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A PERTURBATIVE APPROACH TO THE POMERON

I. THE MULTIFIREBALL EXPANSION*

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ABSTRACT

We present a perturbative S-matrix description of the pomeron motivated by the two-component model of multiparticle production. The latter is taken as a first approximation and corrections are generated by repeated pomeron exchange. Each term in the expansion is associated with a well-defined class of scattering processes, and the convergence properties are related to the smallness of pomeron couplings. The present paper emphasizes the basic ideas, a general formalism, and the interrelationship of various Regge singularities. A following paper discusses a simplified model in more detail.

I. INTRODUCTION

The notion that pomeron couplings are in some sense weak has for years been employed in a variety of theoretical considerations [1], but concrete connections of this idea with experiment have up to now been elusive. Recently a two-component model of high energy multiparticle production has been successfully applied to experiment, and in this paper we show how this model suggests a perturbative S-matrix approach to the pomeron in terms of distinct and physically measurable quantities. The perturbations are expected to be small except at enormous energies, the approach is free from renormalization ambiguities, and a number of experimental predictions emerge.

We begin with a brief summary of the two-component (or "hybrid") model first proposed by Wilson [5] and developed by a number of others [6]. High energy scattering events are grouped into two categories: a short-range correlation or SRC component plus diffractive scattering, usually taken to be at least predominantly into low mass states. The SRC component is characterized by (inclusive) factorizability; specifically, a fixed finite correlation length in rapidity space. In Regge models this means the dominant asymptotic behavior is that of isolated factorizable poles, and the usual Mueller analysis [7] is applicable. The diffractive component comprises elastic scattering, diffractive resonance production (more generally, diffraction into low mass states), and perhaps a small amount of high mass diffraction. Such processes are characterized by pomeron exchange, and give Regge cut contributions to $C_{\text{tot}}$. They are not factorizable in the inclusive sense and contain long-range rapidity correlations.

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[Notes: Parts of this work have appeared in refs. [2-4].]
This picture is in quite good agreement with experiment as regards multiplicity distributions,* with approximately 80% SRC in inelastic events and the remainder mostly diffraction into low masses. The strongest evidence in favor of the two-component picture at present is the observation of striking short-range rapidity correlations in inelastic cross sections [8] together with the constant ratio of charged multiplicity to dispersion [9] \( \frac{n}{n^2 - (n)^2} \frac{1}{2} \approx 2. 

In a pure SRC model the latter ratio would increase with energy, and these two facts seem to rule out models based on either component alone.

The basis of our analysis is the identification of the two-component model with the leading terms of a systematic "fireball" expansion [10] defined in terms of large rapidity gaps in the distribution of final particles in an inelastic collision. A "single-fireball" event has no large gap in the final state and is depicted in fig. 1.1a. A "two-fireball" event has one large gap, and so on. At a given energy only a finite number of fireballs is kinematically possible. It will be necessary for us to distinguish between fireballs of "low" and "high" mass (where the dividing line will be specified more fully below), so that there are four types of double fireball events involving either two low masses (fig. 1.1b), one low and one high mass (fig. 1.1c and d), and two high masses (fig. 1.1f). The other diagrams of fig. 1.1 involve two or more large gaps.**

We now make contact with the two-component model by identifying the single fireball cross section, fig. 1.1a, with the SRC component, fig. 1.1b with the low mass diffraction, and all other possible diagrams with various forms of high mass diffraction. We thus define diffraction to be a process with at least one large rapidity gap, where "large" may now be defined as large enough for pomeron exchange to be a good approximation to the amplitude. The wiggly lines in fig. 1.1 are then to represent pomerons. If we square, integrate over phase space, and sum over the appropriate final states, we assume the single fireball cross section to be given for high fireball mass by the leading SRC Regge pole, to be referred to as a "bare" pomeron \( \tilde{P} \) (see fig. 1.2a). Alternatively, one could define a "high-mass" fireball as one whose energy dependence is controlled by the bare pomeron \( \tilde{P} \).

Applying the same procedure to the other processes shown in fig. 1.1 leads, mutatis mutandis, to fig. 1.2. We have assumed here a kind of pomeron universality: that the same \( P \) and \( \tilde{P} \) exchanges always occur, once or repeatedly, in production processes with any number of fireballs. This universality allows one to speak usefully of a scattering amplitude with "external" pomerons, although we emphasize that it may not be necessary for the pomeron to be purely a factorizable pole.**

It is of course a nontrivial assumption that an absorptive part with no large rapidity gaps in the intermediate state is given approximately by Regge poles alone and that cuts are unimportant. Note that, for clarity we are neglecting secondary Regge poles.

