Optimal Bidding, Scheduling, and Deployment of Battery Systems in California Day-Ahead Energy Market

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Abstract—An optimal supply and demand bidding, scheduling, and deployment design framework is proposed for battery systems. It takes into account various design factors such as the day-ahead and real-time market prices and their statistical dependency, as well as the location, size, efficiency, lifetime, and charge and discharge rates of the batteries. Utilizing second-life / used batteries is also considered. Without loss of generality, our focus is on the California Independent System Operator (ISO) energy market and its two available bidding options, namely self-schedule bidding and economic bidding. While the formulated stochastic optimization problems are originally nonlinear and difficult to solve, we propose a methodology to decompose them into inner and outer subproblems. Accordingly, we find the global optimal solutions within a short amount of computational time. All case studies in this paper are based on real market data.

Keywords: Battery systems, supply-demand bids, economic and self-schedule bids, charge and discharge schedules, second-life batteries, stochastic optimization, California ISO energy market.

I. INTRODUCTION

With the recent advancements in battery technologies [1], grid-scale battery systems are gradually becoming practical energy solutions. Accordingly, there has been a growing interest in finding new and efficient applications for batteries in power systems. For example, a recent study in [2] has listed several applications for grid-scale battery systems, such as bulk energy resource support, voltage support, synchronous reserve, non-synchronous reserve, and frequency regulation.

In this paper, our focus is on utilizing battery systems as bulk energy resources. In this scenario, the battery system enters the wholesale electricity market as a supplier during its discharge cycles and as a consumer during its charge cycles. While the general idea of using battery systems as bulk energy resources is mentioned in the literature before, e.g., see [2], [3], the optimal operation and market participation of battery systems under this application scenario is yet to be investigated. More specifically, we still need to answer the following fundamental questions: Given the wholesale market conditions and the characteristics of the battery system technology, what is the best supply and demand bidding strategy for a battery system? Also, what is the best charge and discharge schedule? Finally, what is the best location to install the battery system?

A. Summary of Technical Contributions

In this paper, we seek to answer the questions that we raised in the previous subsection. Without loss of generality, our focus is on the California energy market, that is operated by the California Independent System Operator (ISO). Please refer to Section II-A for a detailed description of this market. The contributions in this paper can be summarized as follows:

- We propose an optimal bidding, scheduling, and deployment design framework for bulk battery systems. It takes into account design factors such as the day-ahead and real-time market prices and the location, size, efficiency, lifetime, and charge and discharge rates of the batteries. Utilizing second-life / used batteries is also considered.
- Three design scenarios are considered based on the available bidding options in the California energy market. First, the case where the bid includes only energy quantity. Second, the bid also includes price quantity, and we assume that the day-ahead and real-time market prices are
While the formulated stochastic optimization problems are originally nonlinear and difficult to solve, we propose a methodology to decompose them into inner and outer subproblems. Accordingly, we find the global optimal solutions within a short amount of computational time.

Case studies are based on real market data in California.

In this paper, we assume that the battery system is price-taker, i.e., it is not large enough to be price-maker. This is a reasonable assumption given the typical sizes of battery systems in California and elsewhere. For example, currently, the largest battery project in Southern California is located in Tehachapi, CA and has a capacity of 8 MW × 4 hour (32MWh) [4]. This capacity is practically negligible compared to the 30 GW peak energy that is traded in the California day-ahead energy market [5]. The case of large and price-maker battery systems, together with a few other directions to extend the analysis in this paper, are discussed in Section V.

B. Comparison with Related Work

We can classify the literature on battery system operation into two groups. First, the studies that optimize the use of battery systems to improve efficiency and reliability at transmission level [6] or at distribution and microgrid level [7], [8], but without taking into account the profitability and market participation aspects. In contrast, in the second group, profitability is a major concern. Accordingly, the focus is on market participation and offering various market products, such as energy, reserve, and ancillary services. A large number of papers in this second group tend to combine and co-locate batteries with other energy resources, such as wind farms [9], [10], solar farms [11], or demand response aggregators [12].

Different from [9]–[12], the focus in this paper is on the profitability and wholesale market participation of independent battery systems, where the battery system is not combined or co-located with any other energy resource. As a result, our work is in part comparable with the recent studies in [13]–[15], with at least three key distinctions. First, no prior work has discussed the optimal choice of price bids for independent bulk battery systems. Note that, while generators and loads select their price bids in relation to their marginal costs [16], batteries are concerned with the price differential across their charge and discharge cycles. Second, the studies in [13]–[15] do not address battery lifetime, per-cycle profit valuation, and battery charge and discharge efficiency. Finally, those prior studies mostly use simulation-based market data, e.g., for an IEEE 24 bus system as in [14]. However, here, we use one year of real market data from the California ISO market.

Another line of related work is on optimal bidding and market participation, but not for batteries. Rather, the focus is on market participation of conventional generators [17], renewable generators [18], and loads [19], [20]. Finally, this paper is also related to the recent studies on reusing second-life batteries, e.g., see [21], [22]. However, those studies do not discuss optimal bidding and bulk energy market participation.

II. Problem Formulation

A. California Energy Market

Energy trading in the California ISO energy market is done in two settlements through day-ahead and real-time markets. Buyers and sellers participate in these markets by submitting demand and supply bids, respectively. The bids include energy quantities and possibly price quantities [23]. If a demand bid includes both energy and price quantities, then it indicates that the buyer is willing to purchase the given quantity of energy only if the price is equal to or below the price bid. If a supply bid includes both energy and price quantities, then it indicates that the seller is willing to sell the given quantity of energy only if the price is equal to or above the price bid. In the California ISO energy market, the bids with price quantities are called Economic bids [23]. The bids that do not include price quantities are called Self-Schedule bids [23]. A self-schedule demand bid indicates that the buyer is willing to purchase the given quantity of energy, regardless of the price. Similarly, a self-schedule supply bid indicates that the seller is willing to sell the given quantity of energy, regardless of the price. Supply bids can be Economic or Self-Schedule, whether they are submitted to the day-ahead market or real-time market. However, only the demand bids that are submitted to the day-ahead market can be Economic. That is, the demand bids that are submitted to the real-time market must be Self-Schedule. In this paper, we assume that the battery system submits Economic and Self-Schedule bids to the day-ahead market and Self-Schedule bids to the real-time market.

