The Economics of Water Project Capacities under Optimal Water Inventory Management

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Abstract

This paper investigates the economic relation between water storage capacities and the alternative adaptation or mitigation approaches to improving the water resource sustainability. We establish and analyze a new stylized model for the determination of optimal dam capacities, which incorporates stochastic inflows, optimal water inventory management, and conservation efficiency. We prove the following three results:

1) We first show that if improvement in water allocation efficiency shifts the marginal water release benefit (or the water release demand) proportionally up, then the dam designer should optimally choose larger but not smaller dam capacities.

2) By assuming an isoelastic water release demand, we then show that dam capacities and conservation technologies won’t be substitutes, but complementary to each other, in the sense that larger dam capacities will increase water users’ incentive to adopt conservation technologies, and that the adoption will also induce larger optimal dam capacities, if and only if the water release demand elasticity is larger than one.

3) Alternatively, by assuming a linear water release demand, we show that dam capacities and conservation technologies will always be complementary to each other, as long as the adoption doesn’t reduce overflows much. Comparing with Result 2 suggests that the specification of the water release demand is crucial in determining the relation between dam capacities and conservation technologies.

All of the results imply that to improve the water resource sustainability, there could be shared ground between the apparently competing policies encouraging respectively water storage capacity expansions and the alternative adaptation or mitigation approaches.

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1 Introduction

Usually regarded as the source of life, water is arguably the most important natural resource for social and economic development. The endowment of water resources, however, is subjected to large variability across time, and special inequality between regions. To overcome and even utilize the variability and inequality, people build dams, reservoirs, canals, and other water projects to transfer water intertemporally and interregionally. Across the globe, water projects have been major public work with substantial social, economic, and environmental impacts, so their designs have been a source of major policy debates.

There are also alternative approaches rather than expanding water project capacities to improving the water resource sustainability. The alternative approaches include improving the efficiency in water inventory management, water allocation and institution, and water use, recycling, and conservation, among others. Understanding the relation between the alternative approaches and water project capacities is one of the keys to understanding the debates about water projects. The reason is simple: if the alternative approaches are perceived substitutes to expanding water project capacities, then the capacity expansion and the policies encouraging the alternative approaches are expected to bitterly compete for the usually limited policy resources.

1 The most recent studies on the significance of water and water policies include Debaere (2014), Granados and Sánchez (2013), Stone (2013), and Back et al. (Forthcoming), among others.

2 For a general review of the significance of water projects, see the World Commission on Dams (2000). For specific examples, water projects are perceived by Wittfogel (1957) as the key in the formation of the authoritarian tradition in East Asia; Reisman (1989)’s Cadillac Desert starts the debate about the major dams and water management in the western United States; Fischhendler and Zilberman (2005) discuss the political implications of the United States Central Valley Project Improvement Act of 1992; Diao and Pandie (2006) examine the productivity and distributional effects of large irrigation dams in India; Mertha (2008) analyzes three water project controversies in China. Among others, recent and ongoing examples are about the series of dams in the Amazon Basin in South America and along the Yangtze River in China, with the Belo Monte Dam in Brazil and the Three Gorges Dam in China topping the lists, respectively. Examples of the media coverage include Kennedy (2001), Lyons (2012), The Economist (2013a, 2013b), and Cheng (2013), among others. Jackson and Sleigh (2000) detail the social-economic impacts of resettlement for the Three Gorges Dam in China. The most recent example specifically about water scarcity and water projects is the proposed canal from the Paraíba do Sul to the Cantareira Water System, initiated by the Governor of São Paulo, Brazil. For media coverage, see Carvalho (2014) and de Araujo (2014a, b).

3 Schwabe and Connor (2012) discuss several alternative approaches of drought adaptation and mitigation. The World Commission on Dams (2000) highlights the significance of the alternative approaches, including some institutional arrangement.
The significance of understanding the relation between water project capacities and the alternative approaches has been underscored even more by the policy debate related to the recent devastating drought in western US and especially California. The drought is expected to heavily hit the farms in the Central Valley, endanger fish species in the Sacramento–San Joaquin River Delta, exacerbate groundwater scarcity and salinization across the State, worsen drinking water quality in cities, and cause other severe social, economic, and ecological problems in related areas.\(^4\) In response to the devastating drought, as the huge benefit from water storage and conveyance capacities in reducing the drought impact in the western United States have been documented in many studies (e.g. Zilberman et al. (2011), Howitt et al. (2011), and Hansen et al. (2011, 2013)), through federal legislation, several bills have been introduced to the Congress to authorize and fund expansions of water storage and conveyance capacities.\(^5\) Through state policies, Californian lawmakers have also been discussing issuing water bonds to finance more water projects.\(^6\) People who aren’t fans of new dams and reservoirs, however, think that the hope of more sustainable water supply for California relies on funding recycling projects and conservation technology adoption, and that expanding water storage and conveyance capacities could severely cost environment and discourage improvement in conservation efficiency.\(^7\) The relation between water project capacities and

\(^{4}\) As a result of the drought, in January 2014, the California Department of Water Resources announced the first zero allocation of the water from the California State Water Project, in the 54-year history of “the largest state-built, multipurpose, user-financed water project” in the United States, according to the Department (1958–2013). In January 2014, Jerry Brown, the Governor of California, “proclaimed a State of Emergency and directed state officials to take all necessary actions to make water immediately available.” Examples of the media coverage on the drought consequence include Alexander (2014), Freking (2014), Lochhead (2014), Rogers (2013), and Serna (2014), among others. Some early estimates about the loss caused by the drought are presented in Goodhue and Martin (2014).

\(^{5}\) The bills include at least the California Emergency Drought Relief Act of 2014, the Upper San Joaquin River Storage Act of 2014, the Shasta Dam Expansion Act of 2014, the San Luis Reservoir Expansion Act of 2014, the Sacramento Valley Water Storage and Restoration Act of 2014, and the Sacramento–San Joaquin Valley Emergency Water Delivery Act of 2014.

\(^{6}\) The call for the expansions of water storage capacities becomes even more urgent given climate change, since as recognized by Schwabe and Connors (2012), warming could reduce the natural storage capacity of the Sierra Nevada snowpacks, which will make precipitation increasingly fall as rains and flow into the California water system. VanRheenen et al. (2014) also imply that water infrastructure improvement should be considered for the Sacramento–San Joaquin River Basin to cope with climate change.

\(^{7}\) The water bond act is also known as the Safe, Clean and Reliable Drinking Water Supply Act of 2014. It is on the November 4, 2014 ballot in California, and if it gets approved, it will authorize the issuance of the 11-billion-dollar bonds to finance a drinking water and water supply reliability program, among which
the alternative approaches then emerges as the core of the policy debate.

This paper attempts to theoretically analyze the relation between water project capacities and some of the alternative approaches. More specifically, we ask how should the optimal capacities of water projects be determined, what is the impact of improvement in water allocation efficiency on the optimal capacities, and how do the capacities interact with water users’ adoption of conservation technologies? To answer the questions, we first establish a stylized model for the determination of the optimal capacity of a dam, which incorporates the stochastic inflows to the dam and the conservation efficiency in water use. Moreover, we allow the dam to dynamically optimally manage water inventories, just as the lesson learned in the Genesis story about Pharaoh’s dreams and Joseph’s storage solution. This model helps us show three results, which are all rich in insights and policy implications:

Result 1. By analyzing the marginal benefit of dam capacities, we show that if the marginal water release benefit (or the water release demand) shifts up proportionally, then the optimal dam capacity should become larger. The result implies that many integrated improvements in water allocation efficiency, for example the United States Central Valley Project Improvement Act of 1992, could optimally require more or larger water storage projects.

Result 2. By assuming an isoelastic water release demand, we show that the dam capacities and conservation technologies won’t be substitutes, but complementary to each other, if and only if the water release demand is elastic; in other words, larger dam capacities will increase

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3 billion would be for water storage projects and 1.25 billion for water recycling and conservation projects. Examples of discussions on the recent Californian water policy issues include Calefati (2014), Dunning and Machtinger (2014), Ewers (2014), Freking (2014), Garamendi (2014a, b), Nirappil (2014), and White (2014), among others.

8 In Genesis 41, Pharaoh’s dreams suggested seven years of famine would come after seven years of harvest abundance. Joseph’s policy solution for Egypt was to store the food produced in the years of abundance, and release them in the years of famine. Note in this story, Joseph’s optimal inventory management problem was almost deterministic, since he knew from Pharaoh’s dreams almost exactly what would happen. The water inventory management problem in this paper, however, isn’t deterministic but stochastic.

9 In this paper, the terms “marginal water release benefit” and “water release demand” are used interchangeably.
water users’ incentive to adopt conservation technologies, and the adoption will also induce larger optimal dam capacities, if and only if the water release demand elasticity is larger than one. This result can be extended to show that trade policies can affect the optimal dam capacity decisions, since they affect the elasticity of the demand for the water-produced commodities, which is correlated with the water release demand elasticity.

Result 3. By assuming a quadratic water release benefit function and therefore a linear water release demand, we show that the dam capacities and conservation technologies are always complementary to each other, as long as a small increase in conservation efficiency doesn’t reduce overflows much. The comparison between this result and Result 2 suggests that the specification of the water release demand is crucial in determining the relation between dam capacities and conservation technologies: as the irrigation water demand is usually perceived as inelastic, an isoelastic specification and a linear specification could suggest opposite relations between conservation technologies and the capacities of irrigation dams.

Interestingly and counterintuitively, all of the results imply that there could be shared ground between the policies expanding public or private water storage capacities, and the policies encouraging the alternative adaptation and mitigation approaches, for example the subsidies for adopting drip irrigation and other conservation technologies and the reforms enhancing the role of water markets in the allocation of water resources.

The analysis in the paper is accompanied by numerical illustrations, in which we specify our model to the irrigation water inventory management problem of the California State Water Project. According to the California Department of Water Resources (1963–2013), in 2010, the Project is “the largest state-built, multipurpose, user-financed water project” in the United States, and its water benefits “approximately 25 million of California’s estimated 37 million residents” and “irrigates about 750000 acres of farmland.” Since the Project is significant in Californian agriculture, which plays an important role in the United States and
the global agricultural market, there would be few alternative cases with similar significance for us to illustrate our model and show its practical significance. Our specification mainly use the information in the annual management reports (1963-2013) and operation reports (1976-2014) of the Project by the California Department of Water Resources. We use three specifications of the water release benefit in the illustrations: 1) the first one is isoelastic and elastic, with the elasticity being -1.21 as estimated by Frank and Beattie (1979); 2) the second one is isoelastic and inelastic, with the elasticity being -0.79 as estimated by Schoengold et al. (2006); 3) the last one is linear and has the same elasticity as the second isoelastic, inelastic demand when the demand is equal to 936098 acre-feet, which is the 1975-2010 mean of the annual water deliveries from the Project to agricultural use. The three specifications help to confirm our theoretical results and show their empirical relevance. For the technical detail of the specification of the Project’s irrigation water inventory management problem, see Appendix A.

