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REGGE POLE FORMULAE FOR DIFFERENTIAL CROSS SECTIONS OF QUASI TWO-BODY \( \pi N \) AND \( NN \) INTERACTIONS

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REGGE POLE FORMULAE FOR DIFFERENTIAL CROSS SECTIONS OF QUASI TWO-BODY $\pi N$ AND $NN$ INTERACTIONS

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Regge pole formulae for differential cross sections of some quasi two-body \( \pi N \) and \( NN \) interactions, which can have one or two meson trajectories exchanged, are given in this paper. The reactions treated are: 

\begin{align*}
\pi N &\rightarrow \pi \Delta (1236), \pi N \rightarrow \omega \Delta (1236), \pi N \rightarrow A_2N, \pi N \rightarrow \eta N, \\
\pi N &\rightarrow \eta \Delta (1235), \pi N \rightarrow \rho N, \pi N \rightarrow \rho \Delta (1236), \NN \rightarrow N \Delta (1236), \NN \rightarrow \NN^*(1500, \frac{3}{2}^-), \\
\NN &\rightarrow \NN^*(1480, \frac{1}{2}^+). 
\end{align*}
I. INTRODUCTION

The Regge pole model has given quite good fits\(^1\) and interesting predictions\(^2,3\) for certain simple high-energy two-body to two-body interactions which can have single Regge pole exchange. There remain many interesting quasi two-body interactions, involving complications of high spins and unequal mass kinematics, which have not been treated theoretically. With the fast accumulation of high-energy experimental data, it is of interest to know the theoretical formulae which are suitable for phenomenological fits. In this paper we shall give the Regge pole formulae for the differential cross sections of some quasi two-body \(pN\) and \(NN\) interactions which can have one or two meson trajectories exchanged.

According to the established method of Reggeization\(^4\), including the deduction of residue functions free from kinematic singularities\(^5\), the following results will be obtained in a routine fashion.
II. \( \pi N \) INTERACTIONS

A. \( \pi N \rightarrow \pi \Delta \) (1236)

Due to the conservation of isotopic spin and G-parity, only \( \rho \) can be exchanged among the experimentally established trajectories. The differential cross section without polarization is

\[
\frac{d\sigma}{dt} = \frac{1}{4\pi s_{\pi N}^2} \frac{1}{(2 \times \frac{1}{2} + 1)} \times 2 \times \frac{1}{4} \times \left\{ \left[ \sin \theta_t \left[ 1 - \exp(-i\alpha) \right] \right] \frac{2\alpha + 1}{\sin \pi \alpha} \beta_{00; \frac{3}{2} \frac{1}{2}}(t) E_{01}^{1/2} (\cos \theta_t) \right\}^2 + \left\{ \left[ \sin \theta_t \left[ 1 - \exp(-i\alpha) \right] \right] \frac{2\alpha + 1}{\sin \pi \alpha} \beta_{00; \frac{1}{2} \frac{1}{2}}(t) E_{00}^{1/2} (\cos \theta_t) \right\}^2 + \left\{ \left[ \exp (-i\alpha) \right] \frac{2\alpha + 1}{\sin \pi \alpha} \beta_{00; \frac{1}{2} \frac{1}{2}}(t) E_{01}^{1/2} (\cos \theta_t) \right\}^2 + \left\{ \left[ \sin \theta_t \left[ 1 - \exp(-i\alpha) \right] \right] \frac{2\alpha + 1}{\sin \pi \alpha} \beta_{00; \frac{1}{2} \frac{1}{2}}(t) E_{00}^{1/2} (\cos \theta_t) \right\}^2 \right\},
\]

where \( P_{\pi N} \) is the c.m. momentum of the \( \pi N \) system, \( s \) is the total energy squared, \( \theta_t \) is the t-reaction angle, 7 \( \alpha(t) \) is the \( \rho \)-trajectory, and the \( \beta \)'s are the residue functions of the partial-wave helicity amplitudes which have definite parity. The kinematic factors of the residue functions \( \beta \) are 8, 9

\[
\beta_{00; \frac{3}{2} \frac{1}{2}}(t) \alpha(\alpha + 1) \frac{k^2}{(t - 4m_\pi^2)(t - (m_\Delta - m_N)^2)} \frac{1}{2} (p_{\pi N})^{\alpha - 1}, \quad (2a)
\]
\[
\beta_{00}; \frac{3}{2} - \frac{1}{2} \left( t \propto \left[ \alpha(\alpha + 1) (\alpha - 1) (\alpha + 2) \right]^{\frac{3}{2}} \left[ t - \left( m_{\Delta} + m_{N} \right)^{2} \right]^{\frac{1}{2}} \times \left( t - 4m_{\pi}^{2} \right) \left( p_{\pi} p_{\Delta N} \right)^{\alpha - 2} \right),
\]

\[
\beta_{00}; \frac{1}{2} \left[ t \propto \left[ t - \left( m_{\Delta} + m_{N} \right)^{2} \right]^{-\frac{3}{2}} \left[ t - \left( m_{\Delta} - m_{N} \right)^{2} \right]^{-1} \left( p_{\pi} p_{\Delta N} \right)^{\alpha}, \right.
\]

\[
\beta_{00}; \frac{1}{2} - \frac{1}{2} \left( t \propto \left[ \alpha(\alpha + 1) \right]^{\frac{3}{2}} t \propto \left[ t - \left( m_{\Delta} - m_{N} \right)^{2} \right]^{\frac{1}{2}} \left( t - 4m_{\pi}^{2} \right) \times \left( p_{\pi} p_{\Delta N} \right)^{\alpha - 1}. \right)
\]

Notice that the kinematic factors of \( \beta_{00}; \frac{3}{2} - \frac{1}{2} \left( t \right) \) and \( \beta_{00}; \frac{1}{2} - \frac{1}{2} \left( t \right) \) are the same. For fitting we can put them together in Eq. (1). Using the asymptotic form of the E-functions and after some rearrangements, we obtain a formula which is suitable for phenomenological fitting:

\[
\frac{d\sigma}{dt} = \frac{1}{2} \left[ \left| 1 - \exp(-i\pi\alpha) \right|^{2} \left( \frac{1}{\Gamma(\alpha + 1) \sin \pi\alpha} \right)^{\frac{1}{2}} \right] \times \left\{ \left| \sin \theta_{t} \right|^{2} \left| \left( 4m_{\pi}^{2} - t \right)^{\frac{1}{2}} \left( \left( m_{\Delta} - m_{N} \right)^{2} - t \right) \right|^{\frac{1}{2}} \right\}^{2}
\]

\[
\times \left\{ \left| \gamma_{00}; \frac{3}{2} - \frac{1}{2} \left( t \right) \right|^{2} + \left| \gamma_{00}; \frac{1}{2} \left( t \right) \right|^{2} \left( \frac{3}{s_{0}} \right)^{2\alpha - 2} \right\} + \left| \left( \sin \theta_{t} \right)^{2} \right|^{2} \left( \alpha^{2} - 1 \right)^{2} \left| \left( t - 4m_{\pi}^{2} \right) \left[ t - \left( m_{\Delta} + m_{N} \right)^{2} \right]^{\frac{1}{2}} \right|^{2}
\]

\[
\times \left| \gamma_{00}; \frac{3}{2} - \frac{1}{2} \left( t \right) \right|^{2} \left( \frac{s}{s_{0}} \right)^{2\alpha - 4}
\]

(Equation continued)
\[ + \frac{1}{\left( t - (m_\Delta - m_N)^2 \right)^{\frac{1}{2}}} \left( t - (m_\Delta + m_N)^2 \right)^{\frac{1}{2}}  \\
\times \left| \gamma_{00;\frac{1}{2} 1}(t) \right|^2 \left( \frac{s}{s_0} \right)^{2\alpha} \right\} , \]

where \( s_0_{\pi N} = 4s p_{\pi N}^2 = [s - (m_N + m_\pi)^2] [s - (m_N - m_\pi)^2] \). The \( \gamma \)'s are defined through Eqs. (1) and (3). Each is a product of the corresponding \( \beta \) without those kinematic singularities, \((2\alpha + 1) \times \Gamma(\alpha + \frac{1}{2})/\pi^{\frac{1}{2}}\) of the \( \beta \)-functions and constants. The poles of \((2\alpha + 1) \times \Gamma(\alpha + \frac{1}{2})\) at \(-\frac{3}{2}, -\frac{5}{2}\), etc. are cancelled by the zeros of \( \beta \) at these points. So the \( \gamma \)'s are analytic functions of \( t \) for \( t < 0 \). Notice that \( 1/\Gamma(\alpha + 1) \sin \pi \alpha \) is finite at all negative integral values of \( \alpha \). Therefore the odd signature factor in Eq. (3) gives zeros at negative even integers of \( \alpha \). In the rest of the paper, the \( \gamma \)'s will be similarly defined. The \( s_0 \) is an arbitrary positive scale constant. Usually \( s_0 \) is chosen so as to minimize the rate of variation of the \( \gamma \)'s as functions of \( t \). The same notation \( s_0 \) will be used throughout the paper for all interactions, even though the values of \( s_0 \) need not be the same. Notice that \( \sin \theta_t = 0 \) and \( |\cos \theta_t| = 1 \) in the forward direction of the \( s \)-channel. We assume that Regge asymptotic behavior holds in the \( s \)-channel forward direction for the amplitudes which are free of kinematic singularities. That is why we keep the kinematic factors of \( \sin \theta_t \) as they stand and approximate the functions \( F_{\lambda \mu}^{\alpha^+}(\cos \theta_t) \)'s in Eq. (1) by their leading asymptotic terms according to Ref. 10. The same assumption is applied throughout this paper.
B. \( \pi N \rightarrow \omega \Delta(1236) \)

