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Exponential Extrapolation of Fourier Transformed Potentials in 2.5-D dc Resistivity Modeling

Hee Joon Kim*, Yoonho Song** and Ki Ha Lee***

ABSTRACT

In 2.5-D dc resistivity modeling which allows for subsurface current and potential electrodes, numerical errors depend on the coarseness of discretization and increase with spatial wavenumbers. In this regard, the exponential extrapolation of Fourier transformed potentials is useful for the evaluation of inverse Fourier transform integral in the outmost range of wavenumbers. This paper presents an accurate integral scheme in the outmost range using an exponential function of which coefficient is represented by a ratio between the modified Bessel functions of order 1 and 0. The effectiveness of this scheme has been confirmed using a homogenous half-space model and a vertical fault model.

Key words: 2.5-D dc resistivity modeling, discretization error, spatial wavenumber

1. INTRODUCTION

The dc resistivity response in the 2-D earth due to a 3-D (point) current source is described by Poisson's equation:

\[-\nabla \cdot [\sigma(x, z) \nabla \phi(x, y, z)] = i_s(x, y, z), \quad (1)\]

where \(\sigma(x, z)\) is the electrical conductivity, \(\phi(x, y, z)\) the electrical potential, and \(i_s(x, y, z)\) the source current distribution. By taking the Fourier transform of equation (1) with respect to the \(y\) coordinate, one obtains

\[-\nabla \cdot [\sigma(x, z) \nabla \Phi(x, \lambda, z)] + \lambda^2 \sigma(x, z) \Phi(x, \lambda, z) = I_s(x, \lambda, z), \quad (2)\]

where \(\lambda\) is the Fourier transform variable (spatial wavenumber).

If the Fourier transformed potentials \(\phi(x, y, z)\) are obtained for several values of \(\lambda\), then the electrical potential \(\phi(x, y, z)\) can be evaluated using the inverse Fourier transform. When both current and potential electrodes are located along \(y = 0\), the inverse Fourier transform becomes (Dey and Morrison, 1979)

\[\phi(x, 0, z) = \frac{2}{\pi} \int_0^\infty \Phi(x, \lambda, z) d\lambda. \quad (3)\]

Dey and Morrison (1979) performed this integration by fitting the envelope of \(\Phi(\lambda)\) in each subsection \([\lambda_i, \lambda_{i+1}]\) by an exponential function and using the analytic form

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\[ \int_{\lambda_i}^{\lambda_{i+1}} \Phi(\lambda) \, d\lambda = \int_{\lambda_i}^{\lambda_{i+1}} c \exp(-a\lambda) \, d\lambda \]

\[ = \Phi(\lambda_{i+1}) - \Phi(\lambda_i) \]

\[ \frac{a}{\lambda_{i+1} - \lambda_i} \cdot \frac{\ln(\Phi(\lambda_{i+1})/\Phi(\lambda_i))}{a}, \tag{4} \]

where \( a = \frac{\ln(\Phi(\lambda_{i+1})/\Phi(\lambda_i))}{\lambda_{i+1} - \lambda_i} \), \tag{5}

and \( c \) is a constant.

To reduce errors associated with the Fourier integral (3), Fujisaki et al. (1992) suggested a logarithmic interpolation of transformed potentials for small values of \( \lambda \), where \( r \) is the distance between current and potential electrodes, and an exponential interpolation for large values. The optimum value of \( \lambda \) to switch from the logarithmic function to the exponential function was given as about 0.8 by Kim et al. (1995). In the innermost interval \([0, \lambda_1]\), a logarithmic extrapolation is useful to minimize the error associated with the singularity at \( \lambda = 0 \) (Fujisaki et al., 1992; Haryu, 1996). In the outermost interval \([\lambda_{N-1}, \lambda_N]\), on the other hand, an exponential extrapolation is effective to reduce a truncation error associated with a finite value of \( \lambda_N \) (Kim et al., 1995; Haryu, 1996).