** A concrete model where the pomeron can be universal without being a simple pole, based on pion exchange, has been discussed by Shankar [11].

* Although such distributions do not yet constitute a decisive test because of the experimental uncertainties and the large number of parameters available for fitting.

** We have omitted processes involving just one low mass (e.g., \( a + b \) resonance) which are not present at energies of interest to us.
since the single fireball term is positive definite, it cannot contain a negative two-"physical" pomeron cut (with $\alpha_c(0) > \alpha_p(0)$). A two-$P$ cut could be present but we are assuming it is negligible. We are unable to present a proof of this assertion, although imprecise and model dependent arguments exist. In multiperipheral models, for example, the SRC component is associated with a "low-energy kernel" which does not possess branch points of high intercept. The best justification is perhaps that the absence of cuts in the SRC component is the natural mathematical realization of the physical assumption of short range order. We remark that a similar assumption is made in Abarbanel's study of the two-pomeron cut [14], which is formally similar to the present work.**

The bare pomeron $\bar{P}$ is not the same singularity as the "physical" pomeron $P$ that appears, say, in the elastic amplitude and the bare pomeron must satisfy the inequality $\alpha_{\bar{P}}(0) < \alpha_P(0)$. Such a constraint follows because only a subset of the intermediate states which contribute to the total cross section, and thus to $P$, are allowed to contribute to $\bar{P}$. More explicitly, $\bar{P}$ may be repeated any number of times in sufficiently complicated diagrams and it is well known (and will be demonstrated in section 5) that a new singularity is thereby generated with intercept above that of $\bar{P}$. On the other hand we must assume, in diagrams such as fig. 1.2b (which includes ordinary elastic scattering), that the physical pomeron is exchanged since fig. 1.1b from which this diagram originates, depicts

---

** This realization of the bare pomeron appears in refs. [12].**

For further discussion of the singularities of the single fireball cross section, see ref. [13].

Notice that the processes in figs. 1.2c, d, e, …, which represent diffractive corrections to the two-component model, all involve one or more powers of the internal coupling of two pomerons to a fireball of low or high mass and we expect these quantities to be small. The coupling to low mass states such as two pions or the $f$ resonance is known to be weak experimentally [15], and the diffractive production of a single large mass, which receives positive contributions from these terms, is not large [15a]. A concise restatement is that the "physical" triple pomeron vertex (i.e., $PPP$ rather than the bare vertex $\bar{PPP}$), to which both internal vertices contribute, is small. Thus the terms in the fireball series besides those of the two-component model are, in a sense to be made precise below, small corrections. This circumstance suggests treating the two-component model as the lowest order terms in a systematic expansion of the total cross section in powers of pomeron couplings. The lowest order terms constitute a good approximation at energies so far accessible, and only a finite number of higher order terms contribute at a finite energy. Furthermore each term corresponds uniquely to a well-defined subset of reactions, so there are no ambiguities of renormalization.
2. PARAMETERIZATION OF THE POMERON AND ITS SCATTERING AMPLITUDES

To proceed in the analysis of the fireball sum, a more precise specification of some quantities is required. We shall, in the spirit of a first approximation, treat the physical pomeron as a simple pole. The singularities of the absorptive part as calculated through the fireball series would then represent a corrected pomeron, and in principle this procedure can be iterated. Since the pomeron appears both in the (input) production amplitudes and as the leading singularity of the (output) sum, we have the eventual possibility of imposing self-consistency, although no such attempt will be made here. In particular we will work only at $t = 0$, with the forward absorptive part, while a bootstrap calculation would require the absorptive part for all (negative) $t$.

One might ask why, in view of this approximation, we have not simply used $\tilde P$ in the elastic and other amplitudes and iterated thereupon. Our answer is that our decomposition scheme avoids ambiguity by being based primarily on phase space considerations rather than a perturbation expansion. Where large rapidity gaps occur we are logically required to employ the set of singularities that represent the physical pomeron as closely as possible, whereas $\tilde P$ is required by the structure of the scheme to be different.\footnote{An alternative possibility, not discussed here, is that it is the bare pomeron which is the best pole approximation to the physical pomeron singularity (see J. Dash, to be published). This point of view is related to the "schizophrenic pomeron" model discussed earlier by one of us [16].}

A practical supporting consideration is that we require the bare pomeron only at $t = 0$ while the physical pomeron is needed for all negative $t$.

Figure 1.2 implies that a typical multiperipheral recursion relation connects the $N + 1$ fireball cross section to that for $N$ fireballs. Our approach assumes, in other words, that the multifireball cross section is expressible as a product of factors, each of which involves a single fireball. Physical pomerons separate the fireballs and occur an unlimited number of times as the total energy increases, with all the usual consequences that attach to such indefinite recurrence. Do these consequences depend on our having assigned a simple-pole status to the physical pomeron?

