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On the Buyability of Voting Bodies

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Abstract

We study vote buying by competing interest groups in a variety of electoral and contractual settings. While increasing the size of a voting body reduces its buyability in the absence of competition, we show that larger voting bodies may be more buyable than smaller voting bodies when interest groups compete. In contrast, imposing the secret ballot—which we model as forcing interest groups to contract on outcomes rather than votes—is an effective way to fight vote buying in the presence of competition, but much less so in its absence.

We also study more sophisticated vote buying contracts. We show that, regardless of competition, the option to contract on both votes and outcomes is worthless, as it does not affect buyability as compared to contracting only on votes. In contrast, when interest groups can contract on votes and vote shares, we show that voting bodies are uniquely at risk of being bought.

JEL #s: D71, D72, D78.

Keywords: Vote buying, lobbying, corruption, elections.
1 Introduction

In 1757, George Washington ran for a seat on the Virginia House of Burgesses, the colony’s main legislative body at that time. Concerned about the effects of drink on his soldiers, Washington ran an upstanding campaign on the platform of temperance. He was soundly defeated by 270 to 40 votes. The following year, Washington changed his platform and his tactics in another run for the same seat. To aid his chances, Washington offered voters an average of one and a half quarts of various alcoholic beverages in exchange for their votes. The difference in outcome was impressive. Against the same opponent, Washington won by 310 to 45 votes. (Ford, 1896.)

Since Washington’s times, there have been considerable changes in the structure of elections and voting bodies. One notable change is in their size. The total number of voters in Washington’s elections was only about 350. However, the expansion of the voting franchise (perhaps most dramatically with the passing of the 19th Amendment in 1920) has lead to considerably larger numbers in modern elections—even when voter turnout is low. Furthermore, the growth in the size of the U.S. population has led to a considerable expansion of the size of federal legislative bodies. For instance, the House of Representatives now numbers 435 members, whereas it had only 65 at the time of the first Congress of 1789. Similarly, with the admission of numerous states during the nineteenth century, the U.S. Senate has expanded from 26 members to its current total of 100.

One may wonder whether an increase in the size of a voting body makes that body more or less susceptible to vote buying. If the cost per vote remained fixed, then, clearly, the direct effect of expanding the voting body makes vote buying more costly. This, however, ignores the strategic effect of competition in vote buying. In the face of competition, the scale of vote buying needed to secure the desired outcome depends on the response of a rival group. As Groseclose and Snyder (GS, 1996) have shown, the optimal way to blunt competition is to buy a supermajority of voters. However, the magnitude of the optimal supermajority varies with the size of the voting body and, indeed, it may be possible to economize on payments made for deterrent purposes as the size of the voting body grows. Thus, there may be a countervailing strategic effect which is cost-reducing. The question is whether this strategic effect can be sufficiently strong as to outweigh the direct effect.

An electoral policy change explicitly introduced to counteract vote buying was the imposition of the secret ballot. Motivated by Chartist principles and worried about the corruption endemic to its electoral process, in 1856, the Australian state of Victoria was the first to adopt the secret ballot in general elections. Britain and the U.S. soon followed. Yet, the effectiveness of the secret ballot as a deterrent to vote buying is debatable. Several studies indicate that the process of vote buying

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1 We use the term voting body as a generic description for a wide range of institutions where decisions are made by voting. For example, legislatures, committees, and voters in general elections all constitute voting bodies.
has simply shifted from simple schemes such as that employed by Washington to more intricate ones (see, e.g. Cox and Kousser, 1981 and Heckelman, 1998). A recent example of how the process of vote buying has adapted to the secret ballot can be seen in the Taiwanese Presidential election of 2000. In that election, the ruling National Party subsidized betting parlors to offer extremely favorable odds on the event that the party’s candidate, Lien Chen, was elected (August, 2000). This way, the ruling party managed to circumvent the secrecy of the ballot by offering a vote buying contract that was contingent on the outcome rather than on the vote itself. A central question is under what circumstances such schemes can succeed, as well as the cost-effectiveness of outcome-contingent vote buying.

Washington’s scheme, as well as that of the National Party in Taiwan, are quite simple in the sense that only a single contingency—vote or outcome—is contracted upon. There are other vote buying schemes that are more sophisticated and involve multiple contingencies. For example, in the IOC scandal of the 2002 Salt Lake City Olympic Winter Games, it was reported that certain members of the IOC were paid money in exchange for their votes, as well as a “bonus” conditional on the outcome of the vote—i.e., the success of the city’s Olympic bid. Thus, the contracts depended both on votes and outcomes. While the IOC offers a modern example of a sophisticated vote buying contract, contracts where payments are contingent on an individual’s vote as well as some aggregate measure can be found as far back as nineteenth century Great Britain. For instance, Seymour (1915, p.167) details how in elections held in Liverpool in the 1830s the price paid for votes rose and fell like a stock, depending on the current vote share of the candidates. It is an open question as to how the use of such sophisticated contracts affects the buyability of voting bodies.

To study these questions, we reexamine the model of Groseclose and Snyder (GS, 1996) while varying both the contractual sophistication of vote buying contracts as well as the structure—i.e., size and secrecy—of the voting body. First, we study the buyability of voting bodies under simple vote buying schemes. We look at what happens when payments can be tailored to the preferences of an individual voter (i.e., are discriminatory) as well as when they cannot. We then study how the cost of successful vote buying varies with the size of the voting body. Next, we turn to the secret ballot. Here, we model vote buying as being contingent on outcomes, rather than on individual votes, and study the effect this has on the buyability of voting bodies. Following this, we turn to sophisticated contracts. We analyze two variations—contracts contingent on votes and outcomes, and contracts contingent on votes and vote shares. Throughout the paper, we study how the presence or absence of competition between interest groups affects the buyability of the voting body.

Our main results are as follows. We show that competition between interest groups significantly affects both the buyability of voting bodies and the appropriate policy response to it. Absent competition, we show that increasing the size of the voting

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body provides effective protection against vote buying. The same is not true in the presence of competition. In this case, larger voting bodies may be more buyable than smaller voting bodies.

Next, we show that the introduction of the secret ballot has little effect on the buyability of voting bodies in the absence of competition. That is, it does not affect the cost of legislative capture but does, perhaps, reduce the likelihood of capture through equilibrium multiplicity. In the presence of competition, however, the beneficial effect of the secret ballot is unambiguous; the cost of capture is increased and the likelihood decreased relative to the GS case where votes are directly contractible.

Turning to sophisticated contracts, we show that the ability to contract on votes and outcomes has no effect whatsoever on the cost of vote buying, as compared to the case where only votes may be contracted upon. This is true independent of competition. However, the irrelevance of additional contractual contingencies does not generalize. When interest groups can contract on votes and vote shares, vote buying becomes extremely cheap. This leaves the voting body uniquely at risk of being bought.

The remainder of the paper proceeds as follows. In section 2, we describe the model. Our model is exactly that of GS but for variations in the contractual environment. In section 3, we analyze contracts where only votes are contractible and we compare discriminatory with non-discriminatory vote buying. Section 4 examines how policy responses to vote buying affect the buyability of voting bodies. Specifically, we study the effect of changes in size of the voting body and changes in the secrecy of individual votes. In section 5, we analyze sophisticated contracts—those which allow for more than one contractual contingency. Section 6 places the results of this paper in the context of the broader literature. Finally, Section 7 concludes. Appendix A contains proofs of the results presented in the text, while Appendix B studies the robustness of results pertaining to non-discriminatory vote buying.

2 The Model

Our model is identical to that of GS save for variations in the contractual environment. To ease exposition, we slightly modify the notation of GS. Readers desiring additional details or justification for the model may want to consult their paper.

There are an odd number, \( n \), of voters choosing between two policies. The policies, which one could also think of as candidates or party platforms, are labeled \( a \) and \( b \). The policy receiving the majority of votes is adopted.

