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REVIEW

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Indexing cardiopulmonary variables

Indexing cardiovascular and respiratory variables: allometric scaling principles

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Abstract

Objectives To describe the allometric scaling principles underlying appropriate indexing of cardiovascular and respiratory measurements obtained in adult mammals, and to propose guidelines for indexing experimental cardiovascular and respiratory data.

Database used PubMed, using the terms ‘allometry’, ‘allometric’, ‘indexing’, ‘cardiovascular’ and ‘respiratory’.

Conclusions Indexing of cardiopulmonary variables is commonly used in attempts to account for the effects of body size on measurements and to standardize them. Some cardiopulmonary variables have been indexed using various functions of body mass in a process that often ignores the underlying relationship between the variable of interest and body size, as described in the allometry literature. This can result in a failure to ideally reduce the effect of body size on

measurements in a manner that highlights differences. We review how commonly measured cardiopulmonary variables are related to body mass in mammalian species according to the allometry literature, and offer suggestions on how this information can be used to appropriately index cardiopulmonary variables in a simple and informative manner.

Keywords cardiac index, cardiovascular, respiratory.

Introduction

Numerous variables of interest to the anesthesiologist are known to vary as a function of body size. Many variables increase as body size increases (e.g. cardiac output, minute ventilation), whereas others decrease as body size increases (e.g. heart rate, respiratory rate) and some do not vary systematically with body size (e.g. blood pressure, hematocrit). Many researchers have recognized the utility of expressing cardiopulmonary variables known to vary with body size as values indexed to another size-related variable; for example, cardiac index may be calculated as the quotient of cardiac output and body surface area (BSA). Such expressions reduce the variability of measurements, may artificially eliminate the effect of body size on the variable and give a clearer indication of how an individual animal's values compare with those expected for a typical animal of its body size. However, a variety of methods for such indexing are found in the literature. The simplest, and possibly most common adjustment, is to divide the measurement by body mass and present it mass-specifically. Although this is appropriate for some variables, as will be described, many cardiopulmonary variables increase or decrease in a manner that is not directly proportional to body mass, and indexing them to body mass will under- or overestimate, respectively, the true size-adjusted value; this error will be amplified as body size difference

increases (Table 1). Therefore, the actual function of body mass with which the variable changes should be used for indexing. Examining the veterinary anesthesia literature confirms that incorrect indexing of cardiopulmonary measurements is common; for example, of 10 articles recently published in this journal that reported both cardiac index and stroke index, four used methods of indexing unsupported by allometric principles. In addition, in these four manuscripts, cardiac output and stroke volume were incorrectly indexed using the same function of body mass, which suggests that heart rate is not expected to be affected by body size (see below), a supposition known to be incorrect.

Scaling describes the structural and functional consequences of changes in size or scale among otherwise similar organisms (Schmidt-Nielsen 1984). Allometric scaling refers to the scaling of bodies or functions that do not vary in direct proportion to their size (Schmidt-Nielsen 1984). Many morphological and physiological variables have been shown to depend on body mass according to the general allometric equation $Y = Y_0 \times M^b$, where Y_0 is a constant, characteristic of the type of organism and equal to the value of the variable of interest in a 1 kg (assuming that M is expressed in kg) animal, b is the allometric exponent, and M is body mass (West et al. 1997). The value of b usually ranges from − 1 to 1 for cardiopulmonary variables; negative values indicate that the variable decreases as body size increases, and positive values indicate that the variable increases as body size increases. When the exponent is 0, the variable is 'size-independent': it does not vary as a function of body size (see below). A simplified interpretation of the effects of different exponents in allometric equations is to consider the magnitude of the effect of changes in size, based on the logarithms of the exponent. For instance, if a variable scales as $M^{1/4}$, the variable will change by a factor of 10^1 (= 10-fold) whenever body mass changes by 10^4. Because mammals span a size range of 10^8, this indicates that variables that

scale with mass exponents of 1/4 (e.g. circulation time) or − 1/4 (e.g. resting heart rate) differ by approximately 100-fold (10 × 10) between the smallest and largest mammals (Fig. 1). The theoretical basis for the allometric equation $Y = Y_0 \times M^b$ and the common allometric exponents has been discussed (West et al. 1997; Dodds et al. 2001; Glazier 2005, 2010; Chaui-Berlinck 2006; Clemente 2007; West &West 2013; White & Kearney 2014), but is beyond the scope of this review.