It is clear from the original pion exchange multiperipheral model, as emphasized by Shankar [11], that unlimited pomeron recurrence may result from mechanisms that allow arbitrary properties for the physical pomeron, and in our multifireball scheme such a mechanism arises from the assumption of short-range order within each large-mass fireball. Short-range order means that inclusive cross sections are dominated by isolated Regge poles with factorizable residues (led by the bare pomeron), and factorizability of any link in a multiperipheral chain permits indefinite recurrence of any structure that happens to stand between these links.

The multifireball idea is to be illustrated in this paper by a model that, in order to maximize mathematical simplicity, assumes factorizability of several different links (e.g., the physical pomeron) in addition to large mass fireballs. Because not all of this factorization is realized in nature, the numerical reliability of our simplified model may be doubtful. We believe, however, that most of
the qualitative features of the model require no more than the occurrence somewhere in the chain of one factorization mechanism; a promising candidate is short range order within a large mass fireball.

Finally we must specify the properties of the fireballs, i.e., the form of the pomeron-pomeron and pomeron-particle absorptive parts (without large rapidity gaps). From the theoretical prejudice that pomeron scattering does not differ drastically from that of ordinary particle scattering and from preliminary experimental results [17], we expect a no-gap or single-fireball pomeron-particle cross section to have the form shown in fig. 2.1a. An elastic (delta-function) peak, a resonance region, and a high energy tail controlled by $\hat{\mathcal{F}}$ are anticipated. The analogous pomeron-pomeron cross section is expected to be similar, but without the elastic peak. For computational simplicity we will often idealize this situation to the form shown in fig. 2.1b: a single effective resonant peak plus the high energy tail. This will permit the integral equations obtained below to be solved in closed form.

A potentially awkward feature of fig. 2.1b is that it introduces a sharp threshold for the onset of bare pomeron behavior. As described in the following paper, this threshold introduces oscillations into the total cross section which are, perhaps not to be taken literally since fig. 2.1a, which is presumably more accurate, has no sharp thresholds. An alternative parametrization would have the "tail" starting at the kinematic threshold, $s = m_a^2$. We comment on this point in Appendix C.

3. J-PLANE ANALYSIS

The diagrams of fig. 1.2 are suggestive of multiperipheral models and, as is the case there, we can expect considerable simplification to result from studying the absorptive part in the J-plane. In the present context only a Mellin transform will be required, and we will not make explicit use of the constraints of $t$-channel unitarity. Consider an $N$-fireball amplitude, shown in fig. 3.1. We define a family of subenergies $s_i$ as shown, and the usual momentum transfers $t_i$ and Toller angles $\omega_i$. The $i$th fireball has invariant $(\text{mass})^2 u_i$ and $n_i$ particles with momenta ($p_i$). As described in section 2, we make the initial approximation of treating the pomerons as factorizable poles (the general situation is commented upon below) and write the amplitude as

$$ T_{ab-N} = T_{aP-n_1}(u_1); t_1; (p_1) \left( \frac{-t_1 s}{u_1 s_2} \right)^{\alpha_p(t_1)} t_p(t_1) $$

$$ \times T_{PP-n_2}(u_2; t_1, t_2, \omega_2; (p_2)) \left( \frac{-t_2 s_2}{u_2 s_3} \right)^{\alpha_p(t_2)} t_p(t_2) $$

$$ \times \cdots \left( \frac{-t_{N-1} s_{N-1}}{u_{N-1} s_N} \right)^{\alpha_p(t_{N-1})} t_p(t_{N-1}) $$

$$ \times T_{bP-n_1}(u_1; t_{N-1}; (p_N)) $$  \hspace{1cm} (3.1) $$

$\alpha_p$ is the pomeron trajectory, and $t_p$ is the usual signature factor which we have found it convenient to extract in order to maintain a uniform notation. (The incorporation of signature as in ref. [18] is to be understood.) The factors $(-t_i s_i/u_i s_{i+1})$ are the high energy
forms of the appropriate Lorentz boosts or, alternatively, the cosines of the crossed-channel scattering angles for the appropriate quasi-two-body scattering process. This amplitude appears to vanish as any \( t_1 \rightarrow 0 \), which is familiar from two-body unequal-mass scattering, and we assume the residues contain the appropriate \( t_1 \) dependence to cancel this tendency.