Our paper contributes to several threads of literature, namely on the determination of optimal water project capacities, the stabilization and optimal control of water storage, the factors affecting conservation technology adoption, and the models about storable commodity markets. It can also shed some light on the rebound effect in resource economics and primarily energy economics. We shall leave detailed discussion on the literature along with the full development and analysis of our model and results.

The paper is unfolded as follows. Section 2 builds the model for the optimal capacity determination of a dam with optimal water inventory management. To prepare for further analysis, Section 3 characterizes the solution to the optimal water inventory management problem, and derives a useful expression of the marginal benefit of dam capacities. Using the expression, Section 4 shows Result 1. Section 5 pushes the analysis further and exhibits the conditions under which dam capacities and conservation technologies are complementary to

10We also use other information in the Department’s Economic Analysis Guidebook (2008), its survey on the irrigation methods in California (2010), and the training manual on irrigation scheduling prepared by Brouwer et al. (1989) for the Food and Agriculture Organization of the United Nations.
each other, showing our Results 2 and 3. Section 6 concludes the paper.

2 The Determination of the Optimal Dam Capacity

In this Section we build a model for the determination of optimal dam capacities with stochastic optimal water inventory management. The dam sits upstream, and in each period, it gathers inflows and adds them to the water carried from the past period, stores some of the available water for the future, and releases the rest to generate benefits downstream. Most importantly, the water storage must be nonnegative and smaller than the dam capacity, and the water storage and release is optimally controlled.

Figure 1 illustrates the structure of the water system in our model: in each period $t$, there is an independently, identically distributed stochastic inflow, $e_t \sim e$, to the dam; given the water availability, $a_t$, which is the inflow, $e_t$, plus the water storage carried from the past period net of evaporation, $(1 - d)s_{t-1}$, the dam operator chooses the amount of the water to be stored in the dam for future, $s_t$, which is nonnegative, but cannot be larger than the dam capacity, $\bar{s}$; the operator then releases all the rest of the water, $w_t \equiv a_t - s_t$, into a distribution and allocation system, generating the water release benefit, $B(w_t, \alpha)$, which is a function in the water release, $w_t$, and a parameter of conservation efficiency, $\alpha \in (0, 1)$.

Chakravorty et al. (1995, 2009) have discussed the optimal design of the distribution and allocation system in models without stochastic inflows to the system. As the economics of the distribution and allocation system isn’t our paper’s main focus, we leave the functioning of the system out of the model, and only assume that the water release generates the benefit $B(w_t, \alpha)$. We further assume $B(w, \alpha) \equiv B(\alpha w)$, where $B$ is the benefit generated by the effectively used water. In other words, $\alpha$ measures the proportion of the applied water that

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11 The water release can be used to irrigate farms, generate hydropower, supply urban-used water, help the downstream to prevent salinization, protect biodiversity, and generate recreational benefit, among many other usages. In other words, the function $B(\cdot, \cdot)$ can include agricultural, industrial, environmental, and any other water-related benefit. For a general description of the various benefit generated by dams, see the World Commission on Dams (2000) and Shaw (2007). For models focusing on the role of the competing interests in dam capacity determination, see Houba et al. (2012, 2013), Zhu et al. (2013), and Pham-Do et al. (2014). The water release benefit function has already accounted any downstream economic distortions.
The water source

The i.i.d. inflow, \( e_t \sim e \)

Given the water availability, \( a_t \equiv e_t + (1 - d)s_{t-1} \), the dam of the capacity, \( \bar{s} \), chooses the water storage, \( s_t \in [0, \bar{s}] \).

The water release, \( w_t \equiv a_t - s_t \)

The water distribution and allocation system receives, distributes, allocates, and uses the water release, and generates the water release benefit, \( B(w_t, \alpha) \).

Figure 1: A water system with a dam

is effectively used but not wasted in water use, and adopting conservation technologies would increase \( \alpha \). This assumption follows the idea of Caswell and Zilberman (1986) that modern irrigation technologies increase water conservation efficiency. To make the optimal water inventory management problem solvable, we also have regular conditions like \( B''(\cdot) < 0 \) and \( 0 < B'(\varepsilon) < \infty \), where \( \varepsilon \geq 0 \) is a lower bound of the inflow.

We model the optimal water inventory management problem as the following optimal control program:

\[
V^*(\bar{s}, a_0, \alpha) \equiv \max_{\{w_t\}_{t=0}^{\infty}, \{s_t\}_{t=0}^{\infty}} \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \rho^t B(w_t, \alpha) \right] \quad \text{s.t.}
\]

\[
(1 - d) s_{t-1} + e_t \equiv a_t = w_t + s_t \quad \text{for all } t \geq 1,
\]

\[
w_0 + s_0 = a_0,
\]

\[
0 \leq s_t \leq \bar{s} \quad \text{for all } t \geq 0, \quad \text{and}
\]

\[
w_t \geq 0 \quad \text{for all } t \geq 0,
\]

where \( \rho \) denotes the discount factor.\(^{12}\)

\(^{12}\) The Office of Management and Budget (2011) within the Executive Office of the President of the United States recommends a constant discount factor for project evaluation. For a detailed discussion on the
Given the optimal water inventory management, before the dam is built, the dam designer recognizes the construction and maintenance cost, \(C(\bar{s})\), and the environmental damage cost, \(D(\bar{s})\). The marginal cost functions are assumed positive and increasing, which means \(C'(\cdot) > 0, \ C''(\cdot) > 0, \ D'(\cdot) > 0, \) and \(D''(\cdot) > 0\). The assumption isn’t too unrealistic, since the resource for dam building and maintenance is always limited, and as larger dams make the ecological system more vulnerable to further human actions, it is fair to assume an increasing marginal environmental damage cost. Furthermore, the assumption makes the optimal dam capacity problem have solutions.

We model the optimal dam capacity problem as the following program:

\[
\max_{\bar{s} \geq 0} \quad \frac{\text{Dam-generated value}}{\text{Construction cost}} - \frac{\text{Environmental damage}}{\text{Construction cost}} - D(\bar{s}). \tag{2}
\]

The first-order condition is then

\[
\frac{\text{Marginal benefit of dam capacities}}{\text{Marginal cost of dam capacities}} = \frac{C'(\bar{s}) + D'(\bar{s})}{C'(\bar{s}) + D'(\bar{s})}. \tag{3}
\]

The left-hand side of the condition is the marginal benefit of dam capacities, while the right-hand side is the marginal cost of dam capacities. For simplicity we rule out the cases in which the first-order condition doesn’t have any root or it has only negative roots. The root of the first-order condition, \(\bar{s}^*\), is then the optimal dam capacity with optimal water inventory management. Figure 2 illustrates that \(\bar{s}^*\) should make the marginal benefit and

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13The construction cost of dams is huge. For recent Californian examples, the construction of Diamond Valley Lake costs 1.9 billion dollars for the 800000 acre-foot capacity, with the average being 2375 dollars per acre-foot; the expansion of the Los Vaqueros Reservoir in 2011–2012 from the 100000 to the 160000 acre-foot capacity costs 120 million dollars, with the average being 2000 dollars per acre-foot.

14The environmental damage of dams could be correlated with how water releases are used. In this paper we model the environmental damage that is related to water releases into the water release benefit function, \(B(w, \alpha)\). Moreover, \(B(w, \alpha)\) can be regarded as including all the outcomes of the dams that depend on water storage or releases, including also drought relief and flood control.

15For the readers who might think corner solutions should be emphasized in the optimal dam capacity problem, it could be helpful if we regard the dam capacity in our model as the storage capacity of a huge
the marginal cost of dam capacities intersect with each other, and hints that the key to the
economic analysis of optimal dam capacities is to investigate the property of the marginal
benefit of dam capacities, $V^*_1(s, a_0, \alpha)$, since any shifts, rotations, or other changes in the
marginal benefit will move the intersection of the marginal benefit and the marginal cost,
and therefore change the optimal dam capacity.\footnote{Readers might want to think $a_0 \equiv 0$, as there is no water in the dam when the dam is built. Readers can also think $a_0 \equiv e_0$, as there is no water carried from period $-1$ without the dam. The difference between the interpretations is minor in our analysis, so for simplicity, we leave $a_0$ in the dam-generated value function without specifying it.}

The optimal dam capacity, $s^*$, makes the marginal benefit and the marginal cost of dam capacities intersect with each other. Specification: $B(w, \alpha) = 172.2 \cdot \alpha w - \frac{137 \times 10^{-3}}{2} \alpha^2 w^2$, $d = 0.04$, $\rho = 0.9434$, $e \sim N(969113, 321503^2)$ is discretized into 5 quadrature nodes, $\alpha = 0.75$, $a_0 = 0$, and the marginal cost of dam capacities is $C'(s) + D'(s) = 2s^4 \cdot 10^{-21}$.

Figure 2: The determination of the optimal dam capacity

Our model has the minimal but still necessary complication to investigate the relation
among dam capacities, water allocation efficiency, and conservation technologies. To our
knowledge, it is the first optimal dam capacity determination model that incorporates conservation efficiency, stochastic inflows, and optimal water inventory management. The literature has proposed several dam capacity models. For example, Schoengold and Zilberman (2007, p.2943) model the marginal benefit of dam capacities in a static, black-box style, which is sufficient for their focus on the logic of oversized water projects, but might be difficult to proceed with further serious analysis and implications. In the same Chapter, Schoengold and Zilberman (2007, p.2955) also try to improve the model by incorporating water demand uncertainty and simplifying water release as the dam capacity. They stop their analysis after deriving the first-order condition, and they also ignore the stochastic inflows and conservation efficiency, so the improved model might help little to analyze the impact of conservation technologies on water storage capacities. Substantially contributing to the idea of conjunctive use of ground water and surface water, Tsur (1990) discusses the optimal capacity of a groundwater project as a buffer of uncertain supply of surface water. In the model, the inflow to the groundwater project is still assumed deterministic, and conservation efficiency isn’t involved, so the model might help little to examine the relation between water storage capacities and conservation technologies. An admirable attempt by Fisher and Rubio (1997) models the impact of the variability of the stochastic inflows on dam capacities in a setting of continuous-time dynamic control of water inventories and dam renovation. As inflows to dams are highly seasonal, we use a discrete-time approach to model the optimal water inventory management in our model, which is different to Fisher and Rubio (1997)’s approach. As we focus less on the real-time dam renovation, we simplify the dam capacity determination as a one-shot decision. Our model is also different from Fisher and Rubio (1997) as we incorporate conservation efficiency. The papers by von der Fehr and Sandsbraten (1997) and Haddad (2011) build simple models for the optimal capacity choice of a dam for hydropower generation, subject to a deterministic inflow. The models give good implications of the inflow profile and the industrial organization of the hydropower market on the optimal dam capacities, but stochastic water inventories and conservation consideration
aren’t applicable. Houba et al. (2012, 2013), Zhu et al. (2013), and Pham-Do et al. (2014) extend Haddad (2011)’s two-season model by incorporating other competing water uses. The extensions are useful to investigate the role of competing interests in dam capacity choices, but might help little to analyze the relation between dam capacities and conservation technologies. Another recent work by Xie and Zilberman (2014) investigates the impact of water allocation efficiency, inflow distribution, and overflow losses on water project capacities and the relation between dam capacities and conservation technologies. The dam considered in their model, however, doesn’t control water inventories intertemporally, so the analysis is more valid for the water projects that transfer water mainly from water-abundant areas to water-scarce areas, or from wet seasons to dry seasons within the same year, than for the projects that can intertemporally manage water inventories by adjusting water storage and releases. Our model in this paper allows the dam to transfer water not only interregionally but also intertemporally, so we can provide more implications for the role of flood control and drought relief of water projects.

