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On Sequential Commitment in the Price-Dependent Newsvendor Model

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Abstract

We investigate the effect of sequential commitment in the decentralized newsvendor model with price-dependent demand. Sequential commitment allows the self-profit maximizing parties to commit to contract parameters (e.g., wholesale price, retail price, buyback price and order quantity) sequentially and alternately, and we investigate its effect on the equilibrium profits of the channel and its members. Sequential commitment introduces more flexibility to contracting in the supply chain and our analysis can provide some insight to channel members who follow a bargaining process to determine the values of contract parameters. We show that the introduction of sequential commitment to the price-dependent (PD) newsvendor model with buybacks can significantly improve the manufacturer's and the channel expected profits, but it can also decrease the retailer's expected profit. Finally, we demonstrate that with sequential commitment, under some conditions, the choice of the first mover is endogenized and we identify the unique sequence of commitments by channel members that would arise in equilibrium.

Keywords: Supply chain management, Price-dependent newsvendor model, Buybacks, Equilibrium sequence

1 Introduction

It is essentially exclusively assumed in the vast literature on the decentralized newsvendor model that the decision makers commit to the values of the decision variables under their control *simultaneously.* For example, in the price-dependent (PD) newsvendor model with buybacks (e.g., Kandel (1996), Emmons and Gilbert (1998)), the manufacturer (M), assumed to be the Stackelberg leader, initiates the process by offering a take-it-or-leave-it contract, in which M decides (after taking the retailer's reaction into account), simultaneously, upon the values of a per unit wholesale price w and a per unit buyback rate b, at which she will buy excess items back at the end of the selling season. The retailer (R) then decides upon a per unit retail price p and an order quantity Q. Similarly, when one uses a sales-rebate scheme (Taylor (2002)) to coordinate the channel in a PD-newsvendor model, M simultaneously decides upon w, a per unit rebate r ($r > 0$), which will be given above a fixed threshold T, and then R decides upon p and Q.

Our objective in this paper is to introduce a sequential commitment approach for determining the contract parameters and to analyze its effect on the efficiency of the supply chain and the fortunes of its members. By sequential commitment approach we mean that the parties can decide upon the values of the decision variables sequentially and alternately, and they can also decide upon the order at which they commit to the values of the decision variables under their control. For example, consider the PD-newsvendor model with buybacks, in which M controls w and b and R controls p and Q. Then, we would like to study the effect of an approach wherein, for example, M, after setting up the value of w, expects R to either commit to a retail price p or to an order quantity Q, or both, before she commits to a buyback rate b. After R sets the value of either p or Q, or both, M commits to a buyback rate b, and then, R sets the value of the remaining decision variable under his control. We assume that all commitments by M and R to values of contract parameters under their control are credible and verifiable.

As compared to the traditional approach, in which contracting follows the take-it-or-leave-it paradigm, the sequential commitment approach introduces more flexibility to contracting. Indeed, while the traditional approach does not incorporate any element of bargaining or negotiation between M and R, the sequential approach captures some aspects of a bargaining process by which the values of contract parameters (e.g., wholesale price, buyback rate or order quantity) are determined. As such, our sequential commitment approach is in the spirit of contributions by, e.g., Nagarajan and Bassok (2002), and Iyer and Vilas-Boas (2003), wherein the interaction between M and R, for the purpose of determining the values of contract parameters, is modelled as the Nash bargaining problem (1950). Such an approach to model contracting between the parties should be viewed more realistic than the traditional approach. Indeed, to quote from Nagarajan and Bassok (2002): "Anecdotal evidence and articles in the academic literature have overwhelmingly indicated that relationships between agents in a supply chain are characterized by bargaining over terms of the trade. Sellers and buyers often negotiate price, quantity, delivery schedules, etc."

To gain an insight into the effect of such a sequential commitment approach, we study in this paper its effect on the PD-newsvendor model under linear buyback contracts when M controls w and b and R controls p and Q. Linear buyback contracts, wherein the wholesale and buyback prices are constant and independent of the values of other decision variables, e.g., retail price, are very popular. Indeed, they are prevalent in several industries, and are considered to be one of the most popular contracts after wholesale price-only contracts, see, e.g., Marvel and Peck (1995).

We investigate two different power structures regarding the first mover in the channel. More specifically, we study not only the case where M is the Stackelberg leader but also the case in which R plays the Stackelberg leader role. We note, however, that being the Stackelberg leader does not necessarily imply being more powerful. Indeed, a more powerful firm could, in principle, force the other firm to move first if it is advantageous for it to do so.

The literature on supply chains with buybacks and coordination is quite extensive. In general, a supply chain composed of independent agents trying to maximize their own profits does not achieve channel coordination, see, e.g., Spengler (1950). Pasternack (1985) was the first to show that buybacks can coordinate the basic price-independent newsvendor model, wherein the retail price p is fixed exogenously, and Padmanabhan and Png (1995) have discussed and analyzed the benefits and costs of accepting returns from retailers. Subsequently, other contracts, such as, e.g., quantity-flexibility (Tsay (1999)), sales-rebate (Taylor (2002)), and revenue-sharing (Pasternack (1999), Cachon and Lariviere (2005)), have also been shown to be able to coordinate the basic newsvendor model. See also Lariviere (1999), Tsay et al. (1999) and Cachon (2004) for some excellent reviews of coordination mechanisms for the basic newsvendor model and related models.

As noted by Kandel (1996), the price-dependent (PD) newsvendor model, in which the retail price is determined endogenously by R, is considerably more complicated. Emmons and Gilbert (1998) have studied the PD-newsvendor model wherein the expected demand function is linear and the random component of demand follows a uniform distribution, and they have shown that if the wholesale price is large enough, both M and R would benefit from the introduction of buybacks. Granot and Yin (2004a) have shown that in the PD-newsvendor model studied by Emmons and Gilbert (1998), wherein, e.g., expected demand is a linear function of the retail price, the efficiency of the supply chain with buybacks is precisely 75%, and channel efficiency improvement due to the introduction of buybacks is quite insignificant and is upper bounded by 3.16%. It has been conjectured by Kandel (1996) and Lariviere (1999) that constant wholesale and buyback prices (i.e., independent of other decision variables) alone cannot, in general, lead to coordination in the PD-newsvendor model. By contrast, contracts which do not employ constant wholesale and buyback prices can induce coordination. For example, revenue-sharing contracts and the "linear price discount sharing" scheme, have been shown by Cachon and Lariviere (2005) and by Bernstein and Federgruen (2005), respectively, that they could induce coordination in the PD-newsvendor model.

Our objective in this paper is *not* to investigate channel coordination. Rather, our aim is to investigate the effect of sequential commitment in the PD-newsvendor model, bearing in mind that such an approach introduces more flexibility to the possible interaction between M and R, and that it can provide useful information to the parties who use bargaining to determine the values of contract parameters.

Our main results are:

(I) For a uniformly distributed random component of demand, and linear, exponential and negative polynomial expected demand functions:

  (i) The decision as to who will be the first mover has been endogenized. That is, R would rather have M move first, and M would be pleased to do so.

  (ii) Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$, wherein M first offers a buyback rate b, R then determines the retail price p, M subsequently decides upon the wholesale price w, and finally, R sets Q, is the unique equilibrium sequence. That is, R does not want to be the first mover, and neither party would like to resequence the order at which it has committed to the contract parameters under its control.

(II) Sequential commitment in the PD-newsvendor model can significantly increase (respectively, decrease) M's (respectively, R's) expected profit. For example, when the random component in the demand model follows a uniform distribution and for a linear expected demand function, Sequence 2 can increase (respectively, decrease) M's (respectively, R's) equilibrium expected profit by 79.25% (respectively, 73.51%) when the marginal manufacturing cost is 0.9.

(III) By contrast with the negligible effect of buybacks in the traditional PD-newsvendor model, buybacks, coupled with sequential commitment, can have a significant impact on channel efficiency. For example, for a uniformly distributed random part of demand and for a linear expected demand function, Sequence 2 can increase channel efficiency from 10.90%, for a zero manufacturing cost, to 21.25% when the manufacturing cost is 0.9.

Finally, we note that our sequential commitment approach can be viewed as decision postponement. That is, M and R delay their decisions about the values of decision variables under their control until the counterpart commits to a decision variable under their control. However, in the various postponement strategies, which were extensively studied in the operations research/operations management literature (e.g., Lee and Tang (1997), Aviv and Federgruen (2001), van Mieghem and Dada (1999), and Granot and Yin (2004b)), the decision makers delay some operational decisions (e.g., production or pricing) until additional information, usually about demand, is obtained. In our sequential commitment approach, all decisions are made before demand uncertainty is resolved.

The remainder of the paper is organized as follows. In §2 we recall the traditional PD-newsvendor model with buybacks. In §3 we study the effect of sequential commitment in this model, assuming that the expected demand function is linear in the retail price. For some of the results derived in §3, the random portion of demand can have a general distribution, while for other results, as will be noted, we have to assume that the random part of demand is uniformly distributed. We extend the analysis to other expected demand functions, namely, exponential and negative polynomial, and to a power distribution of the random component of demand in §4. Conclusions and further research are discussed in §5. All proofs are presented in the appendix.

2 The PD-newsvendor Model with Buybacks

Consider the single-period price-dependent (PD) newsvendor model with buyback options, wherein a manufacturer (M) sells a single product to an independent retailer (R) facing stochastic demand from the end-customer market. R must commit to a per unit retail price for the entire selling season and an order quantity in advance of the selling season. The decision sequence is as follows. M, who has unlimited production capacity and can produce the items at a fixed marginal cost c, is a Stackelberg leader. M initiates the process by offering a per unit constant (or linear) wholesale price w, at which items will be sold to R prior to the selling season, and a per unit constant (or linear) buyback rate b, at which she will buy back the unsold items at the end of the selling season. In response to the proposed w and b, R commits to an order quantity Q prior to the selling season, and a per unit selling price p, at which to sell the items during the season. Thereafter, demand uncertainty is resolved. At the end of the season, R returns all unsold inventory to M, receiving a refund of b for each unit returned. It is assumed in this paper that unsatisfied demand is lost, there is no penalty cost for lost sales, and that the salvage value of unsold inventory is zero for both M and R. For feasibility, we always assume: (i) $c \leq w \leq p$ and (ii) $0 \leq b \leq w$.

In this paper, we consider a multiplicative demand model, $X = D(p)\xi$, where $D(p)$ is the deterministic part of X, which decreases in the retail price p, and $\xi \in [0, K]$ captures the random factor of the demand model, and is retail price independent. Let $F(\cdot)$ and $f(\cdot)$ be the distribution and density functions of ξ, respectively.[1] For the multiplicative demand model, M's and R's expected profit functions can be expressed as follows:

$$E\Pi_M(w,b) = (w-c)Q - bE[Q-X]^+ \text{ and } E\Pi_R(p,Q) = (p-w)Q - (p-b)E[Q-X]^+, \quad (1)$$

where $E[Q-X]^+ = QF(\frac{Q}{D(p)}) - \int_0^{\frac{Q}{D(p)}} D(p)\epsilon f(\epsilon|p)d\epsilon$ is the expected unsold inventory. Since $\xi \le K$, we always assume that $Q \le KD(p)$.

If the random component, ξ, follows a power distribution on the interval $[0, K]$, then the density function of ξ is $f(\epsilon) = \gamma \cdot \epsilon^t$ for $t \in [0, \infty)$. To ensure $F(K) = 1$, $\gamma = (t+1)K^{-(t+1)}$. Under a power distribution of ξ, we can simplify M's and R's expected profit functions, given by (1), to:

$$E\Pi_M(w,b) = (w-c)Q - \frac{\gamma \cdot bQ^{t+2}}{D(p)^{t+1}(t+1)(t+2)} \text{ and } E\Pi_R = (p-w)Q - \frac{\gamma \cdot (p-b)Q^{t+2}}{D(p)^{t+1}(t+1)(t+2)}. \quad (2)$$

In the next section we study the effect of sequential commitment in the PD-newsvendor model with buybacks, assuming that the expected demand function is linear in the retail price[2], i.e., $D(p) = 1-p$. For $D(p) = 1-p$, we assume that $c < 1$, since for $c = 1$, both M and R get a zero profit due to the fact that demand is zero in this case.

3 Sequential Commitment in the PD-newsvendor Model

In this section, we introduce the sequential commitment approach and study its effect in the PD-newsvendor model with buybacks, wherein M controls (w,b) and R controls (p,Q). By contrast with the traditional approach, wherein M simultaneously offers w and b, and R, subsequently, commits

[1] It would be interesting and challenging to extend our analysis to the additive demand model wherein $X = D(p)+\xi$. The additive model, which is also commonly used in the literature, would be an appropriate model wherein the variance of demand is unaffected by the expected demand level. By contrast, the multiplicative model is appropriate where the variance of demand increases with expected demand in a manner which leaves the coefficient of variation unaffected. We note, however, that the additive model may lead to qualitatively different results than the multiplicative model (see, e.g., Mills (1959), Emmons and Gilbert (1998), Granot and Yin (2004b), Song et al. (2004), and, in particular, the excellent survey by Petruzzi and Dada (1999)). Moreover, it appears that it is less tractable than the multiplicative model (see, e.g., Padmanabhan and Png (1997), and Wang et al. (2004)). Indeed, even when ξ has a binary distribution and $D(p)$ is linear in p, it is difficult to derive a closed-form expression for, e.g., the equilibrium value of p in the PD-newsvendor problem with an additive demand model (see, e.g., Granot and Yin (2004b)).

[2] Note that the analysis can be easily extended to a general linear expected demand function $D(p) = a(k-p)$, where $a(>0)$ and $k(>0)$ are constant, as was assumed in Emmons and Gilbert (1998). Indeed, for $D(p) = a(k-p)$, let $p = k \cdot p'$, $w = k \cdot w'$, $b = k \cdot b'$, $Q = ak \cdot Q'$ and $c = k \cdot c'$. Then, it is not difficult to verify that the expected profit functions of M and R, given by (1), can be transformed to: $E\Pi_M(w,b,p,Q,c) = ak^2 \cdot E\Pi'_M(w',b',p',Q',c')$ and $E\Pi_R(w,b,p,Q,c) = ak^2 \cdot E\Pi'_R(w',b',p',Q',c')$, where $E\Pi'_M$ and $E\Pi'_R$ are the expected profit functions of M and R, respectively, with respect to the expected demand function $D(p') = 1-p'$ and the marginal manufacturing cost c'. Thus, the analysis in a model with decisions (w,b,p,Q), cost c and $D(p) = a(k-p)$ coincides with that in a model with decisions (w',b',p',Q'), cost c' and $D(p') = 1-p'$. Note that due to this normalization, the performance of the models with and without buybacks and the integrated system is independent of individual values of c and k, but is dependent on $\frac{c}{k}$, which can be viewed as the *normalized marginal manufacturing cost.*

to p and Q, in our sequential commitment approach, M and R can "sequence" their decision variables. Thus, M and R can commit to the decision variables under their control sequentially and alternately, as specified in the sequel.

First, we introduce some new definitions. We will refer to the PD-newsvendor model with buy-backs, wherein M controls (w, b) and R controls (p, Q), as the *traditional* PD-newsvendor model, and to the usual ordering of decisions in the traditional PD-newsvendor model, wherein M first determines w and b, and R, subsequently, decides upon p and Q, as the traditional sequence, to be denoted as $\underline{M{:}w, b;\ R{:}p, Q}$. We will refer further to each possible ordering of decisions in the traditional PD-newsvendor model, resulting from sequential commitment, as a sequence, or, a sequencing instance. In general, the notation of a sequence corresponds to the order at which decisions are being made. Thus, for example, the sequence denoted as $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$ corresponds to a sequential commitment where M, in Stage 1, offers b, then R, in Stage 2, decides on p, M then, in Stage 3, requests w, and finally, in Stage 4, R determines the order quantity Q. Similarly, the sequence $\underline{M{:}w;\ R{:}p, Q;\ M{:}b}$ corresponds to the case where M, in Stage 1, decides on w, R then, in Stage 2, decides simultaneously on p and Q, and finally, in Stage 3, M determines b.

Backward induction is used to solve these multi-stage Stackelberg games. In §3.1 we consider the case when M is the Stackelberg leader, and the effect of sequential commitment when M is the leader is discussed in §3.2. The case when R is the leader is analyzed in §3.3, and in §3.4 we investigate the equilibrium sequence(s).

3.1 The manufacturer is the leader

Recall that in the PD-newsvendor model, M controls the wholesale price w and the buyback rate b. There are, in total, 7 sequencing instances resulting from the sequential commitment approach when M is the leader. We next consider each one of them separately.

The traditional sequence: $\underline{\boldsymbol{M{:}w, b;\ R{:}p, Q}}$. Emmons and Gilbert (1998) have studied the traditional sequence with a linear expected demand function and a uniform distribution of ξ. For a uniform ξ, Granot and Yin (2004a) have derived closed-form expressions for the equilibrium values of decision variables and expected profits, for linear, exponential and negative polynomial expected demand functions. Song et al. (2004) have extended these results to a ξ whose distribution has the increasing failure rate (IFR) property (i.e., $\frac{f(\epsilon)}{1-F(\epsilon)}$ is non-decreasing in ϵ). Let us recall their results for $D(p) = 1 - p$:

$$w^* = \frac{1}{2}(1+c), \quad b^* = \frac{1}{2}, \quad p^* = \frac{1}{2}[1 + \frac{z^* - \Lambda(z^*)}{z^* - \Lambda(z^*) + \int_0^{z^*} \epsilon f(\epsilon) d\epsilon}], \quad Q^* = (1-p^*)z^*, \text{ and} \tag{3}$$

$$E\Pi_M^* = 2E\Pi_R^* = \frac{1-p^*}{2}[z^* - \Lambda(z^*) - cz^*], \tag{4}$$

where z^* is the unique solution to $(1-F(z)) - c(1+\frac{\int_0^z \epsilon f(\epsilon)d\epsilon}{z-\beta(z)}) = 0$, and, $\Lambda(z) = zF(z) - \int_0^z \epsilon f(\epsilon)d\epsilon$.

For example, when ξ has a uniform distribution on $[0, K]$ (i.e., $t = 0$ in the power distribution), which has been analyzed in Granot and Yin (2004a), the equilibrium values of decision variables and expected profits of M and R can be simplified to:

$$w^* = \frac{1+c}{2},\ \ b^* = \frac{1}{2},\ \ p^* = \frac{5+\sqrt{1+8c}}{8},\ \ Q^* = \frac{K(3-\sqrt{1+8c})^2}{16},\ \text{ and} \tag{5}$$

$$E\Pi_M^* = 2E\Pi_R^* = \frac{K(3-\sqrt{1+8c})^3(1+\sqrt{1+8c})}{256}, \tag{6}$$

where, as we recall, $c < 1$. The equilibrium values of the decision variables and expected profits, as a function of c, under a uniformly distributed ξ are presented in the top block in Table A.1 in the appendix. Note that in all tables in the appendix, including Tables A.2, A.3 and A.4, the equilibrium values of Q^*, $E\Pi_M^*$, $E\Pi_R^*$ and the channel expected profit are in units of K, and numbers are presented in scientific format.

