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Publication Date
1986-09-01

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September 1986

Prepared for the U.S. Department of Energy under Contract DE-AC03-76SF00098
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FIELD EFFECT AND JOSEPHSON JUNCTION

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Abstract

The value of the Josephson current in an S-M-S system, where M contains low-dimensional electron gas, can be affected noticeably by an applied voltage (field effect). This effect is directly related to the possibility of building a three-terminal device. The dependence of the current on the carrier concentration is studied. The peculiar situation when the state of lowest subband is described by the "dirty" limit whereas the higher subband represent the "clean" limit appears to be realistic. Intersubband scattering results in a non-monotonic behavior of the current.

Introduction

The field effect as a tool allowing one to change the value of the Josephson current is very promising from the point of view of applications. From this perspective, experimental results obtained for the Nb-InAs-Nb system have attracted considerable interest. InAs is characterized by the presence of a natural inversion layer which provides the weak coupling in the Josephson contact.

An externally applied electric field changes the electron concentration \( n \) in the semiconductor. It is of interest to study the dependence of the current on \( n \). This question was studied by the author, and it has been shown that the use of low-dimensional systems (an inversion layer is an example of a two-dimensional electron system; the use of a one-dimensional system looks even more promising, see ref. \([2]\)) leads to a noticeable increase of the field effect.

In ref. \([2]\) the situation when only the lowest subband is filled has been studied. For InAs this corresponds to \( N_s \leq \text{10}^{12} \text{cm}^{-2} \) (see Ref. \([3]\)); \( N_s = \text{nd} \), where \( d \) is the thickness of the inversion layer. It is interesting to analyze the dependence of the current for greater concentrations. A non-monotonic dependence has been observed. \([1]\)

An increase in \( N_s \) results in the appearance of different groups of electrons in the semiconductor. These groups correspond to different electronic levels and are characterized by different values of the coherence length. The interesting situation when one group is "clean," whereas another is "dirty" becomes possible.

Theory

Consider the system S-M-S (Fig. 1) where \( S \) and \( S_y \) are superconductors and \( M \) contains a size-quantized electron gas. An inversion layer or a thin film are examples of such a system. The finiteness of the thickness leads to the appearance of the quantum number \( \ell \) describing the subbands of the transverse motion. The length \( L \) is assumed to be large (e.g. in ref. \([1]\) \( L \) is taken to be \( \approx 3 \times 10^3 \text{A} \)), so there is no quantization in the \( y \)-direction. If the size in \( y \)-direction is large, then the system appears to be two-dimensional in momentum space. If the size in the \( x \)-direction is small enough, then the system is one-dimensional.

The system of interest differs from a usual Josephson contact. The subsystem \( M \) is not an insulator but contains degenerate electron gas. The non-dissipative current appears as a result of the proximity effect.

Our analysis is based on the method of thermodynamic Green's functions. Let us introduce the function \( G_i(\tau;\tau';\gamma;\gamma') \), where \( \gamma = (x,y,z) \), \( \tau \) is the imaginary time, and \( i \) denotes the transverse quantum number. The current flowing in the system is (in units \( \hbar = 1 \))

\[
J = \frac{1}{\hbar} \sum_{\gamma} G_i(\gamma;\gamma';\omega_n)|\gamma;\gamma'\rangle \langle \gamma;\gamma'|. \tag{1}
\]

Here \( \omega_n = (2n+1)\pi T \). Thanks to the equation of continuity, one can evaluate the current at any point \( x \); usually it is convenient to choose the center of the \( M \) film.