Our model is close to the literature about optimal water inventory management. As in Burt (1964, 1966, 1967, 1970), Harrison (1977), Tsur and Graham-Tomasi (1991), Truong (2012), and many other papers, the literature that incorporates stochastic inflows or dynamic control focuses on the optimal control rule of water releases or inventories, but at the same time, largely increases the difficulty of analyzing the marginal benefit and the comparative statics of dam capacities in the models. Our model, as we shall show, extends the research by calculating and approximating the marginal benefit of dam capacities in a straightforward expression, and analyzing the comparative statics of dam capacities. Another series of studies by Dudley and Musgrave (1988), Freebairn and Quiggin (2006), Brennan (2008), Hughes

17Engineers have a long tradition of studying the determination of water project capacities, as an early effort can be traced back to Rippa (1853). This tradition, however, largely focuses on the minimization of the cost of a dam to satisfy specific engineering and policy constraints, but not the maximization of the dam-generated benefit net of the dam cost, as noticed by the surveys by Yeh (1985) and Simonovic (1992). The cost-minimization focus implies sophisticated investigation on the reliability and therefore the probability theory of water projects, but not their value and marginal value. Significant works include Hurst (1951, 1956), Whitin (1953), Moran (1959), and others, and influential surveys include Gani (1957, 1969), Prabh (1965, 1958, 1998), Lloyd (1957), Phatarfod (1989), and others.
and Goesch (2009a,b), Hughes et al. (2013), and Truong and Drynan (2013), among others, has focused on the optimal design of the water storage right, and they are well surveyed by Hughes (2013). Different to their approach, our paper adopts a macro-perspective by ignoring the mechanism design problem of the water storage right, but assuming a centralized water inventory management. Particularly, the closest model in the literature to our model is Truong (2012). The paper builds a water inventory management model, and analyze the impact of smaller storage capacities on water prices and water-generated values. Our model is different from Truong (2012)’s model in at least three aspects. First, Truong (2012)’s analysis focuses only on the impact of dam capacities on water inventory management, while we also focus on the relation between dam capacities and the alternative mitigation and adaptation approaches. Second, Truong (2012)’s model has another dimension of stochasticity, which is the rainfall and goes into the water release benefit function, while for simplicity, our model has only one dimension of stochasticity, which is the inflow. Our model, however, has another key parameter for our purpose, which is the conservation efficiency and doesn’t appear in Truong (2012)’s model. Last but not least, Truong (2012)’s model assumes that the storage capacity constrains the total amount of water that is available for inventory management, and that the overflows are disposed for free without any economic consequence. Our model, however, assumes that the storage capacity constrains only the dam’s capacity to control the water inventory, and that the overflows go into the economy with a low marginal benefit.

Our model also contributes to the literature about the competitive storage model which has been used to analyze the storable commodity market behaviors. The theoretical tradition follows Working (1933, 1934), Gustafson (1958a,b), Samuelson (1971), Gardner (1979), Newbery and Stiglitz (1981), Wright and Williams (1982), Scheinkman and Schechtman (1983), Williams and Wright (1991), Deaton and Laroque (1992), Chambers and Bailey.

\[18\] Dam models in applied probability theory, for example Moran (1959), sometimes follows Truong (2012)’s setting of free disposal of overflows. Our setting of the overflows causing a low marginal benefit is more realistic, especially in the perspective of a social planner, since there are always flooding damages associated with huge overflows. Appendix H shows that the difference between the settings doesn’t affect our main results much.
(1996), and Bobenrieth et al. (2002, 2012), among others. Truong (2012) is the first work with a theoretical focus on the impact of the upper bound of the total amount of commodities that are available for inventory management on the model equilibrium. Our model extends the literature by calculating, approximating, and analyzing the marginal benefit of storage capacities and focusing on the impact of other parameters on the optimal capacity.

3 The Solution to the Water Inventory Management Problem and the Marginal Benefit of Dam Capacities

The solution to the optimal water inventory management problem can be characterized by a Bellman (1957) equation, which is

\[ V^*(\bar{s}, a_0, \alpha) = \max_{w_0, s_0} \{ B(w_0, \alpha) + \rho E_0 [V^*(\bar{s}, (1 - d)s_0 + e_1, \alpha)] \} \] s.t.

\[-s_0 \leq 0, s_0 - \bar{s} \leq 0, -w_0 \leq 0, \text{ and } w_0 + s_0 - a_0 = 0. \]

The Karush (1939)–Kuhn and Tucker (1951) conditions are then

\[ B_1(w_0^*, \alpha) = -\mu_3^* + \mu_4^*, \]

\[ (1 - d)\rho E_0 [V^*_2 (\bar{s}, (1 - d)s_0^* + e_1, \alpha)] = -\mu_1^* + \mu_2^* + \mu_4^*, \]

\[-s_0^* \leq 0, s_0^* - \bar{s} \leq 0, -w_0^* \leq 0, w_0^* + s_0^* - a_0 = 0, \]

\[ \mu_1^* \geq 0, \mu_2^* \geq 0, \mu_3^* \geq 0, \text{ and } \mu_1^* s_0^* = 0, \mu_2^* (s_0^* - \bar{s}) = 0, \mu_3^* w_0^* = 0. \]
The conditions derive the Euler (in)equations:

\[ B_1(w_0^*, \alpha) \geq (1 - d) \rho E_0 [V_2^* (s, (1 - d)s_0^* + e_1, \alpha)] \text{ if } w_0^* = a_0 \text{ and } s_0^* = 0; \]
\[ B_1(w_0^*, \alpha) = (1 - d) \rho E_0 [V_2^* (s, (1 - d)s_0^* + e_1, \alpha)] \text{ if } 0 < w_0^* < a_0 \text{ and } 0 < s_0^* < a_0; \]
\[ B_1(w_0^*, \alpha) \leq (1 - d) \rho E_0 [V_2^* (s, (1 - d)s_0^* + e_1, \alpha)] \text{ if } w_0^* = a_0 - s \text{ and } s_0^* = s. \]

The (in)equations suggest that there exist two critical levels of the water availability, \( a \) and \( \bar{a} \): when the water availability \( a_t \) is smaller than \( a \), the dam should release all of the availability without any storage (\( s_t^* = 0 \) and \( w_t^* = a_t \)); when \( a_t \) is larger than \( \bar{a} \), the dam should store as much as possible but there are overflows (\( s_t^* = \bar{s} \) and \( w_t^* = a_t - \bar{s} \)).

Figure 3 illustrates the solution to the water inventory management problem. For any given water availability, \( a_t \), the water price under the optimal water inventory management is \( p_t \equiv p(a_t) \equiv V_2^* (s, a_t, x) \), as described by the solid line. The optimal water release, \( w_t^* \), makes the marginal water release benefit be equal to the price, which means \( w_t^* \) solves \( B(w_t, x) = p_t \). The optimal water storage is then the difference between the water availability and the optimal water release, which means \( s_t^* = a_t - w_t^* \). When \( a_t \) is smaller than \( a \), it is optimal to release all of the water and have no storage; when \( a_t \) is larger than \( \bar{a} \), it is optimal to store water at the full capacity, and release all the rest of the availability. The water price function then behaves as we see in Figure 3: when \( a_t \) increases from zero, the water price first decreases along the marginal water release benefit function, then departs from it after a kink at \( a_t = a \), and finally moves over another kink at \( a_t = \bar{a} \), having the same shape of the marginal water release benefit function but being shifted to the right by \( \bar{s} \).

To show the three Results of the paper, we first propose a Lemma which derives a useful expression of the marginal benefit of dam capacities.

**Lemma 1.** The marginal benefit of dam capacities under the optimal water inventory man-
The water price function in water availability under the optimal water inventory management is \( p(a_t) \equiv V^*_2(\tilde{s}, a_t, \alpha) \). Specification: \( B(w, \alpha) = 172.2 \cdot \alpha w - \frac{1.37 \times 10^{-4}}{2} \cdot \alpha^2 w^2 \), \( d = 0.04 \), \( \rho = 0.9434 \), \( e \sim N(969113, 321503^2) \) is discretized into 5 quadrature nodes, \( \alpha = 0.75 \), and \( \tilde{s} = 1502256 \).

Figure 3: The solution to the water inventory management problem

The water price function in water availability under the optimal water inventory management is equal to the net present value of the marginal benefit of overflow reduction:

\[
V^*_1(\tilde{s}, a_0, \alpha) = \sum_{t=0}^{\infty} \rho^t E_0 \left[ \begin{array}{c}
\text{No value without overflow} \\
I_{a_t^* > \tilde{a}}(a_t^*) \\
\text{Store vs. release} \\
(B_1(\tilde{a} - \tilde{s}, \alpha) - B_1(a_t^* - \tilde{s}, \alpha))
\end{array} \right] \geq 0.
\] (7)

Appendix B proves Lemma 1, which comes straightforwardly from the Karush (1939)–Kuhn and Tucker (1951) conditions. The intuition is simple. When the optimal water storage hasn’t reached the full capacity \((a_t^* < \tilde{a})\), a small increase in the dam capacity doesn’t contribute at all, since the capacity constraint isn’t binding and the shadow price of the constraint is zero. Only when there are overflows \((a_t^* > \tilde{a})\), the small increase in the dam capacity allows the water on the capacity margin to be stored and generate a relatively high
net present benefit of $B_1(\bar{a} - \bar{s}, \alpha)$, instead of being released and generating a relatively low current benefit of $B_1(a^*_t - \bar{s}, \alpha)$.\footnote{Readers might suspect that in reality the possibility of overflows were zero when a dam is well managed. On the contrary, overflows often happen, and a structure called spillway, releases floods and makes them under control so that the floods don’t destroy the dam. The design of spillways is important in hydraulic engineering. The extreme form of overflows is flooding, which isn’t rare in the California history, and aren’t ignored in literature. For example, the winter 1996–1997 damaging flood in California has motivated Fisher and Rubio (1997)’s paper on optimal dam renovation.}

4 The Impact of Water Allocation Efficiency on Optimal Dam Capacities

Lemma \ref{lemma:impact} helps to show the impact of water allocation efficiency improvement on optimal dam capacities.

**Proposition 1.** *If improvement in water allocation efficiency shifts the marginal water release benefit function proportionally up, then the optimal dam capacity becomes larger.*

Mathematically and more precisely, consider two functions of the water release benefit, $B_1(w, \alpha)$ and $B_2(w, \alpha)$, and the corresponding optimal dam capacities, $s^{*1}$ and $s^{*2}$. If $B_2(w, \alpha) = \gamma B_1(w, \alpha)$ and $\gamma > 1$, then $s^{*2} > s^{*1}$.