Sequence 1: M:w; R:p; M:b; R:Q. According to the notation previously introduced, in Sequence 1, M initiates the process by proposing a wholesale price w in Stage 1. R then commits to a retail price p in Stage 2, M offers a buyback rate b in Stage 3, and, finally, R commits to ordering Q from M in Stage 4. Figure 1 below represents the timeline in this sequence.

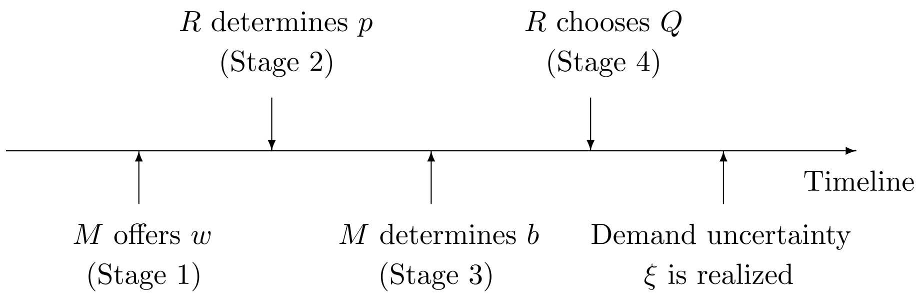

Figure 1: The timeline in Sequence 1: M:w; R:p; M:b; R:Q

Assumption 3.1 *Agents in the supply chain are willing to select an action which improves the performance of the supply chain (i.e., benefit their partners) as long as they are not adversely affected by such an action.*

Note that in Sequence 1, the decision on Q is made after the retail price p is set. Thus, choosing Q is equivalent to choosing z, where $z = \frac{Q}{D(p)}$ is "stocking factor" introduced by Petruzzi and

Dada (1999). See their paper for more details about the advantages of this transformation. For a uniformly distributed ξ, we are able to derive implicit expressions of the equilibrium values in Sequence 1.

Proposition 3.2 *In Sequence 1: M:w; R:p; M:b; R:Q, with a uniformly distributed $\xi \in [0, K]$ and $D(p) = 1 - p$, the equilibrium value of the retail price $p^* \in [\frac{1+\sqrt{1+8c}}{4}, 1)$ is the unique solution to:*

$$(1-p)(p-c)\frac{dH_1(p)}{dp} + (1+c-2p)H_1(p) = 0, \tag{7}$$

where $H_1(p) = \frac{(\sqrt{2p^2-p-c}+\sqrt{p-c})^2}{p}$. Accordingly, the equilibrium values of the other decision variables are:

$$w^* = c + \sqrt{(2(p^*)^2 - p^* - c)(p^* - c)}, \quad b^* = \frac{p^*(3w^* - p^* - 2c)}{w^* + p^* - 2c}, \quad \text{and} \quad Q^* = \frac{K(w^* + p^* - 2c)(1-p^*)}{2p^*},$$

and the equilibrium values of expected profits are:

$$E\Pi_M^* = \frac{K(1-p^*)(w^* + p^* - 2c)^2}{8p^*} \quad \text{and} \quad E\Pi_R = \frac{K(1-p^*)(p^* - w^*)(w^* + p^* - 2c)}{4p^*}.$$

The equilibrium values of the decision variables and expected profits in Sequence 1 for a uniformly distributed ξ, as a function of c, are presented in the top block in Table A.2 in the appendix.

Sequence 2: *M:b; R:p; M:w; R:Q*. For a uniformly distributed ξ, we have derived implicit expressions of the equilibrium decision variables and expected profits in this sequence.

Proposition 3.3 *In Sequence 2: M:b; R:p; M:w; R:Q, with a uniformly distributed $\xi \in [0, K]$ and $D(p) = 1 - p$, the equilibrium value of the retail price $p^* \in [\frac{1+\sqrt{1+8c}}{4}, 1)$ is the unique solution to:*

$$(p-c)\frac{dH_2(p)}{dp} + H_2(p) = 0, \tag{8}$$

where $H_2(p) = \frac{-6p^2+5p+4pc-3c+B_2(p)}{p}$ and $B_2(p) = \sqrt{(2p^2-p-c)(-6p^2+7p+8pc-9c)}$. Accordingly, the equilibrium values of the other decision variables are:

$$b^* = \frac{-3(2(p^*)^2 - p^* - c) + B_2(p^*)}{2(2 + c - 3p^*)}, \quad Q^* = \frac{K(p^* - c)(1-p^*)}{2p^* - b^*}, \quad \text{and} \quad w^* = p^* - \frac{(p^* - b^*)(p^* - c)}{2p^* - b},$$

and the equilibrium values of the expected profits are:

$$E\Pi_M^* = \frac{K(p^* - c)H_2(p^*)}{16} \quad \text{and} \quad E\Pi_R^* = \frac{K(1-p^*)(p^* - b^*)(p^* - c)^2}{2(2p^* - b^*)^2}.$$

The equilibrium values of the decision variables and expected profits in Sequence 2 for a uniformly distributed ξ, as a function of c, are displayed in the top block in Table A.3 in the appendix.

Note that contracting which follows the sequence $\underline{M{:}w;\ R{:}p, Q}$ would result with the traditional wholesale price-only contract. Thus, we will refer to this sequence as the *wholesale price-only contract sequence*. Further, we will say that two sequences *coincide* if their corresponding equilibrium values of decision variables and expected profits are equal.

Next, we consider the following two sequences: Sequence 3: $\underline{M{:}w;\ R{:}p, Q;\ M{:}b}$ and Sequence 4: $\underline{M{:}w;\ R{:}Q;\ M{:}b;\ R{:}p}$.

Sequence 3: $\underline{\boldsymbol{M{:}w;\ R{:}p, Q;\ M{:}b}}$. This sequence is a three-stage problem. In Stage 3, given (w, p, Q), M chooses b to maximize her expected profit in (1). Clearly, for any distribution of ξ and any form of $D(p)$, M would offer a zero buyback rate, i.e., $b^* = 0$. Thus, Sequence 3 reduces to the wholesale price-only contract sequence: $\underline{M{:}w;\ R{:}p, Q}$. We immediately have:

Proposition 3.4 *For an arbitrary distribution of ξ and for an arbitrary expected demand function $D(p)$, Sequence 3: $\underline{M{:}w;\ R{:}p, Q;\ M{:}b}$ coincides with the wholesale price-only contract sequence.*

Sequence 4: $\underline{\boldsymbol{M{:}w;\ R{:}Q;\ M{:}b;\ R{:}p}}$**.** As shown by Proposition 3.5 below, Sequences 4 coincides, under some conditions, with Sequence 3 and the wholesale price-only contract sequence.

Proposition 3.5 *For a power distribution of $\xi \in [0, K]$ with $f(\epsilon) = \gamma(\epsilon)^t$ (where $t > 0$ and $\gamma = (t+1)K^{-(t+1)}$) and when $D(p)$ is a decreasing function of p and satisfies $D(p)\frac{d^2D(p)}{dp^2} - (t+2)(\frac{dD(p)}{dp})^2 \leq 0$, Sequence 4: $\underline{M{:}w;\ R{:}Q;\ M{:}b;\ R{:}p}$ coincides both with Sequence 3 and with the wholesale price-only contract sequence.*

Note that the condition $D(p)\frac{d^2D(p)}{dp^2} - (t+2)(\frac{dD(p)}{dp})^2 \leq 0$ in Proposition 3.5 is satisfied by three commonly used expected demand functions in the operations management and economics literature: linear, $D(p) = 1 - p$, exponential, $D(p) = e^{-p}$, and negative polynomial, $D(p) = p^{-q}$, where $q > 1$.

Let us next study the remaining two sequences when M is the leader, which are shown in Propositions 3.6 and 3.7 below to result with R getting a zero profit.

Sequence 5: $\underline{\boldsymbol{M{:}b;\ R{:}p, Q;\ M{:}w}}$**.** In this three-stage sequence, in Stage 3, for any given (b, p, Q), M would choose w as large as possible, since her expected profit function, given by (1), is strictly increasing in w as long as $Q > 0$. Thus, $w^* = p$, which implies that R's expected profit function, given by (1), is not positive. R, therefore, should select $p^* = b$ if $b \geq c$ or $Q^* = 0$, as in both cases his expected profit is zero. By Assumption 3.1, R would select $p^* = b$ if $b \geq c$, which results with a consignment contract, in which M attains the total expected profit of the integrated channel and R gets a zero profit. M, in Stage 1, will definitely set $b \geq c$ to realize the expected profit of the integrated channel since, otherwise, she will get a zero profit. Thus, we have the following result.

Proposition 3.6 *For an arbitrary distribution of* ξ *and for an arbitrary expected demand function* $D(p)$*, in Sequence 5:* $\underline{M{:}b;\ R{:}p, Q;\ M{:}w}$*,* M *gets the expected profit of the integrated channel and* R *gets a zero profit.*

A similar result is derived for Sequence 6 in the following proposition.

Proposition 3.7 *For an arbitrary distribution of* ξ *and for an arbitrary expected demand function* $D(p)$*, in Sequence 6:* $\underline{M{:}b;\ R{:}Q;\ M{:}w;\ R{:}p}$*,* M *gets the entire expected channel profit, which is strictly less than the expected profit of the integrated channel, and* R *gets a zero profit.*

3.2 The effect of sequential commitment when M is the leader

Having analyzed all possible sequencing instances when M is the Stackelberg leader, we are able to compare the equilibrium profits of M, R and the channel for the different sequencing instances resulting from the sequential commitment approach, and to investigate the effect of such an approach on the equilibrium values of the decision variables and expected profits.

Proposition 3.8 *For a uniformly distributed* ξ *and* $D(p) = 1 - p$*,*

$$E\Pi_M^*(S2) > E\Pi_M^*(S1) > E\Pi_M^*(TS),$$

where "S1", "S2" and "TS" stand for "Sequence 1", "Sequence 2" and "the traditional sequence", respectively.

Tables A.1, A.2 and A.3 in the appendix, respectively, present the equilibrium values of the decision variables and expected profits of M, R and the channel in the traditional sequence: $\underline{M{:}w, b;\ R{:}p, Q}$, Sequence 1: $\underline{M{:}w;\ R{:}p;\ M{:}b;\ R{:}Q}$ and Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$ under a uniform distribution of ξ and $D(p) = 1 - p$. By comparing these values, we can immediately make the following observations:

Observation 3.9 *For a uniformly distributed* ξ *and* $D(p) = 1 - p$*:*

(i) $E\Pi_M^*(S2) > E\Pi_M^*(S1) > E\Pi_M^*(TS)$*.*

(ii) $E\Pi^*_{M+R}(S2) > E\Pi^*_{M+R}(S1) > E\Pi^*_{M+R}(TS)$.

(iii) $E\Pi^*_R(TS) > E\Pi^*_R(S1) > E\Pi^*_R(S2)$.

(iv) $p^*(TS) > p^*(S1) > p^*(S2)$.

(v) $w^*(S2) > w^*(S1)$.

(vi) $b^*(S2) > b^*(S1)$ *and*

(vii) $Q^*(S2) > Q^*(S1)$,

where $E\Pi^*_{M+R}$ *stands for the equilibrium value of the expected channel profit.*

Note that Observation 3.9 (i) is consistent with Proposition 3.8, and implies that in these three sequences, in equilibrium, M will attain the highest expected profit in Sequence 2 and the lowest expected profit in the traditional sequence. Observation 3.9 (ii) suggests that both Sequence 1 and Sequence 2 increase channel efficiency, which, as we recall from Granot and Yin (2004a), is 75% for the traditional sequence. However, it appears that they also adversely affect R's expected profit, as is evident from R's equilibrium expected profits in these three sequences presented in Tables A.1, A.2 and A.3 in the appendix. Indeed, from Tables A.1 and A.2, under a uniform ξ and for $c = 0.9$, as compared to the traditional sequence, Sequence 2 improves M's equilibrium expected profit and channel efficiency by 79.25% and 21.25%, respectively, and decreases R's equilibrium expected profit by 73.51%.

In general, in Sequences 1 and 2, M is delaying one of her two decisions in order to affect R's choice of retail price and order quantity in a manner beneficial to her. Apparently, it is more effective for M to delay her decision on the wholesale price w and offer a more generous buyback rate b rather than delay her decision on b and offer a relatively low w.

By Propositions 3.4 and 3.5, Sequence 3: $\underline{M{:}w;\ R{:}p,Q,\ M{:}b}$ (for any distribution of ξ and for any form of $D(p)$) and Sequence 4: $\underline{M{:}w;\ R{:}Q;\ M{:}b;\ R{:}p}$ (for a power distribution of ξ and for either a linear, exponential or negative polynomial $D(p)$) lead to a wholesale price-only contract. By Propositions 3.6, 3.8, 3.9 and 3.10 in Granot and Yin (2004a), we immediately have the following conclusion:

Corollary 3.10 *For a uniformly distributed* ξ *and for* $D(p) = 1 - p$, *as compared to the traditional sequence, Sequence 3:* $\underline{M{:}w;\ R{:}p,Q,\ M{:}b}$ *and Sequence 4:* $\underline{M{:}w;\ R{:}Q;\ M{:}b;\ R{:}p}$ *lead to a lower expected profit for M, a lower expected channel profit and a higher expected profit for R.*

Recall that in Sequence 5: $\underline{M{:}b;\ R{:}p,Q;\ M{:}w}$, M attains the expected profit of the integrated channel while R gets nothing. Thus, from M's point of view, Sequence 5 dominates all other sequences when she is the leader. However, we note that this sequence may not be reasonable because it forces R to commit, e.g., to an order quantity before the wholesale price is set. Similarly, in Sequence 6: $\underline{M{:}b;\ R{:}Q;\ M{:}w;\ R{:}p}$, M gets the entire expected channel profit and R gets nothing. But, again, it may be difficult for M to enforce Sequence 6 for the same reason that Sequence 5 may not be enforceable.

3.3 The retailer is the leader

The findings in the previous subsections raise a natural question: what can R achieve using the sequential commitment approach if he is the Stackelberg leader? Thus, we consider in this subsection a "power structure" in which R is the leader (see, e.g., Choi (1991), Trivedi (1998), Wang and Gerchak (2003), Gerchak and Wang (2004), Granot and Yin (2004c), and Cachon (2005) for other models in which R can act as the Stackelberg leader). Now, similar to the case when M is the leader, when R moves first, sequential commitment induces a total of seven sequences in the PD-newsvendor model wherein R controls p and Q and M controls w and b.

Let us first consider the sequence $\underline{R{:}p;\ M{:}w;\ R{:}Q;\ M{:}b}$, which will be referred to as Sequence 7. Following our general notation, in this four-stage sequence, R initiates the process by committing to a retail price p in Stage 1. In Stage 2, M sets her wholesale price w, R then commits to a quantity Q in Stage 3, and finally, M offers a buyback rate b. For a uniformly distributed ξ and $D(p) = 1 - p$, we have the following result.

Proposition 3.11 *In Sequence 7:* $\underline{R{:}p;\ M{:}w;\ R{:}Q;\ M{:}b}$*, for a uniformly distributed* ξ *and for* $D(p) = 1 - p$*, the equilibrium values of the decision variables are:*

$$p^* = \frac{1+\sqrt{1+8c}}{4}, \quad w^* = \frac{(1+\sqrt{1+8c})^2}{16}, \quad b^* = 0, \quad \text{and} Q^* = \frac{K(3-\sqrt{1+8c})^2}{16}, \tag{9}$$

and the equilibrium expected profits of M *and* R *are:*

$$E\Pi_M^* = 2E\Pi_R^* = \frac{K(3-\sqrt{1+8c})^3(1-\sqrt{1+8c})}{256}. \tag{10}$$

By comparing the equilibrium values in Sequence 7: $\underline{R{:}p;\ M{:}w;\ R{:}Q;\ M{:}b}$ with those displayed in (5) and (6) for the traditional sequence: $\underline{M{:}w, b;\ R{:}p, Q}$, we immediately have the following conclusion.

Corollary 3.12 *For a uniformly distributed* ξ *and for* $D(p) = 1 - p$*:*

(i) $E\Pi_M^*(S7) = 2E\Pi_R^*(S7) = E\Pi_M^*(TS) = 2E\Pi_R^*(TS)$.

(ii) $Q^*(S7) = Q^*(TS)$.

(iii) $p^*(S7) < p^*(TS)$.

(iv) $w^*(S7) < w^*(TS)$ *and*

(v) $b^*(S7)(= 0) < b^*(TS)$,

where "TS" and "S7" stand for "the traditional sequence" and "Sequence 7", respectively.

Thus, as compared to the traditional sequence, wherein M is the leader and buybacks are implemented, Sequence 7, wherein R moves first, achieves the same equilibrium expected profits for channel members with the same equilibrium quantity, but lower wholesale and retail prices and without an implementation of a buyback policy.

The equilibrium values in the sequence $\underline{R{:}Q;\ M{:}w,b;\ R{:}p}$, which will be referred to as Sequence 8, are presented in Proposition 3.13 below.

Proposition 3.13 *For a power distribution of* $\xi \in [0, K]$ *with* $f(\epsilon) = \gamma(\epsilon)^t$ *(where* $t > 0$ *and* $\gamma = (t+1)K^{-(t+1)}$*) and when* $D(p)$ *is a decreasing function of* p *and satisfies* $D(p)\frac{d^2D(p)}{dp^2} - (t+2)(\frac{dD(p)}{dp})^2 \leq 0$, *Sequence 8:* $\underline{R{:}Q;\ M{:}w,b;\ R{:}p}$, *in equilibrium, results with both* M *and* R *realizing a zero profit.*

Similar to Proposition 3.5, the condition on $D(p)$ in Proposition 3.13: $D(p)\frac{d^2D(p)}{dp^2} - (t+2)(\frac{dD(p)}{dp})^2 \leq 0$, is satisfied by linear, exponential and negative polynomial expected demand functions.

Let us consider another sequence: $\underline{R{:}p;\ M{:}w,b;\ R{:}Q}$. In this sequence, in Stage 3, for any distribution of ξ and any form of $D(p)$, R's expected profit function, given by (1), is concave in Q and the optimal order quantity Q^* satisfies $\frac{dE\Pi_R}{dQ} = 0$, which leads to $p - w = (p - b)F(Q)$. In Stage 2, we work with (b, Q) instead of (w, b) for M's problem. Thus, M's expected profit function in Stage 2 becomes: $E\Pi_M = (p - c - (p - b)F(Q))Q - bE[Q - X]^+$, which can be easily shown to be increasing in b for any given (p, Q). Thus, $b^* = p$, which results with $w^* = p = b^*$, $E\Pi_R = 0$ for any p, and $E\Pi_M = (p - c)Q - pE[Q - X]^+$, which coincides with the expected profit function in the integrated channel. Since R's profit is always zero in Stage 1 regardless of the value of p, by Assumption 3.1, (p, Q) is set to maximize M's expected profit function, and we conclude that in equilibrium, M realizes the expected profit of the integrated channel and R gets a zero profit.

