RESOURCE MANAGEMENT UNDER UNCERTAINTY: 
SOME INFORMATIONAL ISSUES 
by 
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Economic decisions made without sufficient information is the norm rather than the exception. In this paper I will discuss five related topics about information and resource systems. The paper begins with a discussion of how to live with uncertainty, which is the classical approach that many others on this program will speak about, and a discussion of how to make others live with your uncertainty, which is a newer approach. Since the classical answer is to live with uncertainty, I will then discuss with how much uncertainty a forest planner must live. The extended example that I use is uncertainty concerning an important demand-shift variable, though the spotted owl and other forest-planning horrors do make a brief appearance in this section. Given that there is little information, the next section provides an example where it matters very little, at least in a social sense. The example is based on the government's unexpected purchase of the Redwood National Park. A very different strand of the information problem is estimation. The equations used for predictions are typically estimated assuming that the agents possess information that the econometrician does not. The succeeding section examines how lack of information by the decision-making agents changes the econometrician's estimation. In the same vein, the last substantive section shows how economic information can be used to infer information about the natural world.

Two Approaches to Living with Very Limited Information

There are two approaches to making decisions under uncertainty. The classic approach is to make the best decision possible, while the newer approach is to force an agent with improved information to make the decision. Indeed, both of these approaches have such a rich literature that I can do more than mention a few of the important issues and point out where they have been used for resource or environmental problems.

The classical approach to decision making under uncertainty is to write the decision problem as an explicit stochastic dynamic-programming problem. Some objective function, often discounted utility of profits, is maximized, subject to a set of constraints that depends upon random variables. The uncertainty is given and immutable. Many of the other participants in this conference will be speaking to these issues, and Lund and Oksendal give an excellent modern view of much of this field. There is also a now long history of solving actual agriculture and resource problems, and I shall mention a few that seem particularly relevant.

Until very recently, the class of such problems that computers could reasonably solve was quite small and, for these purely practical reasons, most of the literature is problems that are linear in the constraints and
quadratic in the objective function. Rausser and Hochman's volume is a collection of studies (e.g., marketing boards and pesticides) that apply dynamic stochastic programming to real problems in environment resources and agriculture. Many of these studies use the linear-quadratic framework, and the volume explains that framework quite well.

Dixon and Howitt provide a full-scale, linear-quadratic stochastic model for the Stanislaus National Forest. The computational burden was immense. Therefore, in their model, cutting, thinning, planting, etc., are adjusted to keep the actual harvest path of the forest close to an initially completed plan. Models of this sort are also good for finding the value of information as the algebra of the maximization provides an explicit estimate of these costs. The costs of uncertainty in this model are very large: Stock uncertainty costs 60 percent of the total costs of running the forest—a very different result from the simple case that I will outline below.

Even the relatively restrictive linear-quadratic models that dominate the empirical literature make the distinction between open- and closed-loop controls. In an open loop, the policy, say, harvest, is carried out even when later information, say, a blowdown, makes carrying out that policy undesirable. In a closed-loop control, the policy is set using the current information set, which is to say that policy is constantly revised. (More subtly, policy is made with the assumption that it can be revised.) One way to compute the value of information is to find the difference between the value of an open- and closed-loop control.

A completely different way to view natural resource problems is given by Brannan and Schwartz. Their application is to copper mining in Chile, and the question is how much to mine—mothball the mine or abandon the mine. They use computers to approximate the stochastic calculus of their problem, which is a very different approach to solution from the discrete approximations described above. They also force capital-market equilibrium (the rate of return on the mine when operated optimally is the same as the rate on a sure asset), where the other models mentioned would allow a greater or lesser rate of return. For these reasons, the approach is out of the truly classic mold of maximize subject to constraint.

A very different view of decision making under uncertainty comes from distinguishing which agents are certain (more or less) and which are not. While a forest planner is the person who is most informed about a forest, the state public utilities commission is not the group that is most informed about a water utility and the EPA is not the group that knows most about pollution. Water utilities know the most about themselves, and firms know the most about their own pollution. From a regulator’s point of view, getting these entities to comply with his/her wishes could be a classic problem of decision making under uncertainty. There is another possibility; the regulator could structure the incentive system to make the firm or utility both reveal its private information and act close to optimally. Baron and Meyerson solved this problem for a regulated monopolist. In a yet unpublished Ph.D. dissertation at Berkeley, Ellis has applied these ideas to pollution more generally. Wolak uses this framework for the regulation of a water utility. In that case, the utilities cost function is unknown to the regulator. By announcing a schedule of hookup charges and flow charges,
the regulator can enable the utilities to act near optimally. These are all cases where appropriate regulation can cause the production of further information. The uncertainty does not just have to be accepted.

