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Influences of Surface Waves on the Open Ocean Wind Stress Vector

A dissertation submitted in partial satisfaction of the requirements for the degree Doctor of Philosophy in Oceanography

by

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1996
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Abstract of the Dissertation

**Influences of Surface Waves on the Open Ocean Wind Stress Vector**

by

Karl Frederik Rieder

Doctor of Philosophy in Oceanography
University of California, San Diego, 1996

Dr. Richard J. Seymour, Co-Chairman
Dr. Jerome A. Smith, Co-Chairman

Open ocean measurements of the turbulent wind, directional wave field, and wave breaking are used to investigate the effect of waves on the wind stress, the momentum flux between atmosphere and ocean. Data were collected during the Surface Waves and Processes Program (SWAPP), which took place in February and March of 1990 from the research platform *FLIP*, situated in the North Pacific.

Traditional formulations of the drag coefficient are reviewed and tested in swell dominated, open ocean conditions in Chapter II. General trends indicate that drag increases with increasing wind speed, inverse wave age, and wave height, but the existence of significant scatter shows that none provides a stable, accurate estimate of the wind stress. However, significant correlations of the drag coefficient with these
parameters are seen during the onset of three wind events, when waves are being actively generated. Additionally, the rate of increase of the drag coefficient in each of these periods is most closely linked to the turning rate of the wind, indicating that temporal and directional effects must play an important role.

The directional relationship between waves and the wind stress is investigated in Chapter III. Non-zero angles between the direction of the wind stress and the mean wind are measured. In general, the direction of the wind stress lies between the directions of the mean wind and that of the long period swell. Moreover, a significant trend between the variations of the wind stress and swell directions is found for higher wind speeds. Finally, a relation between the wind stress and wave directions can be noticed as a function of frequency, suggesting a close dynamical link.

A possible connection between wave breaking and the wind stress is investigated in Chapter IV. From comparisons of data from 15 half hour periods, the directions of the whitecap motion and the wind stress are found to be generally co-linear. As well, the speed of whitecap motion is shown to correlate with the drag coefficient. These results suggest that information about wave breaking may be used to estimate both the magnitude and direction of the wind stress.
Chapter I

Introduction
The flux of momentum at the atmosphere/ocean interface is the significant driving force for many oceanic phenomena, including the growth of surface gravity and capillary waves, the development of the oceanic mixed layer, and the creation of the oceans' surface currents. Commonly called the wind stress, this momentum flux is of wide importance to oceanographers, atmospheric scientists, and meteorologists. As a component of regional and global atmospheric circulation models, the wind stress plays an important role in weather prediction and the study of scientific topics such as global warming and El Niño.

While the wind provides energy for the growth of waves on the sea surface, the flow of the overlying wind field is also strongly affected by the existence and nature of the waves themselves; thus, the two are actively coupled. The complexity of this system makes the estimation and prediction of the wind stress a difficult task. Operationally, the wind stress is estimated via a bulk formula which states that its magnitude is proportional the square of the wind speed via a constant of proportionality, the drag coefficient. The drag coefficient has been suggested to be dependent on a large number of parameters: wind speed, atmospheric stability, wave age, wave breaking, gustiness, large scale atmospheric features, etc., but since many of these are not mutually independent, a clear, universal parameterization has eluded scientists.

The first studies, like those in the field of wave generation, viewed the problem in the framework of fetch-limited seas, where the parameterization of the state of the sea surface can be reduced to the fewest number of parameters: wind speed and fetch. Such conditions represent an excellent starting point, as they provide a simple, dependable form of the wave spectra which has been well studied. These studies
produced varied results. Most commonly, a wind speed regression was suggested, but the large scatter in these data indicates that wind speed alone does not suffice in providing a sufficiently accurate estimate of the wind stress. Inclusion of other dependencies, such as wave age and wave spectral information, has slightly improved the estimates, but a comprehensive theory has not been accepted and rigorously tested in the open ocean. Recent literature has been based on the premise that the roughness of the sea surface is mainly controlled by short gravity waves, which travel slowly relative to the wind. However, as long waves, gustiness, and wave breaking, for example, significantly alter the magnitude and direction of these short waves, many other parameters may need to be considered in any formulation.

Additionally, these parameterizations all follow the bulk formula's assumption that the wind stress and mean wind are co-linear. Recent research has shown this not to be universally correct. The angle between the wind stress and mean wind directions has been found to vary with the direction of the long surface gravity waves and with the sign of the heat flux between the atmosphere and the ocean. However, these observations of wind stress directionality are few and more are required before directionality can be included in wind stress modeling.

In the open ocean, conditions of the sea surface are far more complicated than those for fetch limited seas. The existence of waves generated by distant storms results in a sea surface which is only partially the result of the existing winds. Waves traveling in different directions and at different periods cause a modulation of the surface roughness and the momentum transfer, both in magnitude and direction, which is not well understood. The lack of a predictable dynamic equilibrium between the wind and the waves makes the solution for the sea surface drag very difficult.
This dissertation looks into the nature of the wind stress vector in open ocean conditions, employing data taken from the Surface Waves and Processes Program (SWAPP), which took place in the North Pacific in February and March of 1990 from the research platform FLIP. In order to concentrate on surface roughness effects, atmospheric stability (heat flux) is not considered, and only near-neutral conditions are analyzed. Chapter II reviews previous drag coefficient parameterizations and investigates how well these models work in complex seas. Attempts are made to understand at what times and for what reasons they perform well and at what times more poorly. The hope is that such information will allow us to better understand the complex dependencies between the waves, wind, and stress. Chapter III investigates the nature of the wind stress direction and its relation to the direction of the surface gravity waves. The directional relationship of the wind stress to the long period swell as well as to the waves on a per-frequency basis is studied. Chapter IV looks into a possible connection between the wind stress vector and wave breaking velocity and direction. While wave breaking has been shown to affect the structure of the atmospheric boundary layer in the laboratory, the use of wave breaking information to estimate the wind stress is a novel approach.
Chapter II

Analysis of Sea Surface Drag Parameterizations In Open Ocean Conditions

Abstract: Data from the Surface Waves and Processes Program (SWAPP) are employed to test current sea surface drag parameterizations in swell dominated, open ocean conditions. General trends in the data indicate that drag increases with increasing wind speed and wave height, and decreases with wave age. However, scatter in the data limits the use of these parameters and other wave dependent parameterizations for modeling efforts. Upon close inspection, it is found that during the onset of three wind events analyzed separately, each of these parameters correlate well with the drag coefficient. However, the dependence of the drag coefficient on each of these parameters varies markedly from event to event. The disparity appears most closely linked to the turning rate of the wind, indicating that temporal and directional effects may play an important role. A temporal lag of O(4) hours between the rise of the wind and subsequent rise in the drag coefficient is also noticed, further pointing out the complexity of the wind stress system.
1. Introduction

As the link between the easily measured wind velocity and the more difficult direct measurement of wind stress, the drag coefficient is the key parameter for the determination of the momentum transfer between atmosphere and ocean. This transfer of momentum drives a wide range of atmospheric and oceanic phenomena, including the oceans' surface currents, the oceanic and atmospheric mixed layers, and surface gravity and capillary waves. Consequently, the drag coefficient plays an important role in coupled atmosphere-ocean circulation models which are used to forecast weather and climate and are often applied to scientific issues such as global warming and El Niño. However, despite the large amount of work over the last decades, research has yet to establish a universal theory by which the drag, and hence the wind stress, may be accurately estimated from indirect measurements.

The wind stress is most often estimated via a simple "bulk formula":

\[
\tau = \rho C_d |\vec{U}| \vec{U}
\]

(1)

where \( \tau \) is the wind stress, \( \rho \) the air density, \( C_d \) the drag coefficient, and \( \vec{U} \) the mean wind velocity. Sea surface drag can be equivalently described by a roughness length, \( z_o \), which for neutrally stable atmospheric boundary layers can be related to the drag coefficient by assuming a logarithmic profile of the mean wind speed with height:

\[
z_o = \frac{z}{\exp \left[ \kappa/\left(C_d \right)^{1/2} \right]}
\]

(2)

where \( \kappa \) is the Von Karman constant and \( z \) is the measurement height from mean sea level. (The roughness length is mathematically the zero-crossing height of an extrapolated logarithmic wind profile).
Over land, the roughness length has been successfully related to the average height of roughness elements. However, the extension of this relation to the roughness length over the sea is conceptually difficult to make since at the atmosphere-ocean interface, the roughness elements are moving (Kitaigorodskii, 1973).

Attempts to model the sea surface drag coefficient and roughness length have both wind and wave dependencies (Geernaert, 1990). It was first suggested by Charnock on dimensional grounds that the roughness length over the sea surface should vary with wind speed (Charnock, 1955). Since that time, many empirical formulas for the drag coefficient have been constructed by researchers who made direct measurements of the wind stress (e.g. Donelan, 1982; Geernaert et al., 1986; Large and Pond, 1981; Wu, 1982). In general, a weak dependence on wind speed was found; however, large variations in the magnitude of the drag coefficient were common, and results from study to study vary significantly.

Several attempts have been made to integrate wave measurements into the formulation of the roughness length. It is not known which scales of waves play the greatest role in the creation of the sea surface roughness. Therefore these formulations vary in derivation. As a rule, they perform uniformly poorly, except under special circumstances. Donelan (1982) proposed that the drag has two forms, one dependent on the form drag due to the long waves and another due to the viscous drag of the short waves; Byrne (1982) postulated that the roughness length was proportional to the integral wave velocity over the wind wave frequencies; Kitaigorodskii (1973) derived an equivalent roughness length in the moving reference frame of the surface gravity waves - the resulting expression puts the roughness length in terms of an exponentially weighted wave energy density; Hsu (1974) related the surface roughness to the steepness of only the dominant waves; and Janssen (1989) expressed the
roughness length in terms of the growth of the waves. The search for a proper
roughness formulation has been based conceptually on wave spectra bereft of swell,
and these models have not been rigorously tested in more complex conditions. While it
is clear that the sea surface roughness must be mainly due to the existence of surface
gravity waves, research has yet to identify a unique set of wave parameters on which
practical models may be based.

Most recently, effort has been put into representing the roughness length via the
wave age \((c_p/u_*)\), the ratio of the celerity of the peak wind wave frequency to the
friction velocity), which is a measure of the state of development of the seas. While
studies during conditions in which swell was absent indicated that the roughness
length varied inversely with wave age (e.g. Donelan et al., 1993; Smith et al., 1992), a
study where swell was present did not indicate a clear relation (Dobson et al., 1993).
Other studies have suggested that for waves which are in dynamic equilibrium,
roughness and wave age may be directly proportional (Toba et al., 1990),
contradicting the above noted trend. Overall, it seems clear that despite the amount of
research done, a consensus has not been found.

To further complicate the issue, it has been recently found that the wind stress and
mean wind are not necessarily co-linear (Geernaert, 1988; Geernaert et al., 1993;
Zemba and Frieha, 1987), invalidating a basic assumption of the bulk formula. Most
commonly, stress estimates calculated by the eddy correlation method have re-enforced
the assumption of wind and stress alignment by not bothering to calculate the
crosswind stress. Stress estimates made using the inertial dissipation method do not
have directionality, a priori alignment with the wind is assumed; the calculated drag
coefficient therefore applies to the magnitude of the whole stress vector (Geernaert,
1990). The data considered here have already been shown to include large angles
between the wind and the stress. These angles have been related to the direction of waves across a wide range of frequencies (Rieder et al., 1994) and to the direction of wave breaking (Rieder et al., 1995). In this work we will simply define the drag coefficient as the coefficient of proportionality between the square of the wind speed and the component of the wind stress in the direction of the mean wind, thus maintaining mathematical sense.

Geernaert (1996) has made a first attempt at modeling the direction independently. However, it has yet to be agreed upon how to make an estimate of the total wind stress vector and apply previous drag coefficient parameterizations to that end.

This paper re-investigates the important parameters which affect the sea surface drag. In particular, we are interested in testing the current theories and roughness parameterizations in open ocean conditions, where a mix of sea and swell exist. Wind and wave measurements taken in the North Pacific 500 kilometers off Point Conception, California will be employed to this end.

2. The Experimental Program and Data Analysis

Data were collected for this study during the Surface Waves and Processes Program (SWAPP), which took place from February 24 through March 18, 1990 approximately 500 km off Point Conception, California in 4000 meters of water. Measurements of the surface gravity waves, mixed layer structure, and air-sea fluxes were made from the research platform FLIP to study air-sea and surface wave-upper ocean boundary layer interactions (Weller et al., 1991).

Sea surface velocities were measured by four 195 kHz Doppler sonars, directed at 45° increments in azimuth. A "quick-look" analysis of the measured velocities provide estimates of the first three Fourier components of the directional spectra every 12.8
minutes within the frequency range of .05-5 Hz (Smith and Bullard, 1995).

Direct calculations of the momentum flux were made possible by high frequency (10 Hz) 3-component measurements of the wind velocity. Two sonic anemometers were deployed on the aft and port booms of FLIP at 6 and 8m above mean sea level respectively. Wind stress could then be calculated directly by the Reynolds's averaging equation:

$$\overline{\tau} = -\rho\left[\langle u'w'\rangle\hat{i} + \langle v'w'\rangle\hat{j}\right],$$  \hspace{1cm} (3)

where $u'$, $v'$, $w'$ are the downwind, crosswind, and vertical fluctuating wind velocities.

The sonic anemometer data were reduced to 2 Hz and wind stress estimates were made every 30 minutes. The estimates were required to satisfy several quality controls, as in Rieder, et al. (1994). First, wind speeds of less than 3 m/s were not considered. Second, to remove any periods for which the wind stress is ill-defined (due to a partial sample, for example), estimates of the downwind stresses were required to be different from zero with a statistical confidence of 99% (assuming a normal distribution). Third, conditions during which the superstructure of FLIP sheltered or shadowed the sonic anemometer were removed. Only data from the sonic anemometer on the aft boom of FLIP ever passed the third test. Finally, the motion of FLIP was analyzed and found to be insignificant in producing possible errors in the wind stress calculation (Smith and Rieder, 1995).

