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in Some Rare-Earth Nuclei

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Abstract: Properties of quasirotational rare-earth nuclei at high spins are studied with continuum γ-ray spectra. It is shown that the height of these spectra at each rotational frequency gives an effective moment of inertia which includes both collective and alignment effects. At high frequencies it is necessary to correct these spectra for incomplete population of the highest spin states. This feeding correction method is presented and studied in detail. It is shown that, in quasirotational nuclei, it can in general be applied successfully. The results are discussed in terms of an interplay between alignment and deformation effects. At high spins, the highly aligned orbitals from the first empty major shell (above the valence shell) play a major role. When they reach the Fermi surface, they produce large alignments visible in our spectra. The effect of the Fermi level position and of the softness toward deformation are discussed.
1. INTRODUCTION

In the last few years, some experimental results\(^1\) as well as theoretical calculations\(^2,3\) have suggested that the structure of nuclei at high spins \((I \gg 40\ h)\) is dominated by an interplay between deformation and alignment effects. Cranking-model type calculations predict various shapes: some are prolate or triaxial, others (near closed shells) become oblate for a certain spin interval. But at the highest spins the nuclei have a tendency towards a large triaxial deformation and sometimes towards very large prolate deformations (superdeformations). Directly involved in their shape changes\(^3\) are aligned orbitals which come down to the Fermi level as the nucleus rotates more rapidly. At a certain frequency these orbitals become populated and cause large alignments. These same orbitals also come down in energy as the deformation increases and therefore constitute a driving force towards large deformations. Experimentally, many paths are populated at high spins, and the observed \(\gamma\)-ray spectra are unresolved, but deformation and alignment effects can still be extracted. In particular, experimental \(\gamma-\gamma\) correlation spectra\(^1\) of quasirotational nuclei show a "valley" along the main diagonal. The width of that valley is related to the collectivity of the motion. "Stripes" parallel to the energy axes have been associated with the alignment of particular orbitals along the rotation axis. Moments of inertia are also sensitive to shapes and alignments. In this paper, we shall focus on a moment of inertia \(\mathcal{J}^{(2)}_{\text{eff}}\), which is proportional to the height of continuum \(\gamma\)-ray spectra obtained in the de-excitation of compound nuclei. The next section will describe how such spectra are obtained experimentally. The third section will show how these shape and alignment effects (known for lower frequencies) appear in the continuum spectra. However, at a frequency above
~0.4 MeV, one has to correct the γ-ray continuum spectra for varying initial populations (feeding) in order to obtain a height proportional to $j_{\text{eff}}^{(2)}$. The fourth section will explain the method used to correct these spectra, the fifth section will examine the conditions of application and the sixth section will present some tests performed with a simulation program. The results obtained at the highest frequencies in some rare-earth nuclei will be given in the seventh section, and interpreted in the eighth section in the light of several recent theoretical calculations. The appendix gives a method to measure the accuracy of the feeding corrections, and shows the effect of various backbends.

2. EXPERIMENTAL PROCEDURE

2.1 Experimental setup

To see detailed features in a continuum spectrum, a selection of initial spins has to be made to reduce the number of paths over which the spectrum is averaged and to avoid smearing out high-spin features by mixing them with low-spin spectra. The technique of limiting the initial spin range by selecting slices of total γ-ray energy de-exciting the states is well known and will not be reviewed here.

Fig. 1 shows a top view of a typical set up used for the experiments reported here. The total γ-ray energy is recorded in the two halves of a "sum crystal" NaI scintillator. They are each 33 cm diameter and 20 cm thick, with their axes vertical. They are centered on the target and their nearest faces are 1.9 cm away from it. In coincidence with the sum spectrometer, the γ-ray continuum spectra are recorded in eight 12.7 cm diameter and 15.2 cm thick NaI scintillators. These are located 1 m away from the target to avoid the
detection of several $\gamma$-rays in one detector (pile-up) per event. This distance also permits the rejection of neutrons which could trigger the NaI detectors, especially in the forward direction. The standard time-of-flight method is used, measuring the time difference between triggers from the sum crystals to the NaI detectors. The latter are located in a horizontal plane containing the beam. Four are packed as close as possible to 90° to the beam, and four are as close as possible to 0° and 180° to the beam. This angle is limited to 163° in the backward direction, so the forward detectors have been placed symmetrically also at 17°: this avoids the higher neutron flux which would hit the detector closer to 0° and also compensates for the forward-backward relativistic solid-angle effect. A Ge(Li) detector, of efficiency around 20%, is also in coincidence with the sum crystals and identifies the products of the reaction. It is placed at 90° to the beam through an opening of the upper sum crystal, allowing a distance of typically 10 cm to 15 cm to the target. With that system, the evolution of the continuum $\gamma$-ray spectra can be followed as a function of total $\gamma$-ray energy, and thereby as a function of spin.

2.2 Treatment of the spectra

To select $\gamma$-ray spectra with different feedings, the spectra in the 12.7 cm x 15.2 cm NaI detectors are sorted in coincidence with different total $\gamma$-ray energy slices as gates. The sum spectrometer has a total efficiency of only $\frac{\Omega}{4\pi} = 75\%$ for 1 MeV $\gamma$-rays so that the energy signal it gives does not correspond to the true total $\gamma$-ray energy. To correct for this, the well known "adding back" technique is used. The total $\gamma$-ray energy is then approximately
where \( \omega \) is efficiency of the NaI detectors \((\omega/4\pi \sim 10^{-3})\) which can be neglected in eq.(1). \( E_\Omega \) is the \( \gamma \)-ray energy detected in the sum spectrometer and \( E_{\Omega+\omega} \) is the sum of the energies \( E_\Omega \) and of the \( \gamma \)-ray energy recorded in the NaI detector. This is not a large correction but it makes changes in the detailed shape of the NaI spectra.

Each sorted spectrum is then unfolded, taking into account the Doppler shift at each angle. Afterwards, it is normalized to its \( \gamma \)-ray multiplicity by dividing by the number of singles events in the same total \( \gamma \)-ray energy slice. The "isotropic" spectrum, as emitted by the nucleus, is then constructed by combining the spectra from each NaI detector, assuming an angular distribution with only a term in \( P_2(\cos \theta) \). A statistical spectrum of the form \( E^3 \gamma e^{-E/\gamma} \) is then subtracted from the isotropic spectrum. By fitting a spectrum of this shape to the tail of the experimental spectra between 2.4 MeV and 4 MeV, a value close to \( T = 0.5 \) MeV has been found in all nuclei studied and has been adopted for the systematic treatment of the data. The feeding correction is applied to the yrast-like part of the isotropic spectra thus obtained.

3. MOMENTS OF INERTIA, DEFINITION AND QUALITATIVE DISCUSSION

3.1 Collective and single-particle effects

Angular momentum can be generated either collectively from many nucleons (rotation) and/or from a few nucleons whose spins are aligned parallel to each other. The contribution from "aligned" particles is dominant in nuclei near the major closed shells, at least at lower spins. The collective contribution is more important near the middle of shells where nuclei are deformed.
Even in well deformed cases, however, the nucleus does not rotate like a rigid body. This is best illustrated in a plot of spin versus rotational frequency (fig. 2) where large irregularities are found. As with a classical rotor, the nuclear rotational frequency is defined as \( \hbar \omega = \frac{dE}{dt} \) (which is about half the collective E2 transition energy). The smooth regular portions in fig. 2 arise from the collective rotation of nucleons occupying a given particle configuration. Those portions form rotational "bands," within which the rate of spin increase with frequency, \( \hbar (dI/d\omega)_{\text{band}} \) is called \( J_{\text{band}}^{(2)} \). This dynamic moment of inertia is related to the collectivity of nuclear motion. If all the nucleons were participating in a rigid rotation, the collectivity would be maximum and \( J_{\text{band}}^{(2)} = J_{\text{rigid}} \). However, it is found that in many nuclei, \( J_{\text{band}}^{(2)} \) is lower than \( J_{\text{rigid}} \). One well-known reason for this is the existence of pair correlations, but this will not be discussed here, especially since these correlations are expected to be quenched at the highest spins. Another reason is that some nucleons have a motion decoupled from the collective motion. They are generally in high-j orbitals, and their angular momentum is approximately aligned with the rotation axis by the Coriolis (and centrifugal) forces. Therefore they contribute directly to the total angular momentum, but not to the collective motion. This is the principal reason why \( J_{\text{band}}^{(2)} \) is smaller than \( J_{\text{rigid}} \) at high spins.

Often, this alignment of a high-j orbital is sudden and shows up as a discontinuity (called a backbend or upbend) in the I versus \( \omega \) plot; it corresponds to a large contribution to the angular momentum at a given frequency. In such a case \( |dI/d\omega| \) becomes very large in that frequency region. For example (fig. 2) the lowest lying state with spin 16 in the nucleus \(^{158}\text{Er}\) has a pair of \( i_{13/2} \) neutrons aligned along the rotation
axis. This state corresponds to a two-quasiparticle configuration on which an approximately unperturbed rotational band can be constructed. The relative amount $i$ of single particle alignment contributing to the angular momentum can be obtained by comparing with the ground band (fully paired). Near the point where the two bands cross $i$ can be estimated as the difference between the spins in the two "unperturbed" bands at a particular frequency.

Under these conditions, the total change in spin $dI_{\text{tot}}$ in a given frequency interval $d\omega$ can be broken into two contributions: $dI_{\text{band}}$ coming from the collective motion and $dI$ coming from alignment of a few specific orbitals. Thus:

$$\frac{dI_{\text{tot}}}{d\omega} = \frac{dI_{\text{band}}}{d\omega} + \frac{dI}{d\omega}$$

(2)

It is often the case that $\frac{dI}{d\omega} \approx 0$ within a band. In this case $\frac{\hbar dI_{\text{tot}}}{d\omega} = J_{\text{band}}^{(2)}$ in the band region. If we assume that $J_{\text{band}}^{(2)}$ varies smoothly in the backbend region, the sharp variation of $\frac{dI_{\text{tot}}}{d\omega}$ comes from the additional contribution $\frac{dI}{d\omega}$ due to alignment. It is clear that $\frac{dI_{\text{tot}}}{d\omega}$ includes both collective and alignment effects. This defines it as an effective dynamic moment of inertia $J_{\text{eff}}^{(2)}$. Thus, the existence of both collective and alignment effects gives rise to two measurable dynamic moments of inertia $J_{\text{eff}}^{(2)}$ and $J_{\text{band}}^{(2)}$. These concepts are particularly useful for high-spin studies because $J_{\text{eff}}^{(2)}$ and $J_{\text{band}}^{(2)}$ can be measured even for unresolved spectra. They will then represent an average value over many decay paths.
3.2 Kinematic moment of inertia

The usual moment of inertia is the kinematic effective moment of inertia \( J^{(1)}_{\text{eff}} \). It can be defined at each frequency by:

\[
J^{(1)}_{\text{eff}} = \frac{\hbar}{\omega}.
\]

This can also be written

\[
J^{(1)}_{\text{eff}}(\omega) = \frac{\hbar}{\omega} \int_0^1 I dI = \frac{1}{\omega} \int_0^\omega \frac{dI}{d\omega} d\omega = \frac{1}{\omega} \int_0^\omega J^{(2)}_{\text{eff}}(\omega) d\omega
\]

The value of \( J^{(1)}_{\text{eff}} \) at a frequency \( \omega \) is an average of \( J^{(2)}_{\text{eff}} \) over the interval 0 to \( \omega \). It is therefore much less sensitive to variations with frequency than \( J^{(2)}_{\text{eff}} \). The concept of \( J^{(1)}_{\text{eff}} \) as a function of \( \omega \) would not be sharply defined if there were large backbends such that \( \omega \) would not be approximately monotonic with \( I \).