**How Much Information is Actually Available?**

It is easy to see that the long-time horizons of natural resource problems lead to great uncertainties. No one expects accurate prediction of price or growth into the next millennium. The problems in the case of resources are much worse. Basic information on the state of resources today is often lacking. Of course, the stock of fish in the sea and oil under the rock and sand is hard to measure—a subject which will be later discussed. There is little known about prices of resources and quantities of trees as well.

Hotelling wrote the pure theory of the economics of exhaustible resources in 1931; approximately three decades later, Barnett and Morse set out to test Hotelling's theory. Simplified, Hotelling stated that natural resource prices should grow at the rate of interest. Barnett and Morse found that few, if any, of them did that. The problem with naive tests of Hotelling's hypothesis is that the correct price is rarely observed. Hotelling's theory refers to fish before they are caught, coal before it is mined, and timber before it is cut. Only timber shows an increasing price trend, and only timber is sold before it is harvested. (Of course, economists have found a myriad of ways around this lack of information—Slade hypothesizes technical progress, Stollery estimates cost functions, and Miller and Upton found data on costs, etc.). Despite these innovative methods, the basic problem remains. Even current resource, *in situ*, prices are not known; only product prices are recorded. The situation for prices in the future is, of course, far worse.

In addition, little is known regarding most of the non-economic inputs needed for resource planning (or profit maximizing). Returning to my favorite example, forestry, there are over 150 National Forests in the United States, and Congress has mandated that each of them have a long-run plan. The plans are made by interdisciplinary teams using a large linear-programming model called FORPLAN (see Johnson and Scheurman). The key biological elements in this program are stock and growth. Growth is very site dependent, and even current growth is not perfectly known. The inputs to calculating growth are measured radial growth, calculated from a sample of tree cores and some notion of how trees should appear. The cores are expensive and the notion of how trees should appear is controversial. For instance, estimating growth curves with random coefficients rather than additive errors makes a big difference in one's projections (Biging). Stock is also hard to pin down. The basic data are samples of 1/10 acre plots, supplemented as the individual forest manager deems necessary. Even if the sampling were a perfect indication of the volume and condition of the forest, which it most assuredly is not, all of the forest would not be available for harvest. The actual land area depends upon the location of Indian burial grounds, the need for stream buffers, set-asides for owl or woodpecker habitat, and so on. These are not entirely known when the plan is produced and will not be known until the ground is very carefully surveyed. Hrubes found that a large fraction of the
Six Rivers National Forest would be unavailable for cutting even though the plans in the beginning stated otherwise. The recent multi-million acre surprise, caused by the listing of the spotted owl as an endangered species, is the most dramatic working out of these uncertainties.

In the remainder of this section, I will discuss the precision of conditional prediction of the price of a particular type of stumpage in California. The predictions are said to be conditional, because they depend upon predictions of an important variable, housing starts, which is widely recognized as being an important economic indicator. In carrying out this exercise, I am mimicking the type of analysis actually done by investment houses and timber companies.

The starting point for this exercise in quantifying uncertainty is estimates of housing starts. Estimates of housing starts for one quarter through eight quarters ahead were made by Chase Econometrics and Data Resources Incorporated (DRI)—two large firms specializing in the prediction of macroeconomic variables. These predictions were made every quarter so that, for each firm, there are predictions made in each quarter from 1971:4 to 1984:1. These predictions were compared to the actual number of private housing starts, and the difference between the predictions and actual outcomes (forecast residual) was calculated. Summary statistics were also computed.

The forecasts of both firms become less accurate as the forecast period increases. The mean square error (MSE) for DRI, for instance, increases by 20-fold between one quarter and eight quarters ahead forecast. One quarter out, DRI's forecast has a root MSE of 128,000 starts and an average absolute error of 100,300 starts. Since housing starts run in the low millions (with a mean of approximately 1.5 million), these errors are on the order of 5 percent to 10 percent of the predicted values. The bias (which is the amount the forecasts and, on average, exceeds the actual values) is also small: -20,000 starts for DRI. The numbers are similar for Chase Econometrics. Table I presents the numbers for both companies and all forecast lengths.