Consider another sequence: $\underline{R{:}Q;\ M{:}w;\ R{:}p;\ M{:}b}$. In this sequence, M's choice of b is always zero, and thus, M's expected profit function becomes $E\Pi_M = (w - c)Q$. Whatever R's decision on p in Stage 3, M prefers a wholesale price w as large as possible in Stage 2, which, in turn, results with $w^* = D^{-1}(\frac{Q}{K})$, due to the assumption that $Q \leq KD(p) \leq KD(w)$. Such a choice for w^* will lead to $p^* = D^{-1}(\frac{Q}{K})$, and R has to choose $Q^* = 0$ in Stage 1, since otherwise, his expected profit is strictly negative. Therefore, in this sequence, both M and R get a zero profit.

In general, except for Sequence 7: $\underline{R{:}p;\ M{:}w;\ R{:}Q;\ M{:}b}$, when R initiates the process by offering either a retail price p, or an order quantity Q, or both, his equilibrium expected profit is always zero. Proposition 3.14 below summarizes this result.

Proposition 3.14 *With the exception of Sequence 7 and for a general distribution of ξ and a general form of $D(p)$, except for Sequence 8, wherein ξ is restricted to a power distribution and $D(p)$ is linear, exponential and negative polynomial, when R initiates the process by offering either a retail price p, or an order quantity Q, or both, his equilibrium expected profit is always zero. Among these six sequences, three induce a complete consignment contract in which M attains the total expected profit of the integrated channel, and in the other three sequences, M gets a zero profit.*

Corollary 3.12, Propositions 3.13 and 3.14 imply that for a uniform ξ and $D(p) = 1 - p$, R can never do strictly better than in the traditional sequence when he moves first.

3.4 The equilibrium sequence

Having considered all possible sequences, resulting from sequential commitment in the PD-newsvendor model, it is natural to investigate which, if any, of these sequences will emerge in equilibrium. For that purpose, let us assume that the first mover (either M or R) has been determined and M and R are free to decide, sequentially, upon the order in which they specify the values of the decision variables under their control. We consider the following two-stage Stackelberg game in order to find an equilibrium sequence.

Set of players: $\{M, R\}$.

Action: Each player chooses which decision variable(s) to decide upon in their first step.

Set of strategies available for M: $\{(w), (b), (w, b)\}$.

Set of strategies available for R: $\{(p), (Q), (p, Q)\}$.

Outcome: a sequencing instance.

Payoff: equilibrium expected profits for M and R.

For example, if M is the leader, who initiates the process, then M could choose either w only, b only or both w and b in her first step (Stage 1), and then, R follows by deciding upon either p only, Q only or both p and Q in his first step (Stage 2). After the first two steps, the decision sequence is determined. We would then say that a sequential ordering of decision variables is an *equilibrium sequence* if neither M nor R can improve their expected profits by resequencing their decision variables. Naturally, if any party elects not to conclude a deal, both parties would realize a zero profit.

In the discussion below, it is assumed that if one of the two parties decides to specify, in any stage, only one of the two decision variables under his/her control, the other party is expected to commit to one or both of the decision variables under its control. Thus, e.g., R cannot insist that M should determine, simultaneously, the values of w and b before R determines the values of p and Q, and M cannot insist that R should specify both decision variables under his control after M has specified the value of one of the two decision variables under her control. Recall that being the Stackelberg leader does not necessarily imply being more powerful.

Now, assume that M is the first mover who offers R, initially, only a wholesale price w. Then, R can either choose a retail price p, or an order quantity Q, or both. To find what is best for R, we need to compare R's expected profit (i.e., payoff) in the following sequences: Sequence 1: $\underline{M{:}w;\ R{:}p;\ M{:}b;\ R{:}Q}$, Sequence 3: $\underline{M{:}w;\ R{:}p,Q;\ M{:}b}$ and Sequence 4: $\underline{M{:}w;\ R{:}Q;\ M{:}b;\ R{:}p}$. According to Propositions 3.4 and 3.5, when ξ has a power distribution and $D(p)$ is either a linear, or exponential or negative polynomial function of p, Sequences 3 and 4 result with a wholesale price-only contract, and it follows from Proposition 3.15 below that R prefers Sequence 1 to Sequences 3 and 4.

Proposition 3.15 *For a power distribution of ξ and for any form of $D(p)$, for any given w, R's equilibrium expected profit derived from Sequence 1: $\underline{M{:}w;\ R{:}p;\ M{:}b;\ R{:}Q}$ is strictly larger than his equilibrium expected profit in a wholesale price-only contract.*

By Propositions 3.4, 3.5 and 3.15, we can conclude that for a power distribution of ξ and when $D(p)$ is either a linear, exponential or negative polynomial function of p, when M offers, at the outset, a wholesale price w, R would then specify only the retail price p.

On the other hand, suppose M offers, initially, a buyback rate b, and consider Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$, Sequence 5: $\underline{M{:}b;\ R{:}p,Q;\ M{:}w}$ and Sequence 6: $\underline{M{:}b;\ R{:}Q;\ M{:}w;\ R{:}p}$. By Propositions 3.6 and 3.7, R's expected profit is always zero in Sequences 5 and 6, and according to the proof of Proposition 3.3, R realizes a strictly positive expected profit in Sequence 2 for a power distribution of ξ and for $D(p) = 1 - p$. Thus, one can conclude that under a power distribution of ξ and $D(p) = 1 - p$, the best response for R is, again, to commit in this stage only to a retail price p.

Thus, whatever M offers first, the best response for R is only to set his selling price p. Knowing this, M would only have to compare her performance in the traditional sequence: $\underline{M{:}w,b;\ R{:}p,Q}$, Sequence 1: $\underline{M{:}w;\ R{:}p;\ M{:}b;\ R{:}Q}$ and Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$, to conclude that her preference, in case she moves first, is to set, in Stage 1, only the buyback rate, and we have:

Proposition 3.16 *When M is the Stackelberg leader in the PD-newsvendor model with buybacks, wherein ξ follows a uniform distribution and $D(p) = 1 - p$, Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$ is the unique equilibrium sequence.*

Let us next consider the sequencing instances wherein R is the first mover. If R initiates the process by offering only a retail price p, then M would get the expected profit of the integrated channel by following, e.g., the sequence $\underline{R{:}p;\ M{:}w, b;\ R{:}Q}$. If R initiates the process by offering only Q, then, again, M would get the expected profit of the integrated channel by following the sequence $\underline{R{:}Q;\ M{:}b;\ R{:}p;\ M{:}w}$. When R initiates the process by offering both p and Q, M and R will both get a zero profit. Therefore, whatever R offers first when he is the first mover, his expected profit is always zero, and thus, he is indifferent between being the Stackelberg leader and not having any deal whatsoever with M. According to the proof of Proposition 3.3, R realizes a strictly positive expected profit in Sequence 2 under a power distribution of ξ and $D(p) = 1 - p$. Thus, our conclusion is therefore:

Proposition 3.17 *In the PD-newsvendor model under a power distribution of ξ and $D(p) = 1 - p$, R prefers not to be the leader and would rather have M move first.*

It follows from Propositions 3.16 and 3.17 that Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$ is the only equilibrium sequential commitment instance in the PD-newsvendor model, and thus, in this case, the first mover is determined endogenously. We observe that this equilibrium sequence is neither the traditional sequence studied, e.g., by Emmons and Gilbert, nor a sequence in which w is proposed before b. In this sequence, M fares better and R fares worse than in the traditional sequence. However, in equilibrium, R is able to prevent M from grabbing the entire expected channel profit, which M would have achieved if she had the power to force R to adopt, e.g., Sequence 5: $\underline{M{:}b;\ R{:}p, Q;\ M{:}w}$.

Finally, it follows from Propositions 3.4, 3.5, and Corollary 3.10, that, under a uniform ξ and $D(p) = 1 - p$, there are two sequences, i.e., Sequence 3: $\underline{M{:}w;\ R{:}p, Q;\ M{:}b}$ and Sequence 4: $\underline{M{:}w;\ R{:}Q;\ M{:}b;\ R{:}p}$, which coincide with the wholesale price-only contract, wherein R would be strictly better off than in the traditional sequence. However, in both sequences, M is the leader and she and the channel are strictly worse off. Thus, unless R can force M to move first, and offer exclusively w, it may be impossible for him to implement these two sequences. Finally, it follows from Propositions 3.6 and 3.7, Observation 3.9 (iii), and Propositions 3.13 and 3.14, that in all other sequences, for a uniformly distributed ξ, R cannot improve his performance beyond that he can achieve in the traditional sequence, and he can even end up being worse off than in the traditional sequence.

4 Extensions and Discussions

We have studied in the previous section the effect of sequential commitment in the PD-newsvendor model with buybacks, in which, in some cases, it is assumed that the random component of demand, ξ, follows a uniform distribution and $D(p) = 1 - p$. In this section, we extend our analysis to more general demand distributions and other expected demand functions. More specifically, in §4.1 we extend the results to two other expected demand functions: exponential and negative polynomial, and in §4.2 we extend the results which have been obtained under the assumption that ξ is uniformly distributed to a power distribution. By Propositions 3.4, 3.6 and 3.7, Sequences 3, 5 and 6, respectively, were analyzed for a general ξ and a general $D(p)$, and by Proposition 3.5, Sequence 4 was studied for a power distribution of ξ and linear, exponential and negative polynomial expected demand functions. Similarly, by Proposition 3.14, all sequences in which R is the leader, except for Sequence 7 and Sequence 8, were analyzed for a general ξ and a general $D(p)$, and by Proposition 3.13, Sequence 8 was analyzed for a power distribution of ξ and linear, exponential and negative polynomial expected demand functions. Thus, to complete the analysis in this section, we only need to consider three sequences: Sequence 1: $M{:}w;\ \underline{R{:}p;\ M{:}b;\ R{:}Q}$, Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$ and Sequence 7: $\underline{R{:}p;\ M{:}w;\ R{:}Q;\ M{:}b}$.

4.1 Extension to other expected demand functions

In this subsection, we extend the results derived in Section 3 for $D(p) = 1 - p$ to exponential, $D(p) = e^{-p}$, and negative polynomial, $D(p) = p^{-q}$, expected demand functions, where $q > 1$. Similar to the linear expected demand function case, the analysis can be easily extended to more general exponential, $D(p) = ae^{-sp}$, and negative polynomial, $D(p) = ap^{-q}$, functions, where $a > 0$, $s > 0$ and $q > 1$. The restriction that $q > 1$ is used to ensure that R's optimal retail price will be bounded. For space consideration, we will focus on two major issues in this subsection: (i) M's (respectively, R's) equilibrium expected profit when M (respectively, R) moves first, and (ii) equilibrium sequence analysis. Note that in the traditional sequence, the equilibrium values of the decision variables and expected profits under a uniform ξ and an exponential or negative polynomial demand function are available in Granot and Yin (2004a) and Song et al. (2004).

4.1.1 Exponential expected demand function

We assume in this subsection that $D(p) = e^{-p}$, and ξ is uniformly distributed on $[0, K]$. Let us first consider the case when M is the leader. Recall from Granot and Yin (2004a) and Song et al. (2004) that in the traditional sequence with an exponential expected demand function, the equilibrium values of the decision variables are:

$$w^* = 1 + c, \quad b^* = 1, \quad p^* = \frac{3 + c + H}{2}, \quad \text{and} \quad Q^* = \frac{K(3 + c - H)}{2} e^{-\frac{3+c+H}{2}}, \tag{11}$$

where $H = \sqrt{c^2 + 6c + 1}$, and the equilibrium values of the expected profits are:

$$E\Pi_M^* = E\Pi_R^* = \frac{K(3 + c - H)(1 - c + H)}{8} e^{-\frac{3+c+H}{2}}. \tag{12}$$

Similar to Proposition 3.8 in the linear expected demand function case, we have the following result for M's equilibrium expected profit with $D(p) = e^{-p}$:

Proposition 4.1 *For a uniformly distributed ξ and for $D(p) = e^{-p}$,*

$$E\Pi_M^*(S2) > E\Pi_M^*(S1) > E\Pi_M^*(TS),$$

where "S1", "S2" and "TS" stand for "Sequence 1", "Sequence 2" and "the traditional sequence", respectively.

Let us next compare Sequence 3: $\underline{M{:}w;\ R{:}p, Q;\ M{:}b}$, Sequence 4: $\underline{M{:}w;\ R{:}Q;\ M{:}b;\ R{:}p}$ and the traditional sequence: $\underline{M{:}w, b;\ R{:}p, Q}$, for a power distribution of ξ and $D(p) = e^{-p}$. Recall from Propositions 3.4 and 3.5 that Sequences 3 and 4 coincide with the wholesale price-only contract sequence, and that it follows from Song et al. (2004) that the equilibrium buyback rate in the traditional sequence satisfies $b^* = 1 > 0$, which implies that M is strictly better off by offering a positive buyback price and her expected profit under buybacks is strictly larger than under a wholesale price-only contract. Thus, we have:

Corollary 4.2 *When ξ has a power distribution and $D(p) = e^{-p}$, in equilibrium, Sequence 3: $\underline{M{:}w;\ R{:}p, Q;\ M{:}b}$ and Sequence 4: $\underline{M{:}w;\ R{:}Q;\ M{:}b;\ R{:}p}$ yield a lower expected profit for M than the traditional sequence.*

We are ready to discuss the equilibrium sequence when M moves first for a power distribution of ξ and $D(p) = e^{-p}$. Recall from the analysis in §3.4 that when M is the leader and initially offers a wholesale price w, R would specify only the retail price p. If M offers, initially, only a buyback rate b, then R can either choose a retail price p, or an order quantity Q, or both. Thus, we compare: Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$, Sequence 5: $\underline{M{:}b;\ R{:}p,Q;\ M{:}w}$ and Sequence 6: $\underline{M{:}b;\ R{:}Q;\ M{:}w;\ R{:}p}$. Recall from Propositions 3.6 and 3.7 that R's expected profit is always zero in Sequences 5 and 6, and according to the proof of Proposition 4.1, R realizes a strictly positive expected profit in Sequence 2. Thus, the best response for R is, again, to set his retail price p. Thus, by Proposition 4.1:

Corollary 4.3 *When M is the Stackelberg leader in the PD-newsvendor model with buybacks, wherein ξ follows a uniform distribution and $D(p) = e^{-p}$, Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$ is the unique equilibrium sequence.*

Next, we consider the case when R is the leader. Since the analysis for all sequences when R is the leader, except for Sequence 7: $\underline{R{:}p;\ M{:}w;\ R{:}Q;\ M{:}b}$, has been done for a general ξ and[3] a general $D(p)$, we only need to study Sequence 7 under a uniform ξ and $D(p) = e^{-p}$.

Proposition 4.4 *For a uniformly distributed ξ and for $D(p) = e^{-p}$,*

(i) $E\Pi_M^*(S7) = 2E\Pi_R^*(S7) = \frac{e}{2}E\Pi_M^*(TS) = \frac{e}{2}E\Pi_R^*(TS)$.

(ii) $Q^*(S7) = \frac{e}{2}Q^*(TS) > Q^*(TS)$.

(iii) $p^*(S7) = p^*(TS) - 1 < p^*(TS)$.

(iv) $w^*(S7) = w^*(TS) - \frac{3+c-\sqrt{c^2+6c+1}}{4} < w^*(TS)$ *and*

(v) $b^*(S7)(= 0) < b^*(TS)$,

where "TS" and "S7" stand for "the traditional sequence" and "Sequence 7", respectively.

Proposition 4.4 implies that the preference of M and R between the traditional sequence and Sequence 7 is not expected demand function invariant. Indeed, for example, by Proposition 4.4 (i), for a uniform ξ and $D(p) = e^{-p}$, M (respectively, R) realizes a strictly higher (respectively, lower) expected profit in Sequence 7 than in the traditional sequence. By contrast, for a uniform ξ and $D(p) = 1-p$, M's and R's expected profits in Sequence 7 coincide with their profits in the traditional sequence. Note, however, that in Sequence 7, for a uniform ξ, the ratio of M's and R's equilibrium expected profits coincides for both linear and exponential expected demand functions, and is equal to 2 : 1.

[3]Except for Sequence 8, for which though, the result is valid for both exponential and negative polynomial expected demand functions.

Finally, we note that for both linear and exponential expected demand functions, whatever R offers initially when he is the leader, his expected profit is always zero. Thus, he is indifferent between being the Stackelberg leader or not having any deal whatsoever with M. However, according to the proof of Proposition 4.1, R realizes a strictly positive expected profit in Sequence 2. Thus, R prefers not to be the leader and would rather have M move first, and the unique equilibrium outcome is Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$.

4.1.2 Negative polynomial expected demand function

We assume in this subsection that $D(p) = p^{-q}$ with $q > 1$, and ξ follows a uniform distribution on $[0, K]$. Recall from Granot and Yin (2004a) and Song et al. (2004) that in the traditional sequence with $D(p) = p^{-q}$, the equilibrium values of the decision variables are:

$$w^* = \frac{qc}{q-1}, \quad b^* = 0, \quad p^* = \frac{q(q+1)c}{(q-1)^2}, \quad \text{and} \quad Q^* = \frac{2K(q-1)^{2q}}{(qc)^q(q+1)^{q+1}}, \tag{13}$$

and the equilibrium values of the expected profits are:

$$E\Pi_M^* = \frac{2K(q-1)^{2q-1}}{c^{q-1}q^q(q+1)^{q+1}} \quad \text{and} \quad E\Pi_R^* = \frac{2K(q-1)^{2q-2}}{(cq)^{q-1}(q+1)^{q+1}}. \tag{14}$$

As revealed by Proposition 4.5 below, M's preference among the traditional sequence and Sequences 1 and 2 is invariant to the type of expected demand function considered in this paper.

Proposition 4.5 *For a uniformly distributed ξ and for $D(p) = p^{-q}$,*

$$E\Pi_M^*(S2) > E\Pi_M^*(S1) > E\Pi_M^*(TS),$$

where "S1", "S2" and "TS" stand for "Sequence 1", "Sequence 2" and "the traditional sequence", respectively.

