Channel Coordination Under Price Protection, Midlife Returns, and End-of-Life Returns in Dynamic Markets

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This paper examines three channel policies that are used in declining price environments: Price protection (P) is a mechanism under which the manufacturer pays the retailer a credit applying to the retailer’s unsold inventory when the wholesale price drops during the life cycle; midlife returns (M) allow the retailer to return units partway through the life cycle at some rebate; and end-of-life returns (E) allow the retailer to return unsold units at the end of the life cycle. Under declining retail prices, if the wholesale prices and the return rebates are set properly, then EM (i.e., midlife and end-of-life returns) achieves channel coordination. However, such a policy may not be implementable because it may require the manufacturer to be worse off as a result of coordination. If P is used in addition to EM and the terms are set properly, then PEM guarantees both coordination and a win-win outcome. If the retail price is constant over time, then EM is sufficient to guarantee both coordination and a win-win outcome.

(Channel Coordination; Supply Chain Management; Incentives; Inventory Management)

1. Introduction
Technology-related industries are marked by short product life cycles and high demand uncertainty. A defining characteristic of these industries is the sharp decline in the prevailing price and underlying value of a product over the life cycle—a phenomenon caused by rapid innovation and the frequent introduction of successively more sophisticated products. For example, the price of personal computers (PCs) and components each fell at a rate of 1% per week in 1998 (Hansell 1998, Graham-Hackett 1998). In such environments, suppliers may extend channel policies that expand the terms of trade beyond the wholesale price. Using a two-period model, we examine three such policies: Price protection (P) is a mechanism under which the manufacturer pays the retailer a portion of the difference in the wholesale price from one period to the next for inventory held at the end of the first period; midlife returns (M) allow the retailer to return units partway through the life cycle at some rebate; and end-of-life returns (E) allow the retailer to return unsold units at the end of the life cycle.

Channel policies are employed simultaneously in a number of industries. Vendors of computer hardware—from computer products and peripherals (Campbell 1998) to networking hubs and routers—offer PEM (i.e., price protection, midlife returns, and end-of-life returns) to their resellers. Similarly, in the electronic components industry, suppliers extend PEM to their distributors (Scheck 1999). In contrast, in environments where the retail price is relatively stable, price protection is not employed. However, EM is commonly used in specific industries where the retail price is relatively stable, such as books and recorded music.

This paper explores how simultaneous channel policies can be used to improve supply chain performance in dynamic markets (i.e., nonstationary
settings). We explore the role of $P$, $M$, and $E$ when they are used in combination or alone to address the following questions: Why do we not generally see subsidies for carrying inventory, analogous to price protection, in environments with static retail prices? Why is price protection used in addition to returns in declining price environments?

We evaluate channel policy combinations by two criteria. First, can the combination guarantee channel coordination (i.e., maximize the profitability of the entire supply chain of both retailer and manufacturer)? Second, can it guarantee “win-win” (i.e., the profit of each party is strictly greater than under no channel policy)? The second criterion is critical. If the combination can guarantee channel coordination but cannot assure win-win, then it is not clear that such a combination will be implementable because achieving coordination may require one of the parties to be worse off.

It is critical that explorations of channel policies take into account the salient characteristics of the industries by which they are motivated. An important dimension of the computer hardware and components industries is that firms have the opportunity to dispose of product both during and at the end of the life cycle. Computer manufacturers dispose of overstock (i.e., excess, new product) including workstations, servers, and PCs through several channels. Manufacturers sell overstock to remarketers such as ReCompute and OnSale, direct to consumers, or through traditional reseller channels (Schwartz 1998). For example, Compaq sells overstock computers on its website, which it calls a “factory outlet” (Halverson 1998). In the component industry, brokers provide an outlet for firms to dispose of excess inventory (Pepe 1999). Hewlett-Packard (HP) sells overstock components and computers through its TradingHubs.com website (Constantino 2000). Recently, the three biggest consumer-PC makers—Compaq, Gateway, and HP—as well as nine major suppliers and subcontract manufacturers announced the creation of a joint-venture company that will operate an online exchange whose purpose, in part, is to facilitate the sale of excess parts (McWilliams 2000). Extending beyond the computer industry, online exchanges and auctions have emerged as mechanisms through which firms generate “significant revenue” by disposing of excess supply or discontinued products (Dalton 1999).

We explore the role of various channel policy combinations when firms can dispose of product during and at the end of the life cycle. Under declining retail prices, if the wholesale prices and the return rebates are set properly, then EM achieves coordination but cannot guarantee win-win. Specifically, the manufacturer may be made worse off under coordination. However, if $P$ is used in addition to EM, and the terms are set properly, then PEM guarantees both coordination and win-win. If the retail price is constant over time, then EM is sufficient to guarantee both coordination and win-win. Channel coordination and win-win cannot be guaranteed under any of the other policy combinations: $PE$, $PM$, $P$, $M$, or $E$.

This paper is organized as follows. Section 2 provides a survey of related research. Section 3 describes the model. Sections 4 and 5 explore prospects for channel coordination under EM and PEM, respectively. Section 6 provides concluding remarks.

2. Literature Survey

Lee et al. (2000) explore the use of price protection with a two-period model, ignoring the possibility of returns or the disposal of unsold inventory at midlife. In this context, they show the effects of price protection on retailer behavior. When disposal is disallowed and the manufacturer is allowed to set her wholesale price at manufacturing cost in the second period, channel coordination is achieved when the wholesale prices and price protection credit are set properly.

This paper makes two contributions vis-à-vis Lee et al. First, as argued in §1, midlife disposal is an integral aspect of the computer hardware industry. Further, in practice, price protection is used together with returns, and these policies interact: Price protection is a subsidy for retaining inventory at midlife, but returns is a subsidy for disposing of inventory. Hence, a full understanding of the role of channel policies requires examining the simultaneous use of both instruments. Here, we allow for disposal of inventory, which changes the structure of the optimal policy, and we explore the simultaneous use of price protection and returns.
Second, the channel coordinating scheme in Lee et al. has the manufacturer pricing at marginal cost in the second period (i.e., the retailer sees the integrated channel margin and the manufacturer margin is zero). By doing so, the manufacturer cedes a portion of total chain profit to the retailer. As a result, a win-win solution cannot be guaranteed. In particular, the manufacturer profit may be necessarily lower under price protection than under a price-only contract, as the numerical example in Lee et al. demonstrates. The fact that in practice manufacturers generally retain positive margins in the later part of the product life cycle suggests that this type of coordination scheme may be difficult to implement. In §5, we identify a channel coordinating PEM scheme that guarantees win-win. In this scheme, consistent with industry practice, the manufacturer’s wholesale prices strictly exceed the manufacturing costs. Hence, Lee et al. present a partial answer to the question: Why do manufacturers offer price protection? Price protection can maximize total chain efficiency; however, it may leave the manufacturer worse off. We present a more complete answer to a more complete question: Why do manufacturers offer price protection and returns simultaneously? If the PEM parameters are properly specified, then total chain profit is maximized, and both parties are better off.

Investigations of returns policies have been based on single-demand occurrence (i.e., newsvendor-type) models and hence do not incorporate nonstationarity. Lariviere and Porteus (1999) explore a price-only contract in a newsvendor setting. Pasternack (1985) explores the role of returns in the context of perishable commodities, i.e., products for which there is a single buying opportunity at the start of the life cycle and a single return opportunity at the end of life. He shows that a properly chosen wholesale price and return rebate coordinate the channel. Kandel (1996) and Emmons and Gilbert (1998) incorporate price-sensitive end consumer demand in a one-period returns model.

Donohue (1998) considers a situation in which there is a single demand occurrence, a single return opportunity at end of life, and a second buying opportunity. She shows that the proper prices and rebate achieve coordination. Tsay (1999) and Brown and Lee (1998) show, respectively, that quantity flexibility and options arrangements can be instruments to achieve channel coordination in a one-demand occurrence environment. These types of arrangements are equivalent to returns policies in which the quantity that can be returned is restricted (see Lariviere 1999).