Table I. Accuracy of Housing Start Forecasts

Fourth Quarter, 1971, to Fourth Quarter, 1983

<table>
<thead>
<tr>
<th>Quarters ahead</th>
<th>MSE</th>
<th>Bias</th>
<th>Variance</th>
<th>Av. abs. res.</th>
</tr>
</thead>
<tbody>
<tr>
<td>one</td>
<td>.02403</td>
<td>-.0331</td>
<td>.02341</td>
<td>.1259</td>
</tr>
<tr>
<td>four</td>
<td>.1935</td>
<td>.04820</td>
<td>.1955</td>
<td>.3647</td>
</tr>
<tr>
<td>eight</td>
<td>.3487</td>
<td>.2127</td>
<td>.3108</td>
<td>.4975</td>
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</table>
Table I—continued.

Fourth Quarter, 1971, to Fourth Quarter, 1983

DRI

<table>
<thead>
<tr>
<th>Quarters ahead</th>
<th>MSE</th>
<th>Bias</th>
<th>Variance</th>
<th>Av. abs. res.</th>
</tr>
</thead>
<tbody>
<tr>
<td>one</td>
<td>.01651</td>
<td>-.02095</td>
<td>.01641</td>
<td>.1003</td>
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<tr>
<td>four</td>
<td>.1476</td>
<td>.1380</td>
<td>.1314</td>
<td>.3070</td>
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<tr>
<td>eight</td>
<td>.3402</td>
<td>.4105</td>
<td>.1759</td>
<td>.4888</td>
</tr>
</tbody>
</table>

Note: Av. Abs. Res. is the average absolute residual.

Source: Computed.

Since the four quarter forward results are intermediate between the one quarter and eight quarter results, it is sufficient to discuss only the eight quarter results. For DRI, the root MSE is 583,000 starts. Compared to the usual number of starts, this is between approximately one-quarter and one-half, a margin of error far too large for this to be a useful forecast. The forecast, however, is biased. On average, it overstates starts by 410,000 starts. Thus, taking DRI's eight quarter out prediction and subtracting 289,000 from it will give an unbiased estimator. The MSE of this new predictor (DRI - 289,000) is listed under "variance," and it is .1759. Its root MSE (square root of .1763) is 420,000 starts, which is still large. For Chase Econometrics, the bias is small and the root MSE is much the same.

These results should be disturbing to a forest planner for two reasons. The large bias, particularly in the DRI forecast, means that a planner would want to subtract an enormous number from the DRI starts. A very good year would be 2 million starts. A DRI forecast of 2 million starts, adjusted downwards by the full bias, would give 1.6 million starts—an entirely average year. A planner would have to wonder whether he/she should adjust the numbers (as well as company's log deck!) or whether DRI had realized its mistake and had already adjusted the numbers. It would be very troubling to work from these numbers. The second reason to be disturbed is less subtle. Industry scuttlebut is that DRI's model predicted closer to the truth than the published forecast. The numbers were adjusted upwards based upon personal beliefs. My own view is that this did happen, and it happened because it is very difficult to sell a client very bad news. And the housing start news is often very bad.

For the purposes of this paper, we need a rough estimate of the uncertainty of the log of housing starts. Using the usual formulas to translate from normal to log-normal gives a mean log-housing start of .312 with a variance of 0.188. While these numbers are not in themselves comforting as predictions, their consequences in actual use is much worse.

Returning to the case of the Redwood, the two variables that a planner might want to predict are the market price and market volume. Presumably, the planner would be engaged in running a program such as
FORPLAN to determine the amount of material available for cutting and would also be asked to help determine how large a log deck the company should hold for the coming building seasons. These planning exercises depend upon predicted prices and to some degree, predicted quantity.

Standard reduced-form regression techniques can be used to produce equations that predict volume and price as functions of demand conditions (primarily housing starts) and supply conditions (primarily remaining inventory). The equations specify the variables of interest as functions of currently observable variables. Given predictions for the observed variables, such as starts and inventory, one has a prediction for price and quantity.

The equations estimated for the Redwood model are fully described in Berck and Bentley. Table II provides a summary of the results.