Only \( \rho \) can be exchanged in this interaction and the \( \rho \) can couple to \( \pi \omega \) only when \( \omega \) is in helicity state one.\(^3\) The differential cross section is

\[
\frac{d\sigma}{dt} = \frac{1}{4\pi s_{\pi N}^{\frac{3}{2}}} \frac{1}{2t \left( \frac{1}{2} + \frac{1}{2} \right)} \times 2 \times \frac{1}{4} \\
\times \left\{2[1 + (\cos \theta_t)^2] \left[1 - \exp(-i\pi \alpha)\right] \frac{2\alpha + 1}{\sin \pi \alpha} \beta \frac{3}{2} \frac{1}{2} (t) E_{11}^{\alpha_{\rho} +} (\cos \theta_t) \right\}^2 \\
+ 2|\sin \theta_t|^2 \left[1 + (\cos \theta_t)^2\right] \left[1 - \exp(-i\pi \alpha)\right] \frac{2\alpha + 1}{\sin \pi \alpha} \beta \frac{3}{2} \frac{1}{2} (t) \\
\times E_{21}^{\alpha_{\rho} +} (\cos \theta_t) \right\}^2 \\
+ 2|\sin \theta_t|^2 \left[1 - \exp(-i\pi \alpha)\right] \frac{2\alpha + 1}{\sin \pi \alpha} \beta \frac{1}{2} \frac{1}{2} (t) E_{01}^{\alpha_{\rho} +} (\cos \theta_t) \right\}^2 \\
+ 2[1 + (\cos \theta_t)^2] \left[1 - \exp(-i\pi \alpha)\right] \frac{2\alpha + 1}{\sin \pi \alpha} \beta \frac{1}{2} \frac{1}{2} (t) E_{11}^{\alpha_{\rho} +} (\cos \theta_t) \right\}^2 \}
\]

The kinematic factors of the residue functions are

\[
\beta \frac{3}{2} \frac{1}{2} (t) \text{ and } \beta \frac{1}{2} \frac{1}{2} (t) \propto \alpha (\alpha + 1) \left[t - (m_\Delta - m_N)^2\right]^{-\frac{1}{2}} \int_{\pi \omega} (p_{\pi \omega} p_{\Delta})^\alpha-1, \\
\beta \frac{3}{2} \frac{1}{2} (t) \propto \alpha (\alpha + 1) \left[(\alpha - 1) (\alpha + 2)\right]^{\frac{1}{2}} t^{-\frac{1}{2}} [t - (m_\Delta + m_N)^2]^{-\frac{1}{2}} \\
\times \int_{\pi \omega} 2 (p_{\pi \omega} p_{\Delta})^\alpha-2, \\
\]

\[\text{(5a)}\]

\[\text{(5b)}\]
\[ \beta_{10; \tilde{N} \Delta}(\pi \rightarrow \tilde{N} \Delta) = \frac{\beta_{00; \tilde{N} \Delta}(\pi \pi \rightarrow \tilde{N} \Delta)}{\beta_{00; \tilde{N}' \Delta'}(\pi \pi \rightarrow \tilde{N} \Delta)}, \quad (6) \]

Therefore in addition to the pure t kinematic factors of the full helicity amplitude given by Ref. 5, the factorizability relation of Eq. (6) can require additional kinematic factors of the residue functions. These additional factors are always analytic. All the \( \beta \) kinematic factors of all the interactions considered in this paper are obtained with consideration of this additional constraint given by the factorizability of the residue functions. The final form of the differential cross section, suitable for phenomenological fitting, is

\[ \frac{d\sigma}{dt} = \frac{1}{g^2_{\pi N}} \left| 1 - \exp(-im\alpha) \right|^2 \left( \frac{1}{\Gamma(\alpha + 1) \sin \pi\alpha} \right)^2 \\
\times \left\{ \left[ 1 + (\cos \theta_t)^2 \right] \alpha^4 \left[ t - (m_\Delta - m_N)^2 \right]^{-\frac{1}{2}} \int_{p_{\pi N}} \right\}^2 \]

(Equation continued)
\[
\begin{align*}
&\times \left[ |\gamma_{10; \frac{3}{2} - \frac{1}{2}}(t)|^2 + |\gamma_{10; \frac{1}{2} - \frac{1}{2}}(t)|^2 \right] \left( \frac{s}{s_0} \right)^{2\alpha - 2} \\
&+ |t^{-\frac{1}{2}} \sin \theta_t|^2 \left[ 1 + (\cos \theta_t)^2 \right] \alpha^4(\alpha - 1)^2 \left| \mathcal{J}_{\pi \omega} \right| \alpha^4(t)^2 \left( \frac{s}{s_0} \right)^{2\alpha - 2} \\
&\times \left[ \gamma_{10; \frac{3}{2} - \frac{1}{2}}(t)|^2 \left( \frac{s}{s_0} \right)^{2\alpha - 4} \right] \\
&+ |t^{-\frac{3}{2}} \sin \theta_t|^2 \alpha^2 \left| t - (m_\omega - m_p)^2 \right| \frac{1}{2} \mathcal{J}_{\pi \omega} \left| \left. \frac{1}{2} \right| \gamma_{10; \frac{1}{2} - \frac{1}{2}}(t)|^2 \left( \frac{s}{s_0} \right)^{2\alpha - 2} \right]\}
\end{align*}
\]

where the \( \gamma \)'s are defined through Eqs. (4) and (7). Notice that a factor \( \alpha \) appears in every term of Eq. (7). Therefore the differential cross section has a minimum where \( \alpha(t) = 0 \).

C. \( \pi N \rightarrow A_2 N \)

For non-charge-exchange \( \pi N \rightarrow A_2 N \), the Pomeranchuk, \( P, P' \), and the \( \rho \) trajectories can be exchanged. Neither can couple to \( \pi A_2 \) system with \( A_2 \) in the helicity state zero.

\[
\frac{d\sigma}{dt} = \frac{1}{4\pi p_\pi N} \frac{1}{(2 \times \frac{1}{2} + 1)} \times 2 \times \frac{1}{4} \times \left\{ 2 \left| (\sin \theta_t)^2 \right|^2 \left[ 1 + \exp(-i\alpha_\rho) \right] \left( \frac{2}{2 \alpha + 1} \right) \right. \\
+ \left. \frac{\beta}{\sin \alpha_\rho} \frac{p(t)}{20 \times \frac{1}{2}} E_{\rho}^\alpha \left( \cos \theta_t \right) \right. \\
+ \left. \frac{\beta}{\sin \alpha_\rho} \frac{\rho(t)}{20 \times \frac{1}{2}} E_{\rho}^\alpha \left( \cos \theta_t \right) \left. \right|^2 \\
+ 2 \left| \sin \theta_t \right|^2 \left[ 1 + (\cos \theta_t)^2 \right] \right\}
\]

(Equation continued)
\[ x [1 + \exp(-i\pi \rho)] \frac{2 \alpha_{\rho} + 1}{\sin \pi \rho} \frac{\rho(t)}{20;\frac{3}{2}} - \frac{1}{2} \alpha_{\rho}^{+} \cos \theta_{t} \]

\[ + [1 - \exp(-i\pi \rho)] \frac{2 \alpha_{\rho} + 1}{\sin \pi \rho} \frac{\rho(t)}{10;\frac{3}{2}} - \frac{1}{2} \alpha_{\rho}^{+} \cos \theta_{t} \]

\[ + 2 |\sin \theta_{t}|^{2} [1 + \exp(-i\pi \rho)] \frac{2 \alpha_{\rho} + 1}{\sin \pi \rho} \frac{\rho(t)}{10;\frac{3}{2}} \alpha_{\rho}^{+} \cos \theta_{t} \]

\[ + [1 - \exp(-i\pi \rho)] \frac{2 \alpha_{\rho} + 1}{\sin \pi \rho} \frac{\rho(t)}{10;\frac{3}{2}} \alpha_{\rho}^{+} \cos \theta_{t} \]

\[ + 2 [1 + (\cos \theta_{t})^{2}] [1 + \exp(-i\pi \rho)] \frac{2 \alpha_{\rho} + 1}{\sin \pi \rho} \frac{\rho(t)}{10;\frac{3}{2}} \alpha_{\rho}^{+} \cos \theta_{t} \]

\[ + [1 - \exp(-i\pi \rho)] \frac{2 \alpha_{\rho} + 1}{\sin \pi \rho} \frac{\rho(t)}{10;\frac{3}{2}} \alpha_{\rho}^{+} \cos \theta_{t} \]

