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Publication Date
1995-03-01
VERTICALLY RELATED MARKETS AND TRADE POLICY 
IN A BARGAINING FRAMEWORK

by

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March, 1995
Vertically Related Markets and Trade Policy in a Bargaining Framework

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Abstract: We analyze the interaction of asymmetric industries in international vertically related markets. Each downstream firm bargains efficiently with its domestic supplier in a first stage and with the foreign supplier in a second stage. The asymmetry in upstream costs leads to interindustry trade. It can also cause vertical integration in the more efficient industry, and possibly vertical foreclosure. The latter occurs if competition in the final goods market is severe (the goods are close substitutes). When the more efficient industry is integrated, a tariff on imports of the final good stimulates interindustry trade of the input, but it may increase or decrease the market share of the domestic upstream firm. The effects of a tariff depend on the industry configuration in the low-cost country.

JEL Classification numbers: C71, C78, F13, L22

Keywords: Multistage bargaining, vertical integration, strategic trade policy, cooperative games

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a We thank, without implicating, Alistair Ulph and Leo Simon. Larry Karp would like to thank the Department of Economics for their continued hospitality.

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1. Introduction

Firms which have a cost advantage in the production of a key intermediate good sometimes restrict exports of these in order to promote exports of the final good. A notable example is the voluntary restriction, by integrated firms such as Toshiba, of exports of DRAM into the US market. Attempts by the low-cost industry to reduce downstream competition by restricting the supply of the input (or vertical foreclosure) provides a motive for strategic government intervention which alters the industry equilibrium [Spencer and Jones (1991, 1992), Rodrik and Yoon (1989)].

In Spencer and Jones' model, producers of the intermediate good set prices. The low-cost supplier commits to a price for the exports of the input before downstream firms compete (in quantity or in price) in the final market. The more efficient industry is assumed to be vertically integrated. It chooses not to export the intermediate good if supply in the high-cost country is sufficiently inelastic. The government located in the high-cost country can impose a tariff on imports of the final good and induce vertical supply.

We analyze the effects of a tariff on imports of the final good for vertically related markets, when exchange of the intermediate good is determined by bargaining\(^1\). This implies a more even distribution of power between upstream and downstream firms than does price setting. It eliminates common motives for integration (e.g. double marginalization) and limit-price policies [as in Ordover, Saloner and Salop (1990)].

\(^{1}\) Other papers in which the equilibrium is determined by bargaining include: Horn and Wolinsky (1988), where upstream and downstream firms bargain over the price of the input; and Hart and Tirole (1990) and Bolton and Whinston (1993), where firms bargain over the gains from trade.
In our model, two industries, each consisting of an upstream and a downstream firm, are located in two different countries. Each downstream firm bargains first with the domestic supplier and then with the foreign one. This timing captures the idea that firms regard the domestic firm as a "natural partner". We could motivate this description by assuming that agents incur different costs to bargaining (and of renegotiating previous agreements) with different agents. A more complete model would endogenize the number of times that agents bargain, and the order of their partners, but we do not attempt this here.

We first study the free-trade equilibrium in the bargaining framework. We then show that the effects of a tariff depend on the degree of competition in the final market and on the industry configuration in the more efficient country.

If upstream firms have the same costs, trade in inputs takes place between domestic partners only. The bargaining solution is equivalent to the solution under vertical integration; the second stage of bargaining is vacuous. If suppliers have different constant marginal costs, the inclusion of the second stage of the game changes the equilibrium: Downstream market shares are reversed, relative to the one-shot game. The downstream firm located in the low-cost country resells some of the input to the foreign upstream firm rather than transforming it into the final good. The reallocation of production tends to increase efficiency. When resale of the input from the downstream to the upstream level (hereafter, simply "resale") is not allowed, the downstream firm located in the low-cost country becomes a Stackelberg leader in the final market, and consumers tend to be better off.

The cost difference may induce vertical merger in the low-cost industry. The conditions under which integration of the more efficient industry occurs depends on whether
resale is allowed. When a merger takes place, the choice between vertical foreclosure and vertical supply by the integrated industry depends upon the degree of competition in the final market but no on the cost difference.

To make our results comparable to Spencer and Jones, we assume that the final market is located in the high-cost country, and we analyze the effects of trade policies there. If goods are perfect substitutes and the more efficient industry is vertically integrated, a specific tariff on imports of the final good induces vertical supply, as is the case with price setting by suppliers. With bargaining, however, an arbitrarily small tariff induces the integrated industry to exit the final goods market and sell all its input to the independent downstream firm. Efficient bargaining, which implies that partners share the gains from trade, enables the foreign integrated industry to reduce the burden of the tariff. The domestic (high cost) supplier of the input may gain or loose from the tariff on the final good, depending on the degree of substitutability. When non-integration prevails in the low-cost country, the tariff has the usual effect of extracting rent from foreign firms. A tariff affects the incentives for vertical integration. Again, the tariff’s effects depend upon whether resale is allowed under non-integration.

The paper is organized as follows. Section 2 points out the implications of efficient bargaining with identical costs. Section 3 studies a model with asymmetric costs, with resale, and section 4 analyzes the case in which resale of the input is not allowed. Section 5 considers the incentives for vertical integration in the low-cost industry. Trade policies are analyzed in sections 6, 7 and 8, and concluding remarks are contained in section 9.
2. Implications of efficient bargaining

We have a two-country model. In country $i$ the downstream firm ($D_i$) acquires its input from the domestic supplier ($U_i$). ($U$ stands for "upstream", $D$ for "downstream"). The amount of the input and the payment are determined through bargaining. We model this using the Nash solution, which maximizes the product of the firms' gains in profits over the disagreement outcome, and splits the surplus. One unit of input is transformed costlessly into one unit of output. Both downstream firms sell in the same market, located in the domestic country. Firm $i$'s inverse demand function is $p(x_i + cx_j)$, where $x_i$ is the quantity sold by firm $i$ and $c \in [0,1]$ determines the degree of substitutability. The parameter $c$ therefore determines the degree of competition in the final market, where the equilibrium is Cournot-Nash.

In this section we assume that suppliers have equal constant marginal costs. Since efficient bargaining avoids the double marginalization problem, the interaction of the two non-integrated industries is equivalent to the interaction of two vertically integrated ones. Equilibrium sales are independent of whether upstream and downstream firms integrate.

The equilibrium does not change if the domestic (foreign) downstream firm can bargain with the foreign (domestic) upstream firm after bargaining with its home partner. The first-stage bargaining is carried out with the knowledge that there will be a second stage, and the equilibrium is subgame perfect. Under simultaneous bargaining, Hart and Tirole (1990) show that firms sell the Cournot-Nash output in the final market after acquiring some of the input from each supplier. In a sequential model, all trade takes place between domestic partners in the first stage, and this game reduces to the previous one-shot game, yielding the
Cournot-Nash outcome. This result would be more obvious if there were a technical reason why the potential surplus between domestic partners was greater than the surplus between foreign ones (e.g. transport costs or increasing marginal costs of production).

The equivalence of the outcomes of the two-stage and of the one-stage games implies that when firms are symmetric, the incentive for vertical integration is not altered by the possibility of inter-industry trade (by which we always mean trade between an upstream firm in one country and a downstream firm in the other country). This result does not hold if firms can commit to strategic policies, as in Ordover, Saloner and Salop's (1990) price-setting framework. In their model the ability to limit-price provides an incentive for individual vertical integration. With bargaining, symmetric firms have no incentive to integrate.

3. The bargaining problem with asymmetric industries

We assume that the foreign supplier \( (U_f) \) has lower costs than the domestic one \( (U_d) \). Their constant marginal costs are \( m \) and \( h \) respectively \( (h > m) \); hereafter we set \( m = 0 \), so \( h \) determines the foreign industry’s cost advantage. When trade occurs between domestic partners only, the outcome in the final market is the same as the Cournot-Nash equilibrium (which we assume is unique) of the game with integrated firms. Total sales in the final market are a function of marginal costs, and we denote by \( G(h) \equiv G \) the low-cost foreign industry’s (greater) Cournot output level, and by \( L(h) \equiv L \) the high-cost domestic industry’s

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2 In the two games (with and without the second-stage bargaining) the Nash solution is computed with reference to different threat points (i.e. in the one-shot game the disagreement payoff for firms is the pair \( (0,0) \), while in each stage of the two-stage game there is a positive outside option because of bargaining with the other firms). The symmetric shift in agreement and disagreement payoffs across the two games guarantees that the difference in threat points does not alter the final outcome.
(lower) Cournot output, with \( G > L \).

Suppose that firms are allowed to bargain with foreigners, and consider the two-stage game where each downstream firm bargains with its domestic supplier in the first stage and with the foreign supplier in the second. We show that total sales of downstream firms in the final market are the same as in the one-shot game, but the market shares are reversed.

To obtain a subgame perfect equilibrium, we solve the model backward, starting from the second stage. Consider the bargaining problem between the foreign supplier \( U_f \) and the domestic buyer \( D_d \). The Nash solution selects the sales quantity \( x \) and the transfer \( T \), the pair \((x_{df}^*, T_{df}^*)\) that maximizes the product of the firms' gains in profits over the disagreement outcome. The first index on variables denotes the nationality of the downstream firm and the second index the nationality of the upstream firm: d for "domestic" and f for "foreign"; a "*" denotes the equilibrium level. In this game, firms take as given the outcome of previous bargains and the outcome of their rivals' concurrent bargain. The foreign downstream firm's profit is the difference between revenue and transfers that the firm has to pay to acquire the intermediate good:

\[
\Pi_{D_d} = (x_{fd} + x_{fd})p[x_{fd} + x_{fd} + c(x_{dd} + x_{dd})] - T_{fd} - T_{fd} 
\]  

where \((x_{dd} + x_{df})\) is the total output for the foreign downstream firm.

The profit for the domestic supplier is the difference between revenues from the sale of the input and the costs of production:

\[
\Pi_{U_d} = T_{dd} + T_{fd} - h(x_{dd} + x_{fd}) 
\]  

If the two parties fail to agree, they obtain a payoff which depends on their agreement
with their national partner, so the disagreement point is \( d_{id} = (\Pi_{id}^*, \Pi_{ld}^*) \):

\[
d_{id} = \{ x_{ff} p[x_{ff} + c(x_{dd} + x_{df})] - T_{ff} ; T_{dd} - hx_{dd} \}.
\]

(3)

The Nash solution of this bargaining game chooses \( x_{id} \) and \( T_{id} \) to maximize the Nash product \([\Pi_{id} - \Pi_{id}^*][\Pi_{ld} - \Pi_{ld}^*]\); this maximizes the surplus that the two firms split. The surplus \( S_{id} \) is the firms' gains over the disagreement payoffs:

\[
S_{id} (x_{id} | x_{ff}, x_{df}, x_{dd}) = (x_{ff} + x_{id}) p[x_{ff} + x_{id} + c(x_{dd} + x_{df})] - x_{ff} p[x_{ff} + c(x_{dd} + x_{df})] - hx_{id}.
\]

(4)

The maximization of this surplus with respect to \( x_{id} \) takes the sales of the rival and the first-stage quantities as given. We denote by \( q_f \) the total sales made by the foreign downstream firm \((q_f = x_{ff} + x_{id})\), and by \( q_d \) those made by the domestic downstream firm \((q_d = x_{dd} + x_{df})\). Substituting these definitions into (4), we re-write the surplus as:

\[
S_{fd} (q_f | q_d, x_{df}) = q_f [p(q_f + c q_d) - h] - x_{ff} [p(x_{ff} + c q_d) - h]
\]

(5)

By symmetry, the surplus for the domestic downstream firm and the foreign low-cost supplier is:

\[
S_{df} (q_d | q_f, x_{df}) = q_d p(q_d + c q_f) - x_{dd} p(x_{dd} + c q_f)
\]

(6)

In the second stage \( q_f \) is chosen to maximize \( S_{fd} \), and \( q_d \) is chosen to maximize \( S_{df} \).