It is shown in Appendix A that the \( N \)-fireball phase space element may be written in terms of individual fireball phase-space elements as

\[
d\phi(N) = \frac{1}{s} d\phi(n_1) \frac{dt_1}{2\pi^2} \frac{dw_1}{w_1^2} d\phi(n_2) \frac{dt_2}{2\pi^2} \frac{dw_2}{w_2^2} \cdots d\phi(n_N) du_N.
\]

In this expression, \( d\phi(n_i) \) is the conventional phase space element for the \( n_i \) particles of the \( i \)-th fireball with momenta corresponding to Fig. 3.1, and \( \omega_1 \) is the overall azimuthal angle. We define single-fireball pomeron absorptive parts as

\[
A_{AB}^{(1)}(u_1; t_1, t_2, \omega_1) = \sum_{n_1} \int d\phi(n_1)
\]

\[
\times \left| T_{PB}^{(1)}(u_1; t_1, t_2, \omega_1) \right|^2.
\]

with an analogous definition for the pomeron-particle absorptive parts \( A_{PB}^{(1)} \). Summing over all numbers of final particles \( (n_i) \), the \( N \)-fireball contribution to the physical \( a-b \) forward absorptive part is

\[
A_{ab}^{(N)}(s) = \sum_{n_1, n_2, \cdots} \int d\phi(N) \left| T_{ab-N}^{(N)} \right|^2
\]

\[
= \frac{1}{s} \int \frac{dt_1 \frac{dw_1}{w_1^2}}{2\pi^2} \frac{ds_1}{s_1^2} \left| s_{P}^{(1)}(t_1) \right|^2 \left( \frac{-t_1 s}{u_1 s_2} \right)^{2\alpha_p(t_1)} \int du_1 A_{PB}^{(1)}(u_1; t_1)
\]

\[
\times \left( \int \frac{dt_2 \frac{dw_2}{w_2^2}}{2\pi^2} \frac{ds_2}{s_2^2} \left| s_{P}^{(2)}(t_2) \right|^2 \left( \frac{-t_2 s_2}{u_2 s_3} \right)^{2\alpha_p(t_2)} \int du_2 A_{PB}^{(1)}(u_2; t_1, t_2, \omega_2) \cdots \int du_N A_{PB}^{(1)}(u_N; t_{N-1}) \right).
\]

The limits of integration are

\[
t_1 : -\infty \rightarrow 0
\]
\[
\omega_1 : 0 \rightarrow 2\pi
\]
\[
u_1 : u_1 \rightarrow \infty
\]
\[
s_1 : \frac{u_1 s_1 + 1}{-t_1} e^\Delta \rightarrow \infty.
\]

The lower limit on the \( s_1 \) ensures a rapidity interval of at least \( \Delta \) between fireballs, where \( \Delta \) is the minimum rapidity gap for which pomeron exchange is a good approximation. The kinematic requirement \( t_1 < 0 \) is automatically included in the \( s_1 \) constraint: \( t_1 = 0 \) requires infinite energies.

The use of a constant minimum rapidity interval can be criticized; see T. L. Neff, LBL-1767 (to be published in Nuclear Physics B).
Taking a Mellin transform and rearranging the limits gives

\[ A_{ab}(j) = \int_{\Delta} ds \left( \frac{s}{m_{ab}} \right)^{a_{ab}} A_{ab}(s) \]

\[ = \left( \frac{m_{ab}}{m_{a} m_{b}} \right)^{\frac{1}{2}} \int_{t_{1}}^{\infty} dt_{1} \frac{d\omega_{1}}{32\pi^{2}} \left| \varepsilon_{p}(t_{1}) \right|^{2} \int_{0}^{\infty} du_{1} A_{Pb}(u_{1}; t_{1}) u_{1}^{2} \frac{2\alpha_{p}(t_{1})}{2\alpha_{p}(t_{1})+\alpha_{p}(t_{1})} \]

\[ \times \left[ \int_{0}^{\infty} \frac{du_{2} A_{Pb}(u_{2}; t_{1}, t_{2}, u_{2})}{32\pi^{2}} \left| \varepsilon_{p}(t_{2}) \right|^{2} \int_{0}^{\infty} du_{3} A_{Pb}(u_{3}; t_{2}, u_{3}) \frac{2\alpha_{p}(t_{2})}{2\alpha_{p}(t_{2})+\alpha_{p}(t_{2})} \right] \]

\[ \times \left[ \int_{0}^{\infty} \frac{du_{4} A_{Pb}(u_{4}; t_{3}, u_{4})}{32\pi^{2}} \left| \varepsilon_{p}(t_{3}) \right|^{2} \int_{0}^{\infty} du_{5} A_{Pb}(u_{5}; t_{3}, u_{5}) \frac{2\alpha_{p}(t_{3})}{2\alpha_{p}(t_{3})+\alpha_{p}(t_{3})} \right] \]