Barnes-Schuster et al. (1998) study the role of options in a two-demand occurrence environment where demand is correlated. The retail and wholesale prices are stationary, but other parameters are nonstationary. At the start of Period 1, the retailer places firm orders for both periods at the same per unit price and purchases options to purchase additional units in Period 2. After Period 1 demand occurs, the retailer exercises options and receives her Period 2 firm order. Barnes-Schuster et al. establish sufficient conditions for channel coordination to be achieved under this arrangement. When prices are linear and two-part tariffs are excluded, the manufacturer makes zero profit under coordination; when a two-part tariff is employed (e.g., through a quantity discount), the manufacturer can capture the total chain profit. The contract they consider is equivalent to a single purchase opportunity and single return opportunity arrangement. However, Barnes-Schuster et al. is distinct from newsvendor returns models in that there is a second demand opportunity and returns are accepted at mid life rather than at the end of life. Our model is distinct from Barnes-Schuster et al. in that we have purchase opportunities both at the start of and during the life cycle, return opportunities both during the life cycle and at the end of life, and price protection.

Eppen and Iyer (1997) and Milner and Rosenblatt (1997) each use two-demand occurrence models to focus on the retailer’s behavior under, respectively, backup agreements and per-unit penalty arrangements. Both papers assume all parameters are stationary, with the exception that Eppen and Iyer (1997)
allow nonstationary holding costs. Excellent reviews of supply chain contracting research are provided by Anupindi and Bassok (1999), Cachon (1999), Lariviere (1999), and Tsay et al. (1999).

3. The Model

To examine simultaneous channel policies in a nonstationary context, we consider a two-period, two-party model. All exogenous parameters are known by both parties at the start of the first period. The retail market is competitive so that the retailer faces fixed market/retail prices in both periods. First, the terms of the channel policy(s) (e.g., wholesale prices, return rebates) are specified, and the manufacturer commits to the terms of trade. Then the retailer chooses her order and disposal quantities. The assumption that the terms of trade over the time horizon are fixed before the firm facing those terms acts is common in the literature (cf. Anupindi and Bassok 1999, Cachon 1999, Donohue 1998, Lee et al. 2000). The implications of relaxing the assumption that the manufacturer must commit to her wholesale prices in advance are discussed in §5.

Channel coordination is achieved when the performance of the fully integrated company is replicated by the decentralized supply chain. To achieve this, the terms of trade must be specified to induce the retailer to behave in the way that is optimal for the integrated firm. Our interest is determining under what channel policy combinations channel coordination can be achieved. When coordination can be achieved, we are interested in how the profit is split between the two parties because this indicates whether coordination can be feasibly implemented. First, we introduce the following notation.

- \( p_i = \) selling price at the market per unit in period \( i = 1, 2 \) (i.e., retail price)
- \( w_i = \) manufacturer’s price to the retailer per unit in period \( i = 1, 2 \) (i.e., wholesale price)
- \( c_i = \) manufacturing cost per unit in period \( i = 1, 2 \) (i.e., marginal cost)
- \( v_i = \) salvage value per unit for units disposed at end of period \( i = 1, 2 \)
- \( b_i = \) rebate paid by manufacturer to the retailer per unit for units returned at end of period \( i = 1, 2 \)
- \( \beta = \) price protection magnitude parameter, \( \beta \in [0, 1] \)
- \( \xi_i = \) random variable denoting the demand in period \( i \) having density \( \phi_i(\cdot) \), distribution \( \Phi_i(\cdot) \), and mean \( \mu_i; \ i = 1, 2 \)

We make the following assumptions.

- **Assumption A1.** \( 0 < c_i < w_i < p_i, v_i \leq b_i \leq w_i \) for \( i = 1, 2 \); \( v_2 \leq v_1 \leq c_2 \leq c_1, w_2 \leq w_1, p_2 \leq p_1 \).

- **Assumption A2.** \( p_i, c_i, v_i, \) and \( \Phi_i(\cdot) \) are exogenous; \( w_i, b_i, \) and \( \beta \) are endogenous.

- **Assumption A3.** No lump sum side payment is allowed.

We assume \( \phi_i(\xi_i) > 0 \) for all \( \xi_i \geq 0 \); the analysis can be extended to any support \([l, u]\), where \( 0 \leq l < u \leq \infty \). Further, we assume all retailer orders can be filled (i.e., manufacturer capacity is infinite). Consistent with the technology-related industries that motivate this work, we restrict our attention to situations in which the retail and wholesale prices, manufacturing cost, and salvage value are declining over time. The assumption of decreasing manufacturing cost is reasonable because of decreasing material costs and potential learning effects. Because the retail price and demand distribution are each nonstationary and exogenous, the relationship between these two factors can be captured.

3.1. The Integrated Channel and the Independent Retailer Under No Channel Policy

We begin by examining the scenario in which the manufacturer and retailer are under the same ownership. This centralized control setting serves as the benchmark for the context in which the retailer is independent, which is explored subsequently. At the end of period \( i = 1, 2 \), the integrated channel may dispose of unsold product for \( v_i \).

Let \( \pi_i = \) the integrated channel’s total expected profit over the two periods, and \( \pi_i(x) = \) the integrated channel’s expected profit in Period 2, given that the integrated channel’s ending stock in Period 1 before salvaging is \( x \).
The integrated channel’s problem is described in the following dynamic program:

\[
\pi_1 = \max_{y \geq 0} \left\{ p_1 E \min(y, \xi_1) - c_1 y + E \pi_2 \left( (y - \xi_1)^+ \right) \right\}, \tag{1}
\]

\[
\pi_2(x) = \max_{y \geq 0} \left\{ p_2 E \min(y, \xi_2) - c_2 (y - x)^+ + v_1 (x - y)^+ + v_2 E (y - \xi_2)^+ \right\}.
\]

The formulation in (1) assumes that unmet demand is lost and that holding costs and stockout costs resulting from loss of goodwill are negligible. These assumptions can be relaxed, as discussed in §5. Define

\[
\overline{S}_2 := \Phi^{-1}_2 \left( \frac{p_2 - c_2}{p_2 - v_2} \right) \quad \text{and} \quad \underline{S}_2 := \Phi^{-1}_2 \left( \frac{p_2 - v_1}{p_2 - v_2} \right).
\]

Note that \( \overline{S}_2 < \underline{S}_2 \). It is well known that an order-up-to/dispose-down-to policy is optimal for (1). The optimal policy in Period 1 is to order up to a quantity, call it \( S_1 \). The optimal policy at the start of Period 2 is given by the following: if the stock level is less than \( \underline{S}_2 \), then order up to \( \overline{S}_2 \); if the stock level is greater than \( \underline{S}_2 \), then dispose down to \( \underline{S}_2 \); if the stock level is between these thresholds, then neither order nor dispose. The quantity \( S_1 \), the optimal policy, and the resulting expected profit to the integrated channel are specified precisely in the appendix (see Lemmas A1 and A2).

Consider the setting in which the retailer is independent and the sole terms of trade are the wholesale prices, i.e., no channel policy is used. The retailer’s problem is identical to (1) except that \( w_i \) replaces \( c_i ; \) \( i = 1, 2 \). Because the manufacturer and retailer are assumed to have the same salvage values, the retailer’s Period 2 dispose-down-to quantity is the same as that of the integrated channel (if the assumption of identical salvage values is relaxed, then the quantities will in general differ). The retailer’s Period 2 order-up-to quantity is strictly less than that of the integrated channel. The retailer’s Period 1 order-up-to quantity in general differs from that of the integrated channel. Consequently, when the retailer is independent and no channel policy is employed, the total chain profit is less than that of the integrated channel (i.e., the channel is not coordinated).

### 3.2. Independent Retailer Under PEM

In a price protection policy, a manufacturer commits to paying the retailer a credit applying to the retailer’s unsold inventory when the wholesale price drops during the product’s life cycle. Under PEM, price protection can be exercised when the wholesale price drops; returns can be exercised both at this time and at the end of the life cycle. Specifically, if at the end of Period 1 the retailer has stock on hand, she can either retain the stock and take the price protection credit or return the stock to the manufacturer. In practice, the price protection credit is typically the full amount of the wholesale price drop, but the inventory eligible may be restricted based on time of purchase. We model price protection as follows: For each unit on which the retailer takes price protection, she receives a credit of \( (w_1 - w_2) \beta \) where \( \beta \in [0, 1] \). For each unit she returns in period \( i \), she receives a rebate of \( b_i ; i = 1, 2 \). The retailer under PEM with excess stock at the end of Period 1 is restricted, on any given unit, to either retaining the unit and taking price protection or disposing of the unit through the returns policy.