B. Energy Bids

Let us divide the day-ahead energy market into $T = 24$ hourly time slots. Let $m[t] \in \{0, 1\}$ denote the type of the bid at time slot $t$. If the battery system submits a supply bid, then $m[t] = 1$. If the battery system submits a demand bid, then $m[t] = 0$. Next, let $n$ denote the number of segments in each economic bid. Each bid segment is in form of an energy-price pair [16]. In California, $n$ can be between 1 and 10 [23]. For each $i = 1, \ldots, n$, let $x_i[t]$ and $y_i[t]$ denote the energy component in segment $i$ of the economic bid at time slot $t$, when the battery system submits a supply bid and when it submits a demand bid, respectively. It is required that

\begin{align}
 x[t] & \triangleq \sum_{i=1}^{n} x_i[t] \leq x^\text{max}, \\
 y[t] & \triangleq \sum_{i=1}^{n} y_i[t] \leq y^\text{max},
\end{align}

where $x^\text{max} \geq 0$ and $y^\text{max} \geq 0$ denote the maximum discharge rate and the maximum charge rate of the battery system of interest, respectively. The choices of $x_i[t]$ and $y_i[t]$ at each time $t$ are also bounded based on the value of $m[t]$ as follows:

\begin{align}
 0 & \leq x_i[t] \leq m[t] x^\text{max}, \quad i = 1, \ldots, n, \\
 0 & \leq y_i[t] \leq (1 - m[t]) y^\text{max}, \quad i = 1, \ldots, n.
\end{align}

If $m[t] = 1$, then (3) and (4) reduce to $0 \leq x_i[t] \leq x^\text{max}$ and $y_i[t] = 0$. If $m[t] = 0$, then (3) and (4) reduce to $x_i[t] = 0$ and...
$0 \leq y_i[t] \leq y^{\text{max}}$. Note that, a battery system cannot submit both supply and demand bids at the same time-slot.

### C. Price Bids

For each $i = 1, \ldots, n$, let $p_i[t]$ denote the price component in segment $i$ of the economic bid at time slot $t$, whether the bid is a supply bid or a demand bid. Let $a[t]$ denote the cleared market price at the day-ahead market at time slot $t$. Note that, $a[t]$ is a random variable. Its value is known only after the market is cleared. Since the battery system is price-taker, $a[t]$ does not depend on the battery charge and discharge variables $x_i[t]$ and $y_i[t]$. Suppose $m[t]=1$, i.e., the battery system decides to submit a supply bid at time $t$. Once the day-ahead market is cleared, the total energy that is sold in the day-ahead market by the battery system at time slot $t$ is calculated as

$$\sum_{i=1}^{n} \mathbb{I}(a[t] \geq p_i[t]) x_i[t],$$

where $\mathbb{I}(\cdot)$ is the 0-1 indicator function. If $a[t] \geq p[t]$, then $\mathbb{I}(a[t] \geq p[t]) = 1$; otherwise, $\mathbb{I}(a[t] \geq p[t]) = 0$. The unsold amount of energy is then sold at the real-time market:

$$\sum_{i=1}^{n} x_i[t] - \sum_{i=1}^{n} \mathbb{I}(a[t] \geq p_i[t]) x_i[t].$$

Let $b[t]$ denote the cleared market price at the real-time market at time slot $t$. Again, $b[t]$ is a random variable. Its value is known only after the real-time market is settled. Since the battery system is price-taker, $b[t]$ does not depend on $x_i[t]$ and $y_i[t]$. From (5) and (6), and after reordering the terms, the total revenue that the battery system obtains from selling energy at time slot $t$ is calculated as

$$\sum_{i=1}^{n} \mathbb{I}(a[t] \geq p_i[t]) x_i[t] a[t] + (1 - \mathbb{I}(a[t] \geq p_i[t])) x_i[t] b[t].$$

(7)

Next, suppose $m[t] = 0$, i.e., the battery system decides to submit a demand bid at time slot $t$. Once the day-ahead market is cleared, the total energy that is purchased by the battery system at time slot $t$ is calculated as

$$\sum_{i=1}^{n} (1 - \mathbb{I}(a[t] \geq p_i[t])) y_i[t].$$

(8)

The unmet energy is then purchased at the real-time market:

$$\sum_{i=1}^{n} y_i[t] - \sum_{i=1}^{n} (1 - \mathbb{I}(a[t] \geq p_i[t])) y_i[t].$$

(9)

From (8) and (9), and after reordering the terms, the total cost that the battery system incurs at time slot $t$ is calculated as

$$\sum_{i=1}^{n} (1 - \mathbb{I}(a[t] \geq p_i[t])) y_i[t] a[t] + \mathbb{I}(a[t] \geq p_i[t]) y_i[t] b[t].$$

(10)

Note that, from (3) and (4), the battery system cannot submit supply and demand bids simultaneously. Thus, the expressions in (7) and (10) cannot be non-zero at the same time.

It is worth reemphasizing that for the problem formulations in this paper, the unsold energy of an uncleared supply bid at time slot $t$ in day-ahead market is assumed to be sold in the real-time market at time slot $t$. Similarly, the unmet energy of an uncleared demand bid at time slot $t$ in day-ahead market is purchased in the real-time market at time slot $t$. An alternative for these assumptions is discussed in Section V-B.