The equation for the Green's function in the \( M \) film can be written in the form:

\[
G_{\ell}^{\text{O}}(x,x';p_y;\omega_n) = G_{\ell}^{\text{O}}(0,0;p_y;\omega_n) + G_{\ell}^{\text{O}}(x,0;p_y;\omega_n) a_{\ell}^{\text{O}}(\omega_n) G_{\ell}^{\text{O}}(L,0;\omega_n) \times a_{\ell}^{\text{O}}(\omega_n)
\]

\[
a_{\ell}^{\text{O}}(\omega_n) = G_{\ell}^{\text{O}}(L,x;\omega_n) + G_{\ell}^{\text{O}}(0,L;\omega_n) \times a_{\ell}^{\text{O}}(\omega_n)
\]

\[
a_{\ell}^{\text{O}}(\omega_n) = G_{\ell}^{\text{O}}(0,L;\omega_n) a_{\ell}^{\text{O}}(\omega_n) G_{\ell}^{\text{O}}(0,x;\omega_n)
\]

\[
(2)
\]
We made a Fourier transformation with respect to \( y - y' \); \( \alpha^2 \) and \( \alpha' \) describe \( x \rightarrow x' \) and \( y \rightarrow y' \) transitions, respectively. In addition, the self-energy parts are taken in the local approximation.\(^2,4\)

The self-energy parts \( \Lambda \) describe pairing of electrons with opposite momenta (and spins). It is important to stress that if \( 6E > \varepsilon_0 \) (\( \varepsilon_0 \) is the energy gap and \( 6E \) is the spacing between transverse levels), then the paired electrons belong to the same subband.\(^5\)

An increase in thickness and the corresponding decrease in \( \varepsilon_0 \) results in it becoming necessary to take into account the functions \( \Lambda_{ik}(x) \). In this paper we consider the most interesting cases, when one or two subbands are filled and \( 6E > \varepsilon_0 \).

We are going to analyze various cases which differ in the number of filled subbands and, in addition, in the concentration of impurities. The impurities are playing a double role in the problem of interest. First of all, scattering by impurities leads to the appearance of intersubband transitions. In addition, this scattering affects the value of the current (the usual separation into the “clean” and “dirty” cases is due to this effect).

One Filled Subband

The number of filled subbands is determined by the electron concentration and the spacing \( \varepsilon_0 \). If

\[
N_s < \frac{(m^* \varepsilon_0)}{\pi},
\]

then only the lowest subband is filled (\( N_s \) is the surface concentration, \( m^* \) is the effective mass). The condition (3) can be easily obtained from the relations:

\[
p^2_f/(2m^*) < \varepsilon_0, \quad p_f = (2mN_s)^{1/2} (3')
\]

If only the lowest subband is filled, one can put \( i = 1 \) in Eq. (2). The inequality \( L > \xi_n \), where \( \xi_n = \nu_F/kT \), allows one to solve Eq (2), and we obtain:

\[
G^0_b(x,x';p_y;\varepsilon_n) + \frac{\gamma^2}{2} G^0_b(L,0;-p_y;\varepsilon_n) x + \frac{\gamma^2}{2} G^0_b(0,L;p_y;\varepsilon_n) x
\]

\[
\alpha_{\varepsilon_n}(\varepsilon_n) = G^0_b(L,0;\varepsilon_n) + \frac{\gamma^2}{2} G^0_b(0,L;\varepsilon_n) x + \frac{\gamma^2}{2} G^0_b(0,L;\varepsilon_n) x
\]

The self-energy parts are equal to

\[
\alpha_{\varepsilon_n}(\varepsilon_n) = G^0_b(L,0;\varepsilon_n) + \frac{\gamma^2}{2} G^0_b(0,L;\varepsilon_n) x + \frac{\gamma^2}{2} G^0_b(0,L;\varepsilon_n) x
\]

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\]

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\alpha_{\varepsilon_n}(\varepsilon_n) = G^0_b(L,0;\varepsilon_n) + \frac{\gamma^2}{2} G^0_b(0,L;\varepsilon_n) x + \frac{\gamma^2}{2} G^0_b(0,L;\varepsilon_n) x
\]

\[
\alpha_{\varepsilon_n}(\varepsilon_n) = G^0_b(L,0;\varepsilon_n) + \frac{\gamma^2}{2} G^0_b(0,L;\varepsilon_n) x + \frac{\gamma^2}{2} G^0_b(0,L;\varepsilon_n) x
\]

\[F_{a}(\gamma) = G_{a}(\gamma) \phi_{a}(\gamma)
\]

Here \( u = \varepsilon_{\alpha} \text{m}^* \nu_F \) is the mobility.