It is conceivable that a manufacturer could allow a retailer to both return and order at the end of Period 1. If, in fact, the retailer returns throughout the life cycle. Specifically, if at the end of Period 1 the retailer has stock on hand, she can either retain the stock and take the price protection credit or return the stock to the manufacturer. In practice, the price protection credit is typically the full amount of the wholesale price drop, but the inventory eligible may be restricted based on time of purchase. We model price protection as follows: For each unit on which the retailer takes price protection, she receives a credit of \( (w_1 - w_2) \beta \) where \( \beta \in [0, 1] \). For each unit she returns in period \( i \), she receives a rebate of \( b_i ; i = 1, 2 \). The retailer under PEM with excess stock at the end of Period 1 is restricted, on any given unit, to either retaining the unit and taking price protection or disposing of the unit through the returns policy.

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The retailer’s problem is described in the following dynamic program:

\[
R_1 = \max_{y \geq 0} \{p_1 E \min(y, \xi_1) - w_1 y + ER_2((y - \xi_1)^+)\},
\]

\[
R_2(x) = \max_{y \geq 0} \{p_2 E \min(y, \xi_2) - w_2 (y - x)^+ + b_1 (x - y)^+ + (w_1 - w_2) \beta \min(x, y) + b_2 E(y - \xi_2)^+\}.
\]

Define

\[
\overline{T}_2 := \Phi_2^{-1} \left( \frac{p_2 + (w_1 - w_2) \beta - b_1}{p_2 - b_2} \right).
\]

Note that (2) implies \( T_2 < \overline{T}_2 \). The optimal dispose-down-to quantity is \( \overline{T}_2 \), and from its definition, it is increasing in the price protection credit and decreasing in the midlife return rebate. This is intuitively appealing because price protection is a subsidy for retaining inventory at midlife, while midlife returns is a subsidy for disposing of inventory.

**Lemma 1.** The optimal quantity achieved by ordering or returns \( T_2 \) for the retailer in Period 2 under PEM is given by the following: if \( x < T_2 \), then \( T_2 = T_2; \) if \( T_2 < x < \overline{T}_2 \), then \( T_2 = x; \) if \( x \geq \overline{T}_2 \), then \( T_2 = \overline{T}_2 \).

All proofs appear in the appendix. The policy described in Lemma 1 amounts to the following: If \( x \leq T_2 \), then take the price protection credit on \( x \) units and order \( T_2 - x \) units; if \( T_2 < x < \overline{T}_2 \), then take the price protection credit on \( x \) units and order none; if \( x \geq \overline{T}_2 \), then return \( x - \overline{T}_2 \) units and take the price protection credit on \( \overline{T}_2 \) units.

Define

\[
T_0 := \Phi_1^{-1} \left( \frac{p_1 - w_1}{p_1 - w_2 - (w_1 - w_2) \beta} \right).
\]

The quantity \( T_0 \) is the myopic order-up-to quantity in Period 1 when each unit unsold at the end of Period 1 has value \( w_2 + (w_1 - w_2) \beta \).

**Lemma 2.** The optimal policy for the retailer in Period 1 under PEM is to order up to \( T_0 \), which is given by the following: if \( T_0 \leq T_2 \), then \( T_1 = T_0; \) if \( T_2 < T_0 \leq \overline{T}_2 \), then \( T_1 \) satisfies

\[
p_1 - w_1 - [p_1 - w_2 - (w_1 - w_2) \beta] \Phi_1(T_1) + (p_2 - w_2) \Phi_1(T_1 - T_2) - (p_2 - b_2) \int_{T_2}^{T_1} \Phi_2(T_1 - \xi_1) d \Phi_1(\xi_1) = 0,
\]

and it also holds that \( T_2 < T_1 < T_0 \); if \( T_0 > T_2 \) and \( k \leq 0 \), then \( T_1 \) satisfies

\[
p_1 - w_1 - [p_1 - w_2 - (w_1 - w_2) \beta] \Phi_1(T_1) + (p_2 - w_2) \times \Phi_1(T_1 - T_2) - [p_2 + (w_1 - w_2) \beta - b_1] \Phi_1(T_1 - \overline{T}_2) - (p_2 - b_2) \int_{T_2}^{T_1 - \overline{T}_2} \Phi_2(T_1 - \xi_1) d \Phi_1(\xi_1) = 0,
\]

and it also holds that \( \overline{T}_2 < T_1 < T_0 \).

\( k \) is a constant defined in the appendix; it is the first derivative of the retailer’s Period 1 objective function with respect to the Period 1 order quantity evaluated at \( T_2 \). Equations (4) and (5) are each the first-order condition of the retailer’s Period 1 objective function corresponding to different regions of the function. The sign of \( k \) determines within which region of the retailer’s objective function the optimal solution lies, and hence which first-order condition is satisfied.

Several of the other channel policy combinations are special cases of (3). To model the independent retailer under PM, simply replace \( b_2 \) by \( v_2 \) in the formulation. To model returns policies without price protection, set \( \beta = 0 \). To model end-of-life (midlife) only returns, replace \( b_1(b_2) \) by \( v_1(v_2) \). Modeling price protection without midlife returns requires a slightly modified dynamic program because the retailer can take price protection on units that she subsequently immediately disposes of at the end of Period 1. Define

\[
\overline{U}_2 := \Phi_2^{-1} \left( \frac{p_2 - v_1}{p_2 - b_2} \right).
\]

It is straightforward to show that under PE, the order-up-to and dispose-down-to quantities in Period 2 are \( \overline{T}_2 \) and \( \overline{U}_2 \). The order-up-to quantity in Period 1 is characterized in a way similar to \( T_1 \) in Lemma 2, with straightforward modifications. The next two sections describe the prospects for various channel policy combinations for achieving channel coordination and specify how the profit is split when coordination is achieved.

### 4. Channel Coordination Under EM

Theorem 1 demonstrates that if the wholesale prices and return rebates are properly set, then channel...
coordination is achieved under EM. However, coordination may require the manufacturer to be worse off. Let \( r \) be the retailer profit and \( m \) the manufacturer profit in a decentralized, no channel policy environment; let \( \pi \) be the profit in an integrated channel. The channel is not coordinated, and \( r + m < \pi \). Define
\[
\lambda := (p_1 - p_2) \int_0^{\hat{\xi}_1} \xi_1 d\Phi_1(\xi_1).
\]

**Theorem 1.** Consider the channel policy combination of \( (w_1(e), w_2(e), b_1(e), b_2(e)) \) for \( e \in (0, p_2 - c_2) \): Let \( w_1(e) = p_1 - (p_1 - p_2)\Phi_1(S_1) - \varepsilon[p_1 - c_1 - (p_1 - p_2)\Phi_1(S_1)]/(p_2 - c_2), \) \( w_2(e) = p_2 - e, \) and \( b_1(e) = p_2 - \varepsilon(p_2 - c_2)/(p_2 - c_2) \); \( i = 1, 2. \)

(a) The channel policy combination achieves coordination.

(b) The resulting profit to the manufacturer and retailer is
\[
\hat{m}(e) = [\pi - \lambda][1 - \varepsilon/(p_2 - c_2)],
\]
and
\[
\hat{r}(e) = [\pi - \lambda]e/(p_2 - c_2) + \lambda.
\]

(c) If \( m < \pi - \lambda \), then for
\[
e \in \left( \frac{(p_2 - c_2)(r - \lambda)}{\pi - \lambda}, \frac{(p_2 - c_2)(\pi - m - \lambda)}{\pi - \lambda} \right),
\]
\( \hat{m}(e) > m \) and \( \hat{r}(e) > r \), i.e., win-win is achieved. If \( m > \pi - \lambda \), then \( \hat{m}(e) < m \), i.e., the manufacturer profit is strictly lower under coordination, and win-win cannot be achieved.

There exists a continuum of price/rebate combinations that achieve channel coordination. As the manufacturer offers more generous return rebates (i.e., as \( \varepsilon \) decreases), she also raises the wholesale prices. As the manufacturer offers higher rebates and wholesale prices, her share of the total chain profit increases. These results are consistent with Pasternack (1985), and Theorem 1 essentially extends Pasternack (1985) to the nonstationary, two-period case. Although price protection is based on the wholesale prices, Theorem 1 highlights the importance of the retail prices.

The key insight from Theorem 1 is the relationship between the retail price trajectory and how the total chain profit is split between the two parties. If retail prices are declining over time (i.e., \( p_1 > p_2 \) and hence \( \lambda > 0 \)), then the retailer profit is bounded below by \( \lambda \), and the manufacturer profit is bounded above by \( \pi - \lambda \). Hence, if the manufacturer profit under no channel policy is greater than this upper bound, then the manufacturer is made strictly worse off under coordination.