Appendix \ref{app:proof} proves Proposition \ref{prop:impact}. The intuition is simple and is illustrated in Figure \ref{fig:impact}. If improvement in water allocation efficiency shifts the marginal water release benefit function proportionally up, then the intertemporally relative values of water don’t change. Therefore, there won’t be any change in the optimal water inventory management rule, and the marginal benefit of dam capacities is shifted up proportionally. This shift moves the intersection between the marginal benefit of dam capacities and the marginal cost to the right, and then has the dam designer choose a larger dam capacity.

The logic of Proposition \ref{prop:impact} is similar to that of the result in Xie and Zilberman (2014) about the impact of water allocation efficiency improvement on optimal dam capacities. The main difference between the two results is that in Xie and Zilberman (2014), the dam isn’t
A proportional, upward shift in the marginal water release benefit proportionally shifts up the marginal benefit of dam capacities, which induces a larger optimal dam capacity. Specification: \( B_1(w, \alpha) = 172.2 \cdot \alpha w - \frac{1.37 \times 10^{-4}}{2} \cdot \alpha^2 w^2 \), \( B_2(w, \alpha) = \gamma B_1(w, \alpha) \), \( B_1^2(w, \alpha) = \gamma B_1^1(w, \alpha) \), \( \gamma = 2 \), \( d = 0.04 \), \( \rho = 0.9434 \), \( e \sim N(969113, 321503^2) \) is discretized into 5 quadrature nodes, \( \alpha = 0.75 \), \( a_0 = 0 \), and the arbitrary marginal cost of dam capacities is \( C'(\hat{s}) + D'(\hat{s}) = 2s^3 \cdot 10^{-21} \). The choice of the marginal cost of dam capacities doesn’t matter qualitatively.

Figure 4: The impact of improvement in water allocation efficiency on optimal dam capacities

assumed to transfer water intertemporally, while in this paper, the main role of the dam is to optimally, intertemporally manage water inventories. This difference means many policy implications applicable to the interregional water transfer projects in Xie and Zilberman (2014) could be also applicable to many intertemporal water storage projects. For example, the policies introducing water markets instead of queuing systems like the United States Central Valley Project Improvement Act of 1992 could expand the water release demand, and therefore larger water storage capacities could be optimally required in the western United States. Reallocating water from irrigation to hydropower or environmental sectors
could shift up the water release benefit function, for example in the case of the Murray–Darling Basin, and therefore it could be optimal to have larger water storage capacities in the significant agricultural area of Australia. Other implications are about optimally centralizing conveyance investments and weakening the market power of monopsony in water generation markets, for example the Water Users Associations in the western United States, among others, since those improvements in water allocation efficiency could suggest that having larger water storage capacities might be better for social welfare. The applicability of the implications for our model, however, is contingent on whether the improvement in water allocation efficiency doesn’t change the optimal water inventory management rule much, as required by Proposition 1.

5 The Relation between Dam Capacities and Conservation Technologies

In this Section we analyze the relation between dam capacities and conservation technologies: are they substitutes or complementary to each other? More specifically, we ask two questions: First, will adopting conservation technologies, which enhances conservation efficiency, encourage the dam designer to choose larger or smaller dam capacities? Second, will larger dam capacities increase or decrease water users’ incentive of adopting conservation technologies to enhance conservation efficiency?

The key to the two questions is eventually the sign of the second-order marginal benefit of dam capacities and conservation efficiency, $V_{13}^{\ast}(\bar{s}, a_0, \alpha)$. On the one hand, the sign tells whether an increase in conservation efficiency will shift the marginal benefit of dam capacities up or down, and whether the dam designer should choose larger or smaller dam

\footnote{For more detailed discussion about the examples, see Xie and Zilberman (2013), and the references they cite, which include Burness and Quirk (1979), Gisser and Sánchez (1980), Gisser and Johnson (1984), Howe et al. (1980), Chakravorty et al. (1993), Dinar and Tsui (1993), Brill et al. (1997), Chatterjee et al. (1998), Saleth and Dinar (2001), Zilberman and Schoengold (2002), Chong and Sunding (2006), Quiggin (2006), Schoengold et al. (2008), Chakravorty et al. (2009), Tsui (2010), and Truong (2012).}
capacities. On the other hand, the value of an increase in conservation efficiency, \( d\alpha \), for a representative water user, is approximately \( V_3^*(\bar{s}, a_0, \alpha)d\alpha \), so the sign of \( V_3^*(\bar{s}, a_0, \alpha) \) tells that whether this value, which is the representative water user’s incentive of adopting conservation technologies, is increasing or decreasing in the dam capacity.\(^{21}\)

It is clear, however, that it isn’t easy to sign \( V_1^*(\bar{s}, a_0, \alpha) \), as we have no closed-form expression for it. Lemma 4 doesn’t give us a closed-form expression for \( V_1^*(\bar{s}, a_0, \alpha) \), and it tells us that \( V_1^*(\bar{s}, a_0, \alpha) \) depends on the optimal control rule and the distribution of the water availability under the rule, which suggests that so does \( V_1^*(\bar{s}, a_0, \alpha) \). It is then impossible to analytically show the relation between dam capacities and conservation technologies, with general functional forms of the effectively used water benefit, \( B(\cdot) \), and the water release benefit, \( B(w, \alpha) \).

Fortunately, we can still show ample results with two families of the functions with rich insights and implications. The two families include first, the benefit functions that derive isoelastic marginal benefits (or water demands), and second, the quadratic benefit functions, which derive linear water demands. The two families are also useful representatives of other general functional forms for at least two reasons. First, as Vaux et al. (1981) recognize, the two families derive respectively the log-linear and the linear specifications of the water release demand, which are convenient for empirical analysis. Second, as Caswell and Zilberman (1986) highlight, whether the elasticity of the marginal productivity of the effectively used water (EMP, defined as \( -\frac{EB''(x)}{B'(x)} \)) is larger or smaller than one is important in determining the relation between conservation and water use.\(^{22}\) Along this tradition, the two families well represent general functional forms, since the benefit functions that derive isoelastic marginal benefits have constant EMPs, while the quadratic benefit functions have their EMPs vary

---

\(^{21}\) The literature on technology adoption usually models the adoption as a discrete choice problem with a fixed adoption cost. In this paper we model the adoption with a linear approximation of the incentive, and ignore the fixed cost. The simplifications give us analytical simplicity without losing the spirit of the trade-off between the adoption benefit and cost.

\(^{22}\) Conservation increases water use when EMP < 1, while decreases water use when EMP > 1. We follow Caswell and Zilberman (1980)’s naming of the measure. In other contexts, similar measures are called the coefficient of relative risk-aversion (RRA), or the curvature of \( B(\cdot) \) (e.g. in Williams and Wright (1991)).
from zero to infinity as the effectively used water increases.  

5.1 An Isoelastic Water Release Demand

In this section we show the result with the benefit functions that derive isoelastic marginal benefits. First we have a Lemma about the properties of the specification.

Lemma 2. Given $B_{11}(w, \alpha) < 0$ and $B_1(w, \alpha) > 0$, the following four statements are equivalent:

1) The elasticity of the marginal productivity of effective water is constant, i.e., $EMP \equiv -\frac{xB''(x)}{B(x)} = c$, where $c > 0$.

2) $B(x) = k_1 x^{1-c} + k_2$, where $0 < c < 1$ and $k_1 > 0$, or $c > 1$ and $k_1 < 0$; otherwise, $B(x) = k_1 \ln x + k_2$, where $c = 1$ and $k_1 > 0$; $k_2$ is an arbitrary constant.

3) The marginal water release benefit (or water demand) is downward sloping and isoelastic, i.e., $\mu \equiv \left( \frac{wB_{11}(w, \alpha)}{B_1(w, \alpha)} \right)^{-1} = -c^{-1}$, where $c > 0$.

4) Any change in conservation efficiency shifts the marginal water release benefit proportionally, i.e., $B_1(w, \alpha) = f(\alpha) \cdot g(w)$, where $f(\alpha)$ is positive and doesn’t depend on $w$, and $g(w)$ doesn’t depend on $\alpha$.

Further assume the four statements are true. If $c \neq 1$, then

$$V_1^*(s, a_0, \alpha) = k_1 \alpha^{1-c}(1-c) \sum_{t=0}^{\infty} \rho^t E_0 \left[ I_{a_t^* > \bar{a}(a_t^*)} \left[ (\bar{a} - s)^{-c} - (a_t^* - s)^{-c} \right] \right] \text{, and}$$

$$V_{13}^*(s, a_0, \alpha) = k_1 \alpha^{-c}(1-c)^2 \sum_{t=0}^{\infty} \rho^t E_0 \left[ I_{a_t^* > \bar{a}(a_t^*)} \left[ (\bar{a} - s)^{-c} - (a_t^* - s)^{-c} \right] \right]. \quad (8)$$

For the micro-foundation of the water benefit functions, $B(x)$ and $B(w, \alpha)$, one can first interpret them as the yield function in water use, multiplied by a fixed size of the land that is feasible for irrigation. The fixed irrigation land size is almost valid, since “incrementally altering the irrigated acreage of a particular field can require substantial costs for center pivots,” as recognized by Hendricks and Peterson (2012). Hendricks and Peterson (2012) also confirm the validity with an estimate of the extensive margin response of water use in the water price, as -0.01, which is fairly small.
if \( c = 1 \), then

\[
V^*_1(s, a_0, \alpha) = k_1 \sum_{t=0}^{\infty} \rho^t E_0 [I_{a_t > a}(a^*_t) \left[ (\bar{a} - \bar{s})^{-1} - (a^*_t - \bar{s})^{-1} \right]], \text{ and } V^*_{13}(s, a_0, \alpha) = 0. \tag{9}
\]

Appendix \( \text{D} \) proves Lemma \( \text{2} \). The first part comes from algebra, while the second part follows the logic used in Proposition \( \text{D} \) that when the marginal water release benefit is shifted proportionally, this shift doesn’t affect the optimal control rule and the distribution of the water availability under the rule, which largely simplifies the expression of \( V^*_ {13}(s, a_0, \alpha) \). Lemma \( \text{2} \) helps us to prove the following Proposition.

**Proposition 2.** Given a positive, downward sloping, isoelastic marginal water release benefit (or water release demand), dam capacities and conservation technologies are complementary if the water release demand is elastic, independent if it has a unit elasticity, and substitutes if it is inelastic. In other words, adopting conservation technologies encourages the dam designer to choose larger dam capacities, and larger dam capacities increase water users’ incentive of adopting conservation technologies, if and only if the water release demand is elastic.

Mathematically and more precisely, assume \( B_{11}(w, \alpha) < 0 \) and \( B_1(w, \alpha) > 0 \), and \( \mu \equiv \left( \frac{w B_{11}(w, \alpha)}{B_1(w, \alpha)} \right)^{-1} \) is a constant. Then \( V^*_ {13}(s, a_0, \alpha) > 0 \) if \( \mu < -1 \); \( V^*_ {13}(s, a_0, \alpha) = 0 \) if \( \mu = -1 \); \( V^*_ {13}(s, a_0, \alpha) < 0 \) if \( -1 < \mu < 0 \).