Since first-stage quantities are here fixed, the maximand in (5) is equivalent to that faced by a high-cost vertically integrated industry (minus a constant), and the maximand in (6) is equivalent to that faced by a low-cost industry, when the two industries compete in quantity.
in the final market. The resulting Cournot equilibrium is \((q_a^*, q_t^*) = (G, L)\). The downstream firm located in the high-cost country sells the larger Cournot market share. The second stage transfer, but not the level of final sales, depends on the outcome of bargaining in the first stage.

In the first stage of the game each downstream firm bargains with its domestic supplier. We solve the first-stage problem for the low-cost industry, assuming that the simultaneous bargain between firms in the high-cost country is successful, i.e. \(x_{dd} > 0\). The first-stage agreement profit for \(D_l\) is\(^3\):

\[
\Pi_{D_l} = \frac{1}{2} \left( q_f^* \left[ p(q_f^* + c q_d^*) - h \right] + x_{ff} \left[ p(x_{ff} + c q_d^*) + h \right] \right) - T_{ff}.
\]

(7)

The agreement payoff for the low-cost supplier is:

\[
\Pi_{q_f} = T_{ff} + T_{df} \]

(8)

and the disagreement pair of payoffs is:

\[
d_{ff} = \left\{ \frac{1}{2} q_f^* \left[ p(q_f^* + c q_d^*) - h \right]; T_{df} \right\}.
\]

(9)

The first-stage surplus in the low-cost industry is:

\[
S_{ff}(x_{ff}) = \frac{1}{2} x_{ff} \left[ p(x_{ff} + c q_d^*) + h \right].
\]

(10)

The first-stage surplus for the high-cost industry is:

\[
S_{da}(x_{dd}) = \frac{1}{2} x_{dd} p(x_{dd} + c q_t^*) - hx_{dd}.
\]

(11)

The simultaneous maximization of the first-stage surpluses (10) and (11) yields the

---

\(^3\) The second-stage transfer is \(T_{ld}(q_t^*, q_d^*) = S_{ld}(q_t^*, q_d^*)/2 + h(q_t^* - x_{nl})\). Using the expression for \(S_{ld}\) in (5) and substituting into the expression for \(D_l\)'s profits gives (7).
equilibrium quantities \( x_{df}^* \) and \( x_{dd}^* \). In the low-cost country the timing of the game and the cost difference ensure that the buyer, \( D_n \), acquires an amount of the input greater than its final sales; that is, \( x_{df}^* > q_t^* \), so \( x_{dd}^* < 0 \). In the second-stage \( D_l \) sells the amount \( q_t^* - x_{df}^* = -x_{dd}^* \) to \( U_d \). This trade, represented in Figure 1, may be a paper transaction (as with future markets), rather than a physical one. The paper transactions are as follows: \( U_l \) sells \( x_{ff}^* \) to \( D_f \) and \( x_{df}^* \) to \( D_d \); \( U_d \) sells \( x_{dd}^* \) to \( D_d \) and buys \(-x_{dd}^* \) from \( D_r \). The physical transactions are as follows: \( U_l \) sells \( x_{ff}^* + x_{df}^* = q_t^* \) to \( D_f \); \( U_d \) sells \( x_{dd}^* + x_{tf}^* \) to \( D_d \), and \( U_l \) sells \( x_{df}^* - x_{tf}^* \) to \( D_d \). The difference between the physical and paper transactions provides a way of transferring production from the low-cost to the high-cost producer. By assumption, the two suppliers are unable to bargain with each other. The downstream firm in the low cost country buys more than it intends to use, and the high cost upstream firm promises to deliver more than it intends to produce. In equilibrium there is no need for these excess quantities actually to be delivered, since there are offsetting trades. Nevertheless, the first stage contracts are important, since they determine the second stage outcome.

The foreign downstream firm's behavior can be explained as a form of price discrimination. This firm transforms \( L \) units of the intermediate good and sells them in the final-goods market where it faces a downward-sloping residual demand curve. The rest of the input is sold into a market where the "demand" is perfectly elastic, being represented by the supplier's marginal costs. (\( U_d \) has committed itself to supplying a fixed amount to \( D_n \).) Taking as given \( q_d^* \), at the Cournot equilibrium an increase in \( q_t \) decreases the marginal revenue for \( D_t \), while the marginal benefit from selling to the high-cost supplier is positive (because of the difference between marginal costs).
We summarize the characteristics of equilibrium in the two-stage bargaining in the following proposition. (See the Appendix for proofs.)

Proposition 1: Suppose that downstream firms are able to bargain for supply of inputs twice, first with their national supplier and then with the foreign supplier; the upstream firms have different marginal costs. (i) The equilibrium output shares of the downstream firms are reversed, relative to the one-shot game, but aggregate output is unchanged. (ii) The downstream firm in the low-cost country sells some of the input produced by the low-cost supplier to the high-cost upstream firm.

Total final sales are the same under the two-stage and one-shot bargaining, so the second stage improves efficiency if and only if it transfers production from the high cost to the low cost supplier. This always occurs if demand is given by the linear function:

\[ p(q_i + cq_j) = a - q_i - cq_j \]  

With linear demand the equilibrium quantities are:

\[ q_d^* = G = \frac{a}{2 + c} + \frac{ch}{4 - c^2} \]

\[ q_f^* = L = \frac{a}{2 + c} - \frac{2h}{4 - c^2} \]

\[ x_{df}^* = |x_{fd}^*| = h. \]

Hereafter, we assume that \( a > 5h \); this inequality insures that \( U_d \) produces a positive amount, for all values of \( c \).
In the one-shot game the low-cost supplier $U_f$ produces $G$ units, and in the two-stage game it produces $q_f^* + |x_{fd}^*| + x_{df}^* = L + 2h > G$. Total production by the most efficient supplier is larger when interindustry trade (and resale) is allowed. Since total sales and consumer welfare have not changed, but production has become more efficient, it is clear that aggregate industry profits and social welfare are increased by the second stage of bargaining. We summarize this in

**Proposition 2**: If demand is linear, aggregate industry profit is increased and consumer welfare unchanged by the second stage of bargaining.

4. Prohibition against resale

Resale of the intermediate good reallocates production between suppliers. The supplier $U_f$ is unable to sell these "extra" units directly to $D_d$ in the second stage because $U_d$ has committed itself to sell a certain amount in the first stage. The low-cost supplier, $U_f$, might increase its profits by forbidding resale of the intermediate good. This prohibition decreases $U_d$'s willingness to supply in the first stage; this increases $D_d$'s demand in the second stage, and enables $U_f$ to capture surplus which otherwise would have accrued to $U_d$ and $D_f$. If resale is not allowed, the second-stage bargaining involves only the domestic downstream firm $D_d$ and the foreign supplier $U_f$ (because the non-negativity constraint on $x_{fd}$ is binding). In the first stage firms bargain with local partners: $D_f$ bargains with $U_f$ and transforms all its input into the final good. The impossibility of reselling the input to $U_d$ leads $D_f$ to increase its share in the final market. Bargaining with the low-cost supplier in the first stage confers a first-mover advantage which allows $D_f$ to achieve the Stackelberg-leader
output level for the game in which both duopolists have the low cost technology. The structure of the game is illustrated in Fig. 2. We have

**Proposition 3:** If resale from the downstream to the upstream level is not allowed, then (i) $D_t$ and $D_a$ sell, respectively, the output levels of the Stackelberg leader and follower for the game in which both firms have the low cost technology; (ii) Provided that demand is not "very convex", aggregate final sales are higher if resale is prohibited.

If goods are sufficiently close substitutes, consumer welfare increases with aggregate output (i.e., it is not necessary to consider the distribution of output). Therefore, for $c$ close to 1, consumer welfare is higher when resale is prohibited. This suggests that if both industries and the final market were located in the same country, anti-trust authorities may want to prohibit resale.

In order to determine whether $U_i$ would want to forbid the resale of the input, independent of legal constraints, we use the linear demand function (12). By calculating the equilibrium we can show that $x_{ad}$, the trade between $U_i$ and $D_o$, equals $h$ whether or not resale is permitted; however, the transfer between the firms differs in the two cases. When resale is prohibited, $U_a$'s effective costs are higher, causing it to supply less to $D_o$, which increases $D_o$'s willingness to pay in the second stage. Profits for $U_i$ under resale ($\Pi_{U_t}^R$) and under non-resale ($\Pi_{U_t}^{NR}$) are:
A simple calculation shows that $\Phi_u(a,c,h) = \Pi_{Uf}^{NR} - \Pi_{Uf}^R$ (the benefit to $U_f$ of prohibiting resale) is a decreasing function of $h$ for all values of $c$. Consequently, if $U_f$ ever wants to allow resale, it would only be for large values of $h$. The explanation for this result is that resale leads to a more efficient allocation of production, and $U_f$ captures some of this surplus. This efficiency gain is greater, the greater is $U_f$'s cost advantage, i.e., the greater is $h$. The result means that in order to determine whether $U_f$ would ever permit resale, we need only evaluate $\Phi_u(.)$ at $h = a/5$. (Recall that we require $h \leq a/5$ to insure that $x_{ad} \geq 0$). By plotting $\Phi_u(a,c,a/5)$ against $c$, we can show that $U_f$ always wants to prohibit resale. On the other hand, the foreign downstream firm may not agree to such prohibition. Its profits under the two configurations are:

$$
\Pi_{Df}^R = \frac{3a^2(c - 2)^2 + 2ah(c - 2)(c^2 + 2) + h^2(c^4 - 4c^2 + 12)}{4(c + 2)^2(c - 2)^2}
$$

$$
\Pi_{Df}^{NR} = \frac{a^2(c - 2)^2}{16(2 - c^2)}
$$

Let $\Phi_D(a,c,h) = \Pi_{Df}^R - \Pi_{Df}^{NR}$. A straightforward calculation shows that the benefit from allowing resale is a decreasing function of $h$. Although $D_f$ benefits from the resale of the input, and the surplus arising from this trade is increasing in $h$, a large difference between suppliers' costs makes prohibition of resale more attractive to $D_f$. As $h$ increases, the change
from the Stackelberg equilibrium in the downstream market (where $D_t$ is the leader) to the Nash-Cournot equilibrium (where $D_t$ produces the lower output) harms the downstream firm. This loss is accompanied by a worsening of the firm's bargaining position vis-à-vis $U_r$. We compare $D_t$'s profits for the two extreme cases, where goods are perfect substitutes so that competition in the final market is strong ($c = 1$) and where there is no competition ($c = 0$). When competition is strong, the benefit from being a Stackelberg leader in the downstream market is substantial, and $D_t$ prefers resale not to be allowed provided that $h > a/6$. Thus, at least when $c = 1$, the foreign firms' would agree upon the prohibition against resale when the supplier enjoys a large cost advantage. If $c = 0$, $\Phi_\sigma(a,0,h) > 0$ for all values of $h$. When markets are separated, the benefit from being a Stackelberg leader is reduced, and the gain from interindustry trade becomes more relevant.

