Cooperative Demand Response Using Repeated Game for Price-Anticipating Buildings in Smart Grid

Kai Ma  
School of Electrical and Electronic Engineering  
Nanyang Technological University  
Singapore 639798  
Email: kma@ntu.edu.sg

Guoqiang Hu  
School of Electrical and Electronic Engineering  
Nanyang Technological University  
Singapore 639798  
Email: gqhu@ntu.edu.sg

Costas J. Spanos  
Department of Electrical Engineering and Computer Sciences  
University of California Berkeley, CA 94720 USA  
Email: spanos@berkeley.edu

Abstract—This paper proposes a cooperative demand response program for distributed price-anticipating buildings in smart grid. The cooperative demand response program is formulated as a constrained social optimization problem. We develop a cooperative strategy and obtain a Pareto optimal solution from the constrained social optimization problem. Comparing with the Nash equilibrium obtained from the one-stage demand management game, the Pareto optimal solution reduces the electricity costs to all the building managers. We further align this Pareto optimal solution with the subgame perfect Nash equilibrium of a repeated demand management game and develop an incentive-compatible trigger-and-punishment mechanism to avoid the noncooperative behaviors of the building managers. Numerical results demonstrate that the cooperative demand response program can reduce the electricity costs, lower the electricity prices, and cut down the total energy consumption.

I. INTRODUCTION

Matching supply with demand has been an active topic in operating electricity markets [1]. Traditionally, we need enough generation capacity to meet the peak load, which requires substantial infrastructure to be idle for all but a few hours a year. Recently, demand response has been proposed to control the load instead of providing enough generation capacity. In practice, demand response can be implemented by direct load control or market-based pricing. For the direct load control, energy providers have the ability to remotely shut down consumer equipments on a short notice when needed [2], [3]. For the market-based pricing, energy providers can adjust the load by flexible pricing, such as critical peak pricing (CPP) and real-time pricing (RTP) [4]. An advanced metering infrastructure (AMI) is used for collecting the energy consumption and announcing the electricity price [5].

There are two types of consumers in the literature of price-based demand response program: price-taking consumers [6]–[8] and price-anticipating consumers [9]–[15]. The price-taking consumers assumes that their energy consumption cannot effect the electricity price, whereas the price-anticipating consumers believe that their energy consumption can change the electricity price. In fact, the price-anticipating consumers are usually referred to the large energy consumers such as commercial buildings. It was reported that the commercial buildings have large potential to provide demand response to the smart grid [16], [17].

Recently, game theory has been applied to study the demand response with price-anticipating consumers. For example, the noncooperative game was utilized to study the cost minimization of interactive consumers [9], [10], the charging control of plug-in electric vehicles (PEV) [11]–[13]. Stackelberg game was employed to model the interactions between the consumers and the utility companies [14], [15]. However, neither the Nash equilibrium nor the Stackelberg equilibrium of the these game models are Pareto optimal solutions. Generally, Pareto optimality is an important criterion for evaluating economic systems and public policies. If economic allocation in any system is not Pareto efficient, there is potential for a Pareto improvement—an increase in Pareto efficiency. Nevertheless, few papers are devoted to the Pareto improvement for the demand management system. In this study, we study the cooperative demand response program with price-anticipating commercial buildings and give a Pareto optimal solution for the demand management system such that all the building managers have lower electricity costs at this solution compared with that at the Nash equilibrium.

The novelties of this work are twofold. First, we formulate the cooperative demand response as a social optimization problem and prove that the solution in this optimization problem is a subgame perfect Nash equilibrium of a repeated demand management game. Second, we develop a incentive-compatible trigger-and-punishment mechanism to avoid the noncooperative behaviors of the building managers and establish the conditions on the durations of the punishment. To the best of our knowledge, there is no work in the literature providing rigorous analysis of Pareto improvement in the demand response program.

The rest of the paper is organized as follows. The system model are built in Section II. In Section III, the cooperative demand response strategy is developed, and the social optimal solution is proved to be a subgame perfect Nash equilibrium of a repeated game. A incentive-compatible trigger-and-punishment mechanism is developed to avoid the noncooperative behaviors of the building managers. Numerical results are shown in Section IV, and conclusions are summarized in Section V.

II. SYSTEM MODEL

We consider a demand management system composed of a control center and several buildings, as shown in Fig. 1.
The control center can adjust the energy consumption of the buildings by periodically announcing the prices to the building managers. We assume that the building managers are price-anticipating consumers, i.e., the building managers know that the pricing curve is affected by their energy consumption. According to the updated electricity price, the building managers can adjust the temperature settings of the HVAC system to reduce their electricity costs. The electricity cost is composed of two aspects: the costs caused by the discomforts and the payments. Next, we will give the formulations for these two costs.