The accuracy of a numerical model depends on the coarseness of discretization (Kim et al., 1995; Haryu, 1996). In 2.5-D dc resistivity modeling, the coarseness of discretization is essentially varied with spatial wavenumbers. Haryu (1996) suggested that the mesh size could be decided based on the Nyquist wavenumber. If we do not change the mesh size for all wavenumbers, Fourier transformed potentials \( \Phi(\lambda) \) for large values of \( \lambda \) contain significant errors because the mesh size may be too large to simulate the potentials (see Figure 2 in Kim et al. (1995)). Since this discretization error increases with increasing \( \lambda \) and is most obvious in the outermost interval \([\lambda_{N-1}, \lambda_N]\), Kim et al. (1995) used the following extrapolation formula:

\[ \int_{\lambda_{N-1}}^{\lambda_N} c \exp(-a\lambda) \, d\lambda = \frac{\Phi(\lambda_{N-1}) - \Phi(\lambda_N)}{a}. \tag{6} \]

Here, it is assumed that potentials \( \Phi(\lambda) \) outside of \( \lambda_{N-1} \) are beyond an acceptable level of the discretization error. The coefficient \( a \) can be approximated using known \( \Phi(\lambda_{N-1}) \) and \( \Phi(\lambda_{N-2}) \) as

\[ a = \frac{\ln(\Phi(\lambda_{N-1})/\Phi(\lambda_{N-2}))}{\lambda_{N-1} - \lambda_{N-2}}. \tag{7} \]

The coefficient \( a \) controls the shape of exponential function, and the error of the integration in equation (6) may be small when \( \lambda_{N-2} \) is sampled near \( \lambda_{N-1} \). However, to suppress the amount of computations, Fujisaki et al. (1992) and Kim et al. (1995) used a relatively wide samplings: \( \lambda_{N-2} = 0.32 \) and \( \lambda_{N-1} = 0.64 \). Thus there is a room for improvement in determining \( a \) if we adopt their sampling scheme for an efficient and accurate evaluation of the Fourier integral.

2. EXPONENTIAL EXTRAPOLATION

When \( \lambda \) approaches to \( \lambda_{N-1} \), the mesh size would become enormously large to simulate Fourier transformed potentials \( \Phi(\lambda) \), which largely depend on the resistivity distribution surrounding an observation point accordingly. If the resistivity distribution near an observation point is homogeneous, the potential \( \Phi(\lambda) \) for large \( \lambda \) may have the form

\[ \Phi(\lambda) = A\left[K_0(\lambda r) + K_0(\lambda r_i)\right], \tag{8} \]

where \( A \) is a constant, \( K_0 \) the modified Bessel function of order 0, and \( r_i \) the distance between image source and observation point. Substituting equation (8) into equation (7) yields

\[ a = \frac{\ln\left[\frac{K_0(\lambda r) + K_0(\lambda r_i)}{K_0(\lambda r_i) + K_0(\lambda r_i)}\right]}{\lambda_{N-1} - \lambda_{N-2}}. \tag{9} \]

If \( \lambda_{N-2} \) approaches to \( \lambda_{N-1} \) equation (9) can be rewritten as

\[ a = \frac{\partial \ln\left[K_0(\lambda r) + K_0(\lambda r_i)\right]}{\partial \lambda} \bigg|_{\lambda = \lambda_{N-1}} \]

\[ = \frac{\lambda r K_1(\lambda r) + \lambda r_i K_1(\lambda r_i)}{K_0(\lambda r) + K_0(\lambda r_i)} \]

\[ \frac{\lambda_{N-1} - \lambda_{N-2}}{\lambda_{N-1} - \lambda_{N-2}} \]

\[ = \frac{\lambda r K_1(\lambda r) + \lambda r_i K_1(\lambda r_i)}{K_0(\lambda r) + K_0(\lambda r_i)} \]

\[ \frac{\lambda_{N-1} - \lambda_{N-2}}{\lambda_{N-1} - \lambda_{N-2}} \]

\[ = \frac{\lambda r K_1(\lambda r) + \lambda r_i K_1(\lambda r_i)}{K_0(\lambda r) + K_0(\lambda r_i)} \]

