) and \( \Sigma^-\pi^+ \), the interference occurs only between states of different isospin. Since the \( I = 1 \) D amplitude is small, this quantity comes
almost purely from $\Sigma(1385) - \Lambda(1520)$ interference. Figure 2a displays the data expressed as a ratio $A_3/A_0$ (we do not yet have measurements of the partial cross sections) and the curves corresponding to the two choices of relative sign. The large excursion from positive to negative, passing through zero at 385 MeV/c (just before resonance), requires the $P_{13}$ amplitude to be as indicated in Fig. 1a. Similarly the $I=0$ $\Sigma^0\pi^0$ angular distribution best shows the interference between $\Lambda(1405)$ and $\Lambda(1520)$. Figure 2b shows the $A_2/A_0$ ratio that arises primarily from the S and D states and their interference, since the P amplitudes are small compared with S and D. The curves correspond to the two choices of relative sign of $\Lambda(1405)$ with respect to $\Lambda(1520)$. It is clear from the data that beyond resonance the two amplitudes must be nearly in phase, as is required if both resonances are unitary singlets.

FOOTNOTE AND REFERENCES

* This work was supported under the auspices of the U. S. Atomic Energy Commission.


2. The method of employing relative signs for SU(3) classification was first used by A. Kernan and W. Smart, Phys. Rev. Letters 17, 832 (1966).


4. There are no indications that higher partial waves appear in detectable amounts at the low energies considered here. We have altered slightly the analysis method of Ref. 3 by employing the appropriate c.m. momentum for each channel in the decay widths of the resonant amplitudes,
using a fixed radius of interaction of 1 fermi in the expression for the centrifugal barrier.

5. $\Sigma^+$ polarizations from this experiment have been published in R. Bangerter, et al., Phys. Rev. Letters 17, 495 (1966).

6. We reverse the convention used in Ref. 3 to make it more appropriate for discussions of unitary symmetry.


8. From the known branching fractions of $\Lambda(1520)$ and $\Lambda(1690)$ (a presumed member of the $3/2^-$ octet) there is reasonably good evidence for some singlet-octet mixing between these two states, as noted by Ref. 1 and by G. B. Yodh, Phys. Rev. Letters 18, 810 (1967). In order to alter the sign of the resonant $\Lambda(1520)$ amplitude the mixing angle would have to exceed about 45 deg. The estimate for this angle is about 16 deg. For $\Lambda(1405)$ there is evidence for much stronger SU(3) symmetry breaking, as discussed later in this letter. The conclusion of our experiment concerning $\Lambda(1405)$ is that the symmetry breaking is not so severe as to alter the sign of the resonant amplitude.

9. The ratio $\chi^2/\langle\chi^2\rangle$ (where $\langle\chi\rangle$ is the expected chi-squared) for the constant scattering length parameterization is 1.23, indicating either that detectable differences with this constraining parameterization are beginning to appear, or that the data in their preliminary form have some additional biases. The resonant parameterizations have yielded ratios of 1.38 for $P_{13}$ parameterized as a resonance and 1.85 for $S_{01}$ parameterized as a resonance.


Table I. Coupling coefficients of $Y^*$ for various SU(3) representations.

<table>
<thead>
<tr>
<th>$Y^*$</th>
<th>Decuplet (10)</th>
<th>$g_{NKY^*}$</th>
<th>$g_{\Sigma\pi Y^*}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_1^*$ ((\Sigma))</td>
<td>$-(1/6)^{1/2} g_{10}$</td>
<td>$(1/6)^{1/2} g_{10}$</td>
<td></td>
</tr>
<tr>
<td>$Y_0^*$ ((\Lambda))</td>
<td>$\frac{1}{2} g_{1}$</td>
<td>$(3/8)^{1/2} g_{1}$</td>
<td></td>
</tr>
<tr>
<td>Singlet (1)</td>
<td>$\frac{1}{10}^{1/2} g_{8D} + (1/2)^{1/2} g_{8F}$</td>
<td>$-(3/5)^{1/2} g_{8D}$</td>
<td></td>
</tr>
<tr>
<td>Octet (D and F)</td>
<td>$\frac{3}{20}^{1/2} g_{27}$</td>
<td>$-(1/40)^{1/2} g_{27}$</td>
<td></td>
</tr>
<tr>
<td>27-plet (27)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
FIGURE LEGENDS

Fig. 1. Amplitudes for the reactions $\bar{K}N \rightarrow \bar{K}N$ and $\bar{K}N \rightarrow \Sigma \pi$ shown inside the unitary circles for the three partial waves of interest in this letter. The correspondences are $\Sigma(1385) = P_{13}$, $\Lambda(1405) = S_{01}$, and $\Lambda(1520) = D_{03}$. The latter is parameterized as a resonance whose trajectory in energy is shown as a dashed circle. The overall arbitrary phase for the reaction amplitude $\bar{K}N \rightarrow \Sigma \pi$ is fixed by taking $D_{03}$ to be imaginary at resonance. The $S_{01}$ and $P_{13}$ amplitudes are here parameterized as constant complex scattering lengths. Their phases in the $\Sigma \pi$ channel are left free to be determined by this experiment. It is these phases which show $\Lambda(1405)$ and $\Lambda(1520)$ to have the same relative sign, and $\Sigma(1385)$ and $\Lambda(1520)$ to have the opposite relative sign. Crosses near the heads of the arrows show where the $S_{01}$ and $P_{13}$ amplitudes would reach if parameterized as Breit-Wigner resonances, as described in the text.

Fig. 2. Data from this experiment showing most clearly the interference between $\Lambda(1520)$ and the high-energy tails of $\Sigma(1385)$ and $\Lambda(1405)$.

(a) The difference between the $A_3/A_0$ coefficient of the Legendre polynomial expansion of the $\Sigma^+ \pi^-$ and $\Sigma^- \pi^+$ angular distributions. This arises primarily from $P_{13} - D_{03}$ interference. The solid curve shows the best fit to all the data measured in the experiment; the dashed curve results from reversing the $P_{13}$ phase. (b) $A_2/A_0$ for $\Sigma^0 \pi^0$. This arises primarily from $S_{01} - D_{03}$ interference. The solid curve is the best fit to all data; the dashed curve is the fit with the phase of $S_{01}$ reversed.
Fig. 1.
Fig. 2.
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