Appendix \( \text{E} \) proves Proposition \( \text{2} \). The intuition isn’t complicated. An elastic water release demand is equivalent to a low EMP. On the one hand, the low EMP means that conservation technologies would shift the marginal water release benefit up.\(^{24}\) When the marginal water release benefit is isoelastic, we don’t need to worry about the optimal control of water inventories, and the upward shift in the marginal water release benefit would apply to the marginal benefit of dam capacities, which would induce larger optimal dam capacities.

On the other hand, the low EMP also means that the marginal contribution of conservation

\(^{24}\)To see this point, note \( B_1(w, \alpha) = \alpha B'(\alpha w) \).
efficiency would be larger when more water is released. This relation would still hold when the water inventories are optimally controlled, since when the marginal water release benefit is isoelastic, changes in conservation efficiency don’t change the optimal control rule. As larger dams have generally more water released, they result in a higher marginal contribution of conservation efficiency, which means larger incentive of the adoption.

Figures 5 and 6 illustrate the complementarity between dam capacities and conservation technologies in the California State Water Project’s irrigation water inventory management problem if we assume the water release demand is isoelastic and elastic. In Figure 5, an increase in $\alpha$ shifts the marginal benefit of dam capacities upward, so larger dam capacities are desirable, given any marginal cost of dam capacities. In Figure 6, the marginal benefit of conservation efficiency is increasing in the dam capacity, so the incentive of adopting conservation technologies is increasing in the dam capacity.

Figures 7 and 8 illustrate the substitution between dam capacities and conservation technologies in the California State Water Project’s irrigation water inventory management problem if we assume the water release demand is isoelastic and inelastic. In Figure 7, an increase in $\alpha$ shifts the marginal benefit of dam capacities downward, so smaller dam capacities are desirable, given any marginal cost of dam capacities. In Figure 8, the marginal benefit of conservation efficiency is decreasing in the dam capacity, so the incentive of adopting conservation technologies is decreasing in the dam capacity.

Proposition 2 contributes to at least two threads of literature. First, compared with the literature on modeling the water project capacity determination, including Tsur (1990), Fisher and Rubio (1997), von der Fehr and Sandsbraten (1997), Schoengold and Zilberman (2007, p.2943,2955), Haddad (2011), Houba et al. (2012, 2013), Zhu et al. (2013), Pham-Do et al. (2014), and others, it is a novelty that Proposition 2 identifies conservation efficiency as a potential factor affecting the dam capacity decision, and presents the conditions under which the impact is positive or negative.

25To see this point, note $B_2(w, \alpha) = wB'(\alpha w)$. 

23
An increase in \( \alpha \) shifts up the marginal benefit of dam capacities, which means conservation technologies are complementary to dam capacities, in the sense that higher conservation efficiency induces a smaller optimal dam capacity. For aesthetic consideration we plot the marginal benefits of dam capacities with only three different values of \( \alpha \). Specification: \( B(w, \alpha) = 2.97 \times 10^7 \cdot \alpha^{1-\frac{1}{12}}w^{1-\frac{1}{12}} \), \( d = 0.04\), \( \rho = 0.9434\), \( e \sim N(969113, 321503^2) \) is discretized into 5 quadrature nodes, \( a_0 = 0\), and the arbitrary marginal cost of dam capacities is \( C'(\bar{s}) + D'(\bar{s}) = 25 + 4\bar{s}^4 \cdot 10^{-19} \). The choice of the marginal cost of dam capacities doesn’t matter qualitatively.

Figure 5: The impact of conservation efficiency \( \alpha \) on optimal dam capacities when the water release demand is isoelastic and elastic

The marginal benefit of conservation efficiency is increasing in dam capacity \( s \), so the advantage of higher conservation efficiency over lower conservation efficiency is increasing in \( s \), which means dam capacities are complementary to conservation technologies, in the sense that larger dams encourage conservation technology adoption. For aesthetic consideration we plot the marginal benefit of conservation efficiency with only one value of \( \alpha \). Specification: \( B(w, \alpha) = 2.97 \times 10^7 \cdot \alpha^{1-\frac{1}{21}} \cdot w^{1-\frac{1}{21}} \), \( d = 0.04, \rho = 0.9434, e \sim N(969113, 321503^2) \) is discretized into 5 quadrature nodes, and \( a_0 = 0 \).

Figure 6: The impact of dam capacity \( s \) on conservation technology adoption when the water release demand is isoelastic and elastic

(2006), and Baerenklau and Knapp (2007), among others. The factors in focus include farm sizes, labor availability, tenure systems, market imperfection and learning cost, economic variables (water prices and others), capital constraints, water-related endowments (land quality, well depth, climate, and others), production risk, human capital, resource exhaustibility, water markets, agricultural characteristics, policies, and psychology, among others, and good surveys are also written by Feder et al. (1985), Caswell (1991), Sunding and Zilberman (2001), and Schoengold and Zilberman (2007). Apparently, few studies have
An increase in $\alpha$ shifts down the marginal benefit of dam capacities, which means conservation technologies are substitutes to dam capacities, in the sense that higher conservation efficiency induces a smaller optimal dam capacity. For aesthetic consideration we plot the marginal benefits of dam capacities with only three different values of $\alpha$. Specification: $B(w, \alpha) = -7.19 \times 10^9 \cdot \alpha^{1-\frac{1}{\sigma_w}} w^{1-\frac{1}{\sigma_w}}, \ d = 0.04, \ \rho = 0.9434, \ e \sim N(969113, 321503^2)$ is discretized into 5 quadrature nodes, $a_0 = 0$, and the arbitrary marginal cost of dam capacities is $C'(s) + D'(s) = 75 + 4s^4 \cdot 10^{-19}$. The choice of the marginal cost of dam capacities doesn’t matter qualitatively.

Figure 7: The impact of conservation efficiency $\alpha$ on optimal dam capacities when the water release demand is isoelastic and inelastic discussed the implication of water project capacities, though large public water projects usually affect a large number of water users.\textsuperscript{26} A recent study by Schoengold and Sunding (Forthcoming) investigate the impact of the water price uncertainty. Their investigate could be related to dam capacities, since Truong (2012) implies that the uncertainty is deeply

\textsuperscript{26}One reason for the lack of attention of the impact of water projects on conservation technology adoption could be the difficulty in empirically identifying the impact: on the one hand, as public water projects affect a large number of water users, there is little variation in the water project capacity across these water users; on the other hand, across the water users with different water projects, it is also difficult to argue that there aren’t any confounding factors when we attempt to estimate the impact of water project capacity.
The marginal benefit of conservation efficiency is decreasing in dam capacity $\bar{s}$, so the advantage of higher conservation efficiency over lower conservation efficiency is decreasing in $\bar{s}$, which means dam capacities are substitutes to conservation technologies, in the sense that larger dams discourage conservation technology adoption. For aesthetic consideration we plot the marginal benefit of conservation efficiency with only one value of $\alpha$. Specification: $B(w, \alpha) = -7.19 \times 10^9 \cdot \alpha^{1 - \frac{1}{79}} w^{1 - \frac{1}{79}}$, $d = 0.04$, $\rho = 0.9434$, $e \sim N(969113, 321503^2)$ is discretized into 5 quadrature nodes, and $a_0 = 0$.

Figure 8: The impact of dam capacity $\bar{s}$ on conservation technology adoption when the water release demand is isoelastic and inelastic depending on dam capacities, but the authors don’t work on it further as it isn’t their main focus.

The most interesting contribution of Proposition 2 is perhaps to the intersection between the two threads of literature, which is about the relation between water project capacities and conservation technologies. Intuitively, it is tempting to conclude that water project capacities and conservation technologies should be substitutes to each other: if conservation efficiency is higher, water users seem to be less urgent for water supply and water project
capacities; if water project capacities are larger and water supply is always abundant, water users seem to have less incentive to adopt conservation technologies. Many studies, however, recognize that the relation between water use or availability and conservation should depend on the EMP, and could be complementary. This recognition is first modeled in Caswell and Zilberman (1986), and well recognized in Caswell et al. (1991), Dinar et al. (1992), Dinar and Zilberman (1991a), Shah et al. (1993), Dridi and Khanna (2011), Lichtenberg (2013), Pfeiffer and Lin (2014), and surveys like Feder and Umali (1993) and Lichtenberg (2002), among others. The potential complementarity have also been shown empirically relevant by studies like Ellis et al. (1985), Peterson and Ding (2005), Frisvold and Emerick (2008), Ward and Pulido-Velazquez (2008), Dagnino and Ward (2012), and Pfeiffer and Lin (2013), among others. Xie and Zilberman (2014) introduce the dependence and the potential complementarity to water project capacities by considering dams without water inventory management. Proposition 2 contributes to the analysis by taking the optimal intertemporal water transfer into account, showing the key role of the impact of conservation efficiency on overflows in determining the substitution or complementarity.

Proposition 2 also has important policy implications. When the water release demand is regarded isoelastic and the elasticity is high, first, governments could encourage conservation technology adoption by increasing the capacity of public water storage under the optimal water inventory management; second, subsidizing water users to adopt conservation technology adoption could encourage them to demand private, optimally managed water storage capacity or service; third, in the other way around, the policies encouraging water users to use water storage to fight against drought could also encourage them to adopt conservation technologies. When the isoelastic water release demand is inelastic, the opposite policy implications would follow. After all, to find the correct policy, the governments should have good knowledge about the elasticity of the water release demand.

\[27\] The media coverage on conservation leading to more water use includes Fountain (2008) and Nixon (2013), among others. Some other studies in a more hydrological perspective, for example Huffaker and Whittlesey (1993, 2000, 2003), Whittlesey and Huffaker (1997), Whittlesey (2000), Scheierling et al. (2006b), and Huffaker and Whittlesey (2008), also suggest that subsidies to conservation could increase water use.
As Proposition 2 highlights the water release demand elasticity, it would be worthy to discuss a little bit more about the factors affecting the elasticity and its estimate. The meta-analysis by Scheierling et al. (2006a) finds that the estimate of the irrigation water demand elasticity is more elastic if it comes from mathematical programming or econometric studies, instead of field experiments. The finding implies that by Proposition 2, if we assume the irrigation water demand is isoelastic, then the estimation method of the demand elasticity could be crucial in determining the relation between irrigation dam capacities and conservation technologies.

It is also natural to speculate that the water release demand elasticity is highly correlated with the economic properties of the water-produced commodity.\(^{28}\) We formalize the idea by assuming \(B(x) = l(x)D(l(x))\), where \(l(\cdot)\) is the production function of the water-produced commodity in the effectively used water, and \(D(\cdot)\) is the isoelastic inverse demand function of the commodity. Further assume \(l'(\cdot) > 0, l''(\cdot) < 0\), and the commodity demand elasticity \(\nu \equiv D(l'(x))l'(x)D'(l'(x)) < 0\). Some algebra then gives the water release demand elasticity as \(\mu = \left(\frac{x''(x)}{l'(x)} + \frac{1}{\nu} \frac{x'(x)}{l'(x)}\right)^{-1}\), which means 1) that the absolute values of the commodity demand elasticity and the water release demand elasticity move in the same direction, 2) that the water release demand would be inelastic when the commodity demand is sufficiently inelastic, and 3) that the water release demand would be elastic when the EMP of the commodity, \(\frac{x''(x)}{l'(x)}\), is smaller than one and the commodity demand is sufficiently elastic. The following Corollary is then straightforward.