3.3 Measurement of dynamic moments of inertia

The \( \gamma-\gamma \) correlation technique by which \( J^{(2)}_{\text{band}} \) is measured has been known for several years\(^7\) and will not be reviewed here.

Fig. 3 illustrates how \( J^{(2)}_{\text{eff}} \) can be measured in a quasirotational nucleus. The solid line is a plot (similar to that of fig. 2) of the calculated spin \( I \) as a function of \( \hbar \omega = E/2 \) in \(^{158}\)Er. From that curve, the change in spin \( \Delta I_{\text{tot}} \) in a given frequency interval \( \hbar \Delta \omega \) can be deduced.

Since, in a quasirotational nucleus, the \( \gamma \)-ray transitions are mostly of stretched quadrupole type, \( \Delta I_{\text{tot}} = 2\Delta N \), where \( \Delta N \) is the number of \( \gamma \)-rays in the frequency interval \( \hbar \Delta \omega \). The dashed curve in fig. 3 shows \( \Delta N \) (or \( \Delta N/\hbar \Delta \omega \).
which is proportional to $\Delta N$ for $\hbar \Delta \omega = 0.065$ MeV. But $\Delta N$, as a function of $\hbar \omega$, is just the $\gamma$-ray spectrum. Provided the spectrum is normalized to the $\gamma$-ray multiplicity, and provided the feeding of the decaying states is full, it follows that

$$\frac{\Delta N}{\hbar \Delta \omega} = \frac{1}{2} \frac{\Delta I_{\text{tot}}}{\hbar \Delta \omega} = \frac{1}{2} \frac{\mathcal{J}_{\text{eff}}^{(2)}}{\hbar^2}.$$  

Thus, the height of the $\gamma$-ray spectrum is directly proportional to the effective moment of inertia $\mathcal{J}_{\text{eff}}^{(2)}$, as indicated by the scale at far right.

If $\mathcal{J}_{\text{band}}^{(2)}$ and $\mathcal{J}_{\text{eff}}^{(2)}$ are known, the change $\Delta i$ in alignment in a given frequency interval is

$$\Delta i = \Delta I_{\text{tot}} \left( 1 - \frac{\mathcal{J}_{\text{band}}^{(2)}}{\mathcal{J}_{\text{eff}}^{(2)}} \right)$$  

obtained from eq.(2) and eq.(5).

3.4 Qualitative features of $\mathcal{J}_{\text{eff}}^{(2)}$ as seen from the $\gamma$-ray spectra of $^{160-154}$Er nuclei

An alignment produces very high values of $\mathcal{J}_{\text{eff}}^{(2)}$, if it is sufficiently localized in frequency. It is then readily seen as a peak in the $\gamma$-ray spectrum, as calculated schematically in fig. 3. More generally $\mathcal{J}_{\text{eff}}^{(2)}$ indicates how easily angular momentum is generated at a given frequency. An experimental spectrum samples many pathways which are in general different. Features like backbends, which occur at a definite frequency for one given band, might then be smeared out in the continuum spectrum. So the latter gives access only to values of $\mathcal{J}_{\text{eff}}^{(2)}$ which are averaged over many decay
paths. However some trends are strong or systematic enough to be seen directly in a γ-ray spectrum. At the higher frequencies (\( \geq 0.4 \) MeV), the height of the spectrum does not measure \( \mathcal{J}_{\text{eff}}^{(2)} \) directly, but is lowered because the spectra are not fully populated* at these high frequencies. Before discussing how to correct for the feeding, it is important to realize that certain interesting properties can be recognized already in the uncorrected spectrum. Fig. 4 shows spectra for a series of erbium nuclei from mass 160 to 154, which are the main products formed at the spins selected. Only the yrast-like part of the spectra is plotted here, since it is that part which is proportional to \( \mathcal{J}_{\text{eff}}^{(2)} \). These spectra have been discussed in detail in ref. 8, and we shall here, for the purpose of illustration, briefly review the main results.

The spectra display peaks at the lower frequencies (\( \hbar \omega < 0.5 \) MeV) corresponding to known backbends, as for example for the nucleus \(^{160}\text{Er}\), where the first, blocked and second backbends occur at the frequencies around 0.3, 0.38 and 0.45 MeV. With decreasing neutron number, the Fermi level for these nuclei comes closer to the lower \( \Omega \) orbits of the \( i_{13/2} \) and \( h_{9/2} \) neutron states, and the deformation becomes smaller and softer. Consequently, the neutron alignments will occur at lower frequencies. In the spectra plotted on fig. 4 this shows up as an increase in the effective moment of inertia with decreasing neutron number for frequencies below 0.4 MeV, except for \(^{154}\text{Er}\), in which a low-lying isomer\(^9\) removes some transitions and where the transitions in the lower frequency region are of mixed multipolarity as shown in fig. 5.

*Fully populated (fed) at a given spin in a given nucleus means that all the population going into that nucleus goes through that spin.
Turning now to the high frequency region of the spectra, above 0.5 MeV, the spectra consist almost exclusively of stretched E2 transitions. Here the spectra of $^{158,160}$Er fall off with increasing frequency, and to learn about the moment of inertia, we must apply the feeding correction method. For the lighter nuclei, however, the spectra rise again and peak at 0.6 MeV, and this suggests that a new source of angular momentum has become available. This new source may arise from the next major shell, the proton $i_{13/2}$ and $h_{9/2}$ orbitals, or the neutron $j_{15/2}$ and $i_{11/2}$ orbitals, or from changes in shape. The similarity of all spectra of fig. 4 in the high-frequency region suggests that all these nuclei gain angular momentum in a similar way. The contributions of the two major shells are not separated in the heavier nuclei, probably because the valence backbends are spread more towards higher frequencies.

Fig. 6 shows spectra obtained in the same way for the nuclei $^{166,162,160,158}$Yb which are each the main product of the reactions performed. The same tendency for compression of the valence backbend in the lighter nuclei can be seen. However the separation of the shells is not so clear in the lighter Yb nuclei as it was in the Er case (fig. 4). To obtain values of $J_{\text{eff}}^{(2)}$ in the high-frequency region, we have to correct the $\gamma$-ray spectra for incomplete feeding. The method used to correct them will be discussed in the next section.

4. THE FEEDING CORRECTION METHOD

4.1 Principle of the method

Although this method can usually be applied directly in "frequency" space, it is more easily explained in "spin" space.
The $\gamma$-ray spectrum of a perfect rotor ($\Delta N/\Delta I = 0.5$) as a function of spin is flat, provided all the population goes through all the decaying states. If not, the height of the spectrum is proportional to the fraction of the population $F(I)$ going through the spin $I$ (fig. 7). If the relative amount of direct population of each spin (which we shall call $k(I)$, the feeding curve) is known, the fraction $F(I)$ is just

$$F(I) = \int_1^{\infty} k(I) dI \int_0^{\infty} k(I) dI . \tag{7}$$

It is the integral above $I$ of the normalized feeding curve. The height of the spectrum divided by $F(I)$ is then 0.5. In general the feeding curve is not known experimentally, but it can be approximated\(^4\) as shown in fig. 7. Consider the $\gamma$-ray spectra (fig. 7b) corresponding to two slightly shifted feeding curves (fig. 7a). The difference $D$ of those two spectra (fig. 7c) is not exactly equal to either of the original feeding curves, but is close to their average, so that it can be used to correct the average of the two spectra of fig. 7b.

4.2 Experimental determination of the feeding curve

The experimental $\gamma$-ray spectra are obtained as a function of $\gamma$-ray energy (or frequency). The next paragraph will show that their difference for two slightly different feedings is also a feeding curve. The different feedings are obtained here by changing the total $\gamma$-ray energy window. The windows can be adjacent or overlapping. One then has a series of $\gamma$-ray spectra sampling the de-excitation of different feeding regions and the method described above can be applied.
4.3 Mathematical formulation for one decay path

The notations of paragraph 4.1 are used, with the assumption that the feeding curve is normalized to 1, i.e. \( \int_0^\infty k(I) \, dI = 1 \). It is further assumed that the \( \gamma \)-ray spectrum is composed of stretched quadrupole transitions so that there is half a transition per spin unit. This is in general true for quasirotational nuclei which are of interest here. The height of the \( \gamma \)-ray spectrum as a function of spin is, at a spin \( I_1 \)

\[
g(I_1) = \frac{dN}{dI} = \frac{1}{2} \int_{I_1}^\infty k(I) \, dI .
\]  

(8)

The height of the \( \gamma \)-ray spectrum as a function of frequency, per unit frequency interval, is then, for the frequency \( \omega_1 \) corresponding to the spin \( I_1 \):

\[
h(\omega_1) = \frac{1}{\hbar} \left( \frac{dN}{d\omega} \right)_{\omega_1} = \frac{1}{\hbar} \left( \frac{dN}{dI} \right)_{\omega_1} \left( \frac{dI}{d\omega} \right)_{\omega_1} = \frac{1}{\hbar} \frac{1}{2} g(I_1) J_{\text{eff}}^{(2)}(\omega_1) = \frac{1}{2\hbar} J_{\text{eff}}^{(2)}(\omega_1) \int_{I_1}^\infty k(I) \, dI
\]

(9)

For a spin feeding curve, slightly shifted from the previous one by \( \Delta I \), the corresponding \( \gamma \)-ray spectrum is

\[
h(\omega_1) = \frac{1}{2\hbar} J_{\text{eff}}^{(2)}(\omega_1) \int_{I_1-\Delta I}^\infty k(I) \, dI .
\]

(10)

The difference of these two spectra is:
If there is, on the average, a monotonic relationship between spin and frequency, then the population above a certain spin \( I \) is also above the corresponding frequency \( \omega \), so that there is a unique correspondence between the two feeding curves, which can be written:

\[
\begin{align*}
  k(I_1) &= f(\omega_1) \frac{d\omega}{dI}(\omega_1) = \frac{h}{J_{\text{eff}}^{(2)}(\omega_1)} f(\omega_1) \\
  \Delta h(\omega_1) &= \frac{1}{2h^2} \Delta I f(\omega_1) \tag{11}
\end{align*}
\]

where \( f(\omega_1) \) is the feeding curve as a function of frequency. Then

\[
\Delta h(\omega_1) = \frac{1}{2h^2} \Delta I f(\omega_1) = \frac{h}{J_{\text{eff}}^{(2)}(\omega_1)} f(\omega_1) \tag{12}
\]

The difference of the two \(\gamma\)-ray spectra at a frequency \( \omega_1 \) is directly proportional to the feeding curve as a function of frequency \( f(\omega_1) \).