The contribution from the \( P' \) trajectory gives exactly the same form as that from the \( P \) trajectory. The kinematic factors of the residue functions are

\[ \beta_{\frac{1}{2} \frac{3}{2}} P(t) \quad \text{and} \quad \beta_{\frac{1}{2} \frac{3}{2}} \rho(t) \propto \left[ \alpha(\alpha + 1)(\alpha - 1)(\alpha + 2) \right]^{\frac{1}{2}} \pi_{\rho}^{\alpha}(t - \frac{1}{2})^{\frac{1}{2}} \]

\[ \times (\pi_{\rho}^{\alpha} \pi_{\rho}^{\alpha})^{\alpha-2}, \quad (9a) \]

where \( \alpha \) is \( \alpha_{\rho} \) (or \( \alpha_{\rho} \)) for \( P \) (or \( \rho \));

\[ \beta_{\frac{1}{2} \frac{3}{2}} P(t) \propto \left[ (\alpha_{\rho} - 1)(\alpha_{\rho} + 2) \right]^{\frac{1}{2}} t^{\frac{1}{2}} \pi_{\rho}^{\alpha}(t - \frac{1}{2})^{\frac{1}{2}} \]

\[ \times (\pi_{\rho}^{\alpha} \pi_{\rho}^{\alpha})^{\alpha-2}, \quad (9b) \]
\[ \beta_{20; \frac{1}{2} - \frac{1}{2}} \frac{\rho(t)}{\rho} = \alpha_{\rho} (\alpha_{\rho} + 1) [(\alpha_{\rho} - 1) (\alpha_{\rho} + 2)]^{\frac{1}{2}} t^{\frac{1}{2}} \int_{\pi A} (t - \frac{\mu_{N}}{2})^{\frac{1}{2}} \times (p_{\pi A} p_{NN})^{-\alpha_{\rho}^{-2}} \]  
\tag{9c}

\[ \beta_{10; \frac{1}{2} - \frac{1}{2}} \frac{P(t)}{P} \rho(t) \propto [\alpha(\alpha + 1)]^{\frac{1}{2}} (p_{\pi A} p_{NN})^{-\alpha_{p}^{-1}} \]  
\tag{9d}

\[ \beta_{10; \frac{1}{2} - \frac{1}{2}} \frac{P(t)}{P} \propto t^{\frac{1}{2}} (p_{\pi A} p_{NN})^{-\alpha_{p}^{-1}} \]  
\tag{9e}

\[ \beta_{10; \frac{1}{2} - \frac{1}{2}} \frac{\rho(t)}{\rho} \propto \alpha_{\rho} (\alpha_{\rho} + 1) t^{\frac{1}{2}} (p_{\pi A} p_{NN})^{-\alpha_{\rho}^{-1}} \]  
\tag{9f}

The difference of a factor of \( \alpha(\alpha + 1) \) between Eqs. (9b), (9c); and between Eqs. (9e), (9f) arises from the assumption that the \( P \) trajectory couples to nonsense-nonsense channels \(^{14}\) (the amplitudes do not have a pole at \( \alpha_{p}(t) = 0 \)), while the \( \rho \) trajectory couples to sense-sense amplitudes at \( \alpha = 0 \). After some rearrangement we obtain

\[ \frac{d\sigma}{dt} = \frac{1}{\sqrt{\pi N}} \left\{ \left[ (\sin \theta_{t})^{2} \right]^{2} \left| \int_{\pi A} (t - \frac{\mu_{N}}{2})^{\frac{1}{2}} \right|^{2} \right. \]

\[ \times \left[ 1 + \exp(-i\pi \alpha_{p}) \right] \frac{\alpha_{p}(\alpha_{p} - 1)}{\Gamma(\alpha_{p} + 1) \sin \pi \alpha_{p}} \gamma_{20; \frac{1}{2} - \frac{1}{2}} \frac{P(t)}{P}(\frac{s}{s_{0}}){\alpha_{p}^{-2}} \]

\[ + \left[ 1 - \exp(-i\pi \alpha_{p}) \right] \frac{\alpha_{p}(\alpha_{p} - 1)}{\Gamma(\alpha_{p} + 1) \sin \pi \alpha_{p}} \gamma_{20; \frac{1}{2} - \frac{1}{2}} \frac{\rho(t)}{\rho}(\frac{s}{s_{0}}){\alpha_{\rho}^{-2}} \]

\[ + \left| \sin \theta_{t} \right|^{2} \left[ 1 + (\cos \theta_{t})^{2} \right] t^{\frac{1}{2}} \int_{\pi A} (t - \frac{\mu_{N}}{2})^{\frac{1}{2}} \right\}^{2} \]

(Equation continued)
\[ x \left[ 1 + \exp(-i\pi\alpha_p) \right] \frac{\alpha_p(\alpha_p - 1)}{(\alpha_p + 1)\Gamma(\alpha_p + 1) \sin \pi\alpha_p} \gamma_{20;\frac{1}{2}; -\frac{1}{2}}^P(t) \left( \frac{s}{s_0} \right)^{\alpha_p - 2} \]

\[ + \left[ 1 - \exp(-i\pi\alpha_p) \right] \frac{\alpha_p^2 (\alpha_p - 1)}{\Gamma(\alpha_p + 1) \sin \pi\alpha_p} \gamma_{20;\frac{1}{2}; -\frac{1}{2}}^P(t) \left( \frac{s}{s_0} \right)^{\alpha_p - 2} \]

\[ + |\sin \theta_t|^2 \left[ 1 + \exp(-i\pi\alpha_p) \right] \frac{\alpha_p}{\Gamma(\alpha_p + 1) \sin \pi\alpha_p} \gamma_{10;\frac{1}{2}; -\frac{1}{2}}^P(t) \left( \frac{s}{s_0} \right)^{\alpha_p - 1} \]

\[ + [1 + (\cos \theta_t)^2] |t| \]

\[ x \left[ 1 + \exp(-i\pi\alpha_p) \right] \frac{\alpha_p}{(\alpha_p + 1)\Gamma(\alpha_p + 1) \sin \pi\alpha_p} \gamma_{10;\frac{1}{2}; -\frac{1}{2}}^P(t) \left( \frac{s}{s_0} \right)^{\alpha_p - 1} \]

\[ + \left[ 1 - \exp(-i\pi\alpha_p) \right] \frac{\alpha_p^2}{\Gamma(\alpha_p + 1) \sin \pi\alpha_p} \gamma_{10;\frac{1}{2}; -\frac{1}{2}}^P(t) \left( \frac{s}{s_0} \right)^{\alpha_p - 1} \]

where the \( \gamma \)'s are defined through Eqs. (8) and (10). The \( \gamma \)'s are related to those of \( \pi N \rightarrow \pi N \) scattering by [15]

\[ \frac{\beta_{A0;\frac{1}{2}; \frac{1}{2}} (\pi A_2 \rightarrow N\bar{N})}{\beta_{A0;\frac{1}{2}; -\frac{1}{2}} (\pi A_2 \rightarrow N\bar{N})} = \frac{\beta_{00;\frac{1}{2}; \frac{1}{2}} (\pi \pi \rightarrow N\bar{N})}{\beta_{00;\frac{1}{2}; -\frac{1}{2}} (\pi \pi \rightarrow N\bar{N})} \]
The contribution of the $P'$ trajectory is exactly the same as that from the $P$. One can see how hopeless it is to do fitting with non-charge-exchange $\pi N \rightarrow A_2 N$ interaction. However there is one interesting point. Notice that in the first and the second term of Eq. (10) there is a factor $(\alpha_p - 1)$. Since $\alpha_p(0) = 1$, their contribution in the s-channel forward direction will be very small. Also notice that every term in Eq. (10) has kinematic factors of sines and cosines of $\theta_\pi$ which do not increase with energy $s$ (i.e., $\sin \theta_\pi = 0$ and $|\cos \theta_\pi| = 1$) in the forward direction of s-channel; therefore the Regge trajectory, especially the $P$, can not contribute with full strength in the forward direction of s-channel and thus the production of $A_2$ in very high energies will be more limited than if $P$ could contribute with full strength. 16 For the charge exchange $\pi N \rightarrow A_2 N$ interaction, only $\rho$ contributes. Since every term in Eq. (10) has a factor $\alpha_\rho$, the differential cross section has a minimum at $\alpha_\rho = 0$.

D. $\pi N \rightarrow \eta N$

Of the known trajectories, only that of $A_2$ can be exchanged here. 17

$$\frac{d\sigma}{dt} = \frac{1}{4\pi s p_{\pi N}} \frac{1}{2} \times 2 \times \frac{1}{4} \left[ \left[ 1 + \exp(-i\alpha) \right] \frac{2\alpha + 1}{\sin \pi \alpha} \frac{\beta}{00;2} \frac{\bar{1}}{2} \right]
$$

$$\times E_{00} \alpha_{\rho} \langle \cos \theta_\pi \rangle \right|^2 + \left[ \sin \theta_\pi \left[ 1 + \exp(-i\alpha) \right] \frac{2\alpha + 1}{\sin \pi \alpha} \frac{\beta}{00;2} \frac{\bar{1}}{2} \right] \frac{E_{01} \alpha_{\rho} \langle \cos \theta_\pi \rangle \right|^2 \right].$$

(11)
The kinematic factors of the residue functions are:

\[ \beta_{00;\frac{1}{2},\frac{1}{2}}(t) \propto \alpha(\alpha + 1) (t - 4m_N^2)^{-\frac{3}{2}} (p_{\pi \eta} p_{NN})^\alpha \]  (12a)

\[ \beta_{00;\frac{1}{2},\frac{1}{2}}(t) \propto [\alpha(\alpha + 1)]^{\frac{1}{2}} \int_{\pi \eta} (p_{\pi \eta} p_{NN})^{\alpha-1} \]  (12b)