Although the high-technology industries that motivate our research are marked by sharp declines in the retail price over the product life cycle, in other industries retail prices may be reasonably stable over time. If the retail price is stationary (i.e., \( p_1 = p_2 \) and hence \( \lambda = 0 \)), then the manufacturer profit simplifies to \( \hat{m}(e) = \pi[1 - \varepsilon/(p_2 - c_2)], \) \( \hat{m}(e) \) and \( \hat{r}(e) \) are monotone in \( e \); as \( e \rightarrow 0, \hat{m}(e) \rightarrow \pi \) and \( \hat{r}(e) \rightarrow 0; \) as \( e \rightarrow p_2 - c_2, \hat{m}(e) \rightarrow 0 \) and \( \hat{r}(e) \rightarrow \pi. \) Hence, the total chain profit can be split arbitrarily between the two parties. By setting the policy parameters appropriately, both coordination and win-win are guaranteed.

Declining retail prices hamper prospects for achieving win-win by placing an upper bound on the manufacturer profit. To see why this occurs, first recall that the manufacturer profit is increasing in the wholesale prices and the return rebates. In the static retail price setting, the manufacturer profit approaches the total chain profit as the wholesale prices and return rebates approach the retail price (i.e., as \( e \rightarrow 0, w_i(e) \rightarrow p \) and \( b_i(e) \rightarrow p; i = 1, 2. \)).

In the declining retail price setting, the channel coordinating Period 1 wholesale price and return rebate are less than and (uniformly) bounded away from the Period 1 retail price. In particular, as \( e \rightarrow 0, w_1(e) \rightarrow p_1 - (p_1 - p_2)\Phi_1(S_1) < p_1 \) and \( b_1(e) \rightarrow p_2 < p_1. \) (In contrast, as \( e \rightarrow 0, w_2(e) \rightarrow p_2 \) and \( b_2(e) \rightarrow p_2, \) as in the static retail price case.) The Period 1 return rebate is bounded above by the Period 2 retail price because otherwise the retailer would have no incentive to carry inventory into Period 2, and hence channel coordination could not be achieved (because \( S_2 > 0, \) any scheme with \( T_2 = 0 \) implies \( S_2 \neq T_2). \) Because the Period 1 return rebate is strictly less than the Period 2 retail price, the Period 1 wholesale price is less than and bounded away from the Period 1 retail price. If the Period 1 wholesale price were not bounded away from the Period 1 retail price, then the retailer would have no incentive to order in Period 1 because the average cost is bounded away from zero, but the
underage cost is not; hence, coordination could not be achieved.

To summarize, the presence of declining retail prices constrains how high the manufacturer’s wholesale price in Period 1 can be and still achieve coordination. Hence, the manufacturer must give to the retailer a margin, \( p_1 - w_1 \), that is bounded away from zero, and the retailer profit is consequently bounded away from zero as well. Essentially, declining retail prices force the channel coordinating EM manufacturer to set a lower wholesale price and consequently garner lower profit.

5. Channel Coordination Under Price Protection and Returns

A critical limitation of the channel coordinating EM policy is that under declining retail prices the manufacturer may do worse under the coordinating EM policy than under no channel policy. The widespread existence of declining retail price environments motivates us to identify a policy that guarantees coordination and win-win. Theorem 2 demonstrates that PEM is such a policy.

**Theorem 2.** Consider the channel policy combination of \((w_1(\epsilon), w_2(\epsilon), \beta(\epsilon), b_1(\epsilon), b_2(\epsilon))\) for \( \epsilon \in (0, p_2 - c_2) \):

Let \( w_1(\epsilon) = p_1 - \epsilon(p_1 - c_1)/(p_2 - c_2) \), \( w_2(\epsilon) = p_2 - \epsilon \), \( b_1(\epsilon) = p_1 - \epsilon(p_1 - v_j)/(p_2 - c_2) \) \((i = 1, 2)\), and

\[
\beta(\epsilon) = \begin{cases} 
(p_1 - p_2)(p_2 - c_2 - \epsilon) \\
/[((p_1 - p_2)(p_2 - c_2 - \epsilon) + \epsilon(c_1 - c_2))] & \text{if } p_1 > p_2, \\
0 & \text{else.}
\end{cases}
\]

(a) The channel policy combination achieves coordination.

(b) The resulting profit to the manufacturer and retailer is

\[ \bar{m}(\epsilon) = \pi[1 - \epsilon/(p_2 - c_2)], \]

and

\[ \bar{r}(\epsilon) = \pi \epsilon/(p_2 - c_2). \]

(c) For \( \epsilon \in ((p_2 - c_2)r/\pi, (p_2 - c_2)(\pi - m)/\pi) \), \( \bar{m}(\epsilon) > m \) and \( \bar{r}(\epsilon) > r \), i.e., win-win is achieved.

Similar to the EM case, the manufacturer profit is increasing in the wholesale prices, return rebates, and price protection credit (as \( \epsilon \) decreases, each quantity increases). If the retail price is declining and the manufacturing cost is stationary, then full price protection (i.e., \( \beta = 1 \)) is used. If the retail price is stationary, then no price protection (i.e., \( \beta = 0 \)) is used and the policy reduces to the EM policy of Theorem 1.

The key result is that if the retail price is declining, then—in contrast to the EM case—by properly setting the PEM terms, the manufacturer’s portion of the total chain profit can be made arbitrarily large (as \( \epsilon \to 0 \), \( \bar{m}(\epsilon) \to \pi \)). This has two implications. First, under declining retail prices, the maximum profit the manufacturer can capture under PEM is larger than the maximum profit she can capture under EM (strictly speaking, the supremum of profit is greater under PEM than under EM). Hence, by properly setting the PEM terms, the manufacturer is made strictly better off than under any EM scheme. Second, a properly designed PEM policy guarantees channel coordination and win-win—even if prices are decreasing and manufacturer profit under no channel policy is relatively high.

To understand why PEM is superior to EM in a declining retail price environment, we first summarize why EM fails to guarantee a coordinating, win-win outcome. Achieving coordination requires inducing the retailer to (1) order a sufficiently large quantity in Period 1 and (2) carry unsold inventory into Period 2. Under declining retail prices, using a large midlife return rebate helps to achieve the first objective, but compromises achieving the second. Achieving both objectives requires setting a low Period 1 wholesale price. If the wholesale price is too high, it will not be attractive for the retailer to purchase units in Period 1 and carry them into Period 2 where the retail price is low. (In contrast, if the retail price is static, then both objectives can be attained with a high Period 1 wholesale price because units purchased in Period 1 and sold in Period 2 suffer no erosion in the revenue they generate.) Setting a low wholesale price limits the manufacturer’s share of the total profit and the prospects for a win-win outcome.

By subsidizing the retailer for carrying units into Period 2, price protection encourages the retailer to...
buy in Period 1 and carry inventory into Period 2. Under a coordinating PEM policy, the Period 1 wholesale price can be set very high—arbitrarily close to the retail price—because by properly subsidizing the retailer for retaining unsold inventory, the PEM policy induces the retailer to order a sufficiently large quantity in Period 1 and carry the proper quantity into Period 2. By eliminating the constraint imposed by declining retail prices on how high the wholesale price in Period 1 can be and still achieve coordination, price protection allows the manufacturer to capture an arbitrarily large portion of the total chain profit.

To be precise, recall that under EM the manufacturer’s Period 1 wholesale price is less than and bounded away from the Period 1 retail price. Under PEM, this constraint is lifted. Specifically, as \( \varepsilon \to 0 \), \( \lim_{\varepsilon \to 0} \frac{\pi_i}{\pi_i} \) is not dominated by \( \lim_{\varepsilon \to 0} \frac{\pi_i}{\pi_i} \) and if \( p_1 > p_2 \), then \( \beta(\varepsilon) \to 1 \); \( i = 1, 2 \). How does price protection lift the constraint on the Period 1 wholesale price and return rebate? Recall that under EM, the Period 1 return rebate is bounded above by the Period 2 retail price, as otherwise the retailer would have no incentive to carry inventory into Period 2. Under price protection, the Period 1 return rebate can be raised above the Period 2 retail price because price protection subsidizes the retailer to carry inventory into Period 2. To see this, note that if the retailer returns a unit at the end of Period 1, she receives \( b_i(\varepsilon) \); if the retailer holds on to the unit and sells it in Period 2, she receives \( p_2 + [w_1(\varepsilon) - w_2(\varepsilon)\beta(\varepsilon)] > b_i(\varepsilon) \) for any \( \varepsilon \). Thus, even if the Period 1 return rebate is arbitrarily close to the retail price, the retailer has incentive to carry stock into the second period. By lifting the constraint on the Period 1 return rebate, price protection also lifts the constraint on the wholesale price.