The "true" spectrum \( H(\omega_1) \) corresponding to full feeding is then

\[
H(\omega_1) = h(\omega_1) \frac{\int_{\omega_1}^{\infty} f(\omega) d\omega}{\int_{\omega_1}^{\infty} f(\omega) d\omega} = \frac{1}{2h^2} J_{\text{eff}}^{(2)}(\omega_1) \tag{14}
\]

In practice, the integration is done in the interval of frequency from \( \omega_{\text{min}} \) to \( \omega_{\text{max}} \) where the feeding curve \( f(\omega) \) is non zero.

\[
H(\omega_1) = h(\omega_1) \frac{\int_{\omega_{\text{min}}}^{\omega_{\text{max}}} f(\omega) d\omega}{\int_{\omega_1}^{\omega_{\text{max}}} f(\omega) d\omega} = \frac{1}{2h^2} J_{\text{eff}}^{(2)}(\omega_1) \tag{15}
\]
4.4 Feeding correction for many decay paths

Carrying out this procedure [eq.(15)] for the more realistic case of many decay paths, each of which has a monotonic relationship between angular momentum and frequency, we obtain:

\[ H(\omega_1) = \frac{1}{2} \sum_v p_v J_v^{(2)}(\omega_1) \frac{\int_{\mathcal{I}_v(\omega_1)}^\infty k(I) dI}{\int_{\mathcal{I}(\omega_1)}^\infty k(I) dI} \]  

(16)

Here the decay path is denoted by the letter \( v \). The probability for following a certain decay path is denoted by \( p_v \), and \( I_v(\omega_1) \) and \( J_v^{(2)}(\omega_1) \) denote the spin and moment of inertia labelled by the decay path. \( \mathcal{I}(\omega_1) \) denotes the average spin corresponding to frequency \( \omega_1 \):

\[ \int_{\mathcal{I}(\omega_1)}^\infty k(I) dI = \sum_v p_v \int_{I_v(\omega_1)}^\infty k(I) dI \]  

(17)

It is seen that the corrected height of the spectrum with many decay paths gives the moment of inertia averaged over decay paths but the weighting factor is slightly redefined relative to the correct weighting factor, which is the probability for following the decay path. Since the spin in general will be higher for a path with a larger moment of inertia, the redefinition of the weighting factors in eq.(16) will result in a slightly too low moment of inertia.
5. CONDITIONS FOR THE APPLICATION OF THE METHOD

In this section we shall address the two main questions concerning the application of the method: (i) Actual experimental feeding functions are not just displaced by a certain constant amount of angular momentum as assumed in the derivation. (ii) In actual nuclei there is generally not a monotonic relationship between spin and frequency as assumed in the derivation.

We shall start with the simpler, more technical, question (i).

5.1 The feeding curve

5.1.1 Experimental derivation of feeding curves

Experimentally, the spin feeding can be varied by gating on a series of intervals of total-$\gamma$-ray energy (sum-energy "slices"). One then obtains a series of $\gamma$-ray spectra corresponding to the de-excitation of different feeding regions. Subtracting the spectra obtained by gating in neighboring sum-energy slices, frequency feeding curves $f(\omega)$ [eq.(13)] can be obtained.

5.1.2 Condition on the spin step

In realistic cases, the spin (frequency) step $\Delta I$ ($\Delta \omega$) between the two "slightly shifted" feeding curves should be small compared to their widths ($<20\%$). For example (see details in 6.2.1) simulation calculations show that for a spin step of 5-10\%, the corrected spectrum is within about the same percentage of the exact value. This condition is easy to fulfill by properly combining the total $\gamma$-ray energy slices.
5.1.3 Change in width of the spin feeding curve

The simplest mathematical derivation assumes a constant shape for the feeding curve as the sum slice is varied. However, many experiments and calculations show that this is not the case under our present experimental conditions. The tendency of the feeding curves to become narrower at higher spins is rather clear. The best available example is probably that of $^{160}$Er, whose moment of inertia $I_{\text{eff}}^{(2)}$ is roughly constant, so that the difference spectra [eq.(11)] are to a large extent directly proportional to the spin feeding curve. Fig. 8a shows that the difference curves are getting narrower as the spin increases. The shape of the spin feeding curves for the same compound nucleus reaction under the same conditions has also been estimated from a statistical type of calculation. (See appendix for more details.)

Fig. 8b shows the spin feeding curves calculated for total $\gamma$-ray energy slices comparable to those shown in fig. 8a. They have the same tendency to get narrower at higher spins. Notice that a detailed comparison of fig. 8a and 8b should not be made since $I_{\text{eff}}^{(2)}$ contributes to the difference spectra in fig. 8a.

To obtain a quantitative estimate of the effect of a changing width in the spin-feeding curve on the feeding correction, the normalized spin feeding curve, $k^{(N)}(I)$, for a total $\gamma$-ray energy slice $N$, is taken to be a Gaussian, centered at a spin $I_0(N)$, and of standard deviation $\sigma_I(N)$:

$$k^{(N)}(I) = \frac{1}{\sqrt{2\pi} \sigma_I(N)} \exp \left( -\frac{(I - I_0(N))^2}{2\sigma_I(N)^2} \right) \tag{18}$$

As shown in section 4.3, the height of the $\gamma$-ray spectrum for the slice $N$ is, denoting again for convenience the height at full feeding $\frac{1}{2h^2}I_{\text{eff}}^{(2)}$ by $H(\omega)$:
\[ h^{(N)}(\omega) = H(\omega) \int_{I(\omega)}^{\infty} k^{(N)}(I) dI \]  

(19)

The difference of the spectra corresponding to two sum slices \( N \), and \( N + 1 \) is, neglecting the factor \( \Delta I/2h \) which cancels out in the normalization:

\[ f^{(N)}(\omega) = h^{(N+1)}(\omega) - h^{(N)}(\omega) = H(\omega) \int_{I(\omega)}^{\infty} \frac{dk^{(N)}(I)}{dN} dI \]

Using the property \( \frac{\partial k}{\partial I} = -\frac{\partial k}{\partial I} \) for a Gaussian, we find:

\[ f^{(N)}(\omega) = H(\omega) \left\{ -\frac{dI_0}{dN} \int_{I(\omega)}^{\infty} \frac{ak^{(N)}(I)}{\partial I} dI + \frac{d\sigma I}{dN} \int_{I(\omega)}^{\infty} \frac{ak^{(N)}(I)}{\partial \sigma I} dI \right\} \]

\[ = H(\omega) \left\{ \frac{dI_0}{dN} k^{(N)}(I(\omega)) + \frac{d\sigma I}{dN} \int_{I(\omega)}^{\infty} \frac{ak^{(N)}(I)}{\partial \sigma I} dI \right\} \]  

(20)

In the last expression, the factor \( H(\omega)k^{(N)}(I(\omega)) \) in the first term gives the proper feeding curve [eq.(13)], which we here shall denote by \( f^{(N)}(\omega) \), and the second term induces an error in the determination of the second moment of inertia:

\[ H(\omega)_{\text{calc}} = h(\omega) \frac{\int_{0}^{\infty} f^{(N)}(\omega') d\omega'}{\int_{0}^{\infty} f^{(N)}(\omega') d\omega'} = H(\omega) \frac{\int_{0}^{\infty} f^{(N)}(\omega') d\omega' / \int_{0}^{\infty} f^{(N)}(\omega') d\omega'}{\int_{0}^{\infty} f^{(N)}(\omega') d\omega'} \]

\[ = H(\omega) \frac{1 + \frac{d\sigma I}{dI_0} \int_{0}^{\infty} dI' \int_{I'}^{\infty} \frac{ak^{(N)}(I'')}{\partial \sigma I} dI'' / \int_{0}^{\infty} f^{(N)}(\omega') d\omega'}{1 + \frac{d\sigma I}{dI_0} \int_{I(\omega)}^{\infty} dI' \int_{I'}^{\infty} \frac{ak^{(N)}(I'')}{\partial \sigma I} dI'' / \int_{0}^{\infty} f^{(N)}(\omega') d\omega'} \]

(21)
For the case of the Gaussian feeding function [eq.(18)], the integral in the numerator vanishes, and the integral in the denominator becomes a simple function. Defining the dimensionless variable

\[ x = \frac{(I(\omega) - I_0)}{\sigma_I} \]

(22)

one obtains

\[ H(\omega)_{\text{calc}} = H(\omega) \frac{1}{1 + \frac{d\sigma_I}{dI_0} \exp\left(-\frac{x^2}{2}\right) / \int_x^\infty \exp\left(-\frac{x'^2}{2}\right) dx'} \]

(23)

We notice that the error depends upon the ratio between the change in standard deviation (width) and the shift of the centroid of the feeding function between neighbouring sum-energy slices. The relative error

\[ L(x) = \frac{H(\omega)_{\text{calc}}}{H(\omega)} - 1 = \frac{-\frac{d\sigma_I}{dI_0} \exp\left(-\frac{x^2}{2}\right) / \int_x^\infty \exp\left(-\frac{x'^2}{2}\right) dx'}{1 + \frac{d\sigma_I}{dI_0} \exp\left(-\frac{x^2}{2}\right) / \int_x^\infty \exp\left(-\frac{x'^2}{2}\right) dx'} \]

(24)

is plotted in fig. 9 as a function of the multiplication factor

\[ M_F = \frac{H(\omega)_{\text{calc}}}{h(\omega)} \]

(25)

for two values of \( \frac{d\sigma_I}{dI_0} \). If the width of the feeding curve tends to narrow towards higher spins (e.g. \( \frac{d\sigma}{dI_0} = -0.2 \)), applying the simple formula [eq.(15)] will give a value of \( H(\omega) \) about 15% too high around the 50%
correction point. This error tends to be compensated if, instead of correcting the average spectrum, one corrects the lower γ-ray spectrum. Examples will be given in section 6.2. This problem is further analysed in the appendix.

5.1.4 Effect of fractionation of the angular momentum

There are systematic effects in the difference spectra due to so-called "channel effects" which refer to the change with spin of the respective proportions of the different residual nuclei. The two most important ones are

(i) an odd–even effect: there are fewer (quadrupole) transitions in the odd nucleus (due to a non-zero final spin) compared to the even neighbor. For example, if the "even" channel increases for the higher total-energy slice, the difference of the spectra will be artificially increased. However, in rotational nuclei, these effects occur at low frequency, and they can therefore be separated from the feeding region if the latter is high enough in frequency. This can be achieved by choosing a high-spin slice. (ii) a threshold effect: the experimental spectra are cut-off below a frequency of 150 keV, so that the transitions below that frequency are lost, and they do in general differ from one product to another. This problem can also be eliminated by choosing a high-spin slice.

5.2 Relation between spin and frequency

We shall now turn to the question (ii) posed at the beginning of this section, namely that the mathematical derivation of the feeding correction assumed stretched E2 transitions and a one to one relation between spin and frequency for each decay path.
5.2.1 Multipolarity of transitions

A considerable mixture of dipole transitions in the region of the feeding curve \( f(\omega) \) will destroy the method, since they will correspond to a region of spin different from that of the quadrupole \( \gamma \) rays. Moreover, the dipole transitions are in general uncorrelated and violate the condition that the spin should be, on the average, a monotonic function of frequency. However, if the dipole transitions are located in a frequency region below the feeding region, it is still possible to extract values of \( g_\text{eff}^{(2)} \) in the feeding region by the method we describe.

5.2.2 Monotonic relationship between spin and frequency

It is clear that there is a unique correspondence between the feeding curve as a function of \( I \) and the feeding curve as a function of \( \omega \) if \( \omega \) is a monotonic function of \( I \). Fig. 10 illustrates how this need only be true on the average. It shows two different paths, each having a backbend, in a given \((I, \omega)\) domain. The position of the backbends relative to \( \omega_0 \) has been chosen for convenience. In such a case, there would be transitions at a frequency greater than \( \omega_0 \) for spins lower than \( I_1 \) and transitions at a frequency lower than \( \omega_0 \) for spins higher than \( I_2 \), and these two effects tend to compensate. When many decay paths are considered, and if these backbends are not coherent* in the feeding region, these local effects would tend to compensate, so that \( f(\omega) \) would still represent the feeding. A detailed calculation of the effect of backbends is presented in the appendix.

*Backbends are coherent if they occur in many decay paths in the same narrow frequency and spin range.
5.2.3 Frequency spread within a band or among bands

There is a remaining "smearing" effect of the backbends within a band, and of the spread in moments of inertia or in alignments between different bands. It amounts to a broadening of the frequency feeding and can be corrected in the same way as the feeding. This will be shown for Gaussian distributions. Fig. 11 represents a realistic relationship between $I$ and $\omega$.