Also measured were air temperature, sea temperature, and humidity, which were used to determine atmospheric stability (c.f. Large and Pond, 1981). Atmospheric stability has been shown by previous research to have dramatic effects on the mean wind profile, and to be of significant importance in the determination of both the magnitude and direction of the wind stress (Geernaert, 1988). Here, we will
investigate only near-neutral conditions, so that we may avoid the use of stability corrections on the profile of the mean wind. Conditions are considered near-neutral if $L_c/L < 0.05$, where $z$ is the measurement height and $L$ is the Monin-Obukhov length, a scale height for balancing buoyancy flux against wind stress. This generally excluded data just prior to the onset of the three wind events.

3. Results

The wind speed, root-mean-square wave and sea height, inverse wave age, and drag coefficient, $C_d$, over the course of the ten day period of SWAPP are plotted in Figure 2.1. The drag coefficient is found by extrapolating the wind speed to 10 m equivalent height (scientific standard) using the direct measurements of wind stress and the assumption of a logarithmic profile. The rms wave height is found by summing over the total wave height spectrum, and taking the square root. The rms sea height is determined from a sum over only the high frequency sea waves, where the low frequency cut-off is taken as the peak in the velocity spectrum. The celerity of this wave frequency is used in the calculation of wave age. We will see that this choice does not qualitatively affect the results.

The peak of the velocity spectrum produces the best estimate of the lower limit of the saturation region. To illustrate this, Figure 2.2 shows the wave height spectra for four periods, each separated by three hours, starting on March 7, 1990 at 20:00 when strong winds began. As seen in the first period, low frequency swell existed as the high frequency sea waves began to grow. As the sea waves developed, the peak of wind wave spectrum steadily moved to lower frequencies. The arrows indicate the low frequency cut-offs of the wind wave spectrum, as chosen by the peak in the velocity
spectrum.

Three strong wind events occurred over a ten day period (see Figure 2.1a). Non-shaded regions indicate the beginning of each event: these will be discussed further later. The first, starting on March 4, 1990, began with a strong increase of the wind from calm conditions (the wind speed rising to 10 m/s). Growth of the waves can be seen clearly following this wind increase (Figure 2.1b). The root mean square wave height reached a peak of nearly 1.0 m (approximately 4 m significant wave height). The onset of the wind occurred with little accompanying swell, indicated by the large inverse wave age (Figure 2.1c), and the small difference between wave and sea heights. During the second event, the wind speed increases very abruptly, doubling in speed in minutes, resulting in strong wave growth. In contrast, the third event shows a more gradual wind speed increase during conditions of pre-existing swell.

The variation of $C_d$ over the course of the ten day period can be seen in Figure 2.1d. Only data passing all criteria are shown. There are long time periods during which no wind stress estimates passed these criteria, commonly due to the shadowing of FLIP. General trends shown in Figure 2.1 indicate that drag increases with increasing wind speed, rms wave height, and inverse wave age (decreasing wave age). Furthermore, individual peaks in the drag coefficient are seen to be mirrored in the other time series, particularly in the inverse wave age. A secondary peak in the $C_d$ time series can be seen for each of the three wind events (following a primary peak which accompanied the initial rise of the wind). Similar peaks are seen in the inverse wave age for the first and second events and in the wind speed record for the first and third events.

Despite the fact that these general trends seem apparent by examination of the time series, a closer inspection shows that high drag coefficients do not always correspond
to high wind speeds, high wave height and/or small wave age. Also, there are periods when $C_d$ varies strongly from half-hour to half-hour, while there are other periods where the drag coefficient steadily increases or steadily decreases. This can be seen clearly during the long second wind event, from March 7, 12:00 to March 9, 12:00. During this forty-eight hour period, wind speeds remained relatively strong, rapidly increasing over a six hour period and then slowly decreasing during the remaining 42 hours. This caused a quick increase and slow decrease of the wave energy. (However, note that the waves continue to grow following the peak of the wind speed). During the increase of the wind and the quickest growth of the waves, there can be seen a distinct increase in $C_d$. Then, as the wind slowly decreases in speed, the drag coefficient drops and then varies wildly. This variation will surely complicate the modeling problem.

A final note about the time series concerns the apparent temporal lag between the increases and decreases of the wind and the resulting increases and decreases in the drag coefficient. Three clear instances of this can be seen: the beginning of the first wind event, the beginning of the second wind event, and the secondary wind increase during the third event. During the first two wind events there was a sudden increase in the wind, and an accompanying increase in the drag coefficient. However, the peak in the wind speed occurs prior (on the order of 4 hours) to the peak in the drag coefficient. Similarly, as the wind speed is falling and quickly begins to increase again during the third wind event, near the end of March 11, a falling and quick rebound of the drag can be seen. But again there is a temporal lag of several hours.

We next look at the wind speed, sea state, and wave dependencies of the drag coefficient. This will be done by examining the current drag coefficient parameterizations and testing how well they match its variability. The variance and
temporal lag issues will then be investigated in greater detail to understand better the relationship between the wind and waves and the roughness of the sea surface.

3.1 Wind Speed Dependence

The calculated drag coefficient is plotted vs. the 10 meter wind speed in Figure 2.3. Most noteworthy is the large amount of scatter in the points, typical in such plots. In the present case, the correlation is insignificant. This low correlation is remarkable since clear general trends were apparent in the time series. The wind speed alone does not reduce the variance of the drag coefficient over the range of wind speeds here. The mean drag coefficient is equal to $1.21 \times 10^{-3}$ with a standard deviation of $0.36 \times 10^{-3}$ or 30% of the mean.

3.2 Sea State Dependence

The non-dimensional roughness length ($z_{o*} = g z_o / u_*^2$) versus wave age has been plotted in Figure 2.4. A trend of decreasing drag with increased wave age is weakly suggested in the data. A line indicating a power law of $n=-1$, as suggested by Smith et al. (1992) and Donelan et al. (1993), is drawn for comparison with previous results. The data here does not support these findings. There is significant scatter, and no significant correlation. Dobson et al. (1993) saw similar scatter in their data from the Grand Banks experiment during times which swell dominated, despite attempts at its removal. The implied spectral similarity assumed in the use of wave age may not be appropriate for swell dominated seas, such as those seen during SWAPP and the Grand Banks experiment.
3.3 Wave Dependence

The models of Hsu (1974), Byrne (1982), and Kitaigorodskii (1973) were previously examined by Geernaert et al. (1986) using a data set collected in the North Sea. Hsu (1974) postulated that the non-dimensional roughness length varies with the wave steepness of the dominant wind waves. Expressing this in terms of the phase velocity of deep water surface gravity waves, the roughness length can be expressed as:

\[ z_o \approx \frac{1}{2\pi} \frac{H}{(c/u_*)^2}, \]

where \( H \) and \( c \) are the height and phase speed of the dominant wind wave. Byrne theorized that the roughness length was proportional to the integral wave orbital velocity over the wind wave frequencies:

\[ z_o = \int_{\omega_0}^{\omega} S(\omega) \omega^2 d\omega, \]

where \( S(\omega) \) is the wave energy density at frequency \( \omega \). By assuming that the wave height can be interpreted as some scale for the protrusions of a completely rough immobile wall, Kitaigorodskii derived an equivalent roughness length in the moving reference frame of the surface gravity waves. The resulting expression puts the roughness length in terms of an exponentially weighted wave energy density:

\[ z_o = \left[ 2 \int_0^\infty S(\omega) \exp\left(-\frac{2k}{\omega u_*}\right) d\omega \right]^{1/2}. \]

Geernaert et. al found that comparing to the North Sea data set that formulations of Hsu and Kitaigorodskii performed better than a simple wind speed dependent drag law, while Byrne’s model performed poorly.
As the models of Hsu and Byrne are based on the height and velocity of the wind waves, it is necessary to separate the wind wave spectrum from the swell. This was done by analyzing the waves above a low frequency cut-off determined as the peak of the velocity spectrum (as in calculation of the rms sea height and the wave age). This separation, however, cannot remove the effect of the long period swell on the wind waves. We are interested to find whether the existence of swell invalidates the use of these models, and whether the trends in the data are maintained. We find correlation coefficients of $r^2=.01$ for both the Hsu and Kitaigorodskii wave parameters, indicating insignificant correlation and a clear failure of these two models for this data set. The correlation coefficient between the measured roughness length and the Byrne modeled roughness length of $r^2=.03$ is poor as well, as seen by the large amount of scatter in Figure 2.5. The measured roughness length varies by several orders of magnitude, but no such variation is seen in the modeled roughness, due to the lack of variability in the sum of the wind wave velocity spectrum in the open ocean. (No frequency band of the 1-d wave spectrum varies by the many orders of magnitude necessary.) The resulting modeled drag coefficients can not vary much from a constant level, in contrast to the large deviations seen in the data.

3.4 Variation in Drag

The hope to explain $C_d$ variations by traditional parameterizations is so far discouraging. By inspection of the time series and the general trends in the data, one would expect that the surface drag could be estimated via the wind speed, wave energy, and/or the wave age. However, when analyzing this open ocean data set, no strong correlations are found. While these models have been shown to perform successfully in fetch-limited or wind wave dominated conditions, the existence of
swell apparently invalidates the results.

The above analysis tested whether there is a one-to-one correlation between high winds (for example) and large drag, the answer being no. But do changes in the wind correspond to changes in the drag? Or at the very least, are the periods of variability in the wind (in direction and magnitude) the same periods when the drag is varying? To test this, we have plotted the variability of the wind, downwind and crosswind components, over the course of the ten day period, along with the variability of $C_d$ (Figure 2.6). The variability is measured by the magnitude of the first difference in the vector wind and drag coefficient, smoothed by a running mean for plotting. We see that often drag variability corresponds to wind variability, but not always. Periods of more constant wind correspond to periods when the drag is varying less and visa-versa. In fact, peaks in the wind's variability are often mirrored by peaks in the drag's. The fact that the crosswind variability is important indicates that periods of turning winds are periods of change in the drag as well. This is of interest, because the previously analyzed models contained no directional or temporal information. But this clearly does not explain the whole story.

The drag coefficient is most steady during conditions when the seas are actively growing due to a strong increase in the wind. Therefore, to look further, we inspect the "beginning period" of each of the three wind events to determine to what extent the short term variability limits the potential to model the drag coefficient. The "beginning period" of each wind event extends from the initial wind increase to the peak in the rms sea height (see the non-shaded areas in Figure 2.1). Within these periods, the usable data will come from the portions of these periods where all the data has passed the quality criteria.

Figures 3.7, 3.8, and 3.9 show the variation of $C_d$ with wind speed, rms wave
height, and wave age over each of these three periods. The first, second, and third wind events are represented by the circles, stars, and crosses respectively. Considering these three periods together, we see that there is an improved correlation between $C_d$ and each of the three measures. The correlation coefficients are $r^2=0.07$, $r^2=0.17$, and $r^2=0.24$ (negative correlation) between $C_d$ and the 10 m wind speed, rms wave height, and wave age, indicating that surface roughness increases with wind speed and wave height, and decreases with wave age. These correlations are still quite low, but do represent an improvement to those for the entire data set: $r^2=0.01$, $r^2=0.08$, and $r^2=0.15$ respectively. If we now inspect each period individually, we see that the trends are more apparent. This is particularly true for the comparison with the rms sea height, where each data subset can be well represented by a linear best fit. However, the empirical regression with wind speed, wave height, and wave age of each period are different from the others (see Figure 2.8). In particular, the second time period, depicted by the stars, shows a much stronger variation of drag with wind speed and wave height as compared with the other two periods. This is exemplified by the correlation between $C_d$ and the ten meter wind speed for the combination of the first and third wind events, depicted by the regression line through the data. The correlation coefficient of $r^2=0.46$ is a marked increase over any previous comparison. These results indicate that significant variability occurs over a wide range of temporal scales, from the averaging period (half-hour) to synoptic time scales.

To investigate what may account for the large difference in drag between these events, we considered a variety of wind and wave parameters. Firstly, a comparison of the wave height spectra was made; the spectra for the first and last period of each of the three wind events are shown in Figure 2.10. For display, the spectra of the first wind event have been offset downward by a factor of 10, and those of the third have
been offset upward by a factor of 10. Significant wave growth occurred over each of the three periods, as shown by the increase in wave energy above the high frequency saturation region. Comparing the three periods, a large difference is noticed between the first event and the latter two. While little swell accompanied the first event, strong swell was present at the beginning of the second and third, as indicated by the peaks at low frequencies. As well, a contrast between the first two and third events can be made. Each of the first two had a clear double peaked structure, and the largest growth occurred principally between the peaks. In contrast, the third had growth over the entire range of frequencies. While qualities of the first and third events may be identified as unique in comparison with the others, no such quality was found concerning the second. The directional characteristics of the waves were also investigated, but did not yield any significant clue either.

To look at other possibilities, we calculated some of the important wind parameters for the three events (Table 1). While the wind speeds are similar for all of the wind events, both the mean $C_d$ and rate of increase of $C_d$ are much larger for event number 2. The drag increases approximately five times more quickly as compared to the other two wind events. Of the other factors considered, we find that the turning rate of the wind to be most nearly a match. For event #2, the wind was turning quickly, as can be seen in the last column of Table 1. The wind turned steadily 24° in 5.5 hours: a rate more than four times as large as either of the other two events. Interestingly, it is during turning winds of steady speed, as opposed to increasing winds of steady direction, that the surface drag increases the most rapidly. Most importantly, however, these results indicate the importance of directional and temporal characteristics of the wind and the waves in modeling of the drag coefficient. In drag coefficient parameterizations to date, directionality and temporal change in the wind or
waves have not been included.

3.5 Temporal Lag Between the Wind and Drag

We noted earlier that there was a temporal lag observed between the increase of the wind and the increase in the drag coefficient, and that this lag was on the order of several hours. This reinforces the possible importance of temporal information in drag modeling. Also, it raises an interesting notion about which wave frequencies are responsible for creating the drag of the ocean surface. By investigating how the various frequencies of waves respond to an increase in wind during these events (the higher frequencies will respond more quickly than low frequencies), we can see which waves have the same temporal response as the drag.