These dynamics are illustrated in the following numerical example: \( p_1 = 7 \), \( c_1 = 3.5 \), \( v_1 = 2 \), \( p_2 = 4 \), \( c_2 = 3 \), \( v_2 = 0 \), and \( \xi_i \sim \text{Uniform}(0, 1) \); \( i = 1, 2 \). The integrated channel’s optimal order-up-to and dispose-down-to levels are \( S_1 = 0.775 \), \( S_2 = 0.500 \), and \( S_2 = 0.250 \), and the resulting expected profit is \( \pi = 1.554 \). Under a coordinating EM policy, the Period 1 wholesale price and return rebate are less than and bounded away from the retail price: \( w_1(\varepsilon) = 4.675 - 1.175\varepsilon \), \( w_2(\varepsilon) = 4 - \varepsilon \), \( b_1(\varepsilon) = 4 - 2\varepsilon \), \( b_2(\varepsilon) = 4 - 4\varepsilon \) where \( \varepsilon \in (0, 1) \). Consequently, the manufacturer’s share of the profit is bounded away from the total chain profit: \( \hat{m}(\varepsilon) = 0.6527(1 - \varepsilon) \).

Under a coordinating PEM policy, the use of price protection lifts the constraint on the Period 1 wholesale price and return rebate: \( w_1(\varepsilon) = 7 - 3.5\varepsilon \), \( w_2(\varepsilon) = 4 - \varepsilon \), \( b_1(\varepsilon) = 7 - 5\varepsilon \), \( b_2(\varepsilon) = 4 - 4\varepsilon \), and \( \beta(\varepsilon) = (3 - 3\varepsilon)/(3 - 2.5\varepsilon) \). Consequently, the manufacturer’s share of the profit is no longer bounded away from the total chain profit: \( \hat{m}(\varepsilon) = 1.554(1 - \varepsilon) \). Consider, for example, \( w_1(0.3) = 5.95 \), \( w_2(0.3) = 3.7 \), \( b_1(0.3) = 5.5 \), \( b_2(0.3) = 2.8 \), and \( \beta(0.3) = 0.9333 \). Under this scheme, the Period 1 return rebate is very generous—greater than the Period 2 retail price. However, the generous price protection credit and the prospects of selling additional units in Period 2 makes it attractive for the retailer to carry units into Period 2. By offering generous price protection and return rebates, the manufacturer is also able to charge high wholesale prices and capture a large portion of the total chain profit: \( \hat{m}(0.3) = 1.088 \) and \( \hat{r}(0.3) = 0.4661 \). Suppose that under no channel policy, the wholesale prices are \( w_1 = 5.4 \) and \( w_2 = 3.6 \); the resulting profits are \( m = 0.8229 \) and \( r = 0.3704 \). Hence, the manufacturer is strictly worse off under any coordinating EM scheme. However, under a coordinating PEM scheme, with \( \varepsilon \in (0.2384, 0.4704) \) both parties are better off.

Price protection is tied most obviously to the decline in wholesale prices because the timing and magnitude of the wholesale price drop determines the timing and magnitude of the credit. Our analysis demonstrates the important but more subtle link between the retail prices and price protection. In fact, a coordinating, win-win channel policy may involve declining wholesale prices without the use of price protection. In particular, if the retail price is constant and the manufacturing cost is declining, it is easy to check that the wholesale price is declining in the EM policy of Theorem 1; hence, EM alone with declining wholesale prices is sufficient to achieve coordination and win-win. Hence, a manager seeking to identify the channel policies to be used to achieve coordination and win-win should focus on the retail price trajectory, not the wholesale price trajectory. If the retail price is static (or the decline is small) then EM is
(likely to be) sufficient. If the price decline is steep, then PEM is likely to be required.

As discussed above, channel coordination is achieved when the behavior parameters of the independent retailer are equated with those of the integrated channel. Under EM the four degrees of freedom the manufacturer has in specifying her channel policy (the two wholesale prices and the two return rebates) are sufficient to equate the three parameters that govern the retailer’s behavior (the order-up-to levels in Periods 1 and 2 and the dispose-down-to level in Period 2). Hence, in this case, one additional degree of freedom is sufficient to achieve coordination. However, the single additional degree of freedom extant in the EM scheme of Theorem 1 is not sufficient to ensure arbitrary profit splitting. In contrast, under PEM there are two additional degrees of freedom and arbitrary profit splitting is assured. Thus, although from a channel coordination perspective P is redundant, the use of P in addition to EM is critical to ensure that a channel coordinating scheme can be implemented in a declining retail price environment.

One might conjecture that PE and PM could also guarantee channel coordination. In each case, there are four degrees of freedom in specifying the channel policy (the two wholesale prices, the price protection credit, and the return rebate), and three behavior parameters must be equated with those of the integrated channel. This conjecture is not correct; in our two-period context, the additional degree of freedom is not sufficient to guarantee coordination. In the context of our model and the instruments under study, the basic idea is that for coordination to be assured, the additional degree(s) of freedom must be properly placed. For example, under PE, there are two degrees of freedom with respect to the Period 2 order-up-to quantity. That is, two of the PE parameters—the Period 2 wholesale price and return rebate—affect this behavior parameter of the retailer. There are four degrees of freedom with respect to the retailer’s Period 1 order-up-to quantity (all four of the PE parameters affect this quantity). However, there is only one degree of freedom (the end-of-life return rebate) with respect to the dispose-down-to quantity, and the lack of an additional degree of freedom hampers coordination. (Specifically, \( \overline{U}_2 = \overline{S}_2 \) requires \( b_2 = v_2 \); under this \( b_2, T_2 = \overline{S}_2 \), only if \( w_2 = r_2 \), which contradicts Assumption A1.) It is straightforward to show that coordination cannot be guaranteed under PM, P, M, or E. In the cases where coordination cannot be guaranteed, there does not exist an additional degree of freedom with respect to each of the retailer’s behavior parameters. In contrast, in the cases where coordination is achieved (PEM and EM), there exists at least one additional degree of freedom with respect to each of the retailer’s behavior parameters.

It is important to emphasize that the observations regarding the role of additional degrees of freedom in achieving coordination (and win-win) are made in the context of our model, its assumptions, and the specific instruments under study. These observations need not hold in general. For example, Cachon and Lariviere (2000) show that a properly designed two-degree-of-freedom revenue-sharing contract achieves coordination in the two decision variables of an independent retailer; an additional degree of freedom is not required to achieve coordination. Our model assumes, for example, that the midlife return rebate is sufficiently small that it is attractive for the retailer to retain stock at midlife rather than to return her entire inventory and immediately reorder (Equation (2)). Employing distinct assumptions may lead to different results regarding the efficacy of channel policy combinations (and the role of degrees of freedom) in achieving coordination and/or win-win.

We conclude this section by noting that the model presented in §3 can be extended in several ways. It is straightforward to extend the model to include holding costs for carrying inventory, stockout costs at the retailer resulting from loss of goodwill, and discounting. Let \( h_i \) be the holding cost per unit, \( g_i \) be the stockout cost per unit, and \( \alpha_i \) be the discount factor in period \( i; i = 1, 2 \). As a generalization of the assumption of declining retail prices, assume \( p_1 + g_1 \geq p_2 + g_2 \). Under suitable relaxations of the assumptions, the results of Theorems 1 and 2 continue to hold. That is, by properly setting the PEM parameters, coordination and win-win are assured; by properly setting the EM parameters, coordination is assured but win-win is not.
The relaxed assumptions are $w_i < p_i + g_i$, $\alpha_i b_i < w_i + h_i$, and $\alpha_i (w_i - \bar{w}_i) \beta \leq w_i + h_i - \alpha_i \bar{w}_i$; $i = 1, 2$. The first assumption requires that the wholesale price be less than the total penalty to the retailer in forgone revenue and goodwill for stockout. The second (third) assumption ensures that the return rebate (price protection credit) is sufficiently small such that purchasing and subsequently not selling a unit is costly to the retailer. However, the relaxed assumptions allow the wholesale price to exceed the retail price, the return rebate to exceed the wholesale price, and the price protection magnitude parameter to exceed unity. While the formulation in §3 assumes that unmet demand is lost, the model can be easily modified to capture backordering of excess demand in Period 1. Under backordering, excess demand is filled at the start of Period 2 at the price and cost for that period. This is captured by replacing $g_i$ with $g_i - \alpha_i (p_2 - c_2)$. Further, it is straightforward to extend the model to allow for differential salvage values at the manufacturer and retailer, provided the manufacturer’s salvage value is greater than the retailer’s.