Recall from Propositions 3.4 and 3.5 that for a power distribution of ξ and for $D(p) = p^{-q}$, Sequence 3: $\underline{M{:}w;\ R{:}p,Q;\ M{:}b}$ and Sequence 4: $\underline{M{:}w;\ R{:}Q;\ M{:}b;\ R{:}p}$ coincide with the wholesale price-only contract sequence, and that it follows from Song et al. (2004) that the traditional sequence: $\underline{M{:}w,b;\ R{:}p,Q}$ also coincides with the wholesale price-only contract sequence. Thus, M's expected profit in Sequences 3 and 4 and the traditional sequence is identical.

Let us next seek the equilibrium sequence, assuming a power distribution of ξ and $D(p) = p^{-q}$. Consider the case when M is the leader. If M offers, initially, only a wholesale price w, then, recall from the analysis in §3.4 that R would specify only the retail price p. On the other hand, if M offers, initially, only a buyback rate b, then we need to compare: Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$, Sequence 5: $\underline{M{:}b;\ R{:}p, Q;\ M{:}w}$, and Sequence 6: $\underline{M{:}b;\ R{:}Q;\ M{:}w;\ R{:}p}$. Recall from Propositions 3.6 and 3.7 that R's expected profit is always zero in Sequences 5 and 6, and according to the proof of Proposition 4.5, R realizes a strictly positive expected profit in Sequence 2. Thus, R, again, would set only his retail price p when M initially offers b. Thus, by Proposition 4.5:

Corollary 4.6 *When M is the Stackelberg leader in the PD-newsvendor model with buybacks, wherein ξ follows a uniform distribution and $D(p) = p^{-q}$, Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$ is the unique equilibrium sequence.*

Let us next compare Sequence 7 and the traditional sequence under a uniform ξ and $D(p) = p^{-q}$.

Proposition 4.7 *For a uniformly distributed ξ and for $D(p) = p^{-q}$,*

(i) $E\Pi_R^*(S7) < E\Pi_R^*(TS)$.

(ii) $E\Pi_M^*(S7) < E\Pi_M^*(TS)$ *for* $q \in (1, 2)$, $E\Pi_M^*(S7) = E\Pi_M^*(TS)$ *for* $q = 2$ *and* $E\Pi_M^*(S7) > E\Pi_M^*(TS)$ *for* $q \in (2, \infty)$.

(iii) $E\Pi_M^*(S7) = 2E\Pi_R^*(S7)$.

(iv) $Q^*(S7) = \frac{1}{2}(\frac{q}{q-1})^q Q^*(TS) > Q^*(TS)$.

(v) $p^*(S7) = \frac{q-1}{q} p^*(TS) < p^*(TS)$.

(vi) $w^*(S7) = w^*(TS)$ *and*

(vii) $b^*(S7) = b^*(TS) = 0$.

Proposition 4.7 (ii) confirms that the preference of M and R between the traditional sequence and Sequence 7 depends on the form of the expected demand function. Indeed, for a uniform ξ and for $D(p) = p^{-q}$, M would be strictly worse off in Sequence 7 ($1 < q < 2$), as compared to the traditional sequence. However, we note again that for $D(p) = p^{-q}$, as was the case for linear and exponential expected demand functions, when M is the Stackelberg leader, Sequence 2 is the unique equilibrium sequence, and R is indifferent between being the Stackelberg leader and not having any deal whatsoever with M. According to the proof of Proposition 4.5, R realizes a strictly positive expected profit in Sequence 2. Thus, R prefers not to be the leader and would rather have M move first, and the unique equilibrium sequence is Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$.

4.2 Extension to a power demand distribution

In this subsection we maintain the assumption that $D(p) = 1 - p$, whenever necessary, and extend the results derived for a uniformly distributed ξ to a more general power distribution. Note that by Song et al. (2004), the equilibrium values of the decision variables and expected profits are available for the traditional sequence with a power distribution of ξ. For comparison purposes, these values, as a function of c, are presented in Table A.1 in the appendix when the exponent t in the power distribution is equal to 1, 2 and 4. Let us next consider the following three sequences: Sequence 1: $\underline{M{:}w;\ R{:}p;\ M{:}b;\ R{:}Q}$, Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$ and Sequence 7: $\underline{R{:}p;\ M{:}w;\ R{:}Q;\ M{:}b}$. For the proofs of Propositions 4.8, 4.9 and 4.10 below, please refer to the proofs of Propositions 3.2, 3.3 and 3.11, respectively, in the appendix.

Proposition 4.8 *In Sequence 1:* $\underline{M{:}w;\ R{:}p;\ M{:}b;\ R{:}Q}$*, for a power distribution of* $\xi \in [0, K]$ *with* $f(\epsilon) = \gamma(\epsilon)^t$ *(where* $t > 0$ *and* $\gamma = (t+1)K^{-(t+1)}$*), and for* $D(p) = 1-p$*, the equilibrium value of the retail price* p^* *satisfies* $\frac{dE\Pi_M(p)}{dp} = 0$*, where* $E\Pi_M(p) = \frac{\gamma(1-p)pz^{t+2}}{t+2}$*,* $z = [\frac{(t+1)[(t+1)(w(p)-c)+p-c]}{\gamma(t+2)p}]^{\frac{1}{t+1}}$*,* $w(p) = \frac{A+B}{2(t+1)(pt+1)}$*,* $A = -pt^2 + 2p^2t^2 + pt^2c + tc + 2ptc + 2c - pt + 2p^2t$*, and* $B = \sqrt{(t+2)(-ptc-c-p-pt+2p^2+2p^2t)(-pt^2c-2ptc-tc-2c+2p+pt-pt^2+2p^2t+2p^2t^2)}$*.*

It appears unlikely that closed-form expressions for the equilibrium values in Sequence 1 can be derived for any value of t. Thus, let us first consider the case of $t = 1$.

For $t = 1$, $w(p) = \frac{-2p+4p^2+3pc+3c+\sqrt{3(-pc-c-2p+4p^2)(-3pc-3c+2p+4p^2)}}{4(1+p)}$ (and since $w \in [c, p)$, we must have $p \geq \frac{2+c+\sqrt{c^2+20c+4}}{8}$), $z = \sqrt{\frac{2}{3\gamma}}\sqrt{\frac{2w+p-3c}{p}}$ and M's expected profit function becomes:

$$E\Pi_M = \frac{2}{9}\sqrt{\frac{2}{3\gamma}}(1-p)p(\frac{2w(p)+p-3c}{p})^{\frac{3}{2}}.$$

We use Matlab to search over $p \in [\frac{2+c+\sqrt{c^2+20c+4}}{8}, 1)$, to find the unique equilibrium retail price p^* which maximizes M's expected profit function, and accordingly, we can compute the equilibrium values of the other decision variables and expected profits of M and R. We have conducted a similar analysis for $t = 2$ and $t = 4$, but for space consideration, we do not report the detailed analysis here. The equilibrium values in Sequence 1, as a function of c, for $t = 0$, 1, 2 and 4, are presented in Table A.2 in the appendix.

Let us next consider Sequence 2 under a power distribution of ξ.

Proposition 4.9 *In Sequence 2:* $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$*, for a power distribution of* $\xi \in [0, K]$ *with* $f(\epsilon) = \gamma(\epsilon)^t$ *(where* $t > 0$ *and* $\gamma = (t+1)K^{-(t+1)}$*), and for* $D(p) = 1-p$*, the equilibrium value of the retail price* p^* *satisfies* $\frac{dE\Pi_M(p)}{dp} = 0$*, where* $E\Pi_M(p) = \frac{t+1}{t+2}(1-p)(p-c)z$*,* $z = (\frac{(t+1)(p-c)}{\gamma((t+2)p-(t+1)b(p))})^{\frac{1}{t+1}}$*,* $b(p) = \frac{-V+\sqrt{V^2-4US}}{2U}$*,* $U = -(t+1)(pt-tc-c-2+3p)$*,* $V = -pt^2+t^2c+9p^2t+3p^2t^2-3pt^2c-5ptc+6p^2-3p-3c-4pt$*, and* $S = -p(t+2)(-2ptc+tc-pt+2p^2t-p+2p^2-c)$*.*

Similar to Sequence 1, it is difficult to derive closed-form expressions for the equilibrium values in Sequence 2 for any value of t. Thus, we consider below some specific values of t.

For $t = 1$, $b(p) = \frac{4p+c-9p^2+4pc+\sqrt{-15p^4+24p^3-34p^2c+24p^3c+8pc^2-8p^2+8pc+c^2-8p^2c^2}}{4(1+c-2p)}$ (and since $b(p) \in [0, p)$, we must have $p \geq \frac{1}{15}((6-\sqrt{6})(1+c)+\sqrt{3(7-2\sqrt{6})(2c^2+c+2\sqrt{6}c+2)}))$, $z = \sqrt{\frac{2(p-c)}{\gamma(3p-2b(p))}}$ and M's expected profit function becomes:

$$E\Pi_M = \frac{2}{3}\sqrt{\frac{2}{\gamma}}(1-p)(p-c)\sqrt{\frac{p-c}{3p-2b(p)}}.$$

Again, we use Matlab to search over $p \in [\frac{1}{15}((6-\sqrt{6})(1+c)+\sqrt{3(7-2\sqrt{6})(2c^2+c+2\sqrt{6}c+2)}), 1)$, to find the equilibrium retail price p^* which maximizes M's expected profit function, and accordingly, we can calculate the equilibrium values of the other decision variables and expected profits of M and R. Similarly, we have analyzed the cases for $t = 2$ and $t = 4$, but for space consideration, we do not report the detailed analysis here. The equilibrium values in Sequence 2, as a function of c, for $t = 0$, 1, 2 and 4, are presented in Table A.3 in the appendix.

Finally, we examine Sequence 7 under a power distribution of ξ.

Proposition 4.10 *In Sequence 7: R:p; M:w; R:Q; M:b, for a power distribution of* $\xi \in [0, K]$ *with* $f(\epsilon) = \gamma(\epsilon)^t$ *(where* $t > 0$ *and* $\gamma = (t+1)K^{-(t+1)}$*), and for* $D(p) = 1 - p$*: for* $c = 0$*, the equilibrium values of the decision variables are:* $z^* = (\frac{t+1}{\gamma(t+2)})^{\frac{1}{t+1}}$, $p^* = \frac{1}{2}$, $w^* = \frac{1}{2(t+2)}$ *and* $b^* = 0$*; and for* $c > 0$, $p^*(z) = \frac{c}{1-\frac{\gamma(t+2)}{t+1}z^{t+1}}$ *and the equilibrium value of the stocking factor* z^* *satisfies* $\frac{dE\Pi_R(z)}{dz} = 0$*, where* $E\Pi_R = \frac{\gamma c(t+1)[(t+1)(1-c)-\gamma(t+2)z^{t+1}]z^{t+2}}{(t+2)[(t+1)-\gamma(t+2)z^{t+1}]^2}$.

Similar to Sequences 1 and 2, it is difficult to derive closed-form expressions for the equilibrium values in Sequence 7 for any value of t. So let $c > 0$, and consider the case where $t = 1$.

For $t = 1$, $p(z) = \frac{c}{1-\frac{3\gamma}{2}z^2}$ (and since $p \in [c, 1)$, we must have $z \leq \sqrt{\frac{2(1-c)}{3\gamma}}$), and R's expected profit function reduces to: $E\Pi_R = \frac{2\gamma c}{3}\frac{[2(1-c)-3\gamma z^2]z^3}{(2-3\gamma z^2)^2}$, which is unimodal in $z \in [0, \sqrt{\frac{2(1-c)}{3\gamma}}]$. We have used Matlab to search over $z \in [0, \sqrt{\frac{2(1-c)}{3\gamma}}]$, to find the equilibrium stocking factor z^* which maximizes R's expected profit function in Stage 1, and accordingly, we can calculate the equilibrium values of the other decision variables and expected profits of M and R. Similarly, we have analyzed the cases for $t = 2$ and $t = 4$, but for space consideration, we do not report the detailed analysis in these two cases. The equilibrium values in Sequence 7, as a function of c, for $t = 0$, 1, 2 and 4, are presented in Table A.4 in the appendix.

Based on the numerical results derived for the traditional sequence and Sequences 1 and 2, which are displayed in Tables A.1, A.2 and A.3, respectively, in the appendix, we observe that:

Observation 4.11 *For a power distribution of* ξ *with* $f(\epsilon) = \gamma(\epsilon)^t$ *and* $t = 1$*, 2 and 4, and for* $D(p) = 1 - p$*:*

(i) $E\Pi^*_M(S2) > E\Pi^*_M(S1) > E\Pi^*_M(TS)$.

(ii) $E\Pi^*_{M+R}(S2) > E\Pi^*_{M+R}(S1) > E\Pi^*_{M+R}(TS)$.

(iii) $E\Pi^*_R(TS) > E\Pi^*_R(S2)$ *and* $E\Pi^*_R(S1) > E\Pi^*_R(S2)$.

(iv) $p^*(TS) > p^*(S1) > p^*(S2)$.

(v) $w^*(S2) > w^*(S1)$, *and*

(vi) $Q^*(S2) > Q^*(S1)$ *and* $Q^*(S2) > Q^*(TS)$,

where, as we recall, $E\Pi^*_{M+R}$ *stands for the equilibrium expected channel profit.*

Note that Observation 4.11 (i) is consistent with Proposition 3.8 and Observation 3.9 (i), according to which, in equilibrium, M attains the highest expected profit in Sequence 2 and the lowest expected profit in the traditional sequence. Note further that Observation 4.11 (ii) implies that both Sequences 1 and 2 improve channel efficiency, which is consistent with Observation 3.9 (ii). Observation 4.11 (iii) implies that R attains the lowest expected profit in Sequence 2, which is consistent with Observation 3.9 (iii). Based on R's equilibrium expected profit in the traditional sequence and Sequence 1, as displayed in Tables A.1 and A.2, respectively, for $t = 1$, 2 and 4, we observe that R's expected profit in Sequence 1 is larger (respectively, smaller) than in the traditional sequence when the manufacturing cost c is small (respectively, large). For example, when $t = 2$, as compared to the traditional sequence, there is an increase (respectively decrease) of 1.65% (respectively, 14.15%) in R's expected profit for $c = 0$ (respectively, $c = 0.9$). This observation implies that R's preference between the traditional sequence and Sequence 1 is not demand distribution invariant. Indeed, for example, by Observation 3.9 (iii), for a uniform ξ and $D(p) = 1-p$, R always realizes a lower equilibrium expected profit in Sequence 1 than in the traditional sequence.

Let us now seek the equilibrium sequences when M moves first, assuming a power distribution and $D(p) = 1 - p$. If M is the leader who initially offers a wholesale price w in the first step, then, as we recall from the analysis in §3.4, R's best response is to commit only to a retail price p in his first step. On the other hand, if M initially offers a buyback rate b, then, we recall from the analysis in §3.4 that R, again, would prefer to set only his retail price p. Therefore, Observation 4.11 (i) immediately implies that under a power distribution with $t = 1$, 2 and 4 and $D(p) = 1 - p$, Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$ is the unique equilibrium sequence.

By examining the equilibrium values in the traditional sequence and Sequence 7 in Tables A.1 and A.4, respectively, in the appendix, we can make the following observations:

Observation 4.12 *For a power distribution of* ξ *with* $f(\epsilon) = \gamma(\epsilon)^t$ *and* $t = 1$, *2 and 4, and for* $D(p) = 1 - p$:

(i) $E\Pi^*_M(S7) > E\Pi^*_M(TS)$.

(ii) $E\Pi^*_{M+R}(S7) > E\Pi^*_{M+R}(TS)$.

(iii) $E\Pi^*_R(TS) > E\Pi^*_R(S7)$.

(iv) $w^*(TS) > w^*(S7)$.

(v) $p^*(TS) > p^*(S7)$, *and*

(vi) $Q^*(S7) > Q^*(TS)$.

Observation 4.12 (i), (ii) and (iii) imply that for a power distribution and $t = 1$, 2 and 4, in equilibrium, M and the channel (respectively, R) realize a higher (respectively, lower) expected profit in Sequence 7, with a larger order quantity, than in the traditional sequence. This result is different from that derived for the uniform distribution case, wherein M's and R's equilibrium expected profits and order quantity in the traditional sequence and Sequence 7 coincide. Observation 4.12 (iv) and (v) are consistent with Corollary 3.12 (iii) and (iv), respectively, according to which, Sequence 7 results with lower wholesale and retail prices than in the traditional sequence.

Finally, recall from §3.4 that when R is the first mover, he gets a zero profit, and that he realizes a strictly positive expected profit in Sequence 2. Thus, R would rather not be the first mover, and, as was the case for a uniform ξ, Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$ is the unique equilibrium sequence for a power distributed ξ with $t = 1$, 2 and 4, and the first mover in these cases is determined endogenously.

5 Conclusions and Further Research

We have introduced in this paper the sequential commitment approach for determining the values of contract parameters, and have analyzed its effect on the PD-newsvendor model with buybacks. As argued earlier, compared to the traditional approach, the sequential commitment approach introduces additional flexibility to contracting between members in the supply chain. Indeed, while contracting according to the traditional approach follows the take-it-or-leave-it paradigm, the sequential commitment approach allows members, if they so desire, not to commit simultaneously to values of all contract parameters under their control. It also allows them to strategically sequence the order by which they commit to these values. As such, the sequential commitment approach is more in line with other approaches to model contracting in the supply chain (see, e.g., Nagarajan and Bassok (2002), and Iyer and Valis-Boas (2003)), and it can provide some insight, such as who should move first, or which contract parameter should be discussed first, or which pair of contract parameters should be negotiated as a package (e.g., b and p), or which orders of the issues to be negotiated should be avoided since they may lead to an impasse, when the supply chain members engage in a negotiation process for determining the values of contract parameters.

Our analysis has revealed that the sequential commitment approach endogenizes the first mover decision. Indeed, while in the traditional approach it is arbitrarily assumed that one of the parties, usually M, is the leader, in the sequential commitment approach, under certain conditions (e.g., uniform ξ and linear, exponential and negative polynomial expected demand functions), both parties prefer that M will move first. Additionally, it was revealed that Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$ is the unique equilibrium sequence in the sense that both parties prefer that M will move first, and neither party can benefit by resequencing the order at which it commits to contract parameters under its control.

We have further demonstrated that sequential commitment can have a significant effect on the supply chain performance and on the fortunes of its members. Indeed, sequential commitment can significantly increase M's expected profit, as compared to the traditional sequence. For example, based on Tables A.1, A.2 and A.3, for a uniform ξ, $D(p) = 1 - p$ and $c = 0.9$, Sequence 1: $\underline{M{:}w;\ R{:}p;\ M{:}b;\ R{:}Q}$ and Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$ improve M's equilibrium expected profit by 25.19% and 79.25%, respectively, as compared to the traditional sequence. In that respect we note that for an arbitrary distribution of ξ and an arbitrary form of $D(p)$, Sequence 5: $\underline{M{:}b;\ R{:}p,Q;\ M{:}w}$ and Sequence 6: $\underline{M{:}b;\ R{:}Q;\ M{:}w;\ R{:}p}$ result with M attaining the entire expected channel profit and R getting nothing. However, Sequences 5 and 6 require, e.g., that R commits to an order quantity before the wholesale price is set, and thus are not very realistic.