Kleiber's law

In 1883, the German physiologist Max Rubner demonstrated that in dogs spanning an order of magnitude difference in body size, metabolic heat production varied in proportion to BSA, or $M^{2/3}$ (Rubner 1883). In 1932, the distinguished physiologist Max Kleiber reported that metabolic rates (in kcal per day) for mammals and birds spanning a body mass range of 0.15 kg to 679 kg were best described by a relationship proportional to $M^{0.74}$, usually expressed rounded off to $M^{3/4}$ (Kleiber 1932). It should be noted that in the allometry literature, mass exponents derived from empirical data are typically expressed as decimal values (e.g. 0.74), whereas those denoting a generalized underlying relationship are expressed as fractions (e.g. 3/4). A few years later, studies using an even larger range of body sizes (mouse to elephant) reported similar findings (Brody et al. 1934; Benedict 1938). Kleiber (1975) noted that these empirical allometric exponents were not significantly different from 0.75, and $M^{3/4}$ has become the most common scaling exponent relating body size to metabolism for interspecific comparisons. Cardiopulmonary variables scaling this way include cardiac output, minute ventilation and oxygen consumption.

Several authors have pointed out that Kleiber's 'law' is an empirical approximation rather than a law, and that the 3/4 exponent is likely not to be universally applicable (Hayssen & Lacy 1985;

93Glazier 2005, 2010; White et al. 2007; Kolokotrones et al. 2010; Hulbert 2014; White & Kearney
942014). Nevertheless, data analysis of the basal metabolic rate of adult mammals often yields the
953/4 exponent (or exponents that are not statistically different from 3/4), although some authors
96have suggested that the 'true' exponent is closer to 2/3, and that 3/4 is probably related to
97contamination of the dataset with non-basal measurements (White & Seymour 2003, 2005;
98Glazier 2005).

99**Interspecific and intraspecific size effects**

100Kleiber's law was empirically derived by collecting data from a variety of species, including
101both mammals and birds. His conclusion has been criticized based on the fact that, within a
102species, the effect of size on metabolism and cardiopulmonary variables appears to be best
103described by $M^{2/3}$, as originally reported by Rubner (1883) and later reinforced by Heusner
104(1982). It has since been argued that scaling within a species (intraspecific) appears to more
105closely fit a mass exponent of 2/3, whereas scaling between species (interspecific) seems to
106better fit a mass exponent of 3/4; there are experimental and theoretical justifications for the use
107of both allometric exponents (Feldman & McMahon 1983). Similarly to the 3/4 exponent, it has
108been suggested that the 2/3 exponent does not apply in all contexts and that the exponents may
109take a variety of values depending on the measurement being made (Glazier 2005).

110**Allometric scaling of cardiopulmonary variables in mammals**

111Values for Y_0 and b in the allometric equation $Y = Y_0 \times M^b$ for various cardiopulmonary variables
112reported in the literature are presented in Table 2. Some additional relationships can be
113hypothesized based on the variables for which there are experimental data. For example, arterial
114oxygen concentration is considered not to vary with body size (West & Brown 2005), which
115implies that oxygen delivery should vary as a function of body mass similarly to cardiac output.