Suppose we relax the assumption that the manufacturer must commit to the Period 2 wholesale price at the start of Period 1. Clearly, under no channel policy and no commitment, the sum of the retailer profit and manufacturer profit is less than or equal to the integrated channel profit. If PEM terms are properly set and committed to, then channel coordination is achieved and the total chain profit can be split arbitrarily. Thus, by committing to the proper PEM terms, the manufacturer guarantees that the profit of each party is greater than or equal to its level under no channel policy and no commitment.

6. Discussion
Our analysis provides an explanation for why we do not generally see subsidies for carrying inventory, analogous to price protection, in environments with static retail prices: Under static retail prices, the use of returns is sufficient to guarantee coordination and win-win. Our analysis provides an explanation for why price protection is used in addition to returns in declining retail price environments: Under declining retail prices, the use of returns is insufficient to guarantee win-win, but the addition of price protection restores this property.

We provide insight into the role of additional degrees of freedom provided by P, M, and E in achieving channel coordination and facilitating profit splitting between the two parties in a dynamic, two-period context. While in the context of our model it appears that one additional degree of freedom is necessary to assure coordination, we demonstrate that having an additional degree of freedom is not sufficient—the additional degree of freedom must be properly placed. Further, while a single additional degree of freedom may coordinate the channel, arbitrary profit splitting may be impossible. However, a second additional degree of freedom via price protection restores arbitrary profit splitting.

Motivated by the characteristics of technology-related industries, we have assumed that prices decrease over time. In other industries, prices may increase over time. For example, in the pharmaceutical industry, prices of several of the most commonly used drugs increased 10% to 20% over a recent two-year period (Tanouye and Connors 1999). Identifying channel policies that achieve coordination in increasing price and/or cost environments may be a promising area for future research.

Acknowledgments
The author gratefully acknowledges the helpful comments and suggestions of Chris Tang, the associate editor, and the reviewers. The author thanks Warren Hausman, Collin Kessinger, V. Padmanabhan, and Seujing Whang for their comments, and especially thanks Hau Lee for his thoughtful, valuable suggestions.

Appendix
All functions described as convex (concave) are strictly so. The proof of Lemma A1 (Lemma A2) is the same as the proof of Lemma 1 (Lemma 2) with straightforward simplifications.

**Lemma A1.** The optimal quantity achieved by ordering or disposal $S_2$ for the integrated channel in Period 2 is given by the following: if $x \leq \bar{S}_2$, then $S_2 = \bar{S}_2$; if $\bar{S}_2 < x < \bar{S}_2$, then $S_2 = x$; if $x \geq \bar{S}_2$, then $S_2 = \bar{S}_2$.

Define:

$$S_i := \Phi_i^{-1} \left( \frac{p_i - c_i}{p_i - c_i} \right).$$
\[ f := p_1 - c_1 - (p_1 - c_1) \Phi_1(S_1) + (p_2 - c_2) \Phi_1(S_2) - (p_2 - v_2) \int_{S_1}^{S_2} \Phi_1(S_2 - \xi_1) d\Phi_1(\xi_1), \]

and

\[ k := p_1 - w_1 - [p_1 - w_1 - (w_1 - w_2)\beta(x)] \Phi_1(T_1) + (p_2 - w_2) \times \Phi_1(T_2 - \xi_1) - (p_2 - v_2) \int_{\xi_1}^{T_2} \Phi_1(T_2 - \xi_1) d\Phi_1(\xi_1). \]

**Lemma A2.** The optimal policy for the integrated channel in Period 1 is to order up to \( S_1 \), which is given by the following: if \( S_0 \leq S_1 \), then \( S_1 = S_0 \); if \( S_0 < S_1 \leq S_2 \), then \( S_1 = S_0 \). In this case, the profit is

\[ p_1 - c_1 - (p_1 - c_1) \Phi_1(S_1) + (p_2 - c_2) \Phi_1(S_2) - (p_2 - v_2) \int_{S_1}^{S_2} \Phi_1(S_2 - \xi_1) d\Phi_1(\xi_1) = 0, \tag{A.1} \]

and it also holds that \( S_2 < S_1 < S_0 \) if \( S_0 > S_2 \) and \( f \leq 0 \), then \( S_1 \) satisfies (A.1) and it also holds that \( S_2 < S_1 < S_0 \) if \( S_0 > S_2 \) and \( f > 0 \), then \( S_1 \) satisfies

\[ p_1 - c_1 - (p_1 - c_1) \Phi_1(S_1) + (p_2 - c_2) \Phi_1(S_2 - \xi_1) - (p_2 - v_2) \int_{S_1}^{S_2} \Phi_1(S_2 - \xi_1) d\Phi_1(\xi_1) = 0, \tag{A.2} \]

and it also holds that \( S_2 < S_1 < S_0 \).

Define

\[ \Gamma_i(x) := \int_{S_i}^{\infty} \xi d\Phi_i(\xi); \quad i = 1, 2. \]

Applying Equations (A.1) and (A.2) to (I), the expected profit to the integrated channel is

\[ \pi = \begin{cases} 
(p_1 - c_1) \Gamma_1(S_0) + (p_1 - v_1) \Gamma_1(S_1) & \text{if } S_0 \leq S_2, \\
(p_1 - c_1) \Gamma_1(S_0) - (p_1 - c_1) \Gamma_1(S_1 - \xi_1) + (p_2 - v_2) A & \text{if } S_0 < S_1 < S_2, \\
(p_1 - c_1) \Gamma_1(S_0) & \text{or } S_0 \geq S_2 \text{ and } f \leq 0, \\
(p_1 - c_1) \Gamma_1(S_0) - (p_1 - c_1) \Gamma_1(S_1 - \xi_1) + (p_2 - v_2) B & \text{if } S_0 \geq S_2 \text{ and } f > 0,
\end{cases} \]

where

\[ A = \Gamma_1(S_0) + \int_{S_0}^{\infty} \xi_1 \Phi_1(S_0 - \xi_1) d\Phi_1(\xi_1) + \int_{S_0}^{\infty} \xi_1 \Phi_1(S_0 - \xi_1) d\Phi_1(\xi_1), \]

and

\[ B = \Gamma_1(S_0) + \int_{S_0}^{\infty} \xi_1 \Phi_1(S_0 - \xi_1) d\Phi_1(\xi_1) + \int_{S_0}^{\infty} \xi_1 \Phi_1(S_0 - \xi_1) d\Phi_1(\xi_1). \]

**Proof of Lemma 1.** We may rewrite \( R_1(x) = \max_{y \geq x} H_2(x, y) \) where

\[ H_2(x, y) = p_1[\mu_1 - E(\xi_1 - y)] + b_1 x - [w_1 - b_1 + (w_1 - w_2)\beta(x)](y - x) - [b_2 + (w_1 - w_2)\beta(y)] y + b_2 E(y - \xi_1)^{+}. \]

It is easy to verify that \( H_2(x, y) \) is concave in \( y \) for fixed \( x \).

Case 1. \( x \leq T_2 \). For \( y \geq x \),

\[ H_2(x, y) = p_1[\mu_1 - E(\xi_1 - y)] - w_2(y - x) + (w_1 - w_2)\beta y + b_2 E(y - \xi_1)^{+}. \]

The first-order condition is

\[ (\partial/\partial y)H_2(x, y) = p_1 - w_2 - (p_1 - b_2) \Phi_1(\xi_1) = 0, \]

and therefore \( T_2 \) maximizes \( H_2(x, y) \) for \( x \leq T_2 \).

Case 2. \( x \geq T_2 \). For \( y \leq x \),

\[ H_2(x, y) = p_1[\mu_1 - E(\xi_1 - y)] + b_1(x - y) + (w_2 - w_1)\beta y + b_2 E(y - \xi_1)^{+}. \]

The first-order condition is

\[ (\partial/\partial y)H_2(x, y) = p_1 - w_2 - (p_1 - b_2) \Phi_1(\xi_1) = 0, \]

and therefore \( T_2 \) maximizes \( H_2(x, y) \) for \( x \geq T_2 \).