Two Filled Subbands ("clean" case)

An increase in the electron concentration results in a situation, when \( N_s > (m^* \varepsilon_0)/\pi \). Then the second subband becomes occupied. If \( \varepsilon_0 > \varepsilon_s \), electrons from the same subband pair up.\(^5\)

The evaluation for each subband can be carried out by analogy with the preceding section, and the total current is a sum of the contributions of the subbands, that is

\[
J_m = A_1 \left\{ \exp(-L/E_{N1}) \right\} + A_2 \left\{ \exp(-L/E_{N2}) \right\}.
\]

Here

\[
E_{N1} = \nu_F L^*/\pi T, \quad E_{N2} = \nu F^2 L^*/\pi T.
\]

Despite the fact that the current is a simple sum of two terms, the dependence of \( J_m \) upon the electron concentration for one filled subband differs from that in the case of two filled subbands. Indeed, in the latter case one should take into account the following conditions:
and we obtain:

\[ v_{F1} = (\pi N_s + m\epsilon E)^{1/2}/m, \]
\[ v_{F2} = (\pi N_s - m\epsilon E)^{1/2}/m, \]

(we assume \( m_1^* = m_2^* = m \)).

The change of the dependence \( j_m(N_s) \) due to the filling of the second subband becomes apparent if we consider the realistic case when \( v_{F1} \) and \( v_{F2} \) differ noticeably, i.e., \( (\epsilon v_{F1}/v_{F2}) >> 1 \). In this case \( \xi_{N1} \gg \xi_{N2} \) and the contribution of the second term in Eq. (11) is very small. Hence, the main contribution comes from the lowest subband. However, the dependence \( j_m(N_s) \) is different (cf., Eq. (9) and Eq. (10)). In both cases the dependence \( j_m(N_s) \) is determined, mainly, by the dependence \( v_F(N_s) \). One can see directly from Eqs. (3') and (12) that these dependences are different. Calculating the derivative \( dj_m/dN_s \), one comes to the conclusion that the second subband being occupied slows down the rate of increase of the current \( j_m \) as a function of the concentration \( N_s \).

**Intermediate Case**

Consider the case of two filled subbands in the presence of a large concentration of impurities. One can see that this case is characterized by some peculiar features.

As is known, the question of a system being "clean" or "dirty" can be resolved depending on the relation between the coherence length \( \xi_N \) and the mean free path \( \ell \). In the case of interest, there are two coherence lengths \( \xi_{N1} \) and \( \xi_{N2} \) and the situation when \( \xi_{N1} \gg \ell \gg \xi_{N2} \) is perfectly realistic. This means that the lowest subband is "dirty" whereas the second one is "clean". Hence, the single sample cannot be assigned to some definite category. It contains two groups of carriers which differ noticeably in their coherence lengths. The amplitude of the current is equal to:

\[ j_m = j_{m1} + j_{m2}, \]

here

\[ j_{m1} = A^0 \exp\left\{-L/\xi_{N1}^0 \right\}, \quad j_{m2} = A \exp\left\{-L/\xi_{N2} \right\}, \]

\[ \xi_{N1}^0 = \frac{1}{2} (\xi_{N1}^2)^{1/2}, \quad \xi_{N1} = v_{F1}/\pi T, \quad \xi_{N2} = v_{F2}/\pi T. \]