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\[ = \frac{\lambda r K_1(\lambda r) + \lambda r_i K_1(\lambda r_i)}{K_0(\lambda r) + K_0(\lambda r_i)} \]

\[ \frac{\lambda_{N-1} - \lambda_{N-2}}{\lambda_{N-1} - \lambda_{N-2}} \]

\[ = \frac{\lambda r K_1(\lambda r) + \lambda r_i K_1(\lambda r_i)}{K_0(\lambda r) + K_0(\lambda r_i)} \]

where \( K_1 \) is the modified Bessel function of order 1. This scheme uses only one wavenumber, \( \lambda_{N-1} \), and the coefficient \( a \) is expressed by the ratio between the modified Bessel functions of order 1 and order 0. Simple numerical experiments are now conducted to find the validity of equation (10). In the experiments we use the same sampling scheme as Fujisaki et al. (1992) and Kim et al. (1995). If one or both of source and observation points are
located on the surface of a homogeneous half-space model and the distance between them is \( r = 10 \, \text{m} \), for example, the coefficient \( a \) in the interval \([0.64, 1.28]\), \( a_1 \) is obtained from equation (5) as

\[
    a_1 = \frac{\ln \left( K_0 (6.4) / K_0 (12.8) \right)}{1.28 - 0.64} = 10.53.
\]

If we use the exponential extrapolation scheme of Kim et al. (1995), the coefficient \( a \) in the interval \([0.32, 0.64]\), \( a_2 \) is given by

\[
    a_2 = \frac{\ln \left( K_0 (3.2) / K_0 (6.4) \right)}{0.64 - 0.32} = 11.03.
\]

This result shows that \( a_2 \) is 4.75% greater than \( a_1 \), and thus the integrated value with \( a_2 \) will be 4.75% smaller than with \( a_1 \). On the other hand, the new algorithm yields a coefficient \( a_3 \) as

\[
    a_3 = \frac{K_1 (6.4)}{K_0 (6.4)} = 10.75.
\]

Because a difference between the coefficients of \( a_2 \) and \( a_3 \) is only 2.09%, the coefficient \( a_3 \) is much better approximation to \( a_1 \) than \( a_2 \).

Next example is for a vertical fault model, for which the potential \( \phi(\lambda) \) is obtained analytically, as shown in Figure 1. Two media of 10 and 100 \( \Omega\cdot\text{m} \) are horizontally contacted. A current source of 1 A is located on the surface 4 m away from the vertical contact. Seven observation points are horizontally positioned at 2 m in depth near the contact. It is clear that equation (8) is derived by ignoring the effect of inhomogeneity on \( \phi(\lambda) \). However, our scheme works well even in the non-uniform model. From Table 1, we can see that the coefficient \( a_3 \) is much better approximation to \( a_1 \) than \( a_2 \).

3. CONCLUSIONS

We have analyzed discretization errors of 2.5-D dc resistivity modeling that allows for subsurface current and potential electrodes. Since numerical errors depend on the coarseness of discretization and increase with increasing spatial wavenumber \( \lambda \), the exponential extrapolation of Fourier transformed potentials is useful for the evaluation of inverse Fourier transform integral in the outmost range of \( \lambda \). In this paper we have developed an integral scheme in the outmost range using an accurate exponential extrapolation function. The coefficient of the exponential function is represented by the ratio between the modified Bessel functions of order 1 and order 0, and its shape is much better than that used in Kim et al. (1995).

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2.5次元直流比抵抗モデリングにおける
フーリエ交換されたポテンシャルの指数関数外挿

Hee Joon Kim, Yoonho Song and Ki Ha Lee

要 旨
地下に電流および電位電極が存在する場合の2.5次元直流比抵抗モデリングにおいて、数値誤差は離散化の粗さに依存し、空間波数が大きくなるにつれて増大する。このような観点から、フーリエ変換されたポテンシャルの指数関数による外挿は波数の最外側区間における逆フーリエ変換積分の評価に有効である。本論文では、最外側区間において係数がオーダー1および1の修正ベッセル関数の比によって表される指数関数を用いる精度のよい積分スキームを紹介する。この方法の有効性は均質半空間モデルおよび垂直断層モデルに対する簡単な数値実験を通じて確かめられた。