The final form of the differential cross section is

\[ \frac{d\sigma}{dt} = \frac{1}{\sin^\gamma_{\pi N}} \left| 1 + \exp(-i\alpha) \right|^2 \left[ \frac{\alpha}{\Gamma(\alpha + 1) \sin \pi \alpha} \right]^2 \]

\[ \times \left\{ (\alpha + 1)^2 \left| (t - 4m_N^2)^{-\frac{1}{2}} \right|^2 \left| \gamma_{00;\frac{1}{2},\frac{1}{2}}(t) \right|^2 \left( \frac{s}{s_0} \right)^{2\alpha} \right\} + \left| \sin \theta_t \right|^2 \left| \int_{\pi \eta} \right|^2 \left| \gamma_{00;\frac{1}{2},\frac{1}{2}}(t) \right|^2 \left( \frac{s}{s_0} \right)^{2\alpha-2} \right\} \]  (13)

where \( \gamma \)'s are defined through Eqs. (11) and (13). They are smoothly varying functions of \( t \) for \( t < 0 \). Since \( \alpha/\Gamma(\alpha + 1) \sin \pi \alpha \) is finite at negative integers of \( \alpha \), the vanishing of the signature factor \( \left| 1 + \exp(-i\alpha) \right|^2 \) at negative odd integers of \( \alpha \) gives minimums in the differential cross sections at these points.
Here only the $A_2$ trajectory can be exchanged.

\[
\frac{d\sigma}{dt} = \frac{1}{4\pi s p_{\pi N}} \frac{1}{2} \times 2 \times \frac{1}{4}
\]

\[
\times \left\{ \sin \theta_t \left[ 1 + \exp(-i\alpha) \right] \frac{2\alpha + 1}{\sin \pi \alpha} \beta_{00;2} \frac{1}{2} (t) E_{01} \alpha_+(\cos \theta_t) \right\}^2
\]

\[
+ \left( \sin \theta_t \right)^2 \left[ 1 + \exp(-i\alpha) \right] \frac{2\alpha + 1}{\sin \pi \alpha} \beta_{00;2} \frac{1}{2} (t) E_{02} \alpha_+(\cos \theta_t) \right\}^2
\]

\[
+ \left[ 1 + \exp(-i\alpha) \right] \frac{2\alpha + 1}{\sin \pi \alpha} \beta_{00;2} \frac{1}{2} (t) E_{00} \alpha_+(\cos \theta_t) \right\}^2
\]

\[
+ \sin \theta_t \left[ 1 + \exp(-i\alpha) \right] \frac{2\alpha + 1}{\sin \pi \alpha} \beta_{00;2} \frac{1}{2} (t) E_{01} \alpha_+(\cos \theta_t) \right\}^2
\}

(14)

The kinematic factors of the residue functions are

\[
\beta_{00;2} \frac{1}{2} (t) \ \text{and} \ \beta_{00;2} \frac{1}{2} (t) \propto \left[ \alpha(\alpha + 1) \right]^{\frac{1}{2}} t^{-\frac{1}{2}} \left[ t - \left( m_\Delta - m_N \right)^2 \right]^{\frac{1}{2}}
\]

\[
\times \int_{\pi N} (p_{\pi N} p_{\pi N})^{\alpha-1}, \quad (15a)
\]

\[
\beta_{00;2} \frac{1}{2} (t) \propto \left[ \alpha(\alpha + 1) (\alpha - 1) (\alpha + 2) \right]^{\frac{1}{2}} t^{-1} \left[ t - \left( m_\Delta - m_N \right)^2 \right] \left[ t - \left( m_\Delta + m_N \right)^2 \right]^{\frac{1}{2}}
\]

\[
\times \int_{\pi N} (p_{\pi N} p_{\pi N})^{\alpha-2}, \quad (15b)
\]

and
\[
\beta_{00; \frac{1}{2}; \frac{1}{2}}(t) \propto \alpha(\alpha + 1) \left[ t - (m_\Delta - m_N)^2 \right]^{-\frac{1}{2}} \left( p_\pi p_\Delta \right)^\alpha \quad \text{--- (15c)}
\]

After some rearrangements we obtain

\[
\frac{d\sigma}{dt} = \frac{1}{\sqrt{s}} \left[ 1 + \exp(-i\pi\alpha) \right]^2 \left[ \frac{\alpha}{\pi(\alpha + 1) \sin \pi\alpha} \right]^2
\times \left\{ \left| t^{-\frac{1}{2}} \sin \theta_t \right|^2 \left| \frac{1}{2} \left[ t - (m_\Delta - m_N)^2 \right]^{\frac{1}{2}} \sum \pi \right| \right]^2 \left[ \gamma_{00; \frac{1}{2}; \frac{1}{2}}(t) \right]^2
+ \left| \gamma_{00; \frac{1}{2}; \frac{1}{2}}(t) \right|^2 \left( \frac{s}{s_0} \right)^{2a-2}
+ \left| t^{-1}(\sin \theta_t)^2 \right|^2 \left| (\alpha - 1)^2 \left[ t - (m_\Delta + m_N)^2 \right] \left[ t - (m_\Delta + m_N)^2 \right]^{\frac{1}{2}} \sum \pi \right|^2 \left[ \gamma_{00; \frac{1}{2}; \frac{1}{2}}(t) \right]^2
\times \left| \gamma_{00; \frac{1}{2}; \frac{1}{2}}(t) \right|^2 \left( \frac{s}{s_0} \right)^{2a-4}
+ (\alpha + 1)^2 \left[ t - (m_\Delta + m_N)^2 \right]^{-\frac{1}{2}} \left[ \gamma_{00; \frac{1}{2}; \frac{1}{2}}(t) \right]^2 \left( \frac{s}{s_0} \right)^{2a} \right \}. \quad \text{--- (16)}
\]

The \( \gamma \)'s are defined through Eqs. (14) and (16). Notice that the differential cross section has minimums at negative odd integers of \( \alpha \).

F. \( \pi N \rightarrow \rho N \)

For charge-exchange \( \pi N \rightarrow \rho N \) interaction, \( A_2 \) and \( \pi \) can be exchanged. The \( A_2 \) can couple to \( \pi \rho \) only when \( \rho \) is in helicity state one. Due to conservation of G-parity, the \( \pi \) can couple to the \( NN \) system only when they are in the same helicity states. It turns out that there is no interference between the contributions from the two trajectories.
\[
\frac{d\sigma}{dt} = \frac{1}{l_{\pi s} \cdot p_{\pi N}} \frac{1}{2} \times 2 \times \frac{1}{4} \cdot \left\{ \frac{1 + \exp(-\alpha_{\pi} \chi)}{2 \pi \sin \alpha_{\pi}} \right\} \frac{\alpha_{\pi} + 1}{10; \frac{1}{2}} \pi(t) E_{00}^{\alpha_{\pi}^{'}} (\cos \theta_t) \right|^{2} \\
+ 2|\sin \theta_t|^2 \left[ 1 + \exp(-\alpha_{\pi} \chi) \right] \frac{\alpha_{\pi} + 1}{10; \frac{1}{2}} \pi(t) E_{01}^{\alpha_{\pi}^{'}} (\cos \theta_t) \right|^{2} \\
+ 2|\sin \theta_t|^2 \left[ 1 + \exp(-\alpha_{A} \chi) \right] \frac{\alpha_{A} + 1}{10; \frac{1}{2}} A(t) E_{01}^{\alpha_{A}^{'}} (\cos \theta_t) \right|^{2} \\
+ 2[1 + (\cos \theta_t)^2] \left[ 1 + \exp(-\alpha_{\pi} \chi) \right] \frac{\alpha_{\pi} + 1}{10; \frac{1}{2} - \frac{1}{2}} A(t) E_{11}^{\alpha_{\pi}^{'}} (\cos \theta_t) \right|^{2} \right\} 
\] 

(17)

The kinematic factors of the residue functions are

\[
\beta_{00; \frac{1}{2} \frac{1}{2}} \pi(t) \propto t^{\frac{1}{2}} \int_{\pi \rho}^{-1} (p_{\pi \rho} p_{NN}^{-1})^{\alpha_{\pi}} \right, 
\] 

(18a)

\[
\beta_{10; \frac{1}{2} \frac{1}{2}} \pi(t) \propto [\alpha_{\pi}(\alpha_{\pi} + 1)]^{\frac{1}{2}} t^{\frac{1}{2}} (t - \lim_{N}^{2})^{\frac{1}{2}} (p_{\pi \rho} p_{NN}^{-1})^{\alpha_{\pi}^{-1}} \right, 
\] 

(18b)

\[
\beta_{10; \frac{1}{2} \frac{1}{2}} A(t) \propto [\alpha_{A}(\alpha_{A} + 1)]^{\frac{1}{2}} \int_{\pi \rho} (p_{\pi \rho} p_{NN})^{\alpha_{A}^{-1}} \right, 
\] 

(18c)

\[
\beta_{10; \frac{1}{2} - \frac{1}{2}} A(t) \propto t^{\frac{1}{2}} \int_{\pi \rho} (p_{\pi \rho} p_{NN})^{\alpha_{A}^{-1}} \right. 
\] 

(18d)