In our case, \( \xi_{N2} \ll \xi_{N1} \) and, therefore, the major contribution comes from the lowest subband, that is \( j_m = A^0 \exp\left\{-L/\xi_{N1}^0 \right\} \). However, the occupation of the second subband leads to an essential change of the dependence \( j_m(N_s) \). First of all, the dependence \( v_{F1}(N_s) \) differs from the case of one filled subband (see previous section). But more importantly, there is a noticeable change of the relaxation time \( \tau_1 \). The presence of impurities results in an additional relaxation channel, and, a result, in a noticeable decrease of \( \tau_1 \). Namely, intersubband transitions due to scattering become possible. The quantity \( \tau_1 \) is determined by the equation

\[ \tau_1^{-1} = \tau_{11}^{-1} + \tau_{12}^{-1}. \]

(14)

\( \tau_{11} \) and \( \tau_{12} \) describe intrasubband and intersubband transitions, respectively. \( \tau_{11} \) can be evaluated with the use of the equation (see, e.g., ref. 7).

\[ \tau_{11}^{-1} = 2\pi \int v_{eff} f_1(k,k') (1 - \cos k_x x_k x'_k) \, \delta((\epsilon_1(k) - \epsilon_1(k')) \, dk'_x; \]

\[ i = 1, 2; \quad \vec{k} \text{ is the two-dimensional quasimomentum.} \]

Note that \( v_{eff} \) is the matrix element of the \( 1 \leftrightarrow 1 \) transition. If the scattering is due to charged impurities then \( v_{eff} \) is the screened potential. The electrons of both subbands contribute to the screening. \( v_{eff} \) satisfies the equation:

\[ v_{eff}(r,r',\omega) = v_0(r,r',\omega) \int v_{0}(r,r',\omega) \, \gamma_{11}(r_1,r_2,\omega) \times \]
\[ \gamma_{2} v_{eff}(r_2,r'_1) \, dr_1 \, dr'_2 \times \int v_{0}(r,r',\omega) \, \gamma_{22}(r_1,r_2,\omega) \times \]
\[ \gamma_{2} v_{eff}(r_2,r'_2) \, dr_1 \, dr'_2. \]

(16)

Here \( \gamma = \left\{ \alpha, \beta \right\} \), \( v_0 \) is the bare potential. Fourier transforming with respect to \( \vec{r} \), one obtains:

\[ v_{eff}(\ell) = (\gamma_{01} + (1-\gamma_{01}) \gamma_{11}(\ell)) \gamma_{1} \]
\[ \gamma_{01}(q) \gamma_{22}(\vec{q}) \gamma_{2}^{-1}. \]

(17)

Hence, the occupation of the second subband is accompanied by a decrease of \( \tau_1 \) and, as a result, by a decrease of \( \xi_{N1} \). It is important that the two-dimensional density of states is equal to \( 4m^*c^*/\pi \) and does not depend on \( N_s \). Hence, \( \tau_1^{-1} \) can be large even for small fillings.

Hence, if the concentration \( N_s = (m^* e^2)/\ell \) (the second subband becomes occupied; for InAs \( N_s = 10^{12} \text{ cm}^{-1} \)), a sharp decrease of \( \xi_{N1} \) takes place, accompanied by a decrease of the Josephson current. Therefore, in this region, a peak of \( j_m \) should be
observed. Such a peak has been observed experimentally\(^1\) for the Nb-InAs-Nb junction. Similar peaks should be observed if the next subbands become occupied.

Hence, the dependence of \(j_m\) on \(N_s\) is non-monotonic. The amplitude of \(j_m\) oscillates, and these oscillations correspond to fillings of the higher subbands.

Acknowledgment

The author wishes to thank Prof. M. Beasley, Dr. M. Nisenoff, and Dr. S. Wolf for valuable discussions. This work is supported by the Office of Naval Research under Contract No. N00014-86-F0015\(^4\) and carried out at the Lawrence Berkeley Laboratory under Contract No. DE-AC03-76SF00098.

References

This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the Department of Energy.

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