### D. Storage Capacity

Finally, let $C^{\text{min}}, C^{\text{max}}$, and $C$ denote the initial charge level and the minimum and maximum allowed charge levels of the battery, respectively, where $C^{\text{min}} \leq C \leq C^{\text{max}}$. Also let $\delta \geq 1$ and $\sigma \leq 1$ denote the discharge and charge efficiency of the battery system, where $\delta = \sigma = 1$ indicates the ideal case with 100% efficiency. At each time $t$, we must have

$$C^{\text{min}} \leq C^{\text{init}} - \delta \left( \sum_{i=1}^{n} \sum_{i=1}^{n} x_i[t] \right) + \sigma \left( \sum_{i=1}^{n} \sum_{i=1}^{n} y_i[t] \right), \quad (11)$$

$$C^{\text{max}} \geq C^{\text{init}} - \delta \left( \sum_{i=1}^{n} \sum_{i=1}^{n} x_i[t] \right) + \sigma \left( \sum_{i=1}^{n} \sum_{i=1}^{n} y_i[t] \right). \quad (12)$$

Note that, in order to sell $x_i[t]$ MWh of energy to the market, the battery system must discharge $\delta x_i[t]$ MWh of energy, where $\delta x_i[t] \geq x_i[t]$. Similarly, if the battery system buys $y_i[t]$ MWh of energy from the market, it is charged by $\sigma y_i[t]$ MWh of energy, where $\sigma y_i[t] \leq y_i[t]$. The constraints in (11) and (12) assure that the total energy that is stored at the battery system at each time slot $t$ is within its allowed operation range.

### E. Discharge Cycles

The life for a rechargeable battery is often stated in number of discharge cycles. Therefore, one can extend the battery lifetime by limiting the number of discharge cycles. Mathematically, this can be done by using the following constraint:

$$\sum_{t=1}^{T} x_i[t] \leq \gamma (C^{\text{max}} - C^{\text{min}})$$

(13)

where $\gamma > 0$ is a design parameter. For example, if $\gamma = 2$, then the daily operation of the battery is limited to two full discharge cycles. See Section IV-C for more detailed examples.

### F. Economic Bidding

To find the optimal supply and demand economic bids, we need to solve the following stochastic optimization problem

$$\max_{m,x,y,p} \sum_{t=1}^{T} \mathbb{E}\left\{ \sum_{i=1}^{n} \mathbb{I}(a[t] \geq p_i[t]) x_i[t] a[t] + (1 - \mathbb{I}(a[t] \geq p_i[t])) x_i[t] b[t] \right\}$$

$$- \sum_{t=1}^{T} \mathbb{E}\left\{ \sum_{i=1}^{n} (1 - \mathbb{I}(a[t] \geq p_i[t])) y_i[t] a[t] + \mathbb{I}(a[t] \geq p_i[t]) y_i[t] b[t] \right\}$$

s.t. Eqs. (1), (2), (3), (4), (11), (12), (13).

1If the battery is designed to offer multiple services, then $C^{\text{max}}$ denotes the portion of the battery that is allocated to energy market participation.
where the variables are \( m = (m[t], \forall t), x = (x_i[t], \forall i, t), y = (y_i[t], \forall i, t), \) and \( p = (p_i[t], \forall i, t) \); and the expected value \( E[\cdot] \) is with respect to the day-ahead and real-time market prices \( a[t] \) and \( b[t] \). Here, the optimization objective is to maximize the total expected value of the battery system’s daily profit, i.e., the expected value of the revenue in (7) minus the expected value of the cost in (10) across all \( T = 24 \) hourly time slots.

The optimization problem in (14) is a mixed-integer nonlinear program (MINLP). It is mixed integer because \( m \) is a discrete variable while \( x, y, \) and \( p \) are continuous variables. Also, it is nonlinear because of the indicator function \( I(\cdot) \).

G. Self-Schedule Bidding

A self-Schedule bid is a special case of economic bid, where the number of segments is \( n = 1 \), and the price components are \( p_1[t] \rightarrow 0 \) for all supply bids, and \( p_1[t] \rightarrow \infty \) for all demand bids. After replacing these values in (14), the problem to find the supply and demand self-schedule bids becomes

$$\text{Max}_{m, x, y} \sum_{t=1}^{T} E\left\{ a[t] (x[t] - y[t]) \right\}$$  \hspace{1cm} (15)

$$\text{S.t.} \quad \text{Eqs. (3), (4), (11), (12), (13)}.$$

Note that, since \( n = 1 \), constraint (1) is covered by constraint (3) and constraint (2) is covered by constraint (4). That is why we did not include (1) and (2) in optimization problem (15).

We can see that the objective function in (15) is significantly less complex compared to the one in (14). The optimization problem in (15) is a mixed-integer linear program (MILP).

III. OPTIMAL BIDDING SOLUTIONS

Recall from Section II-G that as far as the mathematical formulation is concerned, self-schedule bidding is a special case of economic bidding. Therefore, in this section, we focus on solving problem (14) to obtain optimal economic bids.

After reordering the terms, we can rewrite problem (14) as

$$\text{Max}_{m, x, y, p} \sum_{t=1}^{T} \sum_{i=1}^{n} E\left\{ x_i[t] b[t] - y_i[t] a[t] \right\}$$

$$+ \sum_{t=1}^{T} \sum_{i=1}^{n} E\left\{ (a[t] - b[t]) \right\}$$

$$\text{S.t.} \quad \text{Eqs. (1), (2), (3), (4), (11), (12), (13)}.$$  \hspace{1cm} (16)

The first term in the objective function depends on \( x \) and \( y \). The second term in the objective function depends on \( x, y, \) and \( p \). The constraints in (1), (2), (3), (4), (11), (12), (13) depend on \( m, x, \) and \( y \). Thus, we can rewrite problem (16) as

$$\text{Max}_{m, x, y} G(x, y) + \text{Max}_{p} H(x, y, p)$$

$$\text{S.t.} \quad \text{Eqs. (1), (2), (3), (4), (11), (12), (13)}.$$  \hspace{1cm} (17)

where

$$G(x, y) = \sum_{t=1}^{T} \sum_{i=1}^{n} E\left\{ x_i[t] b[t] - y_i[t] a[t] \right\}.$$  \hspace{1cm} (18)

$$H(x, y, p) = \sum_{t=1}^{T} \sum_{i=1}^{n} E\left\{ (a[t] - b[t]) I(a[t] \geq p_i[t]) \right\}.$$  \hspace{1cm} (19)

Problems (14), (16), and (17) are all equivalent. That is, they are just different ways to write the same problem. The advantage of the formulation in (17) is that it clearly identifies two subproblems for our analysis. First, an inner subproblem over \( p \), which takes \( x \) and \( y \) as constant. The objective function for the inner subproblem is \( H(x, y, p) \). The inner subproblem is unconstrained. Second, an outer subproblem over \( m, x, \) and \( y \) that takes the optimal objective value of the inner subproblem as a function over \( x \) and \( y \). The objective function for the outer subproblem is \( G(x, y) + \max_{p} H(x, y, p) \). The constraints in (1), (2), (3), (4), (11), (12) all belong to the outer subproblem.