A. Discomfort costs

For buildings with HVAC systems, changing the temperature settings will cause discomfort to the occupants. The discomfort costs are defined as the following Taguchi loss function [18]:
\[ V_i^l(T_i^{\text{in}}(t)) = \theta_i(T_i^{\text{in}}(t) - \hat{T}_i(t))^2, \quad i \in \mathcal{N}, \] (1)
where \( \mathcal{N} = \{1, 2, \ldots, N\} \) denotes the set of building managers, \( i \) denotes the index of building managers, \( t \) denotes the index of time slots, and \( \theta_i \) is the cost coefficient. \( T_i^{\text{in}}(t) \) and \( \hat{T}_i(t) \) target the indoor temperature and the actual indoor temperature in time slot \( t \), respectively.

B. Electricity payments

The electricity payments of building manager \( i \) are denoted as
\[ V_i^p = p(l_i), \quad i \in \mathcal{N}, \] (2)
where \( l = \{l_1, l_2, \ldots, l_N\} \). According to the technical report from U. S. Department of Energy, the electricity price can be approximated to a linear function of the total energy consumption [19].
\[ p(l) = \lambda(\sum_{i \in \mathcal{N}} l_i - L) + p_0, \] (3)
where \( L \) is the forecast demand, \( \lambda \) is the pricing parameter to implement elastic pricing, and \( p_0 \) is the base price when the actual total energy consumption is equal to \( L \).

C. Electricity Costs

The electricity cost to building manager \( i \) is defined as
\[ V_i = V_i^l + V_i^p = \theta_i \gamma_i^2(l_i - \hat{l}_i)^2 + (\lambda(\sum_{i \in \mathcal{N}} l_i - L) + p_0)l_i. \] (4)

The discomfort costs and the electricity payments are usually conflict with each other, and the building managers need to make a tradeoff between them.

III. MAIN RESULTS

A. Game Formulation and Cooperative Strategy

From the cost formulation (7), the energy consumption of the building managers will change the electricity price and further effect the electricity costs to the other building managers. Thus, the demand management can be formulated as the following noncooperative game:

Definition 1. (One-stage demand management game) A demand management game is defined as a triple \( G = \{\mathcal{N}, (S_i)_{i \in \mathcal{N}}, (U_i)_{i \in \mathcal{N}}\} \), where \( \mathcal{N} = \{1, 2, \ldots, N\} \) is the set of active building managers participating in the game, \( S_i \) is the set of possible strategies that building manager \( i \) can take, and
\[ U_i = -V_i = -\theta_i \gamma_i^2(l_i - \hat{l}_i)^2 - (\lambda(\sum_{i \in \mathcal{N}} l_i - L) + p_0)l_i \]
is the payoff function.

The solution in the one-stage demand management game is the Nash equilibrium, which can be obtained from \( \partial U_i / \partial l_i = 0, i \in \mathcal{N} \), yields
\[ -2\theta_i \gamma_i^2(l_i - \hat{l}_i) - \lambda(\sum_{i \in \mathcal{N}} l_i - L) - p_0 - \lambda l_i = 0, \quad i \in \mathcal{N}. \] (5)

The coefficient matrix of the above equations is denoted as
\[ A = \begin{bmatrix} -2\theta_1 \gamma_1^2 - 2\lambda & -\lambda & \ldots & -\lambda \\ -\lambda & -2\theta_2 \gamma_2^2 - \lambda & \ldots & -\lambda \\ \vdots & \vdots & \ddots & \vdots \\ -\lambda & -\lambda & \ldots & -2\theta_N \gamma_N^2 - 2\lambda \end{bmatrix}. \] (6)
Following the Gerschgorin theorem [20], the coefficient matrix is nonsingular if
\[ \lambda \leq \frac{2\theta_i \gamma_i^2}{N - 3}, \quad \text{for} \ i \in \mathcal{N}, \] (10)

with which, the Nash equilibrium is unique and can be denoted as
\[ I^\text{NE} = A^{-1} C, \] (11)

where \( C \) is defined as
\[ C = \begin{bmatrix} p_0 - \lambda L - 2\theta_1 \gamma_1^2 \hat{l}_1 \\ p_0 - \lambda L - 2\theta_2 \gamma_2^2 \hat{l}_2 \\ \vdots \\ p_0 - \lambda L - 2\theta_N \gamma_N^2 \hat{l}_N \end{bmatrix}. \] (12)