At a given frequency $\omega$, there is a distribution of possible spins, represented by a Gaussian of width $\sigma_\omega$, centered on the average value $\bar{I}$. For a Gaussian spin feeding of width $\sigma_I$ and centered at spin $I_0$, the observed spectrum will be

$$h(\omega) = \frac{H(\omega)}{2\pi \sigma_I \sigma_\omega} \int_0^\infty \exp - \frac{(\bar{I} - I)^2}{2\sigma_\omega^2} \int_0^\infty \exp - \frac{(I'' - I_0)^2}{2\sigma_I^2} \, dI'' \, dI$$

(26)

It can be shown that

$$h(\omega) = \frac{1}{\sqrt{2\pi}} \frac{H(\omega)}{\sqrt{\sigma_\omega^2 + \sigma_I^2}} \int_\bar{I}^\infty \exp - \frac{(I'' - I_0)^2}{2(\sigma_I^2 + \sigma_\omega^2)} dI''$$

(27)

This derivation involves the approximation that

$$\int_{-\bar{I}}^{\bar{I}} e^{-x^2/(2\omega^2)} \, dx \text{ is equal to } \int_{-\infty}^{\infty} e^{-x^2/(2\omega^2)} \, dx, \text{ where } x = \bar{I} - I$$

The spectrum has the same expression as eq.(19) and therefore can be treated in the same way as presented earlier. The width $\sigma^*$ of the difference spectrum is now $\sigma^* = \sqrt{\sigma_I^2 + \sigma_\omega^2}$. 
6. TEST OF THE FEEDING CORRECTION METHOD WITH A SIMULATION PROGRAM

6.1 Description of the program

In these tests, γ-ray spectra are constructed under certain assumptions and then are treated exactly like the experimental spectra. The two quantities needed for that purpose are the feeding curve and the moment of inertia $I_{\text{eff}}(I)$. The following assumptions are made to construct the spectra:

1) The population of states in the evaporation residues as a function of spin, prior to γ-ray emission, has a triangular shape up to a maximum spin $\hbar'_{er}$, where it is rounded with a function of the form $[1 + \exp((I - \hbar'_{er})/0.05 \hbar'_{er})]^{-1}$. By choosing slices of that curve, one chooses feeding curves for the γ-ray spectra. In our calculations, a variety of feeding curves are obtained by multiplying the population triangle by Gaussians of a variety of centers and widths. (See fig. 13c for example.)

2) Rotational spectra are assumed, with the possibility of varying moments of inertia. At each spin $I$ a moment of inertia $I_{\text{eff}}(I) = \hbar dI(I)$ is chosen, and the γ-ray transition energy can be calculated from $\omega = \int_0^I \frac{d\omega}{dI} dI$, and therefore the γ-ray spectrum height can be constructed. This is only the yrast-like part of the spectrum but it is the part of interest here. For each choice of the function $I_{\text{eff}}(I)$, a set of spectra is calculated for different feeding curves. The feeding correction is then applied to these spectra and the result is compared to the input curve $I_{\text{eff}}(I)$ or $I_{\text{eff}}(\omega)$.

6.2 Examples of tests

6.2.1 Constant moment of inertia $I_{\text{eff}}$

The dynamic moment of inertia $I_{\text{eff}}(2)$ is here chosen to be 74 MeV$^{-1}$. The characteristics of the feeding curves are summarized in table 1. They are chosen to be rather close to the values used in the analysis of our
experimental data (see 6.2.2). Various spectra ($S_N$ in table 1) are corrected, using the corresponding differences $\Delta S$. The corrected spectra, in units of $J_{\text{eff}}^{(2)}$ are plotted in fig. 12, together with the correction factor CF (in %) for spectrum 11. The spectra are not plotted beyond the 25% feeding point (correction of a factor of 4). The feeding correction is always applied to the lower of the two spectra used to calculate the difference. This compensates for the narrowing of the feeding curve towards higher spin feedings ($d\sigma/dI = -0.3$). The corrected spectra are always within 15% of the input value, which we think is acceptable. The corrected spectrum from slice 7 has low values, probably because the spin step is big compared to the width of the feeding (see $\Delta I/W$, table 1). This can be remedied by choosing smaller spin steps.

6.2.2 Varying moments of inertia

a. $J_{\text{eff}}^{(2)}$ slowly increasing

The input values of $J_{\text{eff}}^{(2)}$ are shown as a solid line in fig. 13b. They resemble those found in one of our experimental systems ($^{40}\text{Ar} + ^{124}\text{Sn}$). The spin feedings are also chosen so that the calculated difference spectra (some are shown in fig. 13a) are close to the experimental ones for the same system. The difference curves (and spin feeding, fig. 13c) are slightly narrowing as one moves to higher spins so that again here the lower spectrum is corrected. Up to the 25% feeding point, the agreement between the calculated (full dots) and the input values of $J_{\text{eff}}^{(2)}$ is better than 8%. The difference curves (fig. 13b) appear more irregular than the spin feeding curves (fig. 13c) because they have in them the variation due to $J_{\text{eff}}^{(2)}$ as eq.(11) indicates.
b. $\mathcal{J}_{\text{eff}}^{(2)}$ widely varying. Monotonic relationship between $I$ and $\omega$ retained on average.

The input values of $\mathcal{J}_{\text{eff}}^{(2)}$ are represented by the solid line in fig. 14. Although the spin feeding has the same shape as in fig. 13c, the difference spectrum (dot-dashed line in fig. 14) now shows wide variations due to the factor $\mathcal{J}_{\text{eff}}^{(2)}$. The corrected spectrum (dots) is still very good, since the requirement that the population above a certain spin $I_0$ be also above the corresponding frequency $\omega_0$ is fulfilled.

c. Coherent backbend

The moment of inertia $\mathcal{J}_{\text{eff}}^{(2)}$ varies sharply from 75 MeV$^{-1}$ at spin 44 to 100 MeV$^{-1}$ at spin 46 (fig. 15). Accordingly, the $\gamma$-ray energy changes from 1.16 MeV at spin 44 to 0.894 MeV at spin 46. Therefore there will be two "bands" going through the $\gamma$-ray energy interval 0.894 to 1.16 MeV. The moment of inertia value in that interval should be between 75 and 175 MeV$^{-1}$ depending on where the population comes in. These two limits for the input values of $\mathcal{J}_{\text{eff}}^{(2)}$ in that interval are shown as dashed lines in fig. 15. The corrected spectrum (dots) for the difference curve plotted (dot-dashed line) gives too high values of $\mathcal{J}_{\text{eff}}^{(2)}$ in the backbend region. At a given frequency $\omega_0$ in the backbend region, there are more additional transitions than there should be below it and therefore the correction [eq.(15)] is found to be too big, as expected. This effect will increase as $\omega_0$ approaches the upper edge of the backbend region. The regions above and below the backbend are not affected.

These few examples (and others, not included here) have shown that except when there is feeding in frequency regions where there are coherent backbends, the feeding correction method can be used successfully in realistic cases having mostly stretched E2 transitions.
7. EXPERIMENTAL RESULTS

7.1 Nuclei studied

In this paper we shall consider the systems listed in table 2. The bombarding energy $E_B$ of the projectile in the laboratory frame, the maximum spin $I_{\text{max}}$ brought into the system for which the target and projectile will fuse (obtained from the Bass model) and the main product formed at high spin, identified from the Ge(Li) spectra, are also indicated. We restrict ourselves to a series of rare earth nuclei. We shall probe proton effects by studying isotonic nuclei of different proton numbers, and deformation effects by varying the neutron number from well deformed to less deformed isotopes.

7.2 Analysis of difference spectra

We have already (fig. 8a) shown some difference spectra for a good rotor, namely the system $^{124}$Sn + $^{40}$Ar. They move rather regularly towards higher energies when the spin increases. They also become narrower at higher spins, both because of the so-called "spin cut-off" and also because the yrast line becomes steeper. We have seen that an acceptable solution is to correct the lower of the two spectra from which the difference is calculated and also to use total energy slices of varying width to produce difference spectra of more similar width (reducing $d\sigma/dI$).

Fig. 16 shows difference spectra from comparable total energy slices in the three nuclei $^{160,158,156}$Er. They look rather similar, peaking at a frequency of about 0.7 MeV where the proportion of stretched E2 transitions is practically 100%. Most of the low-frequency structures are probably due to channel effects, except for the nucleus $^{156}$Er where a sharp peak shows up at 0.4 MeV. This peak is also composed of stretched E2 transitions located at
the upper edge of the low-energy bump in this nucleus (see fig. 4). This suggests that it belongs to the feeding curve, and populates the spin region around 40 h, which was "compressed" towards lower frequencies as compared to the heavier Er (see section 3.4). We therefore included it in the feeding curve for that nucleus.

7.3 Example of feeding correction

Fig. 17 illustrates the procedure used to correct for the feeding. Two spectra corresponding to two slightly different (22.5 to 25 MeV and 25 to 27.5 MeV) total energy slices are plotted, together with their difference, which is proportional to the frequency feeding curve f(ω). The lower of the two spectra is corrected. Each point is divided by the fraction of the area of the feeding curve at frequencies above its own (see eq.15). The results are thought to be reliable up to the 25% (correction factor of 4) feeding point. They are not plotted beyond this point.

7.4 Consistency of the corrected spectra

Fig. 18 shows three different \( f_{\text{eff}}^{(2)} \) spectra for the system \( ^{126}\text{Te} + ^{40}\text{Ar} \). They correspond to various differences of γ-ray spectra, all with steps 7 to 8% of their widths. Two are deduced from low-spin slices (triangles) and high-spin slices (squares) and the third (circles) covers both ranges. They are in good agreement, although the feeding corrections at a given frequency are quite different for these various spectra. In general, one does not necessarily expect these curves to be identical since the de-excitation pathways might differ and in fact the curves do differ at low frequencies due to the formation of different product nuclei. However, a better consistency
of $\mathcal{J}_{\text{eff}}^{(2)}$ spectra for various spin inputs might be expected for the heavier, more rotational nuclei: the transition energies are lower and the channel effects occur at lower frequency and are better separated from the feeding curve. In addition the temperature probably drops more gradually in the more rotational nuclei. This might help keep the paths more similar and reduce temperature effects.

7.5 Final Experimental Results

Fig. 19 to 22 show the corrected spectra for all the systems studied. The results are presented as $\mathcal{J}_{\text{eff}}^{(2)}$ spectra as a function of frequency. In figs. 19 and 20, the spectra are plotted for a series of Er and Yb isotopes respectively. Figs. 21 and 22 feature some isotones for $N = 88$ and $N = 96$ respectively.

The low-frequency (up to 0.4 MeV) part of the Er and Yb isotopes spectra has already been discussed in section 3.4. The low-frequency region of the spectra in fig. 21 and 22 shows the same tendencies: in the neutron-rich nuclei (fig. 22) the spectra are rather flat (except for the known first backbend), as expected for good rotors. The moment of inertia $2\mathcal{J}_{\text{eff}}^{(2)}/\hbar^2$ values are around 150 MeV$^{-1}$. In contrast, for the neutron-deficient isotopes (fig. 21), the valence alignments are compressed at rather low frequency ($\hbar\omega < 0.5$ MeV) and the values of $2\mathcal{J}_{\text{eff}}^{(2)}/\hbar^2$ reach up to 300 MeV$^{-1}$ in that region.