By contrast, sequential commitment could adversely affect significantly R's performance. For example, for a power distribution of ξ with $t = 0$ (uniform), and $t = 1$, 2 and 4, and $D(p) = 1 - p$, R can never do better than in the traditional sequence when he moves first, and when M is the first mover, sequential commitment can significantly decrease R's equilibrium expected profit. For example, based on Tables A.1 and A.3, for a uniform ξ and $D(p) = 1 - p$, R is always worse off in Sequence 2 than in the traditional sequence, and, e.g., Sequence 2 decreases R's equilibrium expected profit by 73.51% for $c = 0.9$.

We can further conclude from Tables A.1, A.2 and A.3 in the appendix that buybacks, coupled with sequential commitment, can increase significantly channel efficiency. For example, Sequence 2, for a uniform ξ and $D(p) = 1 - p$, increases channel efficiency from 10.90%, for $c = 0$, to 21.25%, for $c = 0.9$. This result should be compared to the relatively insignificant effect of introducing buybacks in the PD-newsvendor model. Indeed, as it was shown by Granot and Yin (2004a), for a uniform ξ and $D(p) = 1 - p$, buybacks increase channel efficiency by at most 3.16%.

Finally, our results demonstrate that the sequential commitment approach could have a significant effect in the PD-newsvendor model, and it would be interesting to investigate the robustness of our results for different distributions of ξ, other than the power distribution, as well as for other expected demand functions. It would also be interesting to extend the sequential commitment approach to other operations management models as well as to the additive demand model (i.e., $X = D(p) + \xi$) of the PD-newsvendor problem. However, as suggested in Footnote 1 in §2, (see also Emmons and Gilbert (1998), Mills (1959), Petruzzi and Dada (1999), and Granot and Yin (2004b)), the additive model could produce results which are qualitatively different from those derived for the multiplicative demand model.

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Appendix

Proof of Proposition 3.2. We use backward induction to solve Sequence 1: $\underline{M{:}w;\ R{:}p;\ M{:}b;\ R{:}Q}$, which is a four-stage Stackelberg game, assuming that $\xi \in [0, K]$ follows a general power distribution $f(\epsilon) = \gamma(\epsilon)^t$, where $\gamma = (t+1)K^{-(t+1)}$, and $D(p) = 1-p$. Recall that the expected profit functions of M and R are given in (2).

Stage 4: Given (w, p, b), R chooses an order quantity Q to maximize his expected profit, given by (2). Note that choosing Q is equivalent to choosing z, where $z = \frac{Q}{D(p)}$. One can easily verify that $E\Pi_R(z)$ is concave in z. Thus, $\frac{dE\Pi_R(z)}{dz} = 0$ gives us the unique optimal z^*, which satisfies $bz^{t+1} = pz^{t+1} - \frac{t+1}{\gamma}(p-w)$, and R's expected profit function reduces to: $E\Pi_R = \frac{t+1}{t+2}(p-w)D(p)z^*$.

Stage 3: Given (w, p), M chooses her optimal b to maximize her expected profit, given by (2). We work with z instead of b for M's problem (see Lariviere (1999)). Taking into account z^* from Stage 4, M's expected profit function becomes: $E\Pi_M = D(p)[(w-c)z + \frac{(p-w)z}{t+2} - \frac{\gamma p z^{t+2}}{(t+1)(t+2)}]$, which is concave in z. Thus, $\frac{dE\Pi_M(z)}{dz} = 0$ gives us the unique optimal $z^* = [\frac{t+1}{\gamma(t+2)}]^{\frac{1}{t+1}} \cdot [\frac{(t+1)(w-c)+p-c}{p}]^{\frac{1}{t+1}}$, and M's expected profit function reduces to: $E\Pi_M = \frac{\gamma}{t+2}D(p)p \cdot (z^*)^{t+2}$.

Stage 2: Given w and knowing z^*, R chooses p to maximize his expected profit function, which reduces to $E\Pi_R = \frac{t+1}{t+2}D(p)(p-w)z^*$, where $D(p) = 1-p$. The first-order condition (F.O.C.) yields $\frac{dE\Pi_R(w,p)}{dp} = A \cdot \frac{t+1}{\gamma(t+2)^2p^2(z^*)^t}$, where $A = (t+1)(1-2p+w)p[(t+1)(w-c)+p-c] + (1-p)(p-w)[c-(t+1)(w-c)]$. Since $A(p=w) > 0$ and $A(p=1) < 0$, we have $\frac{dE\Pi_R(p)}{dp}(p=w) > 0$ and $\frac{dE\Pi_R(p)}{dp}(p=1) < 0$, and the optimal retail price is an inner solution (i.e., $w < p^*(w) < 1$) which satisfies $\frac{dE\Pi_R(p)}{dp} = 0$, i.e., $A(p) = 0$.

Stage 1: We work with p instead of w for M's problem in Stage 1. Note that A can be written as a function of w as follows: $A(w) = (t+1)(1-2p+w)p[(t+1)(w-c)+p-c] + (1-p)(p-w)[c-(t+1)(w-c)]$, which is quadratic in w, and there is a unique $w^*(p) \in [c,p)$, $w^*(p) = \frac{-pt^2+2p^2t^2+pt^2c+tc+2ptc+2c-pt+2p^2t+\sqrt{(t+2)(-ptc-c-p-pt+2p^2+2p^2t)(-pt^2c-2ptc-tc-2c+2p+pt-pt^2+2p^2t+2p^2t^2)}}{2(t+1)(pt+1)}$, which satisfies $A(w^*(p)) = 0$. M's problem in Stage 1 is to choose p to maximize $E\Pi_M = \frac{\gamma}{t+2}(1-p)pz^{t+2}$, where $z = [\frac{t+1}{\gamma(t+2)}]^{\frac{1}{t+1}} \cdot [\frac{(t+1)(w^*(p)-c)+p-c}{p}]^{\frac{1}{t+1}}$.

To complete the proof of Proposition 3.2, we need to consider the case where ξ is uniformly distributed, i.e., $t = 0$. For $t = 0$, $w^*(p) = c + \sqrt{(p-c)(2p^2-p-c)}$ (and since $w \in [c,p)$, we must have $p \geq \frac{1+\sqrt{1+8c}}{4}$), $z^*(w,p) = \frac{w+p-2c}{2\gamma p}$ and M's expected profit function in Stage 1 becomes:

$$E\Pi_M = \frac{1}{8\gamma}\frac{1-p}{p}(w^*(p)-c+p-c)^2 = \frac{1}{8\gamma}(1-p)(p-c)H_1, \tag{A.1}$$

where $H_1 = \frac{(\sqrt{2p^2-p-c}+\sqrt{p-c})^2}{p}$. One can verify that for $p \in [\frac{1+\sqrt{1+8c}}{4}, 1)$, $\frac{d[(1-p)(p-c)]}{dp} < 0$, $\frac{d^2[(1-p)(p-c)]}{dp^2} < 0$, $\frac{dH_1}{dp} > 0$ and $\frac{d^2H_1}{dp^2} < 0$. Thus, $E\Pi_M$ in (A.1) is concave in p, and the F.O.C. yields the unique equilibrium retail price p^* in Sequence 1. Accordingly, it is easy to derive the equilibrium values of the other decision variables: $w^* = c + \sqrt{(2(p^*)^2-p^*-c)(p^*-c)}$, $b^* = \frac{p^*(3w^*-p^*-2c)}{w^*+p^*-2c}$, $z^* = \frac{K(w^*+p^*-2c)}{2p^*}$ and $Q^* = (1-p^*)z^*$, and the expected profits: $E\Pi_M^* = \frac{K(1-p^*)(w^*+p^*-2c)^2}{8p^*}$ and $E\Pi_R^* = \frac{K(1-p^*)(p^*-w^*)(w^*+p^*-2c)}{4p^*}$, where we recall that $K = \frac{1}{\gamma}$ for $t = 0$. □

Proof of Proposition 3.3. We analyze Sequence 2: M:b; R:p; M:w; R:Q, assuming that $\xi \in [0, K]$ has a power distribution $f(\epsilon) = \gamma(\epsilon)^t$, where $\gamma = (t+1)K^{-(t+1)}$, and $D(p) = 1-p$.

Stage 4: R's problems in Stage 4 in Sequences 2 and 1 coincide. Thus, the unique z^* satisfies $w = p - \frac{\gamma\cdot(p-b)z^{t+1}}{t+1}$, and R's expected profit function becomes: $E\Pi_R = \frac{\gamma}{t+2}D(p)(p-b)(z^*)^{t+2}$.

Stage 3: Knowing (b,p) and $z^*(b,p,w)$, we solve M's problem in Stage 3 by working with z instead of w. M's expected profit function as a function of w becomes: $E\Pi_M = D(p)[(p-c)z - \frac{\gamma z^{t+2}}{(t+1)(t+2)}[(t+2)p-(t+1)b]]$, which is concave in z. Thus, $\frac{dE\Pi_M(z)}{dz} = 0$ yields $z^*(b,p) =$

$(\frac{(t+1)(p-c)}{\gamma\cdot[(t+2)p-(t+1)b]})^{\frac{1}{t+1}}$, and M's expected profit function becomes: $E\Pi_M = \frac{t+1}{t+2}D(p)(p-c)z^*$.

Stage 2: Given b and taking into account $z^*(b,p)$ in Stage 3, R chooses p to maximize $E\Pi_R(p) = \frac{\gamma}{t+2}D(p)(p-b)(z^*(b,p))^2$, where $D(p) = 1-p$. The F.O.C. yields: $\frac{dE\Pi_R(p)}{dp} = \frac{A}{\gamma[(t+2)p-(t+1)b]^2 z^{t-1}}$, where $A = (t+1)(p-c)(1+b-2p)[(t+2)p-(t+1)b] + 2(1-p)(p-b)[(t+2)c-(t+1)b]$. Since $A(p = \max(b,c)) > 0$ and $A(p = 1) < 0$, $\frac{dE\Pi_R(b,p)}{dp} > 0$ and $\frac{dE\Pi_R(b,p)}{dp} < 0$, we conclude that the optimal retail price is an inner solution (i.e., $\max(b,c) < p^*(b) < 1$) satisfying $\frac{dE\Pi_R(p)}{dp} = 0$, i.e., $A(p) = 0$. Note that $E\Pi_R = \frac{\gamma}{t+2}(1-p)(p^*(b)-b)(z^*(b,p^*(b)))^{t+2}$, which is strictly positive since $p^*(b) > \max(b,c)$ and $z^*(b,p^*(b)) > 0$.

Stage 1: Again, we work with p instead of b for M's problem in Stage 1. Note that A can be written as a function of b as follows: $A(b) = (t+1)(p-c)(1+b-2p)[(t+2)p-(t+1)b] + 2(1-p)(p-b)[(t+2)c-(t+1)b] = U\cdot b^2 + V\cdot b + S = 0$, where $U = -(t+1)(pt-tc-c-2+3p)$, $V = -pt^2+t^2c+9p^2t+3p^2t^2-3pt^2c-5ptc+6p^2-3p-3c-4pt$ and $S = -p(t+2)(-2ptc+tc-pt+2p^2t-p+2p^2-c)$, which is quadratic in b, and there is a unique $b^*(p) = \frac{-V+\sqrt{V^2-4US}}{2U} \in [0,p)$, satisfying $A(b^*(p)) = 0$. M's problem in Stage 1 is to choose p to maximize $E\Pi_M = \frac{t+1}{t+2}(1-p)(p-c)z$, where $z = (\frac{(t+1)(p-c)}{\gamma((t+2)p-(t+1)b^*(p))})^{\frac{1}{t+1}}$.

To complete the proof of Proposition 3.3, we need to consider the case where ξ is uniformly distributed, i.e., $t = 0$. For $t = 0$, $b^*(p) = \frac{-3(2p^2-p-c)+\sqrt{(2p^2-p-c)(-6p^2+7p+8pc-9c)}}{2(2+c-3p)} \in [0,p)$ (thus, $p \geq \frac{1+\sqrt{1+8c}}{4}$), $z^*(b,p) = \frac{p-c}{\gamma(2p-b^*(p))}$ and M's expected profit function in Stage 1 becomes:

$$E\Pi_M = \frac{p-c}{16\gamma}\cdot\frac{8(1-p)(p-c)}{2p-b^*(p)} = \frac{p-c}{16\gamma}H_2, \tag{A.2}$$

where $H_2 = \frac{8(1-p)(p-c)}{2p-b^*(p)} = \frac{-6p^2+5p+4pc-3c+\sqrt{(2p^2-p-c)(-6p^2+7p+8pc-9c)}}{p}$. Using some algebra, one can show that for $p \in [\frac{1+\sqrt{1+8c}}{4}, 1)$, $E\Pi_M$ in (A.2) is concave in p. Thus, the F.O.C. results with a unique equilibrium retail price p^* in Sequence 2. Accordingly, we can easily derive the equilibrium values of the other decision variables: $b^* = \frac{-3(2(p^*)^2-p^*-c)+\sqrt{(2(p^*)^2-p^*-c)(-6(p^*)^2+7p^*+8p^*c-9c)}}{2(2+c-3p^*)}$, $z^* = \frac{K(p^*-c)}{2p^*-b^*}$, $Q^* = (1-p^*)z^*$ and $w^* = p^* - \frac{(p^*-b^*)(p^*-c)}{2p^*-b^*}$, and the equilibrium expected profits are: $E\Pi_M^* = \frac{K(p^*-c)H_2(p^*)}{16}$ and $E\Pi_R^* = \frac{K(1-p^*)(p^*-b^*)(p^*-c)^2}{2(2p^*-b^*)^2}$, where we recall that $K = \frac{1}{\gamma}$ for $t = 0$. □

Proof of Proposition 3.5. We study the four-stage problem in Sequence 4: M:w; R:Q; M:b; R:p, assuming a power distribution of ξ with $f(\epsilon) = \gamma(\epsilon)^t$ and that $D(p)$ is decreasing in p and satisfying $D(p)\frac{d^2D(p)}{dp^2} - (t+2)(\frac{dD(p)}{dp})^2 \leq 0$.

Stage 4: Given (w,Q,b), R chooses p to maximize his expected profit, given by (1), $E\Pi_R(p) = (p-w)Q-(p-b)E[Q-X]^+$, where $E[Q-X]^+ = QF(\frac{Q}{D(p)}) - \int_0^{\frac{Q}{D(p)}} D(p)\epsilon f(\epsilon)d\epsilon = \frac{\gamma Q^{t+2}}{(t+1)(t+2)}D(p)^{-(t+1)}$, which is the expected unsold inventory. Since $D(p)$ is decreasing in p and satisfying $D(p)\frac{d^2D(p)}{dp^2} -$

$(t+2)(\frac{dD(p)}{dp})^2 \le 0$, it is not difficult to show that $\frac{dE[Q-X]^+}{dp} \ge 0$ and $\frac{d^2E[Q-X]^+}{dp^2} \ge 0$ (i.e., the expected lost sales increase in the retail price p at an increasing rate), and that $\frac{dE\Pi_R}{dp} = Q - E[Q-X]^+ - (p-b)\frac{dE[Q-X]^+}{dp}$ and $\frac{d^2E\Pi_R}{dp^2} = -2\frac{dE[Q-X]^+}{dp} - (p-b)\frac{d^2E[Q-X]^+}{dp^2} \le 0$, which implies that $E\Pi_R$ is concave in p. Thus, the F.O.C. results with a unique stationary point, $p^0(Q,b)$, which satisfies: $b\frac{dE[Q-X]^+}{dp} = p\frac{dE[Q-X]^+}{dp} + E[Q-X]^+ - Q$. Note that $p^0(Q,b)$ is independent of w. Taking derivative of the F.O.C. equation under $p = p^0(Q,b)(> b)$ with respect to b and simplifying yields: $\frac{dE[Q-X]^+}{dp} = (2\frac{dE[Q-X]^+}{dp} + (p^0(Q,b) - b)\frac{d^2E[Q-X]^+}{dp^2})\frac{\partial p^0(Q,b)}{\partial b}$. Since $E[Q-X]^+$ is increasing and convex in p and $p^0(Q,b) > b$, we conclude that $\frac{\partial p^0(Q,b)}{\partial b} \ge 0$, and $p^0(Q,b)$ increases in b. Therefore, the optimal p^* for R in Stage 4 is either p^0 or it is attained at one of the extreme points[4], w or $D^{-1}(\frac{Q}{K})$, which are independent of b.

Stage 3: Given (w,Q) and knowing p^* from Stage 4, which is either increasing or independent of b, we conclude that M's expected profit function, given by (1), is decreasing in b. Thus, $b^* = 0$, which implies that Sequence 4 coincides with the wholesale price-only contract sequence. □

Proof of Proposition 3.7. Consider Sequence 6: M:b; R:Q; M:w; R:p, assuming a general distribution of ξ and a general form of $D(p)$.

Stage 4: Given (b,Q,w), R sets p to maximize $E\Pi_R = (p-w)Q - (p-b)E[Q-X]^+$, given by (1). Assume that $p^0(b,Q)$ satisfies the F.O.C.: $Q - E[Q-X]^+ - (p-b)\frac{dE[Q-X]^+}{dp} = 0$, which is independent of w. Since $E\Pi_R$ is continuous in p, the optimal retail price p^* is either equal to $p^0(b,Q)$ or it is one of the two extreme points, w and $D^{-1}(\frac{Q}{K})$, i.e.,

$$p^* = \begin{cases} \max(w, p^0(b,Q)) & \text{if } p^0(b,Q) \le D^{-1}(\frac{Q}{K}), \\ D^{-1}(\frac{Q}{K}) & \text{if } p^0(b,Q) \ge D^{-1}(\frac{Q}{K}). \end{cases}$$

Stage 3: Given (b,Q) and knowing p^*, M chooses w to maximize $E\Pi_M = (w-c)Q - bE[Q - D(p^*)\xi]^+$, given by (1). Consider two scenarios: (A) When $p^0(b,Q) \ge D^{-1}(\frac{Q}{K})$, p^* is independent of w, and thus, $E\Pi_M$ is increasing in w. Therefore, $w^* = p^* = D^{-1}(\frac{Q}{K})$. (B) When $p^0(b,Q) \le D^{-1}(\frac{Q}{K})$, M has two options regarding w: (B1) If $w \ge p^0(b,Q)$, then M sets $w(\ge p^0(b,Q)))$ to maximize $E\Pi_M = (w-c)Q - bE[Q - D(w)\xi]^+$ and $w^* = p^*$. (B2) If $w \le p^0(b,Q)$, then $w^* = p^* = p^0(b,Q)$. Since the optimal w^* in Option (B2) is on the edge of the feasible region, we conclude that M would choose option (B1), i.e., $w \in [p^0(b,Q), D^{-1}(\frac{Q}{K})]$ is chosen to maximize $E\Pi_M = (w-c)Q - bE[Q - D(w)\xi]^+$, and at optimality, $w^* > \max(b,c)$. Thus, for any (b,Q), $p^* = w^*$.