**Table II**

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Real price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable name</td>
<td>Coefficient</td>
</tr>
<tr>
<td>LINV</td>
<td>-1.60</td>
</tr>
<tr>
<td>LRTBIL</td>
<td>0.13</td>
</tr>
<tr>
<td>LHS</td>
<td>0.48</td>
</tr>
<tr>
<td>LADMAINR</td>
<td>0.25</td>
</tr>
<tr>
<td>LGSTOCK</td>
<td>1.27</td>
</tr>
<tr>
<td>CONSTANT</td>
<td>-11.42</td>
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<tr>
<td>R² = .91</td>
<td>d.w. = 1.8</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Dependent variable (billion bd ft)</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable name</td>
<td>Coefficient</td>
</tr>
<tr>
<td>LINV</td>
<td>0.48</td>
</tr>
<tr>
<td>LRTBIL</td>
<td>0.02</td>
</tr>
<tr>
<td>LHS</td>
<td>0.27</td>
</tr>
<tr>
<td>LADMAINR</td>
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<tr>
<td>LGSTOCK</td>
<td>-0.41</td>
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<tr>
<td>CONSTANT</td>
<td>0.71</td>
</tr>
<tr>
<td>R² = .82</td>
<td>d.w. = 2.3</td>
</tr>
</tbody>
</table>
Table II—continued.

Note:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>LINV</td>
<td>Log of remaining old-growth stumpage, inventory.</td>
</tr>
<tr>
<td>LRTBIL</td>
<td>Log of the t-bill rate</td>
</tr>
<tr>
<td>LHS</td>
<td>Log of lagged housing starts</td>
</tr>
<tr>
<td>LADMAINR</td>
<td>Log of additions and maintenance expense</td>
</tr>
<tr>
<td>LGSTOCK</td>
<td>Log of Douglas fir growing stock</td>
</tr>
<tr>
<td>CONSTANT</td>
<td>A constant term</td>
</tr>
</tbody>
</table>

These equations appear fairly good from an estimator's viewpoint. Inventory and starts which should matter do matter, and they have good t-statistics. The Durbin-Watson statistic does not indicate autocorrelation—a frequent outcome in economic time series. The $R^2$ shows that most of the variance in price is accounted for by the equation. On these econometric grounds, the equation appears to be fairly good.

From the point of view of forecast error, the situation is not so good. Consider just the problem of predicting price for a year near the sample mean. The sample mean price is $60/th (per thousand board-foot lumber tally)—real 1973 dollars. Using the usual formulas for forecast error—accounting for the error term of the equation and the uncertainty in the parameter estimates—and converting from log-normal to normal gives a standard deviation for the prediction of $13/th. Including the uncertainty in the prediction of starts gives a standard deviation of $18/th. Assuming that additions and maintenance expense is no better predicted than starts, and has the same temporal pattern, gives a standard deviation of $24/th. The most realistic case (though even this understates the real variability because the other variables are not perfectly forecast!) is the last one: The standard deviation of a price forecast is 40 percent of that forecast.

Forecasting quantity gives slightly better results. The quantity at the same mean is 961 million board feet. Assuming starts and additions and maintenance are known quantities, the standard deviation in prediction is 60 million board feet. With uncertainty in starts accounted for, it is 130 million board feet and, with uncertainty in starts and additions and maintenance accounted for, it is 311 million board feet which is 32 percent of the predicted quantity.

It is hard to imagine that a planner would be satisfied with this information. In essence, very little is known about the market two years ahead. Decision making under uncertainty appears to be more of a game of dice and less of an exercise of planning under conditions of uncertainty.

The Value of Information: An Example

Since even very basic inputs to the planning process are quite difficult to forecast, we now turn to the consequences of making a large mistake in planning. The forests of the Pacific Northwest and Northern California
were largely old-growth forests until quite recently. The old-growth habitat was very much taken for granted; preservation efforts centered on the world's tallest and fattest trees, not on a collection of reclusive small animals, such as spotted owls and marbled murrelets. There was even a belief that the old-growth canopy denied light to the forest floor and created a biological desert.

This indifference to the old-growth habitat changed quite dramatically throughout the late 1960s and 1970s. In 1968, Redwood National Park, the first sizeable park in quite some time, was created. In 1978 close to one billion dollars was spent to add to that park, primarily to protect the world's tallest tree. A decade later, the much larger fight to preserve old-growth in the Pacific Northwest was joined over the spotted owl. American law protects species on the verge of extinction, so the method chosen to protect the old-growth habitat was to list the spotted owl as an endangered species. Since it is criminal to disturb the owl, once listed, the owl quickly made the logger an endangered species. As of this writing, about 8 million acres of land are to be set aside for owl habitat. The effect of this and other owl actions is probably on the order of 10,000 jobs lost (though the defenders of the loggers use a number closer to 30,000). There are many angry wood's workers, and the issue is still being worked out in the political arena.