At frequencies above 0.5 MeV, the spectra of all nuclei (except that of $^{170}$W) exhibit increasing values of $\mathcal{J}_{\text{eff}}^{(2)}$, some of them becoming very large at the highest frequencies. This means that a large amount of angular momentum is being generated in that region.
In the low-frequency region the "source" of angular momentum is the high-j orbitals of the valence shell. The logical source of angular momentum at higher frequency (spin) is to be found in the next shell, more specifically in its high-j orbitals, mainly the $i_{13/2}$ and $h_{9/2}$ proton orbitals. Two ways of generating angular momentum can be conceived in connection with these orbitals. There can be large alignments of the angular momentum of these high-j orbitals, or they can trigger a large increase of the deformation in these nuclei. Whereas the former hypothesis would result in small values of $j_{\text{band}}^{(2)}$, with large alignments producing the large values of $j_{\text{eff}}^{(2)}$, the latter hypothesis would show highly collective rotational bands where large values of $j_{\text{band}}^{(2)}$ would be the main contribution to $j_{\text{eff}}^{(2)}$. These possibilities will be discussed in more detail in the next section.

8. INTERPRETATION AND DISCUSSION OF THE RESULTS

8.1 Alignment of high-j orbitals and effect of the Fermi level

As shown for example in cranked-shell-model calculations\textsuperscript{12} (see fig. 23), highly aligned orbitals come down in energy when the frequency increases. When they become populated, an alignment occurs which generates a lot of angular momentum and therefore an increase of $j_{\text{eff}}^{(2)}$, in general at the frequency where they cross the Fermi level. Therefore, if the Fermi level were sharp, the crossing frequency would be characteristic of the aligned orbital. The crossing frequencies may be smeared out when averaging over decay paths, as is done in continuum spectra, if the deformation varies between the different decay paths. Also, the decay may follow bands containing the aligned orbit down in frequency below the crossing. However, only a few aligned orbitals from the next shell are expected at high frequencies so that there is still hope to identify them. Fig. 23 shows
single proton energy levels calculated by Dudek\textsuperscript{12}) as a function of frequency for a Woods-Saxon potential and at a given deformation ($\beta = 0.2$, $\gamma = -4^\circ$). In the rare-earth region of interest here, only the $i_{13/2}$ and $h_{9/2}$ proton orbitals are aligned around a frequency of $\sim 0.5$ MeV. The symbols indicate at which frequency the lowest aligned orbital crosses the Fermi level for different nuclei. Measurable differences in crossing frequencies are therefore expected, at least for nuclei with similar deformations and different proton numbers. The $\mathcal{J}^{(2)}$ spectra of the two isotones $^{160}$Er and $^{162}$Yb (fig. 24) are indeed different at high frequencies. We choose to compare first the heavier nuclei since their deformation is more stable and similar and therefore is not expected to play a major role in the observed differences in the spectra. $\mathcal{J}^{(2)}_{\text{eff}}$ starts increasing at $\sim 0.5$ MeV in $^{162}$Yb whereas it hardly begins to increase at $\sim 0.6$ MeV in $^{160}$Er. This is consistent with the proton alignments coming sooner in ytterbium nuclei which have two more protons than erbium nuclei (see fig. 23). The occurrence of large deformations could also produce a large increase of $\mathcal{J}^{(2)}_{\text{eff}}$, but they should be associated with the observation of heavily populated rotational bands. Early results from $\gamma$-$\gamma$ correlation experiments\textsuperscript{1,13}), although rather tentative due to poor statistics, do not seem to show such a strong structure, thus suggesting that large alignments are more likely to occur, and indeed, the "collective" moments of inertia found are rather low ($2\mathcal{J}_{\text{band}}/\hbar^2 \sim 50$ to $100$ MeV$^{-1}$). This interpretation is actually supported by some recent cranking-model calculations done by T. Bengtsson and I. Ragnarsson\textsuperscript{3}). By removing artificially the "virtual crossings" between orbitals, they can follow a given configuration up to the highest spins and therefore identify the crossing orbitals. At each spin, they locate the lowest configuration of
each parity and signature, and can therefore construct a plot of spin versus $E_\gamma$ for each of them, knowing the orbitals involved at each spin. From plots I as a function of $E_\gamma$, the spectra $\mathcal{J}^{(2)}_{\text{eff}}(\omega)$ can be deduced (see section 3.3). The crosses in fig. 24 show an $\mathcal{J}^{(2)}_{\text{eff}}(\omega)$ spectrum deduced from T. Bengtsson and I. Ragnarsson calculations\(^3\) averaged over the four lowest trajectories in the nucleus $^{166}$Yb. The overall agreement with the experimental spectrum is reasonable. The calculated spectrum shows at high frequencies two components which cannot be distinguished experimentally. The first "bump" arises from alignment of $i_{13/2}$ protons, whereas the second "bump" has its origin in the onset of large triaxial deformation of $\epsilon = 0.48$, $\epsilon_4 = 0.04$ and $\gamma = 23^\circ$. The experimental data are more "smeared," probably because many more than the four lowest trajectories are contributing to the observed spectrum.

The $N = 96$ spectra (fig. 22) show a dramatic change between $Z = 72$ and 74. Whereas $^{168}$Hf is rather similar to $^{166}$Yb, the $\mathcal{J}^{(2)}_{\text{eff}}$ spectrum of $^{170}$W never gets very high at low frequencies and shows low values at $\hbar \omega \sim 0.6$ MeV. Several factors can contribute to such a behavior: the proton valence shell becomes fuller and therefore contributes less to the alignment at low frequency. But the $h_{9/2}$ proton orbitals reach the Fermi level at $\hbar \omega \sim 0.4$ to 0.5 MeV, giving rather high values of 150 MeV\(^{-1}\). This is already known from the discrete yrast sequence\(^{14}\). As illustrated in fig. 23, the $i_{13/2}$ proton orbitals cross the Fermi level at higher frequency for $Z = 74$, perhaps producing the rise beginning at $\hbar \omega \sim 0.6$ MeV. Between these two values, there is no alignment and $\mathcal{J}^{(2)}_{\text{eff}}$ is low. In Yb and Hf nuclei, these two orbitals are close to each other, giving high values of $\mathcal{J}^{(2)}_{\text{eff}}$ at $\hbar \omega \sim 0.55$ MeV. The calculated spectrum\(^{15}\) for $^{168}$Hf agrees
with this idea. Also it is estimated from the Bass model that the compound nucleus holds 7 \( \hbar \) less in the \(^{126}\text{Te} + ^{48}\text{Ti}\) system than in the \(^{124}\text{Sn} + ^{48}\text{Ti}\) system. This could also partly explain the low values of \( J^{(2)}_{\text{eff}} \) at high frequencies in \(^{170}\text{W}\) since spins above ~59 \( \hbar \) will not contribute to the spectrum.

8.2 Softness of deformations

The interpretation of our experimental data does not appear as simple as one moves towards lighter, less deformed and softer nuclei. On the one hand, in less deformed nuclei, the highly aligned orbitals coming from the "empty" shell are located at higher energy than in more deformed nuclei, as seen from Nilsson diagrams, and this would tend to delay these alignments towards higher frequencies. On the other hand, less deformed nuclei are generally softer than well deformed ones. Therefore a way for them to generate angular momentum at high rotational frequencies could be to evolve towards large triaxial deformations or even superdeformations, which would increase the values of \( J^{(2)}_{\text{eff}} \). This would involve the same high \( \hbar \) orbitals, but would lead to a more collective motion, at least for the superdeformed shape. The softness of the nucleus makes accessible different possible deformations. We think the lighter erbium nuclei might exhibit such a behavior (fig. 19). The nucleus \(^{158}\text{Er}\), which is less deformed than \(^{160}\text{Er}\) at low spins, exhibits a rise in \( J^{(2)}_{\text{eff}} \) at about the same frequency as \(^{160}\text{Er}\). \(^{156}\text{Er}\) is still less deformed than \(^{158}\text{Er}\) at low spins and \( J^{(2)}_{\text{eff}} \) rises as early and even more steeply at high frequencies in this nucleus. In \(^{166,162}\text{Yb}\), the \( ^{1}_{13/2} \) or \( ^{1}_{9/2} \) protons align at a frequency around 0.5 MeV. The lighter \(^{160,158}\text{Yb}\) are less deformed and therefore these proton alignments are
expected to come later\textsuperscript{16}, as indicated by the much smaller rise of $J_{\text{eff}}^{(2)}$. An increase in deformation, which would tend to increase $J_{\text{eff}}^{(2)}$, does not seem to occur here, suggesting that the ytterbium nuclei are more rigid than the erbium nuclei. Fig. 25 compares $J_{\text{eff}}^{(2)}$ spectra calculated by T. Bengtsson and I. Ragnarsson\textsuperscript{15,17} for $^{166,162}\text{Yb}$ (top) and $^{160,158}\text{Er}$ (bottom) as explained above. In $^{160}\text{Er}$, the "bump" at 0.5 MeV contains some $i_{\frac{13}{2}}$ proton weight, but does not become as large as in $^{166}\text{Yb}$. The rise at 0.8 MeV seems to come from non-collective states. In $^{162}\text{Yb}$, the small bump at $-0.45$ MeV involves $h_{\frac{11}{2}}$ proton and $i_{\frac{13}{2}}$ neutron orbitals only. At $-0.75$ MeV, large triaxial shapes occur.

Experimentally, $^{162}\text{Yb}$ is more similar to $^{166}\text{Yb}$ than in the calculation but the structure calculated in $^{162}\text{Yb}$ could be characteristic of that we observe in $^{160,158}\text{Yb}$. In $^{158}\text{Er}$, the oblate non-collective configurations are calculated to be lowest in energy,* but the curve shown corresponds only to the collective configurations. These involve $i_{\frac{13}{2}}$ proton orbitals and triaxial shapes. The results for N = 88 isotones (fig. 21) could be explained through the same arguments of interplay between deformation and softness effects. The values of $J_{\text{eff}}^{(2)}$ start increasing at about the same frequency in $^{158}\text{Yb}$ and $^{156}\text{Er}$ as a result of a balance between a higher Fermi level in $^{158}\text{Yb}$ and a greater softness of $^{156}\text{Er}$. Dy isotopes should align later than Er isotopes, and the $^{154}\text{Dy}$ spectrum does rise later than the $^{156}\text{Er}$ spectrum. This seems to indicate that Dy isotopes in that vicinity are not appreciably softer than the Er ones.

*This illustrates a rather general discrepancy between the calculations which often predict non-collective oblate shapes at high spins and the experiments which show the presence of stretched E2 transitions suggestive of more collective behavior. This could be a real discrepancy in the structure calculated, or it could be that the "theoretical path" and the "experimental path" are different. Whereas the "yrast" path is calculated up to the highest spins, the "experimental path" is well above it at the highest spins.
8.3 Superdeformations

The very high values of \( J_{\text{eff}}^{(2)} \) reached at high frequencies in \(^{156}\text{Er} \) could tentatively be explained by a tendency towards very large deformations in that region. In fact calculations by Sven Åberg\(^{18} \) also seem to indicate such a tendency. Using a cranked-modified-oscillator model with a Strutinsky renormalization, he calculates, for a series of nuclei, the spin at which large triaxial or superdeformations become lowest in energy (see fig. 26). The lowest spins where this is expected to occur are in the gadolinium nuclei near the neutron closed shell, but in \(^{156}\text{Er} \) superdeformations are expected to arise at spin \( 55 \hbar \), a value below the highest spins observed experimentally. \(^{156}\text{Er} \) seems to be one of the most favorable cases in fig. 26 for observing super-deformations. In the ytterbium nuclei, these large deformations are not expected before spin \( 65 \hbar \), which is around the experimental maximum spin observed. Our experimental results (fig. 20) seem to agree with these predictions.

