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Structure and dynamical balance of the Antarctic Circumpolar Current in Drake Passage

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Structure and Dynamical Balance of the Antarctic Circumpolar Current in Drake Passage

A dissertation submitted in partial satisfaction of the requirements for the degree
Doctor of Philosophy

in

Oceanography

by

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2012
The dissertation of Yvonne L. Firing is approved, and it is acceptable in quality and form for publication on microfilm and electronically:

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Chair

University of California, San Diego

2012
DEDICATION

To the memory of June Brasington Firing.
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PUBLICATIONS


ABSTRACT OF THE DISSERTATION

Structure and Dynamical Balance of the Antarctic Circumpolar Current in Drake Passage

by

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Doctor of Philosophy in Oceanography

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Teresa K. Chereskin, Chair

This thesis investigates the structure and dynamics of the Antarctic Circumpolar Current (ACC) in Drake Passage using observations that resolve spatial scales from 100 m to 1000 km and temporal scales from inertial to interannual. The structure and variability of the current, the eddy and mean contributions to the vorticity balance, and the patterns of internal wave activity are examined. The two primary sources of data are a long time series (2005-present) of upper ocean currents from the ARSV Laurence M. Gould (LMG) shipboard acoustic Doppler current profiler (SADCP), and a four-year process study (cDrake) providing time series of near-bottom currents, bottom pressures, and bottom-surface sound travel times as well as bathymetry, lowered ADCP, and CTD data from five yearly cruises.
The vertical structure in the upper 1000 m is equivalent barotropic, with variable vertical length scale. The mean transport in the upper 1000 m is 95±2 Sv. Transport variability is approximately equally divided between shear and depth-mean components. Eddy kinetic energy decreases with depth faster than mean kinetic energy, reinforcing the view of the ACC as a barrier to mixing.

Using empirical relationships determined from historical hydrography, travel time data from the cDrake array in the PFZ can be converted to baroclinic streamfunction. The near-bottom current and bottom pressure measurements provide the barotropic reference velocity. Streamfunction derivatives can be computed by objective mapping. We used independent measurements and simulated idealized fields to validate the objectively mapped fields and error estimates.

Mean and eddy nonlinear vorticity advection and bottom pressure torque dominate the mean vorticity balance. The residual is first order. SOSE has the same balance and similar scales, with the residual accounted for by sub-grid-scale dissipation. In the southeastern Pacific a Rossby-wave-like balance between mean relative vorticity advection and planetary vorticity advection is observed.

Downward-propagating internal wave energy and shear-strain ratios consistent with near-inertial frequencies predominate over deep waters and in the surface layer. Over shallower topography upward-propagating energy and supra-inertial frequencies dominate. The seasonal cycles in wind stress and internal wave energy south of the Polar Front are aligned; the seasonal cycle north of the Polar Front matches that in surface-layer stratification.
Chapter 1

Introduction

The Antarctic Circumpolar Current (ACC) is the largest current in the world in terms of volume transport, historically estimated at 134±11 Sv [Whitworth and Peterson, 1985]. With no full-depth meridional boundaries, the dynamics of the ACC are unique. The momentum input by the strong westerly winds drives an eastward current strong enough to suppress adjustment by westward propagation of baroclinic Rossby waves [Hughes, 1996], with the result that the eastward flow extends to the bottom. Above the level of the topography, the wind-input momentum cannot be balanced by net zonal pressure gradients, nor the wind-input vorticity by mean meridional flow as in the Sverdrup-balanced gyres. Despite the current’s deep extent, flow is not strong enough for bottom friction to balance the wind-input momentum and vorticity; instead, as described by Munk and Palmén [1951], bottom form stress (the net effect of pressure gradients across topography) and bottom pressure torque (the vortex-stretching effect of flow over topography) provide the balancing forces. While wind stress and wind stress curl are nearly-uniform along the course of the ACC, the balancing terms depend strongly on topography, which is highly variable from place to place (Figure 1.1).

The mean ACC flow is organized into multiple frontal jets (Figure 1.1), associated with (from north to south) the Subantarctic Front (SAF), the Polar Front (PF), and the weaker Southern ACC Front (SACCF). The current is highly variable, with frontal jets repeatedly meandering, spinning off mesoscale eddies, and splitting into multiple filaments which then merge together further downstream.
[Sokolov and Rintoul, 2009b], possibly subject to topographic control [Lu and Speer, 2010; Thompson et al., 2010; Thompson and Richards, 2011]. Variability is highest in the Polar Frontal Zone (PFZ), between the mean positions of the SAF and the PF. Mesoscale eddies, which are generated by instability of the jets, also transfer momentum and vorticity into, out of, or between jets, as well as downward through interfacial form stress. Eddy stirring and transfer across the ACC may be tempered by the barrier effect of the strong jets; the extent and distribution of eddy stirring is a topic of active research [Ferrari and Nikurashin, 2010; Abernathey et al., 2010; Klocker et al., 2012] as is the eddy-mediated response of the current to changes in the wind forcing [Farneti et al., 2010; Hallberg and Gnanadesikan, 2006]. The extent of eddy saturation or eddy compensation—the stabilization of the mean transport under increased forcing, by increased instability and eddy fluxes—will determine the response of the ACC to wind changes; thus understanding the role of eddies in the ACC is an essential part of understanding the ACC response to climate change.

The ACC and its dynamics, meanwhile, are intimately involved in the meridional overturning circulation. Eddies spawned by the meandering fronts play a vital role in the meridional transport of heat. The strong ACC flow over topography also generates internal waves [Nikurashin and Ferrari, 2011] which contribute energy for the diapycnal mixing that transforms water masses as they pass through the ACC.

A number of mooring and ship-based studies of the ACC have provided evidence of the large transport and high eddy variability and shed light on important processes [Bryden and Pillsbury, 1977; Whitworth and Peterson, 1985; Phillips and Rintoul, 2000; Cunningham et al., 2003; Sprintall, 2003; Tracey et al., 2006; Chereskin et al., 2010; Lenn et al., 2011; Naveira Garabato et al., 2011; Ferrari et al., 2012; Waterman et al., 2012a]. Satellite-, drifter-, and float-based data have extended lateral coverage and provided new information about the dynamics [Gille, 2003a,b; Hughes, 2005; Elipot and Gille, 2009; Sokolov and Rintoul, 2009a,b; Hughes, 2005; Falco and Zambianchi, 2011]. Observations in the Southern Ocean, however, remain sparse compared to those available in many parts of
the northern hemisphere. At the same time, the deep-reaching and highly variable ACC demands higher vertical, horizontal, and temporal coverage.

This thesis investigates the structure and dynamics of the ACC, focusing on several observational data sets and on Drake Passage. Drake Passage is one of the regions of high topographic relief likely to be significant in the total momentum and vorticity balances, as well as being a region of particularly strong, concentrated jets and high eddy energy. Observations of the vertical structure and transport of the ACC in Drake Passage, from a time series of direct current measurements, are described in Chapter 2. Methods used to obtain estimates of velocity, relative vorticity, and vorticity gradients from a four-year experiment involving moorings across Drake Passage are elaborated in Chapter 3. In Chapter 4, these estimates along with the Southern Ocean State Estimate are used to investigate the vorticity balance of the ACC in Drake Passage and in a smoother region in the southeastern Pacific. Observations of internal waves and the distribution of internal wave energy in Drake Passage are presented in Chapter 5. Chapter 6 contains a summary of the thesis and overview of future work.
Figure 1.1: Map of the Southern Ocean, with bathymetry contours [Smith and Sandwell, 1997], mean ACC fronts [Subantarctic Front, Polar Front, and Southern ACC Front Orsi et al., 1995] in green, and box outlining Drake Passage.
Chapter 2

Vertical structure and transport of the Antarctic Circumpolar Current in Drake Passage from direct velocity observations

Abstract. The structure of the Antarctic Circumpolar Current (ACC) in Drake Passage is examined using four and a half years of shipboard Acoustic Doppler Current Profiler (ADCP) velocity data. The extended 1000-m depth range available from the 38 kHz ADCP allows us to investigate the vertical structure of the current. The mean observed current varies slowly with depth, while eddy kinetic energy and shear variance exhibit strong depth dependence. Objectively mapped streamlines are self-similar with depth, consistent with an equivalent-barotropic structure. Vertical-wavenumber spectra of observed currents and current shear reveal intermediate-wavenumber anisotropy and rotation indicative of downward energy propagation above 500 m and upward propagation below 500 m. The mean observed transport of the ACC in the upper 1000 m is estimated at 95±2 Sv or 71% of the canonical total transport of 134 Sv [Whitworth and Peterson, 1985]. Mean current speeds in the ACC jets remain quite strong at 1000 m, 10 to 20 cm s$^{-1}$. Vertical structure functions to describe the current and extrap-
olate below 1000 m are explored with the aid of full-depth profiles from lowered ADCP and a three-year mean from the Southern Ocean State Estimate (SOSE). A number of functions, including an exponential, are nearly equally good fits to the observations, explaining >75% of the variance. Fits to an exponentially decaying function can be extrapolated to give an estimate of 154±38 Sv for the full-depth transport.

2.1 Introduction

The Antarctic Circumpolar Current (ACC) is unique among major ocean currents in its lack of complete meridional boundaries as well as its low stratification and large depth extent. Its role in connecting the Pacific, Atlantic and Indian Oceans and in the meridional overturning circulation means that it is a crucial feature for models of global circulation and climate to reproduce accurately. Model predictions of the degree and distribution of eddy stirring depend on the current’s structure and large-scale dynamical balances, but observations that can be used to quantify its transport and structure are sparse. In particular, there are few datasets with the depth range and resolution necessary to define the vertical structure. Satellite measurements provide valuable coverage of the surface ACC, and a better understanding of its vertical structure could allow mapping from these surface observations to metrics such as transport.

Drake Passage, bounded by South America and the Antarctic Peninsula, has been the site of a number of attempts to quantify and describe ACC transport. Whitworth [1983] and Whitworth and Peterson [1985] estimated transport from International Southern Ocean Studies (ISOS) mooring and cruise data to give the canonical ACC transport of 133.8 Sv with a standard deviation of 11.2 Sv. They calculated relative transport using temperature and salinity moorings on either side of the passage between 500 m and 2500 m depth and historical hydrography between 500 m and the surface. Pressure sensors at 500 m on either side of the passage were leveled using direct current measurements from three hydrographic cruises, and after adjustments to remove contributions from the slope regions above
they provided a reference velocity at 500 m. The year-long average total transport above 2500 m was found to be 124.7±9.9 Sv [Whitworth and Peterson, 1985], of which 87 Sv was due to shear above 2500 m and 37.7 Sv to the velocity at 2500 m [Whitworth, 1983]. An estimated 9.1±4 Sv of transport below 2500 m [Whitworth, 1983] brought the total to 133.8±11.2 Sv. Cunningham et al. [2003] revisited the ISOS dataset and the sources of uncertainty to arrive at an upper bound of 27 Sv for the measurement uncertainty on the mean total transport.

Whitworth and Peterson [1985] used the correlation between transport and current derived from the cross-passage pressure difference at 500 m to extend the ISOS transport time series to five years, and found a range of 95 to 158 Sv. Cunningham et al. [2003], however, calculated that only 65% of the total variability in the ISOS transport was barotropic, leaving a significant remaining fraction in the baroclinic component. This is consistent with the results of Sprintall [2003] and Olbers and Lettmann [2007], who found large baroclinic transport variability from upper ocean XBT measurements and numerical model results, respectively.

Other velocity observations point to significant deep structure in both time-varying and mean velocities. Instantaneous measurements have shown large near-bottom currents: from a shipboard Acoustic Doppler Current Profiler (SADCP) and hydrography, Donohue et al. [2001] inferred speeds of 4 to 10 cm s\(^{-1}\) in the Subantarctic Front (SAF) in the Pacific, while Cunningham et al. [2003] reported lowered ADCP-measured velocities of as much as 10 to 20 cm s\(^{-1}\) in Drake Passage. Mean speeds are also substantial; one-year mean near-bottom velocities from an array of current meters in Drake Passage were in excess of 10 cm s\(^{-1}\) in the northern part of the passage [Chereskin et al., 2009]. Using moored current meter records from the SAF south of Australia, Tracey et al. [2006] found mean speeds of 4 to 6 cm s\(^{-1}\) at 3500 db and deep-reaching baroclinic shear (to at least 2000 m).

Observational constraints on the vertical structure of the ACC are essential. The strength of deep currents has implications for the dominant dynamical balance: the interaction of near-bottom currents with topography leads to vortex stretching, momentum, energy, and vorticity dissipation through bottom friction and torque, and ultimately to turbulence production resulting in mixing. Mean-
while, validation of the vertical shear structure is important for models which use shear to parameterize mixing. Definition of the vertical structure of the ACC could also allow single-level observations, such as middepth float data or satellite observations of the surface, to be mapped to the full ACC. Many observations and models of the ACC are consistent with an equivalent-barotropic (EB) vertical structure; that is, that streamlines are parallel at all depths, unless velocity goes to zero. Hughes and Killworth [1995] and Killworth and Hughes [2002] showed how EB structure results from geostrophic flow with a relatively weak planetary vorticity gradient. Sun and Watts [2001] found that first empirical modes calculated by the Gravest Empirical Mode (GEM) method, in which an EB structure for the baroclinic current is inherent, explain 97% of the density variance in the ACC at all longitudes. In both the Fine Resolution Antarctic Model [Killworth, 1992] and the global OCCAM [Killworth and Hughes, 2002], velocity in the ACC has a separable EB structure, such that it is a function of the horizontal coordinates alone multiplied by a function of the vertical coordinate alone. Exponential decay with depth is one explored EB form: Gille [2003a] calculated an average vertical e-folding scale of 700 m from float velocity and atlas hydrography, and Karsten and Marshall [2002] found that climatological buoyancy data are well-described by exponential decay with depth, but that length scales vary from under 500 m to over 1000 m from south to north, which implies a non-separable EB structure for velocity. Observations of quasi-stationary short barotropic Rossby wave-like meanders by Hughes [2005], Tracey et al. [2006], and Chereskin et al. [2010] also support an EB vertical structure.

Understanding the structure of the ACC depends upon understanding the eddy dynamics which both forces and is forced by the mean current [e.g. Bryden and Cunningham, 2003; Karsten et al., 2002]. The horizontal and vertical distributions of eddy momentum fluxes set the locations and structures of the mean jets [Lenn et al., 2011], which are highly time-variable [Sokolov and Rintoul, 2007, 2009b]. The interaction between eddies and jets also influences the rate and distribution of heat and salinity transport across the current. Intensification of eddy kinetic energy (EKE) around complex topographic features is evident in both al-
timeter data and numerical models [Gille, 1997; Williams et al., 2007]. Lenn et al. [2007], from measured currents down to 250 m, found high EKE in the Polar Frontal Zone (PFZ) in Drake Passage. Multiple studies have found evidence that the observed eddies, which have horizontal scales of up to 100 km, are generated by baroclinic instability [Phillips and Rintoul, 2000; Smith, 2007; Smith and Marshall, 2009], although barotropic instability also has been seen [Nowlin and Klinck, 1986; Tracey et al., 2006]. The locations of instability growth and eddy stirring depend on the mean current structure; strong potential vorticity (PV) gradients associated with strong fronts can act to inhibit eddy stirring and therefore mixing.

Smith [2007] computed PV gradients from a hydrographic atlas and inferred multiple locations of baroclinic instability growth down to 1500 m, while Smith and Marshall [2009] and Abernathey et al. [2010] found a deep (≥1000 m) maximum in eddy stirring below the core of the ACC, with stirring inhibited in the surface layer within the current. The analysis of Thompson et al. [2010], however, suggests that this effect is not uniform throughout the ACC.

The studies described above, which suggest important roles for both barotropic and baroclinic components of transport, as well as mesoscale variability and deep eddy stirring, point to the need for direct current measurements with high spatial resolution to confirm inferences drawn from climatology and models. In this paper we use a new dataset of direct velocity observations collected by a deep-profiling sonar over a 4.5 year period to examine the vertical structure of the ACC in Drake Passage. In Section 3 we describe the mean current and find that the direct current observations are generally consistent with an equivalent-barotropic vertical structure. In Section 4 we examine the vertical structure of velocity, EKE, and Reynolds stresses with an eye to illuminating the distributions of energy and eddy stirring, and we search for vertical structure functions to describe the mean and time-varying currents. We calculate top 1000-m transport in Section 5, and use the vertical structure from the previous section to extrapolate in depth. In Section 6 we summarize our findings and discuss the implications for ACC dynamics and transport. We describe the SADCP dataset and other datasets in the following section.
2.2 Data

The ARSV Laurence M. Gould (LMG) crosses Drake Passage repeatedly in all seasons in the course of supplying Palmer Station on the Antarctic Peninsula and conducting research in the area. *Lenn et al.* [2007] described processing and analysis of data from a 153.6 kHz SADCP on the LMG. Since late 2004, the LMG has also been outfitted with a 38 kHz RD Instruments ADCP. The 38 kHz ADCP has a transducer depth of 6 m, a blanking distance of 16 m, and samples in narrowband mode with a 24 m bin and pulse, so that the first depth bin is centered at 46 m and every other bin is independent. Returned ping data are averaged over 5-minute ensembles, which are screened using amplitude, error, and percent good criteria. An Ashtech GPS is used to correct heading from the ship’s gyrocompass, with the difference between instrument and ship coordinates corrected for by bottom tracking (see Appendix). Data are converted to earth coordinates using the corrected heading and the ship’s GPS.

Data from 105 crossings (tracks shown in Figure 2.1) from November 2004 through June 2009 have been processed and edited using the CODAS3 software package (http://currents.soest.hawaii.edu/software/codas3). The 38 kHz instrument records data from as deep as 1222 m; in this dataset just over 50% of ensembles at 1030 m contain good data. Velocities are further averaged to 15-minute bins (covering approximately 5 km along-track) and the barotropic tide from the TPXO6.2 tidal model [*Egbert et al.*, 1994] is removed. Baroclinic tidal predictions vary substantially; however, *Lenn et al.* [2007] analyzed ISOS mooring records from mid-passage and found a maximum baroclinic semidiurnal and first harmonic peak in kinetic energy of no more than 15 cm² s⁻², so we expected the baroclinic tidal currents to be negligible in most of the passage and did not attempt to remove them from our data. A correction for heading-error-induced velocity bias is described in the Appendix.

Calculation of geostrophic current shear using density profiles and the thermal wind balance sheds light on the geostrophic vs. ageostrophic and baroclinic vs. barotropic components of the SADCP-measured total velocity. The LMG also hosts the high resolution expendable bathythermograph (XBT)/expendable
conductivity-temperature-depth (XCTD) sampling program [Sprintall, 2003]. These data are made available by the Scripps High Resolution XBT program (http://www-hrx.ucsd.edu). XBTs are dropped on six crossings each year at intervals of 6 to 15 km, and generally obtain temperature measurements to about 850 m depth. On the same crossings, twelve XCTDs deployed at a spacing of 25 to 50 km measure temperature and salinity to around 1000 m. A position-dependent T-S-z relation constructed from historical hydrography gives a salinity profile for each XBT temperature profile; where XCTD temperature-salinity data are available, they are used to correct the derived salinities (J. Sprintall, pers. comm.).

To give spatial and temporal context to the irregularly sampled LMG datasets, optimally interpolated satellite altimeter sea level anomalies (SLA) were obtained from the Archiving, Validation and Interpretation of Satellite Oceanographic data (AVISO) project [Ducet et al., 2000]. The AVISO mapped SLA product is derived from multiple satellite altimeters. We recalculated anomalies with respect to the November 2004 to June 2009 mean.

To extend the top 1000-m SADCP observations in depth, we examined full-depth current profiles from a lowered ADCP (LADCP) from four Drake Passage cruises (cDrake) in November-December of four consecutive years, with 56 casts in 2007, 47 in 2008, 67 in 2009, and 62 in 2010. The LADCP is a 153.6 kHz broadband RD Instruments ADCP, with a 30° beam angle and a 16-m vertical bin, pulse, and blank before transmit. Data were processed following Fischer and Visbeck [1993] using the UH CODAS LADCP software developed by Eric Firing. LADCP measurements of water velocity relative to the instrument are differenced to obtain vertical shear, which is gridded into 20-m depth bins and integrated to obtain the baroclinic velocity profile. Measured velocity is also integrated over the cast and added to the ship drift to get the depth-averaged, or barotropic, velocity. Errors in the baroclinic (barotrophic) component of velocity with this instrument, sampling scheme, and processing method are 4 cm s\(^{-1}\) (1 cm s\(^{-1}\)) [Chereskin et al., 2010]. For this study, we averaged up- and down-cast velocity profiles.

We also considered the Southern Ocean State Estimate [SOSE, Mazloff et al., 2010], an eddy-permitting state estimate which iterates via the adjoint
method to minimize misfit between the estimate and observations, subject to uncertainties in the inputs. Pressure, temperature, salinity, velocity, and other variables are calculated on a $1/6^\circ$ grid with 42 depth levels. Here we used daily mean velocity output from a run over 2005 through 2007. Neither the LMG SADCP nor the cDrake LADCP data are included in the constraints on SOSE, although the LMG XBT/XCTD dataset is.

Multibeam bathymetry data collected on the RVIB N.B. Palmer were combined with the dataset of Smith and Sandwell [1997] to give high-resolution bathymetry in the Drake Passage area.

In addition to geographic coordinates, we use a coordinate system $(x_p, y_p)$ rotated $23.7^\circ$ counter-clockwise from zonal and meridional directions, so that $x_p$ runs through-passage towards the Atlantic and $y_p$ across-passage towards Tierra del Fuego. For some analyses, velocities from each crossing were vector-averaged to a horizontal grid with a grid spacing of either 5 or 25 km; the results are referred to as “gridded velocities”. Once gridded, velocity measurements may be averaged over all cruises. Following Lenn et al. [2008], we subtracted geostrophic surface current anomalies calculated from AVISO SLA from gridded SADCP currents at each depth before averaging to produce “improved mean” currents with reduced aliasing. We note that while the altimeter-derived surface current anomalies are an overestimate at depth, they are an underestimate at the surface [see Lenn et al., 2007, Figure 9]. Depth sections come from the most commonly visited line (54 to 97 crossings through each 25 km by 25 km grid box), marked by the 25 km-wide gray box in Figure 2.1. For calculation of transport we assumed slab motion in the top 46 m.

### 2.3 Mean Structure and Streamfunction

The 46-1030 m depth-mean ACC (Figure 2.2a) has a strong resemblance to the 26-250 m depth-mean calculated by Lenn et al. [2007] (see their Figure 5), with slightly lower speeds and energies. Although the instantaneous current field is a complex mixture of eddies and jets with an often-multivalued transport
streamfunction, the SAF, PF, and Southern ACC Front (SACCF) jets are clearly visible in the Eulerian mean (Figure 2.3a). Mean speeds over the top 1000 m are 30 cm s\(^{-1}\) in the SAF and PF, and 15 cm s\(^{-1}\) in the SACCF. Standard deviations are 10 to 30 cm s\(^{-1}\) and variance ellipses are oriented along or slightly to the right of the mean current direction in the SAF; in the SACCF and PF they are oriented across the mean current direction. Eddy kinetic energy (EKE = \(\frac{1}{2} \langle u'^2 + v'^2 \rangle\), \(\vec{u}' = \vec{u} - \langle \vec{u} \rangle\)) is highest in the PFZ (between the SAF and PF), consistent with the surface-layer analysis of Lenn et al. [2007] and with EKE from altimetric SLA. The “improved mean” (see Section 2, above) has similar speeds but lower standard deviations (8 to 20 cm s\(^{-1}\)) and less directional variation in the jets. Variance ellipses relative to the “improved mean” are generally anisotropic and oriented across the mean flow over most of the area. We consider the “improved mean” to be a better representation of the long-term mean than can be attained from only 4.5 years of SADCP data, and therefore use it as the mean field in the rest of this paper.

We objectively mapped improved mean current vectors to streamlines. The mapping algorithm uses Gaussian covariance functions that satisfy geostrophic continuity as derived in Gille [2003a] and incorporates error estimates on the data to produce a mapping error [Bretherton et al., 1976]. We used an isotropic decorrelation scale of 50 km determined from track data and a data fractional error of 0.2. We found the results to be insensitive to changes of ±10 km in the decorrelation length scale as well as to the size and structure of the data error within a reasonable range. We tested several options for the background mean streamfunction, and found that the mapped streamfunctions and currents are not sensitive to the background mean as long as it is geostrophic, has some gradient across the passage, and is defined beyond the data points. For the results presented here we used the surface dynamic topography of Maximenko and Niiler [2005]. We show and consider only results with a mapping error of ⩽0.3, as determined by the objective mapping procedure based on data locations and the decorrelation function. Residual velocities are relatively small (rms 4.5 cm s\(^{-1}\) over all depths) and apparently randomly oriented, lending confidence in the assumption of geostrophy.
Maps at selected depth levels are shown in Figure 2.2. The objectively mapped depth-mean field (Figure 2.2a) has large meanders both in the PFZ and in the SACCF area. The SAF is aligned to the north-northeast; the PF turns from northeast entering the passage to nearly due east exiting the passage, while the SACCF has a large northward meander. While the SAF deviates north following the bathymetry, the other meanders do not appear to be related to local bathymetry. The PF strengthens slightly (streamlines converge) going alongstream through our sampling area. The widths of the Eulerian-mean SAF, PF, and SACCF are approximately 100, 200, and 75 km, respectively, while the SAF and PF are separated by about 200 km and the PF and SACCF by over 200 km; we note that the instantaneous jets are narrower. The objective maps are consistent with EB structure in most locations: mapped streamlines are strongly self-similar with depth (Figure 2.2), with gradients decreasing only gradually and features including the three major fronts and the meander in the PFZ visible at all depths. The streamline shapes in the area of the SACCF are the most variable with depth. The position of the PF jet, as represented by the intersection of the highest-gradient streamline in the PF area and $x_p = 0$, has a tilt of 81 km towards South America over 1000 m of depth. The SAF and SACCF positions do not have significant tilts over this depth range.

From the objectively mapped streamfunctions at each depth we constructed a set of local mean stream coordinates, with $x_\psi, u_\psi$ along the local mean streamline and $y_\psi, v_\psi$ across it. These coordinates are used in the following sections.

### 2.4 Vertical Structure

In this section we examine the vertical structure of the top $\sim 1000$ m of the ACC in Drake Passage. We look for a simple function or set of functions to describe the low-mode vertical structure of the current and use spectra to explore higher-wavenumber processes. We also investigate the distribution of eddy fluxes and eddy kinetic energy and their implications for instability and eddy stirring.

All three Eulerian-mean jets (Figure 2.3a) extend to the limits of our depth
range below 1000 m, and the current speed in the center of each jet at 1000 m is approximately half its maximum at 46 m. The mean jets are all somewhat skewed, having higher horizontal shear on their northern sides and higher vertical shear on their southern sides, although the latter gradient is more pronounced for the PF and SACCF. With the exception of the surface-intensified coastal current observed at the northernmost point, current speed decreases slowly as a function of depth (Figures 2.4 and 2.5a) and orientation varies little (Figure 2.5b), especially within the mean jets. Mean-current vertical shear, which is of order $1 \times 10^{-4} \text{s}^{-1}$ except at the northernmost point, is generally positive in the SAF and PF, and weakly positive to moderately negative outside the jets. Shear in the gridded currents, even when smoothed, can be a factor of two or more stronger than that in the mean currents. Shear variance below the surface layer is around $(2-4) \times 10^{-6} \text{s}^{-2}$, which implies a 95% confidence interval on mean shear estimates of $\pm (3-4) \times 10^{-3} \text{s}^{-1}$, larger than any individual shear value. As suggested by the success of the geostrophic objective mapping procedure (see Section 3), mean geostrophic shear (not shown) from XBT/XCTD data and derived salinities (see Section 2) is in general agreement with directly-measured mean vertical shear in across-track currents, especially in the northern part of the passage. The mean SAF and PF jets are clearly identifiable in geostrophic shear, with the same depth-average values as directly measured shear in these jets, and coincident with their directly measured locations. The SACCF is less well defined in geostrophic shear and in velocity from the XBT-assimilating SOSE (Figure 2.3b) than in the SADCP observations (Figure 2.3a).

Given the strength and proximity of the PF and SAF, it is not surprising that EKE (Figures 2.5c and 2.6b) is by far strongest in the PFZ at all depths. The maximum EKE is significantly larger than the maximum kinetic energy of the mean flow (Figure 2.6a), and EKE in the PFZ drops off faster with depth than does KE in the jets. EKE is also unevenly distributed between along- and across-stream contributions (Figure 2.6c, d). The ratio of KE to EKE in the jets is large enough that by the criterion of Ferrari and Nikurashin [2010] they would act as mixing barriers all the way down to 1000 m; however, the derivation of Ferrari and
Nikurashin [2010] is not expected to apply to the constrained and highly nonzonal fronts of Drake Passage, so it remains uncertain to what extent the jets in this area impede mixing.

The across-stream eddy flux of along-stream momentum, $\langle u'_\psi v'_\psi \rangle$, is relatively depth-independent in sign and even in amplitude (Figure 2.6e). From this dataset we could only calculate statistically significant divergences on a rather large scale; we found divergence of $\langle u'_\psi v'_\psi \rangle$, corresponding to deceleration, on the northern side of the PF jet, and convergence, corresponding to acceleration, on the southern edge of the SAF jet. This pattern is similar to the divergences calculated by Lenn et al. [2011] with a 7-year timeseries in the upper 250 m, suggesting that their results for the pattern of acceleration by eddies may be extendable to a greater depth range.

