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EFFECT OF VECTOR AND TENSOR FORCES IN THE EXCITATION OF 
$f_{7/2}^{-d_{3/2}}$ STATES IN THE $^{40}$Ca(p,p$'$$^*$) reaction†

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ABSTRACT

The results of microscopic distorted wave approximation calculations
for $f_{7/2}^{-d_{3/2}}$ excitations in the $^{40}$Ca(p,p$'$)$^{40}$Ca$^*$ reaction are presented. A
"realistic" effective interaction with central, tensor, and spin-orbit com-
ponents is considered. Some evidence for the tensor force is found in the
data from a recent experiment.

The $f_{7/2}^{-d_{3/2}}$ configuration in $^{40}$Ca gives rise to 8 states $J = 2^-$,
$3^-, 4^-$, and $5^-$ with $T = 0$ and 1. The excitation of these levels by inelastic
proton scattering is quite interesting in that it provides a test of the four
independent components of the effective interaction which appears in the

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microscopic model for this reaction [1-4]. These states were excited in a recent experiment [5] using protons with energies between 25 and 40 MeV. The data were analyzed in calculations with a central effective interaction and some discrepancies were noted. We present here the results of calculations with a more complete interaction which includes central, spin-orbit [6,7] and tensor [8] components. In addition to seeing if the results for $^{40}\text{Ca}$ can be improved, we wish to gain some estimate as to just how reasonable are the interaction parameters used in other recent calculations [7,9].

The calculations have been made with the computer code DWBA70 [10] which not only allows the inclusion of vector and tensor forces, but also allows an exact treatment of "knock-on" exchange [1-9]. With this code we are restricted to Yukawa radial forms for the central and spin-orbit components of the interaction and $r^2x$ Yukawa radial forms for the tensor components. We write the interaction

$$t(1,2) = V^S_0(r_{12}) + V^S_1(r_{12})\vec{S}_1 \cdot \vec{S}_2 + V^T(r_{12})S_{12} + V^V(r_{12})\vec{L}_{12} \cdot \vec{S}_{12}$$

where $S$, $V$, and $T$ denote scalar, vector, and tensor, respectively, and

$$S_{12} = 3(\vec{S}_1 \cdot \vec{r}_{12})(\vec{S}_2 \cdot \vec{r}_{12}) - \vec{S}_1 \cdot \vec{S}_2$$

$$\vec{L}_{12} = (\vec{r}_1 - \vec{r}_2) \times (\vec{p}_1 - \vec{p}_2)$$

$$\vec{S}_{12} = \vec{S}_1 + \vec{S}_2$$

For the central part of the force we take

$$V_k^S(r_{12}) = (V^S_{k0} + V^S_{k1}\vec{r}_1 \cdot \vec{r}_2) e^{-r_{12}/\mu}$$

with $V^S_{00} = -15.0$ MeV, $V^S_{10} = 3.6$ MeV, $V^S_{01} = 6.4$ MeV, $V^S_{11} = 5.0$ MeV, and $\mu = 1.42$ fm. This is an even state force which gives a rough reproduction of the cross sections obtained with the G matrix interactions proposed in refs. 3,4. The parameters for
the tensor part of the force have been matched to the low momentum components of the tensor part of the Hamada–Johnston (HJ) potential [11,12] set to zero inside 1 fm. The resulting interaction is quite similar to the tensor part of the OPEP potential [12]. With

\[ V^T(r_{12}) = (V^T_0 + V^T_1 \tau_1 \cdot \tau_2) r_{12}^2 \frac{1}{r_{12}^2/\mu}, \]

we have \( V^T_0 = -0.33 \) MeV · fm\(^{-2} \), \( V^T_1 = 7.07 \) MeV · fm\(^{-2} \), and \( \mu = 0.857 \) fm. The central and tensor interactions given here are quite similar to those used in the calculations of ref. 9. The spin-orbit interaction we have used is the same as that of ref. 9, so it is not necessary to repeat the parameters here. The parameters have been matched to the low momentum components of the spin-orbit part of the HJ potential set to zero inside the hard core [12]. The strength of the interaction is roughly the value required to reproduce, in a first order calculation with exchange, the correct strength for the spin-orbit part of the optical potential [7,9]. In addition the interaction is nearly triplet odd in character, so it is about 3 times stronger for \( T = 0 \) excitations than for \( T = 1 \) excitations. Note that this is opposite to the tensor force which is strongest for \( T = 1 \) excitations.