Since for some parameter values there is a conflict of interest amongst the foreign firms, it is difficult to tell which industry configuration arises in equilibrium. The outcome may depend on lobbying, or on a government decision led by antitrust motives. Because of this ambiguity, the following discussion on integration and trade will consider both configurations.

5. Issues of vertical integration

This section discusses the effect of vertical integration by the low-cost industry, and gives conditions under which this is an equilibrium outcome. We denote by $F$ an integrated low-cost (foreign) firm. The game is now played as follows: In the first stage $F$ chooses total production of the input $x_F$, and domestic firms bargain over the suppliers' production level $x_d$ and the transfer $T_d$. In the second stage, the integrated firm and the domestic downstream firm
bargain over the quantity of the input $x_{df}$ and the transfer $T_{df}$. The pattern of trade is represented in Fig. 3.

The integrated structure (unlike the non-integrated upstream firm) internalizes the rivalry in the final market when bargaining with the downstream firm in the second-stage.

Vertical foreclosure arises when the second-stage surplus to be split between the integrated industry and the downstream firm equals zero. This occurs if the goods are perfect substitutes, i.e., where downstream competition is strong. The second-stage surplus between $F$ and $D_d$ is simply the change in aggregate revenue caused by reallocating $x_{df}$ units from $F$ to $D_d$, or:

$$S_{df}(x_{df}|x_d,x_F) = (x_d + x_{df})p[x_d + x_{df} + c(x_F - x_{df})]$$
$$+ (x_F - x_{df})p[x_F - x_{df} + c(x_d + x_{df})] - x_d p(x_d + cx_F) - x_F p(x_F + cx_d) \tag{16}$$

When goods are perfect substitutes, $S_{df}$ in (16) is 0 for all values of $0 \leq x_{df} \leq x_F$, and the following is obvious.

**Proposition 4**: When $c = 1$, there is a continuum of bargaining equilibria, with $0 \leq x_{df}^* \leq x_F$, and $T_{df}^* = x_{df} p(x_F + x_d)^*$. When goods are perfect substitutes, $F$ is willing to supply inputs to $D_d$, but it charges the entire revenue that $D_d$ obtains: essentially, it forecloses supply. The second stage is now vacuous. It is also clear from the structure of this game (with $c = 1$) that in equilibrium $x_F^* =$
We are left with the question of whether vertical integration in the more efficient industry is an equilibrium outcome. This requires a comparison of industry profits under the different industry configurations. In order to compute profits, we use the linear demand function (12). Profits of the low-cost vertically integrated industry are:

\[ \Pi_F^* = q_F^*(a - q_F^* - cq_d^*) + T_dF^* \]  

where:

\[ q_F^* = x_F^* - x_{dF}^* = \frac{2a - h}{2(2 + c)} = q_d^* = x_d^* + x_{dF}^* \]

\[ x_F^* = \frac{a}{2 + c} + \frac{h(c + 1)}{2(2 + c)} \]

\[ x_d^* = \frac{a}{2 + c} - \frac{h(c + 3)}{2(2 + c)} \]

\[ x_{dF}^* = \frac{h}{2} \]

\[ T_{dF}^* = x_{dF}^*(a - x_d^* - x_F^*) \]

The equilibrium inter-industry trade, \( x_{dF}^* \), is independent of \( c \). (It equals half the level traded when the foreign industry is not integrated.) We observed in Proposition 4 that for \( c = 1 \), there is a continuum of Nash bargaining equilibria, so (18) is consistent with the proposition.

We also note from equation (18) that final sales are the same in both markets. This property requires only that the markets are symmetric and that the marginal revenue curve slopes

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4 When goods are perfect substitutes, the assumption of efficient bargaining implies that integration by the low-cost industry does not induce integration of the rival one, unlike in the case where upstream firms set prices.
down. (If $F$ and $D_3$ can bargain in the last stage, surplus is maximized by selling the same amount in each market.) We now consider the incentives for integration, and the welfare effects, for the two cases where resale is or is not allowed.

5.a: Resale allowed. Using equations (13) and (18), we see that aggregate final sales are the same with and without integration. The utility of consuming (and the revenue from selling) a given aggregate quantity is maximized when that quantity is split evenly between the two markets. This occurs under integration, but not (for $c > 0$) when the firms are non-integrated. Therefore, for $0 < c < 1$, consumer welfare and sales revenue are higher under integration; for $c = 0$ or $c = 1$, consumer welfare and revenue are the same under both market structures. However, industry costs are strictly higher under integration, even for $c = 0$ or $c = 1$. In the absence of vertical integration, and with resale, we saw that for linear demand $U(t)$ produces $L + 2h$, which is greater than the quantity $x^p$ given by (18). Integration causes the low-cost firm to produce a smaller share of a fixed quantity, resulting in higher aggregate costs. Since the gain in consumer welfare and revenue is negligible for $c = 0$ or $c = 1$, integration leads to a decline in social welfare, and a loss in industry profits whether the goods are very weak or very close substitutes. For $0 < c < 1$ integration benefits consumers, because it leads to a more equal distribution of quantities across the two markets, but it has an ambiguous effect on industry profits and social welfare.

We assume that integration occurs if and only if profits of $F$ are greater than those of the non-integrated industry. Profits of the low-cost industry when firms are independent and resale is allowed are:
Using equations (17) - (19) we conclude that \( \Pi_{\text{c}} > \Pi_{\text{dr}} + \Pi_{\text{ur}} \) if and only if:

\[
a > \frac{h (4c^3 - 9c^2 - 5c + 18)}{2c(c^2 - 5c + 6)} \equiv \beta(h,c).
\] (20)

Integration occurs if and only if \( \beta \) is sufficiently small; \( \partial \beta / \partial c \) can be positive or negative. If \( c = 1 \), the inequality in (20) is always satisfied, given our assumption that \( a > 5h \). If competition in the downstream market is sufficiently fierce, integration occurs. Thus, we see that integration definitely occurs in one circumstance where it results in a loss in social welfare. As \( c \) approaches 0, integration never occurs.

We obtain another perspective by considering the incentives to integrate as a function of \( h \). It is easy to establish that \( \beta(0, c) < a \), so that integration occurs when the cost advantage is small. When the cost advantage is large, we have \( \beta(a/5, c) \geq a \) for \( c \in (0, c^*) \), and \( \beta(a/5, c) < a \) for \( c \in (c^*, 1) \), where \( c^* \) is the degree of substitutability at which (given \( h \)) firms are indifferent between integration and non-integration [i.e. \( c^* \) is defined by the equality \( \beta(a/5, c^*) = a \)]. These inequalities imply that when the cost advantage is large the outcome depends on the degree of competition in the downstream market.

Inter-industry trade generates a surplus due to the reallocation of production. Integration by the low-cost industry reduces this surplus. However, integration confers a benefit on \( U_r \) and \( D_r \), since it allows them to internalize the effect, on the final market, of second-stage sales. \( U_r \) and \( D_r \) capture all of the benefit from this internalization, whereas they capture only part of the increased surplus resulting from more efficient allocation of
production. When \( c = 0 \), there is no competition in the final goods market, and therefore no benefit of integration. When the cost advantage is small and \( c > 0 \), the surplus due to inter-industry trade is small; the benefits from internalization overwhelm the loss of \( U_f \) and \( D_f \)'s share of the efficiency gain. When the cost advantage is large, the outcome depends on the degree of competition in the final market. If competition is strong, firms are better off by integrating because of the benefit from the internalization of the effect of final sales. When competition is weak, this benefit is relatively small, and firms prefer not to integrate because of the large surplus due to interindustry trade.

In Section 2, where we considered identical costs, we saw that there was no incentive for vertical integration. From the linear example we see that even the introduction of a moderate cost difference provides such an incentive, as long as final goods are substitutes. However, firms do not vertically integrate if the cost difference is large and downstream competition is weak.

5.b: Resale not allowed. If resale is prohibited under non-integration, then integration results in a change from the Stackelberg outcome associated with two low-cost firms, to the Cournot outcome associated with one low-cost and one high-cost firm. This change leads to lower aggregate final sales and lower consumer welfare. Without resale, aggregate profits of the non-integrated low cost industry are

\[
\Pi_{U_t}^{NR} + \Pi_{D_t}^{NR} = \frac{(2 - c)^2 a^2}{8(2^2 - c^2)} + \frac{h^2}{2}.
\]

(21)

We can compare this quantity to \( \Pi_f^* \) to determine whether integration would occur. Let the benefit to the industry of integrating be \( \Delta(a,c,h) = \Pi_f^* - (\Pi_{U_f}^{NR} + \Pi_{D_f}^{NR}) \). If \( c = 1 \), the change
from the Stackelberg to the Nash-Cournot equilibrium caused by integration tends to harm the foreign industry. However, integration provides $U_r$ with a credible commitment to reduce its supply to $D_a$, thus shifting the latter's costs to $h$. When $h$ is large, the advantage of this credible commitment is larger than the loss caused by the move from the Stackelberg to the Nash equilibrium, so integration is profitable. For $h$ close to its maximum value, $\Delta(a,c,a/5) < 0$ if and only if $c$ is close to zero. For large $h$, integration occurs only if competition is strong, because then the benefit of internalization of the downstream rivalry is great. When $h$ is close to zero, $\Delta(a,c,0) < 0$ for all values of $c$. Integration does not occur because of the loss due to the change from the Stackelberg to the Nash-Cournot equilibrium. For $c$ close to 0, integration reduces joint profits in the foreign industry, whether or not resale is permitted.

When markets are separated, as with $c = 0$, there is no rivalry in the final market, and thus no scope for internalizing the effect of intermediate sales on profits from final sales.

Results are summarized in Table 1, which gives sufficient conditions to determine whether integration occurs for the two assumptions about resale. For example, the entry in row I and column I gives sufficient conditions for the foreign industry not to integrate given that resale is allowed. We see that if resale is prohibited and there is a small cost advantage, integration does not occur. However, if resale is allowed, integration occurs. In this case, the prohibition against resale discourages integration.
Table 1
Sufficient Conditions to Determine Equilibrium
with Linear Demand

<table>
<thead>
<tr>
<th></th>
<th>I. non-integration</th>
<th>II. integration</th>
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<tbody>
<tr>
<td>I. resale allowed</td>
<td>c = 0;</td>
<td>c = 1;</td>
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<tr>
<td></td>
<td>or c &lt; c* and h</td>
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<td>large</td>
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<td>or c &gt; c*</td>
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<td>and h large</td>
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<tr>
<td>II. resale prohibited</td>
<td>c = 0;</td>
<td>c &gt; c'</td>
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<tr>
<td></td>
<td>or c &gt; 0</td>
<td>and h large</td>
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<td></td>
<td>and h small;</td>
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<td></td>
<td>or c &lt; c'</td>
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<tr>
<td></td>
<td>and h large</td>
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Entries give sufficient condition for market structure in column heading to emerge in equilibrium, under the maintained assumption that market structure is as shown in row heading.

6. Trade policies in the presence of vertical integration

In this section we study tariff policies in the high-cost country (where the final market is located), under the assumption that the foreign industry is vertically integrated. In common with most of this literature, we assume that the government moves first in setting a tariff on the final output. We begin with the simple case of $c = 1$, where we know (by Proposition 4) that vertical foreclosure occurs. Then we consider the more general case.