Here we assume that the \(\pi\) trajectory couples to the sense-sense channels at \(\alpha_{\pi} = 0\).
\[
\frac{d\sigma}{dt} = \frac{1}{\sin \pi N} \left\{ \left[ 1 + \exp(-i\pi A) \right]^2 \left( \frac{1}{\Gamma(\alpha + 1) \sin \pi \gamma} \right)^2 \right. \\
	imes \left[ |t^{1\over 2} \int_\pi \rho -1|^2 |\gamma_{00;{1 \over 2} {1 \over 2}} | \pi(t)|^2 \left( \frac{s}{s_0} \right)^{2\alpha_\pi} \right. \\
+ |\sin \theta_t|^2 \alpha_\pi^2 |t^{1\over 2}(t - \omega N)^2|^{1\over 2} |\gamma_{10;{1 \over 2} {1 \over 2}} \pi(t)|^2 \left( \frac{s}{s_0} \right)^{2\alpha_\pi-2} \\
+ \left[ 1 + \exp(-i\pi A) \right]^2 \left( \frac{\alpha_A}{\alpha_A + 1} \right)^2 \left( \frac{\alpha_A}{\alpha_A + 1} \right) \sin \pi \gamma \right. \\
\times \left[ |\sin \theta_t|^2 (\alpha_A + 1)^2 |\int_\pi \rho|^2 |\gamma_{00;{1 \over 2} {1 \over 2}} A(t)|^2 \left( \frac{s}{s_0} \right)^{2\alpha_A^2} \right. \\
+ \left[ 1 + (\cos \theta_t)^2 \right] |t^{1\over 2} \int_\pi \rho|^2 |\gamma_{10;{1 \over 2} -{1 \over 2}} A(t)|^2 \left( \frac{s}{s_0} \right)^{2\alpha_A^2} \right. \} \\
\right\} \, (19)
\]

where \( \gamma \)’s are smoothly varying functions of \( t \) for \( t < 0 \). Notice that the contribution from \( \pi \) vanishes as the \( \pi \) trajectory passes \(-1\). For high enough energy, the contribution from \( \pi \) trajectory may be neglected. The \( \gamma^A \)'s here are related to that of the \( \pi N \rightarrow \eta N \) interaction through the following equation

\[
\frac{\beta_{10;{1 \over 2} {1 \over 2}} A(\pi \rho \rightarrow NN)}{\beta_{10;{1 \over 2} -{1 \over 2}} A(\pi \rho \rightarrow NN)} = \frac{\beta_{00;{1 \over 2} {1 \over 2}} A(\pi \eta \rightarrow NN)}{\beta_{00;{1 \over 2} -{1 \over 2}} A(\pi \eta \rightarrow NN)} \, (20)
\]

For non-charge-exchange \( \pi N \rightarrow \rho N \) interaction, the \( \omega \) trajectory can also be exchanged. It appears coherently with the \( A_2 \).
G. \( πN → αΔ (1236) \).

Again in this case, \( A_2 \) and \( π \) can be exchanged. Even though the \( π \) trajectory is lower than the \( A_2 \), there is still a possibility that the nearness of the \( π \)-pole to the s-channel physical region makes the contribution from \( π \) important up to moderately high energies. In that case we have to consider both \( π \) and \( A_2 \) exchange. However as energy becomes really high, eventually \( A_2 \) will take over. Here we consider only \( A_2 \) exchange. The kinematics here are similar to those for \( πN → ωΔ (1236) \). The differential cross section is in a form exactly the same as Eq. (4) except that the signature factor is \([1 + \exp(-iπα)]\), and the \( α \) and \( β \)'s refer respectively to the trajectory and the residue functions of \( A_2 \). The kinematic factors of the \( β \)'s for \( A_2 \) are the same as given by Eqs. (5a), (5b), and (5c), except for an additional \( α(α + 1) \) factor, assuming as usual that \( A_2 \) couples to nonsense-nonsense amplitudes. The final form of the differential cross section is

\[
\frac{dσ}{dt} = \frac{1}{8\pi N} \left| 1 + \exp(-iπα) \right|^2 \left[ \frac{α}{(α + 1) \Gamma(α + 1) \sin πα} \right]^2 \times \left[ [1 + (\cos θ_t)^2] \left| t - (m_Δ - m_N)^2 \right|^{1/2} \sum_{πω} |2 \left[ |γ_{10}^{3/2} \frac{1}{2} (t) |^2 + \left( \frac{8}{s_0} \right)^{2α-2} \left| \frac{1}{2} - \frac{1}{2} \right|^2 \right] + \left| t^{1/2} \sin θ_t \right|^2 [1 + (\cos θ_t)^2] (α - 1)^2 \left| \sum_{πω} \left[ t - (m_Δ + m_N)^2 \right]^{1/2} \right|^2 \right] \right.
\]

(Equation continued)
\[ x \frac{\gamma \left( \frac{\alpha}{s} \right)^{2 \alpha - 4}}{10 \frac{\alpha}{2} \frac{1}{2}} \left( \frac{s}{s_0} \right)^2 \]

\[ + \left| t^{\frac{3}{2}} \sin \theta_t \right|^2 (\alpha + 1)^2 \left| t - \left( m_\Delta - m_N \right)^2 \right|^{-\frac{3}{2}} \int_0^\infty \pi \omega \left| \gamma \frac{1}{10 \frac{\alpha}{2} \frac{1}{2}} \right|^2 \left( \frac{s}{s_0} \right)^{2 \alpha - 2} \]

where the \( \gamma \)'s are defined through Eqs. (4) and (21) and they are smoothly varying functions of \( t \) for \( t < 0 \). The \( \gamma \)'s here are related to those of \( \pi N \rightarrow \eta \Delta \) in Eq. (16) through the following equation:

\[
\frac{\beta_{10; \Delta} (\pi \rho + \Delta \bar{N})}{\beta_{10; \Delta'} (\pi \rho + \Delta \bar{N})} = \frac{\beta_{00; \Delta} (\pi \eta + \Delta \bar{N})}{\beta_{00; \Delta'} (\pi \eta + \Delta \bar{N})}
\]

Evidently many of the formulae of \( \pi N \) interactions considered here can be used for \( K N \) scattering.
III. NN INTERACTIONS

The NN scattering formulae are more complicated than those of \( \pi N \) scattering due to the presence of more particles with spin and more trajectories which can be exchanged. Even though we can write down formulae for all quasi two-body NN interactions, there will in general be so many arbitrary parameters as to make phenomenological fitting hopeless. We consider only three especially simple cases and for each keep only the highest trajectory that can be exchanged:

NN \( \rightarrow \) N \( \Delta \) (1236) with \( \rho \) exchange; and NN \( \rightarrow \) NN* (1500, \( \frac{3}{2}^- \)) and NN \( \rightarrow \) NN* (1480, \( \frac{1}{2}^+ \)) with P exchange.

A. NN \( \rightarrow \) N \( \Delta \) (1236)

An addition to \( \rho, \pi \) can also be exchanged. (For the same reasons as given for the reaction \( \pi N \rightarrow \rho \Delta \) (1236), the \( \pi \) contribution may be very important at intermediate energies.) Even with \( \rho \) alone there are eight residue functions. We shall see that we can combine some of the residue functions and reduce the number of arbitrary functions from eight to five in practical fitting. The unmodified differential cross section is

\[
\frac{d\sigma}{dt} = \frac{1}{4\pi s p_{NN}^2} \times \frac{1}{(2 \times \frac{1}{2} + 1)(2 \times \frac{1}{2} + 1)} \times 2 \times \frac{1}{4}
\]

\[
x \left\{ 2 |\sin \theta_t|^2 \left[ 1 - \exp(-\text{i}\alpha) \right] \frac{2\alpha + 1}{\sin \pi \alpha} \beta_3 \frac{1}{2^2}, \frac{1}{2} \right( t ) E_0, \alpha_t^+, (\cos \theta_t) \right\}^2
\]

\[
+ 2 |(\sin \theta_t)^2|^2 \left[ 1 - \exp(-\text{i}\alpha) \right] \frac{2\alpha + 1}{\sin \pi \alpha} \beta_3 \frac{1}{2^2}, \frac{1}{2} \right( t ) E_2, \alpha_t^+, (\cos \theta_t) \right\}^2
\]

(Equation continued)
\[
+ 2 \left| 1 - \exp(-i\alpha) \right| \left| \frac{2\alpha + 1}{\sin \pi \alpha} \beta_{1,1,1}(t) E_{00}^{\alpha}(\cos \theta_t) \right|^2 \\
+ 2 |\sin \theta_t|^2 \left| 1 - \exp(-i\alpha) \right| \left| \frac{2\alpha + 1}{\sin \pi \alpha} \beta_{1,1,1}(t) E_{01}^{\alpha}(\cos \theta_t) \right|^2 \\
+ 2[1 + (\cos \theta_t)^2] \left| 1 - \exp(-i\alpha) \right| \left| \frac{2\alpha + 1}{\sin \pi \alpha} \beta_{1,1,1}(t) E_{11}^{\alpha}(\cos \theta_t) \right|^2 \\
+ 2 |\sin \theta_t|^2 [1 + (\cos \theta_t)^2] \left| 1 - \exp(-i\alpha) \right| \left| \frac{2\alpha + 1}{\sin \pi \alpha} \beta_{1,1,1}(t) E_{12}^{\alpha}(\cos \theta_t) \right|^2 \\
+ 2 |\sin \theta_t|^2 \left| 1 - \exp(-i\alpha) \right| \left| \frac{2\alpha + 1}{\sin \pi \alpha} \beta_{1,1,1}(t) E_{10}^{\alpha}(\cos \theta_t) \right|^2 \\
+ 2[1 + (\cos \theta_t)^2] \left| 1 - \exp(-i\alpha) \right| \left| \frac{2\alpha + 1}{\sin \pi \alpha} \beta_{1,1,1}(t) E_{11}^{\alpha}(\cos \theta_t) \right|^2 \\
\right\}.
\]