There were three particular time periods when this temporal lag was evident: during the onset of the first and second wind events, and the secondary wind increase during the third wind event. For these three periods, we have plotted the magnitudes of the 6-7, 4-5, and 2-3 second waves, the wind, and the drag coefficient. The wave magnitudes have been normalized by the saturation value given by the Phillips (1958) spectra, where the Phillips constant is set to $5.5 \times 10^{-3}$.

Figure 2.11 depicts the first wind event, when winds doubled in just over an hour, slowed slightly in the following hour, and then increased again and maintained speed over the course of the next day. The 2-3 second waves respond quickly by nearly reaching saturation during the initial wind increase, while the 4-5 second waves do not reach saturation until just after the secondary peak in the wind. The lowest frequency waves respond slower still, reaching a peak level more than 4 hours after the final peak in the wind. Looking at $C_d$, we see that its magnitude is more variable. It reaches its highest level just following the secondary peak in the wind, at
approximately the same time as the 4-5 second waves. As well, the variation in $C_d$ mirrors most closely that of the 4-5 second waves, flattening during the decrease of the wind between the first and second peaks.

How the waves and drag respond to an abrupt increase in the wind is shown in Figure 2.12. During the second event, the wind increases rapidly and then maintains a nearly constant value. The highest frequency waves are saturated prior to the wind increase, while the 4-5 and 6-7 second waves undergo notable growth. The 4-5 second waves reach saturation approximately 3 hours after the peak in the wind, and the 6-7 second waves reach a peak approximately 5 hours after. $C_d$ increases steadily to a peak approximately 4 hours after the peak in the wind, in between the saturation of the 4-5 and 6-7 second waves.

Figure 2.13 shows the response of the waves and drag to the sudden re-intensification of the wind during the third wind event. While a strong increase of the drag occurs approximately 6 hours after the ramp-up of the wind, similar to the other two events, it is not accompanied by any notable increase in saturation level of any of the three wave frequencies.

In these three periods, there seems to be a nearly-constant time lag of 4–6 hours between the wind speed and the response of the drag coefficient. While it is not obvious that the increase of the drag is consistently linked with any particular wave frequency, the response of the drag is most similar to that of waves with periods between 4 and 6 seconds. The existence of this time lag further underscores the complexity of the wave and wind stress system.

4. Conclusions and Discussion

Several conclusions arise from this open ocean study:
• the drag coefficient, $C_d$, exhibits large temporal variability, which occurs over varied time scales: from fractions of hours to several days.

• the historical drag modeling parameters of wind speed, wave age, and various wave measures do not match variability, despite the fact that each seems to follow the general trends in $C_d$ and at times mirror local peaks and spikes.

• roughness length models are unable to describe the range of variability of $C_d$.

• periods of variability in the downwind and crosswind components of the wind often match periods of variability in $C_d$, indicating that both increasing and turning winds affect surface drag.

• $C_d$ steadily increases and is more correlated to historical parameters during growing seas, but the response of the drag coefficient varies from event to event; the turning rate of the wind appears to be linked to higher drag increase.

• a temporal lag of $O(4)$ hours exists between peak in wind speed and peak in $C_d$; lag and variation of the drag is matched best, but not consistently, by 4–6 second waves: higher frequency waves respond more quickly, lower frequency waves more slowly.

These results suggest that the modeling of the drag coefficient is a far more complicated task than previously assumed. Swell dominated seas add a level of complexity which is not included in the historical formulations of the sea surface drag. Having been based upon spectral similarity found in fetch-limited seas, these models do not have general application; the simplifying assumptions are clearly unjustifiable in the open ocean.

While surface roughness does seem to increase with wind speed (or wave height
or inverse wave age), it does so at a rate and a manner which is apparently highly dependent on the wind and wave history. The resulting effect is that there is not a one to one correlation between a given wind speed, for example, and a given drag coefficient. This, added to the inherent short term variability which appears impossible to model, effectively disables the use of any one of these parameters alone to accurately estimate the surface drag. Other terms which hold temporal and directional information, such as the turning rate of the wind, must play important roles in the determination of the overall surface roughness. These parameters need to be determined and incorporated into the models in order to allow them more universal application. Also, the observed time lag between wind and drag should be further studied; it may hold clues to an improved parameterization.

Drag modeling for the open ocean is clearly more complex than for fetch-limited seas. Fetch-limited or equilibrium conditions provide a far more constant history and therefore result in a more consistent wind and wave dependency. In the open ocean, the solution becomes much more difficult. The existence of waves from different parts of the ocean creates conditions in which the waves often have little to do with the local wind. It is this lack of direct cause and effect which makes open ocean drag coefficient modeling and prediction so difficult. While fetch limited conditions do provide an excellent first start for modeling efforts, open ocean conditions need to be more seriously considered in order to be adapted for use in predictive models. Clearly, these conditions, in which a mixture of waves co-exist, are the most poorly understood; however, these are the conditions which are most common globally and are therefore those of greatest importance.

The overall patterns in this data indicate that increased drag occurs during periods of turning winds and growing seas, particularly when accompanied by large swell.
These periods are also times of probable large standing wave energy: perhaps, standing waves are important to overall surface roughness. Waves traveling downwind are nearly aerodynamically smooth; standing waves, at least conceptually, should represent a more effective roughness surface. As well, standing waves perhaps could be more easily linked to formulations which are used to estimate roughness over the land. This is a subject for future research.

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Figure 2.1. Time series of the (a) wind speed, (b) rms wave and sea heights, (c) inverse wave age, and (d) drag coefficient during the ten days of SWAPP analyzed. General trends of increasing wind speed, wave height, and inverse wave age (decreasing wave age) can be seen to accompany increasing drag. However, large scatter is evident from half-hour period to period. Less scatter occurs during the onset of the three main wind events, shown in the non-shaded regions.
Figure 2.2. Wave height spectra for four periods, each separated by three hours, starting on March 7, 20:00. As winds increase, high frequency sea waves can be seen to grow apart from the low frequency swell. Arrows indicate the low frequency cut-off of the wind wave spectrum as chosen as the peak in the wave velocity spectrum.
Figure 2.3. Drag coefficient versus 10 m wind speed. Large scatter and no correlation can be seen in the data. The mean drag coefficient is equal to $1.21 \times 10^{-3}$. 
Figure 2.4. Non-dimensional roughness length versus wave age. A weak trend of decreasing drag with increased wave age may possibly be seen, in agreement with previous research, which suggested a power law fit with $n=-1$. 
Figure 2.5. Byrne modeled roughness length versus the measured roughness length. The small variability in the Byrne modeled roughness does not match the orders of magnitude of variability observed in the measured roughness length.
Figure 2.6. Variability in the vector wind and drag coefficient. The periods of large variability in the drag coefficient often correspond to those of the vector wind. The matching of the drag's variability with that of the crosswind variability indicates that periods of turning winds are periods of increased drag.
Figure 2.7. The drag coefficient versus 10 m wind speed for the beginning of the three wind events analyzed (circles, stars, and crosses represent the first, second, and third wind events respectively) Significantly higher correlations are seen between these data as compared to the full data set, indicating that during these times, drag increases with increasing wind speed. A significant correlation ($r^2=.46$) for the subset of data which includes the first and third wind events is illustrated by the best-fit regression line.
Figure 2.8. Drag coefficient versus rms wave height for the beginning of the three wind events analyzed (circles, stars, and crosses represent the first, second, and third wind events respectively.) Significantly higher correlations are seen between these data as compared to the full data set, indicating that during these times drag increases with wave height. Moreover, each period individually shows stronger linear correlation, as suggested by the three best-fit lines. In particular, event two shows a stronger increase in drag with wave height.
Figure 2.9. Drag coefficient versus wave age for the beginning of the three wind events analyzed (circles, stars, and crosses represent the first, second, and third wind events respectively) Significantly higher correlations are seen between these data as compared to the full data set, indicating that during these times drag decreases with increasing wave age.
Figure 2.10. Wave height spectra for the beginning and end of the three periods analyzed. Spectra for wind event #1 have been reduced by a factor of 10 and those for event #3 have been increased by 10. It is seen that the second and third events occurred during conditions of strong swell, indicated by the large peaks at lower frequencies. Also noticed is the double-peaked structure of the first and second periods and the resulting strong wave growth of the frequencies between the peaks.
Figure 2.11. (Top) normalized wave energy, (bottom) wind speed, and (bottom circles) drag coefficient during the onset of the first wind event. Wave energies are normalized by the Phillips (1958) saturation spectrum. A distinct temporal lag of about four hours between the peak in the wind and the peak in the drag is noticed. The variation of the drag can be seen to most closely match that of the 4-5 second waves.
Figure 2.12. (Top) normalized wave energy, (bottom) wind speed, and (bottom circles) drag coefficient during the onset of the second wind event. Wave energies are normalized by the Phillips (1958) saturation spectrum. Again, a temporal lag of approximately 4 hours can be seen. The variation of the drag seems to most closely match that of the 6-7 second waves.
Figure 2.13. (Top) normalized wave energy, (bottom) wind speed, and (bottom circles) drag coefficient following the re-increase of wind speed during the third wind event. Wave energies are normalized by the Phillips (1958) saturation spectrum. A temporal lag is again noticed, despite the lack of any noticeable response by the waves.
Table 1. Some of the important wind parameters for the three events analyzed.

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<tr>
<td>Mean $\frac{d\Theta_{wind}}{dt}$</td>
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<td>4.3° / hr</td>
<td>0.8° / hr</td>
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References


Chapter III

Observed Directional Characteristics of the Wind, Wind Stress, and Surface Waves on the Open Ocean

Abstract: Sonic anemometer data were taken during the Surface Waves Processes Program (SWAPP) in March 1990 in the North Pacific. Measurements of the wind stress vector span several strong wind events. Significant angles between the wind stress vector and the mean wind vector are seen. Simultaneous measurements of the directional wave field were made with a surface scanning Doppler sonar. The data suggest that the wind stress direction is influenced by the direction of the surface waves, especially for stronger winds. Overall, the stress vector tends lies between the mean wind and the mean wave directions. At the higher wind speeds (over 8.6 m/s), there is non-zero correlation between the variations in wave directions and stress directions as well. Finally, the stress and wave component directions have similar frequency dependence over the frequency band where wave energy is non-negligible, suggesting a dynamic link.
1. Introduction

The surface flux of momentum, or wind stress, influences all aspects of air-sea interactions. For example, it drives the growth of capillary and surface gravity waves, the development of the mixed layer, and even the large scale circulation of the oceans. An improved understanding of the wind stress vector is of interest to meteorologists, oceanographers, and climatologists alike (Dobson and Toulany, 1991).

Wind stress is often estimated using a "bulk method," in which the stress magnitude is assumed to be proportional to the square of the wind speed:

\[ |\mathbf{\tau}| = \rho C_d U^2, \quad (1) \]

where \( C_d \) is a drag coefficient. In practice, this drag coefficient is taken to depend on atmospheric stability and the height at which \( U \) is measured. The dependence on stability is usually cast in terms of a height dependent stability parameter, \( z/L \), where \( L \) is the Monin-Obukov length (Monin and Obukov 1954, as cited in Businger et al. 1971). Conceptually, \( L \) is a scale height for balancing buoyancy flux against wind stress. In practice, if the stress is to be estimated by the bulk method, so must the buoyancy flux. Corrections for different measurement heights are made assuming a logarithmic profile for the mean wind.

Dependence of the drag coefficient on wave state or surface roughness has also been considered (e.g. Donelan 1982, Hsu 1974, Janssen 1989, Kitaigorodskii and Saslavsky 1974). In most analyses to date, the wind stress is still generally assumed to lie in the direction of the mean wind.
Given high-frequency 3-D wind components, a better estimate of the stress vector is given by

\[ \rho^{-1} \tau = -\langle u'w' \rangle i - \langle v'w' \rangle j, \]

where \( u' \), \( v' \), and \( w' \) are the downwind, crosswind, and vertical fluctuating wind velocities respectively. Direct measurements of the wind stress vector are becoming increasingly common; however, the idea is fairly new, and a large portion of the historical data set is based on bulk formulae.

Somewhat unexpectedly, it has been found that the crosswind component \( \langle v'w' \rangle \) can be non-zero, implying that the wind stress direction is different from that of the mean wind (Geernaert, 1988; Zemba and Friese, 1987). The angle between the stress and mean wind is given by

\[ \theta = \tan^{-1} \left( \frac{\langle v'w' \rangle}{\langle u'w' \rangle} \right). \]

Zemba and Friese (1987) observed large angles between the wind and stress, which they attributed in part to the existence of a coastal jet. Geernaert (1988) attributed 30% of the variance in this wind stress angle to the effect of the heat flux, for the data he considered. He showed that the sign of the angle between the wind stress and the mean wind vectors varied with that of the stability parameter, \( z/L \). The highest correlation with the wind stress angle (0.58) was with a “temperature flux” term, \( U_{10} (T_0 - T_{10}) \), where \( U_{10} \) is the wind speed at 10 m elevation, \( T_0 \) is the sea surface temperature, and \( T_{10} \) is the 10 m air temperature. He suggested this is related to veering of the wind with height, and the redirection of the resulting bursts of momentum. Geernaert (1993) also suggested that some of the remaining variance is due to the influence of the surface gravity wave field, finding that in general the wind stress lies between the mean wind direction and the direction of the long waves.
While Geernaert was able to propose this hypothesis, his data set was limited and did not provide statistically significant results.

Here we consider further the influence of the surface wave directionality on wind stress direction with a larger data set and more extensive statistical analysis. Data were collected during the Surface Waves Processes Program (SWAPP), from February 24 through March 18 1990, on the research platform FLIP. During SWAPP, many instruments were deployed, including two 3-component sonic anemometers to measure the turbulent wind field, and a surface scanning Doppler sonar to measure surface wave directional spectra.