Two interest groups, labeled \( A \) and \( B \), are trying to affect the policy choice. Group \( A \) prefers policy \( a \) while group \( B \) prefers policy \( b \). In a setting where the voters are legislators, the interest groups can be thought of as lobbyists or political action committees. In a setting where voters are citizens voting in an election, the interest groups may be thought of as political parties. In this interpretation, the policy options refer to which party gets to form the government.
Excluding the cost of buying votes, group A enjoys a payoff $W_A > 0$ when policy $a$ is adopted and zero when $b$ is adopted. Group B, on the other hand, enjoys a payoff $W_B > 0$ when policy $b$ is adopted and zero when $a$ is adopted. Thus, groups A and B have diametrically opposed policy preferences. To induce voters to vote for its preferred policy, each group can offer enforceable contracts.

We will vary, however, the contingencies on which these contracts can be based. The net payoff to a group is its payoff associated with the adopted policy less any vote buying costs. Throughout, we assume that $W_A$ is sufficiently large, such that offering contracts that successfully induce the adoption of policy $a$ (if at all possible) is always preferred by group A over doing nothing.

Voters, indexed by $i = 1, 2, ..., n$, care about their actual votes and any transfers from the interest groups. Specifically, voter $i$’s payoff is

$$U_i(c_i, t_i) = u_i(c_i) + t_i$$

where $c_i$ indicates voter $i$’s vote (choice), $a$ or $b$; while $t_i$ denotes any monetary transfers received from an interest group as a consequence of entering into a contract.

Of relevance is the change in a voter’s payoff from switching his vote from $b$ to $a$. Hence, define

$$v_i = u_i(c_i = a) - u_i(c_i = b)$$

and suppose that all voters have strict preferences over $a$ and $b$; that is, $v_i \neq 0$ for all $i$. Next, almost without loss of generality, assume that the indices of voters are ordered such that $v_i$ is a strictly decreasing function of index $i$. And, for future reference, define $v^{-1}(x) \equiv \min \{i|v_i \leq x\}$. Furthermore, suppose that the median voter, $M \equiv \frac{n+1}{2}$, prefers policy $b$; that is, $v_M < 0$. Hence, in the absence of interest group A policy $b$ would be adopted, while policy $a$ is only adopted when interest group A manages to buy the vote. All this is identical to the GS model.

To illustrate many of their results, GS often resort to a model with a continuum of voters and continuous relative preferences. Since they are only concerned with contracting over individual votes, pivotality plays no role in their analysis and, hence, the continuum model is perfectly adequate. In this paper we are also interested in circumstances where (aggregate) outcomes are contractible. This makes pivotality important again. Therefore, throughout, we adopt a setting with a discrete number of voters.

For obvious reasons, the preferences of voters in the neighborhood of the median voter are of special interest. To capture the flavor of GS’s continuous preference model in our discrete voter setting, we assume that:

**Assumption 1.** $v_M - v_{M+1} \leq \frac{W_M}{M}$.

Assumption 1 merely rules out large jumps in the relative preference for policy $a$ versus policy $b$ between the median voter and the voter just to the right of the median voter.

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3We will sometimes refer to vote buying schemes as “bribes.” This is for succinctness and not an expression of the legality (or lack thereof) of a particular scheme.
median. Most of the results contained in the paper do not rely on Assumption 1. Where a result does rely on this assumption, it will be explicitly invoked in the proof.

The extensive form of the game is as follows. First, A offers contracts to all voters, consisting of a non-negative transfer and a set of contingencies under which the transfer is made. Note that the contract offered to a particular voter may be the null contract, a promise of a zero transfer under all contingencies. Next, after observing A’s offers, B offers contracts to all voters. Following this, voters simultaneously opt for one of the two contract offers they have received and vote. The policy outcome is then determined and payoffs are realized.

We assume that if B can do no better than to propose the null contract then it opts for this strategy. Also, if a voter is indifferent between accepting the contract offered by A and that offered by B, he is assumed to accept A’s contract.

Multiplicity of equilibria for a given set of contracts does not arise in the setting proposed by GS, because the payoffs to each voter are independent of the actions of the other voters. In general, however, this will not be the case. In contractual environments where contingencies based on outcomes are permitted, the contract itself creates interactive incentives. To address this issue, we shall take a conservative view about the cost to A of successful vote buying by adopting the following definition.

**Definition 1** A vote buying contract is **successful** if and only if it guarantees adoption of policy a.

Formally, a vote buying contract is successful if and only if all subgame perfect equilibria following the contract lead to the adoption of policy a.

Next, we define **coalitions** and **outside options**, concepts frequently used in the analysis.

**Definition 2** A coalition for policy A consists of a set of voters who weakly prefer to accept A’s contract and vote for policy a given the contracts they have been offered.

Note that any voter who is not in A’s coalition is in B’s coalition. Also note that a winning coalition is a coalition with cardinality at least #M.

**Definition 3** Consider any pair of contracts and the resulting coalitions of voters. The **outside option** for a voter in the B coalition is the payoff that the voter would receive if he unilaterally accepted A’s contract.

### 3 Simple Contracts

In this section, we study “simple” contracts. These are contracts where only a single contractual contingency can be specified. In particular, we study contracts where transfers are made contingent on an individual’s vote, and contracts where transfers are made contingent on the policy outcome.
3.1 Contracting on Votes

3.1.1 Discriminatory Vote Buying

In their seminal paper, Groseclose and Snyder consider the case where contracts are contingent only on votes, and where the offers made are specific to each voter. That is, they consider the case of discriminatory vote buying schemes. While this analysis clearly makes sense in situations where there are relatively few voters with preferences known to the lobbying groups, even in large elections, discriminatory schemes are sometimes observed. For instance, the vote buying strategies of major parties in general elections in the Philippines prescribe variable payments depending on the identity of the voter being bribed. As Quimbo (2002, pp. 1-2) writes:

The amounts [of the payments] may vary among supporters, the undecided, and those on the other side...Undecided voters sometimes get three times as much as supporters. Key supporters from the other side receive even more if they switch sides.

Here, we briefly recapitulate the main result of Groseclose and Snyder. Fix some coalition size $m$, such that $n \geq m \geq M$. Next, define $K(m)$ to be the minimum expected payoff earned by any voter $i = \{1, 2, \ldots, m\}$, where payoffs include transfers. Group $A$ will choose bribes that induce a $K(m)$ such that group $B$ will (just) not wish to “invade” $A$’s coalition in order to implement policy $b$. See GS for details.

For $B$ to obtain its desired policy it must re-bribe at least $m - M + 1$ voters. Thus, $B$ needs to offer transfers that exceed the voters’ expected net payoffs under the vote buying scheme proposed by $A$. By definition, this amount is at least $K(m)$. For $A$ to be successful, re-bribing must cost $B$ at least $W_B$. This implies that for fixed $m$,

$$K(m) = \frac{W_B}{m - M + 1}$$

Conditional on $m$, $K(m)$ implicitly describes the least-cost successful vote buying scheme available to $A$. GS show that for given $m$ the least-cost successful contract is:

For $v^{-1}(K(m)) \leq i \leq m$,

$$t_i = \begin{cases} 
K(m) - v_i & \text{if } c_i = a \\
0 & \text{if } c_i = b
\end{cases}$$

For $i < v^{-1}(K(m))$ or $i > m$, the null contract is offered.

For future reference, we refer to a contract of this form as a $K(m)$ contract. The cost of such a contract is

$$C_A(m) = \sum_{i=v^{-1}(K(m))}^{m} t_i$$
Without proof, we offer the following proposition which follows directly from Grose-close and Snyder.

**Proposition 1** \( m^* \in \arg \min C_A(m) \). Then a \( K(m^*) \) contract is a least-cost successful contract under discriminatory vote buying when only individual votes are contractible.