Stage 2: Given b, R determines Q to maximize $E\Pi_R = -(w^*-b)E[Q - D(w^*)\xi]^+$. Let us

[4]Note that $Q \le KD(p)$ since $\xi \le K$. Thus, $p \le D^{-1}(\frac{Q}{K})$.

consider two scenarios: (A) When $b < c$, we immediately have $w^* = p^* > b$ and $E\Pi_R < 0$ except for $Q = 0$. Thus, R would choose $Q^* = 0$ to avoid a negative expected profit. (B) When $b \geq c$, R has three choices: (B1) If $Q = KD(b)$ and $p^0(b, Q) \geq D^{-1}(\frac{Q}{K})(= b)$, then $w^* = p^* = b$ and $E\Pi_R = 0$. (B2) If $Q \neq KD(b)$ and $p^0(b, Q) \leq D^{-1}(\frac{Q}{K})(> b)$, then $E\Pi_R < 0$ except for $Q = 0$. Thus, $Q^* = 0$. (B3) If $p^0(b, Q) \leq D^{-1}(\frac{Q}{K})$, then $w^* > b$ and $E\Pi_R < 0$ except for $Q = 0$. Thus, $Q^* = 0$. R gets a zero profit in all three options in Scenario (B). Thus, for Scenario (B), the choice of either $Q^* = KD(b)$ or $Q^* = 0$ depends, by Assumption 3.1, on M's expected profit, which is $E\Pi_M = D(b)\{K(b-c) - E[K-\xi]^+\}$ (strictly less than the expected profit of the integrated channel for any value of b since $Q^* = KD(b) \neq Q^I$) and $E\Pi_M = 0$, respectively.

Stage 1: M's decision on b in Stage 1 has two options: (A) If $b \leq c$, then $Q^* = 0$ and $E\Pi^*_M = 0$. (B) If $b \geq c$, then b is determined to maximize $E\Pi_M = \max(D(b)\{K(b-c) - E[K-\xi]^+\}, 0)$ and $E\Pi_R = 0$. □

Proof of Proposition 3.8. Assume a uniformly distributed ξ and $D(p) = 1 - p$.

$E\Pi^*_M(S2) > E\Pi^*_M(S1)$. From the analysis in the proof of Proposition 3.3, in Stage 4, in Sequence 2: M:b; R:p; M:w; R:Q, M decides upon $p \in [\frac{1+\sqrt{1+8c}}{4}, 1)$ to maximize: $E\Pi_M(S2) = \frac{p-c}{16\gamma p}(-6p^2 + 5p + 4pc - 3c + \sqrt{(2p^2 - p - c)(-6p^2 + 7p + 8pc - 9c)})$. Similarly, from the analysis in the proof of Proposition 3.2, in Stage 4, in Sequence 1: M:w; R:p; M:b; R:Q, M chooses $p \in [\frac{1+\sqrt{1+8c}}{4}, 1)$ to maximize: $E\Pi_M(S1) = \frac{(1-p)(p-c)}{8\gamma p}(\sqrt{2p^2 - p - c} + \sqrt{p-c})^2$. Next, we show that $E\Pi_M(S2) > E\Pi_M(S1)$ for any $p \in [\frac{1+\sqrt{1+8c}}{4}, 1)$, which is a sufficient condition for $E\Pi^*_M(S2) > E\Pi^*_M(S1)$. Note that $E\Pi_M(S2) > E\Pi_M(S1)$ is equivalent to $S21 \equiv -6p^2 + 5p + 4pc - 3c + \sqrt{(2p^2 - p - c)(-6p^2 + 7p + 8pc - 9c)} - 2(1-p)(\sqrt{2p^2 - p - c} + \sqrt{p-c})^2 > 0$, and it is not difficult to show that, indeed, for any $p \in [\frac{1+\sqrt{1+8c}}{4}, 1)$ and any $c \in [0, 1)$, $S21 > 0$.

$E\Pi^*_M(S1) > E\Pi^*_M(TS)$. Recall that the equilibrium value of the retail price and M's expected profit in the traditional sequence: M:w, b; R:p, Q are: $p^* = \frac{5+\sqrt{1+8c}}{8}$ and $E\Pi^*_M(TS) = \frac{K(3-\sqrt{1+8c})^3(1+\sqrt{1+8c})}{256}$, where $K = \frac{1}{\gamma}$. Evaluating $E\Pi_M(S1)$, given by (A.1), at $p^* = \frac{5+\sqrt{1+8c}}{8}$ and simplifying yields: $E\Pi_M(S1)(p^*) = \frac{(-3+\sqrt{1+8c})(-5-\sqrt{1+8c}+8c)(\sqrt{3+3\sqrt{1+8c}-12c} + \sqrt{10+2\sqrt{1+8c}-16c})^2}{1024\gamma(5+\sqrt{1+8c})}$. Now, one can easily verify that $E\Pi_M(S1)(p^*) > E\Pi^*_M(TS)$ for any $c \in [0, 1)$, implying that $E\Pi^*_M(S1) > E\Pi^*_M(TS)$. □

Proof of Proposition 3.11. We analyze Sequence 7: R:p; M:w; R:Q; M:b, assuming that ξ has a power distribution and $D(p) = 1 - p$.

Stage 4: M's problem in this stage is pretty straightforward, i.e., M would always choose $b^* = 0$.

Stage 3: Given (p, w) and $b^* = 0$, R determines $z = \frac{Q}{D(p)}$ (since Q is chosen after p) to maximize his expected profit function, given by (2), which can be rewritten as: $E\Pi_R = D(p)[(p-w)z - \frac{\gamma p z^{t+2}}{(t+1)(t+2)}]$, and is concave in z. Thus, z^* satisfies the F.O.C. $p - w - \frac{\gamma p z^{t+1}}{t+1} = 0$, and R's expected profit function becomes: $E\Pi_R = \frac{\gamma}{t+2} D(p) p z^{t+2}$.

Stage 2: Given p and knowing z^* and $b^* = 0$, M determines w to maximize $E\Pi_M = (w - c)D(p)z^*$. Again, we work with z instead of w for M's problem to maximize $E\Pi_M = D(p)[(p - c)z - \frac{\gamma}{t+1} p z^{t+2}]$, which is, again, concave in z. Thus, the F.O.C. $p - c - \frac{\gamma(t+2)}{t+1} p z^{t+1} = 0$ yields $z^*(p)$.

Stage 1: We work with z instead of p for R's problem in Stage 1. Consider two cases as follows.

(A) For $c = 0$, from Stage 2, we have $z^* = (\frac{t+1}{\gamma(t+2)})^{\frac{1}{t+1}}$, which is independent of p. Thus, R's expected profit function becomes: $E\Pi_R = \frac{\gamma}{t+2} (\frac{t+1}{\gamma(t+2)})^{\frac{t+2}{t+1}} D(p)p$, where $D(p) = 1 - p$. It is clear that $p^* = \frac{1}{2}$, and the equilibrium values of w^* and the expected profits can be computed accordingly.

(B) For $c > 0$, R chooses z to maximize $E\Pi_R = \frac{\gamma c(t+1)[(t+1)(1-c) - \gamma(t+2)z^{t+1}]z^{t+2}}{(t+2)[(t+1) - \gamma(t+2)z^{t+1}]^2}$. Similar to the analysis of Sequences 1 and 2, it is difficult to solve R's problem in Stage 1 for any value of t. Thus, to complete the proof of Proposition 3.11, we next consider a uniformly distributed ξ, i.e., $t = 0$.

When $t = 0$, $p(z) = \frac{c}{1 - 2\gamma z}$ (and since $p \in [c, 1)$, we must have $z \le \frac{1-c}{2\gamma}$), and R's expected profit function reduces to: $E\Pi_R = \frac{\gamma c(1 - c - 2\gamma z)z^2}{2(1 - 2\gamma z)^2}$, which can be easily shown to be unimodal in z. Thus, the F.O.C. gives us the unique equilibrium $z^* = \frac{K(3 - \sqrt{1+8c})}{4}$. (Recall that ξ is distributed on $[0, K]$ and $\gamma = \frac{1}{K}$.) Accordingly, we can compute the equilibrium values of the other decision variables and expected profits: $p^* = \frac{1+\sqrt{1+8c}}{4}$, $Q^* = \frac{K(3-\sqrt{1+8c})^2}{16}$, $w^* = \frac{(1+\sqrt{1+8c})^2}{16}$, $b^* = 0$, and $E\Pi_M^* = 2E\Pi_R^* = \frac{K(3-\sqrt{1+8c})^3(1+\sqrt{1+8c})}{256}$. □

Proof of Proposition 3.13. R's problem of determining p^* in Stage 3 coincides with R's problem in Stage 4 in Sequence 4: $M{:}w$; $R{:}Q$; $M{:}b$; $R{:}p$, which has been analyzed in the proof of Proposition 3.5. Therefrom, we conclude that p^* is either increasing in b or independent of b and $E[Q - D(p^*)\xi]^+$ increases in p, which implies that M's expected profit function in Stage 2, $E\Pi_M = (w - c)Q - bE[Q - D(p^*)\xi]^+$ is decreasing in b. Thus, $b^* = 0$, and M's expected profit is increasing in w. Thus, in Stage 2, $w^* = D^{-1}(\frac{Q}{K})$ and $b^* = 0$, which leads to $p^* = w^*$ and $E\Pi_R < 0$ except when $Q = 0$. To avoid a strictly negative expected profit, R would choose $Q^* = 0$ and, in equilibrium, $E\Pi_M^* = E\Pi_R^* = 0$. □

Proof of Proposition 3.14. Recall that the sequences $R{:}p$; $M{:}w$; $R{:}Q$; $M{:}b$, $R{:}Q$; $M{:}w, b$; $R{:}p$, $R{:}p$; $M{:}w, b$; $R{:}Q$ and $R{:}Q$; $M{:}w$; $R{:}p$; $M{:}b$ have been previously analyzed. Thus, below we cover the other three sequencing instances when R is the Stackelberg leader. For space consideration, we do not report the details of the analysis.

Sequence: <u>R:p; M:b; R:Q; M:w</u>**.** The analysis in this four-stage sequence under a general distribution of ξ and a general form of $D(p)$ is pretty straightforward. In Stage 4, after p, b and Q have been determined, M would set her wholesale price as high as possible. Thus, $w^* = p$. Knowing $w^* = p$, R's expected profit would be strictly negative if he chooses a strictly positive order quantity and $b \neq p$. When $b = p$, R is actually indifferent regarding the value of Q, since his expected profit will always be zero. Thus, when $b \neq p$, R would choose $Q = 0$, and otherwise, consistent with Assumption 3.1, Q will be chosen to maximize M's expected profit, i.e., $F(\frac{Q^*}{D(p)}) = \frac{p-c}{p}$. Now, it leaves the choice on whether $b = p$ to M in Stage 2. If $b \neq p$, then $Q^* = 0$ and both M and R earn a zero profit. If $b = p$, then M would realize a positive profit. Since R gets a zero expected profit for any p, p is chosen to maximize M's expected profit function, which coincides with the integrated channel problem. Therefore, in this sequence, M would secure the entire expected profit of the integrated channel, while R gets nothing.

Sequence: <u>R:p, Q; M:w, b</u>**.** The analysis in this sequence under a general ξ and a general $D(p)$ is, again, quite simple. After R's decisions on p and Q have been set, M will definitely set a high enough w and a low enough b. Thus, $w^* = p$ and $b = 0$. Taking M's response in Stage 2 into account, R would not order anything in order to avoid a negative profit. Therefore, there will be no contract between M and R and both of them will realize a zero profit.

Sequence: <u>R:Q; M:b: R:p; M:w</u>, and a general ξ and a general $D(p)$. In Stage 4, knowing (Q, b, p), M will set w as high as possible. Thus, $w^* = p$. Given (Q, b) and knowing $w^* = p$, R's expected profit function in Stage 3 becomes: $E\Pi_R = -(p - b)E[Q - X]^+$, where X is the random demand and $E[Q - X]^+$ is the lost sales. Clearly, the lost sales increase in the retail price p. Thus, $E\Pi_R$ decreases in p. Therefore, in Stage 3, R will choose a retail price as small as possible, i.e., $p^* = \max(b, c)$. In Stage 2, given Q and knowing $w^* = p^* = \max(b, c)$, M has two options: (A) $b < c$ or (B) $b \geq c$. (A) If $b < c$, then $w^* = p^* = c$ and $E\Pi_M = -bE[Q - X]^+$, which decreases in b. Thus, $b^* = 0$, and accordingly, $E\Pi_M = 0$, and $E\Pi_R = -cE[Q - D(c)\xi]^+ < 0$ except for $Q = 0$. (B) If $b \geq c$, then $w^* = p^* = b$, which is a complete consignment contract, and accordingly, $E\Pi_M = (b - c)Q - bE[Q - D(b)\xi]^+$ and $E\Pi_R = 0$. M chooses $b(\geq c)$ to maximize $E\Pi_M = (b - c)Q - bE[Q - D(b)\xi]^+$. Thus, R in Stage 1 is indifferent between $Q = 0$ in Option (A) and a complete consignment contract in Option (B). Therefore, by Assumption 3.1, Q, together with b, is used to maximize M's expected profit function $E\Pi_M = (b - c)Q - bE[Q - D(b)\xi]^+$, which coincides with the expected profit of the integrated channel. Thus, in equilibrium, M attains the entire expected profit of the integrated channel and R gets a zero profit. □

Proof of Proposition 3.15. Let us first consider Sequence 1: $\underline{M{:}w;\ R{:}p;\ M{:}b;\ R{:}Q}$, with a power distribution of ξ and a general form of $D(p)$. By the proof of Proposition 3.2 in the appendix, for any given w, in Stage 2, R's problem is to choose $p(\geq w)$ to maximize $E\Pi_R = \frac{t+1}{t+2}D(p)(p-w)z^*$, where $z^* = [\frac{t+1}{\gamma(t+2)} \cdot \frac{(t+1)(w-c)+p-c}{p}]^{\frac{1}{t+1}}$. Note that $p(z^*)^{t+1} - \frac{t+1}{\gamma}(p-w) = \frac{t+1}{\gamma(t+2)}[(t+1)(w-c) + p - c - (t+2)(p-w)] = b(z^*)^{t+1} > 0$. The last inequality is due to the fact that $b > 0$, which can be verified from the analysis in Stage 2 in the proof of Proposition 3.2, and $z^* > 0$.

In the wholesale price-only sequence: $\underline{M{:}w;\ R{:}p, Q}$ with a power distribution of ξ and any $D(p)$, for any given w, in Stage 2, R chooses p and Q to maximize R's expected profit function, $E\Pi_R = (p-w)Q - \frac{\gamma p Q^{t+2}}{D(p)^{t+1}(t+1)(t+2)}$, which is concave in Q for any p. Thus, $Q^* = D(p)(\frac{(t+1)(p-w)}{\gamma p})^{\frac{1}{t+1}}$ and R's expected profit function reduces to $E\Pi_R = \frac{t+1}{t+2}D(p)(p-w)z_w^*$, where $z_w^* = [\frac{(t+1)(p-w)}{\gamma p}]^{\frac{1}{t+1}}$. R is to choose $p(\geq w)$ to maximize his expected profit function in Stage 2.

For a given w, it is easy to show that $z^* > z_w^*$ for any value of p. Thus, for any values of w and p, R's expected profit in Sequence 1 is strictly larger than his expected profit in the wholesale price-only contract sequence. □

Proof of Proposition 4.1. Let us first consider Sequence 1: $\underline{M{:}w;\ R{:}p;\ M{:}b;\ R{:}Q}$. Since the analysis of Stages 4 and 3 in Sequence 1, as carried out in the proof of Proposition 3.2, is valid for any form of $D(p)$, we continue with the analysis in Stage 2, assuming $D(p) = e^{-p}$ and a uniform ξ on $[0, K]$ (i.e., $t = 0$).

Stage 2: From the analysis in Stages 4 and 3, we have: $b^*(w,p) = \frac{p(3w-p-2c)}{w+p-2c}$ and $z^*(w,p) = \frac{K(w+p-2c)}{2p}$, and M's and R's expected profit functions can be simplified to: $E\Pi_M = \frac{K}{8}\frac{e^{-p}(w+p-2c)^2}{p}$ and $E\Pi_R = \frac{K}{4}\frac{e^{-p}(p-w)(w+p-2c)}{p}$, respectively. In Stage 2, R chooses p to maximize $E\Pi_R$ for any given w. The F.O.C. yields $\frac{dE\Pi_R(p)}{dp} = A\frac{Ke^{-p}}{4p^2} = 0$, where $A = (-p^3 + 2p^2c + pw^2 - 2pwc + p^2 + w^2 - 2wc)$, which is a cubic function of p. Since $A(p = w) > 0$ if $w \neq c$ and $A(p \to \infty) < 0$, the optimal retail price $p^*(w) > w$ is an inner solution and satisfies $\frac{dE\Pi_R(p)}{dp} = 0$, i.e., $A(p) = 0$.

Stage 1: We work with p instead of w for M's problem in this stage. Note that A can be written as a quadratic function of w as follows: $A(w) = (1+p)w^2 - 2c(1+p)w + p^2(1+2c-p)$, and there is a unique $w^*(p) = \frac{c+pc+\sqrt{(1+p)(-c-pc+p^2-p)(p-c)}}{1+p}$, satisfying $A(w^*(p)) = 0$. Since $w^* \in [c, p)$, we must have $p \geq \frac{1+c+\sqrt{c^2+6c+1}}{2}$. Substituting $w^*(p)$ into M's expected profit function we have: $E\Pi_M = \frac{K(p-c)(p^2-pc+\sqrt{(1+p)(-c-pc+p^2-p)(p-c)}-c)e^{-p}}{4p(1+p)}$.

Next, we consider Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$. Similar to Sequence 1, the analysis of Stages 4 and 3 in Sequence 2 in the proof of Proposition 3.3 is valid for any form of $D(p)$, and we can continue our analysis in Stage 2, assuming $D(p) = e^{-p}$ and a uniform ξ on $[0, K]$ (i.e., $t = 0$).