The kinematic factors of the residue functions are \(^9\)

\[
\begin{align*}
\beta_{1,1,1}(t), \quad \beta_{1,1,1}(t) \quad \text{and} \quad \beta_{1,1,1}(t) \\
\alpha \left[ \alpha(\alpha + 1) \right]^{1/2} \left[ t - (m_\Delta - m_N) \right]^{-1/2} \left( p_{NN} p_{pN} \right)^{\alpha - 1}, \quad (23a) \\
\beta_{1,1,1}(t) \propto \left[ \alpha(\alpha + 1)(\alpha - 1)(\alpha + 2) \right]^{1/2} \left[ t - \left( \frac{2}{m_N} \right) \right]^{1/2} \left[ t - \left( \frac{2}{m_N} \right) \right]^{1/2} \\
\times \left( p_{NN} p_{pN} \right)^{\alpha - 2}, \quad (23b)
\end{align*}
\]
\[
\beta_1 \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} (t) \propto (t - \frac{m_N}{2})^{-\frac{1}{2}} \left[ t - (m_\Delta + m_N)^2 \right]^{-\frac{1}{2}} \left[ t - (m_\Delta - m_N)^2 \right]^{-1} \\
\times (p_{NN} p_{N\Delta})^\alpha , \quad (23c)
\]

\[
\beta_3 \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} (t) \quad \text{and} \quad \beta_3 \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} (t) \\
\alpha = \alpha (\alpha + 1) \left[ t - (m_\Delta - m_N)^2 \right]^{-\frac{1}{2}} (p_{NN} p_{N\Delta})^{\alpha - 1} , \quad (23d)
\]

\[
\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} (t) \propto \alpha (\alpha + 1) \left[ (\alpha - 1) (\alpha + 2) \right]^{\frac{1}{2}} t^{\frac{1}{2}} (t - \frac{m_N}{2})^{\frac{1}{2}} \\
\times \left[ t - (m_\Delta + m_N)^2 \right]^{\frac{1}{2}} (p_{NN} p_{N\Delta})^{\alpha - 2} . \quad (23e)
\]

The final form of the differential cross section is

\[
\frac{da}{dt} = \frac{1}{s(s - \frac{1}{2} m_N^2)} \left| 1 - \exp(-im\alpha) \right|^2 \left[ \frac{1}{\Gamma(\alpha + 1) \sin \pi \alpha} \right]^2 \\
\times \left\{ |\sin \theta_t|^2 \alpha^2 \left| t - (m_\Delta - m_N)^2 \right|^{-\frac{1}{2}} \right\}^2 \left[ \gamma_3 \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} (t) \right]^2 + |\gamma_3 \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} (t)|^2 \left( \frac{s}{s_0} \right)^{2\alpha - 2} \\
+ |(\sin \theta_t)^2| \alpha^2 (\alpha - 1)^2 \left| t - \frac{1}{2} m_N^2 \right| \left| t - (m_\Delta + m_N)^2 \right|^{\frac{1}{2}} \left[ \gamma_3 \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} (t) \right]^2 \left( \frac{s}{s_0} \right)^{2\alpha - 4} \}
\]

(Equation continued)
where the $\gamma$'s are defined through Eqs. (22) and (24). Notice the $\alpha$ factor in all terms of Eq. (24) except the third term. Thus if $d\alpha / dt$ turns out not too big, we can observe a minimum in $d\alpha / dt$ at $\alpha(t) = 0$. The $\gamma$'s here are related to those of $\pi N \to \pi \Delta$ (1236) and $\pi N \to \pi N$ by

$$\frac{\beta_{NN;\bar{N}\Delta}(\bar{N}N \to \bar{N}\Delta)}{\beta_{NN;\bar{N}'\Delta'}(\bar{N}N \to \bar{N}\Delta)} = \frac{\beta_{00;\bar{N}\Delta}(\pi N \to \bar{N}\Delta)}{\beta_{00;\bar{N}'\Delta'}(\pi N \to \bar{N}\Delta)},$$

(25a)

and

$$\frac{\beta_{NN;\bar{N}\Delta}(\bar{N}N \to \pi \Delta)}{\beta_{NN;\bar{N}'\Delta'}(\bar{N}N \to \bar{N}\Delta)} = \frac{\beta_{NN;\bar{N}\Delta}(\bar{N}N \to \pi \Delta)}{\beta_{NN;\bar{N}'\Delta'}(\bar{N}N \to \pi \Delta)}. $$

(25b)
Here almost all non-strange trajectories can be exchanged. Due to the availability of very high-energy data for this interaction, hopefully the consideration of only the $P$ trajectory will be adequate. The formalism for this case is quite the same as that of $NN \rightarrow N\Delta$ (1236), except that due to the difference in parity the kinematic factors of the residue functions $\beta$'s are different. They are

\begin{align*}
\beta_{\frac{3}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}}(t) & , \ \beta_{\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}}(t) \ \text{and} \ \beta_{\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}}(t) \\
\propto [\alpha(\alpha + 1)]^{\frac{1}{2}} [t - (m_N^2 + m_N^2)]^{-\frac{1}{2}} (p_{NN}^2 p_{NN}^2)^{\alpha - 1} , \quad (26a)
\end{align*}

\begin{align*}
\beta_{\frac{3}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}}(t) & \propto [\alpha(\alpha + 1) (\alpha - 1) (\alpha + \alpha)]^{\frac{1}{2}} (t - 4m_N^2)^{\frac{1}{2}} \\
& \times [t - (m_N^2 + m_N^2)]^{\frac{1}{2}} (p_{NN}^2 p_{NN}^2)^{\alpha - 2} , \quad (26b)
\end{align*}

\begin{align*}
\beta_{\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}}(t) & \propto \alpha(\alpha + 1) (t - 4m_N^2)^{-\frac{1}{2}} [t - (m_N^2 + m_N^2)]^{-\frac{1}{2}} \\
& \times [t - (m_N^2 + m_N^2)]^{1} (p_{NN}^2 p_{NN}^2)^{\alpha} , \quad (26c)
\end{align*}

\begin{align*}
\beta_{\frac{3}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}}(t) \ \text{and} \ \beta_{\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}}(t) \\
\propto t^{\frac{1}{2}} [t - (m_N^2 + m_N^2)]^{\frac{1}{2}} (p_{NN}^2 p_{NN}^2)^{\alpha - 1} , \quad (26d)
\end{align*}
\[
\frac{\partial \sigma}{\partial t} = \frac{1}{s(s - \left( m_N^a \right)^2)} \left[ 1 + \exp(-i\alpha) \right]^2 \left[ \frac{\alpha}{\alpha + 1} \right]^{-1/2} \sin \pi \alpha \left[ \left( \frac{2\alpha - 2}{\sin \pi \alpha} \right)^2 + \left( \frac{2\alpha - 4}{\sin \pi \alpha} \right)^2 \right]^{2\alpha - 2}\]

where we use \( N \) to denote \( N^* \) for the convenience of notation.

The final form of the differential equation is

\[
\frac{\partial \sigma}{\partial t} = \frac{1}{s(s - \left( m_N^a \right)^2)} \left[ 1 + \exp(-i\alpha) \right]^2 \left[ \frac{\alpha}{\alpha + 1} \right]^{-1/2} \sin \pi \alpha \left[ \left( \frac{2\alpha - 2}{\sin \pi \alpha} \right)^2 + \left( \frac{2\alpha - 4}{\sin \pi \alpha} \right)^2 \right]^{2\alpha - 2}\]

(Equation continued)
The $\gamma$'s are related to $\beta$'s by Eqs. (22) and (27).

C. $\text{NN} \rightarrow \text{NN}^* (1480, \frac{1}{2}^+)$.

Here we consider only the contribution from the $P$ trajectory.

$$
\frac{d\sigma}{dt} = \frac{1}{4\pi s P_{\text{NN}}^2} \frac{1}{(2 \times \frac{1}{2} + 1)(2 \times \frac{1}{2} + 1)} \times 2 \times \frac{1}{4}
$$

$$
\times \left\{ 2 \left[ 1 + \exp(-i\alpha) \right] \frac{2\alpha + 1}{\sin \pi \alpha} \beta_{\frac{1}{2} - \frac{1}{2} \frac{1}{2}}(t) \, E_{00}^{\alpha^+} \left( \cos \theta_t \right) \right\}^2
$$

$$
+ 2 \sin \theta_t \left[ 1 + \exp(-i\alpha) \right] \frac{2\alpha + 1}{\sin \pi \alpha} \beta_{\frac{1}{2} - \frac{1}{2} \frac{1}{2}}(t) \, E_{10}^{\alpha^+} \left( \cos \theta_t \right) \right\}^2
$$

$$
+ 2 \sin \theta_t \left[ 1 + \exp(-i\alpha) \right] \frac{2\alpha + 1}{\sin \pi \alpha} \beta_{\frac{1}{2} - \frac{1}{2} \frac{1}{2}}(t) \, E_{01}^{\alpha^+} \left( \cos \theta_t \right) \right\}^2
$$

$$
+ 2 \left[ 1 + (\cos \theta_t)^2 \right] \left[ 1 + \exp(-i\alpha) \right] \frac{2\alpha + 1}{\sin \pi \alpha} \beta_{\frac{1}{2} - \frac{1}{2} \frac{1}{2}}(t) \, E_{11}^{\alpha^+} \left( \cos \theta_t \right) \right\}^2
$$

The kinematic factors of the $\beta$'s are

$$
\beta_{\frac{1}{2} \frac{1}{2} \frac{1}{2}}(t) \propto \alpha (\alpha + 1) \left( t - \frac{2m_N^2}{m_N} \right)^{\frac{1}{2}} \left[ t - (m_N^2 + m_N^2) \right]^{\frac{1}{2}} \times (P_N P_{\text{NN}})^\alpha,
$$

(29a)
The final form of the differential cross section is

\[
\frac{d\sigma}{dt} = \frac{1}{s(s - 4m_N^2)} \left| 1 + \exp(-im\alpha) \right|^2 \left| \frac{\alpha}{(\alpha + 1)^2} \frac{1}{\Gamma(\alpha + 1) \sin \pi \alpha} \right|^2 \\
\times \left\{ (\alpha + 1)^4 \left| (s - 4m_N^2)^{-2} [t - (m_N^2 + m_N^2)^{-2}]^{\frac{1}{2}} \frac{1}{\gamma_1 \gamma_2 \gamma_3 \gamma_4} \right|^2 \\
\times \left( \frac{s}{s_0} \right)^{2\alpha} \right\} \\
+ \left| \sin \theta_t \right|^2 (\alpha + 1)^2 \left| [t - (m_N^2 - m_N^2)]^{\frac{1}{2}} \right|^2 \\
\times \left[ \left| \gamma_1 \gamma_2 \gamma_3 \gamma_4 \frac{1}{2} \right|^2 + \left| \gamma_1 \gamma_2 \gamma_3 \gamma_4 \frac{1}{2} \right|^2 \left( \frac{s}{s_0} \right)^{2\alpha-2} \right] \\
+ [1 + (\cos \theta_t)^2] \left| t^{\frac{1}{2}} [t - (m_N^2 - m_N^2)]^{\frac{1}{2}} \right|^2 \left| \gamma_1 \gamma_2 \gamma_3 \gamma_4 \frac{1}{2} \right|^2 \left( \frac{s}{s_0} \right)^{2\alpha-2} \right\},
\]

(30)

where the \( \gamma \)'s are defined through Eqs. (28) and (30).