Generally, the Nash equilibrium is not a Pareto optimal solution, and there exist possibilities to increase the payoffs of all the building managers simultaneously. Next, we develop a cooperative strategy to improve the Pareto efficiency. Specifically, the building managers negotiate their energy consumption according to the following social optimization problem:

(P1) \[ \text{maximize} \sum_{i \in \mathcal{N}} U_i \]
subject to \[ U_i > U_i^\text{NE}, \quad i \in \mathcal{N}, \]

where \( U_i^\text{NE} \) denotes the payoff of building manager \( i \) obtained from the noncooperative game. Let \( I^c = \{ l_1^c, \ldots, l_N^c \} \) denote the social optimal energy consumption obtained from (P1) and \( U_i^c \) denote the cooperative payoff of building manager \( i \). It is easy to see that \( I^c \) is a Pareto optimal solution and \( U_i^c \) is larger than \( U_i^c \) for all \( i \in \mathcal{N} \). However, the social optimal energy consumption is not achievable in one-stage demand response because all the building managers can improve their payoffs by taking the noncooperative strategy and the noncooperative behaviors will not affect their future payoffs. To make the social optimal energy consumption achievable, we need to give some punishments to the building managers in the future if they adopt the noncooperative strategy. In that case, the building managers will play the one-stage demand management game repeatedly and care more about the long-term electricity costs. The average electricity cost to building manager \( i \) over multiple stages is defined as
\[ \bar{V}_i = (1 - \delta) \sum_{k=1}^{\infty} \delta^{k-1} V_i(k), \] (13)

where \( k \) is the index of the stages and \( \delta \in (0, 1) \) is the discount factor, which represents how the building managers discount their future costs. In that case, the building managers not only value the current electricity costs but also the future electricity costs. Therefore, each building manager will keep a good reputation to avoid the increased cost in the future. Next, we give the definition of the repeated demand management game.

Definition 2. (Repeated demand management game) Suppose the one-stage demand game \( G \) is repeated infinitely and all the building managers can observe the strategies and the payoffs of the others, an infinite repeated demand management game is defined as \( G(\infty, \delta) = \{ \mathcal{N}, (S_i)_{i \in \mathcal{N}}, (U_i)_{i \in \mathcal{N}} \} \), where
\[ \bar{U}_i = \bar{V}_i \]
\[ = -(1 - \delta) \sum_{k=1}^{\infty} \delta^{k-1} (\theta_i \gamma_i^2 (l_i - \hat{l}_i)^2 - (\lambda \sum_{i \in \mathcal{N}} l_i - L) + p_0 l_i) \]

is the average payoff function.

According to the Folk theorem [21], there exits a subgame perfect Nash equilibrium of the repeated demand management game such that the average payoff \( \bar{U}_i \) is equal to the cooperative payoff \( U_i^c \) for all \( i \in \mathcal{N} \) when \( \delta > \delta^* \), where \( \delta^* \) is a threshold to ensure the existence of subgame perfect Nash equilibrium.

B. Trigger-and-Punishment Mechanism

In this section, we will develop a trigger-and-punishment mechanism to avoid the noncooperative behaviors and prove that the social optimal energy consumption is a subgame perfect Nash equilibrium of the repeated demand management game. Before that, we first characterize the impact of the noncooperative behaviors on the demand management system.

1) Impact of the noncooperative behaviors: We assume only one building manager \( i \) takes the noncooperative strategy when the other building managers adopt the cooperative strategy. In that case, the energy consumption of building manager \( i \) is denoted as
\[ t_i^d = \text{arg max} U_i(l_i^c, l_{i-1}^c, l_{i+1}^c, \ldots, l_N^c), \] (14)

from which, we obtain
\[ t_i^d = \frac{p_0 - \lambda L - 2\theta_i \gamma_i^2 \hat{l}_i + \lambda \sum_{j \in \mathcal{N}, j \neq i} l_j^c}{-2\theta_i \gamma_i^2}. \] (15)

The payoff of building manager \( i \) obtained from the noncooperative strategy is denoted as
\[ U_i^d = -\theta_i \gamma_i^2 (l_i^d - \hat{l}_i)^2 - p^d(t_i^d, t_i^d), \] (16)

and the payoff of the other building managers are denoted as
\[ U_j^d = -\theta_j \gamma_j^2 (l_j^d - \hat{l}_j)^2 - p^d(t_j^d, t_j^d), \quad j \in \mathcal{N}, j \neq i. \] (17)

where \( p^d(t_i^d, t_j^d) \) denotes the price in the case of noncooperative behaviors and is defined as
\[ p^d(t_i^d, t_j^d) = \lambda (\sum_{j \in \mathcal{N}, j \neq i} l_j^d + l_i^d - L) + p_0. \] (18)