From the point of view of an economist, the takings for the Redwood Park and sequestering of timber for owl preservation are a laboratory in the economics of information. Price, employment, and all other variables of interest depend upon the harvestable wood stock. For many reasons, that stock is not known exactly. Measurement of standing timber is notoriously complicated and costly. More importantly, a considerable amount of the timber that would have seemed to be available was ultimately reserved for a preservation use. These reservations were not known at the beginning of the harvesting cycle, or even in 1953 just after the Korean Conflict, when our data begin. The decision to reserve these acreages was surprising to the economic actors. It was new information.

Using the rough facts of the Redwood industry, it is possible to find the value of the information. What would it have been worth to know that the park was going to be created back in 1953 rather than being surprised by it 25 years later?

Old-growth redwood is a nonrenewable natural resource. Its value is derived from the red color and close set ring pattern, neither of which are well reproduced in the current crop of much younger second-growth trees. In 1953, there were about 34.1 billion board feet (lumber tally) of old growth. The two takings of the redwood park were 3.1 billion board feet and about 7 billion board feet were left in private hands as of 1978. Thus, nearly 10 percent of the post-war stock disappeared from commercial use in an unexpected fashion.

The theory of exhaustible natural resources says that (in expectation) prices increase at the rate of interest. By regressing real prices on their lags, one can recover the rate of increase for redwood, and it is about 7 percent. This gives a very simple model of price expectations.
Regression techniques also allow the recovery of an estimate of the price elasticity of demand for redwood stumpage. A good estimate is -.6. Indeed there are more complicated, nonconstant-elasticity forms that one can estimate, but this crude estimate will suffice for this purpose.

Using an early 1950s price of $30/th board feet, and projecting future prices by the 7 percent increase rule, gives an approximation to the price path expected for all relevant time. Using the demand elasticity (and intercept) one can then calculate the quantity purchased in each year. Calculating those purchases shows that the stock would have been exhausted in approximately 1992. In actual fact, there is still some old growth uncut, but that is a consequence of a strategic decision made by the Pacific Lumber Co. The price at the putative time of exhaustion would have been $1,000/th (real 1973 dollars), which we take to be the choke price, the price at which other decorative materials would take over the whole of the market.

If the park takings were perfectly anticipated in 1953, that is, if there were perfect information about future conservation imperatives, the price of redwood would have been $39 and the present value of consumer surplus plus profits, a reasonable indicator of welfare, would have been 7.723 billion.

A reasonable approximation of what actually occurred is that the stock was believed to include the park volume in 1953 but, in 1973 (midway between the two takes, an approximation), it was suddenly reduced by 3.1 billion board feet. In this scenario, prices would have started at $34, increased at the rate of interest, and then jumped in 1973. In this, suboptimal scenario, the welfare would have been 7.713 (only slightly different) taken from the vantage of 1953. There are two reasons for this. The first is that this plan, derived from misinformation, is only different from the perfectly anticipated after 20 years. Discounting makes these later losses quite small. The second reason is that in an optimal plan the marginal value (in terms of surplus) of an additional unit of stock is the same in all periods. Thus, to a first-order taylor approximation, there is no penalty to the wrong plan; the penalty is strictly a second-order effect. The plan with less information also has a distinct shift in welfare: Although total welfare is near identical, post-1973, welfare drops from 1.33 billion to 1.125 billion.

To get a feel for the magnitudes of these effects, consider a plan that begins in 1953 with the misapprehension that 10 billion extra board feet are available. Again, the information as to the true stock arrives in 1973. The welfare loss is now $200 million, from the 1953 vantage point, but one-half billion dollars from the 1973 perspective.

Undoubtedly, the numerical results presented here are sensitive to many things, not the least of which is the high real-interest rate and the constant-elasticity specification. Because the first-order effects are always nil, by the design of the model (an exhaustible-resource model maximizes surplus plus profits), I suspect that the costs to missing information are not nearly as great as the distributional consequences. The workers, consumers, and owners in the late period, when the new information about
the value of conservation has become available, lose quite considerably, while the earlier consumers gain.

How Does Lack of Information Change Estimation?