West et al. (1997) developed a general theoretical model in an attempt to explain many of the empirical allometric relationships between cardiovascular and respiratory variables and body size. Their work suggests that the actual (theoretical) value of *b* may be slightly different from the observed value, which is compatible with the statistical derivation of these observed values (Table 3). Variables with a mass exponent of 0 are expected to be size-independent and thus not to vary with body size. Examples of such variables include hematocrit or hemoglobin concentration, and arterial oxygen tension (PaO_2) and carbon dioxide tension ($PaCO_2$). Blood pressure has typically been considered to be size-independent, although a recent study challenges this assumption (White & Seymour 2014). Although the PaO_2 required for 50% hemoglobin saturation (P_{50}) scales in mammals with a negative mass exponent (Schmidt-Nielsen & Larimer 1961958), at sea level at rest, hemoglobin is close to being fully saturated with oxygen at normal PaO_2 in healthy mammals. Because the two determinants of arterial oxygen concentration are hemoglobin concentration and PaO_2, both of which are considered to be size-independent, arterial oxygen concentration is also expected to be size-independent. Variables with a mass exponent of 1 change in direct proportion to body mass. Such variables include blood volume, heart mass, stroke volume, lung volume and tidal volume. The mass exponent for the majority of the remaining variables is 3/4 interspecifically (2/3 intraspecifically) and applies to variables such as oxygen consumption, cardiac output and minute ventilation that increase with body size, but not in direct proportion to it. Most biological frequencies, such as heart rate or respiratory rate, scale with a mass exponent of − 1/4 (or − 0.25), which indicates that they are lower in larger animals. Circulation time scales with an exponent of 1/4 as it is derived from the ratio of blood volume (exponent: 1) and cardiac output (exponent: 3/4). Similarly, vascular resistance, which is related to the ratio of pressure (exponent: 0) and cardiac output (exponent: 3/4), is expected to

scale with a mass exponent of − 3/4. Oxygen delivery is related to the product of arterial oxygen concentration (exponent: 0) and cardiac output, and thus is expected to vary as $M^{3/4}$.

Implications for indexing experimental cardiovascular and respiratory data

The literature on allometric scaling describes the relationships between body size and most common cardiovascular and respiratory variables. This information should be used when interpreting cardiopulmonary data with consideration of the effect of body size. As Table 1 shows, indexing these variables to an incorrect function of body mass would result in an incomplete and inaccurate interpretation. It should also be noted that if body size were fairly homogeneous in the sample being studied, there would be no effect on the variability of the measurements whether one applied the appropriate function of body mass, an inappropriate function of body mass, or used the raw measurements. If all subjects in the sample have similar body mass, correcting for body mass or any function of body mass will be equivalent to dividing the measurements by a constant, which does not affect their relative variability. Nevertheless, in this situation, the only logical approaches are either to not correct for body size or to correct for the appropriate function of body size, and the latter may allow better comparisons with other published data.

Exponents for indexing cardiovascular and respiratory data are reported in Table 3, and exponents for a few additional variables are described in the text above. It should be noted that these exponents are for comparisons between species such as when data are obtained in multiple species, or if the goal for correcting the raw data for body size is to allow for the making of interspecies comparisons or estimates. Because the literature suggests that the slope for metabolic rate, and therefore probably many of the cardiovascular and respiratory variables, is lower when individuals of a single species are examined (Heusner 1982; Feldman & McMahon

1621983; Schmidt-Nielsen 1984), it may be appropriate to modify the exponents in Table 3 for
163comparisons and correction of data within a species. In this case, the 3/4 and − 3/4 exponents
164would become 2/3 and − 2/3, respectively, and the 1/4 and − 1/4 exponents would become 1/3
165and − 1/3, respectively. The existence of two different sets of exponents describing the effect of
166size within and between species may be related to similarities in body shape among animals of
167the same species, compared with systematic changes in dimensions dictated by size in animals of
168different species. Previous authors have suggested that, within a species, geometric similarity
169underlies the effect of body size, whereas elastic similarity may be more important between
170animal species (Feldman & McMahon 1983).
171Table 2 also reports the value of Y_0 in the $Y = Y_0 \times M^b$ allometric equation, or the expected value
172of the variable in a 1 kg animal (log 0 = 1) if the unit of mass is kg. This Y_0 factor is irrelevant for
173the correction of experimental data for the effect of body size. The data should only be corrected
174for (divided by) M^b, using the appropriate value for b; indeed, according to the equation above,
175$Y/M^b = Y_0$, where Y is the measurement, showing that the intercept of the allometric equation Y_0
176is included in the actual measurement.
177Because BSA varies as $M^{2/3}$, as does intraspecific metabolic rate, it is often used to index
178cardiopulmonary data for comparisons within a species. As we have discussed, and as Table 3
179shows, this is only appropriate for variables that vary in proportion to intraspecific metabolic
180rate. In addition, one of the arguments cited as supporting the indexing of cardiopulmonary
181measurements mass-specifically (to M) rather than to BSA, even for variables known to vary as
182$M^{2/3}$, is that formulae for calculating BSA may be inaccurate. It should be apparent from the
183present discussion that this is irrelevant because cardiopulmonary measurements do not actually
184vary as a function of BSA *per se*; rather, BSA just happens [in reflection of geometric similarity