When the option of offering discriminatory contracts is available, group \( A \) optimally tailors the contract offered to each voter to account for that voter’s intrinsic preferences. Voters with intrinsic preferences favoring policy \( a \) receive smaller transfers than those with intrinsic preferences favoring \( b \). Indeed, the size of the transfer is increasing up to the voter with index \( m^* \), who is offered the largest transfer for “switching sides” from \( b \) to \( a \). Group \( A \) optimally gives up on buying voters with intrinsic preferences toward \( b \) that are greater than those of \( m^* \).

The central insight of GS is that, generally, \( m^* > M \). That is, it tends to be optimal for \( A \) to buy a supermajority.

### 3.1.2 Non-Discriminatory Vote Buying

Often times, it may be difficult for lobbying groups to arrange payments in a discriminatory fashion. For instance, determining the exact preferences of individual voters may difficult. Another possibility is that, even if these preference are known, devising variable payment schemes may pose a considerable logistical challenge. Indeed, in many real-world instances of vote buying, interest groups rely on simple, non-discriminatory schemes. For instance, Robert A. Caro (1982) recounts a vote buying strategy undertaken by Lyndon Johnson who, at the time, was working for Maury Maverick in his run for Congress in 1934:

Johnson was sitting at a table in the center of the room—and on the table there were stacks of five-dollar bills. “That big table was just covered with money—more money than I had ever seen,” Jones says. (...) Mexican American men would come into the room one at a time. Each would tell Johnson a number—some, unable to speak English, would indicate the number by holding up their fingers—and Johnson would count out that number of five-dollar bills, and hand them to him. “It was five dollars a vote,” Jones realized. “Lyndon was checking each name against a list someone had furnished him with. These Latin people would come in, and show how many eligible votes they had in the family, and Lyndon would pay them five dollars a vote.”

This vote buying strategy was not unique to the Maverick campaign. Indeed, the practice of distributing fixed cash payments in exchange for votes, was (and perhaps still is) widespread. For instance, on p. 647, Caro writes:
Texas was not the only state in which money was piled on tables to purchase votes, just as Mexican-Americans were not the only immigrants whose votes were purchased. (...) big oak desks of city officials were traditionally cleared on Election Day and covered with piles of cash. In the big cities of the Northeast, votes might cost more than five dollars each.

How do situations where vote buying is non-discriminatory compare to the case analyzed by Groseclose and Snyder? In particular, are voting processes more immune to outside influence under non-discriminatory vote buying than in circumstances where discriminatory contracts are possible? In this section, we address this question by considering competition in vote buying contracts when the contracts themselves are restricted to be non-discriminatory.

Let $t_A$ be the transfer offered by group $A$. Let $m(t_A)$ be the highest index $i$ such that $v_i + t_A \geq 0$. Clearly, if $B$ offers the null contract, then all voters $i = 1, ..., m(t_A)$ will accept the $t_A$ contract offered by $A$. We now characterize the minimal transfer amount which $A$ can offer and still be successful.

**Proposition 2** Let vote buying be non-discriminatory. Under the least-cost successful contract, group $A$ offers payments in the amount $t_A = \frac{W_B}{M} - v_M$. Group $B$ offers the null contract. All voters with indices $i \leq m(t_A)$ are in $A$'s coalition.

It is interesting to contrast the structure of the least-cost successful contract in the non-discriminatory case with the discriminatory case of Groseclose and Snyder. In both cases, transfers can be viewed as consisting of two parts: 1) a compensatory payment to offset intrinsic preferences favoring $b$ and, 2) a surplus payment to deter $B$ from offering any contract other than the null contract. In the case of discriminatory contracts, the compensatory payments, $-v_i$, vary with the strength of preferences of the individual voter, while for non-discriminatory contracts they cannot. In the latter case, the compensatory payment, $-v_M$, is determined by the intrinsic preference of the median voter. Clearly, all voters with indices to the left of the median will be sufficiently compensated as well. Under both types of contracts, the surplus payment does not vary with the identity of a bribed voter. In the case of discriminatory contracts, the surplus payment is $W_B m^* - M + 1$, which reflects the fact that group $B$ can offer contracts to $m^* - M + 1$ selected voters to obtain a bare majority of support for policy $b$. In contrast, the surplus payment offered under non-discriminatory contracts is lower and equal to $\frac{W_B}{M}$. The reason is that group $A$ is able to economize on the surplus transfer by recognizing that $B$ cannot target selected voters to “pick off” $A$’s coalition.

GS show that it is generally optimal for group $A$ to buy a supermajority of voters when vote buying is discriminatory. The next proposition shows that the same result holds under non-discriminatory vote buying.
Proposition 3 Let vote buying be non-discriminatory. Then, under the least-cost successful contract, A always buys a supermajority of voters.

One may worry that this result heavily relies on the modeling assumption that neither of the lobbying groups can ration their transfers. After all, it seems that A would be happy to stop making payments once a bare majority coalition was obtained. However, this ignores the strategic effect of B’s response in the presence of rationing. In Appendix B, we show that adding rationing to the model does not change the basic conclusion that buying a supermajority is optimal.

Discriminatory versus Non-Discriminatory Vote Buying

Absent competition, it is clear that discriminatory vote buying is cheaper than non-discriminatory vote buying. With competition, however, it is not so clear which scheme is cheaper for A, owing to differences in what is required to deter B under the two cases. The ability to discriminate has both advantages and disadvantages from the point of view of interest group A. The advantage is that A does not have to pay its strongest supporters at all and can pay its weaker supporters less than it pays those who oppose policy a. The disadvantage is that B can also discriminate and therefore specifically target the “weakest links” in A’s coalition. When discrimination is not possible, such a targeted counter attack is not feasible. Which effect dominates depends on the shape of the preference function $v_i$.

Circumstances where (non-)discriminatory contracts are cheaper may be readily identified through a simple graphical analysis. In the figures below, we fix the preferences of the median voter and change the shape of the preference function around this point.

The case were discriminatory vote buying is more cost-effective is shown in Figure 1. Illustrated in the figure are the preferences of voters, $v_i$, as well as the transfers made by A under discriminatory and non-discriminatory vote buying. Since $m^*$ is close to $n$, the transfers paid by A to voters located to the right of the median are almost the same under the two schemes. Where the schemes differ is in the transfers made to voters located to the left of the median, most of whom intrinsically prefer policy a. In the discriminatory case, few of these voters receive transfers whereas in the non-discriminatory case, all of them receive transfers. Thus, the discriminatory scheme is clearly cheaper.

The opposite case—where discriminatory vote buying is less cost-effective—is illustrated in Figure 2. Since $m^*$ is close to $M$, the transfers to deter B are considerably higher in the discriminatory case than in the non-discriminatory case. Moreover, owing to the steepness of the preference function to the right of the median, those receiving transfers from A under the two schemes are almost the same. Where the schemes differ is in the size of transfers to voters to the left of the median. The lack of intrinsic support for policy a means that under both schemes, all voters with indices smaller and $M$ receive payments, but these payments are considerably larger in the discriminatory scheme than in the non-discriminatory scheme.
4 Policy Responses to Vote Buying

4.1 The Size of the Voting Body

One common intuition is that a “cure” for vote buying is to expand the size of the voting body. The intuition relies on the direct effect that such an expansion has on the costs of a single, monopsonistic lobbying group. Such a lobbying group will have to bribe a larger number of voters, while the size of the bribes remains the same. Clearly, this increases its expenses. For marginal policies—policies where $W$ is not too large—the lobbying group will therefore optimally refrain from influencing the vote with a large voting body, but will influence it with a smaller voting body. Of course, in the case where competition is absent, this intuition is indeed correct.

The presence of competition, however, introduces a strategic reason for bribing voters on the part of group $A$. Indeed, it is this strategic effect that is responsible for the Groseclose and Snyder result that bribing a supermajority of the voters is optimal.