Stage 2: From the analysis in Stages 4 and 3, we have: $w^*(b,p) = \frac{p+pc-bc}{2p-b}$ and $z^*(b,p) = \frac{K(p-c)}{2p-b}$, and M's and R's expected profit functions can be simplified to: $E\Pi_M = \frac{K}{2}\frac{e^{-p}(p-c)^2}{2p-b}$ and $E\Pi_R = \frac{K}{2}\frac{e^{-p}(p-b)(p-c)^2}{(2p-b)^2}$. R's problem in Stage 2 is to choose p to maximize $E\Pi_R$ for any given b. The F.O.C. yields $\frac{dE\Pi_R(p)}{dp} = A\frac{e^{-p}(p-c)}{(2p-b)^3}$, where $A = -2p^3+3p^2b+2p^2c-3pcb-b^2p+b^2c+2p^2-3bp+2pc-3bc+2b^2$. Since $A(p = \max(b,c)) > 0$ and $A(p \to \infty) < 0$, the optimal retail price $p^*(b)$ is an inner solution, $p^*(b) > \max(b,c)$, satisfying $\frac{dE\Pi_R(p)}{dp} = 0$, i.e., $A(p) = 0$. Note that $E\Pi_R = \frac{K}{2}\frac{e^{-p}(p-b)(p-c)^2}{(2p-b)^2} > 0$ since $p^*(b) > \max(b,c)$.

Stage 1: We work with p instead of b for M's problem in this stage. Note that A can be written as a quadratic function of b as follows: $A(b) = (2+c-p)b^2 + 3(p^2-pc-p-c)b - 2p(p^2-pc-p-c)$, and there is a unique $b^*(p) = \frac{-3(p^2-pc-p-c)+\sqrt{(p^2-pc-p-c)(p^2-pc+7p-9c)}}{2+c-p}$, satisfying $A(b^*(p)) = 0$. Since $b^* \in [0,p)$, we must have $p \geq \frac{1+c+\sqrt{c^2+6c+1}}{2}$, and M's expected profit function becomes: $E\Pi_M = \frac{Ke^{-p}(p-c)(-p^2+pc+5p-3c+\sqrt{(p^2-pc-p-c)(p^2-pc+7p-9c)})}{16p}$.

For any value of $p(\geq \frac{1+c+\sqrt{c^2+6c+1}}{2})$, let us compare M's expected profit in Sequences 1 and 2. Let $S21(p) \equiv E\Pi_M(S2) - E\Pi_M(S1)$, where $E\Pi_M(S1)$ and $E\Pi_M(S1)$ are M's expected profits in Sequences 1 and 2, respectively, for any value of p. Thus, $S21(p) = \frac{Ke^{-p}(p-c)}{16p(1+p)}\{\sqrt{p^2-pc-p-c}[(1+p)\sqrt{p^2-pc+7p-9c} - 4\sqrt{(1+p)(p-c)}] - p^3 + 2pc + p^2c + 5p + c\}$. One can verify that for any $p \geq \frac{1+c+\sqrt{c^2+6c+1}}{2}$ and any $c \geq 0$, $S21(p) > 0$, implying that $E\Pi_M(S2)(p) > E\Pi_M(S1)(p)$ and $E\Pi^*_M(S2) > E\Pi^*_M(S1)$.

Now, by (11) and (12), for $D(p) = e^{-p}$, $p^* = \frac{3+c+H}{2}$ and $E\Pi^*_M(TS) = \frac{K(3+c-H)(1-c+H)}{8}e^{-\frac{3+c+H}{2}}$ in the traditional sequence. Evaluating M's expected profit function in Stage 1 in Sequence 1 at the equilibrium retail price p^* of the traditional sequence, and simplifying yields: $E\Pi_R(S1)(p^*) = \frac{K(3-c+H)(5+c+3H+\sqrt{(5+c+H)(3-c+H))(1+H))}}{4(3+c+H)(5+c+H)}$, where $H = \sqrt{c^2+6c+1}$. By using some simple algebra, it is not difficult to verify that $E\Pi_M(S1)(p^*) > E\Pi^*_M(TS)$ for any value of c, which implies that $E\Pi^*_M(S1) > E\Pi^*_M(TS)$.□

Proof of Proposition 4.4. Consider Sequence 7: R:p; M:w; R:Q; M:b, assuming that ξ has a uniform distribution (i.e., $t = 0$ in the power distribution) and $D(p) = e^{-p}$. Since the analysis of Stages 4, 3 and 2 in Sequence 7 in the proof of Proposition 3.11 was done for an arbitrary $D(p)$, we only need to analyze R's problem in Stage 1 for $D(p) = e^{-p}$. Note that $w^* = p - \gamma pz$ and $E\Pi_M = (w-c)e^{-p}z$. In Stage 1, R sets p to maximize $E\Pi_R = \frac{\gamma}{2}e^{-p}pz^2$, where z satisfies: $p - c - 2\gamma pz = 0$. R's expected profit function can be simplified to: $E\Pi_R = \frac{Ke^{-p}(p-c)^2}{8p}$, which can be easily verified to be unimodal in p, and the unique equilibrium retail price is $p^* = \frac{1+c+H}{2}$, where $H = \sqrt{c^2+6c+1}$. Accordingly, we can compute the equilibrium values of the other decision variables: $z^* = \frac{K(3+c-H)}{4}$, $w^* = \frac{1+3c+H}{4}$ and $b^* = 0$, and the equilibrium expected profits are: $E\Pi_M^* = 2E\Pi_R = \frac{K(3+c-H)(1-c+H)}{16}e^{-\frac{1+c+H}{2}}$. By comparing these equilibrium values in Sequence 7 and those in the traditional sequence, as displayed by (11) and (12), we can derive: $E\Pi_M^*(S7) = 2E\Pi_R^*(S7) = \frac{e}{2}E\Pi_M^*(TS) = \frac{e}{2}E\Pi_R^*(TS)$, $Q^*(S7) = \frac{e}{2}Q^*(TS)$, $p^*(TS) = p^*(S7) + 1$, $w^*(TS) = w^*(S7) + \frac{3+c-H}{4}$ and $b^*(TS) > b^*(S7) = 0$. □

Proof of Proposition 4.5. Consider Sequence 1: $\underline{M{:}w;\ R{:}p;\ M{:}b;\ R{:}Q}$. The analysis of Stages 4 and 3 in Sequence 1 was carried out in the proof of Proposition 3.2 for a power distribution of ξ and an arbitrary $D(p)$. We continue the analysis in Stage 2, assuming $D(p) = p^{-q}$ and a uniform $\xi \in [0, K]$.

Stage 2: From the analysis in Stages 4 and 3, we have: $b^*(w,p) = \frac{p(3w-p-2c)}{w+p-2c}$ and $z^*(w,p) = \frac{K(w+p-2c)}{2p}$, and M's and R's expected profit functions can be simplified to: $E\Pi_M = \frac{Kp^{-q}(w+p-2c)^2}{8p}$ and $E\Pi_R = \frac{Kp^{-q}(p-w)(w+p-2c)}{4p}$. In Stage 2, R sets p to maximize $E\Pi_R$, which is clearly unimodal in p and has a unique optimal p^* satisfying the F.O.C. $\frac{dE\Pi_R(p)}{dp} = 0$, which is equivalent to requiring that $A \equiv -qp^2 + 2qpc + qw^2 - 2qwc + p^2 + w^2 - 2wc = 0$.

Stage 1: We work with p instead of w for M's problem in Stage 1. Note that A can be written as a quadratic function of w as follows: $A(w) = (q+1)w^2 - 2(q+1)cw - qp^2 + 2qpc + p^2$, and there is a unique $w^*(p) = c + \sqrt{\frac{(p-c)(qp-p-qc-c)}{q+1}}$, satisfying $A(w^*(p)) = 0$. Since $w^* \in [c, p)$, we must have $p \geq \frac{(q+1)c}{q-1}$. M's expected profit function becomes: $E\Pi_M = \frac{Kp^{-(q+1)}(p-c)(\sqrt{\frac{qp-p-qc-c}{q+1}}+\sqrt{p-c})^2}{8}$.

Next, consider Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$. The analysis of Stages 4 and 3 in Sequence 2 was carried out in the proof of Proposition 3.3 for a power distribution of ξ and an arbitrary $D(p)$. We continue the analysis in Stage 2, assuming $D(p) = p^{-q}$ and a uniform $\xi \in [0, K]$.

Stage 2: From the analysis in Stages 4 and 3, we have: $w^*(b,p) = \frac{p+pc-bc}{2p-b}$ and $z^*(b,p) = \frac{K(p-c)}{2p-b}$, and M's and R's expected profit functions can be simplified to: $E\Pi_M = \frac{Kp^{-q}(p-c)^2}{2(2p-b)}$ and $E\Pi_R = \frac{Kp^{-q}(p-b)(p-c)^2}{2(2p-b)^2}$. In Stage 2, R determines p to maximize $E\Pi_R$ for any given b. The F.O.C. is $\frac{dE\Pi_R(p)}{dp} = A\frac{Kp^{-(q+1)}(p-c)}{2(2p-b)^3}$, where $A = -2p^3q + 3p^2qb + 2p^2qc - 3pqcb - pqb^2 + qb^2c + 2p^3 - 3p^2b + 2p^2c - 3pcb + 2pb^2$. Since $A(p = \max(b,c)) > 0$ and $A(p \to \infty) < 0$, the optimal retail price $p^*(b)$ is an inner solution, $p^*(b) > \max(b,c)$, satisfying $\frac{dE\Pi_R(p)}{dp} = 0$, i.e., $A = 0$. Note that $E\Pi_R = \frac{Kp^{-q}(p-b)(p-c)^2}{2(2p-b)^2} > 0$ since $p^*(b) > \max(b,c)$.

Stage 1: We work with p instead of b for M's problem in this stage. Note that A can be written as a quadratic function of b as follows: $A(b) = (2p - qp + qc)b^2 + 3p(qp - qc - p - c)b - 2p^2(qp - qc - p - c) = 0$, and there is a unique $b^*(p) = \frac{p((-3(qp-qc-c-p)+\sqrt{(pq-p-qc-c)(pq+7p-qc-9c)}))}{2(2p+qc-qp)}$, satisfying $A(b^*(p)) = 0$. Since $b^* \in [0, p)$, we must have $p \geq \frac{(q+1)c}{q-1}$. M's expected profit function becomes:

$E\Pi_M = \frac{Kp^{-(q+1)}(p-c)(\sqrt{(qp-p-qc-c)(pq+7p-qc-9c)}-pq+qc+5p-3c)}{16}$.

Similar to the linear and exponential expected demand function cases, one can show, using simple algebra, that for any $p \geq \frac{(q+1)c}{q-1}$, $E\Pi_M(S2) > E\Pi_M(S1)$, implying that $E\Pi^*_M(S2) > E\Pi^*_M(S1)$.

Finally, by (13) and (14), in the traditional sequence under $D(p) = p^{-q}$, $p^* = \frac{q(q+1)c}{q-1}$ and $E\Pi^*_M(TS) = \frac{2K(q-1)^{2q-1}}{c^{q-1}q^q(q+1)^{q+1}}$. Evaluating M's expected profit function in Stage 1 in Sequence 1 at the equilibrium retail price p^* in the traditional sequence, and simplifying yields: $E\Pi_R(S1)(p^*) = \frac{K(q-1)^{q-1}(q^2+1)(q-1+\sqrt{q^2+1})^2}{8c^{q-1}(q(q+1))^{q+1}}$. It is not difficult to show that $E\Pi_R(S1)(p^*) > E\Pi^*_R(TS)$ for any value of q and c, implying that $E\Pi^*_R(S1) > E\Pi^*_R(TS)$. □

Proof of Proposition 4.7. Assume that ξ has a uniform distribution and $D(p) = p^{-q}$. The analysis of Stages 4, 3 and 2 in Sequence 7 in the proof of Proposition 3.11 is valid for an arbitrary $D(p)$, and we only need to analyze R's problem in Stage 1 for $D(p) = p^{-q}$. Note that $w^* = p - \gamma pz$ and $E\Pi_M = (w^* - c)D(p)z$. In Stage 1, R sets p to maximize $E\Pi_R = \frac{\gamma}{2}p^{-q+1}z^2$, where $z = \frac{p-c}{2\gamma p}$. It is easy to verify that $E\Pi_R$ is unimodal in p and uniquely maximized at $p^* = \frac{(q+1)c}{q-1}$. Accordingly, we can compute the equilibrium values of the other decision variables: $z^* = \frac{K}{q+1}$, $w^* = \frac{qc}{q-1}$ and $b^* = 0$, and the equilibrium expected profits are: $E\Pi^*_M = 2E\Pi^*_R = \frac{K(q-1)^{q-1}}{2c^{q-1}(q+1)^{q+1}}$. By comparing these equilibrium values in Sequence 7 and those in the traditional sequence, as displayed by (13) and (14), we can easily derive: $E\Pi^*_M(S7) > E\Pi^*_M(TS)$ for $q > 2$, $E\Pi^*_M(S7) = E\Pi^*_M(TS)$ for $q = 2$, $E\Pi^*_M(S7) < E\Pi^*_M(TS)$ for $q < 2$, $E\Pi^*_R(TS) > E\Pi^*_R(S7)$, $Q^*(S7) = \frac{1}{2}(\frac{q}{q-1})^q Q^*(TS) > Q^*(TS)$, $p^*(S7) = \frac{q-1}{q}p^*(TS) < p^*(TS)$, $w^*(S7) = w^*(TS)$ and $b^*(S7) = b^*(TS) = 0$. □.

Power distribution $f(\epsilon) = \gamma(\epsilon)^t$ with $t = 0$ (uniform)										
c	0.000E+00	1.000E-01	2.000E-01	3.000E-01	4.000E-01	5.000E-01	6.000E-01	7.000E-01	8.000E-01	9.000E-01
w^*	5.000E-01	5.500E-01	6.000E-01	6.500E-01	7.000E-01	7.500E-01	8.000E-01	8.500E-01	9.000E-01	9.500E-01
b^*	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01
p^*	7.500E-01	7.927E-01	8.266E-01	8.555E-01	8.812E-01	9.045E-01	9.260E-01	9.461E-01	9.650E-01	9.829E-01
Q^*	2.500E-01	1.719E-01	1.203E-01	8.353E-02	5.648E-02	3.647E-02	2.188E-02	1.161E-02	4.890E-03	1.163E-03
$E\Pi_M^*$	6.250E-02	4.172E-02	2.726E-02	1.717E-02	1.023E-02	5.636E-03	2.758E-03	1.116E-03	3.180E-04	3.833E-05
$E\Pi_R^*$	3.125E-02	2.086E-02	1.363E-02	8.583E-03	5.116E-03	2.818E-03	1.379E-03	5.579E-04	1.590E-04	1.916E-05
Channel	9.375E-02	6.258E-02	4.089E-02	2.575E-02	1.535E-02	8.453E-03	4.137E-03	1.674E-03	4.770E-04	5.749E-05
Power distribution $f(\epsilon) = \gamma(\epsilon)^t$ with $t = 1$										
c	0.000E+00	1.000E-01	2.000E-01	3.000E-01	4.000E-01	5.000E-01	6.000E-01	7.000E-01	8.000E-01	9.000E-01
w^*	5.000E-01	5.500E-01	6.000E-01	6.500E-01	7.000E-01	7.500E-01	8.000E-01	8.500E-01	9.000E-01	9.500E-01
b^*	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01
p^*	7.500E-01	7.845E-01	8.147E-01	8.423E-01	8.679E-01	8.922E-01	9.153E-01	9.375E-01	9.589E-01	9.797E-01
Q^*	2.500E-01	1.957E-01	1.530E-01	1.182E-01	8.922E-02	6.492E-02	4.463E-02	2.795E-02	1.471E-02	5.043E-03
$E\Pi_M^*$	8.333E-02	6.117E-02	4.381E-02	3.031E-02	1.998E-02	1.231E-02	6.861E-03	3.261E-03	1.156E-03	2.000E-04
$E\Pi_R^*$	4.167E-02	3.059E-02	2.191E-02	1.515E-02	9.989E-03	6.154E-03	3.430E-03	1.630E-03	5.782E-04	1.000E-04
Channel	1.250E-01	9.176E-02	6.572E-02	4.546E-02	2.997E-02	1.846E-02	1.029E-02	4.891E-03	1.735E-03	3.000E-04
Power distribution $f(\epsilon) = \gamma(\epsilon)^t$ with $t = 2$										
c	0.000E+00	1.000E-01	2.000E-01	3.000E-01	4.000E-01	5.000E-01	6.000E-01	7.000E-01	8.000E-01	9.000E-01
w^*	5.000E-01	5.500E-01	6.000E-01	6.500E-01	7.000E-01	7.500E-01	8.000E-01	8.500E-01	9.000E-01	9.500E-01
b^*	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01
p^*	7.500E-01	7.815E-01	8.102E-01	8.371E-01	8.626E-01	8.871E-01	9.108E-01	9.339E-01	9.564E-01	9.784E-01
Q^*	2.500E-01	2.047E-01	1.667E-01	1.339E-01	1.052E-01	7.985E-02	5.761E-02	3.823E-02	2.173E-02	8.430E-03
$E\Pi_M^*$	9.375E-02	7.109E-02	5.256E-02	3.757E-02	2.565E-02	1.643E-02	9.579E-03	4.810E-03	1.837E-03	3.590E-04
$E\Pi_R^*$	4.688E-02	3.554E-02	2.628E-02	1.879E-02	1.283E-02	8.214E-03	4.789E-03	2.405E-03	9.186E-04	1.795E-04
Channel	1.406E-01	1.066E-01	7.885E-02	5.636E-02	3.848E-02	2.464E-02	1.437E-02	7.216E-03	2.756E-03	5.384E-04
Power distribution $f(\epsilon) = \gamma(\epsilon)^t$ with $t = 4$										
c	0.000E+00	1.000E-01	2.000E-01	3.000E-01	4.000E-01	5.000E-01	6.000E-01	7.000E-01	8.000E-01	9.000E-01
w^*	5.000E-01	5.500E-01	6.000E-01	6.500E-01	7.000E-01	7.500E-01	8.000E-01	8.500E-01	9.000E-01	9.500E-01
b^*	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01
p^*	7.500E-01	7.790E-01	8.063E-01	8.326E-01	8.579E-01	8.827E-01	9.069E-01	9.306E-01	9.540E-01	9.772E-01
Q^*	2.500E-01	2.125E-01	1.790E-01	1.485E-01	1.206E-01	9.493E-02	7.128E-02	4.961E-02	3.002E-02	1.288E-02
$E\Pi_M^*$	1.042E-01	8.108E-02	6.154E-02	4.519E-02	3.175E-02	2.099E-02	1.270E-02	6.668E-03	2.704E-03	5.828E-04
$E\Pi_R^*$	5.208E-02	4.054E-02	3.077E-02	2.259E-02	1.588E-02	1.050E-02	6.348E-03	3.334E-03	1.352E-03	2.914E-04
Channel	1.563E-01	1.216E-01	9.231E-02	6.778E-02	4.763E-02	3.149E-02	1.904E-02	1.000E-02	4.057E-03	8.742E-04