In section 2 the experimental program, data collection, and measurements are described. Data analysis is discussed in section 3. Results are presented in section 4. The effect of swell direction on the wind stress vector is studied, and the analysis is extended to comparisons on a frequency-by-frequency basis.

2. Experimental Program and Data Collection

The motivation for the Surface Waves Processes Program (SWAPP) centers on understanding wave breaking and the interaction between surface waves and the upper ocean boundary layer. For SWAPP, the research platform FLIP was moored about 500 km west of Point Conception, at 35N 127W (Figure 3.1). Measurements of the surface gravity waves, mixed layer structure, and air-sea fluxes were made from FLIP, involving a variety of investigators (Weller et al., 1991).

To measure the surface gravity waves, a specialized surface scanning Doppler sonar was deployed from the hull of FLIP (Figure 3.2; Pinkel and Smith 1987, Smith 1989). Four 195 kHz beams, each having 3 m resolution and reaching to 400 m range, were directed at 45° increments in azimuth. A "quick look analysis" (Smith,
1994) provides accurate directional wave spectra for wave periods between about 2 and 14 seconds. This method provides robust estimates from less than 15 minutes' worth of data, and is most suitable for the present study.

Suitable measurements were made for estimating air-sea fluxes by both bulk and (for stress) direct methods. A vector-averaging wind recorder (VAWR) was mounted atop FLIP (22 m above mean sea level) to measure mean wind velocities. Measurements of air temperature, relative humidity, and barometric pressure were also made. These data were averaged over 56.25 second intervals (1/64 hour), recorded, and used to make bulk estimates of the vertical flux of momentum and heat.

For direct measurement of the vertical flux of momentum, and to assist in estimates of heat and buoyancy fluxes, three-component sonic anemometers were deployed at the ends of both the port and aft booms, at 8 and 6 meters above mean sea level respectively. For example, an estimate of the stability parameter using $<T'w'>$ from a sonic anemometer is given by Large and Pond (1981). Measurements of U, V, and W components of wind, and of sound speed (temperature), were taken ten times per second.

For this study, we use data from the sonic anemometer and accompanying instrumentation on the aft boom. This site was chosen since it was out of the lee of FLIP during the ten days examined here. The estimates of wind stress, velocity, and atmospheric stability all refer to a 6 meter elevation above the mean ocean surface.

3. Data Analysis

The sonic anemometer data were averaged to 2 Hz sample rate to reduce data size. Stress estimates were made every 30 minutes. To verify that the frequencies
important to the stress estimate are included, we examined the integral of the $u'v'$ co-
spectra (Figure 3.3) using data from a few representative segments. Most of the
contribution is from frequencies between 0.01 and 1 (periods of 100 and 1 second
respectively). The error incurred by reducing the data to 2 Hz is about 1% or less.
Conversely, 30 minute averaging times provide more than 15 degrees of freedom at
the lowest contributing frequencies.

A series of selection criteria was applied to the data. First, conditions during
which wind speeds were less than three meters per second are not considered.
Second, to ensure reliable estimates of the angle between the mean wind and wind
stress directions, estimates of the downwind stresses are required to be different from
zero with a statistical confidence of 99%. Third, sheltering by FLIP's structure was
considered. Paulson et al. (1972) found a maximum error of about 5% in wind speed
for anemometers mounted approximately 15 meters from the hull of FLIP. However,
stress measurements need to be analyzed further, and a criterion for selecting
acceptable wind directions is needed. For all data satisfying the first two criteria,
measured angles between the wind and stress directions are plotted in Figure 3.4
against the angle between the wind direction and the aft boom of FLIP (0° on the
horizontal axis corresponds to the boom pointing directly upwind; -90° represents
winds perpendicular to the aft boom). When the sonic anemometer is partially
sheltered by FLIP, large angles between the wind and stress directions are seen,
particularly during low wind speed conditions (circles). These angles are so large as
to be unphysical, with nearly opposing wind and stress directions. Therefore, for low
wind speeds, only conditions with wind directions less than 70° off the boom were
accepted. For higher winds, the criterion was relaxed to include conditions with
relative directions less than 100°.
Finally, effects produced by the motion of FLIP and FLIP's boom were analyzed and found to be insignificant. Tilts and velocities at the sites of the anemometers were estimated using a FLIP motion model, together with accelerometer, fluxgate, and gyroscope (heading) measurements. It was found that the induced wind stress is typically two to three orders of magnitude smaller than the stress estimates from the sonic anemometer data.

Figure 3.5 shows wind speed, downwind and crosswind stresses, wind and wave directions, and FLIP's heading for the portion of the observation period used in this study. Three significant wind maxima occurred during this period. In the first event, the wind picked up quickly from calm conditions (including little swell), and veered only slightly in direction over the course of 30 hours. The second and third events were accompanied by varying sea conditions, with swell present at varying angles from the wind direction.

4. Results

One current hypothesis is that the surface gravity wave spectrum influences the direction of the mean stress vector by creating an anisotropic roughness field at the surface. This effect has been attributed to the redistribution of energy and slope density of the short waves, which are theorized to be the significant supporters of momentum flux from the atmosphere to the ocean (Byrne, 1982; Geernaert et al., 1986). The long waves strain the short ones, turning them (and hence the wind stress) toward the direction of the long waves or swell (Geernaert, et al., 1993). Here, we shall also consider hypotheses that this influence varies with wind speed (overall surface roughness has been theorized to increase with wind speed (Charnock, 1955)), and that the influence can be seen in a detailed examination by frequency.
4.1 Stress Direction versus Swell Direction

Figure 3.6 shows a plot of angles between the stress and wind directions (or "stress angles") versus angles between swell and wind directions (or "swell angles"). All directions are calculated using the oceanographic convention (toward). Data are plotted only for neutral stability (-.01<z/L<.01, z=6 meters), to reduce the buoyancy flux effect described by Geernaert (1988). The data are further divided into high wind speed and low wind speed regimes, using the median observed wind speed of 8.6 m/s (yielding 58 samples in each regime). The data were already selected for winds above 3 m/s, and only one case lies below 5.8 m/s, so the low wind interval is roughly 6 to 8.6 m/s. The high wind interval is roughly 8.6 to 12 m/s (all at 6 m elevation). If the stress vector has a linear bias toward the swell direction, the overall trend would be from lower left to the upper right, through the origin. The centroid of the low wind data points is consistent with this, but there is no significant trend in the variations about the centroid. The high wind data support this in both mean and trend: for the high wind cases, the variations in stress angle about the mean is correlated with the corresponding variations in swell angle. The correlation squared is $r^2=0.21$, which is well above the 95% significance level of 0.05 (from 58 samples). In contrast, the low wind data yields a correlation squared of only $r^2=0.01$, which is not significantly different from zero. To illustrate this correlation, a line is drawn through the centroid of the high wind cases along the major axis of the joint variations. Error bars are provided by drawing dashed lines parallel to the major axis but displaced up and down by the RMS distance of the data from this line.

Caution should be exercised in deriving meaning from overall mean values, since these may be determined by effects such as the large scale structure of the wind
field. However, the relatively good correlation between stress and swell angles for
the high wind cases lends support to the hypothesis that the swell direction
influences the stress.

4.2 Wind Speed Dependence

There is a slight suggestion in Figure 3.6 that the low wind speed cases (crosses)
lie lower in the plot than the high wind speed cases, on the average. To investigate
this further, we divide the data into three bands of swell angles: -45° to -35°, -35° to
-25°, and -25° to -15°. This should reduce the effect of varying swell directions on
the direction of the stress, and so bring to light any modulation of this influence by
the strength of the wind. Figure 3.7 shows the stress angle versus wind speed for
these fixed bands of swell angles. The data suggest a dependence on wind speed for
the middle band of angles, -35° to -25°. The correlation squared is 0.455, which is
well above the 95% confidence level of 0.092 for an estimate based on 31 points.
However, no significant correlation is seen in either the -45° to -35° band or the -25°
to -15° band: the measured correlations squared are 0.072 (from 29 samples) in the
high swell angle band, and 0.069 (from 23 samples) in the lowest swell angle band.

4.3 Frequency Analysis

In the above, the turning of the stress vector from the wind direction was
compared with a “peak wave” or “swell” direction. Here, we pursue spectral
descriptions of the stress and waves, hoping to shed light on further interdependencies. The direction of the stress as a function of frequency is estimated as
\[ \theta(f) = \tan^{-1}\left( \frac{C_{v'w'}}{C_{u'w'}} \right) \]

where \( C_{v'w'} \) and \( C_{u'w'} \) are the co-spectra (the real parts of the cross-spectra) between the crosswind and vertical fluctuating velocities and the downwind and vertical fluctuating velocities, respectively. The wave directions are from the "quick analysis" of Doppler sonar data (Smith, 1994). Figures 3.8 and 3.9 show plots of the directions and magnitude of the stress and surface gravity waves versus frequency. The two time periods (1000-1600 UTC March 8, and 0800-1400 March 11, 1990) were chosen because each has a nearly constant angle between the swell and wind directions, and each occurred during periods of near neutral stability. The March 8 period had nearly constant swell and wind directions over the course of 6 hours, while the March 11 period had slow turning of both the wind and swell directions. Both periods comprise several hours, lending statistical confidence to the results. Horizontal to vertical velocity co-spectra were calculated using 52 minute segments, which were then averaged together to form estimates of the stress angle versus frequency for each of the two periods. Wave directional spectra were formed from 12 minute segments and averaged over the same time periods to obtain wave directions. For display, a least-squares-fit fifth-order polynomial is drawn through the data, regressed against the logarithm of frequency. This effectively averages the data over logarithmically increasing intervals as the frequency increases. Only the data within the frequency band containing significant wave energy is included in the fits (0.05 to 0.5 Hz).

For the March 8 case, the wind is directed toward 193°, the net stress direction is toward 182°, and the swell direction is toward 140°. Thus, the stress direction lies between the swell and wind directions, as expected. Note that the higher frequency waves are more closely aligned with the mean wind direction. This is also as
expected, since the higher frequency waves have a quicker response time to turning winds (Masson, 1990), and are locally generated. Somewhat unexpectedly, the variation with frequency of the stress direction mimics that of the surface gravity waves: the direction of the stress in the low frequency "swell regime" is aligned more closely with the direction of the waves at that frequency, and the direction of the wind stress at the high frequency "sea regime" is more closely aligned with the waves at those frequencies. (The scatter of points about the 5th order fit lines is a reasonable indicator of the statistical variability of the directional estimates.) This suggests that the fluctuations in the wind field at each frequency may be coupled to the fluctuations in the sea surface at the same frequencies.

The March 11 case exhibits similar results (Figure 3.9). Again, the stress direction lies between the wind and swell directions (132° versus 163° and 132°, respectively). However, in this case the stress is aligned more closely with the swell. Examining the directions versus frequency, a closer alignment of the wave and stress directions is seen. Surprisingly, the directions of the stresses lie even farther from the wind than the wave directions, over the whole "wave band" of frequencies. Again, the scatter of points about the polynomial fit is a good indicator of the statistical variability. As indicated in the lower portion of Figures 3.8 and 3.9, most of the energy in the wind stress is contained in frequencies lower than those of the waves; however, there is strong evidence of an additive stress at the peak of the wave spectrum. Direct correlations between wind and wave components have been shown previously (e.g. Dobson, 1971; Elliott, 1972). Here, we are not showing direct correlations, but just a similarity in their directionality and spectral shapes.
5. Summary

Wind and wave data from SWAPP are used to explore relationships between the directions of the wind stress, mean wind, and wave field. These comparisons suggest that the direction of the wind stress vector is influenced by the direction of the surface gravity waves. The wind stress vector generally lies between the wind direction and the direction of the long waves, as found also by Geernaert (1993). For winds over 8.6 m/s, there is a significant trend in the observed stress angles versus swell angles, in a sense consistent with such an influence. Finally, the influence of the waves on the wind stress direction can be seen as a function of frequency: the stress component directions as a function of frequency mimic the wave component directions, with both deviating from the mean wind.

The tracking in frequency of the wave and stress directions, in particular, suggests a dynamic link between the waves and stress. We hope to explore this further in future work, including data from different wind and wave conditions.