\[ \times \left[ \int_{0}^{\infty} \frac{du_{6} A_{Pb}(u_{6}; t_{4}, u_{6})}{32\pi^{2}} \left| \varepsilon_{p}(t_{4}) \right|^{2} \int_{0}^{\infty} du_{7} A_{Pb}(u_{7}; t_{4}, u_{7}) \frac{2\alpha_{p}(t_{4})}{2\alpha_{p}(t_{4})+\alpha_{p}(t_{4})} \right] \]

\[ \times \cdots \]

\[ \times \int_{0}^{\infty} \frac{du_{N-1} A_{Pb}(u_{N-1}; t_{N-1}, u_{N-1})}{32\pi^{2}} \left| \varepsilon_{p}(t_{N-1}) \right|^{2} \int_{0}^{\infty} du_{N} A_{Pb}(u_{N}; t_{N}, u_{N}) \frac{2\alpha_{p}(t_{N-1})}{2\alpha_{p}(t_{N-1})+\alpha_{p}(t_{N-1})} \]

\[ \times \left( -t_{1} \right)^{2\alpha_{p}(t_{1})} \]

\[ \times \left[ \int_{0}^{\infty} \frac{du_{N-1} A_{Pb}(u_{N-1}; t_{N-1}, u_{N-1})}{32\pi^{2}} \left( s_{N-1} \right)^{2\alpha_{p}(t_{N-1})-2\alpha_{p}(t_{N-2})} \right] \]

\[ \times \left[ \int_{0}^{\infty} \frac{du_{N-2} A_{Pb}(u_{N-2}; t_{N-2}, u_{N-2})}{32\pi^{2}} \left( s_{N-2} \right)^{2\alpha_{p}(t_{N-2})-2\alpha_{p}(t_{N-3})} \right] \]

\[ \times \cdots \]

\[ \times \int_{0}^{\infty} \frac{du_{1} A_{Pb}(u_{1}; t_{1})}{32\pi^{2}} \left( s_{1} \right)^{2\alpha_{p}(t_{1})-1-J} \]

(3.4)

The quantity in braces is equal to

\[ \left( -t_{1} e^{-\Delta} \right) A_{Pb}(t_{1}) \]

\[ \times \left( -t_{2} e^{-\Delta} \right) A_{Pb}(t_{2}) \times \cdots \times \left( -t_{N} e^{-\Delta} \right) A_{Pb}(t_{N}) \]

If we define Mellin transforms of \( A_{Pb}^{(1)} \) and \( A_{Pp}^{(1)} \) as

\[ A_{Pb}^{(1)}(J; t_{1}) = \frac{1}{m_{a}} \int_{0}^{\infty} du_{1} \left( \frac{u_{1}}{m_{a} \sqrt{t_{1}}} \right)^{-J-1} A_{Pb}^{(1)}(u_{1}; t_{1}) \]

\[ A_{Pp}^{(1)}(J; t_{1}, t_{2}, u_{2}) = \int_{0}^{\infty} du_{2} \left( \frac{u_{2}}{\sqrt{t_{1} t_{2}}} \right)^{-J-1} A_{Pp}^{(1)}(u_{2}; t_{1}, t_{2}, u_{2}) \]

and so on, we can write the entire expression (3.4) as

\[ A_{ab}(j) = \int \frac{dt_{1} d\omega_{1}}{32\pi^{2}} A_{Pb}^{(1)}(j; t_{1}) S(j, t_{1}) \]

\[ \times \int \frac{dt_{2} d\omega_{2}}{32\pi^{2}} A_{Pp}^{(1)}(j; t_{1}, t_{2}, u_{2}) S(j, t_{2}) \]

\[ \times \cdots \]

\[ \times \int \frac{dt_{N-1} d\omega_{N-1}}{32\pi^{2}} A_{Pp}^{(1)}(j; t_{N-2}, t_{N-1}, u_{N-1}) \]

\[ \times S(j, t_{N-1}) A_{Pb}^{(1)}(j; t_{N-1}) \]

(3.5)

where
We can now obtain integral equations for the absorptive part.