### 2.4.1 Spectra

The energy-containing vertical and horizontal length scales of the ACC are of basic interest and are essential to theoretical models of the current system. Spectra can also reveal information about rotation at internal wave scales and, therefore, about the dominant direction of energy propagation in different regions [Leaman and Sanford, 1975].

We computed vertical wavenumber spectra over several depth ranges using the 15-minute averaged profiles (see Section 2.2). We averaged spectra over cruises and over location points within five regions (from south to north: SACCF, Antarctic Zone (AZ), PF, PFZ, and SAF) defined by the positions of the main fronts, and calculated confidence intervals from the resulting degrees of freedom assuming a 50 km decorrelation scale (see Section 2.3). Spectra of $u_p$, $v_p$ and $u_\psi$, $v_\psi$ are red in all regions (Figure 2.7) and depth ranges (not shown). Low-wavenumber (1000-m wavelength) energy is greater in the northern passage than in the SACCF, but the difference is not significant. Intermediate-wavenumber anisotropy in mean stream coordinates, with cross-stream energy greater than along-stream, is significant at the 95% confidence level in the PF below about 400 m at wavelengths of 150 to 350 m; this anisotropy is also significant in shear spectra (not shown).
In this wavelength range. In the PF cross-stream energy at these wavelengths is ∼50-100% greater than along-stream. Shear spectra in the AZ, PF, and PFZ have significant peaks at about 350 m for \( \partial u_\psi / \partial z \) and about 250 m for \( \partial v_\psi / \partial z \); in the SAF there is no significant peak, but energy decreases at wavelengths shorter than these, while in the SACCF the shear spectra are nearly white. Energy in spectra computed from profiles gridded to the estimated decorrelation scale of 50 km is significantly lower at all wavenumbers, implying that motions with vertical scales of up to ∼1000 m have horizontal and/or time scales shorter than 50 km and/or 2.5 hr.

Rotary spectra (not shown) reveal a small but significant preference for counterclockwise (CCW) rotation with increasing depth above 500 m and clockwise (CW) rotation with increasing depth below 500 m, at wavelengths of ∼200-450 m. The shallow signal is more consistent and is found in the three fronts and the PFZ, while the deeper signal is significant only in the PF and PFZ. The predominance of CCW rotation above 500 m is as expected; in the Southern Hemisphere CCW rotation with increasing depth is associated with internal waves with downward group velocity. The only significant rotary anisotropies that extend over the entire SADCP depth range are found at the longest wavelengths (400-500 m): CW in the PFZ and CCW in the AZ and SACCF regions. In the SACCF region the anisotropy derives from just south of the mean SACCF jet.

We calculated horizontal wavenumber spectra (not shown) from the set of individual transects as in Lenn et al. [2007], extending their calculation to greater depth. Data gaps of up to 50 km were filled by linear interpolation. Energy peaks at 350 km for both velocity components. Lenn et al. [2007] found energy in the across-passage velocity \( v_p \) peaking at a lower wavenumber than that in the along-passage velocity \( u_p \), but the difference was not significant. The spectral energy level decreases with depth, but neither the location of the peak nor the spectral slopes change significantly with depth. As in Lenn et al. [2007], the spectra are anisotropic, with \( u_p \) having significantly more energy at all but the lowest and highest wavenumbers. Very similar results exist for spectra of velocity anomalies relative to the gridded (improved) depth mean, with the exception that the peak
energies for both $u_p$ and $v_p$ decrease, and the peak wavelength for $u_p$ is lower, around 230 km; in Lenn et al. [2007] the energy of $v_p$ stayed the same in the surface layer.

### 2.4.2 Vertical Structure Functions

Visual inspection of the mean SADCP currents (e.g. Figures 2.3a, 2.4, and 2.8) reveals fairly smooth, gently-sloping profiles; non-time-averaged currents (gridded SADCP or LADCP) are less smooth but still show significant low-wavenumber structure, suggesting large depth-attenuation length scales. Empirical orthogonal function (EOF) analysis of gridded currents supports this impression. We divided gridded currents on the most commonly sampled line into five regions defined by the positions of the mean fronts as in Section 2.4.1, removed the depth mean from each profile, and calculated the EOFs over all the profiles in each region. The first empirical mode explains 47% of the variance in gridded currents in regions south of the PF; in the PF and SAF 62% and 65%, respectively; and in the PFZ 70%. Each region’s first mode is a good approximation of a straight line, with a slight decrease in shear around 150 m in all regions, and again below 600 m in and south of the PF and below 900 m in the PFZ and SAF (Figure 2.8). The second modes, which have two zero-crossings, explain only 9 to 16% of the variance. These results echo those of Inoue [1985], who found that at most of the ISOS moorings the first EOF captures $> 92\%$ of the total energy in the year-long current profile timeseries.

The EOFs show that certain characteristics of the mean jets also apply to the time-varying current. The dominance of the first modes, particularly in the jet regions, confirms the result of Section 3 that the current in the SADCP depth range is generally equivalent-barotropic. Differences in the shapes of the mean jets visible in Figure 2.3a also appear in the EOFs. The SAF is deeper than the PF, while the PF has stronger vertical shear in the upper part of the SADCP range (Figure 2.8). The SACCF has higher near-surface vertical shear relative to the size of the current. The EOF results also illuminate the sources of EKE in the PFZ: around 40\% of individual profiles reconstructed from the first mode plus
the depth-mean components are westward in this region, as compared to $\leq 15\%$ in the PF and SAF; westward flow likely indicates mesoscale eddies as opposed to a meandering jet.

Encouraged by the high proportion of variance explained by the first modes, we sought simple functions to describe the vertical structure of the velocity data. We investigated a number of smoothly-varying functions that might match well with the EOF results, including linear, exponential, or hyperbolic tangent (as in Killworth and Hughes [2002]), and the first few flat-bottom dynamical modes from different parts of Drake Passage. We also tested the thermal wind profile resulting from exponential buoyancy profiles with length scale a function of the across-stream coordinate [Karsten and Marshall, 2002]. Vertical profiles of complex velocity $u_p(z)+iv_p(z)$ at each grid point (or cast location) were least-squares fit to a function of $z$ with parameters allowed to vary from grid point to grid point. Constraints on the fit parameters were included in order to limit our search to “appropriate” fits. We used two criteria for “appropriateness”: 1) the observed profile should not be a residual of much larger terms; and 2) any nonlinear term should be distinguishable from a straight line over the depth range of the fit. We applied the first criterion by limiting the depth-maximum value of each component of the fit to $1 \text{ m s}^{-1}$. To apply the second criterion, for instance, we limited the length scale of an exponential term to twice the depth range. For linear fits, confidence intervals were derived from estimates of the data covariance matrices; for nonlinear fits parameter covariances were estimated by bootstrapping [Efron and Gong, 1983].

All of the fits had both a mean and a depth-varying component. Most tested shapes were able to explain over 60% of the gridded or mean SADCP variance on average, with rms residuals of the order of or less than the data standard deviations. No one shape stood out as significantly better than the others for describing the vertical structure of the SADCP velocity over the whole sampling area; however, two of the shapes, summarized in Table 2.1, found some support in the LADCP and SOSE datasets as descriptors of the full-depth structure, and we explored these further.
Linear velocity

We first investigated the simplest possible description for the velocity profiles, the line. Ferrari and Nikurashin [2010] explored the ACC as a surface quasi-geostrophic (SQG) system and showed how a linear mean velocity profile and exponential EKE profile result from a constant surface buoyancy gradient. Although the SQG model should only be a good description above about 500 m [Lapeyre and Klein, 2006], the SADCP EKE (Figure 2.6b) is well described by an exponential shape with length scales of 300 to 900 m: the best fit profiles explain 97% of the variance and have rms differences of 16 cm$^2$ s$^{-2}$ (compare to rms profile standard deviations of around 100 cm$^2$ s$^{-2}$). Linear fits ($\vec{u} = \vec{u}_0 + \vec{a}z$) to SADCP velocity explain 65% of the variance and have rms differences of 1.9 cm s$^{-1}$ on average. Simple linear fits to SOSE or LADCP velocity profiles leave quite a bit of the full-depth structure unexplained; however, piecewise linear fits with slope allowed to change at 1030 m approach the performance of the more complex shapes discussed below. In the LADCP dataset the mean shear above 1030 m is on average about twice as large as that below 1030 m, while in SOSE it is three times as big.

Exponential velocity

We fitted velocity profiles to the function

$$\vec{u} = \vec{u}_0 + \vec{a}e^{z/L},$$

(2.1)

where $\vec{u}_0$, $\vec{a}$, and $L$ are the fit parameters and $z$ increases upwards from $z = 0$ at the free surface. SOSE mean velocity profiles are very well-described by (2.1), with 98% of the variance explained and rms errors of 10% the size of data standard deviations. Length scales $L$ are mostly between 1100 and 1700 m (± standard deviations of 10 to 30 m), and increase slightly from south to north (Figure 2.9b). Fitting of (2.1) to SADCP mean velocity produces “appropriate” fits at 79% of SADCP grid points, and at these points the fits explain 76% of the variance with a rms error of 2.0 cm s$^{-1}$—about half of the rms of the original profile standard deviations. However, half of the “appropriate” fits have length scales (Figure 2.9a) at the upper limit (twice the depth range) imposed by the fit constraints, indicating
that the best exponential fit is no better than a straight line fit; another 10% have indeterminate length scales (standard deviation of $L \geq L$). Fits to gridded SADCP velocity give similar results: at the 84% of points where an “appropriate” fit could be made, the rms error is 4.9 cm s$^{-1}$ (as compared to rms profile standard deviations of 9 cm s$^{-1}$), while about half the length scales are at the upper limit. Exponential decay with depth tends to be a better fit to SADCP velocities in the southern part of the passage. Both SADCP and SOSE $L$ are noisy in the alongstream as well as across-stream direction.

The SADCP depth range appears generally insufficient to determine the length scales of best-fit exponential decay in the full-depth ACC, particularly if our goal is not only to describe the upper 1000 m but also to extrapolate below them. We therefore turned to the full-depth SOSE and incorporated the length scales $L_{SOSE}(x_p, y_p)$ (Figure 2.9b) from nonlinear fits to SOSE mean profiles (gridded in the same way as the SADCP data) into fits to mean SADCP profiles, with $\vec{u}_0$ and $\vec{a}$ determined by the linear least-squares fit. The results (e.g. Figure 2.8) explain 65% of the SADCP variance on average and have similar rms error, 1.9 cm s$^{-1}$. Fits to (2.1) using $L_{SOSE}$ thus seem promising for extrapolating observed currents in the upper 1000 m to deeper levels. However, as can be seen in Figures 2.3 and 2.4, there are some significant differences between SADCP and SOSE, especially in the jets (Figure 2.3c, d). SOSE underestimates the strength of the PF and SACCF and overestimates the strength of the SAF (Figure 2.3b). Most relevantly for the problem of determining the vertical structure, SOSE appears to overestimate shear in both the SAF and PF and underestimate the length scale in and just south of the PF.

**Thermal wind from exponential buoyancy**

Karsten and Marshall [2002] described climatological buoyancy profiles in the ACC decaying exponentially with depth below the mixed layer, with length scales that increase equatorward. We found that buoyancy profiles from the XBT/XCTD dataset are well-described by a decaying exponential $b_1 e^{z/L}$, where $z = 0$ at the base of the mixed layer, plus a constant offset term $b_0$; both terms
and all three parameters \((b_0, b_1, \text{and} \ L)\) vary across the current. Along the most commonly sampled line (see Figures 2.1 and 2.2a), mean buoyancy length scales increase from just over 100 m in the south of the passage to 600-1000 m in the PF and 500 m in the PFZ (depth coverage in the SAF was insufficient to determine a length scale). Given that a large portion of the mean velocity, at least, is geostrophic, we might expect that velocity observations would be describable by the corresponding geostrophic velocity profile. The functional form resulting from exponential buoyancy does describe both LADCP and SADCP currents quite well in a least-squares sense—unsurprisingly enough, given the additional degrees of freedom relative to (2.1). However, the best-fit velocity coefficients do not show the expected relationships to the buoyancy fit coefficients from which they should derive, so we cannot justify using this more complex form.

**Combination of functional forms**

No single functional form appears satisfactory for describing the SADCP data at all locations (or even all locations within a particular region). For mean SADCP velocity, (2.1) is best at 30% of points and a linear fit at 70%; for gridded SADCP the proportions are 55% and 45%. There is no clear spatial pattern to which fit works best, and the rms error is not improved by combining the two forms. Given this, in Section 2.5.1 we use (2.1) to extrapolate below the SADCP depth range, while keeping in mind its shortcomings.

**2.5 Transport**

We estimated mean transport in the top 1042 m from the most commonly sampled section (see Figure 2.1). The result, 95.1 Sv, is a sizeable fraction of the canonical value for time-mean full-depth total transport [134 Sv, *Whitworth and Peterson*, 1985]. Mean transport from this dataset increases nearly linearly with depth and is consistent with the 27.8±1 Sv in the top 250 m calculated by *Lenn et al.* [2007] over a longer time period. Of the mean 0-1042 m transport, the SAF carries 35.6 Sv, the PF 48.4 Sv, and the SACC 11.1 Sv (Figure 2.4b). The
proportions of transport carried by each front in the top 1000 m are within error bars of the proportions calculated by Cunningham et al. [2003] from baroclinic transport relative to the deepest common level at WOCE SR1b.

We also calculated time-varying transport by year and from individual transects (see Appendix 2.A.1 for description of gap-filling). Yearly-mean transport in the top 1042 m ranges from 77.6 Sv (2008) to 104.5 Sv (2006). The standard deviation of transport from 53 crossings is 15.5 Sv (Figure 2.10b), giving a standard error of the mean of 2.1 Sv. As a fraction of the mean value, this standard deviation is twice as large as that of the five-year net transport record constructed by Whitworth and Peterson [1985] using the correlation between transport and cross-passage pressure difference at 500 m. The calculation of Whitworth and Peterson [1985] requires the assumption that baroclinic variability below 500 m is negligible, an assumption supported by the finding of Sokolov and Rintoul [2009b] that variability in total baroclinic transport is small. In the SADCP timeseries, however, the variability in transport due to the depth-mean component over the top 1000 m is comparable to the variability due to shear, both above and below 500 m (Figures 2.10c and 2.11).

The cross-passage distribution of transport (Figure 2.10a) changes from crossing to crossing. Transport in the northern part of the passage is more variable than in the southern, and variability is maximum in the mean PFZ. This pattern likely reflects the distribution of eddies—the northern jets provide more energy to generate eddies—as well as variability in the PF and SAF positions (meandering or filamenting). While the Rossby radius is smaller in the southern part of the passage, the smaller envelope of transport in this area suggests that unresolved variation is also likely to be small. Mean transport is insensitive to subsampling transects to as much as 100 km resolution, and variability is insensitive to subsampling to $>50$ km.

### 2.5.1 Extrapolation to Full-Depth Transport

We used the vertical structure functions discussed in Section 2.4.2 to extrapolate from the observations in the top 1030 m to a value for full-depth transport by
integrating the fitted functions. Our estimate of the error on mean transport from a fitted function has two components: that associated with our estimate of the mean, and that associated with the misfit between the fit and the actual current. For the first we used the standard deviation of transport from the fit, obtained by propagating the parameter standard deviations. For the second, in order to account for the misfit over the whole depth range of integration, we used the minimum data standard deviation as an estimate of misfit at all depths below our data range.

As might be expected from the change in shear in the LADCP data above and below 1000 m, fits involving linear trend terms result in unrealistically large or unrealistically small full-depth transport estimates (although similarly large error bars put them within error of the canonical value). Meanwhile, (2.1) using length scales from SADCP gives $98 \pm 10$ Sv in the top 1042 m and $154 \pm 38$ Sv over the full depth. With length scales from SOSE, the full-depth extrapolated transport is larger, at $164 \pm 23$ Sv, but not significantly so (and note that the error bar on this latter estimate does not take into account the failure of SOSE to accurately reproduce the vertical length scales of the real current). Of the 154 Sv of full-depth extrapolated transport, 64 Sv, 63 Sv, and 26 Sv are carried in the SAF, PF, and SACCF, respectively. These proportions are within 10% of the observed partitioning in the top 1042 m, and are thus also similar to those observed by Cunningham et al. [2003].

Both extrapolations from (2.1) are consistent with the canonical $134 \pm 11$ Sv of Whitworth and Peterson [1985]. We also used the observed upper-ocean transport to put bounds on the full-depth transport in a simpler manner. Figure 2.12 shows the area-weighted observed mean velocity profile in the top 1000 m, and the transport resulting from a linear fit to this profile; then it shows two possible methods of linearly extrapolating. The first, extrapolating with the same slope as in the top 1000 m until the current goes to zero, at around 1800 m, and then letting the current be zero below that, gives a value of 117 Sv. We consider this a lower bound, based on numerous observations of eastward flow at depth. For the second estimate we extrapolate linearly at each location from the current at
1000 m to zero at the bottom, giving 220 Sv, which we consider an upper bound.

2.6 Discussion and Summary

The LMG SADCP observations confirm that the mean ACC in Drake Passage is equivalent-barotropic (streamlines are self-similar with depth), at least over the sampled 1000-m depth range; the same is true of the low-wavenumber part of the time-varying current. The vertical structure does not appear, however, to be separable equivalent-barotropic: vertical length scales vary across the passage. Shear is on average lower in the region of the SAF than in the PF region, but there is significant time variability in vertical scale as well.

Both mean and time-varying vertical shear are observed to be small all the way down to 1000 m; this extends the observations of Lenn and Chereskin [2009] in the surface layer. The direct current measurements reveal that mean currents at 1000 m are quite strong (20 cm s$^{-1}$ in northern Drake Passage). The mean 0-1042 m transport of 95±2 Sv (standard error) is 71% of the canonical total transport of 134±11 Sv [Whitworth and Peterson, 1985] in ~30% of the depth range. Given the observations of eastward mean bottom currents discussed in the introduction, we expect transport to be positive at all depths. If we are to reach a total transport of 134 Sv under this condition, the mean shear below 1000 m must be only slightly smaller than that above (see Figure 2.12). The LADCP data, however, indicate a more pronounced decrease in shear (a factor of two on average), as do the results of Inoue [1985] on the vertical structure of currents measured by the ISOS moorings and the exponential or exponential-like vertical structure found by Karsten and Marshall [2002] and other studies of measured or modeled density profiles. Alternatively, the 71% ratio might reflect the large time variability observed in this dataset as well as in studies by Cunningham et al. [2003] and others. We note, however, that the 4.5-year 38 kHz SADCP timeseries is a relatively long one in this region, and that the longer dataset available for the upper ocean [Lenn et al., 2007, 2008] shows no significant difference between upper ocean transport over this time period and over the full decade of that record.
It is also possible that the barotropic component of the current is larger than previously thought and that the total transport of the ACC through Drake Passage is greater than the canonical value. Both the ISOS estimates of Whitworth [1983] and Whitworth and Peterson [1985] and additional estimates of Cunningham et al. [2003] relied on just a few synoptic sections to infer the barotropic component of the transport. From the present analysis we produce several different estimates of total transport, and conclude that 154±38 Sv is a reasonable estimate of the mean, with 117 and 220 Sv representing lower and upper bounds.

The results of our calculation of transport from direct velocity measurements reinforce the conclusion of Cunningham et al. [2003] that the baroclinic component of transport variability is significant. The depth-mean and shear components of transport variability in the top 1000 m are of similar size; since variability in the depth-mean over the top 1000 m can result from barotropic variability or from variability in sub-1000 m shear, the baroclinic component likely makes an even larger contribution to the full-depth transport variability.

An equivalent-barotropic structure permits the ACC to act as a waveguide for Rossby waves and could allow the eddies observed in satellite altimetry to be interpreted as Rossby waves advected downstream by the mean current [e.g. Hughes, 1996]. Such an interpretation is necessary for the critical layer theory elaborated by Smith and Marshall [2009] as an explanation of a deep maximum in eddy stirring–although Abernathey et al. [2010] showed that linear critical layer theory is not necessary to explain a subsurface mixing maximum. The conclusions of Hughes [2005] about the ACC vorticity balance also rely on an equivalent-barotropic vertical structure to extrapolate surface dynamic topography to the current as a whole. In general, the confirmation of equivalent-barotropic structure, at least in Drake Passage, is promising for extrapolation of the better-observed surface to properties and dynamics of the full water column. However, we were not able to find a simple functional form to reliably describe the vertical structure; other observations and analysis are still needed to better determine length scales and structure in Drake Passage and elsewhere.

The resolution of the LMG SADCP dataset allows us to observe three dis-
tinct jets in the mean, while the multiple filaments observed by Sokolov and Rintoul [2007] and Sokolov and Rintoul [2009a] are observable in individual transects. The consistency of the large-scale vertical structure in each frontal region, despite variability in jet position and number, matches the finding of Sokolov and Rintoul [2009a] that given fronts are consistently associated with certain SSH contours. Meanwhile, the SADCP observations are compatible with the role of the ACC as a partial mixing barrier [Marshall et al., 2006]. One indication that mixing across the current may be restricted is that EKE in Drake Passage is concentrated between the mean PF and SAF regions and is much smaller than mean KE in the jets themselves, possibly indicating that the PF and SAF are barriers to mixing [Ferrari and Nikurashin, 2010]. While the Reynolds stress $\langle u' \psi' v' \psi' \rangle$ is nearly depth-independent over the 1000-m range of the SADCP, the ratio of mean KE to EKE increases over this depth range, suggesting that the maximum in effective diffusivity described by Smith and Marshall [2009] and Abernathey et al. [2010] at 1000 to 1500 m below the core of the stream-averaged ACC would, in Drake Passage, have to occur at the deeper end of this range. However, the KE:EKE criterion of Ferrari and Nikurashin [2010] may not apply to Drake Passage, where scale separation between jets and eddies is doubtful and the shifting and meandering of the jets may reduce their effect as mixing barriers [Thompson, 2010; Thompson et al., 2010]. In addition, the high EKE in the PFZ does indicate significant eddy stirring above 1000 m in the center of the ACC in Drake Passage.

The observed current structure also has implications for numerical modeling. There are clear differences in the details of the vertical structure between the SADCP and LADCP observations and the model output of SOSE, and between observations and other climate models, but accurate representation of the mean current structure is a precondition for accurate parameterizations of mixing. Comparison with observations is important for testing numerical models used in climate studies. Further measurements over a more extensive depth range would enable us to refine the vertical structure and length scales of the current, and provide a better estimate of the full-depth transport and eddy activity of the ACC, while the extension of the LMG SADCP timeseries will provide insight into their
time variability. In addition, future runs of SOSE will archive profile data with sampling matching that of the LMG SADCP dataset, allowing both a more accurate comparison and an evaluation of the possibility of assimilation of the SADCP dataset into SOSE or another assimilating model.

2.A Velocity Bias Correction

When we initially examined transport from the 38 kHz ADCP (os38) on individual crossings, a systematic offset between transport from southbound legs and that from northbound legs was revealed: the mean transport over 27 southbound cruises was 24 Sv less than the mean over 26 northbound cruises. Upon investigation, this large transport offset appears to be due to a very small velocity bias (see Donohue et al. [2001]). The source of this bias and our attempt to correct for it are described here.

2.A.1 Transport Calculation

Most LMG Drake Passage crossings are accomplished in around 2 days and are thus suitable for calculation of transport. We calculated transport between the surface and 970 m for crossings that met the following criteria: 1) crossing covers less than 4 days; 2) data extend to at least the 300 m depth contour in the north and the 700 m contour in the south; and 3) after filling as described next, data coverage is at least 90%. The 15-minute data were vector-averaged to a 25-km along-track grid. Profile (partial-depth) gaps up to 50 km (150 km) were filled using an objective interpolation procedure with Gaussian covariance with horizontal and vertical scales of 50 km and 480 m, respectively. The horizontal decorrelation scale was determined from lagged autocorrelations of 15-minute data, and the vertical scale reflects the low shear of the mean profiles. We assumed slab motion above 46 m (the first depth bin). Velocities were transformed into along- and across-track components (where track orientation varies by grid point) to calculate the flux of water across the sampling line and give transport as a function of along-track distance and time.
The resulting transport time series has a bimodal distribution; ignoring two outliers, values from the 26 southbound cruise legs are 24.3 Sv less on average than those from the 25 northbound legs (Figure 2.A.1). The southbound series has a standard deviation of 12.3 Sv and the northbound series of 8.0 Sv (outliers again excluded). The disparity in transport appears at all depths and all locations across the passage (Figure 2.A.1), and differences in transport do not correspond to differences in track longitude, season, coverage, range, or depth extent. Thus we turned to the velocity.

2.A.2 Velocity Distributions

Probability density functions (PDFs) constructed from the 15-minute velocities show the same bias as transport: the distribution of across-track velocity from northbound legs is shifted higher than that from southbound legs. The two distributions are significantly different at the 99% confidence level by the Kolmogorov Smirnov test ($P_{KS} < 10^{-3}$), with means that differ by 3 cm s$^{-1}$. No bias is evident in along-track velocity; its distributions are not significantly different ($P_{KS} > 0.3$). The across-track bias has the same sign independent of location. A constant bias in across-track but not along-track velocity is consistent with a misalignment angle error, as described below.

To help rule out the possibility of ship navigation errors or of high-frequency motion correlations that affect the average water motion relative to the ship, we also examined velocities from the other ADCP on the LMG, a 150 kHz RDI. Its distributions of across- and along-track velocity both have means that differ by less than 0.5 cm s$^{-1}$. (The along-track velocity distributions are different at the 98% confidence level; a small difference in the along-track component is expected because this instrument requires an amplitude correction). We infer that neither navigation errors nor high-frequency motion correlations are an issue because the misalignment angle of the 150 kHz ADCP has been fully accounted for by the normal processing with bottom-tracking; the 38 kHz velocity bias seems to be peculiar to that instrument.
2.A.3 Angle Correction

The standard SADCP data processing procedure includes a correction for misalignment angle, the angle between the ADCP beam 3 and fore. This angle can be determined by either water-track or bottom-track, as described in Joyce [1989]; bottom track values are generally more stable and have been used for the LMG data. Where Ashtech GPS or bottom track data are not available the appropriate correction for a given cruise leg is estimated using corrections to adjacent legs. The 7 of 54 os38 legs on which this estimation was required are not associated with anomalous transport values.

The size of the misalignment angle correction to the across-track component of velocity is approximately

\[ \Delta u \simeq |u_{\text{ship}}| \sin \theta_{\text{mis}}, \]

(2.2)

where \( u_{\text{ship}} \) is the ship velocity and \( \theta_{\text{mis}} \) the misalignment angle. Assuming an average ship speed of 5.5 m s\(^{-1}\), and keeping in mind that the mean velocity difference of 3 cm s\(^{-1}\) corresponds to a bias of 1.5 cm s\(^{-1}\) (up for northbound, down for southbound), we have an unaccounted-for misalignment angle of about 0.16°. The bottom-track misalignment angle corrections used for the os38 have a mean of 0.416°, a standard deviation of 0.046°, and a range of 0.200°. Given this, it is puzzling that a bias of as much as 0.16° should remain, and we are not able to explain why it is not accounted for by the normal bottom-track processing. Nevertheless, we can still correct for it; first we calculated the remaining misalignment angle more carefully.

We used the assumption that the time-mean transports from northbound and southbound legs should be the same. Using the 5-minute velocities, we calculated transport \( U \) on each suitable leg (see Section 2.A.1), and then calculated the constant velocity offset necessary to change \( U \) to \( U_{\text{mean}} = 90 \) Sv, the mean of all the transport estimates. From (2.2) and the leg-mean ship speed, we then obtain the corresponding \( \sin \theta_{\text{mis}} \). Excluding the two outliers, the mean of this series corresponds to an angle of \( \theta_{\text{mis}} = 0.16° \), and the standard deviation to 0.15° (standard error of 0.02°).
We applied the additional angle correction to the 5-minute data by

\[ \vec{u}(t) = u_{\text{meas}}(t)e^{-i\theta_{\text{mis}}} + u_{\text{ship}}(t), \]  

(2.3)

where the measured velocity \(u_{\text{meas}}\) has already been corrected by the bottom-track-derived misalignment angle. As expected, on average the resulting change in cross-track velocity is approximately +1.5 cm s\(^{-1}\) for southbound and −1.5 cm s\(^{-1}\) for northbound data. By propagating the standard error of the additional misalignment angle through the calculation, we obtained an error bar on the velocity correction of less than 2 mm s\(^{-1}\), well within the expected noise.

Southbound and northbound distributions of corrected velocity are not significantly different (\(P_{ks} > 0.3\)) in either component. Due to the small velocity correction and the fact that approximately equal numbers of southbound and northbound legs pass through a given grid box, the mean velocities are barely affected. EKE is reduced by a small amount in most locations; large-scale patterns remain the same. Transport between 0 and 970 m from the corrected velocity has a mean of 90.5 Sv as opposed to 90.2 Sv from the uncorrected records. The standard deviation for the corrected transport time series is 15.5 Sv (standard error 2.1 Sv). The misalignment angle correction rectifies not only total transport (by definition) but \(U(y, z)\) as well (compare Figure 2.10 to Figure 2.A.1).