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part by the California Resources Agency. The U.S. Government is authorized to reproduce and distribute for governmental purposes.
Figure 3.1. Location of the Surface Waves and Processes Program (SWAPP). The research platform FLIP was moored approximately 500 km west of Point Conception at 35N, 127W. Contours in meters.
Figure 3.2. Schematic view of R/P FLIP during SWAPP. Instruments deployed included: (A and B) high frequency sonic anemometers; (C) vector-averaging wind recorder (VAWR) and (D) surface scanning Doppler sonars.
Figure 3.3. The cumulative sum of $u'w'$ co-spectra from high to low frequencies, from a typical sonic anemometer data run. Virtually all energy is contained between frequencies of .01 and 1 (periods of 100 and 1 seconds respectively).
Figure 3.4. The angle between mean wind direction and the wind stress direction versus the wind direction relative to the aft boom of FLIP (0° means the aft boom points upwind). For cases in which the sonic anemometer is at least partially in the lee of the FLIP superstructure (particularly for low wind speeds), large (unphysical) absolute values of the angle between the mean wind and stress directions are seen.
Figure 3.5. Wind speed, downwind and crosswind stress, and mean wind and wave directions along with FLIP heading, during the ten day period of the SWAPP cruise used in this study. All wind measurements were made at 6 meter elevation. Three significant wind events can be seen starting on March 4, 7, and 10.
Figure 3.6. Angles between the stress and wind directions versus angles between swell and wind directions. Almost all points are in the third quadrant, indicating a tendency for the stress vector to align toward the swell direction from the wind direction. The data are divided into high wind (circles) and low wind (crosses) regimes. The major axis of the joint variations for the high wind cases only is illustrated by the straight line; the corresponding correlation squared is 0.21 (for the variations about the mean values). Error bounds (RMS distance from the line) are indicated by dashed lines.
Figure 3.7. Stress angle versus wind speed (measured at 6 m height) for cases in which the swell angles are between -45° and -35° (top), between -35° and -25° (middle), or between -25° and -15° (bottom). A statistically significant trend with wind speed is seen in the middle plot.
Figure 3.8. Direction (top) and magnitude (bottom) of stress and waves versus frequency for March 8, 1000-1600. Over a band of frequencies containing the waves (0.05 to 0.5 Hz), a 5th order polynomial was fit to the directions against the log of the frequency (solid line). This line effectively averages the data over logarithmically spaced intervals. The directional variations of stress and waves versus frequency are similar. Note that the low frequency stresses also lie between the wind direction (193°) and the direction of the long waves (140°). Waves with frequencies greater than .24 Hz have component wave ages less than 1.
Figure 3.9. Direction (top) and magnitude (bottom) of wind stress and waves versus frequency for March 11, 0800-1400. The curve and averaging are as for Figure 3.8. The variations with frequency of the wind stress and wave directions are again similar. In this case, the low frequency stresses are closely aligned with the long waves (near 132°). Waves with frequencies greater than .19 Hz have component wave ages less than 1.
References


The text of Chapter III, in full, is a reprint of the material as it appears in the
Journal of Geophysical Research. The dissertation author, Karl F. Rieder, was the
primary researcher. The co-authors listed in that publication, J. A. Smith and R. A.
Weller, directed and supervised the research which forms the basis for this chapter.
Chapter IV

Some Evidence of Co-Linear Wind Stress and Wave Breaking

Abstract: Data collected during the Surface Waves and Processes Program (SWAPP) are employed to investigate a possible interrelation between wind stress and surface wave breaking. From comparison of data from 15 half-hour long time segments, the directions of the wind stress and the whitecap motion are observed to be generally co-linear, with both lying between the mean wind and the swell. As well, a non-dimensionalized whitecap speed is found to correlate with the drag coefficient. These results suggest that the magnitude and direction of the wind stress might be estimated from wave breaking information.
1. Introduction

Energy for the growth of surface waves is provided by shear flows in the overlying atmospheric boundary layer. Recent work has shown that these shear flows are also altered by the underlying surface gravity waves; hence, the two are coupled (Geernaert 1990). In particular, wave direction has been shown to relate to the direction of the wind stress, which, in turn, does not necessarily align with the mean direction of the wind. The wind stress often veers toward the direction of the oblique swell (Geernaert et al. 1993, Rieder et al. 1994).

It has been found in laboratory experiments that the existence of wave breaking has a significant effect on the atmospheric boundary layer as well (Banner and Melville 1976, Melville 1977). Breaking waves are the location of air flow separation from the water surface and hence increase the phase shift between the air pressure at the surface and the wave elevation. Thus, wave breaking gives rise to significant "bursts" of momentum transfer (Banner 1990). An issue which has not been addressed in the lab, however, is whether the wind stress might relate to the direction of wave breaking versus the direction of the wind. This directional issue adds another dimension to the already perplexing uncertainty in estimating the stress magnitude.

A combination of data was collected during SWAPP (the Surface Waves and Processes Program), including measurements of the wind stress, surface waves, and wave breaking. These provide an opportunity to look into possible relations between breaking waves and wind stress. The surface wave and wave breaking data were previously analyzed by Ding and Farmer (Ding and Farmer 1994). They found that the mean direction of whitecap motion was generally aligned between the wind and waves; for cases in which the wind direction was significantly different from the direction of
the waves, wave breaking was more closely aligned with the wind. Also, various measures of the degree of whitecapping (breaking event spacing, speed, dimension, and duration and active acoustic coverage) increased with wind speed. Here, we introduce eddy correlation measurements of the wind stress, allowing us to address more directly interrelations of the wind, waves, breaking, and stress.

2. The Experimental Program and Data Analysis

SWAPP was designed for studies of inter-relations between the wind, the heat flux, the waves, and the upper most layers of the ocean (Weller et al. 1991). Data were collected from February 24 through March 18, 1990, from the research platform FLIP. The site was approximately 500 km off Point Conception, California, in 4000 m of water. Measurements of the surface gravity waves, mixed layer structure, and air-sea fluxes were made. Other measurements, including those used to estimate wave breaking directions, were made from the Canadian Vessel CSS Parizeau.

From FLIP, a four-beam, 195 kHz Doppler sonar system was used to estimate the directional wave spectra. A "quick-look" analysis of the measured surface velocities provides estimates of the first three Fourier components every 12.5 minutes within the frequency range of .05-.5 Hz (Smith and Bullard 1995).

Two sonic anemometers were deployed on the aft and port booms of FLIP at 6 and 8 meters above mean sea level respectively. These measured the three components of wind speed and air temperature ten times per second, permitting calculation of the wind stress by the eddy correlation method:

\[
\bar{\tau} = -\rho \left[ \langle u'w' \rangle \hat{i} + \langle v'w' \rangle \hat{j} \right],
\]

(1)
where $\rho$ is the water density and $u', v', w'$ are the downwind, crosswind, and vertical fluctuating wind velocities. The sonic anemometer data were averaged and decimated to 2 Hz and wind stress estimates were made every 30 minutes.

Air and sea temperatures and humidity were used to determine atmospheric stability (Large and Pond 1981). Atmospheric stability has been shown to influence the direction of the wind stress (Geernaert, et al. 1993). For this study, only near-neutral conditions are considered, to separate the sea state influences from stability effects. Conditions are considered near-neutral here if $|z/L|<.05$, where $z$ is the measurement height and $L$ is the Monin-Obukhov length, a scale height for balancing buoyancy flux against wind stress.

To ensure well defined wind and wind stress directions, the sonic anemometer data were required to satisfy several quality controls, as in Rieder et al. (1994): wind speeds of less than 3 m/s were not considered; estimates of the downwind stresses were required to be different from zero with a statistical confidence of 99%; and conditions during which the superstructure of FLIP sheltered or shadowed the sonic anemometer were removed. Only data from the sonic anemometer on the aft boom ever passed this last criterion.

The effect of sheltering is illustrated in Figure 4.1. The directions relative to the aft boom of the wind (circles) and wind stress (crosses) are plotted versus the relative wind direction as given by a vector-averaging wind recorder (VAWR), mounted atop FLIP and not influenced by the superstructure. (Zero degrees indicates that the sonic anemometer is directly upwind of FLIP). For wind directions such that the sonic anemometer was even partially in the lee of FLIP, unphysical stress directions are calculated. It is interesting to note that the stress direction is more sensitive to sheltering than the wind direction, which appears reasonable for all cases shown here.
The motion of FLIP was analyzed and found to be insignificant in producing errors in the wind stress calculation. For example, vertical and horizontal root mean square velocities of the sonic anemometer due to motion of FLIP are estimated at .03 and .15 m/s respectively, for a representative sample period (March 11, 20:29). However, the tilt and rotation of FLIP are independent, and the induced vertical and horizontal velocities are poorly correlated (correlation coefficient of \( r^2 = .03 \)). The resultant wind stress contribution is estimated to be \( 8.8 \times 10^{-4} \) N/m\(^2\), more than two orders of magnitude smaller than the measured wind stresses.

Wave breaking data were collected and analyzed by Ding and Farmer (1994) from the Parizeau, stationed in the vicinity of FLIP. A drifting array of four hydrophones of span 8.5 m was deployed at 20 m depth to measure ambient sound produced by large scale spilling breakers. Cross-correlations between hydrophones allowed construction of two-dimensional images, from which instantaneous locations of individual whitecaps were determined. These breaking events were tracked through time, and breaking speed, direction, duration, and spacing were estimated. Mean values of these parameters were calculated over half hour periods from between 300 and 500 breaking events (L. Ding, personal communication, 1994).

Since each of these data sets (waves, wind, and breaking) were collected independently, it was necessary to find the time periods during which all of the measurements were made, and in which all data passed the selection requirements. Of the 23 half hour time periods in which the wave breaking data are available, three had no sonic anemometer data, three failed the statistical confidence criterion, and one had sheltering of both sonic anemometers behind FLIP's superstructure. One of these also failed the near-neutral stability criterion. The remaining 15 time periods are shown in Figure 4.2. The 6 m wind speed and wind and wave directions are plotted during the
10 day period in which the sonic anemometers were operational. The lines indicate the
15 periods for which all data were reported.

3. Wind Stress and Wave Breaking

Figure 4.3 shows the directions of the mean wind, wind stress, swell, and wave
breaking for the 15 available half hour time periods (numbered chronologically). The
breaking direction is the mean direction of all the breaking events (velocity weighted)
for the period. The breaking event directions have a well-defined mean direction due to
the large number of independent events (300-500 each half hour). For the 15 cases
analyzed here, mean breaking event directions could be determined to 99% confidence
within ±1° on average and ±3° at worst (c.f. Zar 1984). Similarly, mean stress
directions were determined typically to ±1° and at worst to ±2°. Stress direction errors
were made by first calculating the 99% confidence limits in the wind stress component
perpendicular to the measured stress direction. The errors were then defined as the arc
tangent of the ratio of these limits to the magnitude of the wind stress. Directions are
plotted with the oceanographic convention, so 180° (for example) represents wind
directed toward the South. As shown in Figure 4.3, the wind was directed
predominantly toward the SSE, and the swell to the ESE, with the two separated by 10
to 50°.

For all time periods but number 8, the wind stress was aligned between the mean
wind and the swell, as reported by Rieder et al. (1994). For all but the eighth and ninth
time periods, the mean direction of wave breaking was also between wind and swell.
When there was a big angle between the wind and swell (time periods 1-7, 14, and 15)
both the wind stress and wave breaking were more closely aligned with the mean
wind, whereas for small angles (time periods 8-13) the directions were more varied.
The overall mean wind stress and wave breaking directions are very well aligned (Figure 4.3, far right). The mean directions exhibit approximately $37^\circ$ between the wind and swell, $11^\circ$ between the wind and wind stress, and only $2^\circ$ between the stress and wave breaking directions.

If we exclude time period 8, we find even better agreement between the mean wind stress and wave breaking directions, aligned within the accuracy of the measurements. Additionally, the mean square difference between wave breaking and wind stress directions is significantly reduced, from 132 to 75 deg$^2$. This can be compared to the mean square difference between wind and wind stress directions: 147 and 144 deg$^2$, including and excluding time period 8 respectively. The few hours surrounding period 8 are characterized by exceptionally large variability in the directions of wind, wind stress, and even waves. This might explain the anomalous results for this case.

With direct stress measurements, we can investigate also how the magnitude of the wind stress varies with wave breaking. Ding and Farmer (1994) showed that the degree of breaking, as measured by breaking event speed, dimension, spacing, or duration, increases with wind speed. Stress also increases with wind speed, as reflected in the concept of the drag coefficient: $C_d$ is the coefficient of proportionality between the total wind stress magnitude and the square of the wind speed (this definition is consistent with inertial dissipation-method results). Thus, to reduce the overlap of information, we look for correlations between suitably normalized wave breaking parameters and the drag coefficient. Figure 4.4a shows the measured 6 m drag coefficient versus non-dimensional mean breaking event speed, defined as the mean whitecap speed divided by the 6 m height wind speed. A suggestion of a trend is seen in the data; however, the correlation coefficient squared of $r^2=.18$ is below the
95% confidence level of $r^2=.26$, based on the sample size of 15. However, if we exclude the anomalous time period 8, depicted by gray circles in Figure 4.4, the correlation coefficient squared increases to $r^2=.33$, well above the 95% confidence level. We also investigated breaking duration, dimension, and density; these display no stronger correlations with the drag coefficient. For comparison, the drag coefficient is also shown versus the 6 m wind speed (Figure 4.4b) and versus wave age (Figure 4.4c). The wave age is a measure of the state of development of the seas. In order to avoid a spurious correlation with friction velocity, the wave age is defined here as the ratio of the phase speed of the peak sea frequency to the 6 m wind speed. Even though wind speed and wave age are the most commonly used parameters to model variations of the surface roughness, neither are found to correlate with the drag coefficient here ($r^2<.11$), either including or excluding time period 8. Using just the downwind stress component in the calculation of the drag coefficient (as opposed to the total magnitude of the wind stress) slightly reduces the correlation with the non-dimensional event velocity.

An alternate formulation is to ask whether wind stress can be estimated as well or better using just breaking event speed, as compared to using just wind speed. Indeed, from this limited data set, we find a higher correlation of the wind stress with the square of the breaking event speed ($r^2=.69$) than with the square of the wind speed ($r^2=.42$). Testing whether these two correlations are statistically different from each other, we find (with the usual assumptions of normality, etc.) about two to one odds that breaking event speed does provide better estimates of the wind stress magnitude than wind speed, based on this small sample.
4. Discussion

Ding and Farmer (1994) noted that the wave breaking showed a general alignment between the wind and the waves. We now see that the wave breaking direction is more closely aligned with the wind stress direction than with either the wind or the waves alone. Also, Ding and Farmer showed correlations between measures of wave breaking intensity and wind speed; here we note a correlation with the wind stress, as well.

The overall alignment of wind stress and wave breaking directions and the correlation of the drag coefficient with a non-dimensional breaking event speed suggest a connection between wave breaking and the wind stress. However, it does not suggest cause versus effect. On one hand, the input of momentum into the sea surface in a given direction may lead to preferential energy dissipation and wave breaking in that direction. As a large majority of the momentum transferred from the wind eventually goes to wave breaking, it should not be surprising for the wind stress and whitecapping to be aligned. On the other hand, the existence of wave breaking may produce a directional roughness effect, and hence alter wind stresses, for example via the mechanism described by Banner (1990). Finally, the observed alignment between the wind stress and wave breaking may be due to independent influences of the wind and waves on both these directions, and there may not be any direct causal relationship between them. We emphasize that these results are purely observational, and that the comparison with (e.g.) the works of Banner and Melville is only suggestive.