What does the presence of the strategic effect do to the intuition that larger voting bodies are less buyable than smaller voting bodies? To study this formally, we need a way to scale the preferences of voters such that the relative strength of preferences does not vary with the size of the voting body. To do this, we introduce a continuous and strictly decreasing preference function $v(\cdot)$ on $[0, 1]$ and impose a grid of size $n$ (odd) such that voter $i$’s relative preference $v_i$ for $a$ is given by $v\left(\frac{i-1}{n-1}\right)$. Notice that the median voter, who has index $i = \frac{n+1}{2}$, always has the relative preference strength $v\left(\frac{1}{2}\right)$ independent of $n$. To ensure that the intrinsic preference of the median voter favors policy $b$, we assume that $v\left(\frac{1}{2}\right) < 0$.

We first consider the case analyzed in Groseclose and Snyder—where bribes are discriminatory. Our main result here is to show that the strategic effect can be sufficiently strong that it overcomes the direct effect. As a result, larger voting bodies may be more buyable than smaller voting bodies. Indeed, as we demonstrate below, the cost of bribing a voting body may be non-monotonic in its size.

To see this, consider the following simple example. Suppose that $W_B = 1$ and that the preference function is

$$v(x) = \begin{cases} 
\frac{1}{3} & \text{if } x < 0.5 \\
-\frac{1}{1000} & \text{if } 0.5 \leq x \leq 0.6 \\
-2 & \text{if } x > 0.6
\end{cases}$$

That is, the voters are divided into three groups: 1) supporters of policy $a$ (those voters with indices such that $\frac{i-1}{n-1} < 0.5$), 2) moderates (those voters with indices such that $\frac{i-1}{n-1}$ lies between $0.5$ and $0.6$) and, 3) strong supporters of policy $b$ (those voters with indices such that $\frac{i-1}{n-1} > 0.6$).\(^4\)

\(^4\)Of course, this preference function is not strictly decreasing in the index. Changing the example
In that case, it is a simple matter to show that the least-cost optimal contract entails group A optimally bribing all its supporters as well as up to three moderates. Since strong supporters of policy b dislike policy a intensely ($v_i = -2$), it is never cost effective for group A to bribe these voters. When there are three or fewer moderates, group A economizes on its overall payments by bribing all of them. Once there are three moderates in the coalition, however, group A no longer pays its supporters anything and, therefore, further expansion of the supermajority generates no savings.

Figure 3 displays A’s total costs for this example as the size of the voting body varies. It is interesting to note the points in the figure where the costs jump. These jumps occur when the number of moderates increases by exactly one voter—which happens when the size of the voting body increases by 10 voters—until there are three moderates (which occurs when the voting body consists of 21 voters).

At these jump points, the strategic effect is operative. Consider the first jump point, which occurs when the voting body grows from 9 to 11 members. In that case, group A is able to cut by half the amount of the surplus, $K(m^*)$, it has to guarantee each of the voters in its coalition in order to deter B. This economization occurs for the standard supermajority reasons. Since this payment was previously being made to all intrinsic supporters of policy a as well as all the moderates, its reduction more than offsets the increasing costs associated with the direct effect of having to bribe two more voters.

The next jump point, which occurs when the voting body grows from 19 to 21 members, illustrates the same effect. Here, the amount of the surplus required to deter B falls to $\frac{1}{3}$; hence group A no longer has to pay its intrinsic supporters at all while it continues to save on payments to moderates. Beyond this point, there are no further economies from recruiting ever larger supermajorities. Hence, group A’s costs are flat thereafter.

Where does the common intuition fail in this example? While the payments made to reverse the intrinsic preferences of those recruited into A’s coalition scale with the size of the voting body, the size of the optimal supermajority does not. Regardless of the size of the electorate, it is optimal to bribe three moderates and not more. When the size of the voting body is sufficiently small (i.e., there are fewer than three moderates), growth in the size of the voting body can lead to cost reductions owing to economies from the strategic effect of expanding the size—in levels not percentages—of the supermajority. Once the number of moderates is three or more, there is no additional scope for economies due to the strategic effect, as it is never optimal to choose a supermajority of more than three moderates. Hence, the direct effect dominates. But in terms of the example, the direct effect is zero owing to the
to exactly fit this assumption is just a matter of adding tiny amounts of slope and making the preference function continuous at the jump points. This can all be readily done while affecting the costs by only an infinitesimal amount. We opted not to do this here, because it obscures the fundamental intuition of the example without adding any economic content. Detailed notes for a fully-fledge continuous example are available from the authors upon request.
uniformity of intrinsic preference of supporters.

What happens when bribes are non-discriminatory? Again, the strategic effect allows group $A$ to economize on its transfers as the number of voters increases. This may readily be seen from the fact that the additional payment to deter, $\frac{W_B}{M}$, decreases with the number of voters. As with discriminatory vote buying, the strategic effect may dominate and the costs of successfully buying the voting body may be non-monotonic in its size.

To see this, consider the case where the preference function is linear. Let $v (\frac{i-1}{n-1}) = c - d \frac{i-1}{n-1}$, where $c$ and $d$ are parameters of the model and $d > 2c$ such that $v (\frac{1}{2}) < 0$. When $c = 1$, $d = 2.1$, and $W_B = 20$ the cost to $A$ of executing the contract in Proposition 2 are shown in Figure 4.

Note that, when the size of the voting body is small, the least-cost successful vote buying contract is accepted by all voters. This is the case for voting bodies consisting of up to 37 voters. Beyond this point, the surplus needed to deter $B$ declines sufficiently that voters with the highest indices prefer to reject $A$’s contract and vote according to their intrinsic preferences. As a consequence, the relative supermajority recruited by $A$ declines with the size of the voting body. This may be readily seen in Figure 5. More importantly, the combination of smaller relative supermajorities and smaller transfers dominates the extra costs of bribing larger numbers of voters over a large region of voting body sizes. Indeed, as the figure shows, it is more expensive for group $A$ to bribe a voting body consisting of 3 voters than it is to bribe a voting body consisting of 259 voters.

Eventually, as the size of the voting body continues to increase, having to bribe more voters starts to dominate the savings from the strategic effect and costs increase. However, it is not until the size of the voting body reaches 759 that the costs to $A$ are as high as they were when faced with a voting body of only 37 voters. In short, if there are costs to expanding the size of the voting body (as there undoubtedly are), there may be a “sweet spot” in terms of legislative size that insulates the voting body from interest group pressure as much as possible. In our example, the sweet spot is 37.

One may wonder if the non-monotonicity in costs is always present under non-discriminatory vote buying. The answer is no. To see this, simply amend the example such that $W_B = 1$. In that case, it is straightforward to verify that the cost of an optimal bribe scheme is strictly increasing in the size of the voting body.

To summarize, we have shown.

**Proposition 4** Under both discriminatory and non-discriminatory vote buying, it may be cheaper for group $A$ to bribe a larger voting body than a smaller voting body.

### 4.2 The Secret Ballot

A common strategy to deter vote buying is the imposition of the secret ballot. Clearly, the idea is that making individual votes unobservable severely limits the ability of
lobbying groups to contract (formally or informally) on individual votes. An early expression of this idea is found in the Chartist Petition of 1838, which states:

The suffrage, to be exempt from the corruption of the wealthy and the violence of the powerful, must be secret. (Webster, 1920, p. 145).

Indeed, the infusion of Chartist ideas is widely credited with the decision of various Australian territories to implement the secret ballot in the 1850s, with the English and several American states adopting the practice later in the 19th century (Newman, 2003).