2.B Shear

In the examination of velocity distributions and transport as a function of time, an apparent northbound-southbound difference in os38 velocity shear near the surface also came to our attention (Figures 2.10 and 2.A.1). PDFs reveal that the bias is even larger in the along-track component of velocity shear, \(du_a/dz\). This component differs significantly at all depths and in all water depths, while the across-track component \(du_c/dz\) is different only between 46 and 118 m. Above 118 m, the difference between the northbound and southbound mean shears is \(3 \times 10^{-4}\) s\(^{-1}\) in the across-track direction and \(1 \times 10^{-3}\) s\(^{-1}\) in the along-track direction. The standard error of the mean is only about \(8 \times 10^{-5}\) s\(^{-1}\). The 150 kHz instrument also gives northbound and southbound \(du_a/dz\) distributions that are
significantly different at all depths, but the differences between the means are much smaller.

The disparity in shear distributions leads to reduced confidence in the near-surface data from the os38 in particular. Recalculation of the mean transport using a slab assumption from 118 m to the surface gives a value 0.55 Sv (from individual crossings) to 0.88 Sv (from the average velocity on the most common track) less than that with a slab starting at 46 m. As the difference in transport is well within our error bars we have continued to use near-surface data for this calculation.

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Table 2.1: Performance of two types of fits to vertical profiles of time-mean velocity. Fit parameters are $\vec{u}_0, \vec{c}, \vec{a}, L,$ and $L_{SOSE}$; fits are explained in more detail in Section 2.4.2. For comparison, standard deviations of observed $\vec{u}(z)$ are about $4.3$ cm s$^{-1}$.

<table>
<thead>
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<th>rms difference (cm s$^{-1}$)</th>
<th>% var. explained</th>
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</tr>
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<td>2.0</td>
<td>76</td>
</tr>
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<td>1.9</td>
<td>65</td>
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<tr>
<td>SOSE L</td>
<td></td>
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<tr>
<td>combined best</td>
<td></td>
<td>100</td>
<td>2.0</td>
<td>67</td>
</tr>
</tbody>
</table>
Figure 2.1: Left: LMG tracks with 38 kHz data (black lines) over Orsi et al. [1995] fronts (gray lines) and bathymetry (described in text). Top right: most commonly sampled line (gray box) and cDrake LADCP profile locations (black dots), with Orsi et al. [1995] fronts in gray. Bottom right: larger view; the box outlines the study area shown at left.
Figure 2.2: Objectively mapped streamfunction $\psi$ contoured at 5 cm intervals: (a) depth-mean streamfunction overlaid on bathymetry $H$, with the most commonly sampled line in white; (b)-(d) streamfunction and current vectors at several depths, with only currents larger than 15 cm s$^{-1}$ plotted.
Figure 2.3: Time-mean speed from SADCP (a) and SOSE (b) on the most commonly-repeated section (see Figures 2.1 and 2.2a). Difference SADCP speed - SOSE speed, scaled by the SADCP standard deviation, for means from all available data (c) and from SOSE data coincident in time with SADCP data during 2005-2007 (d). The black lines on each plot are the 20 cm s$^{-1}$ contour of SADCP speed and indicate the locations of the mean observed fronts, from south to north, the SACCF, PF, and SAF.
Figure 2.4: Time-mean across-track velocity $u_\perp$ on the most commonly sampled line. SADCP (solid) and SOSE (dashed) profiles relative to the depth mean $\bar{u}_\perp$ are shown in the top panel. The bottom panel shows the SADCP depth mean-velocity in cm s$^{-1}$ (colored line; line color connects profiles and across-passage locations) and depth integrated 0-1042 m transport in Sv km$^{-1}$ (black line), as a function of distance along the line.
Figure 2.5: At 118 m (black lines) and 886 m (gray lines): (a) current speed $\langle u_\psi \rangle$, (b) current angle $\langle \theta_\psi \rangle$, and (c) $EKE = \frac{1}{2}\langle u'^2 + v'^2 \rangle$ on the most commonly-sampled line (Figures 2.1 and 2.2a). Filled areas indicate standard error about the mean at 118 m (dark gray) and 886 m (light gray).
Figure 2.6: Average section along the most commonly-repeated transect (see Figures 2.1 and 2.2a): (a) mean KE, (b) EKE, (c) \( \langle u'_\psi u'_\psi \rangle \), (d) \( \langle v'_\psi v'_\psi \rangle \), (e) \( \langle u'_\psi v'_\psi \rangle \). In (c), (d), and (e), only values significantly different from zero are colored. Note the nonlinear colorbar in (a)-(d), selected to show more detail at low values and specific values at color transitions.
Figure 2.7: Vertical wavenumber spectra of $u_\psi$ (solid) and $v_\psi$ (dashed) from 46 to 1030 m, averaged over regions defined relative to the mean fronts, as indicated. 95% confidence intervals for each region are indicated by thin vertical lines.
Figure 2.8: Mean SADCP speed profile in each frontal region (symbols) with the first EOF in each region (black lines) and fits to (2.1) using mean length scales $L_{SOSE}$ determined from fits to SOSE profiles (gray lines). The EOF first modes were scaled to match the mean speed profiles. From north to south $L_{SOSE} = 1635, 2126, 1355, 1289, \text{ and } 1355$ m.
Figure 2.9: Length scales of best-fit exponentials determined by fitting (1) to (a) SADCP mean velocity profiles and (b) SOSE mean velocity profiles.
Figure 2.10: Transport: (a) cumulative from 0-970 m (southbound gray solid line, northbound black dashed line); (b) total from 0-970 m (southbound circles, northbound squares); (c) per unit depth (southbound gray solid line, northbound black dashed line). The black bars in (a) indicate the frontal zones, delimited by the points where mean transport per unit distance falls to $e^{-1}$ of its maximum value in each front.
Figure 2.11: Transport per unit depth relative to 502 m (southbound gray solid line, northbound black dashed line).
Figure 2.12: Schematic showing mean SADCP transport in the top 1000 m (black line, dark gray shaded area) and two possible linear extrapolations for full-depth transport: extrapolating the transport profile with the same shear until it reaches zero (light gray line); or extrapolating each velocity profile linearly from its value at 1000 m to zero at the bottom at each location (gray shaded area). The full-depth transports corresponding to these two extremes are 117 and 220 Sv, respectively.
Figure 2.A.1: Uncorrected transport: (a) cumulative from 0-970 m (southbound gray solid line, northbound black dashed line); (b) total from 0-970 m (southbound circles, northbound squares); (c) per unit depth (southbound gray solid line, northbound black dashed line).
Chapter 3

Methods for computation of geostrophic streamfunction and its derivatives and error estimates from a two-dimensional array of CPIES and a GEM

Abstract. We describe a procedure for calculating barotropic and baroclinic geostrophic streamfunction and its first, second, and third derivatives by objective mapping of current, pressure, or geopotential height anomaly data from a two-dimensional array of Current and Pressure-recording Inverted Echo Sounders (CPIES). CPIES were deployed in an eddy-resolving local dynamics array (LDA) in the eddy-rich Polar Frontal Zone in Drake Passage as part of the cDrake experiment. An empirical look-up table (the Gravest Empirical Mode, GEM) based on regional hydrography is used to translate bottom-to-surface round-trip sound travel time to vertical profiles of hydrographic properties such as geopotential height anomaly. We explain modifications to previous methods that reduce the error on velocity and higher streamfunction derivatives. Simulations are used to investigate the ability to reproduce higher streamfunction derivatives by objective
mapping from an array like the cDrake LDA, and to verify mapping error estimates. The velocity fields calculated from cDrake are compared to lowered ADCP velocity measurements and satellite-derived surface currents. Comparisons between cDrake temperature and velocity fields and moored current and temperature observations in the LDA help validate the cDrake method and results.

3.1 Introduction

As part of the cDrake experiment, an array of 42 Current and Pressure-recording Inverted Echo Sounders (CPIES) was deployed across Drake Passage from November-December 2007 to November-December 2011 (Figure 3.1, Chereskin et al. [2009], Chereskin et al. [2012]). The 3x7 Local Dynamics Array (LDA) in the Polar Frontal Zone (PFZ) provides two-dimensional sampling and the potential to estimate velocity and vorticity in this eddy-rich area. An empirical look-up table for hydrographic property profiles indexed by sound travel time between the surface and a pre-selected reference depth, known as a Gravest Empirical Mode [GEM, Meinen and Watts, 2000; Watts et al., 2001b] table, was constructed from nearly 600 CTD and Argo profiles in the cDrake region [Cutting, 2010], and provides the ability to convert travel time to hydrographic profiles. Here we explain the procedure for computing geostrophic barotropic streamfunction and its derivatives from the cDrake LDA bottom pressure and current, and the procedure for computing geostrophic baroclinic streamfunction from the bottom-to-surface round-trip sound travel time and a GEM.

Two-dimensional arrays of inverted echo sounders, together with current measurements, have been used to compute geostrophic streamfunction and velocity in several experiments, including studies in the Gulf Stream [SYNOP, Tracey et al., 1997; Watts et al., 2001a], Subantarctic Front [SAFDE, Meinen and Luther, 2002; Tracey et al., 2006], and Kuroshio Extension [KESS, Donohue et al., 2010]. Some of these studies have used objective mapping [Bretherton et al., 1976] with the assumption of non-divergence [Qian and Watts, 1992; Watts et al., 2001a] to produce barotropic streamfunction and velocity from bottom pressure and velo-
ity data, and baroclinic streamfunction and velocity from the IES-measured travel times in combination with a GEM. Optimal interpolation parameters (covariance length scale and noise) have been determined by fitting correlations from the PIES data to an a priori Gaussian covariance function for streamfunction. Iterative mapping is used to capture variance at multiple time and space scales.

In recent studies [Meinen and Luther, 2002; Donohue et al., 2010] baroclinic velocity has been computed by mapping travel time data to travel time and the horizontal gradients of travel time, then converting to velocity by using derivatives of geopotential anomaly with respect to travel time. These derivatives are estimated by finite differencing of a look-up table for geopotential anomaly as a function of travel time and pressure. Baroclinic velocity profiles computed by this method have been found to compare well with directly measured currents in SAFDE [Watts et al., 2001b]; in the Southern Ocean, 97% of density and temperature variance can be captured by a GEM [Sun and Watts, 2001], making GEM-based prediction of baroclinic velocity profiles in the cDrake area a promising prospect. Howden and Watts [1999] objectively mapped spatial derivatives of velocity in the Gulf Stream by applying a linear operator to the correlation functions [Bretherton et al., 1976].

In previous experiments where objective mapping was used to compute streamfunction and velocity from an array of CPIES [e.g. Tracey et al., 1997; Meinen and Luther, 2002; Donohue et al., 2010], the array has been centered under the jet. The cDrake LDA, in contrast, was located in the high-variability PFZ, between the mean positions of the two main ACC jets (the Subantarctic Front to the north and Polar Front to the south). A main focus of this work is on resolving the strong mesoscale variability in the PFZ and making accurate estimates of uncertainty in a nonlinear regime, which requires attention to higher-order terms. We build on previous work to extend the objective mapping procedure to third derivatives of geostrophic streamfunction, with variable Coriolis parameter $f$ (Appendix). We discuss the advantages of converting travel time, $\tau$, to geopotential, $\Phi$, for time series at each observation site (using the GEM look-up tables), and mapping directly to velocities and their derivatives without finite differencing. We describe the data sets used (Section 3.2), the construction of the cDrake GEM
(Section 3.3) and the objective mapping process as implemented for the cDrake LDA (Section 3.4). We investigate the assumptions involved in the CPIES data processing and the GEM and mapping procedure, and test the results using simulations (Section 3.5) and comparisons with three independent data sets in the LDA area (Section 3.6). In Section 3.7 we summarize our methods and conclusions.

3.2 Data

The CPIES provide time series of bottom pressure, near-bottom current, and bottom-to-surface round-trip sound travel time. CPIES current meters are cabled 50 m above the bottom, a level chosen to measure geostrophic current outside the bottom boundary layer. Sampling is 30 minutes for bottom pressure, hourly for current, and 4 pings every 10 minutes for travel time. Daily-averaged data were downloaded via acoustic telemetry on annual telemetry cruises, allowing for servicing of malfunctioning instruments. Here we use the full data records extracted directly from the instruments after recovery.

We followed the procedures described in Tracey et al. [1997] and Donohue et al. [2010], with some experiment-specific modifications. Bottom-to-surface round-trip travel time \( \tau \) and bottom pressure and velocity \( p, u, v \) are quality-controlled by removing \( \tau \) outliers and pressure spikes and steps. The \( \tau \) outliers and pressure spikes are associated with strong bottom currents causing an instrument to tip over for a period of time (all instruments self-righted when currents decreased). Some instruments were also dragged down-slope by strong currents, causing an abrupt offset to higher pressure (pressure “step”). Travel time data are windowed and averaged to hourly values [Kennelly et al., 2007; Donohue et al., 2010]. Currents are measured with an Aanderaa acoustic Doppler current sensor, and two corrections were applied. First, current directions were corrected for local magnetic declination. Second, current speeds were multiplied by a sound speed scale factor, the ratio of the local sound speed to the nominal value of 1500 m s\(^{-1}\) used by the instrument in situ. Bottom pressure is averaged to hourly, detided using response analysis [Munk and Cartwright, 1966], and leveled and dedriffed using...
geostrophic streamfunction from geostrophic objective mapping of bottom velocity [Donohue et al., 2010]. Bottom-to-surface round-trip travel time \( \tau \) is adjusted for (a) path-length changes as determined by bottom pressure, (b) the estimated inverted barometer effect of atmospheric pressure, (c) the effect of latitude on gravity in converting between geometric height and pressure, and (d) the seasonal cycle.

It is then converted to travel time, \( \tau_{\text{index}} \), between the surface and an index level, here 2000 dbar, by fitting to \( \tau_{\text{index}} = A\tau^2 + B\tau + C \), where coefficients \( A, B, \) and \( C \) are depth-dependent and determined empirically from historical hydrography, with additional adjustments to \( C \) based on CTD profiles taken at each site during CPIES sampling [Meinen and Watts, 1998; Donohue et al., 2010]. All data are then low-pass filtered using a 4th-order Butterworth filter with a 3-day cutoff, run forward and backward, then sub-sampled to 0.5-day (with the first and last day removed to avoid transients). At each site for each variable, a single 4-year-long time series is constructed. For instruments that were replaced during the experiment, a 4-year-long time series is patched together from multiple deployments at the same site. These time series of twice-daily \( \tau_{\text{index}} \) and bottom \( p, u, v \) are the inputs to objective mapping (in the following sections we will use \( \tau \) for \( \tau_{\text{index}} \)).

Available CTD and Argo profiles within Drake Passage have been used to compute the cDrake GEMs for temperature, salinity, specific volume anomaly, and geopotential anomaly. The GEM used here and described below incorporates 599 profiles extending to at least 2000 dbar, including 227 of the CTD casts from the five cDrake cruises. Two hundred ninety-one XCTD profiles and 118 of the CTD and Argo profiles were used to estimate the seasonal cycle [Cutting, 2010].

A total of 284 lowered acoustic Doppler current profiler (LADCP) casts were made on the five cDrake cruises, including calibration CTD casts performed at each CPIES site after deployment. In the LDA area, 56 LADCP casts fall within half a day of available CPIES mapped data (this excludes a number of the deployment and all of the recovery cruise casts, performed when only some LDA CPIES were in the water). The LADCP is a 153.6 kHz broadband RD Instruments ADCP, with a 30° beam angle and a 16-m vertical bin, pulse, and blank before transmit. Data were processed following Fischer and Visbeck [1993] using the University of Hawaii
CODAS software developed by Eric Firing. Vertical shear measurements obtained by differencing LADCP-measured water velocity relative to instrument are gridded to 5-m depth bins, sub-sampled to 20-m, and integrated to obtain the baroclinic velocity profile, which starts at 60 m depth and extends to the bottom. The depth-averaged, or barotropic, velocity is obtained by adding the ship drift to the integrated measured velocity over the cast. Errors in the baroclinic component of velocity using this instrument, sampling scheme, and processing method are 4 cm s$^{-1}$; those in the barotropic component are 1 cm s$^{-1}$ [Chereskin et al., 2010]. Profiles of standard deviations of the gridded total velocity are also calculated, and up-and down-cast profiles are averaged. For this work the barotropic tide as predicted by the TPXO7.2 tidal model [Egbert et al., 1994] was removed.

Several current meter moorings deployed across Drake Passage as part of the DRAKE experiment [Ferrari et al., 2012] overlapped with the cDrake experiment in space and time. Mooring M2, located in 3800 m depth approximately 16 km from cDrake site C03 (Figure 3.1), overlapped with cDrake for 11 months. Mooring M4, located in 4100 m depth approximately 5 km from cDrake site E02 in the LDA (Figure 3.1), overlapped with cDrake for 16 months; M4 was turned around with new instruments 4.5 months into the overlap period. Each mooring collected current, temperature, and pressure observations at three depths. M2 had two Aquadopps and an RCM-8 at minimum pressures (representing full mooring extension) of 420 dbar, 930 dbar, and 2000 dbar, respectively. For the first deployment M4 had an RCM-7, RCM-8, and RCM-11 at minimum pressures of 430 dbar, 960 dbar, and 2450 dbar, respectively; for the second deployment RCM-11s were located at minimum pressures of 520 dbar, 1020 dbar, and 2540 dbar. Data from co-located microcats and from an Aquadopp nominally at 500 m on M4 during the first deployment are not discussed here. Processing, described by Ferrari et al. [2012], includes correcting velocity data for local magnetic declination, and daily averaging of both current and temperature data (collected at 2-hourly or greater frequency); daily current and temperature were also corrected for the effects of mooring motion using the method of Cronin and Watts [1996]. We applied a 3-day low-pass filter to the daily current and temperature time series, to match the treat-
ment of cDrake data (see above). Current outliers with either component larger than 1 m s$^{-1}$ were removed and replaced by linear interpolation before 3-day low-pass filtering. We compare both the mooring-motion-corrected and uncorrected moored current and temperature records with cDrake current and temperature. The most direct comparison, between the original pressure-varying moored records and cDrake currents and temperatures at the same varying pressures, validates the cDrake fields. Comparisons at fixed pressure levels using mooring-motion-corrected data give an estimate of additional variance from the correction scheme.

We also compare cDrake surface currents with satellite surface currents derived from the mean absolute dynamic topography (MADT) produced by Ssalto/Duacs (http://www.aviso.oceanobs.com/en/data/product-information/duacs.html) and distributed by AVISO with support from CNES (http://www.cnes.fr). We use the weekly delayed-time MADT, which incorporates sea level anomaly from multiple satellites (up to four at a time) with the CNES-CLS09 mean dynamic topography, on a $\frac{1}{3}^\circ$ Mercator grid.

### 3.3 cDrake GEM

Five hundred ninety-nine historical hydrographic casts extending to at least 2000 dbar were used to compute temperature ($T$), salinity ($S$), and specific volume anomaly ($\delta$) GEMs as functions of $\tau_{2000}$, as described in Meinen and Watts [2000] and Donohue et al. [2010]. The specific volume anomaly ($\delta$) profiles were computed from the CTD temperature and salinity profiles using the Seawater toolbox for MATLAB [Morgan, 1994]. GEM curves were computed by spline-fitting data at a suite of pressure levels ranging from 0 to 3800 dbar (231 of the casts extend to at least this depth; pressure levels in dbar were every 20 from 10 to 310, every 50 from 360 to 1010, every 100 from 1100 to 1300, at 1600, every 200 from 2200 to 2400, at 2800, at 3350, and every 200 from 3600 to 3800). The GEM curves were then interpolated to 10-bar pressure resolution using cubic splines. The GEMs were extended below 3800 dbar, where available data are sparse, by offsetting the
curves at 3800 dbar to agree with the mean at each deeper level. A GEM for geopotential height anomaly $\Phi$ relative to 4000 dbar was computed by integrating $\delta_{GEM}$ from a reference pressure, $p_r = 4000$ dbar (see Section 3.4.4 below), to each pressure level:

$$\Phi_{GEM}(p, \tau) = \int_{p_r}^{p} \delta_{GEM}(p', \tau) dp'.$$

(3.1)

The $T, S, \delta$, and $\Phi$ GEMs are shown in Figure 3.2. The $T, S$, and $\delta$ fields exhibit characteristics similar to the GEMs calculated by Sun and Watts [2001] for the Drake Passage as functions of geopotential height rather than travel time.

We linearly interpolate the $\Phi(p, \tau)$ GEM to the observed $\tau(x, y, t)$ and a pressure grid, obtaining $\Phi(x, y, p, t)$. Except where otherwise specified the pressure grid spans 0 dbar to 4000 dbar with 200 dbar resolution. We then objectively map (Section 3.4) $\Phi(x, y, p, t)$ to baroclinic streamfunction (relative to 4000 dbar), and its derivatives. We estimate smaller uncertainties (by an order of magnitude) when we interpolate $\tau$ to $\Phi$ and objectively map to baroclinic streamfunction and its derivatives than when we map $\tau$ and then convert to $\Phi$ and its derivatives using finite difference estimates of $\partial \Phi/\partial \tau, \partial^2 \Phi/\partial \tau^2$, and $\partial^3 \Phi/\partial \tau^3$. The comparisons discussed in this paper indicate that the smaller error estimates are more accurate. We discuss the estimation of the $\Phi$ GEM contribution to error in Section 3.4.3.

### 3.4 Computation of streamfunction and its derivatives by objective mapping

We objectively map from geopotential height anomaly $\Phi$ to baroclinic streamfunction and its derivatives, assuming geostrophy and a Gaussian covariance function for streamfunction $\psi$. Similarly, we map bottom $p, u, v$ to barotropic streamfunction and its derivatives. The relations necessary to map to various derivatives of $\psi$ are set out in the Appendix. Some details of the implementation of objective mapping and calculation of vertical profiles of velocity and its derivatives for the cDrake LDA are laid out here.
3.4.1 Overview and motivation: resolution and error estimation

We wish to calculate terms of the momentum and vorticity balances, including terms up to first derivatives of the relative vorticity $\zeta = v_x - u_y$, which require third derivatives of the streamfunction. The algorithms for objective mapping to streamfunction, velocity, vorticity, and vorticity gradients are described in the Appendix and below. Tests on a sample field meant to simulate the main eddy scale observed in the cDrake LDA (field described in Section 3.5) validated these algorithms. At 10 km horizontal resolution and with the decorrelation scales given below, finite differencing from mapped velocity fields produced results not significantly different from directly-mapped derivatives (differences were only noticeable for small residual quantities such as divergence estimated as $u_x + v_y$). The direct mapping method, however, by accounting for correlations, gives much smaller error estimates, more in line with the actual differences between analytic and mapped fields in the simulations. Similarly, comparison of cDrake velocity fields with moored velocity fields (Section 3.6.2) indicated that the direct mapping error estimates corresponded to the differences between cDrake and moored velocities and shears; the several-times-larger uncertainties resulting from use of propagation of error in finite-differenced $\frac{d\Phi}{d\tau}$ overestimated the actual errors. For this reason, we interpolate to $\Phi$ at selected pressure levels (here every 200 dbar) and then map $\Phi$ to baroclinic velocity and its derivatives.

3.4.2 Background means, multiple scales, and mapping parameters

The fields we wish to map contain multiple time and space scales; we follow a multi-step, multi-scale procedure in order to capture as much of the variance as possible. We first list the steps, followed by the method used to obtain the covariance length scales ($L$) and data noise:signal variance ratios ($E$) given in Table 3.1. Each step operates on streamfunction (geopotential $\Phi$ or scaled bottom pressure $p$) and velocity scaled by the Coriolis parameter ($fu, fv$) for the bottom
level. For $\Phi$ we map $\psi(x, y, t) = \Phi(p_i, x, y, t)$ separately for each pressure level $p_i$. For bottom $p, u, v$, $p$ is converted to streamfunction $\psi = p/\rho_0$, where $\rho_0 = 1050 \text{ kg m}^{-3}$ is a reference density, and $\psi, f u, f v$ are mapped. The objective mapping procedure assumes input fields with zero mean, so at each mapping step we remove the spatial mean from the streamfunction before mapping. In cases where velocity is an input to mapping, we also remove the spatial means $\bar{u}, \bar{v}$ from $u, v$, and the corresponding plane $f \bar{u} x - f \bar{u} y$ from streamfunction. After each mapping step the spatial mean(s) (and plane, if applicable) are added back to the mapped streamfunction and velocity. The baroclinic ($\Phi$) and barotropic (bottom) fields are combined after mapping (see Section 3.4.4 below). The mapping steps are as follows:

1. Calculate the time mean(s) and map at a large scale to produce the broad idealized mean (bim) streamfunction.
2. Remove the bim from the half-daily data and filter to produce the low-pass filtered (lp) time series.
3. For each time step map the lp data.
4. Remove the bim and the mapped lp fields from the half-daily data to produce the high-pass filtered (hp) time series.
5. For each time step map the hp data.
6. Sum the mapped hp, lp, and bim fields.

For the time filtering we use a Butterworth filter with a cutoff period of 61 days, chosen to keep the low-pass (lp) and high-pass (hp) variances approximately equal. For the bim we use a decorrelation scale of $L = 200 \text{ km}$ and data noise of $E = 0.05$. For both lp and hp we determined optimal $L$ and $E$ by calculating correlations between the series being mapped at the sampling sites as a function of site separation $r$, and fitting a Gaussian

$$c_g = (1 - E)e^{-r^2/L^2}$$

(3.2)
to correlations $c_{bin}(r)$ averaged to 10-km $r$ bins (Figure 3.3). To obtain small-$r$ information for this calculation we included time series from duplicate and near-neighbor instruments at a small number of sites; these instruments have separations $0 < r < 15$ km, much closer than the typical 35-40 km site spacing. The final mapping uses only a single time series at each site (Section 3.2). Since $hp$ depends on the mapped $lp$, determining the optimal $L$ and $E$ is an iterative procedure. The parameters for mapping bottom $p$ and travel time $\tau$ are given in Table 3.1. The correlation lengths $L$ obtained for bottom $p$ were used for $u, v$ in their respective derivative correlation functions. The noise parameters $E$ for $p, u, v$ are described in the next paragraph. Since $\Phi$ is generated from $\tau$ using the GEM look-up table, we use $L_\tau$ and $E_\phi = E_\tau + E_{GEM}$ to incorporate the errors introduced by the GEM in the calculation. The calculation of $E_{GEM}$ is described below.

As discussed in Section 3.2, CPIES processing includes an attempt to diagnose and remove and/or correct for pressure spikes caused by instrument tipping, and level changes (steps) caused by down-slope sliding. The screening is done conservatively to avoid removing real signal, so some ambiguous tip-step noise may remain. Six sites (out of 33) in the LDA area, referred to as tip-step sites (Figure 3.1), were found to be especially subject to tipping and stepping. For the $hp$ mapping of bottom $p, u, v$ we use a vector $E$. A larger value of $E$ was fitted to observed $p$ correlations when tip-step sites were included in estimating $c_{bin}$, and this was applied to $p$ inputs from the six tip-step sites. A smaller value of $E$ was fitted when tip-step sites were excluded in estimating $c_{bin}$, and this was applied to $p$ inputs at all other sites. The smaller $E$ was applied to all $u, v$ inputs because the current measurements at 50 m above the bottom were relatively unaffected by tip-steps (consistent with the assumption of negligible deep shear, see Section 3.4.4).

Upon examination of the velocity records and of high-resolution bathymetry obtained during the project, we found that site B02 (Figure 3.1) was situated in a narrow canyon, and has elongated variance ellipses parallel to the canyon axis, suggesting that near-bottom flow was constrained by the local bathymetry rather than representing the larger-scale pressure gradients. We have excluded currents from this site from the geostrophic objective mapping of bottom $p, u, v$. 
3.4.3 Φ GEM-derived contribution to noise:signal variance for objective mapping

The specific volume anomaly GEM ($\delta_{\text{GEM}}$) is constructed by fitting a spline to a set of CTD-derived specific volume anomaly values $\delta_{\text{CTD}}$ at each of a number of selected pressure levels (see Section 3.3), then interpolating the fitted splines to 10 dbar vertical resolution. To estimate the contribution to Φ error from this process, we calculate $\Phi_{\text{CTD}}$ by integrated $\delta_{\text{CTD}}$. We take the deviation from the GEM to represent the noise and the GEM itself to represent the signal, so that we have

\[\sigma_n^2(p) = \frac{1}{N_{\text{CTD}}(p)-1} \sum_{i=1}^{N_{\text{CTD}}}(\Phi_{\text{CTD},i}(p) - \Phi_{\text{GEM}}(p, \tau_{\text{CTD},i}))^2, \quad (3.3)\]

\[\sigma_s^2(p) = \frac{1}{N_{\text{GEM}}(p)-1} \sum_{j=1}^{N_{\text{GEM}}} (\Phi_{\text{GEM}}(p, \tau_j) - \Phi_{\text{GEM}}(p))^2, \]

\[E_{\text{GEM}} = \frac{\sigma_n^2}{\sigma_s^2},\]

where $N_{\text{CTD}}$ is the number of CTD samples at each pressure level, $\tau_{\text{CTD},i}$ is the $\tau_{index}$ of the $i$-th CTD profile, and $N_{\text{GEM}}$ is the number of $\tau_{index}$ bins in the GEM. We included only the 524 CTD profiles with $\tau_{index}$ within the range observed in the LDA over the 4 years of cDrake. We take $E_{\text{GEM}}$, which varies from 0.02 at the surface to 0.30 at 3800 dbar, to be the GEM contribution to the noise-to-signal ratio for mapping Φ.