6.a: Perfect substitute goods and vertical foreclosure. Our results here can be compared to Spencer and Jones'. In their price-setting model, an increase in the high-cost firm's production of the input at the foreclosure price increases the low-cost firm's incentive
for vertical foreclosure. This is because the increase in domestic supply reduces $D_a$'s willingness to pay for imports, thus reducing F's profit margin from the exports of the final good, relative to the profit margin due to the exports of the input. Therefore, a large final-goods tariff is required in order to induce vertical supply.

The effect of a final-goods tariff is very different with bargaining. We summarize the characteristics of a tariff-ridden equilibrium in

**Proposition 5:** (i) When goods are perfect substitutes, an arbitrarily small specific tariff, $t$, on F's exports of the final-good induces vertical supply, and eliminates F from the final market. (ii) The equilibrium production of inputs by F and $U_a$ are equal to Nash-Cournot equilibrium outputs of duopolists with costs $t/2$ and $h$.\*  

The proof of 5.ii is in the Appendix. However, 5.i is straightforward. From equation (16) we know that the surplus from second stage bargaining is identically 0 without a tariff. With a specific tariff $t$, each unit that is exported as an input rather than a final good reduces costs by $t$. There is no change in aggregate industry revenue, since the input is converted to a final good by $D_a$. Hence, the surplus generated by interindustry trade of $x_{df}$ is $tx_{df}$. This is maximized by setting $x_{df} = x_F$, the upper bound, so F exits the final-goods market. Even a small tariff has a large effect on the market shares for the final good.

Proposition 5.ii has three implications. First, because the surplus is shared equally, firm F behaves as if its unit costs have increased by only half of the tariff: bargaining reduces the impact of the tariff. Second, whether or not the tariff is present, the equilibrium output equals the Nash-Cournot equilibrium to some game. The tariff merely shifts the equilibrium
by increasing one firm's costs. If the tariff is small, it has a negligible effect on industry profits and consumer welfare. Thus, contrary to first appearances, a small tariff has little effect on welfare. Third, the tariff increases U$_i$'s (the high cost domestic upstream firm) market share, even though the tariff causes imports of the input to change from 0 to $x^*_F$. It might seem that, because U$_4$ sells intermediate goods, it would object to a final-goods tariff which increases imports of intermediate-goods. However, by improving the power of its downstream partner (D$_d$), the tariff allows U$_i$ to capture more market share. This third implication is due to the assumption that final-goods are perfect substitutes, as we show below.

6.b: Imperfect substitute goods and vertical supply. We use linear demand to study the case where $c \neq 1$. The equilibrium quantities are

\begin{align*}
x^*_F &= \frac{a}{2 + c} + \frac{h(c + 1)}{2(2 + c)} - \frac{t}{4(2 + c)} \\
x^*_d &= \frac{a}{2 + c} - \frac{h(c + 3)}{2(2 + c)} - \frac{t}{4(2 + c)} \\
x^*_df &= \frac{h}{2} + \frac{t}{4(1 - c)}.
\end{align*}

\[5\] A large tariff would, of course, generate non-negligible changes, for the usual reasons. Note that a subsidy on domestic production of the input, in the absence of a final-goods tariff, does not induce vertical supply. However, a subsidy to foreign exports of the intermediate good would have the same effect as the specific tariff on final imports.

\[6\] In performing the calculations leading to (22) we assume that $c \neq 1$ and we ignore non-negativity constraints on quantities. The formulae in (22) are only valid if the parameters a, h, and t are consistent with the non-negativity constraints.
From (22) we see that an increase in the tariff leads $D_d$ to increase its purchases from the integrated firm $F$ and decrease its purchases from the domestic supplier, $U_d$. When final goods are imperfect substitutes, the tariff decreases the domestic supplier’s market share. We noted above that the tariff increases $U_d$’s market share when the goods are perfect substitutes. It is somewhat ironic that the final-goods tariff increases the domestic upstream firm’s market share in the situation where the tariff induces a large increase in imports of the input; and the supplier’s market share is reduced by the tariff in situations where it induces a smaller increase in imports of the input. The reason for this apparent anomaly is that the tariff increases $D_d$’s demand for the input to a much greater degree when the final goods are close substitutes.

Using (22), we calculate final sales as

$$q_d^* = \frac{2a - h}{2(2 + c)} + \frac{t(1 + 2c)}{4(2 + c)(1 - c)}$$

$$q_F^* = \frac{2a - h}{2(2 + c)} - \frac{3t}{4(2 + c)(1 - c)}$$

(23)

From (23) we see that an increase in the tariff: increases $q_d^*$, decreases $q_F^*$, and decreases aggregate final sales. When goods are not perfect substitutes, a small tariff does not eliminate $F$ from the final market. The smallest prohibitive tariff $t^p$, which results in $q_F^* = 0$, is $t^p = 2(1-c)(2a-h)/3$. A smaller degree of substitutability increases $t^p$. With less competition in the final goods market (smaller $c$), a larger tariff is needed in order to exclude imports of final goods. An increase in $h$ increases $F$’s incentive to produce, but its incentive to export the input is increased by an even greater extent. Therefore, a larger value of $h$ reduces the level
of the smallest prohibitive tariff.

For tariffs slightly larger than \( t' \), the non-negativity constraint on \( q_r \) is binding and the formulae in equations (22) and (23) are not valid. However, for such tariffs, firm F still has a credible option to export the final good if it fails to reach an agreement with \( D_d \) in the second stage. If, however, the tariff were so large that F would never want to sell in the final market, the second stage bargaining game makes sense only if we assume that F is able to delay production until it bargains with \( D_d \). Under that assumption, we can show that \( D_d \) sells in the final market the quantity \( a/2 \), the monopoly level associated with zero costs. \( D_d \) buys \( (a-2h)/2 \) from \( U_d \) and \( h \) from F. \( D_d \) uses the high cost domestic supplier to increase the amount of rent it is able to extract from the low cost supplier. If we change the timing of the game, so that \( D_d \) bargains simultaneously with \( U_d \) and F, then in equilibrium \( D_d \) buys \( a/2 \) from F, and nothing from \( U_d \). Clearly \( U_d \) is better off under sequential bargaining (provided that it bargains in the first stage). \( D_d \) is also better off under sequential bargaining, since it would be able to achieve the simultaneous bargaining outcome simply by refusing to buy from \( U_d \) in the first stage. Industry revenues are the same under the two outcomes, but costs are higher under sequential bargaining: Since \( U_d \) and \( D_d \) are both better off under sequential bargaining, the low cost firm F is necessarily worse off.

7. Trade policies when firms are independent

If \( U_f \) and \( D_f \) are not integrated, \( U_f \) does not restrict exports of the intermediate good in order to promote final exports. Regardless of the degree of substitutability between goods, the tariff increases costs for \( D_f \), reduces its supply in the final market, and increases \( D_f \)'s final sales. When resale of the intermediate good is allowed, the downstream firm \( D_f \) sells both in
the final market and to the high-cost supplier, as long as the tariff is not set at the prohibitive level. The effect of a tariff is described in the following proposition. (Recall that we have normalized firm $U_f$'s production costs to 0, and $U_d$'s costs are $h$.)

**Proposition 6:** When the foreign industry is not integrated, a specific tariff $t$ on imports of the final good: (i) Leads to the Nash-Cournot output level with costs $h+t$ (for the foreign industry) and 0 (for the domestic industry), when resale is permitted. (ii) Leads to the output level of the Stackelberg leader with costs $t$ for the foreign industry, and the output level of the Stackelberg follower with 0 costs for the domestic industry, when resale is prohibited. (iii) With or without resale, the tariff reduces final sales of $D_f$ and increases final sales of $D_d$. When demand is not "very convex", the tariff decreases aggregate final sales. (iv) With resale, $D_f$ continues to sell a positive quantity of the intermediate good to $U_d$.

Proposition 6 describes the tariff's effect on final-goods sales by the foreign and domestic firms. With linear demand, we can show that the tariff decreases $U_f$'s aggregate production, and increases that of $U_d$. This holds whether or not there is resale\(^7\). Both with and without resale, the equilibrium second stage trade is independent of the tariff (under linear demand). In both cases we have $x_{df}(t) = h$; when resale is permitted we also obtain $|x_{df}(t)| = h$. The tariff does not alter the cost advantage, which is what drives second-stage trade, and it therefore does not alter the level of exchange.

---

\(^7\) $U_f$'s production is $L(h+t) + 2h$ under resale. With non-resale, it is $h +$ the output of the Stackelberg leader with costs $t$. $U_d$'s production is $G(t+h) - 2h$ under resale; with non-resale, it is $-h +$ the output level of the Stackelberg follower with zero costs.
We see that the effect of the final-goods tariff on $U_d$'s market share depends critically on whether the foreign industry is integrated. In Section 6 we showed that when firms are integrated (and demand is linear and $c < 1$) a non-prohibitive tariff always reduces $U_d$'s market share [equation (22)]. With non-integrated foreign firms, the opposite occurs (footnote 7). The reason is simply that when the foreign firms are integrated, $D_d$ tends to reduce its demand for the input.

We also note from Proposition 6(ii) that the tariff decreases aggregate final supply, which tends to reduce consumers welfare. This, and the analysis above, suggests that when the foreign industry is not integrated, a final-good tariff has the usual effect of shifting rents from foreign to domestic downstream firms, at a cost to consumers, and possibly to domestic upstream firms.

8. Trade Policy and the Incentive to Integrate

Finally, we investigate the effect of the tariff on incentives for foreign integration under linear demand and perfect substitute goods. We first consider the case where resale is permitted. Facing a tariff, the equilibrium profits of a non-integrated foreign industry are

$$\Pi_{D_r} R(t) + \Pi_{U_r} R(t) = \frac{a^2 - a(h + 4t) + 7h^2 + 2ht + 4t^2}{9}.$$  \hspace{1cm} (24)

If firms are integrated, their profits are:
Using (24) and (25), we have $\Pi_F^*(t) > \Pi_{DF}^R(t) + \Pi_{UI}^R(t)$ if and only if

$$a > \frac{6h^2 + 4th + 3t^2}{3h + 2t} \equiv p^R(h, t).$$

The tariff weakens the incentives to integrate, as we see from the relation $\partial p^R/\partial t = 6t(3h + t)/(3h + 2t)^2 > 0$. In the absence of the tariff, the primary incentive to integrate is that it allows $U_i$ to internalize the effect of its sales to $D_i$. We showed above that (for $c = 1$) a tariff eliminates the integrated industry from the final market, increasing the effective cost of the foreign supplier by $t/2$. The sales of the input become less valuable to $F$ as $t$ increases, since $F$'s bargaining power decreases. On the other hand, under non-integration, a tariff reduces $D_i$'s demand of the input, but it does not affect interindustry trade. As $t$ increases, this trade becomes more valuable, thus reducing the low-cost supplier's incentives to integrate.\(^8\)

When resale is prohibited, a tariff has the opposite effect on the incentive to integrate. Without resale, industry profit is:

$$\Pi_{DF_{NR}}(t) + \Pi_{UI_{NR}}(t) = \frac{a^2 + 4t^2 - 4at + 4h^2}{8}$$

and $\Pi_F^*(t) > \Pi_{DF_{NR}}(t) + \Pi_{UI_{NR}}(t)$ if and only if

---

\(^8\) Since $\partial p/\partial h > 0$, a large cost difference weakens the incentives to integrate. As we discussed in Section 5, for $c = 1$, under free trade the benefit from the internalization of the final rivalry always overwhelms the loss due to diminished interindustry trade - whatever is the cost difference - so integration occurs for all values of $h$. With a tariff, the difference in suppliers' costs may alter the decision to integrate, even if $c = 1$. 
Since \( \partial p^\text{NR}/\partial t > 0 \), an increase in the tariff reinforces the incentives to integrate. The tariff reduces the benefit of having the foreign downstream firm become a Stackelberg leader by reducing \( D_t \)'s final sales and the demand for the input faced by the low-cost supplier\(^9\). Thus, the effects of a tariff on the incentives for foreign integration critically depend on whether resale is allowed.