**Corollary 1.** Given a positive, downward sloping, isoelastic marginal water release benefit (or water release demand), an isoelastic demand for the water-produced commodity, and a commodity production function in water with its EMP being smaller than one, dam capacities and conservation technologies are complementary if the commodity demand is sufficiently elastic, and substitutes if it is sufficiently inelastic.

\(^{28}\)For example, the meta-analysis by Scheierling et al. (2006a) finds that the irrigation water demand is more inelastic if the irrigated crop is high-valued.
Corollary \(\Box\) carries important policy implications, too. On the one hand, many small, developing countries are exporting agricultural commodities, and the sector is important for the economy. When their production is small in the world market, the commodity demand they face is almost perfectly elastic. In this case, improvements in conservation efficiency, which could be resulted from international aid, could optimally lead to larger dams for the irrigation needed for the commodity production. This point suggests that the aid that tackling water challenges in developing countries should have a joint perspective about international trade, conservation, and water infrastructure. On the other hand, for the dams for the purposes of producing non-exported commodities or commodities with low demand elasticity, for example, electricity and staple food, dam capacities and conservation could be substitutes to each other. This point suggests that the joint policy about conservation and water infrastructure should critically depend on the property of the water-produced commodity.

5.2 A Linear Water Release Demand

Now we turn to the quadratic effectively used water benefit function, which means \(B'''(\cdot) = 0\) and the derived water release demand is linear. We denote the discounted, expected sum of overflows, which is \(\sum_{t=0}^{\infty} \rho^t E_0 [\max \{a_t - \bar{a}, 0\}]\), as \(X(s, a_0, \alpha)\), and its conservation efficiency elasticity, \(\frac{\alpha}{X(s, a_0, \alpha)} \cdot \frac{\partial X(s, a_0, \alpha)}{\partial \alpha}\), as \(\epsilon\). We then proceed with another Lemma, which expresses the marginal benefit of dam capacities using the discounted, expected sum of overflows and its elasticity with respect to conservation efficiency.

**Lemma 3.** If \(B''(\cdot) = 0\), then \(V_1^*(s, a_0, \alpha) = \alpha^2 |B''(\cdot)| X(s, a_0, \alpha)\), and

\[
V_{13}^*(s, a_0, \alpha) = |B''(\cdot)| \alpha X(s, a_0, \alpha) (2 + \epsilon).
\]  

(10)

Appendix \(\Box\) proves Lemma \(\Box\). Lemma \(\Box\) highlights the two main channels of the impact of conservation efficiency on the marginal benefit of dam capacities. On the one hand, when
there are overflows, as we have argued in Lemma I, the marginal increase in the dam capacity allows the water on the capacity margin to be stored instead of being released. Higher conservation efficiency means that the gain from storing instead of releasing increases. On the other hand, higher conservation efficiency will change the optimal water management rule, which will further change the likelihood and amount of overflows. We can then predict that if higher conservation efficiency reduces overflows a lot, then this channel might dominate the first channel, and make higher conservation efficiency shift the marginal benefit of dam capacities down. This prediction establishes the following Proposition.

Proposition 3. Given a linear marginal water release benefit (or water release demand), dam capacities and conservation technologies are complementary to each other, if and only if a small increase in conservation efficiency doesn’t reduce overflows much. In other words, adopting conservation technologies encourages the dam designer to choose larger dam capacities, and larger dam capacities increase water users’ incentive of adopting conservation technologies, if and only if a small increase in conservation efficiency doesn’t reduce overflows much.

Mathematically and more precisely, assume \( B'''(\cdot) = 0 \). Then \( V_1^*(\bar{s}, a_0, \alpha) \) is increasing in \( \alpha \), and \( V_3^*(\bar{s}, a_0, \alpha) d\alpha \) is increasing in \( \bar{s} \) when \( d\alpha > 0 \), if and only if \( \epsilon > -2 \).

Appendix C proves Proposition 3. Proposition 3 suggests that it is possible that dam capacities and conservation technologies are complementary given a linear water release demand, and the complementarity requires that the conservation efficiency elasticity of the discounted, expected sum of overflows isn’t too negative.²⁹

²⁹ Some more analysis can show that it could be more than possible that dam capacities and conservation technologies are complementary given a linear water release demand. Consider the following three cases. 1) Consider the case in which \( \bar{s} = 0 \). In this case, an increase in \( \alpha \) won’t change the discounted, expected sum of overflows at all, since the overflow at \( t \) is always equal to \( e_t \). Then \( V_{13}^{**}(\bar{s}, a_0, \alpha) > 0 \) for sure when \( \bar{s} = 0 \). As it isn’t too wild to assume that \( V_{13}^{**}(\bar{s}, a_0, \alpha) \) is continuous when \( \bar{s} \) is small, we can then see \( V_{13}^{**}(\bar{s}, a_0, \alpha) > 0 \) when \( \bar{s} \) is small. 2) Consider another case in which \( \bar{s} \) is so large that overflows are impossible. In this case, \( V_{13}^{**}(\bar{s}, a_0, \alpha) = 0 \), and an increase in \( \alpha \) won’t decrease the discounted, expected sum of overflows, since the amount of overflows is nonnegative. Then \( V_{13}^{**}(\bar{s}, a_0, \alpha) \geq 0 \) for sure when \( \bar{s} \) is sufficiently large. 3) The case with a moderate \( \bar{s} \) is a little bit more complicated, and we discuss the impact of a small increase in conservation efficiency on overflows in this case in more detail. First, consider a given
Figures 9 and 10 illustrate the complementarity between dam capacities and conservation technologies in the California State Water Project’s irrigation water inventory management problem with the linear water release demand. In Figure 9, increases in conservation efficiency shift up the marginal benefit functions of dam capacities, so the intersections between the marginal benefit functions and any marginal cost function moves to the right. This pattern suggests that given any marginal cost function, the dam designer should choose larger dam capacities when conservation efficiency is higher. In Figure 10, all of the marginal benefit functions of conservation efficiency are increasing in the dam capacity, so the incentive of adopting conservation technologies is increasing in the dam capacity.

Proposition 3 has some similar contribution to literature and policy implications as those of Proposition 2, namely identifying the relation between dam capacities and conservation, and suggesting potential sharing ground between the policies respectively encouraging conservation and water project capacity expansion. Moreover, Proposition 3 highlights the role of the optimal water inventory control in the result. Caswell and Zilberman (1986) imply that if the water benefit function is quadratic, then the EMP crosses unity as the applied water increases. The implication means that water use and conservation should be complementary when water is abundant, while substitutes to each other when water is scarce. Xie and Zilberman (2014) find the described relation would still be valid, if the water use current availability $a_t$. The small increase in $\alpha$ will change the storage supply, $B(a_t - s_t, \alpha)$, and the storage demand, $(1 - d)\rho E_t [V(\bar{s}, (1 - d)s_t + c_{t+1}, \alpha)]$. The change in the storage supply should dominate that in the storage demand, since there is a discount factor, $(1 - d)\rho < 1$, in the storage demand. Second, the increase in $\alpha$ should shift the storage supply upward around the intersection between the storage supply and the storage demand, when the horizontal coordinate of the intersection is smaller than a critical level $\bar{s}$, which means $a_t$ isn’t very large, while downward otherwise. Third, when $a_t > \bar{a}$, the storage capacity will still be binding, so $s_t^* = \bar{s}$ still holds; when $a_t < \bar{a}$, $s_t^* = 0$ still holds, too; only when $\bar{a} < a_t < \bar{a}$, $s_t^*$ will change, and it will decrease and derive smaller $a_{t+1}$ if $a_t$ isn’t large, while increase and derive larger $a_{t+1}$ otherwise. Therefore, the change in overflows really depends on which scenarios dominates, the scenario with a relatively small $a_t$, or that with a relatively large $a_t$. When the former dominates, the small increase in $\alpha$ will be more likely to decrease $a_{t+1}$ and reduce overflows; when the latter dominates, it will be more likely to increase $a_{t+1}$ and overflows. Fourth, one can imagine that when $\bar{s}$ is large, the scenario with a relatively large $a_t$ is much more likely to dominate, than when $\bar{s}$ is small. Therefore, if the considered dam capacities are fairly large, we can almost say that a small increase in conservation efficiency is more likely to increase but not decrease overflows.

We can also infer from the figure that $\epsilon > -2$, since it is easy to show that $\epsilon = \alpha \frac{V_2^*(\bar{s}, a_0, \alpha)}{V_2^*(\bar{s}, a_0, \alpha)} - 2$ when the water release demand is linear.
An increase in $\alpha$ shifts up the marginal benefit of dam capacities, which means conservation technologies are complementary to dam capacities, in the sense that higher conservation efficiency induces a larger optimal dam capacity. For aesthetic consideration we plot the marginal benefits of dam capacities with only five different values of $\alpha$. Specification: $B(w, \alpha) = 172.2 \cdot \alpha w - \frac{1.37 \times 10^{-4}}{2} \alpha^2 w^2$, $d = 0.04$, $\rho = 0.9434$, $e \sim N(969113, 321503^2)$ is discretized into 5 quadrature nodes, $a_0 = 0$, and the arbitrary marginal cost of dam capacity is $C'(\hat{s}) + D'(\hat{s}) = 2s^4 \cdot 10^{-21}$. The choice of the marginal cost of dam capacities doesn’t matter qualitatively.

Figure 9: The impact of conservation efficiency $\alpha$ on optimal dam capacities in the irrigation water inventory management of the California State Water Project if the water release demand is linear is replaced by water project capacities, and if the project is for water transfer or diversion, without optimal control of water inventories. Proposition 3 suggests that the relation becomes more subtle if we allow the water inventories to be optimally controlled, and whether conservation reduces overflows matters in determining the relation.

Putting Propositions 2 and 3 together also suggests that the choice of the specification of the water benefit function and the water release demand is critical in determining not only the relation between water use and conservation, as suggested in Caswell and Zilberman (1980),
The marginal benefit of conservation efficiency is increasing in dam capacity \( s \), so the advantage of higher conservation efficiency over lower conservation efficiency is increasing in \( s \), which means dam capacities are complementary to conservation technologies, in the sense that larger dams encourage conservation technology adoption. For aesthetic consideration we plot the marginal benefits of conservation efficiency with only five different values of \( \alpha \). Specification: 

\[
B(w, \alpha) = 172.2 \cdot \alpha w - \frac{1.37 \times 10^{-4}}{2} \cdot \alpha^2 w^2, \ d = 0.04, \ \rho = 0.9434, \ e \sim N(969113, 321503^2) \]

is discretized into 5 quadrature nodes, and \( a_0 = 0 \).