3.4.4 Adjustment for deep shear in leveling pressure and mapping barotropic fields

To dedrift and level the bottom pressure data, measured pressure records are compared to mapped pressure based on objective mapping of near-bottom currents. Mapping near-bottom currents, measured at depths ranging from 3600 m to 4300 m in the LDA, as though they were all measured at one level relies on the assumption of negligible deep shear. Mapped Φ gradients in the 3600 to 4300 dbar range are consistent with this assumption, with small vertical shear with rms $< 2 \times 10^{-4} \text{ s}^{-1}$.
Although deep shears are small, we account for them by an iterative procedure. Bottom currents are first mapped as though they are all measured at the same level, and the maps are used to dedrift and level pressure records; the resulting dedrifted, leveled pressures are used to correct $\tau$, as described in Section 2. Shear derived from this $\tau$ is then used to adjust near-bottom current measurements from the estimated instrument depths to 4000 dbar before mapping, and the pressure dedrifting and leveling is redone using these adjusted current maps. References to “bottom” $p, u, v$ from Section 3.4.2 onward are to these adjusted values. The “barotropic” fields produced by mapping the 4000 dbar-adjusted $p, u, v$ are then used to reference the baroclinic fields at 4000 dbar.

### 3.4.5 Mapping error estimates

The objective mapping algorithm also provides for estimation of the percent error variance, $e_\hat{\theta}(x, y, t)$, for each estimate $\hat{\theta}(x, y, t)$ of a quantity $\theta(x, y, t)$; $e_\hat{\theta}(x, y, t)$ depends on the relative location of all input data and on the covariance function and its length scale $L$ and input noise:signal ratio $E$. In order to combine errors from mapping at different scales and to estimate propagation of error when computing other quantities from mapped fields we seek the dimensional error variance [which for the unbiased estimator is equivalent to the variance of the estimated quantity $\sigma^2_\hat{\theta}$; see Bretherton et al., 1976]. The percent error variance is normalized by the signal variance $\sigma^2_\theta = \overline{\theta^2(x, y, t)}$, the diagonal of the covariance matrix used in the mapping (Appendix), so the dimensional error variance is just $e_\theta(x, y)\sigma^2_\theta$. We estimate the signal variance from the mapped fields, as $\sigma^2_\theta \simeq \langle \overline{\theta(x, y, t)^2} \rangle$, where the overbar indicates a space mean and the angle brackets a time mean (recall that $\overline{\theta(x, y, t)} = \overline{\theta(x, y, t)} = 0$). Anisotropy between different derivatives of the same order is small, so we also average the variance estimates from different derivatives ($\sigma^2_u$ and $\sigma^2_v$, for instance) and use a single signal variance factor for each derivative order.
3.4.6 Sensitivity to mapping parameters

We have tested the sensitivity of the objective mapping to variations of 20\% in $L$ and $E$ by comparing streamfunction first derivatives ($u,v$), second derivatives ($\zeta$), and third derivatives ($\nabla \zeta$) with modified parameters to the same quantities from mapping with the parameters given in Table 3.1. Comparisons were made over the area defined by the 20\% percent error contour for $\psi$ mapped from bottom $p,u,v$. Results of the comparisons, in terms of normalized rms differences and correlations, are given in Table 3.2 (mean differences are not shown but are negligible in all cases). All maps produced with modified parameters are well-correlated ($r \geq 0.80$) with maps produced with original parameters. Percent rms differences between modified and original maps from $p,u,v$ are $< 10\%$ for variations of 20\% in $E_{lp}$ or $E_{hp}$ and $\leq 30\%$ for variations in $L_{lp}$. Maps from $\Phi$ show variations of up to 30\% in velocity and up to near 100\% in third derivatives, from a 20\% decrease in $L_{hp}$. Both maps from $p,u,v$ and maps from $\Phi$ are more sensitive to variations in $L_{hp}$, with the largest effects (listed in Table 3.2) coming from decreased $L_{hp}$, which produces maps with significantly larger extrema. This exaggerated sensitivity is not surprising given that the LDA instrument spacing is approximately 35 km, not much shorter than the optimal $L_{hp}$ (50 km, see Table 3.1).

3.5 Mapping idealized fields

We describe the mapping of idealized analytic fields in order to validate the capability of the cDrake objective mapping procedure described in Section 3.4 to produce higher derivatives of the input fields, and to check whether the dimensional error estimates (Section 3.4.5) accurately capture the uncertainty as determined by deviation from the analytic fields. We have chosen the analytic fields to model observations in the LDA, using a streamfunction characterized by near-isotropic highs and lows with wavelength approximately the length of the array, propagating through the array over a period of 30 days in approximately the same direction as the mean flow.

The analytic streamfunction and velocity combine a constant jet with me-
ander or “eddy” variability,

\[
\psi = \frac{a k_e}{k_j} \sin(k_j(y - y_{0j})) + a \cos(k_e(y - y_0)) \cos(k_e(x - x_0(t))),
\]

(3.4)

\[
f_0 u = a k_e \cos(k_j(y - y_{0j})) + a k_e \sin(k_e(y - y_0)) \cos(k_e(x - x_0(t))),
\]

(3.5)

\[
f_0 v = -a k_e \cos(k_e(y - y_0)) \sin(k_e(x - x_0(t))),
\]

(3.6)

with \( a = U_0 f_0 / k_e, U_0 = 0.3 \text{ m s}^{-1}, f_0 = 2 \Omega \sin(57^\circ), \Omega = 7.292 \times 10^{-5} \text{ s}^{-1}, k_e = \frac{2\pi}{200 \times 10^3} \text{ m}^{-1}, k_j = \frac{2\pi}{220 \times 10^3} \text{ m}^{-1}, y_{0j} = 60 \times 10^3 \text{ m}, y_0 = 10 \times 10^3 \text{ m}, x_0(t) = \frac{2\pi t}{Tk_e}, T = 30 \text{ days}. \) The analytic fields were sub-sampled on a 40-km, \(3 \times 7\) grid (Figure 3.4) over \( t = 1, 2, ..., T/2 \) days. We added to each variable \((\psi, u, v)\) noise from a random, zero-mean normal distribution with variance \((n \sigma^2_\psi, n \sigma^2_u, n \sigma^2_v)\), where \( \sigma^2_\psi = a^2 k_e^2 k_j^{-2}/2 + a^2/4, \) and \( \sigma^2_u = a^2 k_e^2/4 \) are the space-variances of the analytic \( \psi \) and \( v \) fields (note that \( \sigma^2_v \) is also the variance of the “eddy” part of the \( u \) field, and that the \( u \) and \( v \) fields were assigned equal noise variance, proportional to this “eddy” variance), and \( n \) values ranged from 0.05 to 0.35. To model noise correlated over \( T_n \) days we linearly interpolated from the random noise fields every \( T_n \) days to the intervening days (if any); we tested \( T_n \) from 1 to 15 days. At each time we objectively mapped the sub-sampled \( \psi \) or \( \psi, u, v \), as described in Section 3.4 and the Appendix, to \( \psi \) and its first, second, and third derivatives on a 10-km grid. The mapping noise-to-signal variance \( E \) was set to \((n, n \sigma^2_u / \sigma^2_\psi, n)\), where \( \sigma^2_u = a^2 k_e^2/4 + a^2 k_j^2/2, \) for \((\psi, u, v)\). The decorrelation length was \( L = 60 \text{ km.} \) A mean over the 21 input sites was subtracted from \( \psi \) before mapping each day, and in the case of mapping from \( \psi, u, v \), means were subtracted from \( u, v \) and the corresponding plane from \( \psi \). We also estimated the dimensional error variance as in Section 3.4.5, using the analytic field variances \( \sigma^2_\theta \) to redimensionalize percent error variance \( e_\theta \).

We compare both day-by-day and time-averaged fields and error fields. Time averages were calculated over \( T/2 \), assuming each day to be independent. To improve statistics of the error comparisons we used multiple ensembles of \( T/2 \) days. We wish to test the accuracy of both the mapped fields themselves (Section 3.5.1) and the estimates of error in the mapped fields deriving from the objective mapping algorithm (Section 3.5.2).
3.5.1 Mapped-analytic comparisons

Plan view plots of 15-day means (Figure 3.4) show that objective mapping from \( \psi, u, v \) is able to reproduce analytic mean velocity, relative vorticity, and even relative vorticity gradients in the center of the array. Mapped and analytic quantities are centered around a 1:1 line, with relatively low scatter (rms differences for each quantity are indicated in Figure 3.4). In the regions shown, mean mapped quantities are larger than their error bars (not shown; see Section 3.5.2) for all but the smallest values. As the input noise:signal variance and/or the noise correlation time is increased (up to \( n = 0.35, T_n = 15 \)), the scatter becomes somewhat larger, but the objectively mapped fields still reproduce the analytic fields in the central area to within error (see below). Mapping from \( \psi \) alone produces similar results, with rms analytic-mapped differences approximately twice as large as those from mapping from \( \psi, u, v \).

Mapped time series of zonal momentum balance terms (Figure 3.5) from points near the center of the array reproduce the signs, slopes, curvatures, and relative sizes of the analytic time series, despite the noise (\( n = 0.20, T_n = 7 \)) in the mapped data. With smoothing by a 4th order Butterworth with a 7-day cutoff period, the correspondence is even clearer. When this smoothing is incorporated, the comparison between mapped and analytic momentum balance terms in the central area is as good from mapping from \( \psi \) (not shown) as it is from mapping from \( \psi, u, v \) (Figure 3.5).

Vorticity balance terms (Figure 3.6) are noisier, but after smoothing with a 7-day Butterworth filter, mapped time series reproduce the signs and relative amplitudes of \( u \zeta_x, v \zeta_y, \) and \( \zeta_t \) with \( n \) up to 0.35 and \( T_n \) up to 15 if mapped from \( \psi, u, v \), or with \( n \) up to 0.10 and \( T_n \) up to 7 if mapped from \( \psi \) only (with higher \( n \) or \( T_n \) the mapped data reproduce the analytic data to within error, but the error is large enough that the signals in various terms are not preserved).

3.5.2 Error estimate comparisons

As described in Section 3.4.5, for each mapped quantity \( \theta \) and mapping estimate \( \hat{\theta} \), the dimensional error variance for the objective maps can be computed
as
\[ \sigma_{\hat{\theta}}^2 = c_{\hat{\theta}}(\bar{x})\sigma_{\theta}^2, \]
(3.7)
where \( \sigma_{\hat{\theta}}^2 \) is the analytic signal variance. We can also estimate the simulated field error variance directly from the mapped \((\hat{\theta})\) versus analytic \((\theta)\) simulation fields:
\[ \sigma_{\hat{\theta}}^2 = \langle (\theta(\bar{x}) - \hat{\theta}(\bar{x}))^2 \rangle = \delta_{\hat{\theta}}^2(\bar{x}), \]
(3.8)
where the angle brackets indicate averaging over realizations. We compared (3.7) and (3.8) for a range of input parameters, averaging over 10\(T\) (or 20 \(T/2\) ensembles), for \(\psi\) and its first and second derivatives as well as \(\zeta, \zeta_x, \zeta_y\). The mean-square values correspond well, with the rms mapping error (rmse, black lines in Figure 3.7) a good approximation to the standard deviation of the scatter of the mapped values around the analytic line (rmsd in Figure 3.7). Grid points with larger mapped-analytic difference tend to have larger mapping error as well (Figure 3.7).

We also compare estimates of error on mean quantities: the standard error of the mean estimated by propagation of mapping error-based variances,
\[ s_m = \frac{\langle \sigma_{\hat{\theta}} \rangle}{\sqrt{N}}, \]
(3.9)
with \(N\) equal to the number of estimates, should be the standard deviation of the distribution of
\[ \Delta = \langle \theta \rangle - \langle \hat{\theta} \rangle. \]
(3.10)
We computed 20 sets of 15-day averages for various input parameters for the fields discussed above. The widths of the best-fit zero-mean Gaussians for \(\Delta\) are a factor of 1 to 2 times the \(rms(s_m)\) for all variables, confirming the mapping error estimates.

### 3.6 Comparison with independent data sets

Having investigated the internal consistency of the cDrake mapping method, we now investigate the validity of the cDrake mapping and GEM-derived fields by comparison with independent observations. LADCP velocity profiles from the five cDrake cruises help us confirm the GEM-derived vertical structure. Comparison
with coincident moored currents validates the time variability of the currents and current shear. We also compare the cDrake GEM-derived surface currents with mean satellite altimeter MADT-derived surface currents over the same time period.

3.6.1 Comparison with LADCP currents

For comparison with the low-vertical-resolution GEM-derived depth profiles we bin-averaged LADCP velocity and shear profiles to 500-m resolution. The number of available LADCP profiles drops off precipitously below 3700 m, so only bins centered at 300 m to 3300 m were considered. We interpolated cDrake maps to LADCP cast positions and times, excluding profiles for which CPIES data were not available on at least three sides of the cast position within half a day before and after the cast, and bin-averaged the 56 qualifying profiles to the same vertical grid. We also compare LADCP bottom currents (from the deepest 20-m bin) with cDrake 4000 dbar mapped currents and cDrake near-bottom site currents.

LADCP and cDrake velocities are correlated with $r^2 \geq 0.61$. The rms velocity difference over all profiles and depths is 15 cm s$^{-1}$; the amplitude of the mean difference is 4 cm s$^{-1}$. Differences decrease with depth, paralleled by the standard deviations. LADCP and cDrake velocity standard deviations follow each other closely and are approximately 1.5 times the rms differences. The rms differences between LADCP bottom velocity and cDrake 4000 dbar or site near-bottom velocity are 12 cm s$^{-1}$ and 13 cm s$^{-1}$, respectively, indicating that most of the difference is in the bottom current and derives from the comparison between LADCP velocities averaged over minutes to hours and cDrake 3-day low-pass filtered data.

The LADCP shear standard deviation is 1.5 to $2.5 \times 10^{-4}$ s$^{-1}$, whereas cDrake shear standard deviation is 0.4 to $1.6 \times 10^{-4}$ s$^{-1}$; both increase from the first to the second bin, then decrease below that, with the LADCP shear standard deviation jumping back almost to its surface value in the deepest bin. The difference in shear variances is consistent with large-vertical-scale internal waves or other ageostrophic energy. LADCP shear angle distributions do not vary significantly from 300 to 3300 m, implying that the low-mode baroclinic currents are indeed equivalent-barotropic.
3.6.2 Comparison with moored currents and temperature

We compared daily mean currents and temperatures from cDrake mapped data and the GEM with daily mean moored currents and temperatures from DRAKE moorings M2 and M4 (Figure 3.1). To compare with uncorrected mooring velocities, we linearly interpolated the cDrake objectively mapped velocity profiles from the 10-km horizontal, 200-dbar vertical grid to the nominal location of each mooring and the moored pressure time series $p(t)$ from each instrument. To compare with pressure-corrected mooring data at pressures $p_c$, we computed $\Phi(x, y, p_c, t)$ and mapped to $(u, v)(x, y, p_c, t)$, then linearly interpolated to the mooring location. (In the case of M4, $p_c$ also varies with time, from the first deployment to the second deployment; we will use the $p_c$ from the second deployment to refer to the pressure levels of the combined M4 time series, although the cDrake fields were interpolated to the appropriate $p$ at each time.) To compare with temperature we interpolated the $T$ GEM to $p(t)$ or $p_c$ and $\tau(t)$ interpolated from cDrake maps to the mooring site. For comparisons involving time means, we estimated the standard error of the mean for moored quantity $\theta$ as $\sqrt{\frac{\sum(\theta-<\theta>)^2}{N-1}}/\sqrt{N_{dof}}$, and for cDrake quantity $\theta$ as $\sqrt{\sum \sigma^2_\theta} / \sqrt{N_{dof}}$, where $N_{dof}$, the number of degrees of freedom, in both cases is the number of days in the time series divided by 11 days (the decorrelation time scale for cDrake mapped velocity), and $\sigma_\theta$ for cDrake is the error on the quantity $\theta$ derived from the dimensional mapping error field(s).

At M2, moored and cDrake temperatures (Figure 3.8) are in very good agreement. Mean differences in temperature are small. Standard deviations of both temperature time series range from 0.12°C (at 2000 dbar) to 0.71°C (at 420 dbar); moored and cDrake temperature standard deviations are essentially the same (Figure 3.8). The non-pressure-corrected rms differences range from 0.05°C (at 2000 dbar) to 0.37°C (at 420 dbar), and correlations from $r^2 = 0.83$ to $r^2 = 0.87$ (Figure 3.8); the pressure-corrected rms differences are slightly higher (up to 0.46°C) and correlations slightly lower ($r^2 = 0.80$ to 0.85, not shown).

Since M2 is located outside the cDrake LDA, cDrake only gives information on the component of shear normal to the local CPIES line. We used the three closest CPIES, C02, C03, and C04, to determine the orientation of the line, and
rotated moored and cDrake mapped currents into line-normal and along-line coordinates in order to compare the normal component of current. The time-varying normal currents (Figure 3.8) are moderately well-correlated \((r^2 \geq 0.68)\) at 420 dbar and 930 dbar, and moderately correlated \((r^2 = 0.48)\) at 2000 dbar. The rms differences are approximately the same size as the standard deviations of either source. The moored current is systematically larger than the cDrake current, by approximately a factor of two. The mean normal currents differ by 13 cm s\(^{-1}\) ± 1 cm s\(^{-1}\), 9 cm s\(^{-1}\) ± 1 cm s\(^{-1}\), and 5 cm s\(^{-1}\) ± 1 cm s\(^{-1}\) at 420 dbar, 930 dbar, and 2000 dbar, respectively. Examination of nearby mean currents (Figure 3.9) shows that while the cDrake mean speeds to the northeast of M2 are closer to the mean moored speeds, a movement of more than 20 km would be required for the cDrake and moored mean speeds to agree to within error. We note that cDrake surface speeds are also smaller than altimeter speeds around M2 (Figure 3.9).

The M4 moored and cDrake currents and temperatures at time-varying pressures (Figure 3.10) are highly correlated \((r^2 \geq 0.67)\) for currents, \(r^2 \geq 0.81\) for temperature). The rms difference is 8 cm s\(^{-1}\) at 2540 dbar, increasing to 15 cm s\(^{-1}\) at 520 dbar. Both sources have standard deviations of 13 to 29 cm s\(^{-1}\) and near-isotropic variance ellipses. The cDrake and moored time-mean currents (Figure 3.11) are the same size to within error, but show a consistent angle bias, with cDrake mean currents directed 16° ± 16°, 14° ± 10°, and 18° ± 6° clockwise of M4 currents at 520 dbar, 1020 dbar, and 2540 dbar, respectively. The 3-day low-pass filtered moored-cDrake angle difference at 2540 dbar has a standard deviation of \(\sim 21°\), but the standard deviation decreases to 10° and the sign becomes persistent at timescales of 30 days and longer. The angle bias is almost entirely due to the barotropic component (Figure 3.11). The mean measured cDrake near-bottom current at the closest site, E02, is directed 9° clockwise of the mean moored 2500 m current.

Shear between current meters (or between cDrake velocity series at current meter \(p(t)\)) is slightly (not significantly) smaller on average from cDrake than from the mooring (Figure 3.11), and the mean shear angles in the lower bin are the same. In the upper bin the moored mean shear is 42° ± 46° clockwise of the cDrake mean
shear; the mean difference is small compared to the variability. Shear component time series are correlated with \( r^2 > 0.40 \) and shear angles with \( r^2 > 0.62 \) in both depth ranges.

The pressure-corrected M4 mooring data, and cDrake velocities from maps at corrected pressure levels, show qualitatively similar mean flow and shear. The comparisons between pressure-corrected moored and cDrake temperature time series are not quite as good as those between pressure-varying time series, with \( r^2 \geq 0.66 \). The rms differences are also slightly higher for both currents and temperature. The angle bias (which derives principally from the barotropic component) remains nearly the same, approximately 17°-19°.

Both the cDrake and mooring estimates show that the flow at M4 has a large westward component and that velocity increases with depth from 500 dbar to 2500 dbar (and to 4000 dbar for cDrake). This site is located in a cyclonic circulation in the Yaghan Basin [Figure 3.9, Chereskin et al., 2009]. Area maps of mean velocities from cDrake and MADT at the surface and cDrake at the bottom (Figure 3.9) show that the \( \sim 15° \) angle offset between moored and cDrake currents is not due to a small (several-km) error in the mooring location; nor can it be clearly attributed to skewing of the cDrake objective map by one site. The mapped altimeter surface current orientation around M4 generally agrees with or is clockwise of the cDrake mapped surface orientation (that is, even farther from the moored velocity direction at 500 m).

We investigated some other possible causes of the differences between cDrake and moored currents at M2 and M4. The cDrake velocities were not significantly changed by mapping directly to the mooring site, nor by mapping at higher vertical resolution before interpolating to the mooring pressures. Mooring blowover can lead to large instrument angle and inaccurate determination of horizontal currents; however, neither excluding days with \( dp = p(t) - p_{\text{median}} > 200 \) dbar from the average, nor using the pressure-corrected series improves the correlations or mean size or angle differences. Nor does excluding small-magnitude moored currents (for which angle may be poorly determined). The M4 moored amplitude deficit derives almost entirely from the second half of the second deployment; at
M2 the moored amplitude excess is stronger in the second half of the record.

The comparisons between moored and cDrake currents and temperatures at sites M2 and M4 confirm that the cDrake maps reproduce the time variability and shear structure of currents within the array very well. The small mean angle offset at M4 may be due to localized current steering by bathymetry and/or to (possibly ageostrophic) shear between 4000 dbar and 2500 dbar, not accurately represented by the GEM.

### 3.6.3 Comparison of cDrake and MADT

The cDrake mean surface currents compare well with satellite-derived MADT surface currents except in the south-central/southwestern part of the LDA and stations just south (Figure 3.9). In that area the difference in direction is greater than 90°, while currents over most of the area are 5 to 20 cm s\(^{-1}\). The angle difference derives from the barotropic component. As it is supported by multiple current meters (see green arrows in Figure 3.9) and does not appear to be an artifact of the mapping, it may reflect a real feature with a scale too small to be resolved by the MADT. Outside of this area, surface current vectors agree in size and direction; both cDrake and MADT capture the SAF jet and the cyclonic meander in the northeastern part of the array, in the same places and with approximately the same magnitudes.

### 3.7 Summary

We have described a procedure for estimating barotropic and baroclinic geostrophic streamfunction, and its derivatives up to relative vorticity gradients, by objective mapping of time series from an array of CPIES and a GEM. We extend previous work [Donohue et al., 2010; Meinen et al., 2003; Tracey et al., 1997, 2006; Watts et al., 2001a, and others, see Section 1] by mapping directly from bottom \( p, u, v \) or from GEM-derived \( \Phi \) time series to second and third derivatives of streamfunction, thus obtaining improved error estimates relative to methods involving finite differencing, as confirmed by both simulations and comparisons.
with independent data sets. We have discussed the construction of the GEM, including at deeper levels where data are sparse; selection of objective mapping parameters based on the CPIES time series and sensitivity to those parameters; and the dimensionalization of mapping errors using spatial variances. We have also confirmed the assumption of negligible deep shear, both internally using mapped \( \Phi \) profiles from the GEM and observed \( \tau \), and by comparison with shear from a current meter mooring.

Objective mapping of a simulated field (Section 3.5) shows that 1) in the center of the array, with noise:signal ratio \( n \) comparable to those calculated for the actual cDrake \( \psi, u, v \) or \( \psi \), we can reproduce daily values of variables up to second derivatives of \( \psi \) to within error, and 2) the mapping error estimates accurately reflect the deviations of the mapped fields from the analytic values. Time series smoothed with a 7-day Butterworth filter capture both the signs and relative magnitudes of the analytic terms of both the zonal momentum balance and the vorticity balance (including terms up to third derivatives of \( \psi \)). The simulations also show that our mapping error estimates are accurate, giving us confidence that with appropriate smoothing or averaging we can calculate momentum and vorticity balance quantities and their uncertainties from real data. Unlike the simulated data, the real cDrake time series do not have a clear dominant period; the degree of smoothing for a given calculation should be chosen to balance the competing desires for time resolution and reduced noise in higher order terms.

The comparison of cDrake mapped velocity and LADCP velocity profiles collected on the five cDrake cruises confirms that the cDrake mapping and GEM method accurately reproduces baroclinic as well as bottom currents. Comparison of cDrake mapped surface currents with satellite altimeter-derived surface currents also supports the cDrake baroclinic currents. Mapped cDrake temperatures, currents, and current shear compare very well with temperatures, currents, and current shear measured at moorings M4 (in the LDA) and M2 (just north of the LDA). The high correlations and small offsets confirm the cDrake method described here for computing velocities. Both baroclinic and barotropic variability are reproduced well (total current \( r^2 \geq 0.56 \), shear \( r^2 \geq 0.36 \)), although the mean
barotropic current at this site exhibits a $15^\circ \pm 6^\circ$ angle offset. In combination with the simulations, these comparisons give us confidence that we can usefully compute higher order derivatives of geostrophic streamfunction in the interior of the LDA by the method described here.

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3.A Objective Mapping

Following Qian and Watts [1992] and Gille [2003a], we derive the relationships necessary to objectively map to second and higher derivatives of the streamfunction $\psi = p/\rho_0$ or $\psi = \Phi$. We will map to

\begin{align*}
    u_{map} &= -\frac{\partial \psi}{\partial y}, \\
    v_{map} &= \frac{\partial \psi}{\partial x}, \\
    v_{mapx} &= \psi_{xx}, \\
    v_{mapy} &= -u_{mapx} = \psi_{xy}, \\
    u_{mapy} &= -\psi_{yy}, \\
    \zeta_{map} &= v_{mapx} - u_{mapy} = \psi_{xx} + \psi_{yy}, \\
    \zeta_{mapx} &= \nabla^2 v = \psi_{xxx} + \psi_{xyy}, \\
    \zeta_{mapy} &= -\nabla^2 u = \psi_{xxy} + \psi_{yyy}.
\end{align*}
where \((u_{map}, v_{map}) = (fu, fv)\). We can then convert from mapping variables to
the real geostrophic velocity and other variables by

\[
\begin{align*}
    u &= u_{map}/f, \\
    v &= v_{map}/f, \\
    u_x &= u_{mapx}/f, \\
    u_y &= (u_{mapy} - \beta u)/f, \\
    v_x &= v_{mapx}/f, \\
    v_y &= (v_{mapy} - \beta v)/f, \\
    \zeta &= (\zeta_{map} + \beta u)/f, \\
    \zeta_x &= (\zeta_{mapx} + \beta u_x)/f, \\
    \zeta_y &= (\zeta_{mapy} - \beta \zeta + \beta u_y + \beta y u)/f.
\end{align*}
\]

In the rest of this section the subscript \(map\) will be dropped but all variables will
be mapping variables. Some of the relations in URI Tech Report Appendix B are
repeated below for reference.