Case 3. \( T_2 < x < T_2 \). Because \( x > T_2 \), \((\partial/\partial y)H_2(x, y) < 0\) for \( y > x \).

Because \( x < T_2 \), \((\partial/\partial y)H_2(x, y) > 0\) for \( y < x \). Thus, \( y = x \) maximizes \( H_2(x, y) \) for \( T_2 < x < T_2 \).

The following result is helpful in the proof of Lemma 2. Let

\[ H_i(y) = p_1[\mu_1 - E(\xi_1 - y)] - w_2 y + E(y - \xi_1)^{+}. \]

**Lemma A3.** \( H_i(y) \) is concave.

**Proof.** For \( y \leq T_2 \),

\[ H_i(y) = p_1[\mu_1 - E(\xi_1 - y)] - w_2 y + [w_1 + (w_1 - w_2)\beta E(y - \xi_1)]^{+} + p_2[\mu_1 - E(\xi_1 - x)]^{+} - w_1 x + b_2 E(\xi_1 - \xi_2)^{+}. \]

Thus,

\[ (d/dy)H_i(y) = p_1 - w_2 - (w_1 - w_2)\beta \Psi_1(y). \]

Because

\[ (d^2/dy^2)H_i(y) = -[p_1 - w_2 - (w_1 - w_2)\beta] \Psi_1(y), \]

\( H_i(y) \) is concave for \( y \leq T_2 \). For \( T_2 < y \leq T_3 \),

\[ H_i(y) = p_1[\mu_1 - E(\xi_1 - y)] + w_2 y + p_1 \beta y + \int_0^{\infty} ([w_1 + (w_1 - w_2)\beta(y - \xi_1)]^{+} + b_1 E(y - \xi_2)^{+}) d\Phi_1(\xi_1) \]

**Thus,**

\[ (d/dy)H_i(y) = p_1 - w_2 - (w_1 - w_2)\beta \Psi_1(y) + (p_2 - w_2) \Phi_1(y) - (p_2 - b_2) \int_0^{\infty} \Phi_1(y - \xi_1) d\Phi_1(\xi_1). \]

Because

\[ (d^2/dy^2)H_i(y) = -[p_1 - w_2 - (w_1 - w_2)\beta] \Psi_1(y) - (p_2 - b_2) \int_0^{\infty} \Phi_1(y - \xi_1) d\Phi_1(\xi_1) < 0, \]

\[ \]
H_1(y) is concave for \( T_2 < y \leq T_1 \). For \( y > T_1 \),

\[
H_1(y) = p_1[y - (w_1 - w_2)(y - \xi_1)] - w_1 y + p_2 \mu_2 \\
+ \int_0^{T_1} \left[ b_1(y - \xi_1 - T_1) + (w_1 - w_2) y T_2 \right] \\
- p_2 E(\xi_1 - T_1)^p + b_2 E(\xi_2 - \xi_1)^p d\Phi_1(\xi_1) \\
+ \int_{T_1}^{T_2} \left[ (w_1 - w_2) y T_1 \right] d\Phi_1(\xi_1) \\
+ \int_{T_2}^{T_1} \left[ (w_1 - w_2) y T_1 \right] d\Phi_1(\xi_1) \\
+ \left[ (w_1 - w_2) y T_1 \right] d\Phi_1(\xi_1) \\
+ b_2 E(\xi_2 - \xi_1)^p d\Phi_1(\xi_1).
\]

Thus,

\[
\frac{d}{dy} H_1(y) = p_1 - w_1 - (w_1 - w_2) \beta \Phi_1(y) + (p_2 - b_2) \Phi_1 \left( \frac{y - T_1}{y - T_2} \right) \\
- (p_2 - b_2) \int_{T_2}^{T_1} \left[ (w_1 - w_2) y T_1 \right] d\Phi_1(\xi_1) < 0.
\]

Because

\[
\frac{d^2}{dy^2} H_1(y) = - (p_2 - b_2) \int_{T_2}^{T_1} \Phi_2(y - \xi_1) d\Phi_1(\xi_1) < 0,
\]

\( H_1(y) \) is concave for \( y > T_1 \). It remains to show that \( H_1(y) \) is differentiable at \( y = T_2 \) and at \( y = T_1 \). Taking limits as \( y \) approaches \( T_2 \) from below and from above, one can easily confirm that

\[
\lim_{y \to T_2^-} \frac{d}{dy} H_1(y) = \lim_{y \to T_2^+} \frac{d}{dy} H_1(y) = \lim_{y \to T_1} \frac{d}{dy} H_1(y).
\]

Thus, \( H_1(y) \) is differentiable at \( y = T_2 \). By a similar argument, \( H_1(y) \) is differentiable at \( y = T_1 \). \( \square \)

**Proof of Theorem 2.** \( H_1(y) \) is concave (from Lemma A3).

**Case 1.** \( T_2 \leq T_3 \). For \( y \leq T_3 \), the first-order condition is

\[
p_1 - w_1 - (p_1 - w_1) \beta y (y - \xi_1) = 0.
\]

Because \( T_1 \) satisfies the first order condition, \( T_1 = T_0 \).

**Case 2.** \( T_2 < T_0 \leq T_3 \). For \( y < T_2 \), the first-order condition is

\[
p_1 - w_1 - (p_1 - w_1) \beta y (y - \xi_1) = 0.
\]

Because \( T_0 \) satisfies the first order condition, \( T_1 = T_0 \).

Note that \( (d/dy)H_1(y)_{y=T_2} > 0 \) and \( (d/dy)H_1(y)_{y=T_0} < 0 \). Hence, \( T_1 \) satisfies (4) and \( T_2 < T_1 < T_0 \).

**Case 3.** \( T_0 > T_2 \) and \( k \leq 0 \). For \( T_0 < y \leq T_2 \), recall that the first-order condition is given by (A.3) and that \( (d/dy)H_1(y)_{y=T_2} > 0 \). Because \( k = (d/dy)H_1(y)_{y=T_2} \), \( (d/dy)H_1(y)_{y=T_0} \leq 0 \). Hence, \( T_1 \) satisfies (4) and \( T_2 < T_1 \leq T_0 \).

**Case 4.** \( T_0 > T_2 \) and \( k > 0 \). For \( y > T_2 \), the first order condition is

\[
p_1 - w_1 - (p_1 - w_1) \beta y (y - \xi_1) = 0.
\]

Note that \( \int_{T_2}^{T_0} [p_1 - w_1 - (p_2 - b_2) \beta y (y - \xi_1)] d\Phi_1(\xi_1) < 0 \) and by (2) \( - (p_2 - b_2) \int_{T_2}^{T_0} \Phi_1(y - \xi_1) d\Phi_1(\xi_1) < 0 \). Thus, \( (d/dy)H_1(y)_{y=T_0} < 0 \). Hence, \( T_2 \) satisfies (5) and \( T_2 < T_1 < T_0 \).

**Proof of Theorem 1.** (a) It is easy to verify \( b_i(e) < w_0(e), c_i < w(e) < p_i \), and \( v_i < b_i(e) < w_0(e), i = 1, 2 \). Define \( \Omega(e) := (w_0(e), w_2(e), b_1(e), b_2(e)) \). Under \( \Omega(e) \), \( T_2 < T_1 \) and \( T_1 < T_0 \). If \( p_1 > p_2 \), then \( T_0 = T_0 \). If \( p_1 > p_2 \), then \( T_0 = T_0 \). If \( \partial T_0 / \partial e \geq 0 \), \( \lim_{e \to 0} T_0 = S_1 \), and \( \lim_{e \to - \infty} T_0 = S_0 \). Therefore, \( T_0 \in [S_1, S_0] \).

**Case 1.** \( S_0 = S_0 \). Because \( S_0 \leq \bar{S}_2, \bar{S}_1 = S_0 \) (by Lemma A2). Note that \( T_0 = S_0 \), thus \( T_0 \leq T_0 \leq T_1 = T_0 \) (by Lemma A2). Thus, \( T_1 = T_0 = S_0 = S_1 \).

**Case 2.** \( S_0 < S_0 \leq \bar{S}_2, \bar{S}_1 < S_0 \) (by Lemma A2). Thus, \( T_0 < T_0 \leq T_0 \leq T_0 \). Because \( T_0 < T_2 < T_0 \), \( T_1 \) satisfies (4) (by Lemma 2).