185and as has been confirmed empirically in interspecific comparisons (Hemmingsen 1950)] to vary
186as $M^{2/3}$ and hence the relationship between M and BSA has the same exponent (slope of a log–log
187allometric plot) as cardiovascular and respiratory variables that vary with intraspecific metabolic
188rate. Because only the exponent is relevant for indexing, dividing the measurement by a BSA
189that can be read off a chart may be pragmatic, but it is simpler, and arguably more correct, to
190simply divide these measurements by $M^{2/3}$, which avoids any question of the accuracy of the
191BSA calculation and can be simply determined with a hand calculator. For example, if a 5 kg cat
192is found to have a cardiac output of 1 L minute^{-1}, its cardiac index could be calculated as
1931/5$^{0.67}$ = 0.34 L minute^{-1} $M^{-0.67}$.
194It should be noted that because there is some uncertainty regarding the actual values of the
195allometric exponents, and because the values of the exponents may vary with the conditions
196under study, analysis of covariance may be a preferable approach to traditional indexing to
197account for the effect of body size on cardiopulmonary measurements (Packard & Boardman
1981988, 1999). This approach has been utilized to account for the degree of interrelatedness
199between species in the development of phylogenetically informed allometry (Garland et al.
2002005).
201In conclusion, we suggest that when cardiovascular and respiratory variables are indexed to
202account for the effect of body size, this indexing should: 1) use body mass raised to the
203appropriate exponent according to the allometric scaling literature; 2) use different exponents for
204inter- *versus* intraspecific comparisons; and 3) ensure that variables which scale intraspecifically
205with metabolic rate ($M^{2/3}$) are indexed for the effect of body size by dividing the measurements
206by $M^{2/3}$ rather than by BSA.
207

The authors have no conflict of interest to report.

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Table 1 Effect of indexing a hypothetical variable specifically to body mass (M), or to a function of body mass related to metabolic rate [$M^{3/4}$ (interspecific) or $M^{2/3}$ (intraspecific)]. The effect of body size on this hypothetical variable is described by $100 \times M^{0.75}$

M (kg)	Measurement	Measurement indexed to M	Measurement indexed to $M^{3/4}$	Measurement indexed to $M^{2/3}$
1	100	100	100	100
2	168	84	100	106
5	334	67	100	114
10	562	56	100	120
20	946	47	100	127
50	1880	38	100	137
100	3162	32	100	145
500	10,574	21	100	164
1000	17,783	18	100	174

279**Table 2** Values for Y_0 and b in the allometric equation $Y = Y_0 \times M^b$ for various cardiopulmonary