These results imply that a small change in the tariff can affect the industry structure, and thus lead to large changes in output and market shares. For example, suppose that the foreign industry is not integrated and resale is allowed, and the tariff is just below the critical value at which foreign firms are indifferent about integration. In that case, an increase in the tariff induces integration. If, on the other hand, the foreign industry is not integrated but resale is not permitted, a small decrease in a tariff might make vertical integration profitable.

9. Concluding remarks

In vertically-related international markets, a more efficient integrated industry has an incentive to restrict its supply of the input. This may induce the government of the less efficient country to tax imports of the final good in order to stimulate vertical supply. We have investigated this scenario in a duopoly, where upstream and downstream firms bargain sequentially over the exchange of the intermediate good. The effects of a tariff on final

\[ a < 2(4h + 5t) + 6\sqrt{h^2 + 2t^2 + 4ht} \equiv p^\text{NR}(t,h) \]  

\[^9\text{Since } \partial p^\text{NR}/\partial h > 0, \text{ an increase in the cost difference encourages integration. Under integration and } c = 1, \text{ an increase in } h \text{ represents an increase in the costs of the integrated industry's rival in the Cournot-Nash equilibrium.}\]
imports depend on the industry configuration in the foreign country and, in the presence of vertical integration, on whether the integrated structure vertically forecloses or supplies the intermediate good.

Perfect substitutability between final goods is a sufficient condition for vertical foreclosure to occur. By foreclosing the supply of the input, the integrated structure increases the downstream rival’s costs, improving its position in the final market. The introduction of an arbitrarily small tariff on imports of the final good eliminates the integrated structure from the final market and induces vertical supply. In this context, bargaining enables the integrated industry to pass part of the cost of the tariff to the downstream rival. The tariff has a large impact on trade flows, replacing final good imports with intermediate good imports. However, it merely raises one firm’s costs, so a small tariff has small efficiency effects. When the foreign industry does not foreclose the supply of the intermediate good, a tariff stimulates interindustry trade, but a non-negligible tariff is needed to eliminate the integrated structure from the final market. The effects of a tariff on the high-cost supplier’s market share differ in these two situations. In general, the share effects depend on the industry configuration prevailing in the foreign country and, under vertical integration, on the domestic downstream firm’s demand of the input.

When the foreign industry is not integrated, a tariff acts as a simple rent-extracting device, by increasing the costs of the foreign downstream firm, favoring domestic activities, and worsening consumers’ welfare. Its effects on the incentives to integrate depend on the availability of the resale option under non-integration. When resale is allowed, firms integrate whenever the benefit from the internalization of final market rivalry overwhelms the loss of
the surplus due to reallocation of production. The introduction of a tariff weakens the incentives to integrate, since the tariff raises the integrated structure's costs, and (at least in the linear example) it does not affect interindustry trade. When resale is prohibited, integration causes a change from the Stackelberg to the Cournot-Nash equilibrium. This loss is accompanied by an increase in the costs faced by the rival downstream firm, making integration more profitable the larger is the difference in suppliers' costs. In this context, a tariff reinforces the incentives to integrate, because it reduces the primary benefit from being non-integrated, i.e. the downstream Stackelberg leadership.

When goods are perfect substitutes and the foreign supplier has a large cost advantage, both the foreign upstream and downstream firms would agree to resale prohibition. Under resale prohibition, the foreign firms would integrate, and foreclose the supply of the intermediate good. A small tariff would eliminate the integrated structure from the final market, but it might also prevent integration from occurring.
Proof of Proposition 1: (i) The justification for this was given in the paragraph below equation (6) in the text. (ii) This follows from comparing the first order conditions to (5) and (10). The comparison uses the fact that firm \( D_i \)'s marginal revenue is decreasing in its own output (the second order condition), and the assumption \( h > 0 \).

Proof of Proposition 3: (i) If resale is forbidden, we must impose the constraint \( x_{td} \geq 0 \) and \( x_{df} \geq 0 \). From Proposition 1 we know that the first constraint is binding, so \( x_{td} = 0 \). The bargain between \( D_d \) and \( U_f \) in the second stage generates the surplus defined in equation (6).

Maximization of (6) with respect to \( q_{ia} \) implies the best-response function \( q_{ia}(q_i) \); \( q_i \) is determined in the first stage when \( D_r \) and \( U_f \) bargain (since \( x_{td} = 0 \)). Profits for \( D_r \) are

\[
\Pi_{D_r} = q_p[q_f + c q_{ia}(q_{ia})] - T_{af}
\] (A1)

while profits for \( U_f \) are given by equation (8). The disagreement point is \( d_{af} = \{ \Pi_{D_r^o}, \Pi_{U_f^o} \} \):

\[
d_{af} = \{ 0 , T_{af} \}
\] (A2)

and the first-stage surplus is:

\[
S_{af} = q_f p[q_f + c q_{ia}(q_{ia})]
\] (A3)

Maximization of \( S_{af} \) in equation (A3) with respect to \( q_i \) determines the first-stage equilibrium quantity, which is then transformed into final output and sold in the final market. The output levels are the same as in a game where a vertically-integrated firm with zero costs acts as a Stackelberg leader and faces an identical follower.
(ii) We assume that the demand is not "very convex", i.e. that it satisfies:

\[ p_i' < -\frac{1 - c}{2 - c} q_i p_i'' \]  

where \( p_i = p(q_i + cq) \), \( i,j = f,d \), and \( i \neq j \). Under this assumption, \(-1 < dq_i/dq_j < 0^{10}\): An increase in one firm's final sales leads to a smaller decrease in the other firm's sales, so that aggregate sales increase. Consider the symmetric Cournot equilibrium associated with zero costs. A shift from the Cournot to the Stackelberg equilibrium implies an increase in \( q_i \), a smaller reduction in \( q_d \), and an increase in aggregate output. The outcome with resale equals the asymmetric Cournot equilibrium with costs zero and \( h \). Clearly aggregate output in that game is lower than in the symmetric Nash game, since one firm's costs have increased.

*Proof of Proposition 5ii:* From part (i) (in the text) we know that \( x_{df} = x_F \) and the second stage surplus is \( tx_p \). As a result of the bargain \( D_d \)'s revenues increase by \( x_F p(x_d + x_F) \), so the transfer is \( T_{df} = x_F [p(x_d + x_F) - t/2] \). This uses the fact that the Nash cooperative solution splits the surplus. In the first stage, \( F \) takes \( x_d \) as given and chooses \( x_F \) to maximize \( T_{df} \). \( F \) behaves as a "Cournot duopolist" with costs \( t/2 \). If we write the bargaining problem for \( U_d \) and \( D_d \) we see that they take \( x_F \) as given and choose \( x_d \) to maximize the surplus \( x_d [p(x_d + x_F) - h] \).

---

\(^{10}\) This condition is always satisfied when goods are perfect substitutes and reaction functions are assumed to be downward sloping. The "weak stability condition" (Dixit 1986) is met in our model since marginal costs are constant.
Proof of Proposition 6: (i) The surplus to be split between \( U_d \) and \( D_f \) is:

\[
S_{pf}(t) = q_f(t) \{ p[q_f(t) + cq_f(t)] - (t + h) \} - x'_f(t) \{ p[x'_f(t) + cq'_f(t)] - (t + h) \}.
\]

(A5)

Each unit of final exports incurs the cost of the tariff and the opportunity cost of production in \( U_d \).

The second-stage surplus arising from the bargaining between \( D_d \) and \( U_f \) is given by equation (6) (quantities are understood to be functions of \( t \)). The second-stage maximand for firm \( D_f \) is equivalent to that of a vertically-integrated firm with costs \( (h + t) \), while \( D_d \)'s maximand is equivalent to that of a firm with zero costs. Thus, \( q_f^*(t) = L(h + t) \), and \( q_d^*(t) = G(h + t); L(h + t) < L(h) \) and \( G(h + t) > G(h) \).

(ii) When resale is prohibited, the second-stage surplus arising through the bargain between firms \( D_d \) and \( U_f \) is given by equation (6). Maximization of \( S_{df} \) with respect to \( q_d \) determines the second-stage output \( q_d(q_f) \). In the first stage, the bargaining between \( U_f \) and \( D_f \) leads to the surplus:

\[
S_{pf}(t) = q_f(t) \{ p[q_f(t) + q_d(q_f(t))] - (t + h) \}
\]

(A6)

Maximization of \( S_{df} \) with respect to \( q_{df} \) yields the Stackelberg leader's output level associated with costs \( t \), while the follower's costs are zero.

(iii) When condition (A4) is met, \( -1 < dq_d/dq_f < 0 \): A decrease in \( q_f \) due to the introduction of the tariff leads to a smaller increase in \( q_d \), causing aggregate sales to fall.

(iv) This exactly parallels the proof of Proposition 1.ii, except that \( t \) is included in \( S_{df} \) and \( S_{td} \).
REFERENCES


Fig. 1: Resale

Production levels:

\[ x_f = x_{ff} + x_{df} \]
\[ x_d = x_{dd} + x_{fd} \]

Final Sales:

\[ q_f = x_{ff} + x_{fd} \]
\[ q_d = x_{dd} + x_{df} = G(m,h) \]
\[ = L(h,m) \]
Fig. 2: Nonresale

Production levels:

\[ x_f = x_{ff} + x_{df} \]

\[ x_d = x_{dd} \]

Final Sales:

\[ q_f = x_{ff} \]

\[ q_d = x_{dd} + x_{df} \]
Fig. 3: Vertical Integration

\[ q_d = x_d + x_{dF} \]

\[ q_F = x_F - x_{dF} \]
**Computations**

To accompany

Vertically Related Markets and Trade Policy in a Bargaining Framework

by L. Karp and L. Sioli

(1) Bargaining problem with asymmetric industries

2nd stage (bargaining between $D_f$ and $U_d$):

$D_f$’s profit:

$$\Pi_{D_f} = (x_{ff} + x_{fd})p(x_{ff} + x_{fd} + c(x_{dd} + x_{df})) - T_{ff} - T_{fd}$$  \hspace{1cm} (1)

Setting $q_d = x_{ff} + x_{fd}$ and $q_d = x_{dd} + x_{df}$:

$$\Pi_{D_f} = q_f p(q_f + c q_d) - T_{ff} - T_{fd}$$  \hspace{1cm} (2)

$U_d$’s profit:

$$\Pi_{U_d} = T_{dd} + T_{fd} - h(x_{dd} + q_f - x_{ff})$$  \hspace{1cm} (3)