In response to the secret ballot, interest groups have devised a number of clever strategies to continue to buy individual votes. One such strategy is known as the Tasmanian Dodge, which arose in response to the early Australian reform efforts. In this scheme, an interest group steals or forges a single empty ballot before the election. It then fills out this ballot and provides it to a voter. The voter casts the filled-out ballot while receiving a new, blank ballot from the polling station. The blank is then returned to the interest group in exchange for payment and the process is repeated.\(^5\)

Robert Caro describes less subtle strategies used to circumvent the secret ballot in Texas in the 1930s:

(...) election supervisors would, in violation of law, stand alongside each voter in the voting booths to make certain that each vote was cast as paid for. (p.719) Even if the voter was allowed to cast his ballot in secrecy, he had little chance of escaping unnoticed if he disobeyed instruction; each ballot was given a number that corresponded to the number on a tear-off sheet attached to the ballot, and a voter had to sign his name on the sheet before it was torn from the ballot and the ballot cast. (p. 721)

While safeguards have been put in place to counteract practices like these, it is interesting to note that recent initiatives designed to spur voter turnout may actually undermine the secrecy of the ballot. For instance, the state of California recently implemented a policy allowing voters to become “permanent absentee voters,” which saves them the trip to the polling station. As with standard absentee balloting, voters are mailed paper ballots in advance of the election. They fill them out at home and send them back. It would be a simple matter for an interest group to acquire these blank—but signed—paper ballots from permanent absentee voters in exchange for money. The interest group could then fill out the ballots as desired and mail them.

Still another way to circumvent the secret ballot is “negative vote buying”—the practice of paying opposition supporters in exchange for their not voting in an election. Cox and Kousser (1981) offer a thoughtful analysis of the effects of this practice

\(^5\)This practice is called “telegraphing” in Cambodia, and “lanzadera” (Spanish for “shuttle”) in the Philippines. (Shaffer, 2002.)
on voter turnout in New York state by reviewing newspaper articles describing various instances of (positive and) negative vote buying. (See also Heckelman, 1998.) Formally analyzing the case of negative vote buying requires amending the model to allow for a third choice, namely, abstention and specifying payoffs for this choice. Since the spirit of the present paper is to further analyze the model of Groseclose and Snyder, which has no abstention, here, we omit consideration of this case.6

In certain instances, the above schemes to circumvent the secrecy of the ballot may be either infeasible (owing to adequate safeguards) or impractical (perhaps owing to scale, as in a general election). In that case, it may still be possible to circumvent the secret ballot by relying on contracts based on policy outcomes rather than individual votes. Since policy outcomes remain observable, such schemes are feasible in virtually all circumstances, and they scale in quite a practical fashion. A real world example can be found in the 2000 Taiwanese presidential election. Here, the ruling party set up subsidized betting parlors that offered extremely favorable odds on a bet that paid in the event that Lien Chen, the ruling party’s candidate won the race (August, 2000). Thus, a voter accepting such a bet was entering into a contract where the ruling party’s payment to him was dependent on the outcome of the election.

When contracting is possible only over outcomes, is it still the case that lobbying groups can successfully bribe voters? How costly are such schemes to implement relative to contracting on votes directly? To study these questions, we analyze the case where the two interest groups are limited to offering (voter specific) contracts contingent only on the policy outcome. Our first result shows that the introduction of the secret ballot is indeed beneficial. Specifically, when it is the case that, absent vote buying, policy $b$ enjoys supermajority support (formally, $v_{M-1} < 0$) then it is impossible for group $A$ to offer bribes in such a way as to guarantee its most preferred outcome.

**Proposition 5** If $v_{M-1} < 0$, then successful vote buying contracts do not exist when only outcomes are contractible.

One may wonder what goes wrong for group $A$ when it can only contract over outcomes. The problem stems from the fact that incentives are only created in case a voter believes that he is pivotal. But when there is supermajority intrinsic support for policy $B$, there always exists an equilibrium in which voters ignore the contract offered by $A$ and vote according to their intrinsic preferences. Clearly, in such a situation, no voter perceives himself as pivotal and, hence, the incentive effects of $A$’s contract are nullified.7

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6In a companion paper (Morgan and Vardy, 2007) we study negative vote buying when the payoffs from abstention lie halfway between the payoffs from voting for $a$ and the payoffs from voting for $b$. We show that the resultant least-cost contract is qualitatively similar to a $K(m^*)$ contract.

7This result does not in an essential way rely on a voter believing that he is pivotal with zero probability. If a voter ascribed a small but positive probability to being pivotal, group $A$ could pay him a transfer to switch his vote from $b$ to $a$. But the necessary transfer becomes unbounded as the probability of being pivotal goes to zero in the limit.
While the previous result shows that $A$ cannot guarantee its preferred policy outcome under supermajority opposition, does there exist an equilibrium in which $A$ obtains its preferred policy? The next proposition shows that, even with the secret ballot, there exists an equilibrium in which $A$ successfully buys the election. Interestingly, the contract offered by $A$ to achieve this outcome at its lowest possible cost closely resembles the contracts derived by Groseclose and Snyder.

**Proposition 6** When only outcomes are contractible, a $K(M)$ contract is a least-cost contract such that there exists an equilibrium in which policy $a$ is adopted.

Furthermore, if $v_{M-1} > 0$, i.e., policy $b$ only enjoys simple majority intrinsic support, then a $K(M)$ contract is a least-cost successful contract.

Combining the results of Proposition 1 and 6, a cost ranking across simple contracts is possible.

**Corollary 1** When vote buying is discriminatory, it is always cheaper for $A$ to contract on votes than to contract on outcomes.

The bluntness of the outcome based contractual instrument limits $A$ to buying a bare majority rather than a supermajority of voters. The reason is that the incentive effects of the contracts which depend on a voter being pivotal are completely undermined if $A$ tries to buy a supermajority. Ironically, in that case $A$’s preferred proposal would go down to defeat. Buying a bare majority rescues the incentive effects but is generally very expensive to implement if it is to deter group $B$ from re-bribing. The upshot is that the incentives for legislative capture are significantly reduced.

Propositions 5 and 6 suggest that, in the presence of competition, the introduction of the secret ballot offers quite a powerful remedy against vote buying. It is interesting to contrast this result with the effect of the secret ballot in the absence of competition. Let group $A$ offer all voters whose intrinsic preference favor policy $b$ up to the median an outcome contingent contract that pays $-v_i$ in the event that policy $a$ is adopted and pays nothing if policy $b$ is adopted. True, group $A$ still cannot guarantee the adoption of its preferred policy. However, if it is adopted, it costs group $A$ exactly the same as when $A$ can contract on votes directly. Therefore, if $A$ succeeds, its cost under the secret ballot is no more than under an open ballot. Moreover, if $A$ does not succeed, it will not have to pay anything. Hence, the introduction of the secret ballot will not deter group $A$ from at least trying to buy the vote.

Thus, the secret ballot is much more effective as a means to prevent vote buying in the presence of competition than in the absence. Also, note that the secret ballot is not without costs; most notably the loss of accountability in settings such as legislatures. It is arguably very important that constituents be able to hold an elected legislator accountable on the basis of his voting record.

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8 Again, notice that there always exists an equilibrium in which all voters ignore the outcome based contract and vote according to their intrinsic preferences.
5 Sophisticated Contracts

Real world vote buying contracts contingent on the combination of an individual’s vote and the policy outcome can also be found. An example of this type of contract came to light in the course of the bribery investigation into Salt Lake City’s bid for the 2002 Olympic Winter Games. According to press reports, certain members of the IOC received payments ranging from $500,000 to $1 million in exchange for their votes. In addition, a “bonus” of $3-5 million was to be paid if the city won the Olympic bid. Thus, the payment scheme was contingent both on an individual’s vote as well as the overall outcome.9

How does vote buying change when one considers more sophisticated contracts with more than one contractual contingency? Are policy making bodies such as the IOC more or less susceptible to outside influence under these circumstances? To examine this question, we consider two cases of complex contracts: contracts where payments are contingent on individual votes and policy outcomes, and contracts where payments are contingent on individual votes and vote shares.