Most of the numbers generated above was derived from regression estimates. An underlying assumption in these regressions turns out to be that the lumber producers have decent information regarding future prices. In light of the earlier discussion on the quality of estimates, that does not seem reasonable. In this section I will discuss the consequences of poor information for the process of estimation.

Lack of information, which is to say uncertainty, combined with an ability to act after information is known and a need to act before, changes the structure of even a very simple estimation procedure. In the case of a forest in the American West, there are time-consuming bureaucratic hurdles to be overcome before timber may be harvested. In the private sector in California, one must file a timber harvest plan with the California Department of Forestry and have it approved. In the public sector, the forest must be cruised (surveyed for trees) and the stumpage put out to public bid. In all ownerships roads must be built or improved. Rain, mud, fire danger, and snow also create strong seasonal incentives for logging. After logging, there is milling and drying, which are also time consuming. For the better grades of redwood, the air-drying, itself, can take about two years. The sum of all of these processes is a one- to three-year time scale for the provision of lumber. Stumpage owners must commit to cutting their timber well before the state of the market is known. Other aspects of the process, such as shipping, are done after the state of the market is known. The ability to act at two separate times is a long-run short-run model, taught to nearly every entering freshman in economics. Its consequences for estimation are not as well known.

To see these consequences, it is best to abstract the situation somewhat. Consider a resource whose shadow value, *in situ*, is known and nearly constant from year to year. The shadow value should be nearly constant because it depends upon long-term demand conditions—the forecast of which (an average of 1.5 million starts) changes very little with current information. The first input in the production process is chosen when only the distribution of the demand-shift variable, housing starts, is known. It is some amalgam of road building, filing plans, etc. The second input is chosen after the number of starts are known. It is some amalgam of milling and drying and such and cutting from areas that are already permitted and roaded.

If all of the information in this model were known at the beginning, the firms would know the demand and supply curves for lumber. They would equate them and find price. Lumber price would be a function of what shifts those curves—the price of the two inputs and housing starts. An econometrician would have a simple job: regress lumber price on the input prices and housing starts. This type of regression is called a reduced form, and it can be estimated by least squares, simple curve fitting.
When the situation is that the information on starts, for example, is not available, the firms have a much more difficult procedure. As described above, they can set supply equal to demand and solve for price for any level of starts. The uncertainty in the starts then induces an uncertainty in price. The firms must then make a two-stage decision, given that uncertain, and later certain, price. Perhaps a mathematical example will make this more clear, at least to the economists.

The steps to create the example start with a careful consideration of the problem of a representative firm. In the second period, the price of output, \( P \), will be known. The second period problem for the firm is the ordinary one of maximizing profits, given whatever first-period choice, \( x_1 \), it happened to make. After a little algebra, the supply curve of such a firm in the second period is derived. By setting that supply curve equal to demand, one can find the distribution of price given \( x_1 \). As mentioned above, it is the uncertainty in housing starts that induces the distribution in price. The last step is to have the firms maximize expected profits, given the distribution of future prices, and to be sure that the choice of \( x_1 \) accomplishes that maximization.

Let the production function for lumber be Cobb-Douglas with decreasing returns to scale:

\[
(1) \quad y = A x_1^{a_1} x_2^{a_2},
\]

where \( x \) is input and \( y \) is output.

The restricted profit function, profits given \( x_1 \), is

\[
(2) \quad \Pi(x_1, P) = \max_{x_2} (P A x_1^{a_1} x_2^{a_2} - w_2 x_2)
\]

where \( w_i \) is the factor price for \( x_i \) and \( P \) is output price. Restricted profits are the most money that can be made given the prices and given that the level of \( x_1 \) has already been chosen. Here the amount of \( x_2 \) is chosen after the price \( P \) is known.

On taking the derivative and solving for \( x_2 \),

\[
(3) \quad x_2 = \left[ \left( \frac{w_2}{a_2} \right) (PA)^{-a_1} x_1^{-a_1} \right]^{1/a_2 - 1}.
\]