Obviously these formulae for NN interactions can also be used for N\bar{N} interactions.
ACKNOWLEDGMENT

I am deeply grateful to Professor Geoffrey F. Chew for his advice and guidance throughout the development of this work.
In Ref. 5, the author didn't write down explicitly the kinematic singularity and zero-free helicity amplitudes for the case of \( m_a = m_b \) and \( m_c \neq m_d \), which turns out to be the most useful case. We write the result here as a supplement to the paper. In the case of \( m_a = m_b \), \( m_c \neq m_d \); or \( m_a = m_b = m_c \neq m_d \), the following helicity amplitudes are free of kinematic singularities and zeros:

\[
[I_{cd;ab}^s + I_{cd;ab}^{-s}] \left[ s - \frac{1}{4} m_a^2 \right]^{1/2} \alpha_1 \left[ s - (m_c + m_d)^2 \right]^{1/2} \beta_1 \\
\times \left[ s - (m_c - m_d)^2 \right]^{1/2} \beta_2 \frac{1}{s} \alpha'_2 ,
\]

where

\[
\alpha'_2 = \max (\uparrow) \eta_{ab} \text{ of } [J_a + J_b - \frac{1}{2} (v_a + v_b)] + \frac{1}{2} (v_a + v_b) ,
\]

for \( v_a = v_b = 1 \);

\[
\alpha'_2 = \max (\uparrow) \eta_{ab} \text{ of } [J_a + J_b] , \text{ for } v_a = v_b = 0 .
\]

All the notations here are defined the same way as in Ref. 5.
It is an interesting phenomenon that purely due to the presence of some kinematic factor of $\alpha$ in the amplitude, the contribution of a Regge pole to certain amplitude can vanish and it thus produces a minimum in the differential cross section. We illustrate this point by observing the simplest case of no spin. The contribution of a Regge pole to the amplitude is

$$f(s,t) \approx \left[1 \mp \exp(-i\pi\alpha(t))\right] \frac{2\alpha(t)+1}{\sin \pi\alpha(t)} \beta(t) \frac{\sin(\alpha(t))}{\sinh(\alpha(t))} \left(\frac{s}{pt} \frac{p_t}{p}\right)^{\alpha},$$

$$\times \left[1 \mp \exp(-i\pi\alpha)\right] \frac{2\alpha + 1}{\sin \pi\alpha} \beta(t) \frac{\Gamma(\alpha + \frac{1}{2})}{(\pi)^{\frac{1}{2}} \Gamma(\alpha + 1)} \left(\frac{s}{p_t} \frac{p_t}{p}\right)^{\alpha}. \quad (II.3)$$

Now let us focus our attention on the factors containing $\alpha$ in, say, the case of positive signature:

$$\left[1 + \exp(-i\pi\alpha)\right] \frac{(2\alpha + 1) \Gamma(\alpha + \frac{1}{2})}{\Gamma(\alpha + 1)}.$$

At $\alpha = -1, -2, -3, etc.$, the $1/\Gamma(\alpha + 1)$ has zeros that are nonsense (unphysical) values of $\alpha$. These zeros are to cancel the pole of $1/\sin \pi\alpha$ at these nonsense values of $\alpha$. However, at the wrong signature values of $\alpha$, i.e., $\alpha = -1, -3, -5, etc.$, the signature factor $[1 + \exp(-i\pi\alpha)]$ also vanishes. Thus the amplitude vanishes at these values of $\alpha$. Similar arguments apply also to the case of negative signature. We see that the amplitude always vanishes at nonsense and wrong-signature values of $\alpha$. For cases with spins, the details of
deriving this conclusion are more complicated; however, the principle is the same. The interactions considered in this paper provide an illustration. The poles of \((2\alpha + 1) \Gamma(\alpha + \frac{1}{2})\) at \(\alpha = -\frac{3}{2}, -\frac{5}{2}, \ldots\) in Eq. (II.3) are annihilated by zeros in the residue function.

So when the spins of external particles are high, for some amplitudes even \(\alpha(t) = 0, 1, 2, \ldots\) can become non-sense and the amplitudes can vanish at those values of \(\alpha(t)\) with the wrong signature. Of course only those t-channel zeros at \(\alpha(t) < 1\) of the amplitudes will have an effect on the s-channel differential cross-section.
APPENDIX III.

The kinematic factors $\left[\sin\left(\frac{1}{2} \theta_t\right)\right]^{\lambda'-\mu'} \left[\cos\left(\frac{1}{2} \theta_t\right)\right]^{\lambda'+\mu'}$ of $f_{cA};Db^t$ with $\lambda' \equiv D - b$, $\mu' \equiv c - A$, are one or zero in the forward direction of the s-channel for production interactions (i.e., $m_a \neq m_b \text{ or } m_c \neq m_d$; or both), since at this point $\sin \theta_t = 0$ and $\left|\cos \theta_t\right| = 1$. We use $a + b \rightarrow c + d$ to denote an s-channel interaction and $D + b \rightarrow c + A$ to denote t-channel interaction. Thus the contribution of a Regge pole is diminished for those amplitudes with $\lambda' \neq 0$ or $\mu' \neq 0$, or $\lambda' \neq 0$ and $\mu' \neq 0$ compared with those having both $\lambda' = 0$, $\mu' = 0$. For example, if particles $c$ and $c'$ can be produced through the interactions $a + b \rightarrow c + d$ and $a + b \rightarrow c' + d$, respectively, with the same highest trajectory exchange, while at the same time $c$ can couple to the t-channel helicity amplitude with $\lambda' = 0$ and $\mu' = 0$ but $c'$ can not, then the relative production rate of $c$ will be greater than that of $c'$. Whether the particle can couple to the t-channel helicity amplitude with $\lambda' = 0$ and $\mu' = 0$ depends on its spin and parity relative to that of the particle $a$; therefore this may constitute a way of determining the spin and parity of the produced particle, in case the spin and parity of the particle $a$ are known.\textsuperscript{20}

Sufficient conditions that would prevent a Regge pole with $(J \text{-parity}) \times \text{parity} = -$ from coupling to the t-channel helicity amplitude with $\mu' = 0$ are
(a) One of the particles A or c has spin zero. Let us choose this to be the particle a.

(b) The spins and intrinsic parities of A and c are such that
\[ \eta_A \eta_c (-)^{S_A + S_c} = -1. \]
When conditions (c) and (b) are satisfied, the (J-parity) \( \times \) parity = + helicity state of A and c with \( \mu' = 0 \) does not exist. A similar argument applies for the amplitudes with \( \lambda' = 0 \). We see that there is no selection rule at any fermion-fermion vertex. An example of the above is that the relative production rate of \( A_2 (2^+) \) in the interaction \( nN \rightarrow A_2 N \) will be more diminished than that of \( A_1 (1^+) \) in the interaction \( nN \rightarrow A_1 N \). The \( P \) trajectory can not contribute with full strength in the interaction \( nN \rightarrow A_2 N \).

If the highest trajectory exchanged has (J-parity) \( \times \) parity = - , the particle c with \( \eta_A \eta_c (-)^{S_A + S_c} = 1 \) will have a smaller relative production rate at high energies.
FOOTNOTES AND REFERENCES

* Work done under the auspices of the U. S. Atomic Energy Commission.


5. L. Wang, Phys. Rev. 142, 1187 (1966); See also Appendix I. The procedure of obtaining the differential cross section is given very clearly in Ref. (3).
6. Thews has studied this interaction (R. Thews, Regge Pole Analysis of \(N^*(1238)\) Production, Laboratory for Nuclear Science and Physics Department, Massachusetts Institute of Technology Preprint, March 1966). But I do not think that he used the correct theoretical formula. See the remark at the end of the paragraph of \(\pi N \to \pi \Delta(1236)\) in this paper and Ref. 13.

7. \(\sin \theta_t = 2[t\delta(s,t)]^{1/2}/\mathcal{J}_{ac} \mathcal{J}_{bd}\), where \(\delta(s,t) = st(\Sigma m_1^2 - s - t) - s(m_b^2 - m_d^2)(m_a^2 - m_c^2) - t(m_a^2 - m_b^2)(m_c^2 - m_d^2) - (m_c^2 - m_a^2 - m_b^2 - m_d^2)\), \(\mathcal{J}_{ac} = 4t p_{ac}\), and \(\mathcal{J}_{bd} = 4t p_{bd}\), for \(a + b \to c + d\) being the s-channel and \(D + b \to c + A\) being the t-channel.