5.1 Contracting on Votes and Outcomes

The possibility of conditioning bribes on both votes and outcomes would seem to offer strategic opportunities for A to reduce its costs of obtaining its preferred policy. As the next proposition shows, however, this is not the case. The least-cost successful contract costs \( A \) exactly the same as when it contracted solely on votes and ignored the outcome altogether. Formally,

**Proposition 7** When contracts can be contingent on both votes and outcomes, then a \( K (m^*) \) contract is a least-cost successful contract.

Why does the possibility of conditioning on outcomes not help in any way? Notice that, while \( A \) could offer the \( K (m^*) \) contract only under the joint contingency of a vote for \( a \) and policy \( a \) being adopted, this would save no money in equilibrium. In addition, such a contract is vulnerable to exploitation by \( B \). In particular, \( B \) can recruit a supermajority at arbitrarily small cost. As long as voters believe that \( B \)’s supermajority coalition will hold together, there is no upside to switching one’s vote to \( A \). Hence, even though \( A \) could contract not to pay in the event of a loss, it is, in fact, optimal to pay. Indeed, this is essential in precluding \( B \) from attempting to recruit a supermajority.

Of course, the contract in the Olympic vote buying scandal does not correspond to the least-cost successful contract derived in Proposition 7. After all, the bonus payment is outcome-contingent. One possible explanation for the discrepancy is that

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http://news.bbc.co.uk/1/hi/world/europe/233742.stm
the lobbying groups may be budget constrained. Indeed, Salt Lake City found itself with considerably more financial resources after its bid was successful than before, and this may have necessitated the bonus scheme. It is well-known that, even in simpler contractual settings, the introduction of budget constraints creates substantial complications in the analysis (see, for instance, Dekel, Jackson and Wolinsky, 2005). While we think that budget constraints have an important role to play in the analysis, we leave this for future research.

5.2 Contracting on Votes and Vote Shares

As we saw above, the ability to contract on votes and outcomes provides no benefit for group $A$ relative to conditioning only on votes. Of course, if votes are publicly observable, then, instead of conditioning on the outcome, the lobbying group might just as well choose to condition on vote share. In an interesting paper, Dal Bo (2004) shows that, in the absence of competition, this contractual contingency provides a powerful lever for a lobbying group. Here, we examine this class of contracts in the presence of competition.

Let $#a$ denote the number of votes cast for policy $a$. The number of votes for $b$ is then $n - #a$. We show that

**Proposition 8** When contracts can be contingent on votes and vote shares, the following is a least-cost successful contract:

For $v^{-1}(K(M+1)) \leq i \leq M+1$

$$t_i = \begin{cases} 
\max (-v_i, 0) & \text{if } c_i = a \text{ and } #a \geq M+1 \\
K(M+1) - v_i & \text{if } c_i = a \text{ and } #a < M+1 \\
0 & \text{if } c_i = b 
\end{cases}$$

For $i < v^{-1}(K(M+1))$ or $i > M+1$, the null contract is offered.

By contracting on votes and vote shares, group $A$ is able to almost completely deflect the effects of competition by compensating “loyal” voters in the event of a defeat or a narrow victory for policy $a$. That is, voters receive the most compensation from group $A$ when they are most needed, i.e., when the success of adopting policy $a$ is most in doubt. In contrast, when success is assured, that is, when $a$ receives a supermajority of support, the contract calls for minimal compensation for loyalty. In other words, competition from $B$ is deflected by the promise of high payments when $a$ is at risk, while keeping actual equilibrium payments to a minimum owing to a successful “entry deterrence” strategy. Hence, competition has almost no effect on $A$’s cost of successful vote buying. Formally,

**Corollary 2** The least-cost successful contract when $B$ is present costs $A$ the same amount as when $B$ is absent and $M+1$—instead of $M$—votes are required for passage of policy $a$.  

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Discussion

The key for A to successfully defend against an invasion by B is to create strong incentives to vote for a, both in the event that B tries to recruit a bare majority and in the case where B tries to recruit a supermajority. The contract achieves this by having A offer to pay generously in the event that its proposal loses or wins with a minimal majority and the voter remains loyal to A. Since, to successfully invade, B must pay each voter an amount equal to his opportunity cost of foregoing A’s proposal, this strategy dramatically raises B’s cost of entry while not affecting A’s costs in the event that it recruits a supermajority. And, as long as B stays out, recruiting a supermajority is cheap and easy for A.

By conditioning on votes and vote shares, A is in a position to offer deterring incentives without actually having to pay for them in equilibrium. Corollary 2 thus highlights the extreme susceptibility of voting bodies to vote buying in rich contractual environments. The policy prescription here is clear. Contingent contracts along the line specified above must be made extremely costly, perhaps by penalties such as forfeiture of office or heavy fines.

Buying out the Competition

Following Groseclose and Snyder, we have so far assumed that it was impossible for group A to directly contract with B and thereby remove the threat of competition prior to contracting with the voters. Indeed, it is straightforward to show that for the class of simple contracts (and, by extension, contracts contingent on votes and outcomes) contracting directly with B is cheaper for group A than contracting solely with the voters.

However, there are real-world situations in which the no-buy-out assumption does not hold. For example, in the Lebanese parliamentary elections of 1960, the following incident occurred:

[A] candidate ... was offered $7,000 to quit the race for the less than $6,000-a-year Deputy’s [member of Parliament’s] job. With pay so small, why was the bribe so high? Explained one candid hopeful: “Any Deputy is sure to be invited to become a bank director—at $4,000 a year. Also, there’s always the wayward young man whose parents will pay $1,500 to spring him from jail. And then a Deputy gets immunity from police searches of his car. Any time he drives out to the country, he can load up with $1,000 worth of hashish.” (Time, Monday, Jun. 27, 1960)

With this example in mind, let us compare the cost of contracting on votes and vote shares with the cost of first buying out the competition. Clearly, to buy out group B, group A can make a take-it-or-leave-it offer of $W_B + \varepsilon$, for arbitrarily small $\varepsilon$. Group B will accept and, subsequently, A can contract with the voters under
monopsony conditions. The total cost to $A$ of this scheme is

$$C_A = \sum_{i=0}^{M} \min(-v_i, 0) + W_B + \varepsilon$$

In contrast, when $A$ can contract on votes and vote shares, under a least-cost successful contract group $A$ incurs a cost of

$$C'_A = \sum_{i=0}^{M} \min(-v_i, 0) - v_{M+1}$$

Thus, under the mild restriction that interest group $B$ cares more about policy $b$ than individual voter $M+1$ cares about his vote for $b$, it follows that

**Remark 1** When contracts can be contingent on votes and vote shares, it is cheaper for group $A$ to only contract with voters than to buy out the competing interest group $B$.

### 6 Related Literature

Having presented the results of our paper, it is useful to place our analysis in the context of the larger (theoretical) literature on vote buying. Obviously, our paper builds on the seminal work of Groseclose and Snyder and extends their model in various directions, both in terms of the structure of the electoral system as well as the contracting environment.\(^\text{10}\)

There are two other papers that have considered sophisticated vote buying contracts. Dal Bo (2004) studies contracts involving votes and vote shares; however competition is absent in his model. Dekel, Jackson, and Wolinsky (2005) study contracts based on votes and outcomes. However, their model differs from ours in many respects—voters are non-strategic, interest groups are budget constrained, and the process of vote buying is modeled as an alternating offer scheme.

Our paper is also somewhat related to the literature on how the structure of electoral systems affects corruption in government. Notable in this literature is Meyerson (1993) who examines how electoral systems differ in their ability to sort between corrupt and non-corrupt candidates running for office.\(^\text{11}\) In contrast, our concerns are not about sorting among candidates with varying corruption levels. Rather, we are interested in how electoral systems differ in their susceptibility to vote buying.

There is a larger literature on the buying and selling of influence that differs significantly in both its concerns and modeling approach from our work and the papers above. Specifically, voting plays little role in this branch of the literature, as the policy

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\(^{10}\)See also, Banks (2000).