### 3.A.1 Covariance of \(\psi\)

We assume, after verifying for the LDA variability that observed meridional,
zonal, and diagonal correlations are not statistically different, an isotropic Gaussian
covariance for \(\psi\),

\[
\begin{align*}
    F_{\psi \psi} = \langle \psi(x_1, y_1) \psi(x_2, y_2) \rangle &= F = V_\psi e^{-\lambda \gamma^2}, \\
    F' = \frac{dF}{d\gamma} &= -2\lambda \gamma F,
\end{align*}
\]

where \(F = F(\gamma)\) is a function of the separation \(\gamma\), \(\lambda = \frac{1}{L^2}\) with \(L\) the correlation
length scale, and \(V_\psi\) the amplitude of \(F_{\psi \psi}\) at zero separation. In the objective
mapping implementation \(V_\psi = 1 - E \simeq \frac{1}{1+\gamma E}\), where \(E\) is the noise:signal variance ratio.
3. A. 2 Separations

To derive the covariances between other variables given above we will need to take derivatives with respect to the \( x \) and \( y \) positions, for which the following will be useful. (In the following the \( \cos \alpha_x, \cos \alpha_y \) of URI Tech Report Appendix B are replaced by \( r, s \).)

\[
\gamma = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2},
\]

\[
x_2 - x_1 = \gamma r, \quad (3.31)
\]

\[
y_2 - y_1 = \gamma s, \quad (3.32)
\]

\[
r^2 + s^2 = 1. \quad (3.33)
\]

\[
-\frac{\partial \gamma}{\partial x_1} = \frac{\partial \gamma}{\partial x_2} = r, \quad (3.34)
\]

\[
-\frac{\partial \gamma}{\partial y_1} = \frac{\partial \gamma}{\partial y_2} = s. \quad (3.35)
\]

\[
-\frac{\partial r}{\partial x_1} = \frac{\partial r}{\partial x_2} = \frac{1}{\gamma}(1 - r^2) = \frac{1}{\gamma} s^2, \quad (3.36)
\]

\[
-\frac{\partial s}{\partial y_1} = \frac{\partial s}{\partial y_2} = \frac{1}{\gamma}(1 - s^2) = \frac{1}{\gamma} r^2, \quad (3.37)
\]

\[
\frac{\partial r}{\partial y_1} = \frac{\partial r}{\partial y_2} = \frac{1}{\gamma} rs, \quad (3.38)
\]

\[
\frac{\partial s}{\partial x_1} = -\frac{\partial s}{\partial x_2} = \frac{1}{\gamma} rs. \quad (3.39)
\]
3.3.3 Cross-covariances $F_{\psi \psi}$

We calculate the cross-covariances following Gille [2003a]:

\[
F_{\psi u} = -F_{\psi \psi y} = -\frac{\partial}{\partial y_2} F = 2\lambda \gamma s F, \quad (3.40)
\]

\[
F_{\psi v} = F_{\psi \psi x} = \frac{\partial}{\partial x_2} F = -2\lambda \gamma r F, \quad (3.41)
\]

\[
F_{\psi v u} = F_{\psi \psi x x} = \frac{\partial}{\partial x_2} F_{\psi \psi x} = -2\lambda (1 - 2\lambda \gamma^2 r^2) F, \quad (3.42)
\]

\[
F_{\psi v y} = -F_{\psi u x} = F_{\psi \psi y x} = \frac{\partial}{\partial y_2} F_{\psi \psi x} = 4\lambda^2 \gamma r s F, \quad (3.43)
\]

\[
F_{\psi \zeta} = F_{\psi \psi x x} + F_{\psi \psi y y} = 4\lambda (\lambda \gamma^2 - 1) F, \quad (3.45)
\]

\[
F_{\psi \zeta x} = F_{\psi \psi x x} = \frac{\partial}{\partial x_2} F_{\psi \psi x} = 8\lambda^2 \gamma r (2 - \lambda \gamma^2) F, \quad (3.46)
\]

\[
F_{\psi \zeta y} = -F_{\psi \psi y y} = \frac{\partial}{\partial y_2} F_{\psi \psi x} = 8\lambda^2 \gamma s (2 - \lambda \gamma^2) F. \quad (3.47)
\]

As $\frac{\partial}{\partial x_1} = -\frac{\partial}{\partial x_2}$, and similarly for $y_1$, $y_2$, $F_{\psi \psi x} = -F_{\psi x \psi}$, $F_{\psi \psi x x} = F_{\psi x x \psi}$, $F_{\psi \psi x x x} = -F_{\psi x x x x \psi}$, etc.
3.A.4 Cross-covariances $F_{u*}, F_{v*}$

At the sea floor, we measure $u, v$ as well as the streamfunction and can map from the three variables $\psi, u, v$ to each desired variable.

$$F_{u\psi} = -F_{\psi u} = -2\lambda \gamma s F,$$ (3.48)

$$F_{v\psi} = -F_{\psi v} = 2\lambda \gamma r F,$$ (3.49)

$$F_{uu} = -\frac{\partial}{\partial y_1} F_{\psi u} = 2\lambda(1 - 2\lambda \gamma^2 s^2) F,$$ (3.50)

$$F_{uv} = F_{vu} = \frac{\partial}{\partial x_1} F_{\psi u} = 4\lambda^2 \gamma^2 r s F,$$ (3.51)

$$F_{vv} = \frac{\partial}{\partial x_1} F_{\psi v} = 2\lambda(1 - 2\lambda \gamma^2 r^2) F,$$ (3.52)

$$F_{uv_x} = -\frac{\partial}{\partial y_1} F_{\psi v_x} = 4\lambda^2 \gamma s(1 - 2\lambda \gamma^2 r^2) F,$$ (3.53)

$$F_{uv_y} = -F_{u uv} = -\frac{\partial}{\partial y_1} F_{\psi v_y} = 4\lambda^2 \gamma r(1 - 2\lambda \gamma^2 s^2) F,$$ (3.54)

$$F_{uu_y} = -\frac{\partial}{\partial y_1} F_{\psi u_y} = -4\lambda^2 \gamma s(3 - 2\lambda \gamma^2 s^2) F,$$ (3.55)

$$F_{u\zeta} = -\frac{\partial}{\partial y_1} F_{\psi \zeta} = 8\lambda^2 \gamma s(2 - \lambda \gamma^2) F,$$ (3.56)

$$F_{v\zeta} = \frac{\partial}{\partial x_1} F_{\psi \zeta} = -8\lambda^2 \gamma r(2 - \lambda \gamma^2) F,$$ (3.57)

$$F_{uv_x} = -F_{uv} = \frac{\partial}{\partial x_1} F_{\psi u_x} = -4\lambda^2 \gamma r(1 - 2\lambda \gamma^2 r^2) F,$$ (3.58)

$$F_{uv_y} = F_{vu} = \frac{\partial}{\partial x_1} F_{\psi u_y} = 4\lambda^2 \gamma r(1 - 2\lambda \gamma^2 s^2) F,$$ (3.59)

$$F_{v\zeta} = \frac{\partial}{\partial x_1} F_{\psi \zeta} = -8\lambda^2 \gamma r(2 - \lambda \gamma^2) F,$$ (3.60)

$$F_{u\zeta_x} = F_{uuv} = -\frac{\partial}{\partial y_1} F_{\psi \zeta_x} = -16\lambda^3 \gamma^2 r s(3 - \lambda \gamma^2) F,$$ (3.61)

$$F_{u\zeta_y} = -F_{u u\zeta} = -\frac{\partial}{\partial y_1} F_{\psi \zeta_y} = 8\lambda^2(2 - 6\lambda \gamma^2 s^2 - \lambda \gamma^2 + 2\lambda^2 \gamma^4 s^2) F,$$ (3.62)

$$F_{v\zeta_x} = F_{vv\zeta} = \frac{\partial}{\partial x_1} F_{\psi \zeta_x} = -8\lambda^2(2 - 6\lambda \gamma^2 r^2 - \lambda \gamma^2 + 2\lambda^2 \gamma^4 r^2) F,$$ (3.63)

$$F_{v\zeta_y} = -F_{v v\zeta} = \frac{\partial}{\partial x_1} F_{\psi \zeta_y} = 16\lambda^3 \gamma^2 r s(3 - \lambda \gamma^2) F.$$ (3.64)

As explained above, $F_{uu} = -F_{u u}, F_{uu xx} = F_{u u x x},$ etc.
3.A.5 Normalization factors: $F_{**}(\gamma = 0)$

To compute the mapping skill we require the amplitudes of the autocovariances at zero separation. We already have $F_{uu}$ and $F_{vv}$. With $\gamma = 0$ we have

\[
\begin{align*}
F_{uxu} &= \frac{\partial}{\partial x_1} F_{uu} = 4\lambda^2 F(0), \quad (3.65) \\
F_{uyu} &= \frac{\partial}{\partial y_1} F_{uu} = 12\lambda^2 F(0), \quad (3.66) \\
F_{vxv} &= \frac{\partial}{\partial x_1} F_{vv} = 12\lambda^2 F(0), \quad (3.67) \\
F_{vyv} &= \frac{\partial}{\partial y_1} F_{vv} = 4\lambda^2 F(0), \quad (3.68) \\
F_{\zeta\zeta} &= \frac{\partial}{\partial x_1} F_{u\zeta} - \frac{\partial}{\partial y_1} F_{u\zeta} = 32\lambda^2 F(0). \quad (3.69) \\
F_{\zeta\zeta\zeta} &= \frac{\partial^2}{\partial x_1^2} F_{\zeta\zeta} = 192\lambda^3 F(0), \quad (3.70) \\
F_{\zeta\zeta\zeta} &= \frac{\partial^2}{\partial y_1^2} F_{\zeta\zeta} = 192\lambda^3 F(0). \quad (3.71)
\end{align*}
\]
Table 3.1: Mapping parameters for the indicated quantities and time series. For $\Phi$ we use $L_\tau$ and $E_\Phi = E_\tau + E_{GEM}(p)$ (see Section 3.4.3).

<table>
<thead>
<tr>
<th></th>
<th>L (km)</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>bim</td>
<td>200</td>
<td>0.05</td>
</tr>
<tr>
<td>$p, u, v$ lp</td>
<td>60</td>
<td>0.25</td>
</tr>
<tr>
<td>$p, u, v$ hp</td>
<td>50</td>
<td>0.35, 0.46 (p tip-step)</td>
</tr>
<tr>
<td>$\tau$ lp</td>
<td>60</td>
<td>0.04</td>
</tr>
<tr>
<td>$\tau$ hp</td>
<td>50</td>
<td>0.09</td>
</tr>
</tbody>
</table>
Table 3.2: Comparisons of derivative terms obtained by objective mapping from indicated quantities using parameters in Table 1 with fields obtained by mapping with indicated parameters increased or decreased by 20%: [maximum rms difference normalized by standard deviation of original map, minimum correlation]. Φ(0) indicates Φ at the surface; surface $E_\phi \simeq E_\tau + 0.02$. For 1st derivatives $u$ and $v$ were compared, for 2nd derivatives $\zeta$, and for 3rd derivatives $\zeta_x$ and $\zeta_y$. Only mapping grid points at which the percent variance error of bottom $\psi$ mapped from $p, u, v$ was less than 0.20 are included.

<table>
<thead>
<tr>
<th></th>
<th>$L_{lp}$</th>
<th>$L_{hp}$</th>
<th>$E_{lp}$</th>
<th>$E_{hp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st from $p, u, v$</td>
<td>0.10, 1.00</td>
<td>0.19, 0.98</td>
<td>0.01, 0.99</td>
<td>0.03, 0.99</td>
</tr>
<tr>
<td>1st from Φ(0)</td>
<td>0.24, 0.97</td>
<td>0.29, 0.96</td>
<td>0.18, 0.99</td>
<td>0.19, 0.99</td>
</tr>
<tr>
<td>2nd from $p, u, v$</td>
<td>0.16, 0.99</td>
<td>0.36, 0.95</td>
<td>0.01, 1.00</td>
<td>0.05, 1.00</td>
</tr>
<tr>
<td>2nd from Φ(0)</td>
<td>0.36, 0.96</td>
<td>0.50, 0.91</td>
<td>0.28, 0.97</td>
<td>0.31, 0.97</td>
</tr>
<tr>
<td>3rd from $p, u, v$</td>
<td>0.26, 0.99</td>
<td>0.63, 0.91</td>
<td>0.02, 1.00</td>
<td>0.07, 1.00</td>
</tr>
<tr>
<td>3rd from Φ(0)</td>
<td>0.58, 0.93</td>
<td>0.97, 0.81</td>
<td>0.47, 0.94</td>
<td>0.52, 0.94</td>
</tr>
</tbody>
</table>
Figure 3.1: Top left: cDrake area (box) over Orsi et al. [1995] fronts (gray lines). Top right: enlargement of the red boxed region showing cDrake CPIES sites over Orsi et al. [1995] fronts (gray lines) and bathymetry (described in text), with box outlining the LDA area, shown below. The gold rectangle shows the LMG most common track. Bottom: CPIES sites (black dots) and bathymetry (filled contours) in the LDA, with M2 and M4 mooring sites (red circles). Site B02 is indicated by a diamond and the 6 LDA tip-step sites (A02, C03, C05, C19, C07, G03) by squares.
Figure 3.2: GEMs for (top to bottom) temperature ($T$), salinity ($S$), specific volume anomaly $\delta$, and geopotential height anomaly $\Phi$ relative to 4000 dbar, as functions of pressure and $\tau_{2000}$, the round-trip sound travel time from 0 to 2000 dbar.
Figure 3.3: Pressure-pressure (left) and $\tau_{2000}$-$\tau_{2000}$ (right) correlations (gray dots) between pairs of sites, plotted as a function of site separation, for lp (top) and hp (bottom) series. Bin-averaged correlations $c_{bin}$ are plotted as black circles, and the best-fit Gaussian, $c_g$, as a black curve. $\tau_{2000}$ is the round-trip sound travel time between 0 and 2000 dbar.
Figure 3.4 (next page): 15-day mean velocity components, relative vorticity, and relative vorticity gradient components mapped from $\psi, u, v$ with $n = 0.20, T_n = 7$. Left panels: Analytic (contours) and mapped (dots). Right panels: analytic vs. mapped values for the same quantities (gray dots), with 1:1 line shown in green. Colorbars and axes have been normalized to $[-1, 1]$; scale factors and units are given above the colorbars and on the bottom right of each right hand plot. The (normalized) rms differences between mapped and analytic values (rmsd), with the same scale as other plot quantities, are also given. The percent error variance criterion for points plotted in both left and right hand panels is shown in the upper left corner of each right-hand plot.
Figure 3.5 (next page): Analytic (left) and mapped (right) time series of selected zonal momentum balance terms (m s$^{-2}$) at three points in the center of the array ($x = 120, 150, 180, y = 30$, note the central pluses on Figure 3.4 are at $y = 40$). Daily data are shown by dots and data smoothed with a 4th order Butterworth filter with a 7-day cutoff by solid lines. Vertical lines on the right panels indicate standard deviation error bars on the smoothed data. Mapping was from $\psi, u, v$ with $n = 0.20, T_n = 7$ (see text). The mapped tendency term, $u_t$, was estimated by center differencing. Note that the first order balance is between the Coriolis term $-fv$ and the pressure gradient term (identical by construction).
Figure 3.6: As in Figure 3.5, for selected vorticity balance terms (s$^{-2}$).
Figure 3.7 (next page): Scatter plots of 15 days of daily analytic vs. mapped values for (top to bottom) $u, v, \zeta, \zeta_x, \zeta_y$. Only points with daily percent variance mapping error $e_\theta$ meeting the cutoffs given in Figure 3.4 are shown. The shade of each point corresponds to the mapping error $\sigma_\theta$ at that point, as indicated by the colorbar. The black lines are the 1:1 line and 1:1 $\pm$ the rms of mapping error (rmse); the rms of the mapped-analytic difference $\delta$ (rmsd) is also given. Scales in the bottom right corner of each panel apply to the axes and colorbar labels as well as rmse and rmsd.
\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure}
\caption{Comparison of predicted and observed variables.
\textbf{Top panel:} Predicted $u$-velocity (1 m s$^{-1}$) with rmse 0.08 and rmsd 0.05.
\textbf{Middle panel:} Predicted $v$-velocity (1 m s$^{-1}$) with rmse 0.05 and rmsd 0.05.
\textbf{Bottom panel:} Predicted $\zeta$-velocities (1e$^{-05}$ m s$^{-1}$) with rmse 0.50 and rmsd 0.56 for $\zeta_x$, rmse 0.15 and rmsd 0.14 for $\zeta_y$, and rmse 0.17 and rmsd 0.25 for $\zeta_y$.

\textit{Note:} Map error (map err) values are also shown for each variable.}
\end{figure}
Figure 3.8: Daily line-normal (see text) currents and temperature from M2 mooring (red) and cDrake interpolated to mooring $p(t)$ (blue), nominally at (top to bottom) 420 dbar, 930 dbar, and 2000 dbar. Squared correlation coefficients ($r^2$), rms differences (rmsd) and standard deviations of each record (indicated by color) are given on each panel.
Figure 3.9 (next page): Time means of cDrake, MADT, and moored currents in the LDA area. Top: altimeter MADT-derived (gold) and cDrake mapped (blue) surface currents, with top-mooring-level moored (thick red) and cDrake (thick blue) currents at M2 (420 dbar) and M4 (520 dbar). Bottom: cDrake mapped (blue) and site (dark green) bottom currents, with bottom-mooring-level moored (thick red) and cDrake (thick blue) currents at M2 (2000 dbar) and M4 (2540 dbar). The time averages are over the M2 and second M4 deployment period (10 April 2008 through 26 March 2009). At the mooring sites the uncorrected mooring currents and cDrake currents interpolated to $p(t)$ were used. The cDrake mapped currents are sub-sampled from a 10-km grid to a 20-km grid for this plot.
Figure 3.10: Daily zonal and meridional currents and temperature from M4 mooring (red) and cDrake interpolated to mooring $p(t)$ (blue), nominally at (top to bottom) 520 dbar, 930 dbar, and 2540 dbar. Squared correlation coefficients ($r^2$), rms differences (rmsd) and standard deviations of each record (indicated by color) are given on each panel.
Figure 3.11: Mean current vectors from cDrake mapping (solid) and M4 mooring (dashed), and mean shear between instrument levels from cDrake mapping (filled circles) and M4 mooring (open circles; open square is mean shear between M4 2500 m instrument and cDrake bottom map).
Chapter 4

Vorticity balance of the Antarctic Circumpolar Current in Drake Passage from observations and the Southern Ocean State Estimate

Abstract. The vorticity balance of the Antarctic Circumpolar Current (ACC) is examined using observations from Drake Passage (cDrake) and the Southern Ocean State Estimate (SOSE). The streamfunction, velocity, vorticity, and vorticity gradient fields from a two-dimensional array (cDrake LDA) of current meter-pressure recording-inverted echo sounders in the Polar Frontal Zone (PFZ) are dominated by eddies. Although the bathymetry under the cDrake LDA is not shallow, it lies downstream of the Shackleton Fracture Zone (SFZ). Mean streamlines are not parallel with depth, indicating possible topographic influence from upstream. Bottom pressure torque across the passage, computed based on observations from near-bottom current meters, implies sufficient bottom pressure torque forcing in the Drake Passage region to balance the majority of the wind stress curl input over the ACC. Locally in the PFZ the depth-integrated vorticity bal-
ance is dominated by the bottom pressure torque and the mean and eddy relative vorticity advection, which are of the same order. Estimates of ageostrophic and unresolved contributions to the vorticity balance are also of first order, but have very large uncertainties; the cDrake LDA observations are insufficient to close the local vorticity balance. The cDrake vorticity balance in the PFZ is reproduced in SOSE, where the residual of the nonlinear advection and bottom pressure torque terms is balanced primarily by sub-grid-scale dissipation. In SOSE, the eddy relative vorticity advection term becomes second order upon smoothing over scales greater than 100 km, but in Drake Passage, nonlinear advection, bottom pressure torque, and sub-grid dissipation still dominate over planetary vorticity advection. The dominant balance at scales above 100 km in the relatively flat southeastern Pacific, in contrast, is between planetary and mean relative vorticity advection.

4.1 Introduction

The broad outlines of the momentum and vorticity balances in the Antarctic Circumpolar Current (ACC) have long been known: momentum input fairly uniformly over the course of the ACC by the wind stress is ultimately balanced principally by bottom form stress [Munk and Palmén, 1951]; vorticity input by the wind stress curl is correspondingly balanced by bottom pressure torque. These balancing forces are likely to be elevated in regions of enhanced bathymetry such as Drake Passage and the Scotia Sea, Kerguelen Plateau, and Campbell Plateau. However, the details of the contributions of particular regions to the mean momentum and vorticity balances have not previously been well observed; nor have the details of the transfer of momentum and vorticity by linear and nonlinear advection and interfacial form stress from the wind forcing to the bottom where the topographic forcing is applied.

Wells and deCuevas [1995] used the Fine Resolution Antarctic Model (FRAM) to calculate the depth-integrated vorticity budget in the ACC. They found a first-order balance between wind stress curl and bottom pressure torque, with bottom pressure torque primarily originating from the Drake Passage/Scotia
Sea/Argentine Basin region and from south of Tasmania and New Zealand. They also found that in the northern part of the ACC both advection and lateral friction added to wind stress curl, while in the south these two terms tended to compensate for each other. Hughes and de Cuevas [2001] investigated the vorticity balance in the OCCAM model ACC, confirming that the zonally-integrated barotropic vorticity balance is dominated by wind stress curl and bottom pressure torque, with locally large nonlinear terms. Grezio et al. [2005] also found the same dominant balance, with an enhancement of both nonlinear advection and bottom pressure torque near topography in the FRAM, OCCAM, and POP models.

Models of the Southern Ocean including FRAM [Killworth, 1992; Hughes and Killworth, 1995] and OCCAM [Killworth and Hughes, 2002] exhibit a separable equivalent-barotropic vertical structure in the ACC: that is, the streamlines are self-similar and the current does not turn significantly with depth; and (for separability) the vertical structure does not depend on horizontal coordinates. The equivalent-barotropic structure arises because the variation of the Coriolis parameter with latitude and the vertical gradient of vertical velocity are both second order [Killworth, 1992; Killworth and Hughes, 2002]; in the models this condition is violated only in limited locations near topography [Hughes and Killworth, 1995; Killworth and Hughes, 2002]. Climatological buoyancy profiles are consistent with separable equivalent-barotropic structure in the ACC as a whole [Karsten and Marshall, 2002], while velocity observations in the upper 1000 m in Drake Passage suggest an equivalent-barotropic (but non-separable) vertical structure [Firing et al., 2011, see Chapter 2].

Hughes [2005] used the mean surface dynamic topography of Niiler et al. [2003] and an assumption of equivalent barotropic vertical structure to examine the mean nonlinear vorticity balance of the ACC. He found two regimes: a wave-like regime in regions of smooth bathymetry in which the surface planetary and mean relative vorticity advection balanced (with a wavelength consistent with a stationary Rossby-wave-like regime with small net depth-integrated divergence); and in regions of enhanced bathymetry (including Drake Passage) a regime in which planetary vorticity advection dominated over relative vorticity advection.
Chereskin et al. [2010] confirmed the first type of balance, between planetary and relative vorticity advection, from direct velocity observations in the Subantarctic Front in the southeast Pacific. Hughes [2005] suggested that the planetary vorticity advection in the second regime would be balanced by bottom pressure torque, which would likely be elevated in those regions; however, without observations of bottom currents this balance could not be directly confirmed. Hughes [2005] also noted the potential importance of the neglected eddy component of vorticity advection.

Eddy kinetic energy is high in the ACC, and due to the lack of full-depth meridional boundaries, eddy fluxes must provide a large fraction of the meridional transfer of momentum, vorticity, and properties. Hughes and Ash [2001] and Williams et al. [2007] examined eddy forcing of the mean ACC using satellite altimeter sea level anomaly data, and found significant surface eddy vorticity fluxes in areas where the current appears to be topographically steered, including Drake Passage. Lenn et al. [2011] observed significant eddy momentum flux between the ACC jets in a time series of direct current measurements in the upper ocean in Drake Passage; Phillips and Rintoul [2000], in contrast, found only small eddy momentum fluxes using an array of current meter moorings south of Australia.

Work by various authors using satellite sea surface height, hydrographic data, and numerical models has suggested that the ACC jets might suppress cross-ACC eddy diffusivity at the surface and as deep as 1000 m [Abernathey et al., 2010; Ferrari and Nikurashin, 2010; Klocker et al., 2012; Naveira Garabato et al., 2011; Smith and Marshall, 2009], although the scale separation between eddies and the mean flow on which these arguments depend may not apply in all regions of the ACC. In any case, the shift in the relative sizes of jets and eddies below mid depths, along with observations of large near-bottom eddy velocities (O(10) cm s$^{-1}$ [Chereskin et al., 2009]), implies that deep eddy fluxes are likely to be non-negligible.

An observational examination of the subsurface contributions to the ACC vorticity balance could help confirm or refute the dynamics and balance inferred from surface data [e.g. Hughes, 2005; Williams et al., 2007, and others] and from models [e.g. Wells and de Cuevas, 1995; Hughes and de Cuevas, 2001; Grezio et al.,]
In this paper, we investigate the vorticity balance of the ACC, focusing on Drake Passage and using a 4-year time series of measurements by current meter-pressure recording-inverted echo sounders (CPIES) deployed across Drake Passage as part of the cDrake project [Chereskin et al., 2012]; a 5.5-year time series of shipboard acoustic Doppler current profiler (SADCP) velocities in the Drake Passage area; and the Southern Ocean State Estimate (SOSE). We first set out the vertical and quasi-geostrophic vorticity balances in Section 4.2. The data sources and methods for computing vorticity balance terms from each source are described in Section 4.3. We describe the structure of the cDrake velocity field in the cDrake local dynamics array (LDA) in Section 4.4, examining the vertical structure of the mean field as well as the role of eddies. In Section 4.5 we present a direct estimate of bottom pressure torque in Drake Passage from the cDrake data. We examine the relative vorticity and mean and eddy relative vorticity advection fields in Section 4.6 and discuss the contributions to the mean vorticity balance in the cDrake LDA in Section 4.7. In Section 4.8 we use SOSE to investigate the importance of eddy terms at various scales in both Drake Passage and a relatively flat-bottomed region in the southeastern Pacific, and to investigate the potential importance of ageostrophic, sub-grid-scale, and dissipative terms not directly computed from cDrake. We discuss our results in Section 4.9.

4.2 The quasi-geostrophic vertical vorticity balance

The vertical vorticity equation is

$$\begin{align}
\zeta_t + \vec{u} \cdot \nabla \zeta + \beta v &= (\zeta + f) w_z + \vec{u} \times \nabla w + \frac{1}{\rho_0} \left( \frac{\partial\rho}{\partial x} \frac{\partial p}{\partial y} - \frac{\partial\rho}{\partial y} \frac{\partial p}{\partial x} \right) + \frac{1}{\rho_0} \nabla \times \vec{\tau}_z + D, \tag{4.1} \end{align}$$

where $\vec{u} = (u, v)$ is horizontal velocity, $\beta = \frac{df}{dy}$ is the meridional gradient of the Coriolis parameter $f$, $\zeta = v_x - u_y$ is the vertical component of relative vorticity, $w$ is the vertical velocity, $\nabla = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y})$ is the horizontal gradient operator, $\rho$ is
density, \( p \) is pressure, \( \tau \) is stress, and \( D \) is vorticity dissipation. In SOSE (and many other models) \( D \) is parameterized as the sum of the sub-grid-scale eddy ("explicit") dissipation, \( \kappa \nabla^2 \zeta + \kappa_4 \nabla^4 \zeta \), and the turbulent ("implicit") dissipation, \( \langle w' \zeta' \rangle = \kappa_i \frac{\partial^2 \zeta}{\partial z^2} + \kappa_4 \frac{\partial^4 \zeta}{\partial z^4} \), where the eddy diffusivities \( \kappa, \kappa_4, \) and \( \kappa_i \) may vary with location (see Section 4.3.3). Note that bottom frictional dissipation is included in the \( \frac{1}{\rho_0} \nabla \times \bar{\tau} \) term.

Under quasi-geostrophic conditions, \( \zeta \ll f \), and \( \bar{u}_z \times \nabla w \) and \( ((\partial \rho/\partial x)(\partial p/\partial y) - (\partial \rho/\partial y)(\partial p/\partial x))/\rho^2 \) are both negligible, leading to the quasi-geostrophic vorticity balance

\[
\zeta_t + \bar{u} \cdot \nabla \zeta + \beta v = f w_z + \frac{1}{\rho_0} \nabla \times \bar{\tau}_z + D, \tag{4.2}
\]

where the vorticity tendency \( \zeta_t \), nonlinear advection of relative vorticity \( \bar{u} \cdot \nabla \zeta \), and planetary vorticity advection \( \beta v \) are balanced by the sum of the vortex stretching term, forcing, and dissipation.

If we integrate (4.2) over depth, from the bottom at \( z = \eta_b \) to the surface at \( z = 0 \) (making the rigid lid assumption), we obtain

\[
\int_{\eta_b}^{0} \zeta_t dz + \int_{\eta_b}^{0} \bar{u} \cdot \nabla \zeta dz + \beta \int_{\eta_b}^{0} vdz = \frac{1}{\rho_0} \nabla \times \bar{\tau}_w - \frac{1}{\rho_0} \nabla \times \bar{\tau}_b - \frac{1}{\rho_0} J(p_b, \eta_b) + \int_{\eta_b}^{0} Ddz, \tag{4.3}
\]

where \( \bar{\tau}_w \) is wind stress, \( \bar{\tau}_b \) bottom frictional stress, \( p_b \) bottom pressure, \( \rho_0 = 1050 \text{ kg m}^{-3} \) the reference density, and \( J \) is the Jacobian operator. The depth integrals of vorticity tendency, nonlinear relative vorticity advection, and planetary vorticity advection are balanced by the vorticity forcing due to wind stress curl, bottom frictional stress curl, and bottom pressure stress curl (bottom pressure torque), and the integral of vorticity dissipation. Note that bottom pressure torque (bpt) is here defined, on the right-hand side, as \(-\frac{1}{\rho_0} J(p_b, \eta_b) \simeq -\bar{u}_b \cdot \nabla \eta_b = -fw_b \) (many authors use bpt = \( \frac{1}{\rho_0} J(p_b, H) \), where \( 0 < H = -\eta_b \)).

We can average (4.2) in time to obtain

\[
\langle \zeta_t \rangle + \langle \bar{u} \rangle \cdot \nabla \langle \zeta \rangle + \langle \bar{u}' \cdot \nabla \zeta' \rangle + \beta \langle v \rangle = \frac{1}{\rho_0} \nabla \times \langle \bar{\tau}_z \rangle + f \langle w_z \rangle + \langle D \rangle, \tag{4.4}
\]

where angle brackets indicate time averaging over some interval and primes indicate deviations from these means. Note that \( \langle \zeta_t \rangle \) need not be zero, depending on
the length of the averaging interval relative to the timescales of the flow. The nonlinear term, relative vorticity advection, has been split into advection of mean relative vorticity by the mean flow, or “mean relative vorticity advection”, and the Reynolds stress term, or “eddy relative vorticity advection”, the mean of transient relative vorticity advection by the transient flow. Finally, we can derive the depth-integrated, time-averaged quasi-geostrophic vorticity balance,

\[
\int_{\eta_b}^{0} \langle \zeta \rangle dz + \int_{\eta_b}^{0} \langle \vec{u} \rangle \cdot \nabla \langle \zeta \rangle dz + \int_{\eta_b}^{0} \langle \vec{u}' \rangle \cdot \nabla \zeta' dz + \beta \int_{\eta_b}^{0} \langle v \rangle dz
\]

\[
= \frac{1}{\rho_0} \nabla \times \langle \tau_w \rangle - \frac{1}{\rho_0} \nabla \times \langle \tau_b \rangle - \frac{1}{\rho_0} J(\langle p_b \rangle, \eta_b) + \int_{\eta_b}^{0} \langle D \rangle dz. \tag{4.5}
\]

4.3 Data and Methods

The data sources and methods for computation of various terms of the vorticity balance, depending on data source, are described here.