Equation (3.5) may be rewritten, in an obvious notation, as

\[ A_{ab}^{(N)}(J) = \int \frac{dt \, d\omega}{2\pi^2} A_{ab}^{(N-1)}(J; t) S(J, t) A_{ab}^{(1)}(J; t). \]

Summing over \( N \), we may define a complete absorptive part as

\[ A_{ab}(J) = \sum_{N} A_{ab}^{(N)}(J), \]

with an analogous expression for the complete a-pomeron absorptive part, \( A_{ap}(J; \tau) \). This gives

\[ A_{ab}(J) = A_{ab}^{(1)}(J) + \int \frac{dt \, d\omega}{2\pi^2} A_{ab}^{(1)}(J; t) S(J, t) A_{ab}^{(1)}(J; t). \]  

(3.6a)

\[ A_{ap} \] itself clearly satisfies the equation

\[ A_{ap}(J; \tau) = A_{ap}^{(1)}(J; \tau) + \int \frac{dt' \, d\omega}{2\pi^2} A_{ap}^{(1)}(J; t') S(J, t') A_{pp}(J; t', \tau, \omega) \]

(3.6b)

where, in turn,

\[ A_{pp}(J; t', \tau, \omega) = A_{pp}^{(1)}(J; t', \tau, \omega) + \int \frac{dt'' \, d\omega''}{2\pi^2} A_{pp}^{(1)}(J; t', t'', \omega') \]

\[ \times S(J, t'', t', \omega'). \]  

(3.6c)

These equations, depicted in fig. 3.2, were first derived by Abarbanel [14]. Note that the various single-fireball absorptive parts may contain different singularities. For example, \( A_{ab}^{(1)} \) may contain the \( \omega \) Regge trajectory, which could not be present in \( A_{ap}^{(1)} \).

In accordance with the discussion of section 2, we now approximate the fireballs as a single low-mass resonance of \( (mass)^2 u_R \) plus a high energy tail controlled by the bare pomeron starting at rapidity \( \Lambda \). We further assume factorizability of the low-mass coupling in the \( \tau' \)'s after integrating over the Toller angle. Explicitly (see fig. 3.3a) we write

\[ \int_0^{2\pi} d\omega_1 A_{pp}^{(1)}(u_1, t_1, t_1', \omega_1) = 2\pi \tilde{v}(t_1, t_1') S(u_1 - u_R) \]

\[ + 2\pi \tilde{g}(t_1, t_1') \tilde{g}(t_1) \left( \frac{u_1}{\sqrt{t_1 \cdot t_1'}} \right)^{\alpha_p(0)} \]

\[ \times \theta \left( \frac{u_1}{\sqrt{t_1 \cdot t_1'}} - e^\Delta \right). \]

(3.7a)

Here \( \tilde{v} \cdot \tilde{v} \) is the square of an effective \( P-P \) resonance, \( \tilde{g} \) is a \( P-P \) triple-Regge coupling, and \( u_1 \sqrt{t_1 \cdot t_1'} \) is the appropriate Lorentz boost. Taking the Mellin transform, we write

\[ \int_0^{2\pi} d\omega_1 A_{pp}^{(1)}(J; t_1, t_1', \omega_1) = 2\pi s_0 \left[ v(t_1, t_1') \right] \]

\[ \times \left( \frac{\tilde{g}(t_1, t_1') \tilde{g}(t_1)}{J - \alpha_p(0)} \right) \]

(3.7b)
where we have defined \( v \) and \( g \) by absorbing a weakly \( J \)-dependent factor \( u^{-J/2}_R \) into \( \tilde{v} \) and some \( t \)-dependence into both \( \tilde{v} \) and \( \tilde{g} \).

We have also extracted a scale energy \( s_0 = 1 \text{ GeV}^2 \), so that \( g \) and \( v \) are in units of \( \text{GeV}^{-2} \). An analogous expression may be given for the a-pomeron single fireball absorptive part (fig. 3.3b):

\[
A_{ab}(J; t_1) = B_{Pa}(t_1) + \beta_{Pa}(0) g(t_1) \frac{\tilde{g}(\alpha_p(t) - J)}{J - \alpha_p(0)} \tag{3.8a}
\]

where

\[
B_{Pa}(t_1) = \frac{1}{s_0} \sum_{a'} \beta^2_{Pa',a'}(t_1)(m^2_a/s_0)^{-J}. \tag{3.8b}
\]

In the last equation \( \beta_{Pa'} \) is the (dimensionless) ordinary pomeron-\( a-a' \) residue, the sum is over elastic scattering plus diffractively produced resonant states of \( a \), and we have again absorbed a weakly \( J \)-dependent factor.