Define

\[
F(Z) := (p_1 - p_2 + e) \Phi_1(Z) + e \int_{Z_0}^{Z_1} \Phi_1(z - \xi_1) d\Phi_1(\xi_1) / \Phi_1(S_1) - \Phi_1(Z - \xi_1)
\]

for \( Z \geq S_0 \). Substituting \( \Omega(e) \) into (4), using the fact that \( S_0 \) satisfies (A.1), and some algebra yields \( F(T_1) = F(S_1) \). Because \( F(\cdot) \) is strictly increasing, \( T_1 = S_1 \).

**Case 3.** \( S_0 > \bar{S}_2 \) and \( f < 0 \). Because \( S_0 > \bar{S}_2 \) and \( f < 0 \), \( S_0 \) satisfies (A.1) and \( \bar{S}_2 < S_0 \leq S_0 \) (by Lemma A2). Thus, \( T_0 = S_0 < S_0 \leq T_0 \). Using the fact that \( S_0 \) satisfies (A.1), it is easy to verify that \( \partial k / \partial e \leq 0 \) and \( k \lim_{e \to 0} \leq 0 \); hence, \( k < 0 \). Consequently, either \( T_0 < T_0 \leq T_2 \) or \( T_0 > T_2 \) and \( k \leq 0 \); in either case, \( T_2 \) satisfies (4) (by Lemma 2). Thus, by the same argument in Case 2, \( T_2 = T_1 \).

**Case 4.** \( S_0 > \bar{S}_2 \) and \( f > 0 \). Because \( S_0 > \bar{S}_2 \) and \( f > 0 \), \( S_0 \) satisfies (A.2) and \( \bar{S}_2 < S_0 < S_0 \) (by Lemma A2). Thus, \( T_0 = S_0 < S_0 \leq T_0 \). Using the fact that \( S_0 \) satisfies (A.2), it is easy to verify that \( \partial k / \partial e > 0 \) and \( k \lim_{e \to 0} \geq 0 \); hence, \( k > 0 \). Because \( T_0 > T_2 \) and \( k > 0 \), \( T_1 \) satisfies (5) (by Lemma 2).

Define

\[
G(Z) := (p_1 - p_2 + e) \Phi_1(Z) + e \int_{Z_0}^{Z_1} \Phi_1(z - \xi_1) d\Phi_1(\xi_1) + \Phi_1(Z - S_0) / \Phi_1(S_1) - \Phi_1(Z - S_0)
\]
for $Z \geq \overline{S}_2$. Substituting $\Omega(\epsilon)$ into (5), using the fact that $S_i$ satisfies (A.2), and some algebra yields $G(T_i) = G(S_i)$. Because $G(\cdot)$ is strictly increasing, $T_i = S_i$. Thus, $\Omega(\epsilon)$ achieves channel coordination.

(b) It is straightforward to show

$$
\hat{m}(\epsilon) = \begin{cases} 
[w_2(\epsilon) - \epsilon c_2][\Gamma(S_i) + \Gamma(S_2)] 
\times (p_1 - v_1)/(p_2 - c_2) & \text{if } S_i \leq \overline{S}_2; \\
[w_2(\epsilon) - \epsilon c_2][\Gamma_i(S_i) - \Gamma_i(S_1 - S_2)] + A(p_2 - c_2)/(p_2 - c_1) & \text{if } S_2 < S_i < \overline{S}_2, \\
[w_2(\epsilon) - \epsilon c_2][\Gamma_i(S_1 - S_2)] + B(p_2 - c_2)/(p_2 - c_1) & \text{if } S_i \geq \overline{S}_2 \\
0 & \text{if } S_i < S_1.
\end{cases}
$$

Plugging in $w_2(\epsilon)$ yields the result. To see this consider the case in which $S_i \leq \overline{S}_2$. Hence, $\pi = (p_1 - c_2)\Gamma_i(S_i) + (p_2 - c_1)\Gamma_i(S_2)$ and $\lambda = (p_1 - c_2)(S_1 - S_2)$; plugging in $w_2(\epsilon)$ yields $\hat{m}(\epsilon) = [1 - \epsilon/(p_2 - c_1)][(p_2 - c_2)\Gamma_i(S_i) + (p_2 - c_1)\Gamma_i(S_2)]$. The other cases are proved similarly. The retailer profit is immediate from $\tilde{T}(\epsilon) = \pi - \hat{m}(\epsilon)$.

(c) The result is immediate from (b).

Proof of Theorem 2. (a) It is easy to verify $b_1(\epsilon) < w_2(\epsilon) + [w_1(\epsilon) - w_2(\epsilon)]\beta(\epsilon)$, $w_2(\epsilon) = w_1(\epsilon)$, $\beta(\epsilon) \in [0, 1]$, $c_i < w_i(\epsilon) = p_i$, and $r_1 < b_1(\epsilon) < w_1(\epsilon)$; $i = 1, 2$. Define $\Psi(\epsilon) := (w_1(\epsilon), w_2(\epsilon), \beta(\epsilon), b_1(\epsilon), b_2(\epsilon))$. Under $\Psi(\epsilon)$, $T_0 = S_0$, $T_1 = S_1$, and $T_2 = \overline{S}_2$. It is easy to verify that $k = k(\epsilon)(\epsilon - c_2)$, hence, $k$ and $f$ have the same sign.

Case 1. $S_0 \leq \overline{S}_2$. Because $S_1 \leq \overline{S}_2$, $S_2 = S_1$ (by Lemma A2). Thus, $T_0 = S_0 \leq \overline{S}_2$. Because $T_1 = T_2 = T_0$ (by (5)). Thus, $T_1 = T_2 = S_1 = S_2$.

Case 2. $\overline{S}_2 < S_0 \leq \overline{S}_2$. Because $S_1 = \overline{S}_2$, $S_2 = \overline{S}_2$, $S_0$ satisfies (A.1) (by Lemma A2). Because $T_1 > T_2$, $T_1 = T_0$ (by (4)).

Substituting $\Psi(\epsilon)$ into (4) and some algebra yields

$$
p_1 - c_1 - (p_1 - c_2)\Phi(T_1) + (p_2 - c_2)\Phi(T_1 - S_2) \\
- (p_2 - v_1) \int_{T_1 - S_2}^{T_1} \Phi(T_1 - \xi)d\Phi(\xi) = 0. \quad (A.4)
$$

Because (A.1) and (A.4) coincide, $T_1 = S_1$.

Case 3. $S_0 > \overline{S}_2$ and $f \leq 0$. Because $S_0 > \overline{S}_2$, $f \leq 0$, $S_1$ satisfies (A.1) (by Lemma A2). Because $T_0 > T_2$ and $k \leq 0$, $T_1$ satisfies (4) (by Lemma 2). Thus, by the same argument in Case 2, $T_1 = S_1$.

Case 4. $S_0 > \overline{S}_2$ and $f > 0$. Because $S_0 > \overline{S}_2$ and $f > 0$, $S_1$ satisfies (A.2) (by Lemma A2). Because $T_0 > T_2$ and $k \geq 0$, $T_1$ satisfies (5) (by Lemma 2). Substituting $\Psi(\epsilon)$ into (5) and some algebra yields

$$
p_1 - c_1 - (p_1 - c_2)\Phi(T_1) + (p_2 - c_2)\Phi(T_1 - S_2) - (p_2 - v_1)\Phi(T_1 - S_2) \\
- (p_2 - v_1) \int_{T_1 - S_2}^{T_1} \Phi(T_1 - \xi)d\Phi(\xi) = 0. \quad (A.5)
$$

Because (A.2) and (A.5) coincide, $T_1 = S_1$. Thus, $\Psi(\epsilon)$ achieves channel coordination.

(b) It is straightforward to show

$$
\hat{m}(\epsilon) = \begin{cases} 
[w_2(\epsilon) - \epsilon c_2][\Gamma_i(S_1 - S_2)] + A(p_2 - c_2)/(p_2 - c_1) & \text{if } S_i < S_0 \leq \overline{S}_2, \\
[w_2(\epsilon) - \epsilon c_2][\Gamma_i(S_1 - S_2)] + B(p_2 - c_2)/(p_2 - c_1) & \text{if } S_i < S_0 \leq \overline{S}_2 \\
0 & \text{if } S_i < S_0 \leq \overline{S}_2.
\end{cases}
$$

Plugging in $w_2(\epsilon)$ yields the result. The retailer profit is immediate from $\tilde{T}(\epsilon) = \pi - \hat{m}(\epsilon)$.

(c) The result is immediate from (b).

References


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Accepted by Chris Tang; received October 1999. This paper was with the author 5 months for 3 revisions.