4.3.1 cDrake

As part of the cDrake experiment [Chereskin et al., 2012], an array of 42 current meter-pressure recording-inverted echo sounders (CPIES) was deployed across Drake Passage from November-December 2007 to November-December 2011 (Figure 4.1). The CPIES measure current 50 m off the bottom (outside the bottom boundary layer); bottom pressure; and bottom-surface round-trip sound travel time. The 3×7 Local Dynamics Array (LDA, Figure 4.1) was positioned in the eddy-rich PFZ just downstream of the Shackleton Fracture Zone (SFZ), with 37-km instrument spacing and designed to cover one meander wavelength. The cDrake CPIES data and processing are described in Chapter 3. Bottom pressures and near-bottom currents are objectively mapped, using a geostrophic continuity constraint (\( \nabla \cdot (f \vec{u}) = 0 \)), to barotropic geostrophic streamfunction, velocity, relative vorticity, and relative vorticity gradients, while bottom-surface round-trip sound travel times are converted to profiles of geopotential anomaly by use of a Gravest Empirical Mode [GEM, Meinen and Watts, 2000; Watts et al., 2001b] look-up table and then objectively mapped to baroclinic geostrophic streamfunction, velocity,
relative vorticity, and relative vorticity gradients (Chapter 3). The baroclinic fields are (by construction) self-similar with depth; the extent of turning with depth of the total current depends on the relative directions and magnitudes of the barotropic and baroclinic components (Figure 4.2). The objective mapping procedure iterates through three sets of time and space scales, with the shortest mapping decorrelation length scale for cDrake being 50 km. Chapter 3 also contains descriptions of simulations and validation of the cDrake velocity maps by comparison with independent measurements. Based on the results of these simulations, for the computations of vorticity balance terms described here, we use maps based on time series that have been low-pass filtered with a Butterworth filter with a 7-day cutoff, unless otherwise specified. We will refer to the variables computed from the CPIES data by objective mapping and the GEM as cDrake velocity, vorticity, etc.

Multibeam bathymetry data were collected on each of the five cDrake cruises. Multibeam data are averaged to 2-km resolution and gaps are filled by linear interpolation from the Smith and Sandwell [1997] bathymetric database. Gradients are estimated by first-differencing and are then further smoothed by Gaussian-weighted averaging to the objective mapping grid. For computing the bottom pressure torque term, $-f \vec{u}_b \cdot \nabla \eta_b = -f w_b$ (note $\eta_b < 0$), we are interested in bathymetry gradients at the scales that influence the flow outside the bottom boundary layer. Bishop et al. [2012] suggest that relatively small seamounts can affect the geostrophic flow at larger scales, so we tested length scales ranging from 25 km to 50 km (the shortest objective mapping scale) for the bathymetry gradient weighted averaging. Bottom pressure torque from less-smoothed bathymetry has higher variance and a slightly lower mean across the passage. We take the differences between 25 km and 50 km values to represent the uncertainty of the estimate. We also tested both mapped (that is, consistent with geostrophic continuity) and non-mapped bottom currents; the results are not very different, so we use non-mapped bottom currents to represent the full $w_b$.

We calculate wind stress curl from satellite-based surface wind vectors. We obtained 0.25-degree gridded, 3-day averaged Level 3 wind vectors from SeaWinds on QuikScat (from 2007-2009) from Remote Sensing Systems (RSS) at
ftp://ftp.ssmi.com/qscat/qscat_wind_vectors_v04/. The Quikscat wind vectors are calculated using the Ku-2011 algorithm [Ricciardulli and Wentz, 2011]. We obtained ASCAT Level 2 swath data (from 2009-2011) from JPL PODAAC at ftp://podaac-ftp.jpl.nasa.gov/allData/ascat/preview/L2/25km. ASCAT data are provided by EUMETSAT and KNMI. We averaged both QuikScat and ASCAT data to the cDrake objective mapping grid using Gaussian-weighted averaging with a 50-km decorrelation length, and combined them to form one time series for the cDrake time period. Wind vectors were converted to wind stress using the drag coefficient parameterizations of Yelland and Taylor [1996] (below 6 m s$^{-1}$ speed) and Yelland et al. [1998] (above). Gradients were estimated by center differencing.

Planetary vorticity advection, relative vorticity advection, vorticity tendency, bottom pressure torque, and wind stress curl are all computed directly from the cDrake mapped velocity, vorticity, and vorticity gradient fields, along with bathymetry and wind data. While these terms are the ones we expect to dominate, we also estimated other terms. Bottom frictional torque was calculated from the mapped bottom velocity using either a linear [Gill, 1968] or quadratic [Arbic and Scott, 2008] drag parameterization. We estimated the size of dissipation away from the boundaries by finite differencing the mapped vorticity gradients to second derivatives of the vorticity field and applying constant order-of-magnitude diffusivities based on SOSE (see Section 4.3.3). We used the mapped geostrophic streamfunction derivative fields to compute curvature [Watts et al., 1995] and the gradient wind velocity field, then used center differencing to estimate the total nonlinear vorticity advection based on the gradient wind flow, and compared this quantity with the geostrophic nonlinear vorticity advection.

4.3.2 Shipboard ADCP

Shipboard acoustic Doppler current profiler (SADCP) velocity measurements in the top 1000 m have been collected on approximately twice-monthly, all-season Drake Passage crossings by the ARSV Laurence M. Gould (LMG) since November 2004. The 159 crossings (through October 2012) processed to date have been compiled into a mean velocity field on a 25-km by 25-km horizontal grid with
24-m vertical resolution [Firing et al., 2011; Lenn et al., 2007]. The most common track runs just to the west of the cDrake central line and has 80 to 147 crossings through each 25-km segment.

We use the time-mean gridded SADCP velocities to compute mean relative vorticity \( \langle \zeta \rangle = \langle v \rangle_x - \langle u \rangle_y \) and mean relative vorticity advection \( \langle \vec{u} \rangle \cdot \nabla \langle \zeta \rangle \) by center differencing. SADCP mean velocities and their derivatives are transformed from the SADCP grid coordinates to zonal-meridional for comparison with cDrake.

From SADCP data on the most common track, we compute a time series of one component of relative vorticity advection, \( v_{tr} u_{tr,yy} \), where \((u, v)_{tr}\) is across-and along-track velocity and \( y_{tr} \) is the along-track coordinate, rotated approximately 9° from meridional. We compare this quantity to cDrake \( vu_{yy} \) computed from the three-day low-pass filtered mapped data (neglecting the 9° difference in orientation).

Given the differences between the time and space scales represented in the SADCP and cDrake fields, we emphasize that the comparisons between them are intended to validate the statistics of the cDrake higher-derivative fields, rather than particular values at particular times or locations. The comparison with relative vorticity and vorticity gradients computed from a different dataset (direct velocity measurements) by a different method (finite differencing) confirms that the cDrake method does not result in over- or under-estimation of the higher derivatives of streamfunction, as estimates agree to within a factor of two.

### 4.3.3 SOSE

The Southern Ocean State Estimate [SOSE; Mazloff et al., 2010] is an eddy-permitting, data-assimilating state estimate which uses the adjoint method to iterate the MITgcm to minimize misfit between the estimate and observations by modifying model parameters and forcings (atmospheric fluxes and initial and boundary conditions) within their uncertainties. SOSE is calculated on a \( \frac{1}{6} \)° grid at 42 depth levels, with higher resolution near the surface. We use iteration 59, spanning 2005 through 2007. Data from cDrake are not assimilated into this version of SOSE.

We compute vorticity balance terms from SOSE by two methods. For direct
comparisons with the cDrake maps, we put 5-day mean pressure anomalies through the same objective mapping procedure used for the cDrake data (Chapter 3). SOSE allows us to map both over a larger area and from data extending beyond the edges of the mapping grid, thus decreasing the error bars on the mapped higher derivatives. We use data extending 2° (about 220 km) in latitude and 4.3° (about 240 km) in longitude outside of the mapping grid; to reduce computation time we sub-sample to 1/3° in latitude and longitude (approximately 40 km and 20 km, respectively; compare to the cDrake LDA spacing of 37 km). We vary the mapping decorrelation length scales by multiplying each one by the same factor to investigate the balances at multiple scales. These fields will be referred to as SOSE mapped fields.

SOSE balances momentum internally. We compute terms of the SOSE vorticity balance by taking the curl of 30-day average SOSE momentum balance terms (e.g., for the \( u \)-momentum equation, \( u_t, -fv, uu_x + vu_y, \tau_x^z/\rho_0, \kappa\nabla^2 u + \kappa_4\nabla^4 u, \kappa_i\partial^2 u/\partial z^2 \)), obtaining

\[
\zeta_t = \frac{\partial v_t}{\partial x} - \frac{\partial u_t}{\partial y}, \tag{4.6}
\]

\[
-fw_z + \beta v = \frac{\partial (fu)}{\partial x} - \frac{\partial (-fv)}{\partial y}, \tag{4.7}
\]

\[
\vec{u} \cdot \nabla \zeta - w\zeta = \frac{\partial (uv_x + vv_y)}{\partial x} - \frac{\partial (uu_x + vu_y)}{\partial y}, \tag{4.8}
\]

\[
\frac{1}{\rho_0} \nabla \times \tau_z = \frac{\partial \tau^y_x/\rho_0}{\partial x} - \frac{\partial \tau^x_z/\rho_0}{\partial y}, \tag{4.9}
\]

\[
\kappa\nabla^2 \zeta + \kappa_4\nabla^4 \zeta = \frac{\partial (\kappa \nabla^2 v + \kappa_4 \nabla^4 v)}{\partial x} - \frac{\partial (\kappa \nabla^2 u + \kappa_4 \nabla^4 u)}{\partial y}, \tag{4.10}
\]

\[
\kappa_i \frac{\partial^2 \zeta}{\partial z^2} + \kappa_{iz} \frac{\partial^2 v}{\partial z^2} - \kappa_{iy} \frac{\partial^2 u}{\partial z^2} = \frac{\partial (\kappa_i \partial^2 v/\partial z^2)}{\partial x} - \frac{\partial (\kappa_i \partial^2 u/\partial z^2)}{\partial y}, \tag{4.11}
\]

where \( \kappa \) and \( \kappa_4 \) are constant explicit (eddy) diffusivity coefficients and \( \kappa_i \) is the implicit diffusivity used to parameterize the vertical Reynolds stresses (\( \langle uu'v' \rangle = \kappa_i \frac{\partial^2 u}{\partial z^2} \)). We refer to these terms as the SOSE tendency (4.6), geostrophic divergence (4.7), nonlinear (4.8), external forcing (4.9), explicit dissipation (4.10), and implicit dissipation (4.11) terms. The geostrophic divergence term (4.7) is composed of planetary vorticity advection \( \beta v \) and stretching \( -fw_z \). Planetary vorticity advection can be computed from the \( u \)-momentum balance Coriolis term. The
depth-integrated stretching term can be decomposed into bottom pressure torque,
\(-\frac{1}{\rho_0} J(p_b, \eta_b)\), and the depth-integrated effect of ageostrophic stretching, \(\int_{\eta_b}^0 f w_{a,z} dz\) (where \(w_a\) is the ageostrophic \(w\)). Bottom pressure torque (bpt) can be computed either as

\[
bpt = -f \bar{u}_{b,g} \cdot \nabla \eta_b, \tag{4.12}\]

where \(\bar{u}_{b,g}\) is the geostrophic bottom velocity, or, following Hughes and de Cuevas [2001], as

\[
bpt = -\nabla \times \int_{\eta_b}^0 \nabla p \, dz. \tag{4.13}\]

### 4.4 Vertical structure of cDrake mapped streamfunction

The cDrake LDA is located in the region of enhanced variability between the mean Subantarctic Front (SAF) and Polar Front (PF) jet positions (Figure 4.3). The cDrake velocity fields in the LDA are dominated by meander and eddy events like the one discussed in Chereskin et al. [2009] (their Figure 4), where a south-eastward meander of the SAF and a northward meander of the PF led to the development of an anticyclonic and cyclonic eddy, respectively, in the LDA area. During these events, which occur about five times a year over the four-year record, near-bottom daily-mean speeds can exceed 40 cm s\(^{-1}\) for periods of a week at a time. As in the example in Chereskin et al. [2009], patterns in the barotropic and baroclinic components of the flow are often offset in space and time. The barotropic and baroclinic components of the depth-mean velocity have approximately equal variance, consistent with findings on barotropic and baroclinic contributions to Drake Passage transport by Cunningham et al. [2003] and Firing et al. [2011].

Different events have different paths of development through the array, but averaged over four years, the net result of the LDA meander/eddy events on the deep circulation is a standing anticyclonic eddy in the area just downstream of the SFZ, and a stronger standing cyclonic eddy in the Yaghan seafloor depression (Figure 4.3). The mean deep flow appears to be deflected around some of the seamounts in the northwestern part of the array. The northeastward flow of
the SAF is also visible in the deep flow at the northern end. The mean 200-dbar baroclinic streamlines capture parts of the SAF and PF jets at the northwestern-northern and southern edges, respectively, of the mapping area, as well as a northward meander centered to the southwest of the deep cyclonic standing eddy and possibly a southward meander (with a small closed anticyclonic contour) centered slightly to the northwest of the deep anticyclonic standing eddy. Ferrari et al. [2012] found a similar pattern (a southward meander of the SAF and northward meander of the PF) as the first EOF of the 18-year satellite sea level anomaly record; the cDrake 4-year mean baroclinic flow thus appears to be a typical pattern, but not necessarily representative of a longer time mean (which would average over positive and negative phases of this mode). Averaged over the cDrake time period, satellite-derived mean absolute dynamic topography (MADT) is for the most part parallel to the total cDrake streamfunction at 200 dbar (Figure 4.3).

The mean baroclinic and barotropic streamfunctions are not generally parallel; however, the degree of departure from an equivalent-barotropic condition also depends on the relative sizes of the barotropic and baroclinic currents. At the northern end, deep streamlines are close to parallel to surface streamlines from MADT. On the eastern edge of the array, although the barotropic and baroclinic streamlines cross, the total surface streamlines (the sum of the barotropic and baroclinic components) are aligned with the deep streamlines. To test whether the mean current departs significantly from an equivalent-barotropic condition, we look for turning with depth of $O(1)$ rad in the current direction. The angle change between the 200-dbar total current and the bottom (barotropic) current is particularly large in the portions of the SAF and PF captured by the LDA map, where the baroclinic currents are relatively strong, and is greater than 0.5 rad over about half of the map area. Bottom current sensors likely to be affected by very local bathymetry have been excluded from the barotropic maps (Chapter 3), while the total current at 200 dbar is in generally good agreement with the MADT current, even in areas with large turning with depth. Thus the non-equivalent-barotropic condition of the mean flow in the LDA region appears to be real. We note that SADCP time series suggest that both the total and baroclinic mean currents in
the jets are more equivalent-barotropic than those in the PFZ [Firing et al., 2011]. The flow in the LDA may be affected by the nearby SFZ (flow over significant topography is not expected to be equivalent-barotropic).

4.5 Bottom pressure torque in Drake Passage

Extrema in mean bottom pressure torque (bpt) in Drake Passage (Figure 4.4) occur over the Shackleton Fracture Zone (SFZ) and at the boundaries over the continental slopes. Where the cDrake line crosses the SFZ, near-bottom flow is closer to along-slope than across it at most instruments; despite the deflection from directly cross-slope flow, strong near-bottom currents lead to large values of bpt. Because the strength of the upslope and downslope flow is similar, however, the bottom pressure torques on the upstream and downstream sides nearly cancel each other. In the LDA, elevated values of bpt are found where strong bottom currents flow nearly parallel to bathymetry gradients (nearly up- or down-slope). Even larger bpt values over the southern and northern slopes, in contrast, are produced by moderate currents flowing nearly along but slightly up (in the north) or down (in the south) steep topographic slopes (note that bpt is defined as $-\frac{1}{\rho_0}J(p_b, \eta_b) = -f \vec{u} \cdot \nabla \eta_b = -f w_b$, so in the Southern Hemisphere it is positive when the flow is up-slope). The net bpt integrated across the passage is approximately $3 \times 10^{-3}$ to $4 \times 10^{-3}$ m$^2$ s$^{-2}$ (for 25-km or 50-km smoothed bathymetry gradients, respectively). Wind stress curl forcing over the ACC is approximately $2 \times 10^{-10}$ m s$^{-2}$; integrating zonally and over a 10$^\circ$ latitude band centered at 60$^\circ$S gives a total of $4.4 \times 10^3$ m$^3$ s$^{-2}$. The net bpt along the CPIES transect thus could balance the entire ACC wind stress curl if applied over 1100-1500 km of along stream distance. Figure 4.4 covers 350 to 450 km of zonal distance, while the length of the extended Drake Passage, including the Scotia region, is more than twice that.

Variability in bpt (Figure 4.4c) is low over and near the SFZ, indicating a consistent pathway for flow over this obstacle. Elevated bpt variance is found in the southwestern corner of the LDA, and at the northern boundary and under the
SAF. In the southwestern corner of the LDA the variability in bpt corresponds to variability in current speed more than in current direction, while in the north both aspects of bottom current variability play a role.

### 4.6 Relative vorticity and nonlinear advection in the LDA

We now address quantities involving higher derivatives of streamfunction, the relative vorticity and relative vorticity gradient fields. The mean relative vorticity in the LDA (Figure 4.5) is approximately $10^{-5}$ s$^{-1}$ (giving a Rossby number $(Ro = \zeta/f)$ of $O(0.1)$) and reflects the standing eddies visible in the mean streamlines (Figure 4.3) and the dipole pattern found in satellite sea level anomaly by Ferrari et al. [2012]. The mean vorticity field is strongly barotropic, decreasing only slightly with depth and having well-correlated shallow and deep fields ($r = 0.76$ between 400 dbar and 3600 dbar, significant at the 99% level). Mean and eddy relative vorticity advection are $O(10^{-11})$ s$^{-2}$, with the mean contribution about twice as big as the eddy contribution. The shallow and deep relative vorticity advection fields (Figure 4.5) are moderately-correlated with depth ($r = 0.55, 0.44$, both significant at the 99% level, for mean and eddy, respectively) but decrease by a factor of approximately two from 400 dbar to 3500 dbar. Both relative vorticity and relative vorticity advection have significant variance over time (standard deviations of $10^{-5}$ s$^{-1}$ and $10^{-11}$ s$^{-2}$, respectively). Chereskin et al. [2009] described two eddy events in the LDA where $Ro$ from daily mean near-bottom current measurements was $-0.15$ and $0.4$. In four years of 7-day low-pass filtered mapped vorticity we find several such events each year. At 200 dbar $|Ro| \geq 0.2$ 12% of the time and $|Ro| \geq 0.1$ 40% of the time (with a maximum magnitude of 0.75), while at the bottom $|Ro| \geq 0.1$ 6% of the time (with a maximum of 0.35). There is a slight preference for negative (cyclonic) $\zeta$ at all depths; however, this preference is only significant (64% cyclonic) at 4000 dbar. The depth profiles of time-varying relative vorticity and relative vorticity advection (not shown) mirror the depth properties of the mean fields; relative vorticity advection changes sign between the
near-surface and near-bottom approximately 36% of the time, but because of the decrease in magnitude with depth there is little cancellation in the depth integral.

The long term SADCP dataset provides an independent check on the size of the mean and eddy relative vorticity advection terms. We compare rms values of cDrake mapped fields with rms values of the same fields estimated from the SADCP dataset. The cDrake and SADCP rms $\langle \zeta \rangle$ over the LDA area and the top 1000 m are comparable and both decrease near-linearly with depth, from $5.5 \times 10^{-6}$ s$^{-1}$ to $4.2 \times 10^{-6}$ s$^{-1}$ (cDrake), or from $3.8 \times 10^{-6}$ s$^{-1}$ to $2.4 \times 10^{-6}$ s$^{-1}$ (SADCP). The mean relative vorticity advection, $\langle \vec{u} \rangle \cdot \nabla \langle \zeta \rangle$, also decreases near-linearly with depth, from $2.5 \times 10^{-11}$ s$^{-2}$ to $1.4 \times 10^{-11}$ s$^{-2}$ (cDrake), or $2.7 \times 10^{-11}$ s$^{-2}$ to $1.3 \times 10^{-11}$ s$^{-2}$ (SADCP).

The rms values of the SADCP and cDrake track-resolvable mean relative vorticity advection component ($\langle v \rangle \langle u_{yy} \rangle$) in the overlap area of the LDA and the most common track are approximately the same size, with the SADCP values decreasing slightly faster with depth ($1.7 \times 10^{-11}$ s$^{-2}$ to $0.8 \times 10^{-11}$ s$^{-2}$ compared to $1.7 \times 10^{-11}$ s$^{-2}$ to $1.0 \times 10^{-11}$ s$^{-2}$ for cDrake). The rms cDrake track-resolvable eddy component ($\langle v' u_{yy}' \rangle$) decreases from $2 \times 10^{-11}$ s$^{-2}$ to $0.8 \times 10^{-11}$ s$^{-2}$, while the SADCP eddy component decreases from $2.2 \times 10^{-11}$ s$^{-2}$ to $0.4 \times 10^{-11}$ s$^{-2}$ with depth. Both SADCP and cDrake have mean and eddy components of similar size, and a faster decrease with depth in the eddy components than in the mean.

Given the differences between SADCP and cDrake sampling (3-day low-pass filtered data from late 2007 to late 2011, vs. approximately twice-monthly synoptic data from late 2004 to early 2011), spatial smoothing (center differencing the 25-km SADCP grid produces an effective length of 75 km, compared to the cDrake 50-km high-pass mapping scale), the agreement in size and depth pattern between cDrake and SADCP vorticity and vorticity advection components is quite good; the rms values of the SADCP estimates are within a factor of two of the cDrake rms values. Thus the SADCP data appear to support the conclusion from the simulations described in Chapter 3 that we can accurately map both mean and time-varying vorticity and vorticity gradients (and therefore vorticity advection) in the LDA.
4.7 Vorticity balance in the LDA

Selected terms from the depth-integrated time-mean quasigeostrophic vorticity balance (4.5) in the LDA from cDrake data are shown in Figure 4.6. The dominant terms by an order of magnitude are the mean and eddy relative vorticity advection and the bottom pressure torque. Planetary vorticity advection is an order of magnitude smaller and wind stress curl (not shown) two orders of magnitude smaller. Eddy relative vorticity advection is nearly as large as mean relative vorticity advection; while the individual components of eddy advection, $\langle u' \zeta' \rangle$ and $\langle v' \zeta' \rangle$, are generally larger than the corresponding components of the mean relative vorticity advection, they are also more strongly negatively correlated, such that the total eddy relative vorticity advection is smaller than the total mean relative vorticity advection. The total mean relative vorticity advection field, meanwhile, is uncorrelated with the eddy relative vorticity advection field. It is mildly correlated ($r = 0.23$, significant at the 99% level) with the bottom pressure torque field, so there is some cancellation between the two. The residual of the mapped terms, however, is still as large as the largest term (Figure 4.6).

This large residual is significantly different from zero; the few points in the central LDA not significantly different from zero based on the errors computed in the objective mapping algorithm are indicated by gray pluses. Simulations described in Chapter 3 validate the objective mapping error computations, and the comparisons with SADCP-derived fields described in the previous section also indicate that the relative vorticity advection terms are reasonable.

We estimated several other terms that might account for the residual of the terms shown. The bottom frictional torque calculated using the near-bottom mapped currents and either linear or quadratic drag parameterizations (not shown) is four orders of magnitude smaller than bottom pressure torque. The depth integral of dissipation due to sub-grid-scale eddies (the “explicit” dissipation), parameterized using $\int \kappa \nabla^2 \langle \zeta \rangle dz$, with the second derivatives of $\zeta$ estimated by finite differencing of the mapped first derivatives, and a typical SOSE interior coefficient of $\kappa = 200 \text{ m}^2 \text{ s}^{-1}$, is of the same order as the mean relative vorticity advection (Figure 4.6; note that SOSE, like other models, uses a bi-harmonic form for the
eddy diffusivity; however, the cDrake data do not allow us to estimate the required fourth derivatives of $\zeta$). The spatial pattern is uncorrelated with any of the other terms in the balance, or their residual. Depth-integrated turbulent (“implicit”) dissipation, parameterized as $\kappa_i \partial^2 \zeta / \partial z^2$, with the vertical derivatives estimated by finite differencing of mapped $\zeta$, and $\kappa_i = 10^{-2}$ m$^2$ s$^{-1}$ (again, an order of magnitude estimate of a typical coefficient used by SOSE), is four orders of magnitude smaller than bottom pressure torque.

So far we have considered only the fields produced by objective mapping constrained to satisfy geostrophic continuity. Although ageostrophic residual velocities are negligible compared to the mapped geostrophic velocities, the ageostrophic contribution to relative vorticity gradient depends not only on the size of the ageostrophic currents but also on their spatial variability, and thus is not necessarily negligible in comparison to the geostrophic relative vorticity gradient. The elevated values of $Ro$ found in the LDA (see Section 4.6) point to a possibly significant contribution from cyclostrophic flow. We can estimate this contribution from the gradient wind balance:

\[
\kappa_c V^2 + fV = -\frac{1}{\rho} \frac{\partial p}{\partial n} \approx -fV_g, \tag{4.14}
\]

\[
V = \frac{-f \pm \sqrt{f^2 + 4\kappa_c fV_g}}{2\kappa_c}, \tag{4.15}
\]

\[
V_c = V - V_g, \quad \kappa_c < 0, \tag{4.16}
\]

\[
V_c = V_g - V, \quad \kappa_c > 0, \tag{4.17}
\]

where $n$ is the normal direction, $V$, $V_g$, and $V_c$ are the total, geostrophic, and cyclostrophic correction speeds, respectively; the total ($\vec{u} = V e^{i\theta}$), geostrophic ($\vec{u}_g = V_g e^{i\theta}$), and cyclostrophic ($\vec{u}_c = \vec{u} - \vec{u}_g = V_c e^{i\theta}$) velocities are all parallel, and

\[
\kappa_c = \frac{\psi_{xx} \psi_y^2 + \psi_{yy} \psi_x^2 - 2 \psi_{xy} \psi_x \psi_y}{(\psi_x^2 + \psi_y^2)^{3/2}} \tag{4.19}
\]

is the curvature (based on objectively mapped derivatives of geostrophic streamfunction, Watts et al. [1995]). We estimated $\vec{u}$ using the better-behaved of the two possible solutions for $V$ from (4.15). Although differences between $\vec{u}$ and $\vec{u}_g$ are
generally quite small, if $\vec{u}_c$ is highly variable in space, $\zeta_c = v_{c,x} - u_{c,y}$ and $\nabla \cdot \zeta_c$ may be non-negligible, and $\zeta$ may differ significantly from $\zeta_g$. We estimated $\zeta$, $\partial \zeta / \partial x$, and $\partial \zeta / \partial y$, the relative vorticity and relative vorticity gradients from the total gradient wind flow, by finite differencing of $\vec{u}$. We also estimated errors on the resulting relative vorticity advection terms, by propagation of error through the finite differences. The resulting mean and eddy relative vorticity advections (not shown) show the same spatial patterns as the geostrophic terms, and are 10 to 20% smaller. Error bars on the gradient wind terms are larger than those on the geostrophic terms, but not large enough to close the LDA vorticity balance. Thus it appears that the cyclostrophic contribution to relative vorticity advection can explain part but not all of the residual in Figure 4.6.

To more closely examine the characteristics of the vorticity balance terms in the LDA, we present time series of the dominant terms, along with shallow and deep eddy kinetic energy (Figure 4.7), from the middle of the LDA. The terms have been averaged over points adjacent to the three marked sites in Figure 4.6. The time series are dominated by eddy events, and residuals are as large as and correlated with the relative vorticity advection. Bottom pressure torque and relative vorticity advection are negatively correlated with $r = -0.7$; however, the time-varying bottom pressure torque is significantly smaller than the time-varying relative vorticity advection. The depth structure of relative vorticity advection during selected eddy events (not shown) shows a mixture of barotropic and baroclinic components: the magnitude decreases with depth, by a factor of around four to six, but (as noted in Section 4.6) the sign is usually preserved. Planetary vorticity advection (not shown) has a qualitatively similar depth profile.

### 4.8 Vorticity balance from SOSE

We used SOSE to illuminate the vorticity balance in the LDA area as well as investigate the larger-scale vorticity balance. SOSE balances momentum and therefore vorticity, to the limit of the grid resolution (equivalent to about $10^{-11}$ m s$^{-2}$ in the vorticity balance, or $10^{-8}$ m s$^{-2}$ for the depth-integrated vorticity balance).
We examined the SOSE vorticity balance both in Drake Passage and in a region in the southeastern Pacific (Figure 4.8), where, in contrast to Drake Passage, bathymetry is relatively smooth and the ACC relatively wide. We investigated both SOSE’s own vorticity balance and the cDrake-comparable vorticity balance terms derived by objectively mapping SOSE hydrostatic pressure anomalies to streamfunction and its derivatives.

4.8.1 SOSE vorticity balance in Drake Passage

In the depth-integrated SOSE vorticity balance in Drake Passage (Figure 4.9), the first order balance is among the geostrophic divergence term ($5 \times 10^{-8}$ m s$^{-2}$, (4.7), Figure 4.9f), the nonlinear term ($4 \times 10^{-8}$ m s$^{-2}$, (4.8), Figure 4.9c), and the explicit dissipation term ($3 \times 10^{-8}$ m s$^{-2}$, (4.10), Figure 4.9b). The nonlinear and geostrophic divergence terms have a larger dominant spatial scale than the expliciternal dissipation term, and are moderately anti-correlated ($r = -0.64$). The tendency term ((4.6), not shown) is the same order as the residual of the first three terms over 30-day periods ($5 \times 10^{-9}$ m s$^{-2}$), but an order of magnitude smaller in a 900-day mean. Implicit dissipation ((4.11), not shown) is $10^{-10}$ m s$^{-2}$. SOSE balances vorticity to within $O(0.05)$ of the size of the dominant terms (Figure 4.9a).