A. Solving the Inner Subproblem

The inner subproblem in (17) seeks to find the optimal price bids \( p \), assuming that the rest of the variables \( m, x, \) and \( y \) are fixed. We can show the following key theorem.

**Theorem 1:** For any number of segments \( n \geq 1 \), the optimal price bids at time \( t \) in the inner subproblem are obtained as

$$p^*[t] = \ldots = p^n*[t] = p^*[t].$$  \hspace{1cm} (20)

If the day-ahead and real-time market prices are statistically independent, then, at each time slot \( t \), we have

$$p^*[t] = E\{b[t]\}.$$  \hspace{1cm} (21)

Otherwise, i.e., if the day-ahead and real-time market prices are statistically dependent, then, at each time slot \( t \), we have

$$p^*[t] \equiv \arg \max_{\rho \geq 0} E\{ (a[t] - b[t]) I(a[t] \geq \rho) \}.$$  \hspace{1cm} (22)

The proof of Theorem 1 is given in the Appendix. We can make multiple interesting observations from the above results. First, from (20), regardless of the number of segments that are allowed to be included in an economic bid, there is always only one segment in an optimal bid. In other words, unlike generators and loads that submit economic bids with several price segments in relation to their marginal costs [16], battery systems do not need to submit more than one price segment in their economic bids. It must be noted that, in general, the operation cost of a generator depends on its output power. As a result, a single-segment bid is often sub-optimal for a generator. If, similar to a battery, a generator does not have an operation cost, then it too would not need to submit bids with more than one segment. Such generator could then be treated as a special case battery system with an unlimited initial charge level. Second, the optimal solutions in (21) and (22) depend only on price parameters \( a[t] \) and \( b[t] \), but not the rest of the optimization variables \( m, x, \) and \( y \). This will be helpful when it comes to solving the outer subproblem, as we will see in Section III-B. Third, the choice of the optimal price bid in (21) and (22) at time \( t \) does not depend on whether the bid at this time slot is a supply bid or a demand bid.

The optimal price bids in (20)-(22) can be calculated using historical price data. For example, consider the following 31
B. Solving the Outer Problem

Given the optimal price bids in (20)-(22), we can rewrite the objective function in the outer subproblem in (17) as

$$\sum_{t=1}^{T} E\{b[t]\} \left( \sum_{i=1}^{n} x_{i}[t] \right) - E\{a[t]\} \left( \sum_{i=1}^{n} y_{i}[t] \right)$$

$$+ \sum_{t=1}^{T} E\{(a[t]-b[t]) I\{a[t] \geq p^*[t]\}\} \left( \sum_{i=1}^{n} x_{i}[t] \right)$$

$$+ \sum_{t=1}^{T} E\{(a[t]-b[t]) I\{a[t] \geq p^*[t]\}\} \left( \sum_{i=1}^{n} y_{i}[t] \right).$$

The above objective function depends only on the summation of supply energy bids across all segments and the summation of demand energy bids across all segments. This is because, as we explained in Section III-A, we need only one segment in an optimal economic bid, whether it is a supply bid or demand bid. Therefore, we can reformulate the outer subproblem as

$$\max_{m,x,y} \sum_{t=1}^{T} (\theta[t] + \psi[t]) x[t] + (\theta[t] - \phi[t]) y[t]$$

S.t. Eqs. (3), (4), (11), (12), (13),

where $x[t]$ and $y[t]$ are defined in (1) and (2); and we have

$$\theta[t] = E\{(a[t]-b[t]) I\{a[t] \geq p^*[t]\}\},$$

$$\phi[t] = E\{a[t]\},$$

$$\psi[t] = E\{b[t]\}.$$ (27)

The outer subproblem in (24) is an MILP, which is much more tractable than the original MINLP in (14).

It is interesting to compare the outer subproblem in (24) for economic bidding with the optimization problem in (15) for self-schedule bidding. To make the comparison easier, let us first rewrite problem (15) using the notation in (26) as

$$\max_{m,x,y} \sum_{t=1}^{T} \phi[t] x[t] - \phi[t] y[t]$$

S.t. Eqs. (3), (4), (11), (12), (13).

We can see that while problems (24) and (28) are both MILP and have similar structures, the coefficients in their objective functions are very different. As an example, again consider the 31 price data points that we analyzed in Section III-A. We have $\theta[14] = 1.2$, $\phi[14] = 48.9$, and $\psi[14] = 52.9$. In this case, the coefficients of $x[14]$ and $y[14]$ in (24) are $1.2 + 48.9 = 50.1$ and $1.2 - 52.9 = -51.7$, respectively. In contrast, the coefficients of $x[14]$ and $y[14]$ in (28) are 48.9 and $-48.9$, respectively. The differences are major and they can result in different solutions for the energy bids $x[14]$ and $y[14]$.

Problems (24) and (28) can be solved efficiently using various MILP solvers, such as CPLEX [24] or MOSEK [25]. These problems each has $T = 24$ binary and $2 \times T = 48$ continuous variables and a total of $6 \times T = 144$ constraints.

IV. Case Studies

In this section, we compare three bidding approaches. First, Self-Schedule Bidding, based on the solution of problem (28). Second, Economic Bidding - Design I, based on the solution of problem (24), where $p^*[t]$ is as in (21). Third, Economic Bidding - Design II, based on the solution of problem (24), where $p^*[t]$ is as in (22). We use CPLEX [24] to solve the optimization problem in each case. Using a laptop computer with a 2.90 GHz CPU and 8 GB RAM, on average, it took only $< 1$ second, $< 1$ second, and about 4 seconds to obtain the optimal supply-demand bids for Self-Schedule Bids, Economic Bids - Design I, and Economic Bids - Design II, respectively.