8. Notice that the residue functions have the usually assumed threshold behavior at \(t = 4m_\pi^2\) and \(t = (m_\Delta + m_N)^2\); i.e., \(\beta(t) \propto [t - (m_\Delta + m_N)]^L (t - 4m_\pi^2)^{L'}\), where \(L, L'\) is the lowest orbital angular momentum for a fixed \(J_t\) of the \(N\Delta\) and \(\pi\pi\) systems respectively. We find that this is true for all the interactions considered in this paper.

9. At integral values of \(\alpha\) (and their symmetric points about \(\alpha = \frac{-1}{2}\)), nonsense-sense residue functions vanish in a square-root fashion. For example, if \(\alpha = 0\) is such a point, \(\beta_{sn} \propto [\alpha(\alpha + 1)]^{1/2}\), where \(s\) stands for sense and \(n\) for nonsense. The factorizability of the residue function says that \(\beta_{sn}^2 = \beta_{ss} \beta_{nn}\). Therefore either \(\beta_{ss}\) or \(\beta_{nn}\) has a factor \(\alpha(\alpha + 1)\). Which one is chosen is a dynamical question. Throughout this paper our formulae involving the \(\rho\) trajectory are obtained with the assumption that the \(\rho\) couples
to sense-sense channels at \( \alpha = 0 \) \( \beta_{sn} \propto [\alpha(\alpha + 1)]^{1/2} \), \( \beta_{ss} \propto 1 \), \( \beta_{nn} \propto \alpha(\alpha + 1) \). If it couples to nonsense-nonsense channels, then \( \beta_{sn} \propto [\alpha(\alpha + 1)]^{1/2} \), \( \beta_{ss} \propto \alpha(\alpha + 1) \), \( \beta_{nn} \propto 1 \). For example, the \( \beta_{00}^{1/2,1/2} \) of Eq. (2c) should then have a factor
\[ \alpha(\alpha + 1); \beta_{10}^{3/2,1/2} \text{ of Eq. (5a) and } \beta_{20}^{3/2,-1/2} \text{ of Eq. (5b)} \]
should not have a factor \( \alpha(\alpha + 1); \beta_{00}^{1/2,1/2} \) of Eq. (9c) and
\[ \beta_{10}^{1/2,-1/2} \text{ of Eq. (9f) should not have the factor } \alpha(\alpha + 1); \beta_{20}^{1/2,1/2} \]
of Eq. (23c) should have a factor \( \alpha(\alpha + 1) \), and
\[ \beta_{3/2,1/2} \text{ of Eq. (23d) and } \beta_{3/2,1/2} \text{ of Eq. (23e)} \] should not have the factor \( \alpha(\alpha + 1) \). Also the differential cross sections should be changed accordingly. Hopefully by fitting the experimental data, we can determine which is the correct coupling.

10. The asymptotic forms of the E-functions used in this paper are:

\[ E_{00}^{\alpha_{+}}(\cos \theta) \propto \frac{\Gamma(\alpha + \frac{1}{2})}{(\pi)^{1/2} \Gamma(\alpha + 1)} \left( \frac{s}{p_{t}p'_{t}} \right)^{\alpha} \],

\[ E_{01}^{\alpha_{+}}(\cos \theta) \propto \frac{2\alpha}{[\alpha(\alpha + 1)]^{1/2}} \frac{\Gamma(\alpha + \frac{3}{2})}{(\pi)^{1/2} \Gamma(\alpha + 1)} \left( \frac{s}{p_{t}p'_{t}} \right)^{\alpha-1} \],

\[ E_{02}^{\alpha_{+}}(\cos \theta) \propto \frac{4\alpha(\alpha - 1)}{[\alpha(\alpha + 1)(\alpha - 1)(\alpha + 2)]^{1/2}} \frac{\Gamma(\alpha + \frac{3}{2})}{(\pi)^{1/2} \Gamma(\alpha + 1)} \left( \frac{s}{p_{t}p'_{t}} \right)^{\alpha-2} \],

\[ E_{11}^{\alpha_{+}}(\cos \theta) \propto \frac{2\alpha^{2}}{\alpha(\alpha + 1)} \frac{\Gamma(\alpha + \frac{3}{2})}{(\pi)^{1/2} \Gamma(\alpha + 1)} \left( \frac{s}{p_{t}p'_{t}} \right)^{\alpha-1} \],

\[ E_{12}^{\alpha_{+}}(\cos \theta) \propto \frac{4\alpha^{2}(\alpha - 1)}{\alpha(\alpha + 1)(\alpha - 1)(\alpha + 2)]^{1/2}} \frac{\Gamma(\alpha + \frac{3}{2})}{(\pi)^{1/2} \Gamma(\alpha + 1)} \left( \frac{s}{p_{t}p'_{t}} \right)^{\alpha-2} \].
The E-functions have poles at all half integers of \( \alpha \). When a physical trajectory passes a positive half integer, the pole of the E-function in the amplitude will be either cancelled by its compensating trajectory passing \( -\alpha - 1 \) or its residue function vanishes.\(^{11}\) However the leading term at high \( \cos \theta \) in the E has poles only at negative half integers. Thus the vanishing of the residue can give a zero in the asymptotic form of the amplitude. When the leading trajectory passes a negative half integer, there obviously can not be any compensating trajectory. Therefore it is the vanishing of the residue function that cancels the poles at negative half integers. The author would like to thank Professor S. Mandelstam for informing her that the compensating trajectory always exists for a trajectory passing positive half integers in potential theory.

12. See F. Arbab and C. Chiu's paper in Ref. 1.
13. With the assumption of the existence of Daughter Trajectories, D. Z. Freedman and J. M. Wang have shown that this is true. (Regge Poles and Unequal Mass Scattering Processes, Department of Physics, University of California preprint, July 1966. And related references are given in the paper).
14. In this paper all the formulae involving \( P, P', \) and \( A_2 \) are written according to one of the mechanisms proposed by Gell-Mann to cancel the pole at \( \alpha = 0 \) in the amplitudes; these trajectories all couple to nonsense-nonsense channels at \( \alpha = 0 \) and the pole
in the nonsense-nonsense channels is cancelled by another trajectory passing \( \alpha = -1 \), (i.e., \( \beta_{nn} \propto \alpha(\alpha + 1) \), \( \beta_{ss} \propto \alpha^2(\alpha + 1) \), \( \beta_{nn} \propto 1 \)). His alternate mechanism is to have the pole in the nonsense-nonsense channels annihilated by a zero of the residue functions, \( \left[ \text{i.e.,} \beta_{sn} \propto \alpha(\alpha + 1) \right]^{\frac{1}{2}}, \beta_{ss} \propto \alpha^2(\alpha + 1), \beta_{nn} \propto \alpha \right] \).

G. Chew has proposed a different mechanism [G. Chew, Phys. Rev. Letters 16, 60 (1966)], in which the trajectories couple to the sense-sense channel at \( \alpha = 0 \), but \( \beta_{ss} \) vanishes there, i.e., \( \beta_{sn} \propto \alpha(\alpha + 1) \), \( \beta_{ss} \propto \alpha \), \( \beta_{nn} \propto \alpha^2(\alpha + 1) \).

J. Finkelstein and Boris Kayser pointed out that in potential theory the coupled sense-sense residue function can not change sign for \( t < \text{threshold} \). Thus if \( \beta_{ss} \) vanishes, it must have a double zero, \( \left[ \text{i.e.,} \beta_{sn} \propto \alpha(\alpha + 1) \right]^{\frac{1}{2}}, \beta_{ss} \propto \alpha^2 \), \( \beta_{nn} \propto \alpha(\alpha + 1) \). However, for \( t < 0 \) non-relativistic arguments are of dubious value. In the relativistic case, the kinematic singularity free residue functions are real analytic and have right-hand cuts. This does not rule out the possibility of their changing sign for \( t < 0 \). Actually there are experimental indications that the residue functions need to change sign to explain certain aspects of \( \pi N \) and \( N N \) scattering [R. Phillips and W. Barite, Phys. Rev. 139, B1336 (1965); E. Leader and R. C. Slansky, University of California Lawrence Radiation Laboratory Report No. UCRL-16597, 1965 (to be published in Phys. Rev.)]. The author would like to thank Professor Chew and Mr. Finkelstein for discussions.

15. See Ref. 11 of Ref. 3.
16. See Appendix III.

17. R. Phillips and W. Rarita studied this interaction [Phys. Rev. Letters 15, 807 (1965)]. They applied the kinematics of \( \pi N \rightarrow \pi N \) to this case, i.e., instead of \( |\mathcal{A}_{\pi\pi} \sin \theta_t (\pi\pi \leftrightarrow \pi\pi)|^2 \) in the second term of Eq. (13), they used \( |\mathcal{A}_{\pi\pi} \sin \theta_t (\pi\pi \leftrightarrow NN)|^2 \). The author found that numerically the two can differ by a factor of two for small \(|t|\).

18. J. Stack has also observed this phenomenon for the case of no spin.

19. I owe this nice summarizing statement to Prof. G. Chew.

20. I would like to thank Dr. A. Goldhaber for bringing my attention to this selection rule. The same conclusion is obtained in coherent productions in heavy nuclei by Drs. A. Goldhaber and M. Goldhaber [Coherent High Energy Reactions With Nuclei, 'Preludes in Theoretical Physics, edited by A. De-Shalit, H. Feshbach, L. Van Hove (John Wiley and Sons, Inc., New York, 1966)].
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