It is straightforward to see that each building manager can improve its payoff by taking the noncooperative strategy, i.e., \( U_i^d \geq U_i^c \) for all \( i \in \mathcal{N} \). In the cooperative demand response program, the noncooperative behavior of one building manager will change the electricity price, the total energy consumption, and the payoff of the other building managers. Next, we will study the impacts of the nonlinear behaviors on the performances of demand response program.
Proposition 1. Suppose one building manager takes the noncooperative strategy while the other building managers keep cooperative, we have the following conclusions:

- The electricity price is increased.
- The payoffs of all the building managers that keep cooperative are decreased.
- The energy consumption of the noncooperative building manager and the total energy consumption are both increased.

Proof: Following the social optimality of the cooperative payoff in (P1), we have \( \sum_{i \in \mathcal{N}} U^d_i \leq \sum_{i \in \mathcal{N}} U^c_i \). Since there is \( U^d_i > U^c_i \), at least one building manager will have the payoff decrease. Suppose the payoff of building manager \( j \) (\( j \in \mathcal{N}, j \neq i \)) is decreased. Since the energy consumption of the building manager \( j \) are not changed, the decrease of the payoff function (17) only comes from the increase of the payments and thus the increase of the electricity price. Given the increased electricity price and the unchanged energy consumption, the other building managers that keep cooperative will also have the payoff decrease. From the pricing function (6), we see that the increase of the price only comes from the increase of the total energy consumption and thus the increase of the energy consumption of the noncooperative building manager.

In practice, the the amount of changes in the price, the payoffs, and the total energy consumption is affected by the scale of the demand management system (e.g., the number of buildings). In the simulations, we will discuss it in detail.

To avoid the noncooperative behaviors of the selfish building managers, we develop the following trigger-and-punishment mechanism: All of the building managers are assumed to adopt the cooperative strategy in the first stage. In the subsequent stages (i.e., \( k \geq 2 \)), the building manager will maintain the cooperative strategy if all the other building managers adopt the cooperative strategy in the last stage. If any building manager see a noncooperative behavior in the last stage, all of the building managers will enter into the period of punishment and choose the noncooperative strategy for the subsequent \( T \) stages. There are two questions to be answered in the trigger-and-punishment mechanism: How to identify the noncooperative behaviors in the demand management system and what is the punishment strength to avoid the noncooperative behaviors?

2) Noncooperative Behavior Detection: The changes of total energy consumption, the electricity price, and the payoffs can be used for detecting the nonlinear behaviors of the building managers. Next, we choose the change of the total energy consumption from the cooperative energy consumption as the indicator for noncooperative behavior detection. The change of the total energy consumption are defined as

\[
\Delta L = \sum_{i \in \mathcal{N}} l_i - \sum_{i \in \mathcal{N}} l^c_i
\]

To detect the noncooperative behaviors of any building manager, the detection threshold is chosen as

\[
\eta = \min (\Delta l^d_1, \Delta l^d_2, \ldots, \Delta l^d_N).
\]

where \( \Delta l^d_i = l^d_i - l^c_i, i \in \mathcal{N} \), (21)

where \( l^d_i \) is the energy consumption of the building manager that adopts the noncooperative strategy and \( l^c_i \) is the energy consumption of the building manager that adopts the cooperative strategy, given the other the building managers keep cooperative. The detection rule is denoted as

\[
\hat{q} = \begin{cases} 
1, & \text{if } \Delta L \geq \eta, \\
0, & \text{if } \Delta L < \eta 
\end{cases}
\]

where \( \hat{q} = 1 \) denotes that the control center detects noncooperative behavior in the demand management system and \( \hat{q} = 0 \) denotes that the control center does not detect the noncooperative behavior.

3) Punishment Strategy: Assuming all the building managers adopt the social optimal energy consumption, the average payoff of the building manager \( i \) without noncooperative behaviors is denoted as

\[
\bar{U}_i^c = (1 - \delta) \sum_{k=1}^{\infty} \delta^{k-1} U^c_i,
\]

and the average payoff of the building manager \( i \) with noncooperative behaviors at stage \( T_0 \) is denoted as

\[
\bar{U}_i^d = (1 - \delta) \left( \sum_{k=1}^{T_0-1} \delta^{k-1} U^c_i + \delta^{T_0-1} U^d_i + \sum_{k=T_0+1}^{T_0+T} \delta^{k-1} U^c_i \right)
\]

Next, we give the conditions to achieve the social optimal energy consumption in the following proposition:

Proposition 2. The social optimal energy consumption \( l^c \) is a subgame perfect Nash equilibrium of the repeated demand management game, and the trigger-and-punishment mechanism is incentive compatible if

\[
T > \frac{\log(1 + (1 - \delta)(U^c_i - U^d_i) / \delta(U^c_i - U^c_i))}{\log \delta},
\]

where \( \delta \) should satisfy

\[
\delta > \frac{U^d_i - U^c_i}{U^d_i - U^c_i}.
\]

Proof: To align the social optimal energy consumption with the subgame perfect Nash equilibrium and achieve the incentive compatibility of the trigger-and-punishment mechanism, there should be \( \bar{U}_i^c > \bar{U}_i^d \) for all \( i \in \mathcal{N} \), from which, we have

\[
\sum_{k=T_0}^{T_0+T} \delta^{k-1} U^c_i > \delta^{T_0-1} U^d_i + \sum_{k=T_0+1}^{T_0+T} \delta^{k-1} U^c_i.
\]

Since \( U^c_i, U^d_i, \) and \( U^{NE}_i \) are assumed to be constant within different stages, (27) can be transformed to

\[
\delta^{T_0-1} 1 - \frac{\delta^{T_0+1}}{1 - \delta} U^c_i > \delta^{T_0-1} U^d_i + \delta^{T_0} \frac{1 - \delta^T}{1 - \delta} U^{NE}_i.
\]
Then, we obtain the condition (25). The lower bound of $\delta$ is obtained from

$$1 + \frac{(1 - \delta)(U_i^c - U_i^d)}{\delta(U_i^c - U_i^{NE})} > 0,$$

(29)

from which, we obtain the condition (26). From (25), we see that the conditions on the duration of punishment is not related to the stage at which the noncooperative behaviors occur. No matter which stages the game start from, the social optimal energy consumption is still a equilibrium for the subgame and thus a subgame perfect Nash equilibrium of the repeated demand management game.

### IV. Numerical Results

In this section, we evaluate the performance of the cooperative demand response program by using Monte Carlo method. We assume that the target energy consumption of the buildings are uniformly distributed in $[100\text{kW}, 150\text{kW}]$, the cost coefficients $\gamma_i$ are uniformly distributed in $[2, 4]$. The base price $p_0$ is set to 5 cents/kWh, the forecast demand is estimated by $L = \sum_{i \in N} l_i/1.5$, and the pricing parameter $a$ is calculated by $a = 2/N$. Before giving the numerical results, we first define the performance indexes as follows.

To evaluate the cost decrease of the building managers obtained from the cooperative demand response program, we define the average cost decrease (ACD) as

$$ACD = \frac{\sum_{i \in N}(U_i^c - U_i^d)}{\sum_{i \in N} U_i^c} \times 100\%,$$

(30)

To evaluate the cost increase of the building managers $j$ ($j \in N, j \neq i$) when some building manager $i$ takes the noncooperative strategy, we define the average cost increase (ACI) as

$$ACI = \frac{\sum_{j \in N, j \neq i}(U_j^d - U_j^c)/(N - 1)}{\sum_{i \in N} U_i^c/N} \times 100\%,$$

(31)

We compare the cooperative and noncooperative demand response program in Table 1. It is shown that the cooperation reduces the electricity price, the total cost, the average cost, and the total energy consumption effectively. Next, we study the impact of the number of the buildings on the performance of the demand response program. As shown in Fig. 2, both the electricity prices under the noncooperative and cooperative demand response program are almost constant with the number of the buildings. As shown in Fig. 3, the average cost decrease obtained from the cooperation is increased with the number of the buildings and starts to saturate when the number of buildings is larger than 100. Assuming one building manager has the noncooperative behavior and the other building managers adopt the strategy of cooperation, the average cost decrease of the noncooperative building manager is increased with the number of the buildings (Fig. 4) and the average cost increase of the other cooperative building managers are decreased with the number of the buildings (Fig. 5). Both of them saturates when there are more than 100 buildings. It is also shown that the noncooperative building manager has relatively large cost decrease and thus strong motivations to take the noncooperative strategy.

### V. Conclusion

In this study, we formulate the cooperative demand response program as a constrained social optimization problem. It is shown that the cooperative demand response program lowers the electricity price, the total cost, the average cost, and the total energy consumption comparing with the noncooperative demand response program. We use the repeated game to keep cooperative among selfish price-anticipating building managers and develop the trigger-and-punishment mechanism to avoid
the noncooperative behaviors. We establish the conditions on the durations of punishment to ensure a subgame perfect Nash equilibrium and incentive compatibility.

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REFERENCES