To examine the impact of location on the performance of the proposed bidding strategies for battery systems, we use the locational marginal price (LMP) data at three different nodes to represent three geographical regions in California: node DAVIS_1 to represent Northern California, node HURON_6 to represent Central California, and node CHINO_6 to represent...
Southern California [5]. Since the focus in this paper is on solving the bidding and charge and discharge scheduling optimization challenges, rather than price forecasting, we assume that the probability distribution functions for the day-ahead and real-time market prices are accurate, i.e., they are known. Of course, it will be interesting to combine our design with price forecasting methods; however, such analysis is beyond the scope of this paper and can be considered as a future work.

Unless we state otherwise, we assume that $\gamma \gg 1$, and the battery system of interest has the same size as in Southern California Edison battery energy storage facility in Tehachapi, CA [4], where $x^{\text{max}} = y^{\text{max}} = 8 \text{ MW}$ and $C^{\text{max}} = 32 \text{ MWh}$. We again emphasize that the results in this paper are based on the assumption that the battery system is price-taker. If the battery system is large and price-maker then it may generate even more profit through strategic market participation and by relieving or creating congestion on transmission lines.

A. Impact of Location and Season

Consider the case studies in Fig. 2. We can see that Economic Bidding - Design II outperforms Economic Bidding - Design I; and Economic Bidding Design I outperforms Self-Schedule Bidding. For example, on average, daily profit in Summer 2013 and across the three geographical regions is two times higher for Economic Bidding - Design II compared to Self-Schedule Bidding. Another observation is that, in winter, there is little difference in the profit across the three geographical regions. For example, the average daily profit for Economic Bidding - Design II is $1,388 in Northern California, $1,300 in Central California, and $1,391 in Southern California. This is because the power grid in California is typically less congested during the winter when there is less air conditioning load. However, the situation changes drastically in summer, where the average daily profit for Economic Bidding - Design II is $1,891 in Northern California, $1,300 in Central California, and $1,762 in Southern California. We can see that locations that are prone to more price fluctuations, e.g., due to congestion, are more profitable to install batteries.

B. Impact of Battery Efficiency

So far, and for the results in Fig. 2, we assumed that the battery efficiency is 100%, i.e., $\delta = \sigma = 1$. However, in practice, the efficiency of lithium-ion batteries is 85 to
90% [26]. More advanced battery technologies offer 95% efficiency [27]. The annual profit versus battery efficiency based on the price data in Northern and Southern California is shown in Figs. 3(a) and (b), respectively. We can see that the profit decreases as battery efficiency degrades. However, the relative performance remains the same across the three bidding options. That is, regardless of the battery efficiency, Economic Bids - Design II outperform Economic Bids - Design I; and Economic Bids - Design I outperform Self-Schedule Bids.

The optimal supply-demand bids based on the price data in Northern California in Summer 2013 and at 95% battery efficiency are visualized in Fig. 4. The average daily profit corresponding to the three sets of bids in Fig. 4 are $706, $1,442, and $1,676, respectively. We can make several interesting observations. First, the optimal self-schedule bids tend to have fewer charge and discharge cycles than the optimal economic bids. We will address this issue in details later in Section IV-C. Second, there are some similarities between the energy bids under Economic Bidding - Design I and Economic Bidding - Design II; however, the differences between the two designs in terms of price bids are significant. Third, the price bids under Economic Bidding - Design II have more variations during the day. Note that, a price bid equal to zero for an economic demand bid at a time slot $t$ means that the entire energy bid $y[t]$ must be purchased at the real-time market.

C. Battery Life and Impact of Parameter $\gamma$

Recall from Fig. 4 that the optimal self-schedule bids have fewer charge and discharge cycles than the optimal economic bids. This may suggest that, although economic bidding is more profitable, it depreciates the battery at a higher rate. This shortcoming can be fixed by adjusting parameter $\gamma$. The results are shown in Fig. 5. In Fig. 5(a), the annual profit decreases as we decrease $\gamma$. For example, reducing $\gamma$ from 4 to 1.4 lowers the profit under Economic Bidding - Design II by 10% from $420,000 to $378,000. However, in return, decreasing $\gamma$ can significantly increase the battery life, as shown in Fig. 5(b). For example, if the battery can support up to 2000 cycles [28], then reducing $\gamma$ from 4 to 1.4 increases the battery life under Economic Bidding by about 180% from 2.1 to 3.7 years.

We can conclude that when $\gamma$ is not too large, Self-Schedule Bidding does no longer have lifetime advantage over Economic Bidding, yet its achievable profit is still much

![Fig. 4. The details of supply-demand bids across different bidding approaches for the case of Northern California in Summer 2013. The color code is the same as that of Fig. 3. Numbers above or below the bars indicate price bids.](image-url)

![Fig. 5. The impact of changing parameter $\gamma$ on profit and battery life.](image-url)

![Fig. 6. The impact of changing $\gamma$ on Economic Bids - Design II for the case of Northern California in Summer 2013. Battery efficiency is 95%.](image-url)
less. As an example, at $\gamma = 1.4$ and for batteries with 5000 cycles [29], Self-Schedule Bidding results in $228,000 profit per year for 11 years, adding up to $2,510,000 profit over the battery lifetime. Under same operational conditions, Economic Bidding - Design II results in $378,000 profit per year for 9.1 years, adding up to $3,450,000 profit over the battery lifetime.

The optimal supply-demand Economic Bids - Design II for three different values of parameter $\gamma$ are visualized in Fig. 6, based on the price data in Northern California in Summer 2013 and at 95% battery efficiency. We can see that the number of charge and discharge cycles reduces significantly as we decrease $\gamma$. The average daily profit corresponding to the three sets of bids in this figure are $1,623, $1,558, and $1,400, respectively. The bids in this figure are comparable with those in Fig. 4. When $\gamma = 1$, the Economic Bidding - Design II in Fig. 6 results in almost twice more profit compared to the case of Self-Schedule Bidding in Fig. 4, even though the battery is utilized almost equally. In other words, the per cycle profit in this case is almost twice higher for Economic Bidding - Design II. It is interesting to also note in Fig. 6 that changing parameter $\gamma$ only affects the energy bids but not the price bids.

