Title
DETERMINATION OF Ykq AND YsQ COMPONENTS IN THE SHAPES OF RARE EARTH NUCLEI

Permalink
https://escholarship.org/uc/item/6h68f2k7

Authors
Hendrie, D.L.
Glendenning, Norman K.
Harvey, B.G.
et al.

Publication Date
1967-10-01
DETERMINATION OF $Y_{40}$ AND $Y_{60}$ COMPONENTS IN THE SHAPES OF RARE EARTH NUCLEI


October 1967
DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.
DETERMINATION OF $Y_{40}$ AND $Y_{60}$ COMPONENTS IN THE SHAPES OF RARE EARTH NUCLEI


October 1967
DETERMINATION OF $Y_{40}$ AND $Y_{60}$ COMPONENTS IN THE SHAPES OF RARE EARTH NUCLEI

D. L. Hendrie, Norman K. Glendenning, B. G. Harvey, O. N. Jarvis†, H. H. Duhm‡, J. Saudinos††, and J. Mahoney

Lawrence Radiation Laboratory
University of California
Berkeley, California

October 1967

ABSTRACT

Differential cross sections for 50-MeV alpha particles of members of the ground state rotational band up to the $6^+$ or $8^+$ state were measured in a number of even nuclei of the rare earth region. The data was analyzed under the assumption of a perfect rotor description for the nucleus and a deformed optical interaction between alpha and nucleus by solving the resulting coupled equations. Higher order components $Y_4$ and $Y_6$ in the nuclear shapes were determined with precision. A systematic variation of $B_4$ from positive values in the light rare earths to negative values in the heavy ones is established.

*Work performed under the auspices of the U. S. Atomic Energy Commission.
†Permanent address: AEHE, Harwell, Berkshire, England.
‡Permanent address: Max Planck Institut für Kernphysik, Heidelberg, Germany.
††Permanent address: CEN Saclay, Gif-sur-Yvette, S.-et-O., France.
The rotational spectra of nuclei in the rare earth (A=152 to 190) and actinide (A=220 to 250) regions, implying large permanent deformations, have been long known. The electric quadrupole moments of these deformed nuclei have been accurately determined by several methods. It is certainly associated largely with a $Y_2$ component in the nuclear radius. The knowledge about possible higher components $Y_4$ and $Y_6$, however, has been very tenuous up to now. Although calculations have predicted a $Y_4$ component in the uranium region\(^1\),\(^2\), the evidence from alpha decay studies is ambiguous\(^3\). Inelastic deuteron scattering near the Coulomb barrier has provided evidence for the existence of such moments in the rare earth nuclei, but the magnitudes and even signs of the moments were not well determined\(^4\). Here we report a systematic study of the shape of deformed nuclei in the rare earth region through excitation of the ground state rotational band by alpha particles\(^5\). We scattered a well collimated and analysed beam of 50-MeV alpha particles on several metallic foils of isotopically pure elements in the rare earth region. Angular distributions were taken with a cooled lithium-drifted multiple counter array. Special care was taken to keep backgrounds low and to maintain resolutions of about 50 keV.

Alpha particles are a good probe of the nuclear surface. Because they are strongly absorbed in the interior we are assured that most of those that are inelastically scattered are involved in a surface interaction. Moreover, even at moderate energy they carry high enough momentum that direct transfer of large units of angular momentum is possible. This is a desirable feature since we want to measure the higher order terms $Y_L$ ($L > 2$) in the shape, which is possible only when direct excitation of the $J=L$ member of the band.
competes significantly with the cascade transition. In fact if the direct excitation alone were present, the angular distributions to the J=L state would be determined only by the L'th multipole component of the nuclear field. Since we have measured the differential cross sections to the various rotational levels of the ground state up to the 6+ and sometimes 8+ we are therefore in a position to determine, through a careful analysis, the shape up to Y6 of the nuclear field produced by the ground state.

In our analysis of the data we assume that the alpha-nucleus interaction can be represented by a deformed complex optical potential and that the nucleus is a perfect rotor, at least up to the 8+ state. We calculate the cross section to members of the ground state band by solving numerically the coupled differential equations that follow from this picture, without further assumptions. Coulomb excitation effects were found to be significant and were treated on an equal basis with the nuclear excitation. The multipole expansion of the interaction, the number of partial waves, and the number of coupled channels were all carried to convergence so that we have an exact numerical solution of the scattering model.

Since we treat explicitly the rotations, the optical potential should be essentially the same as that in the neighboring spherical region since in both cases it must carry only the effects of the omitted intrinsic excitations. This is discussed in greater detail elsewhere. Therefore, the same optical potential was used with only minor adjustments throughout the region from the spherical nucleus Sm148 through the deformed region to Hf178. This means that essentially only the shape parameters of the optical potential had to be
determined in the analysis. We parameterize the shape according to:

\[ R = R_0 (1 + \beta_2 Y_{20} + \beta_4 Y_{40} + \beta_6 Y_{60}) \].

Figure 1 shows the data and calculation for Sm\(^{154}\). Notice that the only agreement occurs for \(\beta_4 = 0.05\) and that the experimental data is reproduced in detail, including location of diffraction peaks, absolute magnitude of the cross sections, and even depths of the minima of the various states. The dashed and dotted curves show the disagreement with the data that is obtained when \(\beta_4\) is respectively 0.0 and -0.05. We see in this way that the value of \(\beta_4\) is quite precisely determined.