280variables in mammals, with M in kg

Variable	Y_0	b	Reference
Heart rate (beats minute^{-1})	241	− 0.25	(Stahl 1967)
Cardiac output (mL minute^{-1})	187	0.81	(Stahl 1967)
Heart mass (g)	5.8	0.98	(Stahl 1965)
Stroke volume (mL)	0.78*	1.03	(West et al. 199
Blood volume (mL)	65.6	0.995	(Stahl 1967)
Circulation time (seconds)	17.4	0.25	(Schmidt-Nielse
Mean arterial pressure (mmHg)	87	0.026	(Calder 1984)
Systemic vascular resistance (mmHg minute^{-1} mL^{-1})	0.47	− 0.78	(Calder 1984)
Lung mass (g)	11.3	0.99	(Stahl 1967)
Total lung capacity (mL)	53.5	1.06	(Stahl 1967)
Vital capacity (mL)	56.7	1.03	(Stahl 1967)
Functional residual capacity (mL)	24.1	1.13	(Stahl 1967)
Tidal volume (mL)	7.69	1.04	(Stahl 1967)
Respiratory rate (breaths minute^{-1})	53.5	− 0.26	(Stahl 1967)
Minute volume (mL minute^{-1})	379	0.80	(Stahl 1967)
Respiratory system compliance (mL cm H_2O^{-1})	1.56	1.04	(Stahl 1967)
Oxygen consumption (mL minute^{-1})	11.6	0.76	(Stahl 1967)
Hemoglobin concentration (g dL^{-1})	12.9	0	(Schmidt-Nielse
Alveolar surface (m^2)	3.34	0.95	(Gehr et al. 198
Diffusion capacity for oxygen (mL O_2 second^{-1} mmHg^{-1})	0.065	0.99	(Gehr et al. 198
Arterial-to-mixed venous oxygen concentration difference (mL dL^{-1})†	6.2	− 0.05	
Venous oxygen tension (mmHg)	41.3	− 0.005	(Calder 1984)
Venous carbon dioxide tension (mmHg)	42	− 0.03	(Calder 1984)

281*Calculated from Y_0 values for cardiac output and heart rate. †Calculated from values for oxygen

282consumption and cardiac output.

283**Table 3** Theoretical values for *b* in the allometric equation $Y = Y_0 \times M^b$ for various

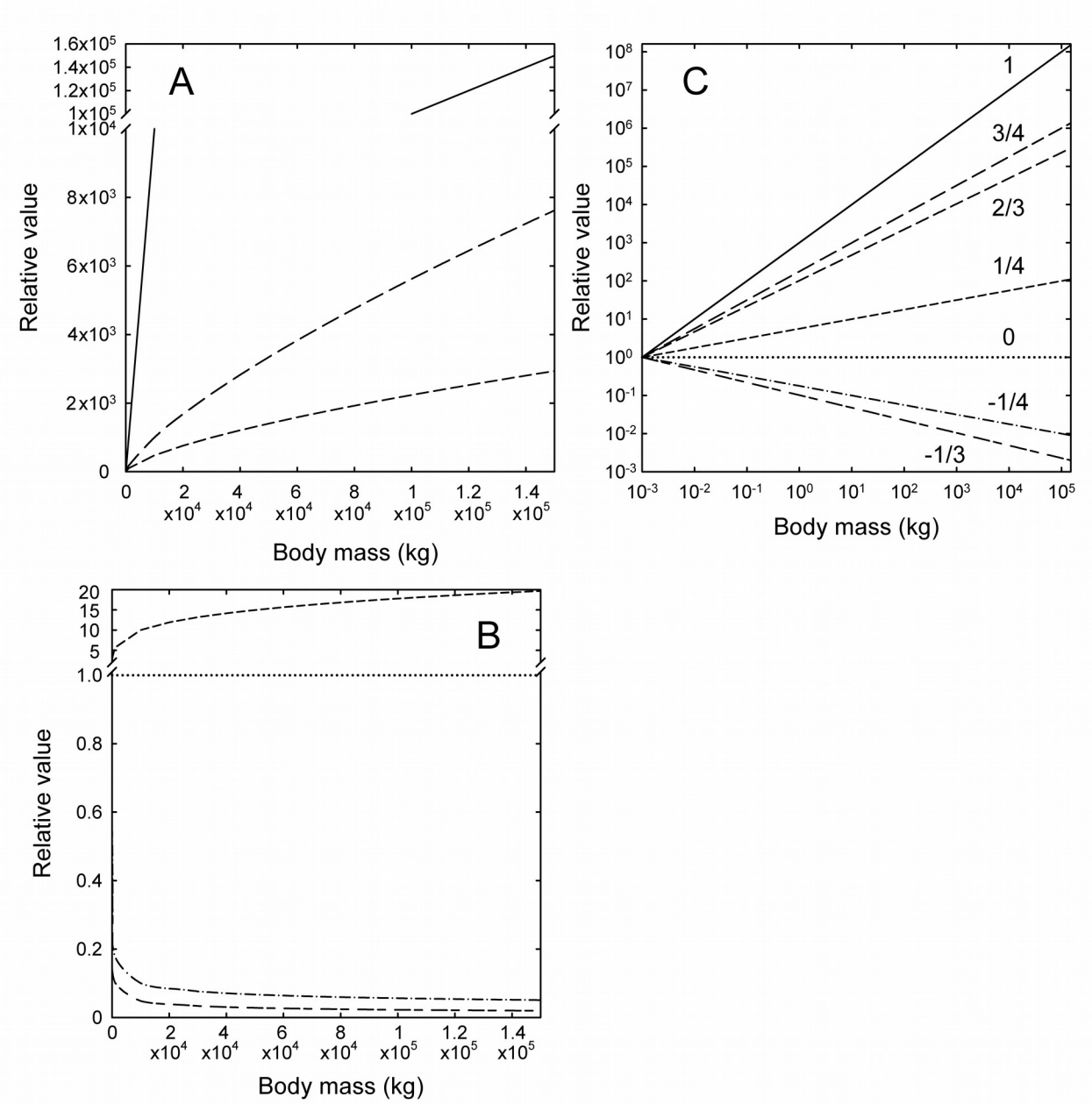
284cardiopulmonary variables in mammals, with *M* in kg