The depth-integrated geostrophic divergence (4.7) is approximately equally split between the bottom bin (the deepest non-land depth bin at each location) and the integral of the bins above, where the primary contributor is ageostrophic stretching (not shown), as opposed to the relatively small planetary vorticity advection (Figure 4.9d). In the bottom bin, the principal contributor is bottom pressure torque ((4.13), Figure 4.9e); the ageostrophic contribution to $fw_b$ (estimated as the difference between the total $fw_b$ and the geostrophic $fw_{bg}$; not shown) is much smaller. This is consistent with cDrake, where $fw_b$ was not significantly different from $fw_{bg}$ (Section 4.3.1). SOSE planetary vorticity advection tends not to change sign with depth: $\int \beta v dz$ has the same sign as $\beta v$ at 200 dbar 94% of the time. The ageostrophic stretching is more likely to change sign with depth, and to be of opposite sign to bpt; the depth-integrated total geostrophic divergence has the same
sign as shallow geostrophic divergence 70% of the time, or 75% of the time with the bottom bin excluded. The integrated ageostrophic $f w_z$ is largely compensated by the bottom pressure torque, such that the total geostrophic divergence term (Figure 4.9f) is smaller than the bottom pressure torque (Figure 4.9e).

Excluding the bottom bin increases the negative correlation between the geostrophic divergence and nonlinear advection terms to $r = -0.80$; given that $\beta v$ is a negligible part of the geostrophic divergence term, this implies a balance between nonlinear advection and stretching (ageostrophic divergence) in the interior. Like the geostrophic divergence term, the nonlinear advective term decreases with depth until it increases again in the bottom bin, and the depth-integrated nonlinear advective term has the same sign as at 200 dbar 75% of the time. The explicit dissipation term is even more strongly dominated by the bottom bin, where values are an order of magnitude larger than in the rest of the water column (reflecting the spatial pattern of $\kappa$ and $\kappa_4$). Dissipation tends to change sign with depth more than either the geostrophic divergence or advective terms; the depth integral has the same sign as the 200-dbar term only 55% of the time (whether the bottom bin is included in the integral or not).

The terms of the vorticity balance derived by taking the curl of terms of the SOSE momentum balance include ageostrophic contributions (see Section 4.3.3), and thus are not directly comparable to the vorticity balance terms derived from objectively mapped geostrophic fields. To compare with terms from cDrake, we objectively mapped SOSE pressure anomaly fields, using the same methods and parameters as for cDrake. Although the SOSE mapped terms in the cDrake LDA area (Figure 4.10) are about one quarter to one half the size of the cDrake terms, the relationships between different terms are preserved. As in cDrake, mean and eddy relative vorticity advection (Figure 4.10 i and j) are the same order of magnitude, with the eddy term being somewhat smaller than the mean term. Bottom pressure torque (Figure 4.10 m) is the same order of magnitude, and planetary vorticity advection (Figure 4.10 l) is an order of magnitude smaller. The residual (Figure 4.10 h) is the same size as the dominant terms. Mapping with decorrelation length scales four to six times longer (not shown) reduces the eddy relative
vorticity advection and increases the planetary vorticity advection (relative to the mean relative vorticity advection); however, the dominant terms are still (mean) relative vorticity advection, bottom pressure torque, and the residual.

The mapped terms (Figure 4.10 k, l, m, n, h) are in general agreement with comparable or near-comparable terms from SOSE’s own vorticity balance (Figure 4.10 c, d, e, f, g, respectively), although the terms smoothed over 50 km are slightly smaller than the terms from the 1/6° grid. The mapped bpt (Figure 4.10 m) is (relatively) smaller than SOSE bpt (Figure 4.10 e); however, since SOSE bpt is largely balanced by the integrated ageostrophic stretching, the total geostrophic divergence (Figure 4.10 f) and mapped $\beta v \cdot \text{bpt}$ (Figure 4.10 n) are of similar size. The SOSE nonlinear term ((4.8), Figure 4.10 c) and SOSE mapped total advection (Figure 4.10 k) are in even better agreement, indicating that ageostrophic contributions to relative vorticity advection, as well as $\int_{mb}^{0} w \zeta dz$, are small in SOSE.

The correspondence between SOSE mapped terms and SOSE non-mapped terms, and qualitative correspondence between SOSE mapped terms and cDrake mapped terms, gives us additional confidence in the cDrake mapped terms in the LDA. In SOSE, the residual of the nonlinear and geostrophic divergence terms (approximately equivalent to the mapped linear and nonlinear advection and bottom pressure torque residual) is largely balanced by the explicit (sub-grid-scale) dissipation term. Numerical models generally assign quite large eddy diffusivities in order to balance, so the large sub-grid-scale dissipation in SOSE may not be representative of the real ocean. On the other hand, we note that 1) when compared to direct SADCP measurements, SOSE underestimates eddy kinetic energy in the PFZ, so while $\kappa$ may be an overestimate, the $\nabla^2 \zeta$ part of $\kappa \nabla^2 \zeta$ is likely underestimated by SOSE; and 2) the cDrake LDA spacing of 37 km and mapping scale of 50 km are two to five times SOSE’s eddy-permitting 1/6° grid resolution, so more “sub-grid” eddy energy would be expected in cDrake than in SOSE.

## 4.8.2 SOSE vorticity balance in the southeastern Pacific

We also examined the vorticity balance in the southeastern Pacific (Figure 4.11), a region of much smoother topography, lacking the lateral constraints
of Drake Passage. The same three terms ((4.7), (4.8), and (4.10)) dominate the depth-integrated balance at $10^{-8}$ m s$^{-2}$, and again planetary vorticity advection is much smaller than geostrophic divergence, nonlinear advection, or explicit dissipation. Depth profiles show the same quality of decrease with depth plus bottom intensification that is present in the Drake Passage area, although the profiles of geostrophic divergence and nonlinear advection are smoother and less likely to change sign (depth-integrated and 200-dbar values have the same sign 80-84% of the time). The dominant horizontal scale of the features in the geostrophic divergence and nonlinear terms is also larger than in Drake Passage. The negative correlations between the depth-integrated geostrophic divergence and nonlinear terms with and without the bottom bin are the same as in the Drake Passage region, and the residual of the three dominant terms is only slightly smaller ($4 \times 10^{-9}$ m s$^{-2}$).

Vorticity balance terms in the southeastern Pacific from SOSE mapped at the same scales as cDrake (not shown) are qualitatively similar to those in Drake Passage (although all terms are smaller, as fields are smoother): mean and eddy relative vorticity advection and bottom pressure torque are first order; planetary vorticity advection is second order; and the residual of the mapped terms is first order. Mapping with decorrelation length scales three or four times larger than cDrake length scales, however, reveals a dramatically different balance. The eddy relative vorticity advection (Figure 4.12 j) becomes second order, as do the residual (Figure 4.12 h) and the bottom pressure torque (Figure 4.12 m), except in the vicinity of isolated seamounts. The dominant balance is between mean relative vorticity advection (Figure 4.12 i) and planetary vorticity advection (Figure 4.12 l), which exhibit negatively correlated wave-like patterns with a wavelength of about 500 km, like those found by Hughes [2005] in mean surface dynamic topography and Chereskin et al. [2010] in LADCP and SADCP sections. Mean relative vorticity advection and planetary vorticity advection are negatively correlated (Figure 4.13 a, $r \leq -0.8$) at all depths, and their relative sizes change with depth, from a two-to-one dominance of mean relative vorticity advection at the surface, to balancing around 1150 m, to a five-to-one dominance of planetary vorticity advection by 3300 m (Figure 4.13 b). This depth dependence is also consistent
with stationary short barotropic Rossby waves in an equivalent-barotropic flow as described by Hughes [2005].

4.9 Discussion and Summary

The cDrake CPIES dataset enables us to make maps of depth- and time-varying streamfunction, velocity, relative vorticity, and relative vorticity gradients in Drake Passage, at horizontal scales down to 50 km. Fields in the PFZ just downstream of the SFZ are dominated by meander and eddy events, producing a four-year mean circulation pattern of an anticyclonic/northward meander and cyclonic/southward meander consistent with the first EOF of sea level anomaly as described by Ferrari et al. [2012]. The Rossby number of the time-varying geostrophic flow is large, up to 0.75 near the surface and 0.35 at the bottom. Although the Rossby number of the four-year mean flow is $O(0.1)$, the current departs significantly from an equivalent-barotropic condition over much of the LDA. In the SAF and at the eastern side of the LDA the mean current does appear to be approximately equivalent-barotropic.

The four-year time series of near-bottom current measurements across Drake Passage, along with high-resolution multibeam bathymetry collected on the cDrake cruises, allowed us to compute the mean bottom pressure torque, the dominant balancing term for the wind stress curl in the ACC-wide vorticity balance. Bottom pressure torque integrated across the passage is $3 - 4 \times 10^{-3} \, \text{m s}^{-2}$; if applied along approximately 800 km of Drake Passage and the Scotia Ridge area, this forcing would balance 60 to 70% of the wind stress curl input over the ACC. The cDrake measurements thus confirm the importance of Drake Passage for the zonally-integrated ACC vorticity balance.

The vorticity balance in the high-$Ro$ PFZ is dominated by the nonlinear advection terms as well as the bottom pressure torque. Depth integrals of both mean and eddy relative vorticity advection are $O(10^{-7}) \, \text{m s}^{-2}$, while planetary vorticity advection is a tenth of that size. SADCP measurements confirm these scalings. Bottom pressure torque in the PFZ is also $O(10^{-7}) \, \text{m s}^{-2}$, and is negatively corre-
lated with nonlinear relative vorticity advection; however, it is approximately one third of the size of relative vorticity advection, so that the residual of these terms is significant, even in the center of the LDA, and resembles both relative vorticity advection and bottom pressure torque. Two additional terms that may be large enough to help close the balance were estimated. The cyclostrophic contribution to nonlinear advection (the difference between geostrophic relative vorticity advection and relative vorticity advection based on the gradient wind velocity and vorticity fields) is correlated with the geostrophic nonlinear advection term, and reduces it (and the residual) by 10 to 20%. An estimate of the dissipation of vorticity due to “sub-grid” (sub-50-km) motions, using harmonic diffusivity with a constant diffusivity parameter, is the same size as the residual; however, uncertainty is high due both to the use of finite differences and to uncertainty in the appropriate eddy diffusivity coefficient. We are thus unable to close the local vorticity balance in the PFZ from the cDrake dataset.

The vorticity balance of SOSE in Drake Passage is also dominated by bottom pressure torque and nonlinear terms. At scales up to about 100 km, eddy relative vorticity advection is of the same order as mean relative vorticity advection; at larger scales the importance of eddy relative vorticity advection is reduced and the mean relative vorticity advection and bottom pressure torque dominate. Objective mapping of SOSE pressure anomaly fields in the same manner as the cDrake pressure or geopotential anomaly fields, but with a more extensive set of input points, and spatial resolution comparable to or higher than that of the cDrake CPIES grid, produces fields that agree qualitatively with the cDrake fields, helping validate the cDrake maps of vorticity advection from the admittedly limited cDrake array. The importance of sub-grid-scale dissipation in the SOSE vorticity balance is suggestive for cDrake, but not conclusive.

Although SOSE vorticity and vorticity balance terms are smaller in a topographically smoother section of the southeastern Pacific, the dominant balance at 1/6° (10-20 km) to 50-km scales is the same as in Drake Passage, with both mean and eddy relative vorticity advection, as well as sub-grid-scale dissipation and bottom pressure torque, being first order. Smoothing over lengths of 150 to 200 km
reveals a balance like that found by Hughes [2005] and Chereskin et al. [2010] in this region: a Rossby-wave-like balance between planetary vorticity advection and mean relative vorticity advection. Bottom pressure torque is still important locally near seamounts.

In contrast to the previous results of [Hughes, 2005] based on a mean surface dynamic topography, both the cDrake dataset and SOSE indicate that nonlinear advection is a dominant term of the vorticity balance in Drake Passage, at scales up to several hundred kilometers. Although the transient eddy contribution to relative vorticity advection becomes negligible at scales over about 100 km, its significance at 50-km scales in both cDrake and SOSE implies that it is non-negligible for any synoptic survey or small process-scale array. In addition, in both SOSE and cDrake, ageostrophic and sub-grid-scale motions appear to be important for the local vorticity balance (although in cDrake the error bars are also large). The remarkably large near-bottom mean and eddy velocities and vorticities observed in cDrake also have implications for the energy balance.

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Figure 4.1: Top left: cDrake area (box) over Orsi et al. [1995] fronts (gray lines). Top right: enlargement of the red boxed region showing cDrake CPIES sites over Orsi et al. [1995] fronts (gray lines) and bathymetry (described in text), with box outlining the LDA area, shown below. The gold rectangle shows the LMG most common track. Bottom: CPIES sites (black dots) and bathymetry (filled contours) in the LDA. Site B02 is indicated by a diamond and the 6 LDA tip-step sites (A02, C03, C05, C19, C07, G03) by squares.
Figure 4.2: Illustration of combination of barotropic current with self-similar baroclinic current from a GEM. In the left-hand example the total current is equivalent-barotropic (net turning with depth < 1); in the right-hand example, the total current is not (net turning with depth > 1).
Figure 4.3: Left panels: mapped cDrake time-mean barotropic streamfunction (cool colors), with baroclinic streamfunction at 200 dbar (warm colors, top), and total streamfunction at 200 dbar (warm colors, bottom), and bathymetry (gray). Right panel: cDrake total 200 dbar streamfunction (warm colors) and surface MADT (gray). Barotropic, baroclinic, and total streamfunctions have arbitrary offsets for visual clarity, and are each contoured at intervals of 0.25 m² s⁻². Bathymetric contour intervals are every 500 m from 4500 m to 0 m. MADT contours are every 5 cm (0.5 m s⁻²). cDrake barotropic and baroclinic streamfunctions are only shown where their objective mapping percent error is less than 30%.
Figure 4.4: Vectors (a) of bathymetry gradient (black) and mean bottom current scaled by $-f$ (teal); mean (b) and standard deviation (c) of bottom pressure torque $-\frac{1}{\rho_0}J(p_b, \eta_b) = -f \bar{u}_b \cdot \nabla \eta_b$, from non-mapped near-bottom currents and 50-km-smoothed bathymetry gradients.
Figure 4.5: Mean relative vorticity $\langle \zeta \rangle$, mean relative vorticity advection $\langle \vec{u} \rangle \cdot \nabla \langle \zeta \rangle$, and eddy relative vorticity advection $\langle \vec{u}' \cdot \nabla \zeta' \rangle$ at 400 dbar and 3600 dbar. Points with absolute values less than the standard error of the mean are covered by gray pluses.
Figure 4.6: Indicated terms of the depth-integrated mean vorticity balance, and the residual of planetary vorticity advection, relative vorticity advection, and bottom pressure torque (middle right panel). Points with absolute values less than the standard error of the mean are covered by gray pluses.
Figure 4.7: Time series of cDrake vorticity balance terms averaged over grid points adjacent to the marked points in Figure 4.6. Four-year 30-day low-pass filtered time series of: (a) shallow eddy kinetic energy (light green) and deep eddy kinetic energy times 10 (dark green); (b) depth-integrated relative vorticity advection (blue), planetary vorticity advection (magenta), bottom pressure torque (red), tendency (gold), and residual (black), with standard deviation error bars; and (c) expanded time series of 7-day low-pass filtered depth-integrated relative vorticity advection (blue) and bottom pressure torque (red) during selected eddy events.
Figure 4.8: Regions used to investigate the SOSE vorticity balance in Drake Passage and the Southeastern Pacific (gray filled boxes). Gray contours are the fronts [Orsi et al., 1995] as in Figure 4.1 and black outlined boxes are the red boxes from Figure 4.1, for reference.
Figure 4.9: Dominant terms of the SOSE vorticity balance in Drake Passage: (b) explicit dissipation (4.10), (c) the nonlinear term (4.8), and (f) the geostrophic divergence term (4.7); two contributors to the geostrophic divergence term: (d) planetary vorticity advection and (e) bottom pressure torque (4.13) with SOSE bathymetry contours (black, every 1000 m from 0 m to 4000 m); and (a) the residual of all SOSE vorticity balance terms (4.6)+(4.7)+(4.8)+(4.9)+(4.10)+(4.11).
Figure 4.10 (next page): Expanded view of terms in Figure 4.9 (a-f): (g) residual of nonlinear (c) and geostrophic divergence (f) terms. Terms of the 50-km SOSE mapped vorticity balance in Drake Passage: (h) relative vorticity advection + planetary vorticity advection − bpt, (i) mean relative vorticity advection, (j) eddy relative vorticity advection, (k) relative vorticity advection (i+j), (l) planetary vorticity advection, (m) -bpt, and (n) planetary vorticity advection - bpt. Note different colorbars for (a-g) and (h-n). Panels (d) and (l), and (e) and (m), are directly comparable. Panels (c) and (k), (f) and (n), and (g) and (h) are approximately comparable [(c) includes ageostrophic advection and \( w\zeta \), (f) includes ageostrophic \( fw_z \)].
Figure 4.11: As in Figure 4.9, for southeastern Pacific SOSE vorticity balance, with bathymetry contoured every 400 m from 5700 m to 2900 m.
Figure 4.12 (next page): As in Figure 4.10, for the southeastern Pacific SOSE vorticity balance and southeastern Pacific 200-km mapped SOSE vorticity balance terms (note different color scales for mapped and non-mapped terms), and bathymetry contoured every 400 m from 5700 m to 2900 m.
Figure 4.13: Relationship between SOSE 200-km mapped planetary vorticity advection, $\beta \langle v \rangle$, and mean relative vorticity advection, $\langle \vec{u} \cdot \nabla \langle \zeta \rangle \rangle$, in the south-eastern Pacific: (left) correlation $r$; (right) magnitude of regression coefficient $c_1$ ($\langle \vec{u} \cdot \nabla \langle \zeta \rangle \rangle = c_0 + c_1 \beta \langle v \rangle$). Only depths at which at least 70% of the mapping grid points are good are shown.
Chapter 5

Geography of internal waves in Drake Passage

Abstract. The distribution of internal wave energy in Drake Passage is investigated using fine-structure velocity, temperature, and salinity profiles from 282 CTD and lowered acoustic Doppler current profiler (ADCP) casts from five annual cruises, 12 years of twice-monthly upper-ocean shipboard ADCP transects, and 383 XCTD profiles distributed evenly throughout the year over eight years. Outside the surface layer, kinetic energy is higher than potential energy. Energy is elevated over the northern and southern continental slopes as well as north of the Polar Front. Energy does not decrease significantly with depth over moderately deep areas including the continental slopes, the Shackleton Fracture Zone (SFZ) ridge, and mid-passage seamounts. Rotary spectra and shear-strain ratios indicate dominant upward energy propagation and supra-inertial frequencies over the southern slope and the SFZ and seamounts. In deeper waters, energy decreases with depth and downward-propagating, near-inertial energy dominates. The seasonal cycle in internal wave energy in the surface layer is significant and varies with latitude; north of the Polar Front it is in phase with the seasonal cycle in upper ocean heat content, while south of the Polar Front it is aligned with the seasonal cycle in wind stress.
5.1 Introduction

The previous chapters have focused on the large- to meso-scale geostrophic circulation of the ACC, without discussing the mechanism of dissipation of the wind-input energy. Meanwhile, diapycnal mixing in the Southern Ocean is crucial both for local biological productivity, by supplying nutrients to the surface layer [Law et al., 2003], and for the global overturning circulation, by transforming intermediate [Sloyan and Rintoul, 2001; Sloyan et al., 2010] and deep [Ito and Marshall, 2008; Sloyan and Rintoul, 2000] water masses. Internal wave generation and breaking are important mechanisms for the transfer of energy from large- and meso-scales to small scales where it is available for turbulent dissipation and diapycnal mixing. In the global ocean, internal tides may provide a large fraction of the necessary mixing energy. However, in the ACC, where internal tides are small [Heywood et al., 2007], more internal wave energy may be found in wind-generated near-inertial waves and in higher-frequency internal lee waves generated by strong geostrophic flow over rough topography [Marshall and Naveira Garabato, 2008; Nikurashin and Ferrari, 2010a,b; Saenko et al., 2012]. The strong deep currents found in the ACC (see Chapter 4) allow for Southern Ocean lee wave generation [estimated at 0.1 TW, Nikurashin and Ferrari, 2011] to dominate the global map and form a significant mechanism for dissipation of the considerable energy input by the wind at the surface.

A number of authors have investigated mixing in the Southern Ocean, often using Garrett and Munk (GM)-like parameterizations [Garrett and Munk, 1975; Cairns and Williams, 1976; Gregg and Kunze, 1991] to estimate turbulent dissipation from vertical wavenumber spectra of finescale measurements of shear and strain; or from strain alone, combined with assumptions about the shear-strain ratio. The GM spectrum is separable in frequency and wavenumber and is dominated by near-inertial waves. Polzin and Lvov [2011], however, surveyed finestructure measurements with both depth and time resolution from a number of experiments in different regions and found geographic variations in spectral shapes and seasonal cycles. They showed how these deviations from the continuous, separable GM spectrum can arise from the importance of nonlinearity and wave-mean flow
Finestructure studies in the ACC paint a fairly consistent picture of downward-propagating internal wave energy in the upper ocean and strong upward-propagating internal wave energy over rough topography. Polzin and Firing [1997] examined 24 LADCP and CTD profiles over and north of the Kerguelen Plateau, finding depth-enhanced shear spectra, a predominance of upward-propagating energy, significant local variation, and depth-integrated (parameterized) dissipation approximately five times larger than the GM spectrum, all implying generation of lee waves by geostrophic flow over topography. Waterman et al. [2012a], from a later study over the Kerguelen Plateau, also found energy to be elevated in the top 1000 m, where polarization consistent with downward propagation and shear-strain ratio associated with near-inertial frequencies dominated; and within 1000 m over rough topography, where upward propagation and supra-inertial frequencies dominated. Naveira Garabato et al. [2004] examined similar data from 346 profiles from five cruises in the southeastern Pacific to the southwestern Atlantic. They found elevated (GM-based) diffusivity values within Drake Passage—up to \(10^{-1} \text{ m}^2 \text{s}^{-1}\) near the bottom—associated with upward internal wave energy propagation. In the upper ocean and in areas of smoother bathymetry, such as upstream of Drake Passage, they found smaller values, associated with downward energy propagation and consistent with the estimated wind energy flux to near-inertial waves. They also verified the connection between distributions of shear variance and strain variance. Studies relying on this connection include that of Sloyan [2005], who also found elevated mixing up to 1500 m over rough topography based on WOCE CTD data in the ACC. Thompson et al. [2007] examined three years of XCTD profiles in Drake Passage and found near-surface GM-based diffusivities of \(10^{-3} \text{ m}^2 \text{s}^{-1}\) north of the Polar Front and \(10^{-4} \text{ m}^2 \text{s}^{-1}\) south, corresponding with both spatial and seasonal patterns in wind stress. Wu et al. [2011] estimated turbulent dissipation from six years of Southern Ocean Argo temperature and salinity profiles and an assumed shear-strain ratio of 7, finding that over regions of smooth topography, GM-based dissipation between 300 and 1800 m totals \(1.8 \times 10^{-3} \text{ W m}^{-2}\), with diffusivity on the order of
10^{-5} \text{ m}^2 \text{s}^{-1}, and has a seasonal cycle extending to 1500 \text{ m} that corresponds with the seasonal wind stress cycle. Over rough topography, including in the Drake Passage, diffusivity is several times larger, increases with depth below 1200 \text{ m}, and has a smaller seasonal cycle. They estimated that the total dissipation rate from bottom-generated internal waves was comparable to wind energy flux.

Some studies have used other methods to estimate turbulent dissipation and/or diffusivity. Thompson et al. [2007] also used density overturning scales to estimate diffusivity, finding the same patterns but larger values than those implied by strain spectra. Ledwell et al. [2011] estimated $1.3 \times 10^{-5} \text{ m}^2 \text{s}^{-1}$ of diapycnal diffusivity near 1500 \text{ m} depth based on the dispersion over one year of a tracer upstream of Drake Passage; this value was approximately twice that derived from summertime measurements of finescale shear using the GM parameterization. In Drake Passage, St. Laurent et al. [2012] used measurements of velocity microstructure to estimate a diapycnal diffusivity of up to $10^{-4} \text{ m}^2 \text{s}^{-1}$ in the ACC fronts, and two orders of magnitude smaller outside the fronts. Waterman et al. [2012a] used both LADCP and CTD finestructure measurements and measurements of microstructure shear, temperature, and salinity from 54 profiles over the Kerguelen Plateau to calculate dissipation. From the microstructure shear they found average dissipation between 250 \text{ m} and 1500 \text{ m} of $1 \times 10^{-9} \text{ W kg}^{-1}$ (corresponding to diffusivity of $6.9 \times 10^{-5} \text{ m}^2 \text{s}^{-1}$); in the surface mixed layer dissipation was an order of magnitude larger, and within about 1000 \text{ m} of the bottom where topography was relatively rough or near-bottom flows strong it was about twice the interior value. They found that the spatial pattern of internal wave energy (discussed above) matched that of dissipation; however, while the observed dissipation in the upper ocean was consistent with the wind energy flux to near-inertial motions, the magnitude of dissipation implied by either the near-bottom internal wave field and the GM parameterization or an estimate of lee wave generation was several times larger than the observed near-bottom dissipation. Waterman et al. [2012b] investigated this discrepancy and suggested wave-mean interactions leading to nonlocal dissipation, as previously suggested by Polzin and Firing [1997] and Polzin and Lvov [2011].
The results of Waterman et al. [2012a] reinforce the general patterns obtained by various studies inferring turbulent dissipation and mixing from fine-structure measurements and GM-like parameterizations, although they cast doubt on the magnitudes derived from GM-like parameterizations. In this study we use finescale observations of shear and strain in Drake Passage, including a 12-year, approximately twice-monthly time series of upper-ocean currents, an 8-year, six-times-yearly time series of upper-ocean stratification, and a set of 282 full-depth current, temperature, and salinity profiles, to compute spectra of velocity, shear, displacement, and strain. We examine the geographic and temporal distributions of internal wave energy, and use the ratio between shear and strain variance, and the ratio between positive and negative polarization from rotary spectra of velocity, to determine the dominant frequency and direction of energy propagation in various depth ranges and regions. This work is preliminary to an investigation of parameterized dissipation in Drake Passage.

5.2 Data

As part of the cDrake project, lowered ADCP (LADCP) and conductivity-temperature-depth (CTD) casts were taken at 41 sites (Figure 5.1) in Drake Passage on each of the five yearly cDrake cruises in November-December of 2007 through 2011, as well as at 30 additional sites on only one or two of the cruises. Casts were performed to within 20 m of the bottom when possible, with a short bottom stop (no bottom soak for bottles). A total of 282 joint LADCP and CTD casts covering at least 700 m of depth range are available.

The CTD was a SeaBird SBE 11-Plus. Up to six bottle samples were taken per cast to verify salinity, and calibrations were performed pre- and post-cruise for each cruise. CTD data were quality-controlled, despiked, and averaged to 2 dbar vertical resolution for both the up- and down-casts. Salinity and temperature profiles were low-pass filtered using a Butterworth filter with a 5-dbar cutoff.

The LADCP, an RD Instruments 153.6 kHz broadband with a 30° beam angle, sampled with a 16-m vertical bin (∆z_r), pulse (∆z_t), and blank before
transmit. Data were processed using the CODAS software from Eric Firing’s group at the University of Hawaii, which implements the method of Fischer and Visbeck [1993]. Instrument-relative measurements of water velocity are differenced to obtain vertical shear profiles, which are bin-averaged into 5-m depth bins, sub-sampled to 20-m \( (\Delta z_g) \), and integrated to obtain the baroclinic velocity profile relative to the surface. The cast-integrated LADCP-relative velocity is added to the cast-averaged ship velocity to obtain the barotropic velocity. Typical errors in the baroclinic (barotropic) component with this instrument, sampling scheme, and processing method are \( 4 \text{ cm s}^{-1} \) \( (1 \text{ cm s}^{-1}) \) [Chereskin et al., 2010].

Shipboard acoustic Doppler current profiler (SADCP) data are collected on approximately 20 yearly Drake Passage crossings by the ARSV Laurence M. Gould (Figure 5.1), made in all seasons. Direct current measurements have been collected by an RDI 150 kHz narrowband ADCP since 1999 and additionally by an RDI 38 kHz phased array ADCP since 2004. Both sample in narrowband mode, averaging over 5 minutes. The 150 kHz instrument (nb150) uses a pulse length of \( \Delta z_t = 8 \text{ m} \) (16 m up to November 2004), bin length of \( \Delta z_r = 8 \text{ m} \), and blank before transmit of 16 m. The first sampling bin for data up to November 2004 is at 26 m, and data from November 2004 onwards are interpolated to the same vertical grid \( (\Delta z_g = 8 \text{ m}) \); good data are returned as deep as 298 m approximately half the time. The 38 kHz instrument (os38) has a pulse length of \( \Delta z_t = 24 \text{ m} \), bin length of \( \Delta z_r = 24 \text{ m} \), and blank before transmit of 16 m. The first bin is centered at 46 m; good data are returned as deep as 1030 m approximately half the time. Data processing and quality control are described in Lenn et al. [2007] and Firing et al. [2011]. There are 6905 hourly-averaged profiles covering at least 600 m from the os38 between November 2004 and October 2012; from the nb150 there are 9449 hourly-averaged profiles covering at least 240 m between September 1999 and October 2012.