The alignment of stress and breaking directions is also interesting in light of previous findings. Geernaert et al. (1993) and Rieder et. al. (1994) showed that the wind stress direction was influenced by the wave directions across a wide range of
frequencies. While the spectral comparison of wave and wind stress directions addresses a continuous, "quasi-linear" interaction between the wind and wave fields, the observed alignment of the wind stress and the wave breaking addresses an interaction which is episodic, and conceptually highly non-linear. This draws attention to both the wide scale and varied nature of wind-wave coupling.

As a final note, these results suggest the possibility that the wind stress might be inferred from a sub-surface, passive acoustic array: the magnitude and direction is obtained by finding the average speed and direction of wave breaking events. Such a system may offer some advantages over current buoy-mounted systems, which are susceptible to motion-induced contamination, weathering, and encounters with surface vessels: the measurements may be carried out from a subsurface buoy or, in shallow water, from the sea floor. The technological development this would entail is worth considering.

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Figure 4.1. Sonic anemometer measured (o) wind and (+) wind stress directions relative to the aft boom versus relative wind directions as measured by a vector-averaging wind recorder (VAWR) mounted atop FLIP (0 degrees indicates the sonic anemometer is directly upwind of FLIP). A strong influence on the wind stress direction can be seen for cases in which the sonic anemometer was even partially in the lee of the superstructure.
Figure 4.2. Wind speed and wind and wave directions during SWAPP. Lines indicate half hour time periods in which good wind stress, wave, and wave breaking data were all available. These 15 periods occurred only during the second and third wind events of the ten day period.
Figure 4.3. Mean wind, wind stress, wave breaking, and swell directions for the 15 half hour periods analyzed. Directions follow the oceanographic convention, e.g. 180 indicates southward motion. For 13 of the 15 periods, both the wind stress and wave breaking are aligned away from the direction of the mean wind and toward the long period swell.
Figure 4.4. Wind stress magnitude versus (a) dimensionless breaking event speed, (b) 6 m wind speed, and (c) wave age for the 15 half hour periods analyzed. Gray circles represent anomalous time period 8. A trend of increased drag with increased dimensionless event speed is seen.
References


The text of Chapter IV, in full, is a reprint of the material as submitted to the
Journal of Physical Oceanography. The dissertation author, Karl F. Rieder, was the
primary researcher. The co-authors listed in that publication, J. A. Smith and R. A.
Weller, directed and supervised the research which forms the basis for this chapter.
Chapter V

Summary / Conclusions
The nature of the wind stress on the open ocean was the subject of this dissertation. Data taken in the North Pacific, 500 km off the California coast, have shown that surface gravity waves have a profound influence on both the magnitude and direction of the wind stress. Long period swell, short period wind waves, as well as wave breaking were seen to have an effect on the momentum flux between the atmosphere and the ocean.

In Chapter II, traditional parameterizations of the wind stress magnitude were studied. For this open ocean data set, sea surface drag varied directly with wind speed, inverse wave age, and wave height, but in a manner highly dependent on the wind / wave history. In the beginning of three wind events studied, as waves were being actively generated, a strong correlation between the drag coefficient and these parameters was found. However, as the wind speed leveled off or began to fall, large variability and no correlation was evident. The rate of increase of the drag coefficient during each of the three events was more closely linked to the turning rate of the wind as compared to any other variable that was inspected. Also, a time lag on the order of several hours between the rise in the wind and the subsequent rise in the drag coefficient was apparent. These results suggest that temporal and directional effects of the wind and waves must be significant factors in the creation of the sea surface roughness. It is the clearly observed complexity of the wind stress system that makes accurately modeling the drag coefficient so difficult.

The relationship between wind stress and wave directions was studied in Chapter III. The direction of the wind stress was clearly influenced by waves across a wide range of frequencies. In general, the wind stress vector tended to lie between the direction of the mean wind and that of the long waves, or swell. Moreover, during
high winds, a significant correlation between the directions of the wind stress and swell was found, suggesting a causal relationship between them. An influence of the wave direction on the wind stress could be seen on a per-frequency basis as well: the directions of the periodic fluctuations in the atmospheric boundary layer which contributed to the wind stress were seen to qualitatively match those in the directional wave field. These results suggest the existence of a dynamic link between the fluctuations at the sea surface and those in the overlying wind field. The modulation and organization of the air flow by the surface gravity waves provides a means by which the magnitude and direction of the wind stress may be altered.

Chapter IV looked at a possible link between the wind stress and wave breaking. For cases in which wave breaking data (whitecap speed, direction, duration, and dimension) were available, the wind stress was found to be better aligned to the direction of wave breaking than with either the wind or the waves. As well, it was suggested that measures of wave breaking intensity, namely breaking speed, perhaps could act as better estimators of the wind stress magnitude than more traditional parameters such as wind speed. As wave breaking is episodic, this observed relation to wind stress is conceptually nonlinear, in contrast to the linear relation observed in the spectral comparison of wind stress and wave directions of the previous chapter. This draws attention to the complex nature and varied scales of the interactions between wind and waves.

This research has clearly shown that the roughness of the sea surface is primarily due to waves and wave breaking, affecting both the magnitude and direction of the momentum flux from the atmosphere to the ocean. The different modes and scales of this interaction and the influence of many factors (including their magnitude, direction,
and temporal history) attest to the complex nature of the relationship between the waves and the wind stress.

For open ocean conditions, current parameterizations, which match surface roughness to wind speed, wave age, or non-directional wave information, have not been able to provide accurate estimates of the wind stress vector. To address the non-equilibrium conditions of a swell dominated region, future modeling efforts will need to address the role of waves across a wide range of frequencies and to include directional and temporal characteristics of both the wind and the waves. The effect of turning or increasing winds and the mis-alignment between wind and waves are but two of the important issues that should be addressed. New, innovative measurements (such as wave breaking speed and direction) may prove to be most useful, providing a distinct approach to estimating the wind stress. Since many geophysical models are sensitive to wind stress input, it is of high importance to make accurate stress estimates. This study has suggested that the errors resulting from the use of indirect measures and current parameterizations are not random, and therefore put bias into the models predicting weather, climate, and ocean circulation.

The mis-alignment of the mean wind and the wind stress substantially complicates the estimation of the wind stress from indirect measures and brings into question some of the basic assumptions of boundary layer modeling. By the balance of forces acting in the atmospheric surface layer, the wind stress is forced to be aligned with the geostrophic wind above; the mean wind turns in accordance with the drag boundary condition to maintain this alignment. It is the surface gravity waves which provide the directionally anisotropic drag. By reinvestigating the problem without the a-priori assumption of mean wind and wind stress alignment, we may both better understand the underlying physics and be able to more accurately estimate the wind stress vector.
Open ocean, swell-dominated conditions provide the most difficult environment for estimating the wind stress, but since these conditions are those that are most common globally, they are of greatest interest to modelers. Before many of the questions raised in the dissertation can be answered, more extensive research programs will need to be carried out. These programs will need to make a wide, inclusive set of measurements such that all factors, including wind, waves, and wave breaking, may be addressed.
Appendix

Wave Induced Motion of FLIP

Abstract: The value of data gathered from R/P FLIP is enhanced if the motion of FLIP is described. At surface wave frequencies, FLIP’s motion can be estimated from measurements of apparent acceleration and heading. This minimal set of measurements is often available in past data sets (e.g., from the Surface Wave Process Program, or SWAPP). In addition, the accelerometer data is often of superior quality, with less noise and greater dynamic range than other available data. The challenge is to partition the motion between true horizontal acceleration and tilt. For this, a quasi-linear dynamic model of FLIP’s response to forcing by surface waves is developed, including added mass and drag terms. The model is refined on the basis of comparisons with more extensive motion data, gathered over a variety of wind and wave conditions: estimates of FLIP’s tilt from measurements of the earth’s magnetic flux vector, combined with gyro-compass heading, and horizontal velocity at 35 m estimated from surface-scanning Doppler sonars. The overall agreement between model estimates derived from accelerometer data and the others is good.
1. Introduction

The research platform FLIP (Floating Instrument Platform) is a 108 m long vessel, which "flips" to a vertical orientation on station. When vertical, it provides a stable platform from which to make oceanographic measurements (Figure A.1). For example, as part of the Surface Wave Process Program (SWAPP), FLIP was moored at 35°N, 127°W, roughly 500 km off the coast of California, from February 24, 1990 through March 18, 1990. The objective of this program was to measure surface waves and wave-related flows in the mixed layer, and to relate these to forcing by buoyancy fluxes and wind stress (Weller et al., 1991).

Although FLIP's motion is generally small (e.g., a few degrees of tilt), knowledge of this motion could help in several ways. During SWAPP, for example, specialized Doppler sonars were deployed to measure the surface water velocity along several lines radiating from FLIP. These permit estimation of both directional surface wave spectra and lower frequency motions at the surface (Smith, 1992; Smith and Bullard, 1995). The directional surface wave estimates could be improved if FLIP's response to the waves could be incorporated. FLIP's motion also affects wind measurements made with sonic anemometers and surface elevation measurements made with resistance wires. The wind measurements are used to calculate the wind stress, or momentum flux from the atmosphere to the ocean, which provides energy for the growth of waves and the development of surface currents. The surface elevation measurements provide independent estimates of the surface waves, which are used for comparison with the sonar results. Analysis of FLIP's motion itself may in theory also provide estimates of directional surface wave spectra, essentially using FLIP as a tilt-and-roll buoy.

A model for FLIP's motion is derived by analysis of the important forces acting on the hull, and the solution of the corresponding equations for translation and rotation. With this model, a three axis accelerometer (located at 35 m depth) together with FLIP's heading (measured by a gyroscope) are sufficient to estimate FLIP's motion. The accelerometers measure a combination of true acceleration plus a component of gravity due to tilt. The model dynamics partition the motion between tilt and acceleration through definition of a frequency dependent center of rotation, assuming forcing by linear deep water surface gravity waves.

At times during SWAPP and previous experiments, only accelerometer and heading data are available for estimating the motion of FLIP. At these times, the dynamic model of FLIP's motion is needed to separate the effects of tilt and acceleration. At other times, more data are available: three-axis "fluxgate" measurements (of the magnetic vector due to the earth's magnetic field), wave wire
measurements (of surface elevation), and Doppler sonar measurements of radial velocity along the surface. The additional measurements provide a basis for comparing and testing the dynamic model.

The motion of FLIP has been studied previously (Regier, 1975; Rudnick, 1967). However, we felt it worthwhile to re-derive the model, to provide results in forms appropriate to the available data, and also to explore the importance of some additional effects. New features of the analysis include treatment of the inertial mass of the water flowing around FLIP (added mass), and of skin friction effects (through a quasi-linear drag term). Inclusion of these effects may improve the performance of the model.

Several comparisons are used to test and refine the model. First, an empirical center of rotation is estimated (from a combination of accelerometer, heading, and fluxgate measurements) and is compared with the model derived center of rotation versus frequency. This comparison allows for tuning of the model. Second, the accelerometer-derived tilts of FLIP are compared to those calculated from fluxgate and heading data. Finally, the velocity at the sonar depth as estimated by the model is compared to the range averaged sonar velocity.

2. FLIP Motion Model

2.1 Formulation

For simplicity, consider first the 2-D case, with wave forcing in just one vertical plane \((x,z)\). Extension to the 3-D case is straightforward. We take \(x\) to be horizontal, and \(z\) vertical, increasing upwards. The horizontal motion of FLIP is described by the position of the center of mass, \(x_f\), and angle of tilt from vertical, \(\theta\) (assumed to be small). In general, the vertical motion of FLIP is less important, and is more easily estimated (e.g., by integration of the vertical component of acceleration; see supplement). The equations for translational and rotational motion are:

\[
\ddot{x}_f = \frac{F}{M} \tag{1}
\]

\[
\dot{\theta} = \frac{T}{I}. \tag{2}
\]

Here \(F\) and \(T\) are the total force and torque on the body, respectively, and are discussed below. \(M\) and \(I\) are the total mass and the pitching moment of FLIP, respectively, given by:
\[ M \equiv \int_{-H}^{+h} m(z)dz = \int_{-H}^{0} \rho A(z)dz, \quad (3) \]
\[ I \equiv \int_{-H}^{h} (z-z_f)^2 m(z)dz = \gamma_f^2 M, \quad (4) \]
where the depth of FLIP's center of mass, \( z_f \), is
\[ z_f \equiv M^{-1} \int_{-H}^{h} zm(z)dz. \quad (5) \]
These correspond to the zeroth, second, and first moments of the vertical mass distribution of FLIP, respectively. In the above, \( H \) is the extent of FLIP in the vertical below the water line, \( h \) is the extent above water, \( A(z) = \pi r^2(z) \) is the cross-sectional area of FLIP versus depth, \( m(z) \) is the mass distribution of FLIP in the vertical, and \( \gamma \) is known as the radius of gyration. FLIP's total mass \( M \) is equal to the displaced water mass; the radius versus depth is specified,\(^1\) so the displacement (and hence \( M \)) is well known. However, the center of mass \( z_f \) and the radius of gyration \( \gamma \) are higher moments of the distribution of mass, \( m(z) \), which are progressively more sensitive to the details of how FLIP is loaded. In general, these are not well known, so adjustments of \( z_f \) and \( \gamma \) are allowed, with some constraints; this is discussed below.

The forcing and motion are more easily described in the frequency domain, so 1 and 2 are Fourier-transformed in time. We adopt the convention of allowing only positive frequencies, so that wave direction is uniquely described by a vector wavenumber. Thus, for example, the time-series of displacement is recovered from the (hatted) frequency coefficients by
\[ x_f(t) = (2/\pi)^{1/2} \int_{0}^{\infty} \hat{x}_f(\omega)e^{-i\omega t} d\omega. \quad (6) \]
We now drop the hats, and consider \( x_f, \theta, F \) and \( T \) to be functions of \( \omega \).