Fig. 7. The impact of discharge rate on annual profit at different locations.

Fig. 8. Annual profit versus battery capacity for second-life batteries.

Fig. 9. Performance comparison with sole real-time market participation based on one year of market data in Southern California.

D. Impact of Charge and Discharge Rates

For all the case studies so far, we assumed that the capacity and the charge and discharge rates of the battery system are based on the major Southern California Edison battery system in Tehachapi, CA [4]. However, it is insightful to also see how the results may change if we increase these battery parameters. For example, the annual profit versus the charge / discharge rate of the batteries is shown in Fig. 7, based on the market price data in Southern California. Here, the capacity of the battery is still fixed at 32 MWH. Under Economic Bidding - Design II, the annual profit increases by 130% from $460,000 to $594,000 if we increase the charge and discharge rate from 8 MW per Hour to 32 MW per Hour. Similar results can be derived by changing the storage capacity of the battery.

E. Second-Life Batteries

We can employ the proposed optimal battery storage system operation framework also to assess the performance when we use second-life battery systems, which support lower storage capacities than their rated capacities. Second-life batteries have received great attention in recent years, particularly due to the increasing penetration of plug-in electric vehicles [21], [22]. The annual profit versus degraded battery capacity is shown in Fig. 8. We can see that market participation is still profitable even for second-life batteries. For example, even after losing half of its storage capacity, the battery system that is installed in Tehachapi, CA can result in over $350,000 annual profit if the Economic Bidding - Design II is being used.

F. Comparison with Real-Time Market Participation

In this section, we compare the performance of our proposed market participation approaches with the case where the battery system participates only in the real-time market. In this case, the charge and discharge schedules are obtained by solving problem (15), but after replacing $a[t]$ with $b[t]$. The results are shown in Fig. 9, where four market participation scenarios are compared with each other. The black bars represent the three scenarios that involve bidding to the day-ahead market. The white bar, however, is the scenario where
market participation is limited to the real-time market. The annual profit is calculated for each scenario based on the market data in Southern California. We can see that the annual profit is less for the case of sole real-time market participation.

V. REMARKS AND EXTENSIONS

In this section, we provide some pointers and remarks about a few directions to extend the analysis in this paper. In all discussions, we assume that \( n = 1, \delta = \sigma = 1, \) and \( \gamma \gg 1. \)

A. Risk Management

While the focus in this paper is on maximizing profit, one can extend the analysis to include risk management. This can be done by incorporating a risk model, such as the conditional value-at-risk (CVaR) [30], into the problem formulation in (14). In this approach, the objective is to maximize

\[
\mathbb{E}\{\text{Profit}\} - \beta \text{CVaR}_\alpha \{ - \text{Profit}\},
\]

where \( \alpha \in [0, 1] \) is a confidence interval and \( \beta \geq 0 \) is a weighting parameter to adjust the importance of risk management. If \( \beta = 0, \) then the design objective reduces back to profit maximization. Here, \( \text{CVaR}_\alpha \) indicates the average profit for the \( (1 - \alpha) \) fractile worst case random scenarios with respect to profit. By maximizing the objective function in (29), we care about not only the overall expected profit but also the average profit in low-profit scenarios. The latter is directly related to risk management and risk aversion.

The new objective function in (29) is generally difficult to handle because the CVaR term adds to the complexity of an already complex problem formulation in (14). In particular, the inner-outer subproblems decomposition approach that we proposed in Section III may no longer be applicable in presence of the CVaR term. Alternatively, we can rather include the CVaR term in the outer subproblem in (24), as we will explain in the next paragraph. Interestingly, although this alternative problem formulation is a deviation from the original problem formulation in (14) where the objective function is replaced with (29), it can still provide us with an effective mechanism to conduct risk management with respect to the obtained profit, as we will see in a case study at the end of this subsection.

Suppose \( K \) denotes the number of random market price scenarios. We can write the expected values in (25)-(27) as

\[
\theta[t] = \frac{1}{K} \sum_{k=1}^{K} \theta_k[t],
\]

\[
\phi[t] = \frac{1}{K} \sum_{k=1}^{K} \phi_k[t], \quad \psi[t] = \frac{1}{K} \sum_{k=1}^{K} \psi_k[t],
\]

where for each random scenario \( k = 1, \ldots, K, \) we have \( \theta_k[t] = (b_k[t] - b_k[t]) \mathbb{I}(b_k[t] \geq \psi[t]), \phi_k[t] = \alpha_k[t], \psi[t] = b_k[t]. \) Notations \( \alpha_k[t] \) and \( b_k[t] \) are the realizations of the day-ahead and real-time market prices under scenario \( k, \) respectively. From (30) and (31), and following the analysis in [30], we can rewrite problem (24) to incorporate risk management:

\[
\begin{align*}
\text{Max}_{\eta \in \mathbb{R}^K} \sum_{t=1}^{T} & \left[ \frac{1}{K} \sum_{k=1}^{K} (\theta_k[t] + \psi_k[t]) x[t] \\
+ & \frac{1}{K} \sum_{k=1}^{K} (\theta_k[t] - \phi_k[t]) y[t] \\
- & \beta (\phi + \frac{1}{1 - \alpha} \frac{1}{K} \sum_{k=1}^{K} \eta_k) \right] \\
\text{S.t.} \quad & \eta_k + \varphi + \sum_{t=1}^{T} (\theta_k[t] + \psi_k[t]) x[t] \\
& + (\theta_k[t] - \phi_k[t]) y[t] \geq 0, \quad k = 1, \ldots, K, \\
& \eta_k \geq 0, \quad k = 1, \ldots, K, \\
& \text{Eqs. (3), (4), (11), (12), (13)},
\end{align*}
\]

where \( \eta_k \) and \( \varphi \) are auxiliary variables. If we choose parameter \( \beta \) to be small, then problem (32) results in solutions that are close to those of problem (24). However, as we increase \( \beta, \) the solutions of problem (32) become more risk averse.