Figure 10: The impact of dam capacity \( s \) on conservation technology adoption in the irrigation water inventory management of the California State Water Project if the water release demand is linear but also the relation between dam capacities and conservation. The two econometrically convenient specifications, namely the isoelastic water release demand and the linear water release demand, could give opposite qualitative predictions of the relation. The comparison between Figures 6 and 8 and Figures 7 and 10 confirms this point, since the two water release demands in the two pairs of Figures both have a water release demand elasticity of -0.79 if the demand is equal to the mean of the 1975–2010 annual water deliveries from the
California State Water Project to agricultural use, but differ in their functional forms: the water release demand in Figures 7 and 8 is isoelastic, while the demand in Figures 9 and 10 is linear. This confirmation implies that on conservation issues, the robustness of the simulation results around the water demand specification should deserve more attention.

6 Conclusion

To analyze the relation between water storage capacities and the alternative approaches to improving the water resource sustainability, we build a stylized model for the optimal dam capacity determination under the optimal water inventory management. To our knowledge, the model is the first optimal dam capacity determination model that incorporates conservation efficiency, stochastic inflows, and optimal water inventory management. It also contributes to the literature on optimal water inventory management and storable commodity markets by analyzing the role of storage capacities in the models.

We show that larger dams could be demanded when improvement in water allocation efficiency proportionally shifts the water release demand up. We also show the conditions under which dam capacities and conservation technologies aren’t substitutes, but complementary to each other, given different specifications of the water release demand: given an isoelastic water release demand, the complementarity will hold if and only if the water release demand is elastic; given a linear water release demand, the complementarity will hold as long as a small increase in conservation efficiency doesn’t reduce overflows much. The conditions suggest the significance of the water release demand specification in the analysis.

From a broader perspective, our results link to the literature on the Jevons (1865) paradox, the Khazzoom (1980, 1987, 1989)–Brookes (1992) postulate, or the rebound effect, primarily in energy economics (e.g. surveys by Greening et al. (2000) and Hertwich (2005)).

Studies find that the irrigation water demand is usually inelastic (e.g. Moore et al. (1994), Schoengold et al. (2000), Hendricks and Peterson (2012), and a meta-analysis by Scheierling et al. (2006a)). For the dams for irrigation water use, by Proposition 2, an isoelastic specification of the water demand suggests that the dam capacities and conservation should be substitutes to each other. A linear specification, however, is very likely to suggest a complementarity, as discussed in Footnote 29. 
In the literature, a positive rebound effect on energy use could partially offset the energy saving effect of the improvement in energy use efficiency. In water economics, the rebound effect of conservation technologies is sometimes even so large that their adoption could induce more water use (e.g. Ward and Pulido-Velazquez (2008) and Pfeiffer and Lin (2014)). Our results extend the investigation on the rebound effect in water economics further from water use to significant water infrastructures like dams, reservoirs, and other water storage projects, which usually have a large number of stakeholders.

As the dam in our model transfers water across time, our results provide important implications for the design of water systems in the areas where water is endowed with huge intertemporal uncertainty and inequality: 1) integrated improvements in water allocation efficiency, for example the United States Central Valley Project Improvement Act of [1992], could also optimally require more or larger water projects; 2) our result also suggests that the policies encouraging public or private water storage could encourage water users to adopt conservation technologies, for example the drip irrigation, the water recycling system, the field monitoring system, and the solar-driven desalination plants, and the policies subsidizing conservation technology adoption could also increase the demand for water storage capacities. All the implications suggest that in water policy discussions, there could be shared ground between the policies encouraging respectively expanding water storage capacities and improving water allocation and conservation efficiency. As the relation between the policies is important in policy debates and could be counterintuitive, it should deserve more serious theoretical modeling and empirical investigations.

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A Appendix: The Specification of the California State Water Project’s Irrigation Water Inventory Management Problem

1996, and 2001 is 0.038, which is approximately 0.04. Given the evaporation rate, the 1974–2010 end-of-calendar-year storage data, and the 1975–2010 delivery data, we can find the corresponding 1975–2010 inflows by calculation, which have a mean of 3486019 acre-feet and a corrected sample standard deviation of 1156486 acre-feet. The Department (1963–2013) also reports that at the end of 2010, the project-wide storage capacity is 5.4038 million acre-feet.

The Department (1963–2013) records the 1975–2010 data of the annual deliveries to agricultural use, which have a mean of 936098 acre-feet, or equivalently, 27.80% of the total delivery. We use this percentage to adjust the inflow distribution and the storage capacity, which means that for agricultural use, the baseline storage capacity is $0.2780 \times 5.4038 \times 10^6 = 1502256$ acre-feet, and the inflow distribution has a mean of $0.2780 \times 3486019 = 969113$ acre-feet and a corrected sample standard deviation of $0.2780 \times 1156486 = 321503$ acre-feet. The mean and the standard deviation are used to generate the normal distribution for the inflows in the illustration.

The Department (2010) shows that in 2010 43.0% of the irrigated land in California uses gravity irrigation, 15.4% sprinkler irrigation, and 38.4% low-volume irrigation. Brouwer et al. (1989) suggest that the conservation efficiency of gravity or flood irrigation is around 0.6, sprinkler irrigation around 0.75, and low-volume or drip irrigation around 0.90. An easy method to estimate the general conservation efficiency is to calculate the average of the conservation efficiencies among the three irrigation technologies, weighted by the acreage percentages. The calculation gives an estimate of 0.743, which is approximately 0.75.\footnote{The easy method implicitly assumes that the per acre water use is the same across the three technologies. The estimate will be an underestimate of the general conservation efficiency if the EMP is smaller than one.}

The estimates of the price elasticity of the water demand for irrigation in California vary in a wide range. An early estimate by Frank and Beattie (1979) is -1.21, while a more recent one by Schoengold et al. (2006) is -0.79 with a sample in which the mean price is $46.49 per thousand cubic meters, which is approximately $57 per acre-foot. We then assume that in our specifications, the water release demand should be 936098 acre-feet if the water price
is $57 per acre-foot and the conservation efficiency is 0.75.\textsuperscript{33} Given this assumption, we specify three water release benefit functions satisfying respectively 1) that the derived water release demand (or marginal water release benefit) is isoelastic and has an elasticity of -1.21, 2) that the derived water release demand is isoelastic and has an elasticity of -0.79, and 3) that the derived water release demand is linear and has an elasticity of -0.79 when the demand is 936098 acre-feet (and the price is $57 per acre-foot). The three water release benefit functions and three derived water release benefit functions are then 1) $B(w, \alpha) = 2.97 \times 10^7 \cdot \alpha^{1-\frac{1}{1.21}} w^{1-\frac{1}{1.21}} + k_2$ and $B_1(w, \alpha) = 29723558.9866 \times (1 - \frac{1}{1.21}) \cdot \alpha^{1-\frac{1}{1.21}} w^{1-\frac{1}{1.21}}$, 2) $B(w, \alpha) = -718961554.6 \cdot \alpha^{1-\frac{1}{0.79}} w^{1-\frac{1}{0.79}} + k_2$ and $B_1(w, \alpha) = -718961554.6 \times (1 - \frac{1}{0.79}) \cdot \alpha^{1-\frac{1}{0.79}} w^{1-\frac{1}{0.79}}$, and 3) $B(w, \alpha) = 172.20 \cdot 202529353 \cdot \alpha w - \frac{0.0001370269}{2} \cdot \alpha^2 w^2 + k_2$ and $B_1(w, \alpha) = 172.20 \cdot 202529353 \cdot \alpha - 0.0001370269 \cdot \alpha^2 w$, respectively, where $k_2$ is an arbitrary constant. For simplicity we set $k_2 = 0$. Table 1 summarizes and Figure 11 plots the three specifications.

Table 1: Specifications of the water release benefit function in the California State Water Project’s irrigation water inventory management problem

<table>
<thead>
<tr>
<th>Water release benefit function</th>
<th>Derived water release demand</th>
<th>Related Figures</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B(w, \alpha) = 2.97 \times 10^7 \cdot \alpha^{1-\frac{1}{1.21}} w^{1-\frac{1}{1.21}}$</td>
<td>Isoelastic, elastic, $\mu = -1.21$</td>
<td>3 and 10</td>
</tr>
<tr>
<td>$B(w, \alpha) = -7.19 \times 10^9 \cdot \alpha^{1-\frac{1}{0.79}} w^{1-\frac{1}{0.79}}$</td>
<td>Isoelastic, inelastic, $\mu = -0.79$</td>
<td>4 and 8</td>
</tr>
<tr>
<td>$B(w, \alpha) = 172.2 \cdot \alpha w - \frac{0.37 \times 10^{-4}}{2} \cdot \alpha^2 w^2$</td>
<td>Linear, equivalent to $\mu = -0.79$</td>
<td>2, 3, 4, 6, 11 and 13</td>
</tr>
</tbody>
</table>

$\mu$ is the price elasticity of the derived water release demand (or marginal water release benefit).

In water project evaluations, the annual discount rate recommended by the California Department of Water Resources (2008) is 0.06. The corresponding discount factor is $(1 + \textsuperscript{33}The number 57 is chosen only for convenience. Note only the relative price but not the absolute price matters, so the choice doesn’t matter for the qualitative results that we illustrate.
The water release demands are $B_1(w_t, \alpha)$, calculated respectively with the three specifications in Table 4 when $\alpha$ is equal to 0.75. The demand elasticity is denoted as $\mu$. The demands are equal to 936098 acre-feet when the price is $57 per acre-feet.

Figure 11: Three specifications of the water release demand in the California State Water Project’s irrigation water inventory management problem

$0.06)^{-1} = 0.9434.$

Table 4 summarizes the specification of the Project’s irrigation water inventory management problem.

**B Appendix: Proof of Lemma 1**

*Proof.* Careful investigation on the [Karush (1939)](karush1939)determination conditions helps:

1) When $a_0 < \bar{a}$: $s_0^* = 0$, so $s_0 - \bar{s} < 0$, and then $\mu_2^* = 0$.

2) When $\bar{a} < a_0 < \bar{a}$: $s_0^* < \bar{s}$, so $s_0 - \bar{s} < 0$, and then $\mu_2^* = 0$. $w_0^* > 0$, so $\mu_3^* = 0$. Then $B_1(w_0^*, \alpha) = \mu_4^* = (1 - d)\rho\mathbb{E}_0[V_2^*(\bar{s}, (1 - d)s_0^* + e_1, \alpha)]$. 

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Table 2: Specification of the California State Water Project’s irrigation water inventory management problem

| Inflow in acre-feet          | $e_t \sim N(969113, 321503^2)$, i.i.d. |
| Evaporation loss rate        | $d = 0.04$                                |
| Discount factor              | $\rho = 0.9434$                           |
| Water release benefit function in $\$ | $B(w, \alpha) = 2.97 \times 10^7 \cdot \alpha^{1 - \frac{1}{1.21}} w^{1 - \frac{1}{1.21}}$ |
| (one in each illustration)   | $B(w, \alpha) = -7.19 \times 10^9 \cdot \alpha^{1 - \frac{1}{0.79}} w^{1 - \frac{1}{0.79}}$ |
|                              | $B(w, \alpha) = 172.2 \cdot \alpha w - \frac{1.37 \times 10^{-4}}{2} \cdot \alpha^2 w^2$ |
| Baseline conservation efficiency | $\alpha = 0.75$                       |
| Baseline dam capacity in acre-feet | $\bar{s} = 1502256$               |

Based on the information from the California Department of Water Resources (1963-2013, 1976-2014, 2008-2010), Frank and Beattie (1979), Brouwer et al. (1989), and Schoengold et al. (2006).