Variable	*b*	Reference
Heart rate (beats minute^{-1})	− 1/4	(West et al. 1997)
Cardiac output (mL minute^{-1})	3/4	(West et al. 1997)
Heart mass (g)	1	(Schmidt-Nielsen 1984)
Stroke volume (mL)	1	(West et al. 1997)
Blood volume (mL)	1	(West et al. 1997)
Circulation time (seconds)	1/4	(West et al. 1997)
Mean arterial pressure (mmHg)	0	(West et al. 1997)
Systemic vascular resistance (mmHg minute mL^{-1})	− 3/4	(West et al. 1997)
Lung volume (mL)	1	(West et al. 1997)
Tidal volume (mL)	1	(West et al. 1997)
Respiratory rate (breaths minute^{-1})	− 1/4	(West et al. 1997)
Minute volume (mL minute^{-1})*	3/4	(West et al. 1997)
Oxygen consumption (mL minute^{-1})	3/4	(West et al. 1997)
Hemoglobin concentration (g dL^{-1})	0	(Schmidt-Nielsen 1984)
Alveolar surface (m^2)†	11/12	(West et al. 1997)
Diffusion capacity for oxygen (mL O_2 second^{-1} mmHg^{-1})	1	(West et al. 1997)
Arterial-to-mixed venous oxygen concentration difference (mL dL^{-1})‡	0	(West et al. 1997)

285*Calculated from respiratory rate and tidal volume. †Calculated from number of alveoli and

286surface of alveolus. ‡Calculated from oxygen consumption and cardiac output.

Figure 1 Allometric relationships of variables with commonly encountered mass exponents discussed in the text. All data have been standardized so that the value for the smallest mammal (0.001–0.0015 kg, Etruscan shrew *Suncus etruscus*) has a value of 1, and the largest mammal (approximately 150–190 tonne, blue whale *Balaenoptera musculus*) scales accordingly. (a) Body mass and relative values of allometric variables are plotted on linear axes for variables scaling with mass exponents of 1 (solid line), 3/4 (long-dash line) and 2/3 (medium-dash line). (b) Body mass and relative values of allometric variables are plotted on linear axes as in (a) but with scales expanded to visualize data for variables scaling with mass exponents of 1/4 (short-dash line), 0 (dotted line), − 1/4 (dash-dot line) and − 1/3 (long- and short-dash line). Note differences in ordinate scaling around breaks in axes in (a) and (b). (c) Body mass and relative values of allometric variables shown in (a) and (b) are plotted on logarithmic axes to show that log-transformation of the data yields a straight line with slope equal to the mass exponent. The intercept value (value of a mammal with body mass of 1 kg because log 1 = 0) equals the value of the variable for a mammal of 1 kg body mass.


A
Relative value
Body mass (kg)
B
Relative value
Body mass (kg)
C
Relative value
Body mass (kg)
1
3/4
2/3
1/4
0
-1/4
-1/3

301