Disagreement point: $d_{fd} = (\Pi_{D_f}^*, \Pi_{U_d}^*)$:

$$d_{fd} = \{ x_{ff} p(x_{ff} + c(x_{dd} + x_{df})) - T_{ff} ; T_{dd} - h x_{dd} \}$$  \hspace{1cm} (4)

Second-stage surplus $S_{fd}$:

$$S_{fd} (q_f | q_d, x_{ff}) = q_f [p(q_f + c q_d) - h] - x_{ff} [p(x_{ff} + c q_d) - h]$$  \hspace{1cm} (5)

Second-stage surplus ($S_{df}$) arising from the simultaneous bargaining between $U_d$ and $D_f$:

$$S_{df} (x_{df} | x_{dd}, q_f, x_{ff}) = (x_{df} + x_{dd}) p(x_{dd} + x_{df} + c(x_{ff} + x_{fd}))))$$  \hspace{1cm} (6)

Using linear demand $p(Q) = a - q_i - c q_j$ (i,j = d,f, i $\neq$ j);

$$S_{fd} = q_f (a - q_j - c q_d - h) - x_{ff} (a - x_{ff} - c q_d - h)$$  \hspace{1cm} (7)
Maximization of $S_{df}$ with respect to $q_d$ (taking $q_d$ as given) and maximization of $S_{df}$ with respect to $q_d$ yield the reaction functions:

$$q_d(q_d) = \frac{a - cq_d - h}{2}$$

$$q_f(q_f) = \frac{a - cq_f}{2}$$

Second-stage equilibrium output:

$$q_d^* = \frac{a}{2 + c} + \frac{ch}{4 - c^2} \equiv G$$

$$q_f^* = \frac{a}{2 + c} - \frac{2h}{4 - c^2} \equiv L$$

First stage (bargaining between $U_f$ and $D_f$):

2nd-stage transfer:

$$T_{df}^* = \frac{S_{df}}{2} = \frac{1}{2} q_d p(q_d + cq_f) - \frac{1}{2} x_{ad} p(x_{ad} + cq_f)$$

Substituting 2nd-stage equilibrium output and transfer into (1), $D_f$’s first-stage profit is:

$$\Pi_{D_f} = \frac{1}{2} \{ q_f^* [ p(q_f^* + cq_d^*) - h] + x_{df} [ p(x_{df} + cq_f) + h] \} - T_{df}.$$
U_t's profit:

$$\Pi_{U_t} = T_{ff} + T_{df}^*$$  \hspace{1cm} (14)

Disagreement point $d_{it} = \{\Pi_{df}^o, \Pi_{ut}^o\}$:

$$d_{ff} = \{ \frac{1}{2} q_f^*[p(q_f^* + cq_d^*) - h]]; T_{df} \}.$$  \hspace{1cm} (15)

First-stage surplus $S_{it}$:

$$S_{ff}(x_{ff}) = \frac{1}{2} x_{ff}^{} [p(x_{ff}^* + cq_d^*) + h].$$  \hspace{1cm} (16)

By symmetry:

$$S_{dd}(x_{dd}) = \frac{1}{2} x_{dd}^{} [p(x_{dd}^* + cq_f^*)] - h x_{dd}.$$  \hspace{1cm} (17)

Using the linear demand:

$$S_{ff} = \frac{1}{2} x_{ff}^{} (a - x_{ff}^* - cG + h)$$  \hspace{1cm} (18)

$$S_{dd} = \frac{1}{2} x_{dd}^{} (a - x_{dd}^* - cL - 2h)$$  \hspace{1cm} (19)

Maximization of $S_{it}$ with respect to $x_{it}$ and of $S_{dd}$ with respect to $x_{dd}$ yield first-stage equilibrium output:

$$x_{ff}^* = \frac{1}{2} (a - cG + h)$$  \hspace{1cm} (20)

$$x_{dd}^* = \frac{1}{2} (a - cL - 2h)$$

and:
\( x_{df}^* = q_{d}^* - x_{dd}^* = h \)
\( x_{jd}^* = q_{j}^* - x_{jj}^* = |h| \)

(2) Prohibition against resale: linear demand

2nd stage (bargaining between \( D_d \) and \( U_f \)):

\[
\Pi_{D_d} = (x_{dd} + x_{df})(a - x_{dd} - x_{df} - cq) - T_{dd} - T_{df}
\]

(22)

\( U_f \)'s profit is given by (14). Disagreement point \( d_{df} = (\Pi_{D_d}^*, \Pi_{U_f}^*) \):

\[
d_{df} = \{ x_{dd}(a - x_{dd} - cq) - T_{dd} ; T_{ff} \}
\]

(23)

Second-stage surplus:

\[
S_{df} = x_{df}(a - 2x_{dd} - x_{df} - cq)
\]

(24)

Maximization of \( S_{df} \) with respect to \( x_{df} \) yields:

\[
x_{df} = \frac{a - cq}{2} - x_{dd}
\]

(25)

Let \( (a - cq)/2 \equiv k \).

Substituting (25) into (24) delivers \( S_{df}^* = x_{df}^2 \) and \( T_{df}^* = x_{df}^2/2 \).

First stage (bargaining between \( D_d \) and \( U_j \)):

\( D_d \)'s profit:

\[
\Pi_{D_d} = k(a - k - cq) - T_{dd} - \frac{(k - x_{dd})^2}{2}
\]

(26)

\( U_d \)'s profit is given by (3).

Disagreement point \( d_{dd} = (\Pi_{D_d}^*, \Pi_{U_d}^*) \):
\[ d_{dd} = \{ k(a - k - cq) - \frac{k^2}{2} : 0 \} \]  

First-stage surplus:

\[ S_{dd} = \frac{k^2}{2} - \frac{(k - x_{dd})^2}{2} - hx_{dd} \]  

Maximization of \( S_{dd} \) with respect to \( x_{dd} \) yields:

\[ x_{dd}^* = k - h \]  

Equilibrium total output is given by the Stackelberg quantities produced by two firms with zero costs (firm f is the leader and firm d is the follower):

\[ q_f^* = \frac{a(2 - c)}{2(2 - c^2)} \]
\[ q_d^* = \frac{a(4 - 2c - c^2)}{4(2 - c^2)} \]  

Substituting \( q_f^* \) into (29):

\[ x_{dd}^* = \frac{a(4 - 2c - c^2) - 4h(2 - c^2)}{4(2 - c^2)} \]

\[ x_{df}^* = q_d^* - x_{dd}^* = h \]  

(3) \( U_f \)'s profit under resale and nonresale

\[ \Pi_{U_f} = T_f + T_{df} = \frac{S_{ff}}{2} + \frac{S_{df}}{2} \]  

Under Resale: \( S_{df} \) is given by (16), and \( x_{df}^* = L + h \). \( S_{df} = x_{df}^2 = h^2 \). Substituting \( x_{df}^* \) and the value of \( G \) into (16) delivers:
\[ \Pi_{\text{NR},R} = \frac{[a(c - 2) + h(c^2 - 2)]^2}{2(c + 2)^2(c - 2)^2} + \frac{h^2}{2} \]  

Under NonResale: using the linear demand function in equation (A3) in the paper:

\[ S_{\text{NR}} = q_f[a - q_f - \frac{a - cq_f}{2}] \]

\[ = \frac{1}{8} \frac{a^2(2-c)^2}{2 - c^2} \]

\[ S_{\text{nr}} = x_{\text{nr}}^2 = h^2, \text{ and} \]

\[ \Pi_{\text{NR},R} = \frac{1}{16} \frac{a^2(2-c)^2}{2 - c^2} + \frac{h^2}{2} \]  

Comparison between \( \Pi_{\text{NR},R} \) and \( \Pi_{\text{NR},R} \):

\[ \frac{\Pi_{\text{NR},R} - \Pi_{\text{NR},R}}{16(2 - c^2)(c + 2)^2(c - 2)^2} = \frac{a^2(c - 2)^2(c^4 - 4c^2 + 8) + 8ah(c - 2)(c^2 - 2)^2 + 4h^2(c^2 - 2)^3}{16(2 - c^2)(c + 2)^2(c - 2)^2} \]

\[ \frac{\partial (\Pi_{\text{NR},R} - \Pi_{\text{NR},R})}{\partial h} = \frac{(2 - c^2)[a(c - 2) + h(c^2 - 2)]}{2(c + 2)^2(c - 2)^2} < 0 \]

If \( c = 1 \), \( \Pi_{\text{NR}} - \Pi_{\text{NR},R} = (5a^2 - 8ah - 4h^2)/144 > 0 \) iff \( a > 2h \).

If \( c = 0 \), \( \Pi_{\text{NR}} - \Pi_{\text{NR},R} = (a^2 - 2ah - h^2)/16 > 0 \) iff \( a > h(1 + \sqrt{2}) \). Since \( a \geq 5h \) by assumption, \( \Pi_{\text{NR}} > \Pi_{\text{NR},R} \) always.

(4) \( D_t \)'s profits under resale and nonresale

\[ \Pi_{D_t} = q_f p(q_f + cq_d) - T_{\text{nr}} - T_{id} \]

Under resale, \( T_{\text{nr}} \) is given by the first term in equation (33), \( T_{id} = -h^2/2, q_t = L, \) and \( q_d = G \). Thus:
\[ \Pi_D^R = \frac{3a^2(c - 2)^2 + 2ah(c - 2)(c^2 + 2) + h^2(c^4 - 4c^2 + 12)}{4(c + 2)^3(c - 2)^2} \]  
\hspace{1cm} (39) 

Under nonresale \((T_n = 0)\):

\[ \Pi_D^{NR} = \frac{a^2(c - 2)^2}{16(2 - c^2)} \]  
\hspace{1cm} (40) 

and \( \Phi_D = \Pi_D^R - \Pi_D^{NR} = \)

\[ \frac{a^2(c - 2)^2(c^4 + 4c^2 - 8) + 8ah(c - 2)(c^2 - 2)(c^2 + 2) + 4h^2(c^2 - 2)(c^4 - 4c^2 + 12)}{16(c^2 - 2)(c + 2)^3(c - 2)^2} \]  
\hspace{1cm} (41) 

with

\[ \frac{\partial \Phi_D}{\partial h} = \frac{a(c - 2)(c^2 + 2) + h(c^4 - 4c^2 + 12)}{2(c + 2)^3(c - 2)^2} \equiv k \]  
\hspace{1cm} (42) 

\( k > 0 \) iff

\[ h > \frac{a(2 - c)(c^2 + 2)}{c^4 - 4c^2 + 12} \equiv u \]  
\hspace{1cm} (43) 

and \( z > 1/5 \). Thus, since by assumption \( h \leq a/5 \), \( \partial \Phi_D / \partial h < 0 \).

When \( c = 1 \):

\[ \Phi_D(a,1,h) = \frac{1}{48}(a^2 - 8ah + 12h^2) \]  
\hspace{1cm} (44) 

\( \Phi(a,1,h) > 0 \) iff \( a > 6h \): \( \Pi_D^R > \Pi_D^{NR} \) for \( h < a/6 \).