3) When $a_0 > \bar{a}$: $s_0^* = \bar{s} > 0$, so $\mu_1^* = 0$. $w_0^* = a_0 - \bar{s} > 0$, so $\mu_3^* = 0$. Then $B_1(w_0^*, \alpha) = \mu_4^*$ and $(1 - d)\rho E_0 [V_2^*(\bar{s}, (1 - d)s_0^* + e_1, \alpha)] = \mu_2^* + \mu_4^*$, so

$$
\mu_2^* = (1 - d)\rho E_0 [V_2^*(\bar{s}, (1 - d)s_0^* + e_1, \alpha)] - B_1(w_0^*, \alpha) = (1 - d)\rho E_0 [V_2^*(\bar{s}, (1 - d)\bar{s} + e_1, \alpha)] - B_1(a_0 - \bar{s}, \alpha) = B_1(\bar{a} - \bar{s}, \alpha) - B_1(a_0 - \bar{s}, \alpha) > 0,
$$

(11)

where the last inequation is by $B''(\cdot) < 0$.

Note $\mu_2^*$ is the marginal contribution of dam capacities to the period-0 dam-generated value. By iterations we can then prove the Lemma.

\[\Box\]

C Appendix: Proof of Proposition \[\square\]

\[\text{Proof.}\] By Lemma \[\square\],

$$
V_1^{*1}(\bar{s}, a_0, \alpha) = \sum_{t=0}^{\infty} \rho^t E_0 [I_{a_t^1 > \bar{a}^1}(a_t^{*1}) (B_1^1(\bar{a}^1 - \bar{s}, \alpha) - B_1^1(a_t^{*1} - \bar{s}, \alpha))] , \text{ and}
$$

$$
V_1^{*2}(\bar{s}, a_0, \alpha) = \sum_{t=0}^{\infty} \rho^t E_0 [I_{a_t^2 > \bar{a}^2}(a_t^{*2}) (B_1^2(\bar{a}^2 - \bar{s}, \alpha) - B_1^2(a_t^{*2} - \bar{s}, \alpha))] .
$$

(12)
If $B^2_1(w, \alpha) = \gamma B^1_1(w, \alpha)$, then $\bar{a}^1 = \bar{a}^2$, and $a^1_t$ and $a^2_t$ share the same distribution. Given $\gamma > 1$, $V^2_1(\bar{s}, a_0, \alpha) = \gamma V^1_1(\bar{s}, a_0, \alpha) > V^1_1(\bar{s}, a_0, \alpha)$ for any $\bar{s} > 0$. Since the optimal dam capacities make the marginal benefit of dam capacities intersect with the marginal cost from the above, $\bar{s}^* > \bar{s}^1$.

\[ \square \]

D Appendix: Proof of Lemma 2

Proof. Note first $B''(\cdot) < 0$ and $B'(\cdot) > 0$ follow $B_{11}(w, \alpha) < 0$, $B_1(w, \alpha) > 0$, and $B(w, \alpha) \equiv B(\alpha w)$.

From 1) to 2): $-\frac{x B''(x)}{B'(x)} = c$ is a second-order ordinary differential equation, whose solution is $B(x) = k_1 x^{1-c} + k_2$ if $c \neq 1$ and $B(x) = k_1 \ln x + k_2$ when $c = 1$, where $k_1$ and $k_2$ are arbitrary constants. $B''(\cdot) < 0$ and $B'(\cdot) > 0$ derive the result about the signs of the constants.

From 2) to 3): $B(w, \alpha) \equiv B(\alpha w)$ derives the result straightforwardly.

From 3) to 1): By $B(w, \alpha) \equiv B(\alpha w)$, $\mu \equiv \left( \frac{w B_{11}(w, \alpha)}{B_1(w, \alpha)} \right)^{-1} = \left( \frac{\alpha w B''(\alpha w)}{B'(\alpha w)} \right)^{-1}$. Since $\mu = -c^{-1}$, where $c$ is a positive constant, then $\text{EMP} \equiv -\frac{x B''(x)}{B'(x)} = c$.

From 2) to 4): $B(w, \alpha) \equiv B(\alpha w)$ derives the result straightforwardly.

From 4) to 1): By $B(w, \alpha) \equiv B(\alpha w)$, $B_1(w, \alpha) = \alpha B'(\alpha w)$. Since $B_1(w, \alpha) = f(\alpha)g(w)$, we have then $\alpha B'(\alpha w) = f(\alpha)g(w)$. Taking the derivative of both sides with respect to $w$ gives $\alpha^2 B''(\alpha w) = f(\alpha)g'(w)$. The last two equations gives $-\frac{\alpha w B''(\alpha w)}{B'(\alpha w)} = -\frac{w g'(w)}{g(w)}$.

Note $\alpha B'(\alpha w) = f(\alpha)g(w)$ gives $B'(w) = f(1)g(w)$, so $g(w) = \frac{B'(w)}{f(1)}$, and then $-\frac{\alpha w B''(\alpha w)}{B'(\alpha w)} = -\frac{w g'(w)}{g(w)} = -\frac{w B''(w)}{B'(w)}$, which means $-\frac{x B''(x)}{B'(x)}$ is a constant. Since $B''(\cdot) < 0$ and $B'(\cdot) > 0$, the constant is positive.

The first part of the Lemma is then proved. Note that when 4) holds, any change in conservation efficiency doesn’t affect the optimal control rule and the distribution of the water availability under the rule. The second part then follows Lemma 4 and some algebra. \[ \square \]
E Appendix: Proof of Proposition 2

Proof. By Lemma 2, the sign of $V_{13}^*(\bar{s}, a_0, \alpha)$ is the same as the sign of $k_1$ when $c \neq 1$, and is zero when $c = 1$. By $\mu = -c^{-1}$, the result follows.

F Appendix: Proof of Lemma 3

Proof. Following Lemma 3, if $B''(\cdot) = 0$, then

$$V_1^* (\bar{s}, a_0, \alpha) = \sum_{t=0}^{\infty} \rho^t \mathbb{E}_0 \left[ I_{a_t^* > a}(a_t^*) B_{11}(\cdot, \alpha) (\bar{a} - a_t^*) \right]$$

$$= \sum_{t=0}^{\infty} \rho^t \mathbb{E}_0 \left[ I_{a_t^* > a}(a_t^*) \alpha^2 B''(\cdot) (\bar{a} - a_t^*) \right]$$

$$= \alpha^2 B''(\cdot) \sum_{t=0}^{\infty} \rho^t \mathbb{E}_0 \left[ I_{a_t^* > a}(a_t^*) (\bar{a} - a_t^*) \right]$$

$$= \alpha^2 |B''(\cdot)| X(\bar{s}, a_0, \alpha). \hspace{1cm} (13)$$

We then have

$$V_{13}^*(\bar{s}, a_0, \alpha) = |B''(\cdot)| \left( 2\alpha X(\bar{s}, a_0, \alpha) + \alpha^2 \frac{\partial X(\bar{s}, a_0, \alpha)}{\partial \alpha} \right)$$

$$= |B''(\cdot)| \alpha X(\bar{s}, a_0, \alpha) \left( 2 + \frac{\alpha}{X(\bar{s}, a_0, \alpha)} \frac{\partial X(\bar{s}, a_0, \alpha)}{\partial \alpha} \right)$$

$$= |B''(\cdot)| \alpha X(\bar{s}, a_0, \alpha) \left( 2 + \epsilon \right). \hspace{1cm} (14)$$

G Appendix: Proof of Proposition 3

Proof. We have two ways to prove. First, following Lemma 3, $V_{13}^*(\bar{s}, a_0, \alpha) > 0$ if and only if $\epsilon > -2$. The result of the Proposition follows. Second, it is also easy to show that
\[ \epsilon = \alpha \frac{V^*_1(s, a_0, \alpha)}{V^*_1(s, a_0, \alpha)} - 2. \] Given \( V^*_1(s, a_0, \alpha) \geq 0, V^*_3(s, a_0, \alpha) > 0 \) is then equivalent to \( \epsilon > -2. \)  

H Appendix: Comparison of Three Dam Models

In this Section we compare the implications of three different dam models, namely the models in this paper, Xie and Zilberman (2014), and Truong (2012).

In Xie and Zilberman (2014)’s model, the main role of the dam in the water system is to transfer water interregionally, but not intertemporally. The dam cannot hold more inflow than how much its capacity allows, and the dam releases all of the held. In other words, the dam capacity limits the water availability. More precisely, the dam-generated benefit in their model is

\[ V(\bar{w}, \alpha) = E_0 \left[ \sum_{t=0}^{\infty} \rho^t B(\min \{e_t, \bar{w}\}, \alpha) \right], \tag{15} \]

where \( \bar{w} \) is the dam capacity. Xie and Zilberman (2014) show that the improvements in water allocation efficiency that shift the marginal water release benefit up will induce larger optimal dam capacities, and that the relation between dam capacities and conservation technologies is non-monotonic: they are complementary when dams are small, while substitutes when dams are large.

In Truong (2012)’s model, the dam optimally managed water inventories, while the dam capacity still limits the water availability, and overflows are disposed freely. More precisely, a simplified version of the dam-generated benefit in his model is

\[ V^*(\bar{s}, a_0, \alpha) \equiv \max_{\{w_t\}_{t=0}^{\infty}, \{s_t\}_{t=0}^{\infty}} E_0 \left[ \sum_{t=0}^{\infty} \rho^t B(w_t, \alpha) \right] \text{ s.t.} \]

\[
(1 - d)s_{t-1} + e_t \equiv a_t \geq w_t + s_t \text{ for all } t \geq 1, \\
w_0 + s_0 \leq a_0, \ w_t + s_t \leq \bar{s} \text{ for all } t \geq 0, \\
w_t \geq 0 \text{ for all } t \geq 0, \text{ and } s_t \geq 0 \text{ for all } t \geq 0. \tag{16}
\]

Define \( w^*(a, \bar{s}) \) as the optimal water releases when the water availability is \( a \), given the dam
capacity \( \bar{s} \). By some algebra, the marginal benefit of dam capacities is then

\[
V_1^*(\bar{s}, a_0, \alpha) = B_1(w^*(\bar{s}, \bar{s}), \alpha) \sum_{t=0}^{\infty} \rho^t E_0[I_{a_t^* > \bar{s}}(a_t^*)] > 0. \tag{17}
\]

This expression suggests that for Truong (2012)'s model, Propositions 1 and 2 in this paper still apply, while Proposition 3 becomes a little more complicated, but at least one of the main messages still applies: if a small increase in conservation efficiency reduces overflows a lot, then dam capacities and conservation technologies could be substitutes to each other.