The significant forces acting on FLIP include: the pressure gradient imposed by the waves; an inertial force proportional to the relative acceleration of fluid flowing around FLIP; a drag force acting per unit area of hull, proportional to the square of the 

\(^1\)The radius of FLIP versus depth is 1.92 m from above the surface to -18.3 m depth, then increases linearly to 3.06 m at -45.7 m, and remains 3.06 m to the bottom of FLIP at -91.4 m.
velocity of the water relative to FLIP's hull; and the vertical balance between buoyancy and weight. We neglect (for example) the wind stress acting on the portion of FLIP extending above water. Hence we write
\[ F = F_p + F_d + F_b, \quad (7) \]
where the right hand terms represent forcing by the wave’s pressure gradient, fluid inertia, drag, and buoyancy anomaly, respectively. The torques are separated similarly.

2.2 Wave Pressure

The pressure gradient \( F_p \) due to the waves is estimated from linear wave theory:
\[ F_p(\omega) = -\rho g \eta x \int_e^{0} e^{kz} A(z) dz, \quad (8) \]
where \( \omega \) is the radian frequency of the waves, \( k = \omega^2 / g \) is the wavenumber magnitude from linear dispersion, and \( \eta_x = i k \eta \) is the (complex) surface slope at frequency \( \omega \).

2.3 Fluid Inertia

If FLIP were accelerating at exactly the same speed as the surrounding water (due to the surface wave pressure gradient), there would be no inertial effect. Only the relative acceleration of water around FLIP’s hull has an effect. The inertial force is often accounted for in the form of added mass, which is physically related to the volume of water distorted in flowing around the object in question. For example, with potential flow around a circular cylinder, the added mass is equal to the mass of water displaced by the cylinder (Lamb, 1932). The conceptual danger here is to neglect the fact that the water surrounding FLIP is already accelerating due to the exterior pressure field, and that, to some extent, FLIP is accelerating with it. Using linear wave theory, and neglecting the diameter of FLIP compared to the wavelength, the inertial force \( \tilde{F}_i \) can be written in the form:
\[ \tilde{F}_i(\omega) = -\rho C_i \int_{-H}^{0} \left[ g \eta x e^{kz} + \left( \ddot{\chi}_f + (z - z_f) \dot{\theta} \right) \right] A(z) dz, \quad (9) \]
where inertial coefficient \( C_i \) is equal to the ratio of added mass to FLIP’s mass. Thus, for potential flow about a cylinder, \( C_i = 1 \). The first term in the integrand is an
additional wave force, due to the fact that the wave-induced flow must accelerate around FLIP. The rest is the added mass term.

It is clear from inspection of 9 that the inertial force changes the net center of mass of the system. At the net center of mass, angular acceleration should not induce translation. This occurs in 9 because the center of mass of the displaced water is at some depth \( z_b \), rather than at the depth of FLIP's center of mass, \( z_f \). The combined mass of FLIP and the added mass of the water forced to flow around FLIP has a net center of mass at depth \( z_c \) somewhere between \( z_f \) and \( z_b \):

\[
z_c = M_i^{-1} \int_{-H}^{H} z(m(z) + \rho C_i A(z))dz = \frac{z_f + C_i z_b}{1 + C_i}, \tag{10}
\]

where

\[
z_b = \rho M^{-1} \int_{-H}^{0} zA(z)dz \tag{11}
\]

is the center of buoyancy,

\[
M_i = M(1 + C_i) \tag{12}
\]

is the net mass including the added mass, and we make the analytic extension \( A(z) = 0 \) from \( z = 0 \) to \(+ h\), \( z_b \). From the specified shape of FLIP’s hull, \( z_b \) is accurately known. At the net center of mass \( z_c \), pure rotation and translation are uncoupled, and the net inertial force simplifies to

\[
F_i(\omega) = -\rho C_i \int_{-H}^{0} \left( g \eta_z e^{kz} + \ddot{x}_c \right) A(z)dz, \tag{13}
\]

where \( \ddot{x}_c \) is the acceleration at \( z_c \).

The added mass also changes the net radius of gyration. The new radius of gyration is defined by

\[
\gamma_c^2 = M_i^{-1} \int_{-H}^{H} (z - z_c)^2 (m(z) + \rho C_i A(z))dz \tag{14}
\]

(again, \( A(z) = 0 \) from \( z = 0 \) to \(+ h\)). This is similar in form to the equation 4 for \( \gamma^2 \), except that the integral includes the mass distribution of the displaced water as well as that of FLIP, and it is referenced to \( z_c \) rather than \( z_f \). This may be expressed in terms of \( z_f \), \( z_b \), \( \gamma_f^2 \), and the radius of gyration of the displaced water volume,
\[ \gamma_b^2 \equiv \rho M^{-1} \int_{-H}^{0} (z-z_b)^2 A(z) dz. \] (15)

In terms of these,

\[ \gamma_c^2 = \frac{\left( \gamma_f^2 + z_f^2 \right) + C_i \left( \gamma_b^2 + z_b^2 \right)}{1 + C_i} - z_c^2 \]

\[ = \frac{\gamma_f^2 + C_i \gamma_b^2}{1 + C_i} + \frac{C_i (z_b - z_f)^2}{(1 + C_i)^2}. \] (16)

Since the volume distribution is known, this reduces the net sensitivity to the less well known mass distribution of FLIP.

The inertial force increases both the apparent mass and forcing roughly in proportion, resulting in little change in the response to waves. This accounts for the reasonable performance of models that neglected these terms. While the added mass should affect the resonant tilting period (discussed below), the error bounds on the center of mass and radius of gyration are large enough to accommodate the discrepancy.

Using the net center of mass and corresponding radius of gyration, 1 and 2 become

\[ \ddot{x}_c = \frac{F}{M_i} \] (17)

\[ \ddot{\theta} = \frac{T}{M_i \gamma_c^2} . \] (18)

2.4 Drag

The drag per unit area is normally assumed to have the form \(-C_d \nu(z) \nu(z)\), where \(C_d\) is a dimensionless drag coefficient and the vector velocity \(\nu\) is a function of time and space. This introduces coupling between frequencies in the above equations, and makes them analytically intractable. However, a quasi-linear solution is possible, with the assumption that the coupling is weak. For small frequency bands "\(d\omega\)" the velocity from an individual band is always small compared to the total velocity.
variance. Thus, the velocity magnitude \( |v(z)| \) is roughly the total rms velocity. If there is negligible phase coupling between frequencies, then the non-linear drag force can be replaced by a quasi-linear drag, where the coefficient \( V(z) \) is a function of depth and also a slow function of time:

\[
D_v(z) = C_d V(z).
\]  

(19)

Here \( C_d \) is the usual drag coefficient, and \( V(z) \) is the root-mean-square velocity of water relative to the hull of FLIP. The rms velocity \( V(z) \) is formed over an appropriate time interval (say, one hour). The simplest model results from setting \( D_v(z) = constant \) (linear drag). This is used first to assess the qualitative effects of drag on FLIP's motion. To form the depth dependent drag, we neglect the motion of FLIP in calculating the relative velocities, and also the details of the flow around FLIP (which would introduce some sensitivity to horizontal direction). \( D_v(z) \) is calculated for each time period from an estimate of the 1-dimensional surface elevation frequency spectrum, using

\[
V(z) = \sqrt{\int_0^\infty \eta(\omega)^2 \omega^2 e^{2k(\omega)z} \, d\omega}.
\]  

(20)

The exponential decay with depth makes the result insensitive to higher frequency waves. This permits the use of spectra derived from the vertical acceleration of FLIP (supplement). Should the drag term prove important, it might be necessary to iterate the model from equation 20, to include the motion of FLIP in an improved estimate of the rms difference velocity. A mean flow can also be included in 20, conceptually as a delta function at zero frequency; this could be important, since means flows over 0.2 m/s have been encountered, and the mean flow component may not decay with depth.

The area per depth increment \( dz \) is \( 2\pi r(z) \). Altogether, the quasi-linear drag force takes the form:

\[
F_d(\omega) = 2\pi \rho \int_{-H}^{0} \left( \frac{\omega \eta_c}{ik} e^{kz} - (\dot{x} + (z - z_c) \dot{\theta}) \right) D_v(z) r(z) \, dz
\]  

(21)

Only the drag force is in quadrature with the surface slope \( \eta_c \).

2.5 Torques

The corresponding torques are found by equations analogous to 8, 13, and 21, but with the additional factor \( (z - z_c) \) inside the depth integrals. In addition, there is a restoring force proportional to tilt, due to the fact that the center of buoyancy is above
the center of mass (otherwise there would be no reason for FLIP to remain vertical). The torque due to the balance of buoyancy versus weight can be written

\[ T_b = gM(z_b - z_f)\sin \theta = gM(z_b - z_f)\dot{\theta}. \] (22)

The balance between weight and buoyancy doesn’t enter directly into the equations for translation at the center of mass. However, it does enter into the equation for vertical displacement (supplement).

2.6 Solution

It is convenient to partition the forcing into a part due to the net forcing by waves, and other parts resulting from FLIP’s own motion. The equations for translational and rotational motion (17 and 18) can be re-written in the form:

\[ \dot{x}_c = \eta_x F_w - d_{xx} \dot{x}_c - d_{x\theta} \dot{\theta}, \] (23)

\[ \ddot{\theta} = \eta_x T_w - d_{\theta x} \dot{x}_c - d_{\theta\theta} \dot{\theta} - b_{\theta} \theta, \] (24)

where \( \eta_x(w) \) is the (complex) wave slope amplitude; \( F_w \) and \( T_w \) are defined as the net forcing and torque (respectively) applied per unit wave slope, normalized by the corresponding inertial moments (including the added mass); the \( d_{\eta m} \) are various quasi-linear drag components; and \( b_{\theta} \) is the buoyant restoring force working against tilting of FLIP. From equations 8, 13, 21, and the torque analogs:

\[ F_w = -\rho g \frac{M_i}{\gamma_c} \int_{-H}^{0} \left\{ A(z)(1+C_i) + \frac{2\pi r(z)D_v(z)}{\omega} \right\} e^{kz} dz \] (25)

\[ T_w = -\rho g \frac{M_i}{\gamma_c^2} \int_{-H}^{0} \left\{ A(z)(1+C_i) + \frac{2\pi r(z)D_v(z)}{\omega} \right\} (z - z_c) e^{kz} dz \] (26)

\[ d_{xx} = \frac{2\pi \rho}{M_i} \int_{-H}^{0} D_v(z) r(z) dz \] (27)

\[ d_{x\theta} = \frac{2\pi \rho}{M_i} \int_{-H}^{0} (z - z_c) D_v(z) r(z) dz \] (28)
\[ d_{\theta x} = \frac{2\pi \rho}{M_i \gamma_c^2} \int_0^1 (z-z_c) D_v(z) r(z) dz = \gamma_c^2 d_{x\theta} \tag{29} \]

\[ d_{\theta \theta} = \frac{2\pi \rho}{M_i \gamma_c^2} \int_0^1 (z-z_c)^2 D_v(z) r(z) dz \tag{30} \]

\[ b_{\theta} = \frac{g(z_b-z_f)}{(1+C_i) \gamma_c^2}. \tag{31} \]

Solutions to these equations of motion are sought for \( x_c \) and \( \theta \) as functions of frequency (i.e., looking for solutions proportional to \( e^{-i\omega x} \)), with the result:

\[ x_c = \eta_x \left( \frac{d_{x\theta} T_w + i\omega^{-1}(\omega^2 + i\omega d_{\theta\theta} - b_{\theta})F_w}{(d_{xx} - i\omega)(\omega^2 + i\omega d_{\theta\theta} - b_{\theta}) - i\omega d_{\theta x} d_{x\theta}} \right) \]

\[ \equiv \eta_x X \tag{32} \]

\[ \theta = \eta_x \left( \frac{d_{\theta x} F_w - (d_{xx} - i\omega) T_w}{(d_{xx} - i\omega)(\omega^2 + i\omega d_{\theta\theta} - b_{\theta}) - i\omega d_{\theta x} d_{x\theta}} \right) \]

\[ \equiv \eta_x \Theta. \tag{33} \]

Within the approximations used, the same formulation applies to the \( y \)-component: the response \( Y \) to a unit amplitude wave traveling in the \( y \) direction is equal to the response \( X \) to another unit amplitude wave traveling in the \( x \) direction. For small tilts, the tilt \( \theta \) in the \( x-z \) plane and \( \phi \) in the \( y-z \) plane (say) may be treated as independent (approximately Cartesian); in this case, the formal equivalence applies between \( \Theta \) and \( \Phi \) as well. In general, the tilts of \( FLIP \) are small enough that this is a good approximation. The equivalence of \( x \) and \( y \) responses also depends on the assumption that the quasi-linear drag is horizontally isotropic.

For real \( \omega \) and no forcing, the solution goes to zero. Physically, damping (the \( d_{nm} \) terms) excludes the possibility of unforced motions having the assumed time-dependence (steady oscillations). Solutions of the unforced (homogeneous but coupled) equations can be found as the zeros of the denominator common to 32 and 33, allowing the frequency \( \omega \) to be complex. This leads to solutions in the form of damped oscillations. For the present purposes, such homogeneous solutions are assumed to have decayed to nothing. There remain near-resonances, yielding large
amplitude responses to forcing at (real) frequencies near these (complex) values. This fact is used later to help refine the ill-known parameters $z_f$ and $\gamma_f$.

When damping is negligible, the solution simplifies considerably. in this case, 32 and 33 simplify to

$$x = -\eta_x \omega^{-2} F_w$$

$$\theta = \frac{-\eta_x T_w}{(\omega^2 - b_\theta)}.$$  

(34)  

(35)

The homogeneous solution for $\theta$ is simple harmonic motion with frequency $\omega_R = \sqrt{b_\theta}$.