By substituting for \( x_2 \) in (2), one derives another expression for restricted profits—this one in terms of the second-period price. Therefore, the restricted profit function is
Equation (4) gives profits as a function of the uncertain price, $P$, and the first-period choice of $x_1$. The first-period choice of $x_1$ is made to maximize expected profits:

$$\max_{x_1} E\Pi(x_1, P) - w_1 x_1.$$ 

Solving the first-order condition gives

$$x_1^* = \left(\frac{w_1}{a_1}\right) \left(\frac{w_2}{a_2}\right)^{-a_2-a_1} E \left[p^{1-a_2}\right]^{-1} \left[A^{1-a_2}\right]^{-1},$$

where $s = a_1 + a_2$. Substituting back into the restricted profit function and subtracting the factor cost for $x_1$,

$$\Pi(P, EP) = \left[\frac{w_1}{a_1} \right]^{-a_1} \left[\frac{w_2}{a_2} \right]^{-a_2} \left[\frac{1}{A^{1-s}}\right].$$

(When $P = EP$, this reduces to

$$\Pi(P) = (1-s)(PA)^{-a_1-a_2} \left[\frac{w_1}{a_1} \right]^{-a_1} \left[\frac{w_2}{a_2} \right]^{-a_2},$$

the familiar form for the Cobb-Douglas profit function.) The short-run supply curve is
Let demand be log-linear and let the demand-shift variable, $h$, be lognormally distributed; the short-run equilibrium is given by

$$
(8) \quad \frac{\partial \Pi(x^*_t, p)}{\partial p} = p^{1-a_2} E \left( \frac{1}{p^{1-a_2}} \right)^{\frac{a_1}{1-s}} K
$$

where

$$
K = \begin{pmatrix}
\frac{v_1}{a_1}^-a_1 & \frac{v_2}{a_2}^-a_2 & \frac{1}{A^{1-s}}
\end{pmatrix}
$$

Solving this for $\ln p^{1-a_2}$,

$$
(9) \quad Gp^{-\gamma} h^\beta = p^{1-a_2} E \left( \frac{1}{p^{1-a_2}} \right)^{\frac{a_1}{1-s}} K,
$$

where $G$, $\gamma$, and $\beta$ are positive constants.

Solving this for $\ln p^{1-a_2}$,

$$
(10) \quad \ln p^{1-a_2} = \frac{1}{a_2 + \gamma - \gamma a_2} \left( \ln G / K + \beta \ln h - \frac{a_1}{1-s} \ln E \left( \frac{1}{p^{1-a_2}} \right) \right).
$$

Since $\ln h \sim N(\mu, \sigma^2)$, $\ln \left( \frac{1}{p^{1-a_2}} \right)$ is also normal. Let $\delta = a_2 + \gamma - \gamma a_2$, which is positive because $a_2 < 1$.

$$
(11) \quad \ln p^{1-a_2} \sim N \left( \frac{\ln G / K + \beta \mu - \frac{a_1}{1-s} \ln E \left( \frac{1}{p^{1-a_2}} \right)}{\delta}, \frac{\beta^2 \sigma^2}{\delta^2} \right).
$$

From the usual formulas, $p^{1-a_2}$ is normally distributed with mean
Solving,

\[
E P^{1-a_2} = \exp \left[ \frac{1}{\delta} \left( \ln \left( \frac{G}{K} \right) + \beta \mu - \frac{a_1}{(1-s)} \ln E \left( \frac{1}{p^{1-a_2}} \right) \right) + \frac{\beta^2 \sigma^2}{2 \delta^2} \right].
\]

Equation (15) is what should be estimated. It differs from the naive specification in including the parameters \((\mu, \delta)\) of the distribution of housing starts as well as including the realized values. From (15) and (14), it is clear that expected price increases with \(\mu\), but realized price decreases in \(\mu\), \(\ln h\) held constant. In (15) \(h\) and \(\mu\) have different signs: A surprise in housing starts—high when \(\mu\) is low—is what gives a high price. Since the two variables have opposite signs, it accounts for a frequent observation: running a regression with current and lagged starts gives opposite signs to the two variables. The lagged variable in that regression is simply a proxy for \(\mu\). It belongs in the regression and the opposite signs are expected. There is more that can be drawn from (15), but we shall desist. When the decision-making agents do not know the values of variables, the information they have must be used to supplement the ordinary variables in a reduced-form equation. Estimation is not so simple after all.

**How Economic Information can Help Develop Biological Information**

At times, it is possible to deduce information about the underlying stock of a resource from market data. Fisheries provide the most useful
example, and I shall make extensive use of Berck and Johns in what follows. It is very difficult to measure the number of fish in the sea, even the number of a specific species, such as halibut. The stock of halibut is used by the International Pacific Halibut Commission to set season length and other economic regulations for the fishery. It is common in fisheries work to make use of economic information (the amount caught and amount of catch effort expended) to help estimate the critical stock numbers. In this section, I will provide an outline of an efficient way to use that economic information.