Expendable conductivity-temperature-depth (XCTD) data are collected on six LMG Drake Passage crossings per year under the Scripps High Resolution XBT/XCTD program (www-hrx.ucsd.edu). Most XCTD profiles extend to approximately 1000 m, with approximately 0.1-m resolution. Three hundred eighty-
three XCTD profiles between December 2001 and October 2009 have been quality-controlled and processed by removing the spectral spikes following Gille et al. [2009]. Processed XCTD data were provided by J. Sprintall. Profiles were bin-averaged to 2 dbar vertical resolution and low-pass filtered using a Butterworth filter with 5-bar cutoff.

We divided velocity and hydrography profiles into topographic regions using high-resolution multibeam bathymetry data collected on the five cDrake cruises, as well as the bathymetric database of Smith and Sandwell [1997]. The latitudinally-varying seasonal cycle in wind speed in Drake Passage was computed based on two years of QuikScat satellite scatterometer winds and two years of ASCAT satellite scatterometer winds (see Chapter 4).

### 5.3 Methods

We wish to examine kinetic and potential energy and polarization (the ratio between counterclockwise and clockwise rotation), as well as the shear-strain ratio and corresponding dominant frequency, in various depth bins. We use the LADCP and SADCP data to compute vertical wavenumber spectra of currents and shear, and the CTD and XCTD data to compute vertical wavenumber spectra of displacement and strain, in various depth ranges and distance-above-local-bottom ranges.

We compute vertical wavenumber spectra using half-overlapping windows of length $N_{\text{win}}$, where $N_{\text{win}}$ covers 600 m for the LADCP and os38 currents and CTD- or XCTD-derived strain intended to be compared with the LADCP or os38 data, and 240 m for the nb150 data and XCTD-derived strain intended to be compared with the nb150 data. Only continuous runs of $N_{\text{win}}$ good data points are used. Each set of $N_{\text{win}}$ points is detrended and a Hanning window $W$ is applied before computing the fast Fourier transform (FFT), $F$. The power spectral density estimate is then computed from the positive wavenumber coefficients:

$$S_i(k_{z,i}) = \frac{2|F_i|^2\Delta z}{\sum_i W_i^2}, \quad k_{z,i} = \frac{i}{N_{\text{win}}\Delta z}, \quad i = 1, 2, ..., \frac{N_{\text{win}}}{2},$$  \hspace{1cm} (5.1)
where $\Delta z$ is the vertical resolution. Spectra from the half-overlapping windows within selected depth ranges are averaged together.

Up- and down-cast LADCP and CTD data are treated separately until after the FFT computation, when their spectra are averaged together. At most sites each half-cast covers about 1.5 hours. SADCP profiles are averaged from their original 15-minute resolution to hourly; SADCP spectra are then further averaged to 3-hourly resolution. Pre-averaging the SADCP data to hourly (in addition to reducing computation time) decreases the average energy by a factor of two (at the low-wavenumber end) to four (at the high-wavenumber end), such that the velocity spectra from hourly data are redder, more like the LADCP spectra.

### 5.3.1 Velocity and shear spectra and rotary spectra

To improve the spectral estimates for both velocity and shear, we prewhiten by computing FFTs of velocity first-differences, $u_{i+1} = u_{i+1} - u_i, v_{i+1} = v_{i+1} - v_i$. Squaring and normalizing the FFTs to obtain $S_{u1u1}, S_{v1v1}$, we then convert to velocity spectra

$$(S_{uu}, S_{vv})(k_z) = (S_{u1u1}, S_{v1v1})(k_z) (|1 - e^{-2\pi i k_z \Delta z}|)^{-2}, \quad (5.2)$$

and shear spectra

$$(S_{uz, uz}, S_{vz, vz})(k_z) = (S_{u1u1}, S_{v1v1})(k_z) \Delta z^{-2}. \quad (5.3)$$

LADCP spectra (both velocity and shear) are corrected for smoothing due to range averaging, finite differencing, and interpolation (the correction for tilting is neglected as it is expected to be small), following Polzin et al. [2002]:

$$S_{\text{corrected}} = \frac{S}{\text{sinc}^2(k_z \Delta z_t) \text{sinc}^8(k_z \Delta z_r) \text{sinc}^2(k_z \Delta z_g)}, \quad (5.4)$$

where $\Delta z_t = 16 \text{ m}, \Delta z_r = 16 \text{ m}, \Delta z_g = 20 \text{ m}$ are the sampling pulse, bin, and grid spacing, respectively. SADCP spectra from the nb150 from November 2004 onwards are corrected for smoothing due to range averaging and interpolation (again neglecting the tilting correction):

$$S_{\text{corrected}} = \frac{S}{\text{sinc}^2(k_z \Delta z_t) \text{sinc}^6(k_z \Delta z_r) \text{sinc}^2(k_z \Delta z_g)}, \quad (5.5)$$
where $\Delta z_t, \Delta z_r$, and $\Delta z_g$ are as given in Section 5.2. SADCP spectra from the os38 and nb150 before November 2004 are corrected for smoothing due to range averaging:

$$S_{\text{corrected}} = \frac{S}{\text{sinc}^2(k_z \Delta z_t) \text{sinc}^2(k_z \Delta z_r)}, \quad (5.6)$$

where $\Delta z_t$ and $\Delta z_r$ are as given in Section 5.2 (these data are not interpolated, so no correction involving $\Delta z_g$ is necessary).

Rotary spectra of velocity are computed from the velocity component spectra and cross-spectra:

$$R_{\text{pos}} = \frac{S_{uu} + S_{vv}}{2} - \text{Im}(S_{uv}), \quad (5.7)$$

$$R_{\text{neg}} = \frac{S_{uu} + S_{vv}}{2} + \text{Im}(S_{uv}), \quad (5.8)$$

where $S_{uu}$ is the autospectrum of zonal velocity $u$, $S_{vv}$ the autospectrum of meridional velocity $v$, $S_{uv}$ the cross spectrum, $R_{\text{pos}}$ indicates counterclockwise rotation with increasing depth, and $R_{\text{neg}}$ clockwise rotation with increasing depth. In the Southern Hemisphere, counterclockwise rotation is associated with downward internal wave group velocity (and energy propagation), and clockwise rotation with upward group velocity (and energy propagation). We will use the polarization ratio, $R_{\text{pol}} = R_{\text{pos}}/R_{\text{neg}}$, as a measure of the dominance of downward-propagating ($R_{\text{pol}} > 1$) or upward-propagating ($R_{\text{pol}} < 1$) energy. To determine the polarization ratio corresponding to an average over multiple profiles or depth ranges, we first determine the average counterclockwise and clockwise energy by averaging $R_{\text{pos}}$ and $R_{\text{neg}}$, then take the ratio $R_{\text{pol}} = \langle R_{\text{pos}} \rangle / \langle R_{\text{neg}} \rangle$.

### 5.3.2 Displacement and strain and their spectra

Following Kunze et al. [2006], we estimate strain

$$\xi_z(z) = \frac{N^2(z) - N_q^2(z)}{N_q^2}, \quad (5.9)$$

where $N^2(z)$ is the buoyancy frequency (computed using the Seawater toolbox for MATLAB [Morgan, 1994]), $N_q^2(z)$ is a quadratic fit to $N^2(z)$ in each profile segment, and $N_q^2$ is the mean of $N_q^2(z)$ in each profile segment. For CTD data
we compute $N_q^2(z)$ from the cast-averaged $N^2(z)$. For both CTD and XCTD data $N^2 < 0$ were excluded from the quadratic fit, and segments with $\bar{N}_q^2 < 10^{-7}$ rad$^2$ s$^{-2}$ were excluded from the spectral calculations. Strain spectra are corrected for first-differencing (in the computation of $N^2$) by

$$S_{corrected} = \frac{S}{\text{sinc}^2(k_z \Delta z)}, \quad (5.10)$$

where $\Delta z = 5$ m is the vertical resolution used to compute $N^2$. We convert strain spectra $S_{\xi_z\xi_z}$ to displacement spectra $S_{\xi\xi}$ by

$$S_{\xi\xi} = \frac{S_{\xi_z\xi_z}}{(2\pi k_z)^2}. \quad (5.11)$$

### 5.3.3 Integrating spectra to compute variances and shear and strain ratio

We compute kinetic energy density spectra from the velocity spectra, $(S_{uu} + S_{vv})/2$, and potential energy density spectra from the displacement spectra and stratification: $S_{\xi\xi} \bar{N}_q^2/2$. (From here on we will refer to the kinetic/potential energy per unit mass as kinetic/potential energy.) Where there are multiple $N_{win}$ segments in a given depth range, the segment-$\bar{N}_q^2$ are averaged to produce one $\bar{N}_q^2$ per depth range per profile. To obtain total kinetic or potential energy ($KE = \frac{1}{2} \langle u^2 \rangle + \langle v^2 \rangle$, $PE = \frac{1}{2} \bar{N}_q^2 \langle \xi^2 \rangle$) we integrate over the resolved spectral range to obtain velocity or displacement variances. We also integrate shear and strain spectra to shear variance $\langle u_z^2 \rangle$ and strain variance $\langle \xi_z^2 \rangle$, and use these to calculate the normalized shear:strain ratio,

$$R_\omega = \frac{\langle u_z^2 \rangle}{\bar{N}_q^2 \langle \xi_z^2 \rangle}, \quad (5.12)$$

where $\langle u_z^2 \rangle = \int (S_{u_z u_z} + S_{v_z v_z}) dk$ is the total shear variance and $\bar{N}_q^2$ is again the depth-range mean stratification. Following Polzin [1995], the dominant frequency $\omega$ is then given by

$$\omega^2 = \frac{1}{2} \left( \bar{N}_q^2 (1 - R_\omega) - f^2 + \sqrt{\bar{N}_q^2 (R_\omega - 1)^2 + 2 \bar{N}_q^2 f^2 (1 + 3 R_\omega) + f^4} \right), \quad (5.13)$$

where $f$ is the Coriolis frequency. As with the polarization ratio, shear-strain ratios (and associated dominant frequencies) corresponding to averages over multiple
profiles and/or segments are computed by first determining $\langle u^2 \rangle$, $\langle \bar{N}^2 \rangle$, and $\langle \zeta^2 \rangle$, then taking the ratio. The distributions of individual-profile $R_\omega$ and $\omega$ in various profile groups are also examined and are consistent with the results of this method.

We chose upper wavenumber integration limits based on the spectra (Figures 5.2 and 5.3). The noise floor for os38 spectra is around 90-m wavelength, and for nb150 spectra around 40-m wavelength. Noise floors for CTD and XCTD-derived spectra begin at much higher wavenumbers; however, for comparing velocity-derived and stratification-derived quantities it is important to use the same limits of integration, so we use the ADCP-based upper limits of integration for both. Similarly, it is useful to be able to compare os38- and LADCP-derived quantities in their common depth ranges, so even though the LADCP noise floor is at slightly higher wavenumber (70-m wavelength) we use $k_z = 1/90$ m$^{-1}$ for the upper limit of integration. We chose lower wavenumber cutoffs, corresponding to 320-m wavelength for spectra from 600-m segments (from LADCP, CTD, os38, and XCTD) and to 200-m wavelength for spectra from 240-m segments (from nb150 and XCTD), in an attempt to capture as much of the well-resolved portion of the spectra as possible.

### 5.4 Spectra

The average spectra of kinetic and potential energy (Figure 5.2) from LADCP and CTD data are of comparable magnitude and slope. The current spectra reach an instrument- and sampling-based noise floor around $k_z = 1/70$ m, while the CTD-derived displacement spectra do not become white until above 30-m wavelength. Shear spectra are close to white.

Energy, kinetic and potential, drops off by about half an order of magnitude between the 0 to 1400 m depth bin and the 3000 to 4400 m depth bin, at all wavelengths (above the noise floor). The polarization ratio ($R_{pol}$, the ratio of counterclockwise to clockwise rotation, or downward to upward energy, see Section 5.3), is close to one overall and above 2400 m, and slightly less than one between 2000 and 4400 m. This is consistent with dominant upward propagation at the deeper
depths, and a mix of directions in the shallower depths.

The SADCP and XCTD spectra parallel the LADCP and CTD spectra in both the 300 to 1000 m and 0 to 1400 m depth ranges (Figure 5.3); although differences between average spectra from the different sources are significant (note the very small confidence intervals indicated on Figure 5.3), they are small, less than a tenth of an order of magnitude at most wavenumbers. The nb150 spectra between the surface and 300 m also line up fairly closely with the deeper os38 and LADCP spectra. The nb150 data show a tendency for positive polarization (dominant downward propagation) in the 0 to 300 m layer which is not visible below 300 m. The XCTD-derived potential energy spectrum, meanwhile, shows an order of magnitude more energy in the surface layer than between 300 and 1000 m.

5.5 Patterns of internal wave energy

To investigate how internal wave energy varies in space and time, we integrated the spectra between the wavelengths marked in Figure 5.3 (see Section 5.3 for explanation of the limits of integration). Integrated kinetic and potential energy and polarization show the same features as the spectra (Figures 5.2 and 5.3). Mean integrated kinetic energy from the LADCP current spectra is approximately a factor of three larger than integrated potential energy from the CTD displacement spectra and stratification (Figure 5.4), in all depth ranges and distance above bottom ranges. Both kinetic and potential energy decrease by approximately a factor of five with depth. As is clear from Figure 5.2, polarization is insignificantly different from one except between 2000 and 4400 m, where it is negative, indicating dominant upward propagation of internal wave energy. The shear-strain ratio is between 3 and 4 at all depths. Near the surface it is very close to the GM value of 3, and the dominant frequency is approximately $1.7 \times 10^{-4}$ rad s$^{-1}$. In deeper depth ranges $R_\omega$ is slightly larger than 3, and the dominant frequency is $1.35 \times 10^{-4}$ to $1.5 \times 10^{-4}$ rad s$^{-1}$ (compare to $f = 1.25 \times 10^{-4}$ rad s$^{-1}$ at 59°S).

Kinetic energy from the LADCP and os38 spectra is elevated, by about
half an order of magnitude, over the southern slope and around and north of the mean position of the Polar Front (PF, approximately 58°S, Figure 5.5), and by even more over the northern slope. The enhanced values over the northern slope are almost an order of magnitude larger than those over the southern slope, and are found over a larger depth range. Kinetic energy in the top 300 m, from the nb150 spectra, is also elevated over the northern slope, but is otherwise higher south of the PF than north. Polarization ratios of the average LADCP spectra are not significantly different from one, with a few exceptions (Figure 5.6); for all of these exceptions $R_{pol}$ is around 0.8, indicating a slight dominance of upward-propagating energy. Polarization from the os38 in both the 300 to 1000 m and 0 to 1400 m depth ranges indicates that on average over the nearly-7-year time series there is about 20% more upward-propagating energy at 59°S and about 20% more downward-propagating energy at the northernmost bin around 55.5°S. The longer nb150 time series shows significant counterclockwise/downward polarization between 0 and 300 m at 60.5°S and 55.5°S and clockwise/upward polarization at 56.25°. The large range of values shown in Figures 5.5 and 5.6 indicate that the energy level and polarization in a given location are highly variable in time; even statistically significant results may be likely to change with more sampling.

To attempt to capture the effect of bathymetric features such as the Shackleton Fracture Zone (SFZ), the 2500 m ridge that runs diagonally across Drake Passage, we divided profiles into the five regions shown in Figure 5.1, and plotted kinetic and potential energy as functions of depth range in each region (Figure 5.7).

Over the northern slope (N), energy is nearly constant with depth; interestingly, kinetic energy is slightly lower in the 300 to 1000 m depth range and the 1000 to 2400 m distance above bottom (mab) range, while potential energy is lower in the 300 to 1000 m depth range only. There is no significant polarization in this region from the LADCP data, and $R_\omega$ is approximately three.

The depth-dependence over the southern slope (S) is similar, except that kinetic energy is elevated (significantly) in the 1000 to 2400 mab range, while potential energy is reduced (not quite significantly). Here polarization is negative in all ranges except the 1000 to 2400 m range (where error bars are particularly
large), indicating that upward propagation predominates even near the surface. The shear-strain ratio is close to three except in the bottom to 1400 mab range, where it is closer to two, consistent with supra-inertial waves with frequencies around $2.1 \times 10^{-4}$ rad s$^{-1}$.

In the mid-passage shallower areas (C, bottom depth < 3100 m, including the SFZ and seamounts), energy is highest above 1000 m and lowest in the bottom depth ranges, 2000 to 3400 m and 1000 to 2400 mab. This is contrary to the elevation of energy over topography that we might expect. Polarization does indicate (insignificant) dominance of upward propagation at all depths. Shear-strain ratios are greater than or equal to three above 2400 m, and slightly less than three in the deepest depth range.

The magnitudes of kinetic and potential energy in the deep regions upstream (W) and downstream (E) of the SFZ are very similar, with slightly more energy downstream. In both regions energy decreases with depth, by a factor of approximately three; the decrease continues even into the distance-above-bottom ranges. A decrease in energy with depth might suggest a predominance of surface-source, downward-propagating energy. In the upstream region overall polarization is (insignificantly) negative; however, in the downstream region polarization is positive, significantly so above 1400 m, consistent with this hypothesis. In both regions shear-strain ratios are four to eight.

The long time series of SADCP and XCTD data in all seasons allow us to examine the time evolution of internal wave energy. Both kinetic and potential energy show seasonal cycles (Figure 5.8) varying with latitude and depth range. A seasonal cycle is significant if its peak and trough are significantly different, based on the 95% confidence intervals (not shown) of the average spectra from each month and latitude bin. In the northernmost latitude bin (north of 55.75°S), the monthly climatology of kinetic energy shows a significant cycle with two peaks, in January and in September, both above and below 300 m. Not enough XCTD data were available to estimate potential energy in this latitude bin. South of 55.75°S and north of approximately 58.5°S, kinetic and potential energies peak in March-April, and are minimal in July-September. The kinetic and total energy seasonal cycles
are significant both above and below 300 m; potential energy seasonal cycles are significant only above 300 m. South of 58.5°S the seasonal cycle becomes smaller. The only significant cycle in this latitude range is found above 300 m, where energy peaks around May-July and has a minimum around December-February.

Winds in Drake Passage are strongest and most variable April through August, and weakest in January to February; the seasonal cycle is stronger in the southern part of the passage than in the northern part, although mean wind speeds are stronger in the north [based on satellite wind data; see also Gille, 2005; Thompson et al., 2007]. Although average internal wave energy is larger in winter than in summer both north and south of the PF (Figure 5.8), consistent with the wind forcing and with the findings of Thompson et al. [2007], only south of the PF do the 12-monthly cycles in internal wave energy and wind stress align. The internal wave energy seasonal cycle in the northern part of the passage aligns with the expected seasonal cycle in upper-ocean stratification (Stephenson et al. [2012] showed an upper-ocean heat content maximum in March-April and minimum in August-October north of the Polar Front); stronger late-summer thermoclines may allow increased transfer of surface-input near-inertial energy from the mixed layer to the thermocline.

No significant secular trend is visible in either kinetic or potential energy. Interannual variability is also insignificant and does not appear to be correlated with climate indices such as the Southern Oscillation Index or the Southern Annular Mode index.

5.6 Summary

Internal wave energy in Drake Passage is strongest over the continental slopes and in the northern part of the passage, around and north of the Polar Front. The enhancement of internal wave energy in the northern half of the passage, where eddy kinetic energy is also higher than in the southern half of the passage, is consistent with the general tendency for positive correlation between eddy kinetic energy and dissipation (based on internal wave energy) from global
Argo profiles found by Whalen et al. [2012]. Over the northern slope, upward and downward propagation appear to be balanced, and energy is near-constant with depth. Over the southern slope, in contrast, significant negative polarization and shear-strain ratio less than three are both consistent with bottom-generated supra-inertial waves. Although in shallower areas in the mid-passage (including the SFZ area) upward propagation also dominates, energy is enhanced near the surface rather than near the bottom. In areas of the passage away from topography, the decrease of energy with depth, neutral or positive polarization, and shear-strain ratios greater than three are all consistent with a predominance of surface-generated near-inertial waves. Surface-layer internal wave energy in the northern part of the passage exhibits a significant seasonal cycle which is aligned with the seasonal cycle in upper ocean heat content [Stephenson et al., 2012] and stratification.

We intend to build on this description of the distribution of internal wave energy in Drake Passage to investigate time and space patterns of internal wave generation and dissipation in Drake Passage. We will further investigate the space- and time-dependences of the shear-strain ratio, compare the observed spectra with the canonical Garret and Munk (GM) spectra, and use the GM parameterizations to estimate dissipation from the available shear and strain data as well as from shear data alone (making use of the extensive SADCP time series) and strain data alone (as is often done when current data are not available). We will also estimate the lee wave generation by flow impinging on topography using the cDrake bottom current time series and high-resolution multibeam bathymetry (see Chapters 3 and 4), and relate it to the observed internal wave levels.

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Figure 5.1: Locations of SADCP profiles (averaged over 3 hours, dots) and LADCP/CTD profiles (open circles). Colors indicate region: northern (dark brown) and southern (light brown) slope areas shallower than 2000 m, central areas shallower than 3100 m (dark green, includes SFZ and seamounts), and deeper areas upstream (light green) and downstream (magenta) of the SFZ.
Figure 5.2: Average spectra of kinetic (KE) and potential (PE) energy, shear, and polarization ratio (> 1 indicates predominance of counterclockwise rotation with depth) from LADCP and CTD, in different depth ranges. 95% confidence intervals are given by vertical bars between \( k_z = 3 \times 10^{-3} \) and \( k_z = 8 \times 10^{-3} \) (barely visible except for polarization). Vertical dashed lines indicate integration intervals.
Figure 5.3: Average spectra of kinetic and potential energy, shear, and polarization from LADCP/CTD (“L”, also shown in Figure 5.2, PE from CTD) and SADCP/XCTD (“os38”, “nb150”, PE from XCTD), in different depth ranges. 95% confidence intervals are given by vertical bars between $k_z = 3 \times 10^{-3}$ and $k_z = 9 \times 10^{-3}$ (barely visible except for polarization). Vertical dashed lines indicate integration intervals for LADCP and os38 (black) and nb150 (gray).
Figure 5.4: Left: kinetic (blue, squares) and potential (red, circles) energy; right: polarization ratio $R_{pol}$ (teal, crosses, $>1$ indicates a predominance of counterclockwise rotation with depth) and shear-strain ratio $R_\omega$ (purple, triangles), from LADCP and CTD spectra. The horizontal axis location of each thick vertical line indicates the value of energy (or ratio) from the average spectrum (or spectra) in the depth range (top panels) or distance above bottom range (bottom panels) corresponding to the line’s vertical axis range. Light gray lines indicate 95% confidence intervals.
Figure 5.5: Kinetic energy as a function of latitude, in indicated depth ranges (mab stands for meters above bottom), from SADCP (light gray dots) and LADCP (dark gray dots), with latitude-bin-averaged values from SADCP (circles) and LADCP (squares) and corresponding 95% confidence intervals (vertical lines, based on the total number of degrees of freedom of the average spectrum in each latitude bin). Note that in the 0-300 m depth range KE is integrated over $k_z = 1/200 \text{ m}^{-1}$ to $k_z = 1/40 \text{ m}^{-1}$, while in other depth ranges KE is integrated over $k_z = 1/320 \text{ m}^{-1}$ to $k_z = 1/90 \text{ m}^{-1}$ (see Figure 5.3 and Section 5.3.3).
Figure 5.6: Polarization ratio $R_{pol}$ as a function of latitude, in indicated depth ranges (mab stands for meters above bottom), from SADCP (light gray dots) and LADCP (dark gray dots), with latitude-bin-averaged (see Section 5.3) values from SADCP (circles) and LADCP (squares) and corresponding 95% confidence intervals (vertical lines, see Figure 5.5 caption). A ratio $> 1$ indicates a predominance of counterclockwise rotation with depth, associated with downward energy propagation.
Figure 5.7 (next page): As in Figure 5.4, for spectra from regions indicated by color in Figure 5.1.
Figure 5.8: Climatologies of kinetic (KE), potential (PE), and total (E) energy as functions of cross-passage distance and month, from 0:300 m (nb150 and XCTD, top), 300:1000 m (os38 and XCTD, middle), and 0:1400 m (os38 and XCTD, bottom).
Chapter 6

Summary

This thesis has attempted to shed light on the dynamics of the Antarctic Circumpolar Current based primarily on observations in Drake Passage. The principal sources of observations were a 4.5-year time series of all-season direct velocity measurements in the upper 1000 m, a 12-year time series of all-season direct velocity measurements in the upper 250 m, and an 8-year time series of all-season XCTD profiles, all from the ARSV Laurence M. Gould (LMG); and the 4-year cDrake experiment, which provided measurements of bottom pressure, near-bottom current, and bottom-surface round-trip sound travel time, as well as over 280 LADCP and CTD profiles. The Southern Ocean State Estimate (SOSE) was also used to provide context to the observations.

In Chapter 2, direct current measurements from a shipboard acoustic Doppler current profiler (SADCP) exhibit a slowly-decreasing vertical structure, consistent with equivalent-barotropic (self-similar with depth) flow in the jets. The mean currents decay slowly with depth over the top 1000 m, with vertical length scales varying laterally and being particularly long between the jets. Mean transport in the top 1000 m is $95 \pm 2$ Sv, and full-depth transport is likely between 117 Sv and 220 Sv. SOSE, which reproduces the observed transport in the top 1000 m, has a full-depth transport in the middle of this range.

Both barotropic and baroclinic contributions to transport variability are significant. The peak in eddy kinetic energy (EKE) in the Polar Frontal Zone (PFZ) decreases more quickly with depth than the mean flow, possibly indicating a
barrier role for the jets in Drake Passage. Eddy momentum fluxes in the top 1000 m are consistently out of the PF and into the SAF. Vertical wavenumber spectra at intermediate wavelengths are consistent with upward energy propagation between 500 m and 1000 m.

Methods for computation of geostrophic barotropic and baroclinic currents, vorticity, and vorticity gradients from the CPIES data and a GEM, using objective mapping, were described in Chapter 3. The current fields were validated by comparison with independent measurements from current meter moorings. Comparisons to LADCP current profiles and satellite-derived surface currents also helped confirm the objective mapping and GEM method. Simulations were used to verify the objective mapping procedures for higher derivatives of streamfunction.

In Chapter 4, the fields produced by the methods of Chapter 3 were examined and used to compute terms of the quasi-geostrophic vorticity balance. In the cDrake LDA, located in the PFZ, the full-depth mean current is not equivalent-barotropic, possibly reflecting the influence of the Shackleton Fracture Zone just upstream. Near-bottom current measurements and high-resolution multibeam bathymetry enabled calculation of bottom pressure torque across Drake Passage, giving an average vorticity forcing integrated across the passage of $3 \times 10^{-3} \text{ m}^2\text{s}^{-2}$. If applied over the 600-800 km length of the Drake Passage-Scotia region, this forcing is large enough to balance the majority of the ACC wind stress curl forcing ($4.4 \times 10^3 \text{ m}^3\text{s}^{-2}$).

The vorticity balance in the highly variable and high-$Ro$ PFZ is dominated by nonlinear advection and bottom pressure torque. In the center of the LDA, time series of nonlinear advection and bottom pressure torque are negatively correlated, although the nonlinear advection signal is significantly larger. Both mean and eddy components of the relative vorticity advection contribute significantly to the first-order balance. The residual of terms directly computable by geostrophic objective mapping is also first order. Ageostrophic (cyclostrophic) relative vorticity advection and sub-grid-scale dissipation are possible balancing terms; however, their estimated errors are too large to provide confidence in the role of either or both of these terms in the vorticity balance.
The vorticity balance in SOSE is dominated by nonlinear advection, vortex stretching, and sub-grid-scale dissipation, providing some evidence in favor of sub-grid-scale dissipation as an explanation for the residual of nonlinear advection and bottom pressure torque in cDrake. The SOSE stretching term is the residual of bottom pressure torque and the integrated ageostrophic divergence. The quasi-geostrophic vorticity balance terms from objective mapping of SOSE in Drake Passage are consistent with the cDrake LDA vorticity balance. The SOSE vorticity balance in a topographically smoother region of the southeastern Pacific features larger dominant spatial scales, and at scales of 150 km and longer is dominated by a Rossby-wave-like balance between mean relative vorticity advection and planetary vorticity advection, as found by Hughes [2005] and Chereskin et al. [2010].

Finescale observations of currents and stratification from both the LMG and cDrake were used to examine the distribution and characteristics of internal wave energy in Drake Passage in Chapter 5. Internal wave energy is elevated over the continental slopes and in the PFZ and SAF. Upward- and downward-propagating energy components are balanced over the northern slope, while over the southern slope and the mid-passage SFZ and seamounts upward propagation dominates. In deeper areas, energy decreases with depth and downward propagation dominates. A seasonal cycle in upper-ocean internal wave energy is aligned with the annual surface-layer heat content cycle north of the PF as described by Stephenson et al. [2012].

The work presented in this thesis leads to a number of related questions of interest for future investigations. More study of the time and space scales of the differences between the LMG SADCP observations and SOSE might illuminate the extent to which SOSE does or does not successfully reproduce different instability processes. The finescale observations can be used to estimate diffusivity and dissipation according to the GM parameterization; the combination of shear and strain observations also allows investigation of the robustness of estimates using only shear or (more commonly) only strain and relying on a constant shear-strain ratio in this region. The generation of internal lee waves can be estimated from the high-resolution cDrake bathymetry data and 4-year time series of near-bottom...
current observations, and compared to the internal wave energy observed in the same area.
References