The \( 3^- T = 0 \) and \( 5^- T = 0 \) members of the \( f_{7/2} - d_{3/2} -1 \) multiplet in \( ^{40}Ca \) are observed at 3.73 and 4.48 MeV, respectively. In these states there is appreciable mixing between the \( f_{7/2} - d_{3/2} -1 \) configuration and other particle-hole configurations. In the present calculations, we have used the R.P.A. wave functions of Gillet and Sanderson [13] to describe these levels. These wave functions provide a good account of the experimental \( (e,e') \) data for the excitation of these levels [14]. The theoretical \( (p,p') \) cross sections we obtain for these excitations are compared with the 25 and 40 MeV data of ref. 5 in fig. 1.
Results obtained with and without the inclusion of the non-central components of the interaction are both shown. The optical model parameters used in the calculations are the same as in ref. 5.

The cross sections for the $3^- T = 0$ excitation are almost unaffected by including the non-central components of the interaction. On the other hand, the cross sections for the $5^- T = 0$ state are increased and their peaks are pushed out in angle slightly. These effects are more pronounced at 40 MeV than at 25 MeV. In these transitions spin-transfer ($S = 1$) to the target nucleus is not important [2-4], so that contributions from the tensor force are negligible for all practical purposes. The effects in fig. 1 are due to the spin-orbit force which does contribute with $S = 0$ transfer to the target [6,7]. There is not clear cut evidence in the experimental data that the spin-orbit force is important. We conclude that the interaction parameters used here, while not unreasonable, may in fact overestimate the strength of the spin-orbit force. It would be interesting to have experimental data in the 60-100 MeV energy region where spin-orbit contributions might show up more clearly.

The results in fig. 1 can be understood, qualitatively, by using Born approximation and treating the interaction (equation 1) in the short range limit [15]. Neglecting spin coupling and derivative coupling ($\vec{p}$ dependence in spin-orbit force) to the target we obtain for the cross section,

$$
\frac{d\sigma}{d\Omega} \sim J_5^2 |\rho(\vec{q})|^2 + J_V^2 q^2 k^2 \sin^2 \theta |\rho(\vec{q})|^2 ,
$$

where $\vec{q}$ is the momentum transfer, $\theta$ the scattering angle, $k$ the wave number of the incident nucleon, and $\rho$ is the scalar nuclear transition density.
The factors \( J_S^2 \) and \( J_V^2 \) give a measure of the strength of the central and spin-orbit parts of the interaction, respectively. The spin-orbit force gives an additive contribution to the cross section which increases with energy and L-transfer because of the weighting factor \( q^2 k^2 \sin^2 \theta \). The multipole dependence results because \( |\rho(\vec{q})|^2 \) peaks at larger values of \( q \) as \( L \) increases.

The remaining members of the \( f_{7/2} - d_{3/2} \) multiplet are thought to be quite pure. There is in fact some small configuration mixing in these states which tend to reduce the transition strengths [13]. A typical reduction factor might be of the order of 0.5 in cross section [16]. We have assumed pure \( f_{7/2} - d_{3/2} \) configurations in the calculations for these remaining states. The theoretical cross sections have been normalized to the experimental data for the \( 2^- T = 0 \) (\( Q = -6.03 \) MeV) and \( 2^- T = 1 \) (\( Q = -8.42 \) MeV) excitations. Calculations were made at 25 and 40 MeV and the same reduction factor was used for all of the states at a given energy. At 25 MeV the required factor was 0.4 which is in good agreement with the theoretical estimate, but at 40 MeV a factor of 0.25 was needed. It is a bit disconcerting that the theoretical results do not exhibit the same energy dependence as the experimental data. The discrepancy is not large, however, and is due to the central components of the interaction - not the non-central components. The results of calculations with a central G matrix interaction and an approximate treatment of exchange [5] exhibit a similar tendency, but not to the same degree. It would be useful to repeat these calculations with exchange included exactly and at the same time investigate the effect of adding odd components to the central interaction.
The theoretical cross sections for these six remaining levels are compared with the 25 and 40 MeV data of ref. 5 in fig. 2. Again we have shown the results with the complete interaction and the central interaction alone. The $4^-$ $T = 0$ ($Q = -5.61$ MeV) level was not resolved from a weak state lying nearby. Contributions from the latter are contained in the experimental cross sections shown. The $4^- T = 1$ ($Q = -7.66$ MeV) and $3^- T = 1$ ($Q = -7.69$ MeV) levels appear in the data as two members of an unresolved triplet. In this case we have shown the individual theoretical results for the $3^-$ and $4^-$ states, as well as the summed theoretical cross sections which are to be compared with the data. In addition, the $5^- T = 1$ ($Q = -8.54$ MeV) level was not resolved from a nearby $2^+$ state. An estimate of the $5^- T = 1$ cross section has been made [17] by adjusting $2^+$ and $5^-$ collective model cross sections to fit the experimental data. The probable cross section for the $5^- T = 1$ level is indicated by the cross hatched area in fig. 2.