2.7 Implementation

We now turn to constraints on FLIP's center of mass $z_f$ and the radius of gyration $\gamma_f$. Both depend on the loading of FLIP. As a starting point, Regier (1975) quoted values of $z_f = 56$ m and $\gamma_f = 25.6$ m. Since 1975, FLIP has undergone some reconstruction, which could change the estimates of $z_f$ and $\gamma_f$. For example, while Regier reported a resonant tilting period of 47 seconds in 1975, the data from SWAPP indicates a resonant period of 58 seconds. As noted above, the resonance can be found from the zeros of the denominator common to both 32 and 33, leading to a cubic equation in $\omega$. However, for the present purpose, the resonant tilting period is specified much more easily from the undamped solution:

$$T_{res} = \frac{2\pi}{\omega_R} = \frac{2\pi}{\sqrt{b_\theta}} = 2\pi \gamma_c \sqrt{\frac{(1+C_i)}{g(z_b - z_f)}}.$$  

(36)

Since the volume moments $z_b$ and $\gamma_b$ are known, and $T_{res}$ can be specified from the data, the result is a constraint on the joint variations of $z_f$ and $\gamma_f$; these must vary together so as to reproduce the observed resonant period. (Another constraint, that $z_b > z_f$, is also enforced by this equation.)

We choose to specify $\gamma_f$ and find $z_f$ from the solution of the quadratic equation resulting from 36 and 16:

$$z_f = z_b - \frac{\left( g \omega_R^{-2} \right) - \sqrt{\left( g \omega_R^{-2} \right)^2 - 4 \left( \frac{C_i}{1+C_i} \right) (\gamma_f^2 + C_i \gamma_b^2)}}{2 \left( \frac{C_i}{1+C_i} \right)}.$$  

(37)
(We take the smaller solution. The other solution corresponds to the unphysical case of \( z_f \) and \( z_b \) so far apart that the increase in \( \gamma_c \) overwhelms the increase in restoring force. Note that \( g \omega R^{-2} \) is of order 1 km, while \( \gamma_f \) and \( \gamma_b \) are 20 to 30 m.)

There are three adjustable parameters: the inertial mass ratio \( C_I \) (which we shall take to be 1.0), the drag coefficient \( C_d \), and the combination of \( \gamma_f \) and \( z_f \) which does not change the resonant period. In fact, fixing the inertial mass ratio to 1.0 is acceptable since, by inspection of the forcing equations, a change in \( C_I \) has an effect equivalent to changes in \( C_d \) and \( \gamma_f \); thus, it does not introduce any new degrees of freedom. The remaining two parameters can be adjusted (within reasonable bounds) to find values providing the best fit between the various kinds of data, over a variety of environmental conditions.

The purpose of this model is to permit estimation of FLIP's motion from a reduced set of measurements; e.g., from just accelerometer measurements at 35m depth and heading. The model partitions the motion of FLIP between tilt and the acceleration measured at any specified depth. A simple way to visualize this partitioning is that the model specifies a center of rotation at each frequency in response to the forcing by a free gravity wave. Thus, at a given depth, the ratio between horizontal displacement and tilt is set, for each frequency, by the distance from the center of rotation.

Conceptually, the center of rotation is the depth at which the centerline of FLIP remains fixed horizontally. In practice, the presence of the drag force makes this complex; however, the concept remains useful, especially since the drag is small. For small tilts, the center of rotation is approximately

\[
z_f(\omega) = z_c - X(\omega)/\Theta(\omega).
\]

(38)

3. Comparisons

3.1 Data and Calibrations

The fluxgate measures the 3 component magnetic flux vector of the earth's magnetic field. Unfortunately, the vertical fluxgate channel was faulty. Fortunately, at 35°N the magnetic flux lines are quite steep (~60° from horizontal), so the tilt in the horizontal plane can still be resolved, given an independent measurement of heading (such as that provided by the gyrocompass). However, the calibration of the flux measurements are in doubt. At low frequencies, the accelerometer readings are
dominated by gravity times tilt, so we use the low-frequency accelerometer data to calibrate the fluxgates.

The accelerometer data was checked first for self-consistency. Two minute averages were formed, and the magnitudes were checked against the known value of gravity. Next, tilts calculated from the accelerometers were compared with tilts estimated from the wave wires, first using the measured distances of each wire from the axis of FLIP, and then using the low-frequency mean (two-minute) tilts over the three wire array, which forms a triangle 1 meter on a side. The latter estimate of tilt has less resolution than the first, but provides a check on the component of tilt perpendicular to the orientation of the boom.

Having confirmed the accelerometer calibrations in some detail, the fluxgate data was then post-calibrated via two models, incorporating both heading and accelerometer-derived tilts: (1) a direct least-squares-fit linear model from the raw fluxgate readings to heading and tilt, and (2) a geometric model, which included the measured geometry of the experimental setup, including boom tilt, orientation, and twist. In the magnetic North-South direction, the resulting data agree well via either model, but in the East-West direction, the geometric model fares somewhat worse than the direct linear fit. This indicates that some unknown effect may be degrading the E-W result. For this reason, only N-S components of the data are used in the comparison with the FLIP motion model.

Sample N-S and E-W accelerometer and fluxgate equivalent spectra are shown in Figure A.2, extending up through the wave frequencies. As enforced by calibration, the accelerometer and fluxgate spectra are nearly identical at low frequencies (particularly at the tilt resonant frequency of .017, corresponding to a period of 58 seconds). However, at high frequencies the effect of waves accelerating FLIP can be seen clearly: at the peak wave frequencies, power in the accelerometer measurements is more than two orders of magnitude greater than that of the fluxgate. As well, the noisy nature of the fluxgate data is shown by the noise floor extending above 0.2 Hz.

3.2 Empirical Motion and Model Tuning

With accelerometer, fluxgate, and gyroscope measurements, an empirical center of rotation can be calculated. The accelerometer measurements combine true acceleration at a known depth ($z_a=35$ m) and a component of gravity due to tilt. For example, the x-component of the accelerometer measurement is

$$a_x = \ddot{x}_a - g \sin \theta = \ddot{\theta}(z_r - z_a) - g \sin \theta$$

(39)
where \( x_a \) is the horizontal displacement at the depth of the accelerometer, \( z_a \), and \( z_r \) is (again) the center of rotation. For small tilts, \( \sin \theta = \theta \), the displacement and tilt can be combined in a manner analogous to 38 to obtain an empirical center of rotation from a combination of accelerometer (\( a \)) and fluxgate (\( \theta \)) measurements:

\[
z_r = z_a + \omega^{-2} \left( g + \frac{a}{\theta} \right).
\]  

(40)

The comparison between empirical and modeled center of rotation (as functions of \( \omega \)) allows a method by which to evaluate the model and set the free parameters.

Within the region of high surface wave energy (above .05 Hz) the effect of drag is minimal. At lower frequencies, increasing the drag (\( C_d = 0.0 \) to \( 0.5 \)) deepens the center of rotation, such that motion of FLIP is more translational than rotational. Also, increasing the drag from zero quickly suppresses the tilt resonance. We deduce from Figure A.2 that the system has little drag. Within the wave band, drag can be ignored.

Variation of the radius of gyration alters the shape and position of the center of rotation curve across a wide range of frequencies. This is seen in Figure A.3, where the empirical and modeled center of rotation (for values of \( \eta = 17, 19, \) and \( 21 \) and zero drag) have been plotted. Increasing \( \eta \) moves the center of rotation away from the center of mass, while decreasing it moves the center of rotation asymptotically closer to the net center of mass, \( z_c \). Also, as the surface wave frequency increases, the forcing becomes localized at the surface, and the center of rotation approaches a fixed level. Within the constraints of the physics included in the model (e.g., a rigid hull forced by surface waves), the center of rotation is always shallower than the net center of mass.

The modeled center of rotation for \( \eta = 19 \) fits the empirical curve best in the range of the most energetic waves (frequencies between .08 and .10 Hz). This value is lower than that suggested by Regier (1975), 25.6 m, but is within reason, especially considering modifications made to FLIP since then.

3.3 Tilt Comparison

Tilt can be obtained from accelerometer data, using the model center of rotation from above:

\[
\theta = \frac{x_a}{z_a - z_r} = \frac{-a_x}{\omega^2 (z_a - z_r) + g}.
\]

(41)

(the same formulation applies to the \( x \) and \( y \)-components). To compare with measured tilt, two time segments are chosen. At these two times (March 11, 1990 to March 13,
1990), all the relevant data are available, and two different conditions are represented. The modeled tilt in the N-S plane (Figure A.4) show good agreement with the magnetic flux tilt measurement in phase, and compare favorably in amplitude as well, with errors of 17.6 % and 24.6% of the total variance respectively. The noise in the fluxgate signal is partially responsible for the calculated error.

3.4 Velocity Comparison

Our intended use of the motion model is to correct velocity estimates from a Doppler sonar system mounted at 35 m depth on FLIP's hull. For comparison, the mean velocity over a suitable range away from FLIP (80 to 300 m) is used to indicate the motion of the sonar (and hence of FLIP at that depth) relative to the water. In SWAPP this velocity is available every 1.5 seconds.

As noted above, the accelerometer measurements correspond to a combination of true acceleration and a component of gravity due to tilt. Making the small angle approximation, and using the center of rotation (38) to convert angle to displacement, the displacement of FLIP's hull at the depth of the accelerometers can be written

\[ x_a = \frac{-a_x}{\omega^2 + g/(z_a - z_r)} \]  

This is most easily done in the frequency domain, since \( z_r \) is a function of frequency. Horizontal displacement can be computed anywhere along FLIP's hull, using the center of rotation as in 42. Displacements can be converted to velocity, via multiplication by \(-i\omega\).

The model and sonar velocities have been filtered by a fourth order Butterworth high pass filter to frequencies above .05 Hz. The resulting time-series (Figure A.5) show good agreement, with errors of 15.7% and 15.0% of the total variance, respectively.

4. Conclusions

The most fundamental aspects of FLIP motion are well described by this linear model. By the calculation of a frequency dependent center of rotation, the model allows the accelerometer measurements to be partitioned between tilt and acceleration, thus allowing the estimation of motion at any point on the FLIP superstructure. Comparisons with independent data have shown that the modeled response of FLIP matches the actual response to within the accuracy necessary for the purposes here.
Sonar and model derived estimates of velocities at the sonar depth show good agreement both in magnitude and phase. The tilting of FLIP is also modeled well, as indicated by the comparison with data from a geomagnetic vector pointer.

The estimates of acceleration and tilt may now be used to determine the effects of FLIP motion on the various measurements taken from the superstructure. One important example is the measurement of the wind stress. For a sample period (March 11, 20:29) the vertical and horizontal root mean square velocities of the sonic anemometer due to FLIP motion are estimated at .03 and .15 m/s respectively. However, since the tilt and rotation resonances are quite different, the induced vertical and horizontal velocities are poorly correlated (correlation coefficient of $r^2=.03$). The resultant wind stress contribution is estimated to be $8.8\times10^{-4}$ N/m$^2$, more than two orders of magnitude smaller than the measured wind stresses. Indeed, FLIP is a stable platform.

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Supplement: Vertical Motion of FLIP

One significant force acting to move FLIP vertically is pressure, in response to the fluctuating sea surface height:

$$ F_z = \rho g k \eta \int_{-H}^{0} e^{kz} A(z) dz. \quad (43) $$

For flow parallel to a cylinder, the inertial force is negligible. The drag force has the same form as in the horizontal case, but the coefficient is not necessarily the same. We shall neglect it here. The other important forces are the restoring force brought about by buoyancy, balanced against the weight of FLIP:

$$ F_b = \rho g \int_{-H+z}^{0} A(z' - z) dz' - M = -z(\rho g A_0) \quad (44) $$
where $A_0$ is the cross-sectional area of FLIP at the mean waterline (FLIP is uniform in cross-section over several meters on either side of the mean waterline).

An estimate of the vertical motion of FLIP is given by the solution of the equation

$$M\ddot{z} = F_z + F_b = \rho g k \eta \int_{-H}^{0} e^{kz} A(z) dz - \rho g z A_0$$  \hspace{1cm} (45)

$$a_z \equiv \ddot{z} = \eta \frac{\rho g k \int_{-H}^{0} e^{kz} A(z) dz}{(M - \rho g \omega^2 A_0)},$$  \hspace{1cm} (46)

where $a_z$ is the vertical component of acceleration measured along FLIP's centerline. Thus, a wave spectrum estimate is given by

$$S(f) \equiv |\eta|^2 = \frac{|a_z|^2}{\omega^4} \left( \frac{M - \rho g \omega^2 A_0}{\rho \int_{-H}^{0} e^{kz} A(z) dz} \right)^2.$$  \hspace{1cm} (47)

The solution 46 has a resonance at $\omega^2 = \rho g A_0/M$. For FLIP, this resonance occurs at a period of about 26 seconds.
Figure A.1. Schematic view of R/P FLIP during SWAPP. Instruments deployed included: (A,B) high frequency sonic anemometers, (C) vector-averaging wind recorder, and (D) surface scanning Doppler sonars.
Figure A.2. North-South and East-West (acc) accelerometer and (flux) fluxgate equivalent spectra. As enforced by calibration, the spectra match at the tilt resonant period of 58 seconds, but at higher frequencies, the effect of waves, which accelerate FLIP, can clearly be seen.
Figure A.3. (Grey) model and (black) empirical center of rotations vs. frequency. The model derived center of rotation was calculated for various radii of gyration (17, 19, and 21 m). The model with radius of gyration = 19 m (solid) is seen to fit the empirical data best in the range of the most energetic waves. Larger radii move the center of rotation away from the center of mass. For low frequency waves, whose effective forcing occurs further below the sea surface, the center of rotation is deeper. In contrast, for high frequency waves, whose effective forcing approaches the sea surface, the center of rotation asymptotically approaches some depth below the center of mass.
Figure A.4. Time-series of (black) accelerometer-derived tilt, via FLIP motion model, and (grey) fluxgate measured tilt, for two time periods: (top) March 11, 1990, 20:29, and (bottom) March 13, 1990, 1:47.
Figure A.5. Time-series of (black) accelerometer-derived velocity at the sonar depth, via \textit{FLIP} motion model, and (grey) the sonar range-mean velocity for two time periods: (top) March 11, 1990, 20:29, and (bottom) March 13, 1990, 1:47.
References


The text of the Appendix, in full, is a reprint of the material as submitted to Ocean Engineering. The dissertation author, Karl F. Rieder, was the secondary researcher.