If we insert (3.7b) and (3.8a) into eq. (3.5) for \( A_{ab}^{(N)}(J) \) we obtain a sum of terms, each involving a product of the factors listed in Table 1. The Table also gives the origin of each factor in terms of a particle production process. The notation is

\[
\rho(J) = \frac{\tilde{g}(\alpha_p(t) - J)}{J - \alpha_p(0)} \tag{3.9a}
\]

for a \( \tilde{P} \) "propagator" and

\[
\langle B_{Pa} v \rangle \cdot \langle v g \rangle \cdot \rho \cdot \beta_{Pa}(0). \]

and so on for the various loop integrals. From the Table it is easy to construct the \( J \)-plane expression for any given scattering process. For example, fig. 1.2h is given by

\[
\langle B_{Pa} v \rangle \cdot \langle v g \rangle \cdot \rho \cdot \beta_{Pa}(0). \]
4. LOWEST ORDER TERMS

We now wish to examine the absorptive part by considering the terms in the perturbation expansion which may be present in currently accessible energy regimes. For convenience, in this section we suppose $\Delta \approx \bar{\Delta}$. First consider the energy range

I. $\Delta \leq \log \frac{s}{s_0} < 2\Delta$.

Only the processes in figs. $1.1a$ and $b$ are kinematically possible, and the simple form of the two-component model should be relevant. We have

$$ A_{ab}^{I}(J) = \beta_{Ra}(0) \rho_{Rb}(0) + (\beta_{Ra} B_{Pb})(J) $$

in the notation of section $3$. Transforming back to $s$ via

$$ A_{ab}(s) = \frac{1}{2\pi i} \int_{C-i\infty}^{C+i\infty} dJ (s/s_0)^J A_{ab}^{I}(J) $$

where $\text{Re}(C)$ is to the right of the singularities of $A_{ab}^{I}(J)$, and using $\sigma_{tot}(s) = s^{-1} A_{ab}(s)$, we see that the first (SRC) term gives a factorizable contribution to $\sigma_{tot}$ which falls as $s^{-1}$, and the second (diffraction) term is a nonfactorizable APS-like branch point and its contribution to $\sigma_{tot}$ falls as $s^{2\alpha_F(0)-2} (\text{const.} + \log s)^{-1}$. Factorization of $\sigma_{tot}$ should hold to an accuracy roughly $\sigma_{SRC}/\sigma_{tot} \sim 2/3$.

Now suppose the energy satisfies

II. $2\Delta \leq \log \frac{s}{s_0} < 3\Delta$.

the processes of figs. $1.1c$, $d$, and $e$ are possible and contribute to $A_{ab}$. We are now using $s_0 = 1 \text{ GeV}^2$ as a scale energy.

$$ A_{ab}^{II}(J) = (\beta_{Ra} B_{Pb}) \rho_{Rb} + \beta_{Ra} \rho (B_{Pb}) + (\beta_{Ra} v)(B_{Pb} v) $$

(4.3)

To get an idea of the energy dependence of these terms we approximate the branch point appearing in the loop integrals by a pole at $J = \alpha_c$:

$$ \int dt f(t) e^{\Delta(\alpha_F(1)-1-J) \over J - 2\alpha_F(t) + 1} \approx f_0 e^{\Delta(\alpha_F-J) \over J - \alpha_c} $$

$\alpha_c$ may be interpreted as $2\alpha_F(\bar{t}) - 1$, where $\bar{t}$ is an average momentum transfer. Then

$$ A_{ab}^{II}(J) \approx c_1 \frac{e^{\Delta(\alpha - \alpha_F(0) - 2J) \over (J - \alpha_c)(J - \alpha_F(0))} + c_2 \frac{2\Delta(\alpha - \alpha_F(0)) \over (J - \alpha_c)^2} $$

where the $c_i$ are constants, and

$$ \sigma^{II}(s) \approx \left\{ \begin{array}{ll}
\frac{c_1}{\alpha_c - \alpha_F(0)} \left[ \alpha_c^{-1} \frac{\Delta(\alpha - \alpha_F(0))}{s} e^{\alpha_F(0)-1} e^{\alpha(\alpha_F(0))} \right] \\
+ c_2 s^{\alpha_c^{-1}} \{ \log s - 2\Delta \} e^{\alpha(\log s - 2\Delta)}
\end{array} \right. $$

This quantity increases at the threshold $\log s = 2\Delta$ but eventually levels off and turns over. The point at which this turnover occurs and the energy dependence of the complete cross section depend on the values of the parameters involved (we will study this quantitative question in a simplified model in the following paper.).
It is of interest to resolve the absorptive part into pole and cut components. Using (3.9) and the method of partial fractions we can write

\[ \Lambda_{ab}^{I+II}(s) = \frac{\zeta(\alpha_p(t)-J)}{J - \alpha_p(t)} \left[ \beta_{Pa}(0) \beta_{Pb}(0) \right. \]

\[ + \frac{s_0}{16\pi} \int \frac{\Delta(2\alpha_p(t)-1-J)}{\alpha_p(t) - 2\alpha_p(t) + 1} \left| s_p(t) \right|^2 B_{Pa}(t)g(t)\beta_{Pb}(0) \]

\[ + (a \leftrightarrow b) \right] \]

\[ + \frac{s_0}{16\pi} \int \frac{\Delta(2\alpha_p(t)-1-J)}{J - \alpha_p(t) + 1} \left[ B_{Pa}(t) B_{Pb}(t) \right. \]

\[ - \frac{\zeta(\alpha_p(t)-J)}{\alpha_p(t) - 2\alpha_p(t) + 1} \left\{ B_{Pa}(t)g(t)\beta_{Pb}(0) - (a \leftrightarrow b) \right\] \]

\[ + (B_{Pa}(v)(B_{Pb}(v)). \quad (4.4) \]