8.4 Gauge plots and deformation changes

The changes in deformation can be explored through the so-called "gauge plots"\(^{19} \) of neutron number vs Fermi energy which are shown in fig. 27 and 28 for Er and Yb isotopes respectively. The curves plotted for the discrete lines are obtained from the procedure of ref. 19. Above spin 25, values from our continuum \( J_{\text{eff}}^{(2)} \) spectra are obtained. The spin \( I \) is just the integral of \( J_{\text{eff}}^{(2)} \) : 

\[
I(\omega) = \int_{0.15}^{\omega} J_{\text{eff}}^{(2)}(\omega) d\omega + I_c,
\]

where \( I_c \) is the spin generated by the \( \gamma \)-ray transitions below our threshold of \( E_\gamma = 300 \text{ keV} \). The excitation energy is then 

\[
E_{ex} = \int_{0.15}^{\omega} \omega J_{\text{eff}}^{(2)}(\omega) d\omega + E_c,
\]

where \( E_c \) is the excitation energy.
generated by the γ-ray transitions below 300 keV. In Er nuclei, the values obtained from the continuum region seem to be in good agreement with the values deduced from discrete lines at the same spin. They also seem to follow the deformed branch up to the highest spins, or to evolve towards even larger deformation for the lightest nuclei. In Yb nuclei, the points at N = 94 have been obtained from the expression \( \lambda(N=94) = \frac{1}{4}[E_{ex}(N=96) - E_{ex}(N=92) - S_{4n}(N=96)] \) where \( S_{4n} \) is the 4 neutron separation energy. The results from the continuum spectra do not seem to follow as clearly one of the branches found from lower-spin, discrete-line data. However the kink in these curves would indicate that the lighter Yb are less deformed.

8.5 Comparison to the rigid-body moment of inertia

The study of \( J_{eff}^{(2)} \) spectra in the rare-earth region suggests that aligned orbitals coming from the next higher shell are largely contributing to the angular momentum at high frequencies. This tends to produce a rise in the spectrum at these frequencies. We have seen that the valence alignments also produce a bump, more or less pronounced, depending on the case, at lower frequencies. The \( J_{eff}^{(2)} \) spectrum thus presents "oscillations" which can be interpreted as shell effects, in this case alignments due to high-j orbitals in each shell. They are of various amplitudes and periods depending on the details of the shell structure. The oscillations occur around an average value which, in the absence of pairing, is expected to be \( J_{rigid} \). One of the best examples is shown for \(^{156}\)Er in fig. 29. The values of \( J_{eff}^{(1)} \) are low at low frequencies (spins) due to the pairing correlations, then increase towards rigid-body values. In the case of fig.
becomes much greater than $J_{\text{rigid}}$ in the regions where alignments occur. At 0.5 MeV, it is below $J_{\text{rigid}}$. Two effects of different nature could explain these low values. On the one hand, one would say that there are no alignments in that frequency region, therefore there is only collective motion and thus $J_{\text{eff}}^{(2)} = J_{\text{band}}^{(2)}$, which is expected from theoretical calculations$^{3,20}$ to be around $2/3 J_{\text{rigid}}$. On the other hand, one could also say that there is a completely non-collective motion in the spin region which would correspond to these frequencies for a quasirotational nucleus. This would result in an uncorrelated component of the spectrum in this spin region, or a lack of transitions which would occur there if the nucleus were rotating. In fact, in this nucleus $^{156}$Er, there is evidence (from discrete line studies$^{21}$) that some non-collective motion contributes in the relevant spin region. These effects are much less pronounced in more collective nuclei as shown for the nucleus $^{160}$Er plotted in fig. 30.

9. CONCLUSION

This paper has shown that, at the highest spins, in the absence of detailed discrete line studies, the generation of angular momentum might nevertheless be understood by looking at continuum $\gamma$-ray spectra, at least in quasirotational nuclei. A moment of inertia $J_{\text{eff}}^{(2)}$ is measured, which indicates how much angular momentum is generated at each frequency. It includes collective effects and alignments. Experimentally, $J_{\text{eff}}^{(2)}$ is generally proportional to the height of the $\gamma$-ray spectrum, provided the latter is fully fed. A method proposed to correct the $\gamma$-ray spectrum for the feeding (in quasirotational nuclei) is studied in detail in this paper, and allows the determination of $J_{\text{eff}}^{(2)}$ up to rather high spins (~50 h). One
sees shell effects in the spectrum, more or less pronounced depending on the Fermi level, the deformation and/or the softness of the nucleus. In a given nucleus, the spectrum, in general, oscillates (as a function of frequency) around an average value close to $f_{\text{rigid}}$. The high values indicate regions where a lot of angular momentum is generated within a small frequency interval, i.e. alignments. In the rare-earth region, the two observed "bumps" are interpreted as alignments coming, first from the valence shell, and then from the next empty proton shell.

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TABLE CAPTIONS

Table 1  Input values used in the simulation program to test the feeding correction for a constant moment of inertia $J_{\text{eff}}^{(2)}$ (see section 6.2.1).

Table 2  List of the systems studied. The bombarding energy, the maximum angular momentum leading to fusion of the two nuclei, and the main evaporation products at high angular momentum are also indicated.

Table 3  Input values, close to experimental values, for the system $^{40}\text{Ar} + ^{124}\text{Sn}$, used to test the feeding correction method in fig. 32.
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FIGURE CAPTIONS

Fig. 1: Top view of the experimental set-up.

Fig. 2: Plots, as a function of frequency, of the relative alignment $i$, total spin $I$, and moment of inertia $\mathcal{J}_{\text{band}}^{(2)}$ (excluding the backbending regions) for the nucleus $^{158}$Er. The solid line shows the value of $\mathcal{J}_{\text{rigid-sphere}}$ obtained from the formula $2\mathcal{J}_{\text{rigid-sphere}}/\hbar^2 = A^{5/3}/35$.

Fig. 3: Calculated spin vs frequency (dots) for the nucleus $^{158}$Er. The solid line is drawn to guide the eye. Deduced (dashed histogram) number of transitions per frequency interval of 65 keV, also rescaled as a moment of inertia $2\mathcal{J}_\text{eff}^{(2)}/\hbar^2$.

Fig. 4: Yrast-like part of the $\gamma$-ray spectra for the nuclei $^{160}$Er (solid line), $^{158}$Er (long-dashed line), $^{156}$Er (short-dashed line) and $^{154}$Er (dotted line).

Fig. 5: Percentage of stretched quadrupole transitions deduced from angular distributions for the same nuclei as in fig. 4. The symbols used are also the same.

Fig. 6: Yrast-like part of the $\gamma$-ray spectra for the nuclei $^{166}$Yb (solid line), $^{162}$Yb (long-dashed line), $^{160}$Yb (short-dashed line) and $^{158}$Yb (dotted line).
Fig. 7: Schematic illustration of the principle of the feeding correction method. (a) The spin feeding curve $k(I)$, solid line, and another one slightly shifted (dashed line). (b) The corresponding spectra as a function of spin. (c) the difference curve $D(I)$ between the two spectra is used as the feeding curve.

Fig. 8: (a) Experimental set of difference curves corresponding to the following total $\gamma$-ray energy slices: (o) 6-4, (•) 8-6, (△) 10-8, (▲) 12-10. The first slice corresponds to the interval 0-2.5 MeV. Each slice is 2.5 MeV wide. (b) Spin-feeding curves calculated from a statistical model (see appendix), with assumptions on the spin input and moment of inertia similar to the experimental conditions of (a).

Fig. 9: The relative error in the feeding correction factor due to the change in width $d\sigma/dI_0$ of the feeding curve is plotted against the multiplication factor $H_{\text{calc}}/h$ (see text).

Fig. 10: Qualitative illustration of the effect of backbends on the frequency feeding curve: for example there are some frequencies lower than $\omega_0$ corresponding to spins higher than $I_2$ on the dashed curve. If the backbends are spread enough, this effect tends to compensate.

Fig. 11: Representation of the spread in spin at a certain frequency $\omega$ by a Gaussian of width $\sigma_\omega$. Only a portion of the spin $I$ vs frequency $\omega$ curve is shown.
Fig. 12: Test of the feeding correction with a simulation program. The input moment of inertia $J_{\text{eff}}^{(2)}$ is constant and shown as a solid line. It is compared to the spectra corrected for the feeding for various total $\gamma$-ray energy slices similar to the experimental ones of fig. 8. Open dots: the spectrum in coincidence with slice 11 is corrected from the difference in spectra in coincidence with slices 11 and 12. Full dots: same for spectrum 9 corrected from the difference 10-9. Squares: same for spectrum 7 corrected from the difference 8-7. The dashed line shows the correction factor (in %) by which spectrum 11 has to be divided.

Fig. 13: (a) Calculated difference curves obtained for feedings and moments of inertia $J_{\text{eff}}^{(2)}$ similar to the experimental ones for $^{160}$Er. (b) The input moment of inertia is shown as a solid line. The dots represent the calculated spectrum 10, corrected from the difference 11-10. (c) Feeding curves as a function of spin $k(I)$ assumed in the program.

Fig. 14: Same as fig. 12 but for a widely varying moment of inertia $J_{\text{eff}}^{(2)}$. The monotonic relationship between $I$ and $\omega$ is preserved. Dots: spectrum 10 corrected from the difference spectrum 11-10, which is shown here as a dot-dashed line.

Fig. 15: Same as fig. 14 for a large coherent backbend characterized by an increase of $2J_{\text{eff}}^{(2)}$ from 150 MeV$^{-1}$ at spin 44 to 200 MeV$^{-1}$ at spin 46. The experimental difference curve (dot-dashed line) is in the backbend frequency region. The value of $2J_{\text{eff}}^{(2)}$ should vary between the limits 150 MeV$^{-1}$ and 350 MeV$^{-1}$ (dashed lines) in the backbend region depending on
the details of the feeding. Instead, it becomes very large near the upper edge of the backbend (the arrow indicates a value off-scale).

Fig. 16: Difference of spectra in coincidence with total $\gamma$-ray energy slices (11-10), for the nuclei $^{160}\text{Er}$ (solid line), $^{158}\text{Er}$ (long-dashed line), $^{156}\text{Er}$ (short-dashed line).

Fig. 17: Illustration of the feeding correction procedure. The spectra in coincidence with the total-energy slices 9 (solid), and 10 (dashed) are plotted, together with their difference and the spectrum 9 corrected (thick solid). The latter is not plotted beyond the 25% correction point.

Fig. 18: Spectra corrected for the feeding, in the system $^{126}\text{Te} + ^{40}\text{Ar}$, for three different feeding regions: a high spin (slices 8 to 10, squares), a low spin (slices 6 to 8, triangles), and a broad spin region (slices 6 to 11, circles). In each case the solid symbols indicate the 50% and 25% correction points.

Fig. 19: Corrected spectra from high-spin slices for $^{160}\text{Er}$ (solid), $^{158}\text{Er}$ (long-dashed) and $^{156}\text{Er}$ (short-dashed). The thin lines indicate values of $\mathcal{J}$ (2) band for $^{160}\text{Er}$ extracted from $\gamma-\gamma$ correlation data 1).

Fig. 20: Corrected spectra for the nuclei $^{166}\text{Yb}$ (solid), $^{162}\text{Yb}$ (long-dashed), $^{160}\text{Yb}$ (short-dashed) and $^{158}\text{Yb}$ (dotted).

Fig. 21: Corrected spectra for the N = 88 isotones: $^{158}\text{Yb}$ (solid), $^{156}\text{Er}$ (dashed) and $^{154}\text{Dy}$ (dotted).
Fig. 22: Corrected spectra for the $N = 96$ isotones: $^{170}$W (solid), $^{168}$Hf (dashed) and $^{166}$Yb (dotted).