The inclusion of the non-central interaction components leads to an increase in the theoretical cross sections by factors which are less than two on the average. The main effect is due to the tensor force. Contributions from the spin-orbit force were noticeable only for the $4^- T = 0$ and $5^- T = 1$ transitions. The most striking feature of the results occurs in the case of the abnormal parity transitions ($\Delta \pi = (-1)^{J+1}$). Here the theoretical cross sections obtained with the complete interaction have a shape characteristic of $L = J + 1$ transfer as compared to the $L = J - 1$ shape obtained with the central interaction alone. For the $T = 0$ transitions this effect is almost entirely due to exchange, because the tensor force makes only a small contribution to the direct amplitude in this case, i.e. $V_0^T = -0.33$ MeV · fm$^{-2}$. 
The shape change is born out quite nicely by the experimental data for the $2^-$ excitations; however, the situation is not so clear for the $4^-$ excitations. More complete data is needed to test the theoretical results for the $4^- T = 1$ excitation. The experimental cross sections for the $4^- T = 0$ excitation do exhibit an $L = 5$ shape, but the data is an order of magnitude higher than the theoretical results. The reason for this discrepancy is not known at present, but it is probably worth mentioning that the theoretical results for the $T = 0$ excitations were indeed quite sensitive to the force parameters.

With the exception of the $4^- T = 0$ transition, the calculations made here give a fair description of the $f_{7/2}d_{3/2}^{-1}$ excitations in $^{40}$Ca. The data provides some evidence for the tensor force and does not rule out the spin-orbit force. We conclude that the "realistic" interaction parameters used here are reasonable. An experiment is underway [18] to obtain cross sections for those states which were not resolved in the experimental study of ref. 5. It is clear that a good measurement of the cross section for the $4^- T = 1$ excitation which gives a measure of the strong isovector part of the tensor force would be important in testing the estimates made here. The $2^-$ and $4^- T = 0$ excitations depend on the small, not well known part of the tensor force in the direct amplitude and on the details of exchange which is sensitive to the precise form of the tensor force. These transitions demand further study.

A detailed qualitative discussion of the role of the tensor and spin-orbit forces in the $f_{7/2}d_{3/2}^{-1}$ transitions will be given in ref. 15.
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Figure Captions

Fig. 1. Differential cross sections for $3^- T = 0$ ($Q = -3.73$ MeV) and $5^- T = 0$ ($Q = -4.48$ MeV) levels in $^{40}$Ca for 25 and 40 MeV incident protons. $S$ denotes results with central interaction and $S + V + T$ refers to results with complete interaction.

Fig. 2. Same as fig. 1 for $2^- T = 0$ ($Q = -6.02$ MeV), $2^- T = 1$ ($Q = -8.42$ MeV), $4^- T = 0$ ($Q = -5.61$ MeV), $4^- T = 1$ ($Q = -7.55$ MeV), $3^- T = 1$ ($Q = -7.69$ MeV), and $5^- T = 1$ ($Q = -8.43$ MeV) levels in $^{40}$Ca. Cross hatching for $5^- T = 1$ level is probable cross section.
Fig. 1
Fig. 2
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