The agreement is somewhat improved for almost all the nuclei by including a small negative \(\beta_6\) term. This term has been included in the best fit curves shown for three of our target nuclei in fig. 2. Here we easily see such changes in the data as shifts in the position of maxima, and smaller amplitudes of oscillation which, we find, imply a change in the sign of \(\beta_4\) with increasing target mass.

We have made a preliminary exploration of deeper potentials from which it is already clear that our deformation parameters scale as \(\beta r_0\), as has been suggested\(^8\). Accordingly in the presentation of our results we quote values of \(\beta r_0\), or more precisely \(\beta (r_0/1.2)\). The latter choice is made, somewhat arbitrarily, so as to achieve an approximate correspondence with our calculated values reported later. In any case the actual value of \(\beta\) used in our calculation can be obtained from those we list by reference to the value of \(r_0\) used for the optical potential. Our results are summarized in Table I and fig. 3.
The deformation $\beta_2$ is consistently smaller throughout the region than has usually been believed on the basis of electric quadrupole moments determined by Coulomb excitation experiments. Two points should be stressed in this connection. There is no reason to believe that the charge and nuclear fields have exactly the same shape. Secondly, the quadrupole moment does not define a unique value of $\beta_2$ as is sometimes supposed. This is because the $Y_4$ and $Y_6$ components in the shape also contribute to the quadrupole moment (and to every other even moment, as does $Y_2$).

To see whether the values of $\beta_4$ determined by our analysis can be understood in terms of the single-particle structure of nuclei, we have calculated this component in the shape by a simple method due to Harada. The change in energy of the single-particle states due to a $Y_4$ component in the shape of the Nilsson potential was computed to first order. The equilibrium value of $\beta_4$ was determined as the one for which the total energy was a minimum. These calculated values are compared in fig. 3 with the values determined from the analysis of the data. It is gratifying that this simple model reproduces very well the observed trend characterized by larger positive $\beta_4$ deformations near the beginning of the deformed region, decreasing to large negative values at the end. A more realistic calculation by Nilsson and his coworkers agrees well with our result. This calculation takes into account pairing, core polarization, and Coulomb effects.

We wish to thank Claude Ellsworth for the fabrication of the targets and Noel Brown for his efforts on the computer program.
References

1. K. Kjällquist, Nucl. Phys. 2 (1958) 163
2. K. Harada, Phys. Letters 10 (1964) 80
5. Some of the experimental results were reported by B. G. Harvey, D. L. Hendrie, O. N. Jarvis, J. Mahoney, and J. Valentin, Phys. Letters 24B (1967) 43; the possibility of establishing a $Y_4$ component by proton scattering was suggested by R. C. Barrett, Phys. Rev. Letters 14 (1965) 535
8. N. Austern and J. S. Blair, Ann. Phys. 33 (1965) 32; it should be noted, however, that quadrupole excitations at energies below the Coulomb barrier must scale as $\beta_2 R_c^2$
Table 1

<table>
<thead>
<tr>
<th></th>
<th>Sm$^{152}$</th>
<th>Sm$^{154}$</th>
<th>Gd$^{158}$</th>
<th>Er$^{166}$</th>
<th>Yb$^{174}$</th>
<th>Yb$^{176}$</th>
<th>Hf$^{178}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_2$</td>
<td>0.246</td>
<td>0.270</td>
<td>0.282</td>
<td>0.276</td>
<td>0.276</td>
<td>0.276</td>
<td>0.246</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>0.048</td>
<td>0.054</td>
<td>0.036</td>
<td>0.0</td>
<td>-0.048</td>
<td>-0.054</td>
<td>-0.072</td>
</tr>
<tr>
<td>$\beta_6$</td>
<td>-0.012</td>
<td>-0.018</td>
<td>-0.018</td>
<td>-0.018</td>
<td>0.0</td>
<td>-0.006</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Deformation parameters, $\bar{\beta} = (r_o/1.2)\beta$ obtained by our analysis of 50 MeV alpha scattering. The optical potential parameters used were about $V = 65.9$ $W = 27.3$ $r_o = 1.44$ $a = 0.637$ with only very slight adjustments to individual nuclei. The Coulomb potential was generated by a uniform charge distribution with a correct quadrupole moment.
Figure Captions

Fig. 1. Differential cross-sections for 50 MeV alpha particles scattered from $^{154}_{\text{Sm}}$. The coupled channel calculations corresponding to values of $\beta_4$ equal to $+0.05$, $0.0$, and $-0.05$ are compared. In the latter two cases the optical potential radius and $\beta_2$ were readjusted so as to achieve the best possible agreement with the $0^+$ and $2^+$ state. These results illustrate the extreme sensitivity to $\beta_4$ especially since differences in the $4^+$ and $6^+$ cross sections of an order of magnitude appear at certain angles.

Fig. 2. Coupled channel calculations of the differential cross sections for 50 MeV alpha particles of three nuclei exhibiting respectively positive, zero, and negative values of $\beta_4$ are compared with the data. The shape parameters in each case are exhibited in the figures.

Fig. 3. Solid lines indicate the values of $\beta_4$ calculated in first order perturbation theory based on the Nilsson scheme. The values of $\beta_4$, multiplied by $(r_0/1.2)$, that were obtained from an analysis of the scattering data are shown by solid dots. The error bars indicate our feeling of the precision with which the parameters can be extracted.
Figure 1
Figure 3

Calculated and measured values of $\beta_4$

Mass

150 160 170 180 190
This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.