Fig. 23: Proton Routhians calculated with a Woods-Saxon potential by Dudek, with deformation parameters $\beta = 0.2$ and $\gamma = -4^\circ$. The approximate location of the Fermi level for the nuclei Er (open circle), Yb (square), Hf (full circle), and W (diamond) is shown. The highly aligned orbitals ($h_{9/2}$, $i_{13/2}$) coming from the higher empty shell are indicated.

Fig. 24: Comparison of the experimental (thick-dashed line) and theoretical (crosses +) $\mathcal{F}_{\text{eff}}^{(2)}$ spectra for the nucleus $^{166}$Yb. The experimental spectra for $^{160}$Er (thick solid line) and $^{162}$Yb (dotted line) are also shown. The thin lines (with the same symbols for the same nucleus) show values of $\mathcal{F}_{\text{band}}^{(2)}$ deduced from $\gamma-\gamma$ correlation measurements$^{1,13}$.

Fig. 25: $\mathcal{F}_{\text{eff}}^{(2)}$ spectra calculated by T. Bengtsson and I. Ragnarsson$^{3,15,17}$ for the nuclei: top, $^{166}$Yb (solid), $^{162}$Yb (dashed); bottom, $^{160}$Er (solid) and $^{158}$Er (dashed).

Fig. 26: Chart of rare-earth nuclei on which are indicated: the spin at which they become superdeformed, the corresponding $\beta$ and $\gamma$ deformation (as defined in the upper left diagram), and the low spin deformation (light or shaded areas). These values are calculated by S. Åberg with a cranked-shell model.
Fig. 27: Gauge plots of neutron number vs Fermi energy for erbium nuclei at different spins. The open symbols show data obtained from discrete lines at spins 0 (○), 10 (◇), 16 (△), 26 (○). The full symbols are from continuum data (see text) for spins 25 (●), 30 (▼), 35 (▲) and 40 (●). The lines are drawn to guide the eye.

Fig. 28: Same as fig. 27 for ytterbium nuclei. The discrete line data are for spins 0 (○), 10 (◇), 16 (△) and the continuum data for spins 25 (●), 30 (▼), 35 (▲) and 40 (●).

Fig. 29: \( \mathcal{J}_{\text{eff}}^{(2)} \) spectrum (solid) and \( \mathcal{J}_{\text{eff}}^{(1)} \) spectrum (dashed) for the nucleus \( ^{156}\text{Er} \).

Fig. 30: Same as fig. 29 for the nucleus \( ^{160}\text{Er} \).

Fig. 31: Relative error \( L \) for the experimental correction factor plotted as a function of the experimental multiplication factor \( M_F \) for the following conditions: the Gaussians in eq.(A5) are of equal width (FWHM = 17 h) for slices A and B; (a) the average of the two spectra is corrected (\( \alpha = 0.5 \)), (b) the lower of the two spectra is corrected (\( \alpha = 1 \)). Each curve corresponds to a particular spin step \( \Delta I \) as indicated.

Fig. 32: Like fig. 31, but for feeding curves (eq.A5) close to the experimental ones (\( l_{er} = 64 \) h, \( \sigma = 12.74 \), \( \Delta \sigma/\Delta I \) and \( \Delta I \) as indicated in table 3). The average spin feeding \( \langle I \rangle \) for each slice is also shown in table 3.
Fig. 33: Schematic representation of a backbend on a spin I vs frequency ω plot. The feeding correction method will be tested for a spin feeding such as $k_A$, or $k_B$ occurring in the backbend region. $ω_c$ is the frequency where $g^{(2)}_{\text{eff}}$ will be calculated.

Fig. 34: Spin versus frequency plot showing: (a) a series of backbends uniformly spread over a frequency interval $Δω$. (b) (c) (d) the construction of the line mn whose slope is the moment of inertia $g^{(2)}_{\text{eff}}$ measured experimentally for full feeding. $Δω$ is the distance between the lines g and h. In (b) $Δω$ is the same as in (a). In (c) $Δω$ is equal to the frequency spread of the backbend itself. In (d) $Δω$ is larger than the frequency spread of the backbend (see text).

Fig. 35: (a) Like fig. 33 for one backbend (thick line) with branches of equal slopes, and for a series of backbends uniformly spread in frequency as drawn. (b) Relative error at $ω_c$ as a function of the number of backbends, spread as shown in (a).
APPENDIX

A1. Statistical calculation of the feeding curve

The feeding curve for each product is obtained from the probability that radiation will compete with neutron emission at each spin and excitation energy. Summing over all possible neutrons will give the total spin-feeding curve. It is then easy to obtain the spin-feeding curves corresponding to a total γ-ray energy. The following assumptions have been used: (i) The initial population of the compound nucleus is at a single energy and has a dependence on spin which is roughly triangular with a realistic rounded cut-off at a maximum $E_{\text{er}}^{10}$. (ii) The probabilities for a given state to radiate or emit a neutron are $P_\gamma = \frac{\Gamma_\gamma}{(\Gamma_\gamma + \Gamma_n)}$ or $P_n = \frac{\Gamma_n}{(\Gamma_\gamma + \Gamma_n)}$ respectively. The total neutron width $\Gamma_n(E_*)$ (where $E_*$ is the excitation energy above the yrast line) is obtained from Lane and Lynn $^{22}$ from the time-reversed process. The total radiative width $\Gamma_\gamma(E_*)$ is the sum of four terms: a Lorentzian corresponding to the Giant Dipole Resonance (GDR) normalized to the $E1$ sum rule, a collective $E_2$ component of 100 Weisskopf units for $E_\gamma \leq 1.5$ MeV, and two statistical components, a magnetic dipole component and an electric quadrupole component of 0.05 and 1 Weisskopf units respectively. (iii) At each emission, the normalized neutron energy spectrum $N(E_n)$ is statistical $^{23}$, and each neutron removes a spin $I_n$ which is 0.03 of the angular momentum of the emitting state. The population $p^{(n)}(E,I)$ in energy-spin space after the emission of the $n^{\text{th}}$ neutron is obtained by multiplying the population $p^{(n-1)}(E + E_n + B_n, I + I_n)$ by
the probability of neutron emission $P_n (E + E_n + B_n, I + I_n) \times N(E_n)$, where $B_n$ is the binding energy of the $n$th neutron. Level densities of Gilbert and Cameron$^{24}$ are used. The probability that a given state is a $\gamma$-feeder could be estimated by the probability that only gammas are emitted after neutron population, but this is an underestimate since the GDR strength function, with significant contribution at $E_\gamma > 1$ MeV, results in some gammas preceding neutrons. Instead, it is found empirically that this probability can be expressed as $P_\gamma x \exp(-U/a)$, independent of the state spin. Here $a = 7$ MeV and $U = E^* - B_n$. The resulting feeding curves are in reasonable agreement with the experimental and theoretical feeding curves of Sarantites et al.$^{25}$. The feeding curve for $(E,I)$ will then be a sum over all neutrons of the products $p^{(n)}(E,I) \times P_\gamma(E,I) \times \exp(-U/a)$. The calculated feeding curves for the system $^{40}$Ar + $^{124}$Sn at 185 MeV are shown in fig. 8b (see section 5.1.3).

A2. Analysis of the feeding correction method

This is an analysis of the accuracy of the feeding correction factor used in the text. The problem is that for an experimental case we do not know the exact feeding curve. Therefore, we estimate it from a finite difference of spectra, which introduces an error into the feeding correction factor. In this section, we calculate this error using an exact model that works for realistic spectra (backbends, multiple paths, etc.), and for any feeding function.
A2.1 Results for a monotonic function of spin versus frequency

A2.1.1 Uniform feeding curves

By uniform feeding curves we mean that all the spin-feeding curves have the same shape. We have seen above that when the frequency being analyzed is in a region fully fed, (i.e., all feeding comes at higher frequencies) the spectrum height is directly proportional to the moment of inertia $\mathcal{J}_{\text{eff}}^{(2)}$.

When the region being analyzed is not fully fed, the height of the $\gamma$-ray spectrum as a function of $I$ is, for a total $\gamma$-ray energy slice $A$, and considering $\Delta I = 2$ for both odd and even $I$,

$$g_A(I) = \sum_{I'=I}^{I_{\text{max}}} k_A(I')$$

where $K_A(I')$ is the normalized spin-feeding curve. To get exactly the corresponding fully fed spectrum, $g_A(I)$ has to be divided by $\sum_{I'=I}^{I_{\text{max}}} k_A(I')$, which is the exact correction factor $C_x$. Experimentally, the normalized correction factor $C$ is obtained from the finite difference of two $\gamma$-ray spectra in coincidence with two total $\gamma$-ray energy slices $A$ and $B$ ($E_B > E_A$):

$$C(I) = \frac{\sum_{I'=I}^{I_{\text{max}}} (g_B(I') - g_A(I'))}{\sum_{I'=I}^{I_{\text{max}}} \sum_{I''=I'}^{I_{\text{max}}} (k_B(I'') - k_A(I''))}.$$  \hspace{1cm} (A2)

Therefore, in this test, one can choose any feeding functions $k(I)$ and calculate the following quantity:

$$L = \frac{C_x(I)}{C(I)} - 1$$

which is a measure of the relative error made in the feeding correction actually used. It will be plotted as a function of $1/C(I)$ which we call the "multiplication factor" $M_F$ (as in section 5.1.3). As a practical limit we
consider only values of \( M_F < 4 \). At the present stage (uniform feeding curves) it is the average spectrum that should be corrected. Fig. 31a. shows the values of \( L \) using Gaussian feeding curves of width (FWHM) 17 h. These errors are compared for different spin steps \( \Delta I \) ranging from about 12\% to 48\% of the width of the Gaussian feeding curve. Up to a multiplication factor of 4, the error \(|L|\) never exceeds 5\%, which is quite satisfactory. The fact that the method is exact for \( M_F = 2 \) is a general result for any symmetric feeding curve. This is easy to see on very simple feeding curves like the rectangular one of fig. 7.

A2.1.2 Non uniform feeding curves

In realistic cases, the feeding curves for slices A and B are not of the same shape, and the above procedure becomes much poorer. In this case a more accurate correction for feeding is obtained by correcting a weighted combination \( \bar{g} \) of spectra from slices A and B, e.g., \( \bar{g} = \alpha g_A + (1 - \alpha) g_B \). The normalized exact correction factor for the spectrum \( \bar{g} \) is:

\[
C_{A}(I) = \sum_{I' = 1}^{I_{\text{max}}} \left( \alpha k_A(I') + (1 - \alpha) k_B(I') \right) \equiv \sum_{I' = 1}^{\infty} \bar{k}(I'). \tag{A4}
\]

We want to choose \( \alpha \) to minimize the error \( L \). Usually, for feeding curves of decreasing width, \( \alpha \) should be larger than 0.5, and generally, for realistic shapes (which tend to fall more steeply on the high side), \( \alpha \) is also greater than 0.5. We take a feeding curve of the form:

\[
k(I) = \frac{1}{1 + \exp((I - I_0)/0.05 \text{er}^2)} e^{-\frac{(I - I_0)^2}{2\sigma^2}} \tag{A5}
\]
This is of the same form as in section 6 (skewed feeding curves of decreasing width), and it matches rather well the experimental feeding curves. Under these conditions, we find that $a = 1$ (correcting the lower spectrum) is generally the best single value. Fig. 32 shows $L$ vs $M_F$ for the case of feeding given by eq. (A5) with parameters listed in table 3. These are close to the parameters that produced fig. 8b, and they work well for $^{160}$Er. In fig. 32 we use $a = 1$, as in our experimental work. The maximum resulting errors are $\approx 18\%$. However, it would be easy to observe experimentally the spread in results from different slices shown in fig. 32. This would be an indication to try a different $a$ for each pair of slices or to choose better-behaved slices, or at least to average the results from different slices, which would give a significantly better result.