When \( c = 0 \):

\[ \Phi_D(a,0,h) = \frac{1}{16}(a^2 - 2ah + 3h^2) \]  
\hspace{1cm} (45) 

\( \Phi(a,0,h) = (1/16)[(a - h)^2 + 2h^2] > 0. \)
(4) Vertical integration

To obtain equation (16) in the paper:

Integrated firm’s profit:

$$\Pi_F = (x_F - x_{df})p[x_F - x_{df} + c(x_d + x_{df})] + T_{df}$$  \hspace{1cm} (46)

Firm D’s profit:

$$\Pi_{D_d} = (x_d + x_{df})p[x_d + x_{df} + c(x_F - x_{df})] - T_{df} - T_{dd}$$  \hspace{1cm} (47)

Disagreement point $$d_{df} = \{ \Pi_{F}^o, \Pi_{D_d}^o \}$$:

$$d_{df} = \{ x_d p(x_A + cx_d) - T_{dd}; x_F p(x_F + cx_d) \}$$  \hspace{1cm} (48)

$$S_{df} = \Pi_F - \Pi_{F}^o + \Pi_{D_d} - \Pi_{D_d}^o$$, as given by (16) in the paper.

Introducing the linear demand:

2nd stage:

$$\Pi_F = (x_F - x_{df})[a - (x_F - x_{df}) - c(x_d + x_{df})] + T_{df}$$  \hspace{1cm} (49)

$$\Pi_{D_d} = (x_d + x_{df})[a - (x_d + x_{df}) - c(x_F - x_{df})] - T_{dd} - T_{df}$$  \hspace{1cm} (50)

$$d_{df} = \{ x_d(a - x_d - cx_p) - T_{dd}; x_F(a - x_F - cx_d) \}$$  \hspace{1cm} (51)

$$S_{df} = 2x_d(1 - c)(x_F - x_{df} - x_d)$$  \hspace{1cm} (52)

Assume c \neq 1.

Maximization of $$S_{df}$$ with respect to $$x_{df}$$ yields the second-stage equilibrium quantity:
First stage:

(a) Bargaining between \( D_d \) and \( U_d \):

Second-stage transfer:

\[
T^*_d(x_F=x_d) = \frac{1}{2}(x_F - x_d)(a - x_F - x_d)
\]  \hspace{1cm} (54)

Substituting equations (53) and (54) into (50), firm \( D_d \)'s profit becomes:

\[
\Pi_{D_d} = \frac{1}{4}(x_F + x_d)[2a - (1 + c)(x_F + x_d)]
\]

\[
- \frac{1}{2}(x_F - x_d)(a - x_F - x_d) - T_{dd}
\]  \hspace{1cm} (55)

\( U_d \)'s profit:

\[
\Pi_{U_d} = T_d + hx_d
\]  \hspace{1cm} (56)

Disagreement point \( d_d = \{\Pi_{D_d}^*,\Pi_{U_d}^*\} \):

\[
d_d = \{ \frac{1}{4}x_d[2a - (1 + c)x_d] - \frac{1}{2}x_d(a - x_F); 0 \}
\]  \hspace{1cm} (57)

First-stage surplus:

\[
S_d = \frac{1}{4}(x_d + x_F)[2a - (1 + c)(x_d + x_F)]
\]

\[
- \frac{1}{2}(x_F - x_d)(a - x_F - x_d) - hx_d
\]  \hspace{1cm} (58)

Maximization of \( S_d \) with respect to \( x_d \) yields:
(b) Profit maximization by the integrated industry:

Introducing (53) and (54) into (49):

\[ x_d^*(x_p) = \frac{2a - 2h - (1 + c)x_F}{3 + c} \]  

(59)

\[ \Pi_F = \frac{1}{4}[(x_F + x_d)[2a - (x_F + x_d)(1 + c)] + \frac{1}{2}(x_F - x_d)(a - x_F - x_d)] \]  

(60)

Maximization of \( \Pi_F \) with respect to \( x_F \) yields:

\[ x_F^*(x_d) = \frac{2a - (1 + c)x_d}{3 + c} \]

(61)

Substitution of (61) into (59) yields the first-stage equilibrium quantities:

\[ x_F^* = \frac{2a + h(1 + c)}{2(2 + c)} \]

\[ x_d^* = \frac{2a - h(3 + c)}{2(2 + c)} \]

\[ x_{df}^* = \frac{h}{2} \]

\[ x_F^* - x_{df}^* = x_d^* + x_{df}^* = \frac{2a - h}{2(2 + c)} \]

(62)

(5) Low-cost industry profits

Non integration and resale:

\[ \Pi_{U_f}^R + \Pi_{D_f}^R = L(a - L - cG) - T_{df} + T_{df} \]  

(63)

\[ T_{U_f} = S_{Uf}/2 - h x_{Uf} = h^2/2 - h^2; \]

\[ T_{df} = S_{df}/2 = h^2/2. \]  

Thus:
\[ \Pi_{u_i}^R + \Pi_{D_i}^R = L(a - L - cG) + h^2 \quad (64) \]

**Non integration and nonresale:**

\[ \Pi_{u_i}^{NR} + \Pi_{D_i}^{NR} = q_j(a - q_f - cq_j) + T_{sf} \quad (65) \]

\[ T_{sf} = S_{sf}/2 = h^2/2. \text{ Thus:} \]

\[ \Pi_{u_i}^{NR} + \Pi_{D_i}^{NR} = \frac{a^2(2 - c)^2}{8(2 - c^2)} + \frac{h^2}{2} \quad (66) \]

**Integration:**

\[ \Pi_P = \frac{4a^2 + 2ach(c + 3) + h^2(c + 3)}{4(c + 2)^2} \quad (67) \]

**Integration and resale:**

\[ \Pi_P - (\Pi_{u_i}^R + \Pi_{D_i}^R) = \frac{h[2ac(c - 3)(c - 2) - h(4c^3 - 9c^2 - 5c + 18)]}{4(c + 2)(c - 2)^2} > 0 \quad (68) \]

iff

\[ a > \frac{h(4c^3 - 9c^2 - 5c + 18)}{2c(c^2 - 5c + 6)} \equiv \beta \quad (69) \]

When \( h = a/5 \): \( \beta(\cdot) = ka/5 \). \( a > ka/5 \) iff

\[ z \equiv 6c^3 - 41c^2 + 65c - 18 > 0 \quad (70) \]

with \( \partial z/\partial c > 0 \). Since \( z \) is monotonic in \( c \), and since \( z(c = 0) < 0 \) and \( z(c = 1) > 0 \), there exists \( c^* \in (0,1] \) such that \( a \leq \beta(a/5,c) \) for \( c \in (0,c^*] \), and \( a > \beta(a/5,c) \) for \( c \in (c^*,1] \).

**Integration and nonresale:**

Let \( \Delta(a,c,h) \equiv \Pi_P^* - (\Pi_{u_i}^{NR} + \Pi_{D_i}^{NR}) \).
When \( h = 0 \):

\[
\Delta(a,c,0) = \frac{-a^2 c^4}{8(c + 2)^2(2 - c^2)} < 0
\]  

(71)

and integration doesn't occur.

When \( h = a/5 \):

\[
\Delta(a,c) = a^2 \frac{41c^4 + 46c^3 - 92c^2 - 92c + 20}{200c^2 - 2(2c + 2)^2}
\]  

(72)

By plotting \( \Delta(a,c) \) against \( c \), \( \Delta(a,c) > 0 \) for \( c > c' \) (integration occurs for high values of \( c \)), and \( \Delta(a,c) \leq 0 \) for \( c \leq c' \).

(6) Vertical integration and tariff; \( c = 1 \)

Second stage (bargaining between \( F \) and \( D_d \)):  

Integrated industry's profit:

\[
\Pi_F(t) = [x_F(t) - x_{df}(t)][p[x_F(t) + x_d(t)] - t] + T_{df}(t)
\]  

(73)

Downstream firm's profit:

\[
\Pi_{D_d}(t) = [x_d(t) + x_{df}(t)][p[x_d(t) + x_F(t)] - T_{df}(t) - T_d(t)
\]  

(74)

Disagreement point \( d_{df} = \{\Pi_{df}^*, \Pi_{ff}^*\} \):

\[
d_{df}(t) = \{ x_d(t)p(x_d(t) + x_F(t)) - T_d(t); x_F(t)[p(x_F(t) + x_d(t)) - t]\}
\]  

(75)

Second-stage surplus:
Second-stage transfer:

\[ T_{af}(t) = x_{af}(t) \left[ p(x_d(t) + x_p(t)) - \frac{I}{2} \right] \quad (77) \]

Set \( x_{af} = x_p \).

**First stage:**

(a) **Bargaining between** \( U_d \) and \( D_d \):

**Downstream profit:**

\[ \Pi_{D_d}(t) = x_d(t)p[x_d(t) + x_p(t)] + \frac{I}{2}x_p(t) - T_d(t) \quad (78) \]

**Upstream profit:**

\[ \Pi_{U_d}(t) = T_d(t) - hx_d(t) \quad (79) \]

**Disagreement point** \( d_d = \{ \Pi_{D_d^0}, \Pi_{U_d^0} \} \):

\[ d_d(t) = \left\{ \frac{I}{2}x_p(t), 0 \right\} \quad (80) \]

where \( x_{af}^0 \) implies \( x_d = 0 \). **First-stage surplus:**

\[ S_d(t) = x_d \left( p[x_d(t) + x_p(t)] - h \right) \quad (81) \]

(b) **Profit maximization by** \( F \):

\[ \Pi_F(t) = x_p(t) \left[ p[x_d(t) + x_p(t)] - \frac{I}{2} \right] \quad (82) \]
(7) Foreign industry’s profits under vertical integration, tariff, and \( c = 1 \): 

From above, assuming linear demand, optimal output level are given by the Cournot interaction of two firms with costs \( t/2 \) and \( h \):

\[
x_p^* = \frac{a - t + h}{3} \\
x_d^* = \frac{a - 2h + t/2}{3}
\] (83)

The integrated firm’s Cournot profit is \( \Pi_p = x_p x_d^* \).

(8) Foreign industry’s profits under nonintegration and resale, tariff, and \( c = 1 \). Linear demand:

2nd stage (bargaining between \( D_f \) and \( U_d \)):

(variables are functions of \( t \))

\( D_f \)'s profit:

\[
\Pi_{D_f} = q_f [p(q_f + c q_d) - t] - T_{ff} - T_{fd}
\] (84)

\( U_d \)'s profit:

\[
\Pi_{U_d} = T_{dd} + T_{fd} - h(x_{dd} + q_f - x_d^*).
\] (85)

Disagreement point: \( d_{fd} = (\Pi_{D_f}^{a}, \Pi_{U_d}^{a}) \):

\[
d_{fd} = \{ x_f [p(x_f + c q_d) - t] - T_{ff} ; T_{dd} - h x_{dd} \}.
\] (86)

Second-stage surplus \( S_{fd} \):

\[
S_{fd} = q_f [p(q_f + c q_d) - (t + h)] - x_f [p(x_f + c q_d) - (t + h)]
\] (87)

Second-stage surplus \( (S_{dt}) \) arising from the simultaneous bargaining between \( U_f \) and \( D_d \):
Using linear demand, final sales are:

\[ S_{df} = q_d \ p[q_d + c q_d] - x_{df} \ p[x_{df} + c q_d] \]  

(88)

Using linear demand, final sales are:

\[ q_d^* = \frac{a + t + h}{3} \]
\[ q_f^* = \frac{a - 2t - 2h}{3} \]  

(89)

First stage (bargaining between \( U_f \) and \( D_f \)):

2nd-stage transfer:

\[ T_{fd}^* = \frac{S_{fd}}{2} - h(q_f - x_{df}) \]  

(90)

Substituting 2nd-stage equilibrium output and transfer into (84), \( D_f \)'s first-stage profit is:

\[ \Pi_{D_f} = \frac{1}{2} \{ q_f^* [p(q_f^* + q_d^*) - (t + h)] + x_{df} [p(x_{df} + q_d^* - t + h)] - T_{df}^* \} - T_{df}^*. \]  

(91)

\( U_f \)'s profit:

\[ \Pi_{U_f} = T_{df} + T_{df}^* \]  

(92)

Disagreement point \( d_{df} = \{ \Pi_{Df}^*, \Pi_{Uf}^* \} \):

\[ d_{df} = \{ \frac{1}{2} q_f^* [p(q_f^* + q_d^*) - (t + h)]; T_{df}^* \} \]  

(93)

First-stage surplus \( S_{df} \):

\[ S_{df}(x_{df}) = \frac{1}{2} x_{df} [p(x_{df} + c q_d^*) - t + h]. \]  

(94)

Using the linear demand:
Maximization of $S_{ff}$ with respect to $x_{ff}$ yields:

$$x_{ff}^* = \frac{a - 2t + h}{3}$$  \hspace{1cm} (96)

and $x_{ld}^* = q_l^* - x_{ff}^* = -h$.