Let $x_t$ be the stock of fish at time $t$, $h_t$ be the amount caught, and $a,b$ be parameters. A convenient oversimplification of the growth rules for fish are

$$x_{t+1} = ax_t - h_t + \epsilon.$$ 

The $\epsilon$ is a random error in fish growth or harvest with variance, $Q$, and zero expectation. The economics of the model are that the harvest of fish depends upon the stock and upon prices of inputs to catching fish and the price of fish caught. For simplicity, all of the prices are suppressed (or folded into $b$),

$$h_t = bx_t + \delta.$$ 

The $\delta$ is a random error with variance, $R$, and zero expectation. The regulator believes this model but does not know $a,b; Q; R; \text{ and } x_t$. The Kalman-Filter maximum-likelihood technique allows the discovery of each of these.

Given an $a,b$ (and estimates for $x$, $Q$, and $R$ at time zero), the Kalman-Filter provides a way to find the values of $x$, $Q$, and $R$ at all other times. The basic procedure is that the best estimate of $x_t$ using all data available until $t - 1$ is called $x_{t|t-1}$. Multiplying that estimate by $b$ gives the best estimate of $h_{t|t-1}$, using only the information until $t - 1$. Actual harvest is observed. This is the information at $t$. The difference between actual and estimated harvest is $\Delta_t$. One can show that $\Delta_t$ and $x_{t|t-1}$ are jointly normal. By using the algebra of conditional expectations, one can find $x_{t|t}$, which is $x_{t|t-1}$ conditional of $\Delta_t$. It is the best estimate of stock given all information available to $t$. Similarly, this can be done for $R$ and $Q$. By repeating this step as many times as needed, one can derive estimates for stock for all periods.

Given the estimates of the stock and the assumption of normality, one can find the likelihood of the observations, $h$. An observation, $h_t$, should be regarded as drawn from the sum of two normals, $\delta$ and $bx_{t|t}$. Indeed, this simply adds an extra term to the variance-covariance matrix in the likelihood expression and provides no real additional numerical problem.

The entire procedure is (1) choose some parameter values; (2) calculate the x's; (3) calculate likelihood; and (4) determine if likelihood is maximized (if not, intelligently vary the parameters and go to step 2).

The procedure is easy to generalize in many different directions, such as non-linearity, making $x$ a vector, etc.
The procedure can potentially be used anywhere economic information reveals something about a poorly measured stock. In Berck and Johns, we suggested using it to measure the capital stock, the stock of illegal aliens, and the stock of an exhaustible resource. In the latter case, one has an estimate of the ultimately discoverable stock of oil. Agents drill and otherwise explore to augment the stock of discovered oil and decrease the stock of oil to be discovered. From their success rate, one infers the ultimately discoverable stock.

Where the stock at hand has few economic consequences, this method will not work. There are those who believe that economic activity is causing a hole in the ozone layer and warming the atmosphere. Neither of these ecological catastrophes has currently impeded economic activity, so there is no way to work backwards from lost production to the magnitude of the ecological problem. More local air pollution does lend itself to these methods. Ozone is a major (10 percent to 15 percent output loss) detriment to crops in the San Joaquin Valley. One might well be able to use productivity to map the ozone concentration in the years before the measurements were made.

Conclusion

Information, or more exactly the lack of it, is pervasive in economic decision making. There is a very well-developed theory of how to make decisions with imperfect information and a newer set of theories about how to force agents with better information to make the decisions. The degree to which information is not known is much less appreciated. While everyone understands how it would have been difficult in 1953 to predict that 9 million acres would be set aside to save the spotted owl in 1992, the degree to which it is possible to predict simple economic variables, such as housing starts, is much less well understood. When those forecasts are combined with estimated equations to produce conditional forecasts of yet other variables, the precision further deteriorates. In the case of natural resources, it is possible that the losses from lack of information are quite low. After all, the market solution to resource allocation is that marginal value is the same in all periods. Allocating a little to the wrong period can have no real welfare effects. Explicitly recognizing uncertainty causes real problems for recovering what little information there is available. Even the simplest regression estimates are not right if the agents are forced to make their decisions based upon estimates. On the brighter side, sometimes the activities of economic agents can be used to infer information about resource stocks that policy makers, even biologists, find useful.
References