As an example, suppose \( \alpha = 0.95 \) and consider the results in Fig. 10, which are based on the Winter 2015 data in Southern California. As we increase the risk management parameter \( \beta \) from 0.001 to 10, the average profit decreases, but the average profit under the worst 5% scenarios rather increases, offering a more risk averse design. Note that, the curve in Fig. 10(a) shows \( \mathbb{E}\{\text{Profit}\}, \) i.e., the first term in (29). Also, the curve in Fig. 10(b) shows \( -\text{CVaR}_\alpha \{ -\text{Profit}\}, \) i.e., the second term in (29), excluding the weighting parameter. Therefore, the above bidding approach based on the solution of problem (32) has been successful in conducting risk management with respect to the obtained profit, even though the problem formulation in (32) is not exactly the same as the problem formulation in (14) where the objective function is replaced with (29).

B. More Flexible Real-Time Market Participation

Recall from Section II-C that an unsold energy at a time slot in the day-ahead market is sold in the real-time market
at the same time slot. Similarly, an unmet energy at a time slot in the day-ahead market is purchased in the real-time market at the same time slot. In this section, we explain how one can change the problem formulation to give the battery system more flexibility about the time of selling an unsold or purchasing an unmet energy in the real-time market.

Let \( z[t] \) denote the amount of energy that is sold to or purchased from the real-time market at time slot \( t \). A positive-valued \( z[t] \) indicates selling energy while a negative valued \( z[t] \) indicates purchasing energy. To comply with the maximum discharge and charge rate requirements, we must have

\[
y_{\max}^t \leq z[t] + \mathbb{I}(a[t] \geq p(l))x[t] - (1 - \mathbb{I}(a[t] \geq p(l)))y[t] \leq x_{\max}, \tag{33}
\]

where the expression inside the two inequality signs is the net amount of energy discharge (when positive) or energy charge (when negative) of the battery system across both the day-ahead market and the real-time market. Note that, if we use the set up in Section II-C, then the above expression reduces to \( z[t] - y[t] \). Accordingly, since \( x[t] \) and \( y[t] \) cannot be non-zero at the same time, the inequality constraints in (33) are dominated by the constraints in (3) and (4). Therefore, they can be removed from the problem formulation.

The storage capacity constraints are revised in this case as

\[
C_{\min}^t \leq C_{\max}^{\text{init}} - \sum_{l=1}^{T} \left(z[l] + \mathbb{I}(a[l] \geq p(l))x[l] - (1 - \mathbb{I}(a[l] \geq p(l)))y[l]\right). \tag{34}
\]

\[
C_{\max}^t \geq C_{\max}^{\text{init}} - \sum_{l=1}^{T} \left(z[l] + \mathbb{I}(a[l] \geq p(l))x[l] - (1 - \mathbb{I}(a[l] \geq p(l)))y[l]\right). \tag{35}
\]

If we use the set up in Section II-C, then (34) and (35) reduce to (11) and (12), respectively. Recall that for all discussions in Section V, we assume that \( n = 1 \) and \( \alpha = \beta = 1 \).

Finally, we can revise the objective function as

\[
\sum_{t=1}^{T} \mathbb{E}\left[ \mathbb{I}(a[t] \geq p(l))x[t]a[t] - (1 - \mathbb{I}(a[t] \geq p(l)))y[t]a[t] + z[t]b[t]\right]. \tag{36}
\]

The above objective function has fewer terms than the original objective function in (14). However, the complexity has now moved to the constraints in (33)-(35) because they now include the 0-1 indicator function \( \mathbb{I}(\cdot) \). As a result, the optimization problem that is formulated under the alternative real-time market bidding scenario in this section involves strong coupling between the price and energy bids in its constraints. Accordingly, the inner-outer subproblems decomposition approach that we proposed in Section III is no longer applicable. Solving such alternative problem formulation is beyond the scope of this paper and can be considered as a future work.

### C. Sub-Hourly Markets

If both the day-ahead and the real-time markets work on a sub-hourly basis but with equal time slots, then the analysis in this paper is still applicable without any changes, except for selecting a new and proper value for \( T \). However, sometimes, the day-ahead market is hourly while the real-time market is sub-hourly. For example, in California, the real-time energy market consists of \( \Gamma = 12 \) five-minute market cycles within each hour [5]. One approach to incorporate a sub-hourly real-time market is to take the average of the \( \Gamma \) price values for each hour so as to represent the real-time market on an hourly basis in the problem formulation. This is what we have done so far. Alternatively, one can increase the resolution in defining the optimization variables for the real-time market.

For each \( t = 1, \ldots, T \) and any \( \tau = 1, \ldots, \Gamma \), let \( z[t, \tau] \) denote the amount of power that is purchased from or sold to the real-time market during the sub-hourly cycle \( \tau \) at hour \( t \). A positive \( z[t, \tau] \) indicates selling energy while a negative \( z[t, \tau] \) indicates purchasing energy. To comply with the maximum discharge and charge rate requirements, we must have

\[
y_{\max}^t \leq \Gamma z[t, \tau] + \mathbb{I}(a[t] \geq p(l))x[t] - (1 - \mathbb{I}(a[t] \geq p(l)))y[t] \leq x_{\max}, \tag{37}
\]

where the expression inside the two inequality signs denotes the net amount of energy discharge (when positive) or energy charge (when negative) of the battery system based on the combined impact of the day-ahead and real-time market participation. Since \( z[t, \tau] \) is discharged or charged during \( 1/\Gamma \) fraction of an hour, \( z[t, \tau] \) is multiplied by \( \Gamma \) to match the hourly rates for \( x[t] \) and \( y[t] \). If \( \Gamma = 1 \), then the constraints in (37) reduce to those of an hourly market in (33).

