Title
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Publication Date
1987-03-01
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March 1987
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Summary

Relativistic electrons passing through two identical magnetic sections generate synchrotron radiation whose spectrum is strongly modulated as the photon energy varies. The modulation is caused by the interference of radiation from each section, and has been observed [1] in the spectrum of spontaneous radiation from transverse optical klystron which utilizes two undulators. In this paper, we analyze and apply another device based on two simple wigglers. The device, which will be called the interference wiggler, can be used for precise diagnostics of electron beam energy; by analyzing the modulated spectrum with a monochromator, the electron energy can be determined up to an accuracy of $10^{-3}$ or $10^{-4}$. In this paper we develop general design criteria for interference wigglers. We also give several example designs to measure the electron energy to an accuracy $10^{-4}$ for the planned electron beam facility at CEBAF [2], and to an accuracy $10^{-3}$ for the 1–2 GeV Light Source at Berkeley [3].

Spectrum of Interference Wigglers

The electron trajectory in an interference wiggler is shown in Fig. 1. The trajectory will be assumed to lie on the horizontal plane. In wiggler approximation [4], the radiation in the direction $(\phi, \psi)$, where $\phi$ and $\psi$ are respectively the horizontal and vertical angles, comes mainly from small segments of electron trajectory about the points where the slope is parallel to $\phi$. For the trajectory in Fig. 1, there are in general four such points labelled 1, 2, 3 and 4. Among these, we will consider only 1 and 2, assuming that either that 3 and 4 are sufficiently separated transversely from 1 and 2 so that they can be considered separately, or that the radiation intensity from 3 and 4 is much weaker due to weaker magnetic field. Computing the electric field from 1 and 2 and squaring it one obtains the angular density of flux. The result, when the effect of the electron beam angular divergence is taken into account, is as follows:

$$\frac{d^2 \varphi}{d\phi d\psi} = 2 \frac{d^2 \varphi}{d\phi d\psi} (1 + f_3 f_4 \cos \alpha),$$

where

$$f_3 = \exp[-\epsilon_3 KL(1 + g)/\gamma_3^2],$$

$$f_4 = (1 + \delta_4^2)^{-1/4} \exp[-k_0 \lambda_0^2/2(1 + \delta_4^2)],$$

$$\delta_3 = \sqrt{\gamma_3 \sigma_3},$$

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$$\alpha = \frac{KL}{2} \left( \frac{1 + g}{\gamma_3^2} - \frac{\delta_3^2}{1 + \delta_3^2} + \frac{\gamma_3^2}{1 + \delta_4^2} \right) + \frac{1}{2} \left( \tan^{-1} k_3^2 + \tan^{-1} k_4^2 \right),$$

and $k = 2\pi/\lambda$, $\lambda$ = radiation wavelength, $\sigma$ = electrons' relative energy spread (rms), $L$ = the distance between two wigglers (see Fig. 1). $\gamma_3$, $\gamma_4$ = the average electron energy in unit of rest energy, $g$ is defined so that $L(1 + g/2\gamma_3^2) = \text{arc length of electron trajectory between the two crests in Fig. 1}$, $\sigma_3$ and $\sigma_4$ are respectively the horizontal and vertical angular spread of electron beam (rms).

In Eq. (1), $d^2 \varphi/d\phi d\psi$ is the angular density of flux from point 1 or 2 alone, which is a smooth function of photon energy represented by the dotted curve in Fig. 2. The term proportional to $\cos \phi$ is due to interference and causes the modulation of the spectrum represented schematically as the solid curve in Fig. 2. An equation similar to Eq. (1) was first derived by Ellaume [1] in the analysis of the spontaneous radiation from transverse optical klystron which is a second undulator system.

For a complete characterization of source, it is necessary to calculate the flux density in phase space known as the brightness by using the method discussed in Ref. 5. The results are in accord with the expectation that the sources at 1 and 3, for example, appear to be separated transversely. In forward direction, the source separation is given by the maximum excursion amplitude $a$ of electron trajectory (see Fig. 1).

Method of Determining $\gamma_0$ and $\sigma$

The modulated spectrum has peaks when $\alpha = 2\pi n$, $n$ being an integer. In this paper, we consider only the forward direction $\phi = \psi = 0$. Using Eq. (5), and neglecting for the moment the last two terms, the location of nth peak $k_n$ is found to be

$$k_n = \frac{\gamma_0^2(4\pi/L(1 + g))}{n}.$$  

From this, it follows for any pairs of integers $(n, m)$ that

$$n = m k_0/(k_{n+m} - k_n).$$

The location of peaks $k_n$ and $k_{n+m}$ can be determined by analyzing the spectrum with a monochromator. The integer $m$ can be determined by counting the number of peaks between $k_n$ and $k_{n+m}$. We can thus determine the integer $n$ associated with $k_n$. The electron energy $\gamma_0$ is then determined from Eq. (6).

To discuss the measurement accuracy, let $\Delta$ indicate the error in the measurement. We obtain from Eqs. (7) and (6) that

$$\frac{\Delta n}{n} = \frac{\Delta k_n}{k_n} + \frac{\Delta(k_{n+m} - k_n)}{k_{n+m} - k_n},$$

$$\frac{\Delta \gamma_0}{\gamma_0} = \frac{1}{2} \left( \frac{\Delta k_0}{k_0} + \frac{\Delta L(1 + g)}{L(1 + g)} \right).$$

For an unambiguous determination of $n$ it follows from Eq. (8) that the monochromator bandwidth $\Delta k_0/k_0$ needs to be smaller than $1/n$ and that the spectrum needs to be observed over a wide range of $k$ so that $k_{n+m} - k_n$ is of order $k_n$. From Eq. (9), it follows that both the monochromator bandwidth and the errors in the magnet parameters should be about the
We also require the following so that the design at critical energy requires monochromators based on grazing incident gratings, which is cumbersome. We shall instead set the wavelength range to be between \( \lambda = 500\,\text{Å} \) and \( 1000\,\text{Å} \), for which normal incidence monochromators with resolution well beyond the required 10^{-3} are readily available. Using Eq. (14), we obtain \( B_1(T)\approx 3.74/\sqrt{L(m)} \). A possible magnet parameters are \( d = 20\,\text{cm}, L = 70\,\text{cm} \) and \( B_1 = 0.223\,\text{T} \). With these, \( \eta \approx 0.9 \) is expected (\( \eta \) is optical efficiency). Accurate diagnostics of electron beam energy should be useful for the machine physics study.

**Acknowledgements**

Discussions with K. Halbach, H. Hogrefe, E. Hoyer and R. Perera are gratefully acknowledged.

This work was done with support from the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

**References**

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