\(^{11}\)See also Persson, Tabellini, and Trebbi (2003) for a useful survey.
is typically determined by a single player. Some of the earliest work in this area (see Tullock, 1972, 1980) models the policy maker as non-strategic and supposes that competition among interest groups takes the form of an imperfectly discriminating all-pay auction, or contest. One of the primary concerns of this literature is how variation in the structure of the auction affects rent-seeking expenditures by lobbying groups. The interested reader should consult Nitzan (1994) for an excellent survey. Another important approach to modelling competition for influence is the use of menu auctions (Bernheim and Whinston, 1986). Unlike the rent-seeking literature, here, the policy maker is modeled as a strategic player. The seminal work along these lines is Grossman and Helpman (1994) who apply this analysis to trade policy. Other notable work in this vein includes Grossman and Helpman (1996, 1999).

7 Conclusions

In this paper, we have studied the buyability of voting bodies such as legislatures, committees, and electorates, under a variety of circumstances. Based on real world evidence, we have first distinguished between discriminatory and non-discriminatory vote buying. While discriminatory vote buying is clearly cheaper in the absence of competition, we have shown that non-discriminatory vote buying may actually be less costly when interest groups compete. Next, we investigated how the cost of successful vote buying depends on the size of the voting body. Here, we found an analogous result: while the cost is clearly increasing in the number of voters in the absence of competition, larger voting bodies may be cheaper to buy than smaller voting bodies in the presence of competition. Of course, both of these results are consequences of the strategic interactions between the competing interest groups.

Competition also plays an important role in the effectiveness of the secret ballot as an anti-vote buying measure. Here, we have modelled the secret ballot as forcing vote buying contracts to be outcome-contingent rather than vote-contingent. While the secret ballot is effective in the presence of competition, we have shown that it is much less so in its absence.

Finally, we have studied the buyability of voting bodies when vote buying contracts can depend on multiple contingencies. Specifically, we have looked at contracts contingent on votes and outcomes, and contracts contingent on votes and vote shares. While the option to add outcome based contingencies to vote based contracts turns out to be worthless, the option to make contracts depend on vote shares as well as on individual votes turns out to be extremely valuable. Availability of such contracts put voting bodies uniquely at risk of being bought, even in the presence of competition.

Taken together, what are the implications for policy makers? First, the presence of competition is by no means a guarantee that underlying preferences of voters will be represented in the form of policy outcomes. Perhaps more surprisingly, the presence of competition does not necessarily even raise the costs of interest groups seeking to influence policy. Indeed, sophisticated interest groups can construct contracts that
nullify competition (almost) completely. Moreover, our paper highlights the fact that the effectiveness of various policy tools designed to curb influence depends crucially on the presence or absence of competition. One such tool, the extension of the voting franchise, is shown to sometimes have the surprising effect of making it cheaper for interest groups to wield influence, but only in the presence of competition. Another tool, the secret ballot, proves a robust deterrent in the presence of competition, but is of little help in curtailing influence in a setting where competition is absent. This suggests the need to think carefully about competitive conditions and motivations of those seeking to wield influence as well as the motives of voters themselves in developing anti-corruption policies.
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A Proofs of Propositions

Proposition 1 Let $m^* \in \arg \min C_A(m)$. Then a $K(m^*)$ contract is a least-cost successful contract under discriminatory vote buying when only individual votes are contractible.

Proof. See GS. □

Proposition 2 The following comprises the least-cost successful contract for $A$ under non-discriminatory vote buying.

Group $A$ offers payments in the amount $t_A = \frac{W_B}{M} - v_M$. Group $B$ offers the null contract. All voters with indices $i \leq m(t_A)$ are in $A$’s coalition.

Proof. First note that, given $A$’s offer, $B$’s unique best response is to choose the null contract. To see this, note that $B$ would have to recruit up to the median voter, $M$, if he wanted to implement policy $b$. To do so, he has to offer at least an amount $t_B = \frac{W_B}{M} + \varepsilon$, $\varepsilon > 0$. This offer will be accepted by all constituents with indices $i \geq M$. Thus, any contract in which $B$ gets its preferred policy costs at least $C_B = W_B + M\varepsilon$

and since this strictly exceeds the value to $B$ of its preferred policy, $B$ is strictly better off offering the null contract.

Thus, we have shown that $A$’s offer constitutes a successful contract. It remains to show that it is also least-cost. Suppose that $A$ offers a transfer $t_A' < t_A$, then by offering $t_B = t_A' - v_M + \varepsilon$, group $B$ will attract all voters with indices $i \geq M$. For $\varepsilon$ sufficiently small, this contract will cost $B$ less than $W_B$ and, hence, $A$’s contract is not successful.

Therefore, the contract described in the proposition is indeed the least-cost successful contract. □

Proposition 3 Let vote buying be non-discriminatory. Then, under the least-cost successful contract, $A$ always buys a supermajority of voters

Proof. For a supermajority of voters to receive payments from $A$, it must be the case that voter $M + 1$ is in $A$’s coalition. This amounts to the condition that $v_{M+1} + t_A \geq 0$

Substituting for $t_A$, we obtain

$v_{M+1} - v_M + \frac{W_B}{M} \geq 0$

Noting that $v_{M+1} < v_M < 0$, it is convenient to rewrite this inequality as $v_M - v_{M+1} \leq \frac{W_B}{M}$

which holds by Assumption 1. □
Proposition 4 Under both discriminatory and non-discriminatory vote buying, it may be cheaper for group A to bribe a larger voting body than a smaller voting body.

Proof. See main text. ■

Proposition 5 If $v_{M-1} < 0$, then successful vote buying contracts do not exist when only outcomes are contractible.

Proof. We will show that for any contract offered by A, there exists a subgame perfect equilibrium where policy $b$ is adopted. To see this, suppose group A offers some arbitrary contract, group B does nothing and voters vote according to their intrinsic preferences. Since A’s contract is only contingent on outcomes, it is payoff relevant to voter $i$ only to the extent that $i$ is in a position to alter the outcome by his vote. Furthermore, since $b$ commands a supermajority of intrinsic support, then, under the putative equilibrium, each voter has zero probability of affecting the policy by changing his vote. At the same time, changing one’s vote from $b$ to $a$ leads to a first order payoff effect in the amount $v_i$. Therefore, voters can do no better than to vote according to their intrinsic preferences. Furthermore, since B obtains its preferred outcome at no cost, it can do no better than to do nothing. ■

Proposition 6 When only outcomes are contractible, a $K(M)$ contract is a least-cost contract such that there exists an equilibrium in which policy $a$ is adopted.

Furthermore, if $v_{M-1} > 0$, i.e., policy $b$ only enjoys simple majority intrinsic support, then a $K(M)$ contract is a least-cost successful contract.

Proof. First note that A cannot successfully buy a supermajority since, were A to do so, no member of the supermajority coalition would be pivotal. In that case, voting according to intrinsic preferences is optimal and policy $b$ would be adopted. Hence, A must be buying a simple majority.

To successfully deter B with a simple majority, it must be the case that the cost to B of recruiting any single member of A’s coalition is at least $W_B$. Thus, all members of the A coalition must obtain surplus of at least $W_B$ when $a$ is adopted.

Next, notice that all voters in A’s coalition who are promised a positive payment in the event that $a$ is adopted must earn the same surplus. (If not, then such a contract is not least cost, since A could successfully offer a cheaper contract by lowering the payments to those receiving the higher surplus.). Thus, it must be the case that, if voter $i$ is in A’s coalition and receives a positive transfer, this transfer must be $W_B - v_i$. Finally, notice that an (outcome based) $K(M)$ contract is a least-cost contract satisfying this property while still deterring B.