Table A.1: Equilibrium values in the traditional sequence: $\underline{M{:}w,b;\ R{:}p,Q}$ for $D(p) = 1 - p$

Power distribution $f(\epsilon) = \gamma(\epsilon)^t$ with $t = 0$ (uniform)										
c	0.000E+00	1.000E-01	2.000E-01	3.000E-01	4.000E-01	5.000E-01	6.000E-01	7.000E-01	8.000E-01	9.000E-01
w^*	4.456E-01	5.230E-01	5.911E-01	6.531E-01	7.105E-01	7.644E-01	8.156E-01	8.644E-01	9.112E-01	9.564E-01
b^*	3.884E-01	4.326E-01	4.694E-01	5.016E-01	5.302E-01	5.563E-01	5.804E-01	6.028E-01	6.239E-01	6.438E-01
p^*	7.016E-01	7.501E-01	7.904E-01	8.255E-01	8.567E-01	8.851E-01	9.112E-01	9.355E-01	9.582E-01	9.797E-01
Q^*	2.439E-01	1.787E-01	1.301E-01	9.289E-02	6.418E-02	4.217E-02	2.567E-02	1.379E-02	5.873E-03	1.411E-03
$E\Pi_M^*$	6.996E-02	4.795E-02	3.193E-02	2.040E-02	1.231E-02	6.848E-03	3.381E-03	1.379E-03	3.956E-04	4.798E-05
$E\Pi_R^*$	3.123E-02	2.030E-02	1.297E-02	8.004E-03	4.691E-03	2.544E-03	1.227E-03	4.902E-04	1.380E-04	1.645E-05
Channel	1.012E-01	6.825E-02	4.490E-02	2.841E-02	1.700E-02	9.392E-03	4.608E-03	1.869E-03	5.336E-04	6.444E-05

Power distribution $f(\epsilon) = \gamma(\epsilon)^t$ with $t = 1$										
c	0.000E+00	1.000E-01	2.000E-01	3.000E-01	4.000E-01	5.000E-01	6.000E-01	7.000E-01	8.000E-01	9.000E-01
w^*	4.604E-01	5.251E-01	5.858E-01	6.435E-01	6.989E-01	7.524E-01	8.043E-01	8.548E-01	9.042E-01	9.526E-01
b^*	3.806E-01	4.103E-01	4.372E-01	4.619E-01	4.851E-01	5.069E-01	5.276E-01	5.474E-01	5.663E-01	5.852E-01
p^*	7.159E-01	7.532E-01	7.872E-01	8.186E-01	8.482E-01	8.762E-01	9.029E-01	9.284E-01	9.531E-01	9.769E-01
Q^*	2.480E-01	2.013E-01	1.614E-01	1.271E-01	9.733E-02	7.168E-02	4.978E-02	3.145E-02	1.668E-02	5.753E-03
$E\Pi_M^*$	9.019E-02	6.725E-02	4.875E-02	3.405E-02	2.262E-02	1.403E-02	7.871E-03	3.761E-03	1.340E-03	2.328E-04
$E\Pi_R^*$	4.224E-02	3.062E-02	2.167E-02	1.484E-02	9.686E-03	5.915E-03	3.271E-03	1.544E-03	5.438E-04	9.334E-05
Channel	1.324E-01	9.787E-02	7.042E-02	4.888E-02	3.231E-02	1.995E-02	1.114E-02	5.304E-03	1.884E-03	3.261E-04

Power distribution $f(\epsilon) = \gamma(\epsilon)^t$ with $t = 2$										
c	0.000E+00	1.000E-01	2.000E-01	3.000E-01	4.000E-01	5.000E-01	6.000E-01	7.000E-01	8.000E-01	9.000E-01
w^*	4.690E-01	5.290E-01	5.866E-01	6.424E-01	6.965E-01	7.494E-01	8.012E-01	8.520E-01	9.021E-01	9.514E-01
b^*	3.777E-01	4.019E-01	4.245E-01	4.459E-01	4.663E-01	4.858E-01	5.046E-01	5.228E-01	5.404E-01	5.577E-01
p^*	7.238E-01	7.571E-01	7.884E-01	8.181E-01	8.466E-01	8.740E-01	9.006E-01	9.264E-01	9.515E-01	9.760E-01
Q^*	2.494E-01	2.096E-01	1.739E-01	1.416E-01	1.125E-01	8.624E-02	6.271E-02	4.189E-02	2.395E-02	9.338E-03
$E\Pi_M^*$	9.963E-02	7.639E-02	5.699E-02	4.103E-02	2.819E-02	1.815E-02	1.063E-02	5.361E-03	2.055E-03	4.028E-04
$E\Pi_R^*$	4.765E-02	3.584E-02	2.630E-02	1.867E-02	1.266E-02	8.062E-03	4.675E-03	2.335E-03	8.877E-04	1.726E-04
Channel	1.473E-01	1.122E-01	8.329E-02	5.970E-02	4.085E-02	2.621E-02	1.531E-02	7.696E-03	2.943E-03	5.754E-04

Power distribution $f(\epsilon) = \gamma(\epsilon)^t$ with $t = 4$										
c	0.000E+00	1.000E-01	2.000E-01	3.000E-01	4.000E-01	5.000E-01	6.000E-01	7.000E-01	8.000E-01	9.000E-01
w^*	4.785E-01	5.346E-01	5.894E-01	6.432E-01	6.960E-01	7.481E-01	7.995E-01	8.503E-01	9.007E-01	9.505E-01
b^*	3.756E-01	3.952E-01	4.140E-01	4.322E-01	4.499E-01	4.670E-01	4.837E-01	5.001E-01	5.162E-01	5.320E-01
p^*	7.322E-01	7.621E-01	7.911E-01	8.191E-01	8.465E-01	8.732E-01	8.994E-01	9.251E-01	9.504E-01	9.754E-01
Q^*	2.502E-01	2.162E-01	1.844E-01	1.545E-01	1.265E-01	1.002E-01	7.563E-02	5.289E-02	3.213E-02	1.383E-02
$E\Pi_M^*$	1.086E-01	8.513E-02	6.499E-02	4.796E-02	3.384E-02	2.245E-02	1.362E-02	7.175E-03	2.917E-03	6.302E-04
$E\Pi_R^*$	5.289E-02	4.099E-02	3.098E-02	2.265E-02	1.586E-02	1.044E-02	6.296E-03	3.296E-03	1.333E-03	2.865E-04
Channel	1.615E-01	1.261E-01	9.597E-02	7.061E-02	4.970E-02	3.290E-02	1.992E-02	1.047E-02	4.250E-03	9.166E-04

Table A.2: Equilibrium values in Sequence 1: $\underline{M{:}w;\ R{:}p;\ M{:}b;\ R{:}Q}$ for $D(p) = 1 - p$

Power distribution $f(\epsilon) = \gamma(\epsilon)^t$ with $t = 0$ (uniform)										
c	0.000E+00	1.000E-01	2.000E-01	3.000E-01	4.000E-01	5.000E-01	6.000E-01	7.000E-01	8.000E-01	9.000E-01
w^*	4.566E-01	5.419E-01	6.151E-01	6.803E-01	7.394E-01	7.935E-01	8.431E-01	8.886E-01	9.301E-01	9.673E-01
b^*	3.894E-01	4.552E-01	5.131E-01	5.668E-01	6.183E-01	6.690E-01	7.202E-01	7.731E-01	8.300E-01	8.951E-01
p^*	6.318E-01	6.940E-01	7.439E-01	7.866E-01	8.242E-01	8.582E-01	8.894E-01	9.185E-01	9.461E-01	9.727E-01
Q^*	2.661E-01	1.949E-01	1.429E-01	1.032E-01	7.239E-02	4.850E-02	3.024E-02	1.674E-02	7.417E-03	1.891E-03
$E\Pi_M^*$	8.407E-02	5.788E-02	3.886E-02	2.510E-02	1.535E-02	8.686E-03	4.375E-03	1.829E-03	5.417E-04	6.871E-05
$E\Pi_R^*$	2.331E-02	1.481E-02	9.202E-03	5.483E-03	3.069E-03	1.569E-03	6.994E-04	2.499E-04	5.921E-05	5.077E-06
Channel	1.074E-01	7.269E-02	4.806E-02	3.059E-02	1.842E-02	1.025E-02	5.074E-03	2.079E-03	6.009E-04	7.378E-05
Power distribution $f(\epsilon) = \gamma(\epsilon)^t$ with $t = 1$										
c	0.000E+00	1.000E-01	2.000E-01	3.000E-01	4.000E-01	5.000E-01	6.000E-01	7.000E-01	8.000E-01	9.000E-01
w^*	4.813E-01	5.457E-01	6.066E-01	6.645E-01	7.197E-01	7.724E-01	8.229E-01	8.710E-01	9.167E-01	9.597E-01
b^*	3.532E-01	4.079E-01	4.621E-01	5.165E-01	5.718E-01	6.285E-01	6.876E-01	7.500E-01	8.177E-01	8.948E-01
p^*	6.277E-01	6.714E-01	7.123E-01	7.512E-01	7.886E-01	8.248E-01	8.601E-01	8.948E-01	9.291E-01	9.637E-01
Q^*	2.719E-01	2.269E-01	1.870E-01	1.513E-01	1.192E-01	9.048E-02	6.498E-02	4.266E-02	2.372E-02	8.737E-03
$E\Pi_M^*$	1.138E-01	8.644E-02	6.387E-02	4.550E-02	3.088E-02	1.959E-02	1.127E-02	5.541E-03	2.043E-03	3.708E-04
$E\Pi_R^*$	2.654E-02	1.900E-02	1.318E-02	8.749E-03	5.478E-03	3.159E-03	1.613E-03	6.773E-04	1.975E-04	2.317E-05
Channel	1.403E-01	1.054E-01	7.704E-02	5.425E-02	3.636E-02	2.275E-02	1.288E-02	6.218E-03	2.240E-03	3.940E-04
Power distribution $f(\epsilon) = \gamma(\epsilon)^t$ with $t = 2$										
c	0.000E+00	1.000E-01	2.000E-01	3.000E-01	4.000E-01	5.000E-01	6.000E-01	7.000E-01	8.000E-01	9.000E-01
w^*	4.938E-01	5.506E-01	6.060E-01	6.600E-01	7.129E-01	7.645E-01	8.147E-01	8.636E-01	9.109E-01	9.565E-01
b^*	3.107E-01	3.670E-01	4.245E-01	4.833E-01	5.438E-01	6.064E-01	6.717E-01	7.406E-01	8.147E-01	8.972E-01
p^*	6.169E-01	6.561E-01	6.948E-01	7.330E-01	7.709E-01	8.085E-01	8.461E-01	8.836E-01	9.214E-01	9.597E-01
Q^*	2.827E-01	2.458E-01	2.106E-01	1.772E-01	1.454E-01	1.152E-01	8.687E-02	6.042E-02	3.622E-02	1.504E-02
$E\Pi_M^*$	1.308E-01	1.025E-01	7.816E-02	5.754E-02	4.044E-02	2.667E-02	1.603E-02	8.320E-03	3.297E-03	6.736E-04
$E\Pi_R^*$	2.608E-02	1.945E-02	1.403E-02	9.693E-03	6.323E-03	3.810E-03	2.042E-03	9.064E-04	2.834E-04	3.672E-05
Channel	1.569E-01	1.220E-01	9.219E-02	6.723E-02	4.676E-02	3.048E-02	1.807E-02	9.226E-03	3.581E-03	7.103E-04
Power distribution $f(\epsilon) = \gamma(\epsilon)^t$ with $t = 4$										
c	0.000E+00	1.000E-01	2.000E-01	3.000E-01	4.000E-01	5.000E-01	6.000E-01	7.000E-01	8.000E-01	9.000E-01
w^*	5.023E-01	5.540E-01	6.054E-01	6.566E-01	7.074E-01	7.579E-01	8.079E-01	8.573E-01	9.060E-01	9.537E-01
b^*	2.306E-01	2.978E-01	3.661E-01	4.356E-01	5.065E-01	5.791E-01	6.538E-01	7.312E-01	8.125E-01	8.998E-01
p^*	5.913E-01	6.310E-01	6.708E-01	7.108E-01	7.509E-01	7.912E-01	8.318E-01	8.728E-01	9.142E-01	9.563E-01
Q^*	3.090E-01	2.753E-01	2.420E-01	2.090E-01	1.764E-01	1.442E-01	1.126E-01	8.171E-02	5.182E-02	2.356E-02
$E\Pi_M^*$	1.522E-01	1.218E-01	9.493E-02	7.153E-02	5.157E-02	3.500E-02	2.175E-02	1.177E-02	4.931E-03	1.105E-03
$E\Pi_R^*$	2.293E-02	1.767E-02	1.318E-02	9.433E-03	6.389E-03	4.009E-03	2.249E-03	1.054E-03	3.524E-04	5.038E-05
Channel	1.752E-01	1.395E-01	1.081E-01	8.096E-02	5.796E-02	3.901E-02	2.400E-02	1.282E-02	5.283E-03	1.155E-03

Table A.3: Equilibrium values in Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$ for $D(p) = 1 - p$

Power distribution $f(\epsilon) = \gamma(\epsilon)^t$ with $t = 0$ (uniform)										
c	0.000E+00	1.000E-01	2.000E-01	3.000E-01	4.000E-01	5.000E-01	6.000E-01	7.000E-01	8.000E-01	9.000E-01
w^*	2.500E-01	3.427E-01	4.265E-01	5.055E-01	5.812E-01	6.545E-01	7.260E-01	7.961E-01	8.651E-01	9.329E-01
b^*	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
p^*	5.000E-01	5.854E-01	6.531E-01	7.110E-01	7.624E-01	8.090E-01	8.521E-01	8.922E-01	9.301E-01	9.659E-01
Q^*	2.500E-01	1.719E-01	1.203E-01	8.353E-02	5.647E-02	3.647E-02	2.188E-02	1.161E-02	4.888E-03	1.164E-03
$E\Pi_M^*$	6.250E-02	4.172E-02	2.726E-02	1.717E-02	1.023E-02	5.636E-03	2.758E-03	1.116E-03	3.180E-04	3.833E-05
$E\Pi_R^*$	3.125E-02	2.086E-02	1.363E-02	8.583E-03	5.116E-03	2.818E-03	1.379E-03	5.579E-04	1.590E-04	1.916E-05
Channel	9.375E-02	6.258E-02	4.089E-02	2.575E-02	1.535E-02	8.453E-03	4.137E-03	1.674E-03	4.770E-04	5.749E-05
Power distribution $f(\epsilon) = \gamma(\epsilon)^t$ with $t = 1$										
c	0.000E+00	1.000E-01	2.000E-01	3.000E-01	4.000E-01	5.000E-01	6.000E-01	7.000E-01	8.000E-01	9.000E-01
w^*	3.333E-01	4.126E-01	4.863E-01	5.564E-01	6.239E-01	6.896E-01	7.537E-01	8.166E-01	8.786E-01	9.397E-01
b^*	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
p^*	5.000E-01	5.690E-01	6.295E-01	6.846E-01	7.359E-01	7.844E-01	8.306E-01	8.750E-01	9.179E-01	9.595E-01
Q^*	2.887E-01	2.259E-01	1.767E-01	1.365E-01	1.030E-01	7.496E-02	5.153E-02	3.228E-02	1.699E-02	5.824E-03
$E\Pi_M^*$	9.623E-02	7.064E-02	5.059E-02	3.499E-02	2.307E-02	1.421E-02	7.922E-03	3.765E-03	1.335E-03	2.310E-04
$E\Pi_R^*$	3.208E-02	2.355E-02	1.686E-02	1.166E-02	7.690E-03	4.737E-03	2.641E-03	1.255E-03	4.451E-04	7.699E-05
Channel	1.283E-01	9.418E-02	6.746E-02	4.666E-02	3.076E-02	1.895E-02	1.056E-02	5.021E-03	1.780E-03	3.080E-04
Power distribution $f(\epsilon) = \gamma(\epsilon)^t$ with $t = 2$										
c	0.000E+00	1.000E-01	2.000E-01	3.000E-01	4.000E-01	5.000E-01	6.000E-01	7.000E-01	8.000E-01	9.000E-01
w^*	3.750E-01	4.473E-01	5.154E-01	5.807E-01	6.440E-01	7.057E-01	7.663E-01	8.258E-01	8.846E-01	9.426E-01
b^*	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
p^*	5.000E-01	5.630E-01	6.205E-01	6.742E-01	7.253E-01	7.743E-01	8.217E-01	8.677E-01	9.127E-01	9.568E-01
Q^*	3.150E-01	2.579E-01	2.100E-01	1.687E-01	1.325E-01	1.006E-01	7.257E-02	4.817E-02	2.738E-02	1.062E-02
$E\Pi_M^*$	1.181E-01	8.956E-02	6.623E-02	4.734E-02	3.232E-02	2.070E-02	1.207E-02	6.061E-03	2.315E-03	4.523E-04
$E\Pi_R^*$	2.953E-02	2.239E-02	1.656E-02	1.183E-02	8.079E-03	5.174E-03	3.017E-03	1.515E-03	5.787E-04	1.131E-04
Channel	1.476E-01	1.120E-01	8.278E-02	5.917E-02	4.040E-02	2.587E-02	1.509E-02	7.576E-03	2.893E-03	5.653E-04
Power distribution $f(\epsilon) = \gamma(\epsilon)^t$ with $t = 4$										
c	0.000E+00	1.000E-01	2.000E-01	3.000E-01	4.000E-01	5.000E-01	6.000E-01	7.000E-01	8.000E-01	9.000E-01
w^*	4.167E-01	4.817E-01	5.440E-01	6.043E-01	6.632E-01	7.211E-01	7.781E-01	8.344E-01	8.901E-01	9.453E-01
b^*	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
p^*	5.000E-01	5.581E-01	6.127E-01	6.652E-01	7.158E-01	7.653E-01	8.138E-01	8.613E-01	9.081E-01	9.543E-01
Q^*	3.494E-01	2.969E-01	2.501E-01	2.075E-01	1.686E-01	1.327E-01	9.962E-02	6.934E-02	4.195E-02	1.800E-02
$E\Pi_M^*$	1.456E-01	1.133E-01	8.601E-02	6.316E-02	4.438E-02	2.934E-02	1.774E-02	9.320E-03	3.780E-03	8.146E-04
$E\Pi_R^*$	2.426E-02	1.889E-02	1.433E-02	1.053E-02	7.396E-03	4.890E-03	2.957E-03	1.553E-03	6.300E-04	1.358E-04
Channel	1.699E-01	1.322E-01	1.003E-01	7.368E-02	5.177E-02	3.423E-02	2.070E-02	1.087E-02	4.410E-03	9.503E-04

Table A.4: Equilibrium values in Sequence 7: $\underline{R{:}p;\ M{:}w;\ R{:}Q;\ M{:}b}$ for $D(p) = 1 - p$