With respect to the storage capacity constraints, at each hourly time slot \( t \) and sub-hourly time slot \( \tau \), we must have

\[
C_{\min}^t \leq C_{\max}^{\text{init}} - \sum_{l=1}^{T} \left( \sum_{\kappa=1}^{\Gamma} z[t, \kappa] + \mathbb{I}(a[l] \geq p(l))x[l] - (1 - \mathbb{I}(a[l] \geq p(l)))y[l] - \sum_{\kappa=1}^{\Gamma} z[t, \kappa] \right), \tag{38}
\]

\[
C_{\max}^t \geq C_{\max}^{\text{init}} - \sum_{l=1}^{T} \left( \sum_{\kappa=1}^{\Gamma} z[t, \kappa] + \mathbb{I}(a[l] \geq p(l))x[l] - (1 - \mathbb{I}(a[l] \geq p(l)))y[l] - \sum_{\kappa=1}^{\Gamma} z[t, \kappa] \right). \tag{39}
\]

where the expression on the right hand side denotes the current state-of-charge and \( b[t, \tau] \) is the random variable that denotes the cleared market price at sub-hourly cycle \( \tau \) of hour \( t \) in the real-time market. If \( \Gamma = 1 \), then the constraints in (38) and (39) reduce to those in (34) and (35), respectively.

Finally, we can revise the objective function as

\[
\sum_{t=1}^{T} \mathbb{E}\left[ \mathbb{I}(a[t] \geq p(l))x[t]a[t] - (1 - \mathbb{I}(a[t] \geq p(l)))y[t]a[t] + \sum_{\tau=1}^{\Gamma} z[t, \tau]b[t, \tau]\right]. \tag{40}
\]
While the above objective function has fewer terms than the one in (14), the complexity has now moved to the constraints in (37)-(39) as they now include the 0-1 indicator function $\mathbb{I}(\cdot)$. The optimization problem that is formulated under this alternative real-time market bidding scenario involves strong coupling between the price and energy bids in its constraints. Accordingly, the inner-outer subproblems decomposition approach that we proposed in Section III is no longer applicable. Solving such alternative problem formulation is beyond the scope of this paper and can be considered as a future work.

D. Large and Price-Maker Battery Systems

While the price-taker assumption in this paper is adequate for the existing battery projects, it is interesting to extend the results to address larger and price-maker battery systems that may become reality in the future. In case of a pool-based market, one can extend the results in [31], [32] and [33] to obtain the optimal self-schedule bids and the optimal economic bids for battery systems, respectively. In a nodal market, one can also benefit from the existing models, e.g., in [34], [35].

VI. CONCLUSIONS

A new framework was proposed for bidding, scheduling, and deployment of battery systems in the California ISO energy market. Both market and battery parameters are taken into consideration. Three design scenarios are presented. First, self-schedule bidding, where the bid includes only energy quantity. Second, economic bidding, with both energy and price quantities, where the day-ahead and real-time market prices are assumed to be statistically independent. Third, economic bidding, where the statistical dependency across day-ahead and real-time market prices is taken into consideration. The originally nonlinear stochastic optimization problems are solved efficiently using a decomposition approach. Several case studies are presented using real market data to assess the impact of location, season, battery efficiency, lifetime, charge and discharge rates, and using second-life / used batteries.

To motivate a few future research directions, we also provided some detailed pointers and remarks with respect to risk management, more flexible real-time market participation, sub-hourly markets, and large and price-maker battery systems.

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where the outmost expected value is now only on $a[t]$ but not on $b[t]$. Let us define $F(\rho)$ as the objective function in (44):

$$ F(\rho) = \frac{1}{K} \sum_{k=1}^{K} f_k(\rho), $$

(45)

where for each scenario $k = 1, \ldots, K$, we have

$$ f_k(\rho) = (a_k[t] - \mathbb{E}\{b_k[t]\}) \mathbb{I}\{ a_k[t] \geq \rho \}. $$

(46)

We need to show that the optimal solution of problem (44) is obtained as in (21). First, consider any $\rho$ such that

$$ \rho > \mathbb{E}\{b[t]\}. $$

(47)

For each $k$, where $a_k[t] < \mathbb{E}\{b[t]\}$, we have

$$ f_k(\rho) = f_k(\mathbb{E}\{b[t]\}) = 0. $$

(48)

For each $k$, where $\mathbb{E}\{b[t]\} \leq a_k[t] < \rho$, we have

$$ f_k(\rho) = 0, \quad f_k(\mathbb{E}\{b[t]\}) = a_k[t] - \mathbb{E}\{b[t]\} \geq 0. $$

(49)

Finally, for each $k$, where $\rho \leq a_k[t]$, we have

$$ f_k(\rho) = f_k(\mathbb{E}\{b[t]\}) = a_k[t] - \mathbb{E}\{b[t]\}. $$

(50)

From (45), (47), (48), and (49), we have

$$ F(\rho) \leq F(\mathbb{E}\{b[t]\}). $$

(51)

Next, consider any $\rho$ such that

$$ \rho < \mathbb{E}\{b[t]\}. $$

(52)

For each $k$, where $a_k[t] < \rho$, we have

$$ f_k(\rho) = f_k(\mathbb{E}\{b[t]\}) = 0. $$

(53)

For each $k$, where $\rho \leq a_k[t] < \mathbb{E}\{b[t]\}$, we have

$$ f_k(\rho) = a_k[t] - \mathbb{E}\{b[t]\} < 0, \quad f_k(\mathbb{E}\{b[t]\}) = 0. $$

(54)

Finally, for each $k$, where $\mathbb{E}\{b[t]\} \leq a_k[t]$, we have

$$ f_k(\rho) = f_k(\mathbb{E}\{b[t]\}) = a_k[t] - \mathbb{E}\{b[t]\}. $$

(55)

From (45), (52), (53), and (54), we have

$$ F(\rho) \leq F(\mathbb{E}\{b[t]\}). $$

(56)

From (51) and (56), we conclude that $p^*[t] = \mathbb{E}\{b[t]\}$ is a maximizer for problem (44). That is, (21) holds.