It appears to us that the feeding correction applied carefully to cases with $I(\omega)$ monotonic should give errors less than 20%, and probably not much larger than 10%. In this case, careful application requires that the feeding curves be examined to ensure that the spin step is reasonably small, and that the widths are decreasing with increasing $I$, having $d\sigma/dI$ values in the range of table 3. Note that using $a = 1$ on spectra generated from feeding curves of constant shape, produces much poorer results (fig. 31b) than would $a = 0.5$. Careful application further requires examining the spectra corrected from different slices. If this internal consistency is not good, the results should be used with caution since the discrepancies could arise from the feeding correction method rather than from physical properties of the nucleus.
A2.2 Effect of a frequency spread within a band or among bands (effect of several paths)

Often, in experimental spectra, a given γ-ray energy $E_\gamma$ is contributed not by a single spin $I$ but by a range of spins with relative probabilities $P(I, E_\gamma)$. If we neglect the variation of $\mathcal{J}_{\text{eff}}^{(2)}$ from one path to another, eq. (A1) and (A2) become

$$g_A(E_\gamma) = \sum_{I=2}^{I_{\text{max}}} P(I, E_\gamma) \sum_{I'=I}^{I_{\text{max}}} k_A(I')$$

$$C(E_\gamma) = \sum_{I=2}^{I_{\text{max}}} P(I, E_\gamma) \sum_{I'=I}^{I_{\text{max}}} \sum_{I''=I'} (k_B(I'') - k_A(I''))$$

(A6)  

(A7)

Eq. (A6) and (A7) are identical to eq. (A1) and (A2) if the feeding $k_A(I)$ and $k_B(I)$ are replaced by the convolution of $P(I, T)$ with $k_A(I)$ and $k_B(I)$, and $I$ is replaced by $T(E_\gamma)$, the centroid spin producing a given $E_\gamma$. Hence, the distribution $P(I, E_\gamma)$ has no effect on the method nor $L(T)$ once the feeding is replaced by the convoluted feeding.

A2.3 Effects of backbends

In this section, we no longer require that $I(\omega)$ be monotonic. Hence we consider backbends, meaning that in some region $\omega$ decreases as the spin increases. Thus $I(\omega)$ is multiple valued in the backbend region. When backbends are present there are two questions: (1) what is the "right" answer and (2) how good is the feeding correction method.
A2.3.1 The "right" answer

Within a backbend, such as that shown in fig. 33, the \( \gamma \)-ray spectrum corresponding to a given frequency, \( \omega_c \), generally gets a contribution from three spin regions, shown as \( I_1 \), \( I_2 \), and \( I_3 \). The moment of inertia \( J^{(2)}_{\text{eff}}(\omega) \) associated with that frequency will no longer correspond to a particular spin, but can still be defined as the sum of the magnitudes of the values for the three individual branches. For full feeding this is still directly proportional to the height of the \( \gamma \)-ray spectrum, and no problem exists. However, imagine a feeding curve where there is no feeding at \( I_3 \), but significant feeding at \( I_2 \) and \( I_1 \). The experiment has no way to "sense" the contribution from the \( I_3 \) branch, and the "right" answer is not clear. We will always take the "right" answer to be the fully fed case, but recognize that there can be cases of low feeding in a backbend where there is no hope to get it. In the following calculations, we shall only consider feeding curves that peak in the region of the higher-spin bend (between \( I_2 \) and \( I_3 \)).

If we consider multiple bands, full feeding is not sufficient to define an obvious "right" answer. In fig. 34a we consider a fixed-shape backbend spread uniformly over a frequency region, \( \Delta \omega \). We want the fully-fed \( \gamma \)-ray spectrum at \( \omega_c \). The result is no longer obvious. In fig. 34b is a simple construction which gives the "right" answer. Conceptually it is equivalent if in fig. 34b we draw the backbend once and draw various \( \omega_c \) from frequency \( g \) to \( n \) separated by \( \Delta \omega \). The three lines \( pq \), \( qr \) and \( rs \) are drawn through and defined by the intersections of the lines \( g \) and \( h \) with the backbend. Those lines \( pq \), \( qr \) and \( rs \) intersect at \( q \) and \( r \) where vertical lines \( i \) and \( j \) are drawn. The intersections of \( i \) and \( j \) with \( pq \) and \( rs \) determine the line \( mn \).
The "right" answer can be shown to be exactly the slope of the line mn. Fig. 34 b-d show how the three-line (pqrs) construction goes smoothly over to a direct construction of mn as the backbends spread over a larger range of omega. Always the fully fed result for the moment of inertia is the slope of the line mn. The result is that for any backbend or region of backbends we have a value of the spectrum height $H_A(\omega_c)$ (equivalently $S_{\text{eff}}^{(2)}(\omega_c)$) at $\omega_c$ which we consider to be the correct one for full feeding. We now proceed to see how the result given by the feeding-correction method compares to that.

A2.3.2 Results of the correction method for backbends

We assume (fig. 33) a path composed of three branches, in a spin I vs frequency $\omega$ plane. The observed spectrum height $h(\omega_c)$ at a frequency $\omega_c$, and corresponding to a feeding curve $k_A(I)$ is:

$$h(\omega_c) = \left[ I_1^{(2)}(\omega_c) \sum_{I'=I_1}^\infty k_A(I') + I_2^{(2)}(\omega_c) \sum_{I'=I_2}^\infty k_A(I') + I_3^{(2)}(\omega_c) \sum_{I'=I_3}^\infty k_A(I') \right]$$

(A8)

where $I_1^{(2)}$ for example is the effective moment of inertia at $I_1$ in fig. 33. The difference of two such spectra corresponding to two feeding curves $k_A(I)$ and $k_B(I)$ is:

$$\Delta h(\omega_c) = \left[ I_1^{(2)}(\omega_c) \sum_{I'=I_1}^\infty + I_2^{(2)}(\omega_c) \sum_{I'=I_2}^\infty + I_3^{(2)}(\omega_c) \sum_{I'=I_3}^\infty \right] (k_B(I') - k_A(I'))$$

(A9)

To obtain the correction factor $C(\omega_c)$ (at a frequency $\omega_c$) by which the experimental spectrum has to be divided, one has to integrate $\Delta h$ (properly normalized) above $\omega_c$. Transforming to spin space, we find
\[ C(\omega_c) = \left[ \sum_{I' = I_1}^{I_2} + \sum_{I' = I_3}^{\infty} \right] \sum_{I'' = 1}^{\infty} \left( k_B(I'') - k_A(I'') \right) \]  \hspace{1cm} (A10)

We can see from fig. 33 that the sum between spins \( I_2 \) and \( I_3 \) is excluded in eq. (A10) since, in that region, \( \omega \) is smaller than \( \omega_c \). The corrected spectrum \( H(\omega_c) = \overline{h}(\omega_c)/C(\omega_c) \) (where \( \overline{h}(\omega_c) \) is eq. (A8) with \( k_A \) replaced by \( \alpha k_A + (1 - \alpha)k_B \)) gives the fully fed spectrum at the frequency \( \omega_c \), as obtained experimentally by the feeding correction method. This can be compared with \( H_X(\omega_c) \) defined in section A2.3.1 to give the relative error, \( L \):

\[ L = \frac{H(\omega_c)}{H_X(\omega_c)} - 1 , \]  \hspace{1cm} (A11)

analogous to the case without backbends [eq. (A3)].

In the following examples, for simplicity each backbend is taken to be three lines of slope unity, i.e., \( |Q| = 1 \) everywhere in each branch. This situation is shown in fig. 35a for one backbend (thick line) and for a variable number of identical backbends (nine in fig. 35a) uniformly spread in frequency as indicated.

The feedings are all taken to be realistic as in eq. (A5), \( \alpha = 1 \) is used, and the spin step is 20\% of the Gaussian width. The feeding curve is centered on the upper bend of the backbend. The resulting relative error, \( L \), is shown in fig. 35b as a function of the number of backbends for the central frequency \( \omega_c \) (\( \omega_c \) is indicated in fig. 35a). The error is greatly reduced as the backbend is spread out (made less coherent). When the backbend is spread over several times its own width, the error is no larger than 20\\%.
Thus non-coherent backbends are not a serious problem, whereas, coherent ones in the feeding region are. However, coherent backbends produce peaks in the spectrum, and the presence of such peaks in the feeding region is a warning that the method may be less accurate or unusable.

A spreading of the backbends in spin rather than frequency should also be considered. Here the error is reduced for more backbends as the feeding is effectively spread out. In this case also, there would be a peak in the feeding region of the spectrum. Such a peak may or may not be a problem, but requires that the data be examined very carefully.

A2.4 Conclusion

Based on these analyses we believe that the errors in $J^{(2)}_{\text{eff}}$ values derived by careful use of the feeding correction method are not likely to exceed 20% and are more probably around 10%. To have such accuracy consistently, however, one must control the step size and to some extent the shape of the feeding curves, ensure that there are no sharp peaks in the spectrum where there is large feeding, and examine the results from various slices for consistency.
Fig. 1

Lead shield

Eight NaI detectors

Sum Spectrometer

Target

Beam

Ge detector

12.5 cm

XBL 8411-10997
Fig. 2
Transitions per \( \Delta n_\omega = 65 \text{ KeV} \)

\[ \eta n_\omega \approx \frac{E_1 - E_{1,2}}{2} \text{ (MeV)} \]

Fig. 3
Fig. 4

Counts per $\Delta \hbar \omega = 20$ KeV

$\hbar \omega \approx E_\gamma / 2$ (MeV)

XBL 834-1544
Counts per $\Delta \omega = 20$ KeV

$\hbar \omega$ (MeV)

Fig. 6
Fig. 7
Fig. 8
Fig. 9

Relative Error

$\frac{d\alpha}{dl} = -0.4$

$\frac{d\alpha}{dl} = -0.2$

Multiplication Factor
\[ \tilde{I} = J^{(1)} \omega \]
Fig. 13

$2J(\ell) \left( \frac{\hbar^2}{\text{MeV}} \right)^{-1}$

$k(l)$ Arbitrary Units

$D(o)$ Arbitrary Units

$\eta \omega$ (MeV)
Fig. 14

\[ 2j_{\text{eff}}^2 m^2 \text{ (MeV}^{-2}) \]

\[ D \text{ (Transitions per } \Delta \omega = 20 \text{ KeV)} \]

\[ \omega \text{ (MeV)} \]
Fig. 15
Fig. 16

D (Transitions per $\Delta \omega = 20$ KeV)

$\hbar \omega$ (MeV)

XBL 8411-6257
Fig. 17

Counts vs. $\hbar \omega$ (MeV)

- D

XBL 8411-6261
Fig. 19
Fig. 21
Fig. 23
Fig. 24
Fig. 25

$\eta\omega$ (MeV)

$\mathcal{J}_{\text{eff}}^2 / \hbar^2$ (MeV$^{-1}$)
Fig. 26

XBL 8411-4958
Fig. 27
Fig. 28

N

\[ \lambda \text{ (MeV)} \]

XBL 8411-10989
Fig. 29
Fig. 31

- Relative Error vs. Multiplication Factor

- Graph a shows the relationship for different values of Δl.

- Graph b illustrates the effect for various Δl values.

- Multiplication Factor ranges from 1 to 6.

- Relative Error ranges from -0.6 to 0.05.
Fig. 33

k_B
k_A

ω_C

I_1
I_2
I_3

ω
Fig. 34

Frequency
Fig. 35
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