To see that there exist equilibria in which policy $a$ is not adopted following the offer of the $K(M)$ contract by A, suppose that B does nothing and that all voters simply vote according to their intrinsic preferences. Then, if $v_{M-1} < 0$, policy $b$ is adopted. Furthermore, since no voter is pivotal, voters can do no better than to vote according to their intrinsic preferences.
Finally, notice that, when \( v_{M-1} > 0 \), then, when \( A \) offers the \( K(M) \) contract and \( B \) offers any rationalizable contract, all voters perceive the probability of being pivotal as being equal to one. Hence, voters can do no better than to accept the \( A \) contract and vote accordingly. Hence, in this case, the \( K(M) \) contract is a least-cost successful contract.

**Proposition 7** When contracts can be contingent on both votes and outcomes, then a \( K(m^*) \) contract is a least-cost successful contract.

**Proof.** For a contract to be successful, it must deter \( B \) from successfully recruiting a majority of any size. To deter \( B \) from recruiting a bare majority it must be the case that, for any bare majority recruited by \( B \), the outside options of those recruited—which consist of the joint event of voting for \( a \) and \( a \) winning—must sum up to at least \( W_B \).

We claim that the cheapest way of doing this entails a \( K(m^*) \) contract. Suppose not. Then there exists a contract such that \( B \) is deterred from recruiting a bare majority which costs less than \( K(m^*) \). Suppose under this alternative contract, \( B \) has to recruit \( m' - M \) voters to obtain a bare majority. Clearly, \( B \) will choose the voters whose outside option under the contingency that they defect from \( B \)’s coalition is the smallest. Therefore, it must be the case that the surplus of these voters under the contingency the joint of voting for \( a \) and policy \( a \) being chosen sums to \( W_B \). Moreover, this must be true of all coalitions of size \( m' - M \).

Next, notice that all voters recruited into \( A \)’s coalition who are promised a positive payment in the event that they vote for \( a \) and \( a \) is chosen must earn the same surplus. Let this surplus amount be \( S \). (If not, then such a contract is not least cost since \( A \) could successfully offer a cheaper contract by lowering the payments to those receiving the higher surplus.). Thus, it must be the case that, if voter \( i \) is in \( A \)’s coalition and receives a transfer, this transfer must be \( S - v_i \) under the contingency that \( i \) votes for \( a \) and policy \( a \) is adopted. But now recall that a \( K(m^*) \) contract is in fact the least-cost contract satisfying this property while still deterring \( B \).

Hence, if \( A \) offers a \( K(m^*) \) contract, it will successfully deter \( B \) from recruiting a bare majority and furthermore, it is the cheapest possible way to do it.

Next, to deter \( B \) from recruiting a supermajority, it must be the case that the outside options, which consist of the joint event of voting for \( a \) and \( a \) losing, must sum to at least \( W_B \). The \( K(m^*) \) contract satisfies this condition. Hence, a \( K(m^*) \) contract is a least-cost successful contract.

**Proposition 8** When contracts can be contingent on votes and vote shares, the following is a least-cost successful contract:

\[
\text{For } v^{-1}(K(M + 1)) \leq i \leq M + 1t_i = \begin{cases} 
\max(-v_i, 0) & \text{if } c_i = a \text{ and } \# a \geq M + 1 \\
K(M + 1) - v_i & \text{if } c_i = a \text{ and } \# a < M + 1 \\
0 & \text{if } c_i = b
\end{cases}
\]

\[
\text{For } i < v^{-1}(K(M + 1)) \text{ or } i > M + 1, \text{ the null contract is offered.}
\]
**Proof.** First, we show that the contract in the proposition is least-cost. Note that, in equilibrium, the cost of the contract is

\[ \sum_{i=1}^{M} \max (-v_i, 0) \]

and recall that, in the absence of competition, the minimum cost of obtaining \#\(a\) = \(m\) votes is

\[ C_A(m) = \sum_{i=1}^{m} \max (-v_i, 0) \]

Therefore, the only contracts with potentially lower costs have \#\(a\) = \(M\). What do successful contracts of that sort look like? To deter \(B\) from re-bribing one voter and obtaining his preferred policy, all voters voting for \(A\) must receive a surplus of at least \(W_B\) in the event that \(A\) is approved with exactly \(M\) votes. Hence, these contracts are strictly more costly than the contract in the proposition.

It remains to show that the contract in the proposition is a successful contract. We claim that if \(A\) offers this contract, \(B\) can do no better than doing nothing, and voters \(i \leq M + 1\) can do no better than voting for \(a\).

First, suppose \(B\) does nothing and consider a deviation by any voter \(i\) currently voting for \(a\). By deviating from \(a\) to \(b\), the voter earns

\[ \Delta U_i = -v_i - t_i \]

For \(i < v^{-1}(K(M + 1))\), \(v_i > 0\) and \(t_i = 0\). Hence this is strictly unprofitable. For \(v^{-1}(K(M + 1)) \leq i \leq M + 1\), \(t_i \geq -v_i\) and, therefore, \(\Delta U_i \leq 0\). Hence this is also unprofitable.

Next, we show that \(B\) has no profitable deviation. Clearly, if \(B\) offers a contract that does not alter the policy, it does not benefit. Suppose \(B\) alters the policy by recruiting \(k \geq 2\) voters from \(A\)’s coalition. To induce these voters to switch, each must be paid the value of his outside option conditional on policy \(b\) being adopted or policy \(a\) being adopted with a bare majority (since these are the two possible contingencies associated with deviating from \(b\) to \(a\) when \(k \geq 2\)). That is, for all \(k\), each voter must be paid an amount at least \(K(M + 1)\) and, by construction

\[ kK(M + 1) \geq W_B \]

for \(k \geq 2\). Therefore, \(B\) has no profitable deviation. This completes the proof. ■

**B Rationing**

One may worry that the supermajority result for non-discriminatory vote buying solely arises from the fact that \(A\) cannot ration the set of voters who take up its
offer. Here, we show that this is not the case. Let us amend the model as follows. Suppose that each of the groups are restricted to offering each constituent either the null contract or a \( t \) contract where \( t \) is a fixed transfer that does not depend on the identity of the constituent. Thus, by offering null contracts to certain voters, a group can ration its transfers.

To obtain the supermajority result for the case of rationing, preferences need to approximate the continuous relative preference model of Groseclose and Snyder. Hence, we extend Assumption 1 to all voters. Specifically, we assume that

**Assumption 2.** For all \( i, v_i - v_{i+1} \leq \frac{W_B}{M} \).

**Proposition 9** Let vote buying be non-discriminatory. Then, under a least-cost successful contract with rationing, \( A \) always buys a supermajority of voters.

**Proof.** Suppose \( A \) successfully recruits a simple majority. That implies that all voters \( i = 1, 2, \ldots, M \) must enjoy a payoff of at least \( W_B \). The cost to \( A \) of this scheme is

\[
C_A^M = (W_B - v_M) (M - v^{-1}(W_B) + 1)
\]

Now suppose \( A \) successfully recruits a supermajority of \( M + 1 \). This implies that all voters \( i = 1, 2, \ldots, M + 1 \) must enjoy a payoff of at least \( \frac{W_B}{2} \). The cost to \( A \) of this scheme is

\[
C_A^{M+1} = \left( \frac{W_B}{2} - v_M \right) \left( M + 1 - v^{-1} \left( \frac{W_B}{2} \right) + 1 \right)
\]

From Assumption 2 it follows that

\[
v^{-1} \left( \frac{W_B}{2} \right) - v^{-1} (W_B) \geq 1
\]

Hence, \( C_A^{M+1} < C_A^M \).
Figure 1: Cheap Discriminatory Vote Buying
Figure 2: Cheap Non-Discriminatory Vote Buying
Figure 3: Cost of Discriminatory Vote Buying as a Function of $n$
Figure 4: Cost of Non-Discriminatory Vote Buying as a Function of $n$
Figure 5: Size of A’s Coalition as a Function of $n$