In the less efficient country, both bargaining stages are the same as under free trade. Thus, $x_{ff}^* = h$.

Profits of the foreign industry:

$$\Pi_{ul}^* + \Pi_{dj}^* = q_f(a - q_f - q_d - t) + T_{af} - T_{jd} = \frac{1}{9}[a^2 - a(h + 4t) + 7h^2 + 4t^2 + 2ht]$$  \hspace{1cm} (97)

$\Pi_F^*$ is given in the above section. $\Pi_F^* - (\Pi_{ul}^* + \Pi_{ld}^*) =

$$\frac{1}{9}(-3t^2 - 6h^2 + a(3h + 2t) - 4ht)$$  \hspace{1cm} (98)

which is $> 0$ iff:

$$a > \frac{6h^2 + 3t^2 + 4ht}{3h + 2t}$$  \hspace{1cm} (99)

(9) Foreign industry's profits under nonintegration and nonresale, tariff, and $c = 1$. Linear demand:

The second stage (bargaining between $D_a$ and $U_d$) is the same as without a tariff.

In the first stage, the bargaining between firms in the foreign country leads to the maximization of

$$S_{ff} = q_f(a - q_f - q_d - t)$$  \hspace{1cm} (100)

which yields:
\[
q_f = \frac{a}{2} - t
\]
\[
q_d = \frac{1}{4}(a + 2t)
\]
\[
x_{df} = h
\]

and

\[
\Pi_{U_f}^{NR}(t) + \Pi_{D_f}^{NR}(t) = \frac{1}{8}(a^2 + 4t^2 - 4at + 4h^2)
\]

Comparing \(\Pi^*_F\) and the profit in (102):

\[
\Pi^*_F - (\Pi_{U_f}^{NR} + \Pi_{D_f}^{NR}) = -\frac{a^2 - 4a(4h + 5t) + 4(7h^2 + 4ht + 7t^2)}{72}
\]

which is > 0 iff

\[
k_1 \equiv \frac{2(4h + 5t) - 6\sqrt{h^2 + 2t^2 + 4ht}}{2(4h + 5t) + 6\sqrt{h^2 + 2t^2 + 4ht}} < a < \frac{2(4h + 5t) + 6\sqrt{h^2 + 2t^2 + 4ht}}{2(4h + 5t) + 6\sqrt{h^2 + 2t^2 + 4ht}} \equiv k_2
\]

with \(k_1 < 5h < k_2\).

(10) Vertical integration, tariff and \(c \neq 1\)

Second stage:

Using the linear demand, firm F's profit is:

\[
\Pi_F = (x_F - x_{df})(a - (x_F - x_{df}) - c(x_d + x_{df}) - t) + T_{df}
\]

where variables are functions of \(t\).

Firm D_f's profit:

\[
\Pi_{D_f} = (x_d + x_{df})(a - (x_d + x_{df}) - c(x_F - x_{df})) - T_d - T_{df}
\]

Disagreement point \(d_{df} = \{\Pi_{D_f}, \Pi_F\}\):

\[ d_{df} = \{ x_d (a - x_d - cx_d) - T_{df} : x_F (a - x_F - cx_d - t) \} \]  

(107)

Second-stage surplus:

\[ S_{df} = x_{df} [2(1 - c)(x_F - x_{df} - x_d) + t] \]  

(108)

Maximization of \( S_{df} \) with respect to \( x_{df} \) yields the second-stage equilibrium quantity:

\[ x_{df}(x_F, x_d) = \frac{1}{2} (x_F - x_d) + \frac{t}{4(1 - c)} \]  

(109)

First stage:

(a) Bargaining between \( U_d \) and \( D_d \):

Second-stage transfer:

\[ T_{df} = \frac{1}{2} S_{df}(x_F, x_d) = x_{df}(x_F, x_d)(a - x_F - x_d - \frac{t}{2}) \]  

(110)

Substitution of (110) and (109) into (106) gives firm \( D_d \)'s first-stage profit. The disagreement payoff is obtained by setting \( x_d = 0 \) into this profit. \( U_d \)'s profit is:

\[ \Pi_{U_d} = T_d - h x_d \]  

(111)

and \( \Pi_{U_d^o} = 0 \).

First-stage surplus:

\[ S_d = \left[ \frac{1}{2} (x_d + x_F) + \frac{t}{4(1 - c)} \right] [a - \frac{1}{2} (1 + c)(x_d + x_F)] - \frac{t}{4} - \left[ \frac{1}{2} (x_F - x_d) + \frac{t}{4(1 - c)} \right] [a - x_F - x_d - \frac{t}{2}] \]

\[ - \frac{1}{2} \left[ \frac{1}{2} x_F + \frac{t}{4(1 - c)} \right] [x_F(1 - c) + \frac{t}{2}] - h x_d \]  

(112)

Maximization of (112) with respect to \( x_d \) yields:

(b) Profit maximization by the integrated industry:
Maximization of (114) with respect to \(x, y\) yields:

Substitution of the reaction functions into each other yields the optimal quantities indicated by equation (22) in the paper.

\[
\text{Derivation of equation (A4)}
\]

Consider firm i's marginal revenue:

\[
MR = p_i + q_i p_i', \quad \text{where } p_i = p(q_i + cq_i).
\]

Assume downward sloping reaction functions:

\[
\partial MR_i / \partial q_j = c(q_i p_i'' + p_i') < 0.
\]

Consider the FOC for profit maximization:

\[
p_i + q_i p_i' - k_i = 0
\]

where \(k_i\) is firm i's constant marginal costs.

From this FOC, compute how a change in rival's output affects firm i's output:

\[
\frac{\partial q_i}{\partial q_j} = - \frac{p_i' + q p_i''}{2 p_i' + q p_i''} \equiv \gamma
\]

We want \(-1 < \gamma < 0\).

The assumption of downward sloping reaction functions and the second-order condition for profit maximization ensure that \(\gamma < 0\). From (116), \(\gamma > -1\) if:

\[
\frac{\partial^2 \Pi}{\partial q_i \partial q_j} = \frac{\partial}{\partial q_i} \left( \frac{\partial \Pi}{\partial q_j} \right) = - \frac{p_i'' + q p_i'''}{2 p_i' + q p_i''} \equiv \gamma
\]
\[- c \frac{p_i' + qp_i''}{2p_i' + qp_i''} + 1 > 0 \]

\[- c \left( p_i' + qp_i'' \right) + 2p_i' + qp_i'' < 0 \]

\[p_i'(2 - c) + qp_i''(1 - c) < 0 \]

\[p_i' < -qp_i'' \frac{1 - c}{2 - c} \]

If $c = 1$, equation (A4) becomes $p_i' < 0$, which always holds (i.e. if goods are perfect substitutes, $\gamma > -1$ is always satisfied).
Vertically Related Markets and Trade Policy in a Bargaining Framework

Nontechnical summary

Countries which have a cost advantage in the production of a key intermediate good sometimes restrict exports of these in order to promote exports of the final good. When a low-cost industry restricts the supply of the input, there is scope for strategic government intervention, which aims at altering the industry equilibrium. This is the issue raised by Spencer and Jones (1991, 1992), who analyze the interaction between two asymmetric industries located in different countries. In the first stage, the more efficient vertically-integrated industry sets the price for the intermediate good to be exported, and in the second stage firms compete in the final market located in the less efficient country. The integrated industry chooses to foreclose the intermediate market when the supply of the input in the high-cost country is sufficiently inelastic. A tariff set by the government located in the high-cost country can induce vertical supply.

We analyze the effects of a tariff on imports of the final good when trade in the intermediate market is determined by bargaining rather than by price setting. Upstream and downstream firms are assumed to bargain over the gains from trade. The assumption of bargaining implies a more even distribution of power between upstream and downstream firms, and it avoids common motives for integration such as double marginalization.

We consider two industries, each consisting of an upstream and a downstream firm, which are located in different countries. Each firm bargains with the domestic partner before bargaining with the foreign one. This timing captures the idea that firms regard the domestic firm as a "natural partner".

If firms are identical, trade in the intermediate good will take place between domestic partners only, and the second stage is vacuous because there are no additional gains from trade. The solution is equivalent to that of vertical integration. If suppliers' marginal costs differ, the second stage of the game is meaningful, and interindustry trade arises in equilibrium. The downstream firm located in the low-cost country acquires some of the input from the efficient supplier and sells part of it to the less efficient upstream firm. Relative to
the one-shot game, downstream market shares are reversed, and the reallocation of production between suppliers (who cannot bargain by assumption) improves efficiency by increasing aggregate industry profits. The low-cost supplier, however, finds it profitable to prohibit the resale of the input, reducing the supply of the rival upstream firm, and increasing the demand of the input in the first stage. When resale is prohibited the downstream firm located in the low-cost country improves its share in the final market. Bargaining with the low-cost supplier in the first stage gives this firm a first-mover advantage, which allows it to behave as a Stackelberg leader in the downstream market.

The asymmetry in suppliers' costs may induce vertical integration by the more efficient industry. When vertically integrated, the industry's choice between foreclosure and vertical supply depends on the degree of competition in the final market. The integrated structure unlike the independent supplier, internalizes the rivalry in the final market. Perfect substitution between final goods is a sufficient condition for vertical foreclosure to occur, and in that case the solution is equivalent to that of the one-shot game.

When analyzing trade policies, we assume that the final market is located in the low-cost country, and consider the effects of a specific tariff set by the local government on the imports of the final good. These effects depend on the industry configuration in the low-cost country and on the degree of competition in the downstream market. When the industry is integrated and forecloses the supply of the input, an arbitrarily small tariff induces vertical supply and eliminates the integrated structure from the final market. The tariff simply raises one firm's costs (so that a small tariff has a negligible effect on welfare). Moreover, the assumption of bargaining implies that these costs are shared equally by the partners. When the integrated industry supplies the intermediate good, the tariff stimulates interindustry trade, but a non-negligible tariff is needed to eliminate the foreign industry from the final market. If the more efficient industry is not integrated, the tariff acts as a mere rent-extracting device, shifting rents from foreign to domestic firms, and negatively affecting consumers' welfare.