The bracketed quantity in the first term is then the pole residue while the second and third terms contain the branch point. The apparent lack of factorization in the pole term may be remedied by simply adding a small \( O(g^2) \) term or, equivalently, by including part of the contribution of fig. 1.2g. This yields a pole residue \( \beta_{Pa}(0) \beta_{Pb}(0) \), where

\[ \beta_{Pa}(0) = \beta_{Pa}(0) + \frac{s_0}{16\pi} \int \frac{\Delta(2\alpha_p(t)-1-J)}{\alpha_p(t) - 2\alpha_p(t) + 1} \left| s_p(t) \right|^2 B_{Pa}(t)g(t), \]

\[ (4.5) \]

an expression which should be interpreted as the pomeron-a residue to first order in \( g \).

It should be noted that the singularities of the individual terms in the fireball series are not identical to those in the sum. Upon summing the series, as will be shown, the bare pomeron pole is no longer present but is replaced by a new pole with higher intercept and a residue differing from (4.5). The strength of the cut is also altered, and new singularities appear. The complete collection of singularities in the sum is, however, equivalent to (4.2) in this energy regime.

We now investigate the sign of the corrections to the two-component model. If \( \alpha_p(0) > 2\alpha_p(0) \) then the pole residue is clearly increased by the high-mass diffraction of figs. 1.1c and d. These terms also subtract from the cut, and, while internal low-mass diffraction can either add or subtract (see below), it is probably a small effect. If the corrections are large enough, the cut discontinuity can even be negative. In the alternative case where \( 2\alpha_p(0)-1 > \alpha_p(0) \), which must hold if \( \alpha_p(0) = 1 \), some care is required. Note that what we have called pole and cut terms in (4.4) are separately complex after transforming to \( s \), although their sum is of course real. The relevant part (i.e., the real part) of the correction terms is given by the principal value of the \( t \) integrals. This can have either sign, but our numerical estimates below suggest a positive principal value of substantial magnitude. Thus we expect that the effect of the first corrections to the two-component model will be to enhance the strength of the pole and diminish the cut.
We would now like to calculate explicitly the contribution of internal low-mass diffraction (fig. 1.1e) to the cut, both for completeness and because the method will be needed later. The J-discontinuity of a function of the form

\[ f(J) = \int_{-\infty}^{\infty} dt \, f(t) \, e^{J(2\alpha_p(t) - I - J)} \]

is given by

\[ \text{disc } F(J) = \frac{1}{2\pi} \int_{\infty}^{0} dt \, f(t) \, \delta(J - 2\alpha_p(t) - 1) \]

\[ = \frac{\pi}{\alpha_p} \theta(\alpha_c(0) - J) \, f(t_c(J)) \]

where

\[ \alpha_c(0) = 2\alpha_p(0) - 1 \quad \text{and} \quad t_c(J) = \frac{J - \alpha_c(0)}{2\alpha_p} . \]

With this definition of the discontinuity, we write the contribution of \( F \) to the inverse Mellin transform as

\[ + \frac{1}{\pi} \int_{\infty}^{0} dJ \frac{J}{s/\sigma} \text{disc } F(J) . \]

The energy threshold \( \theta(\log s - \Delta) \), which gave rise to the factor \( \Delta(2\alpha_p(t) - I - J) \), appears to have been lost from the final expression. Actually, the distortion of the J-contour required to write the inversion as (4.7) is impossible for \( \log s < \Delta \) (since the exponential blows up), and closing the contour to the right in (4.2) gives zero.

Using the identity

\[ \text{disc } [A(J) B(J)] = \left[ \text{disc } A(J) \right] B(J - I\epsilon) + A(J + I\epsilon) \left[ \text{disc } B(J) \right] \]

we have (suppressing the signature and exponential factors)

\[ \text{disc } [\langle B_{Pa}(v) \rangle B_{Pb}(v)] = \frac{s_0}{16\pi} \theta(\alpha_c(0) - J) \left[ B_{Pa}(t_c) \nu(t_c) \right] \]

\[ \times \frac{s_0}{16\pi} \int dt \frac{B_{Pb}(v)(t)}{J - I\epsilon - 2\alpha_p(t) + 1} + (a \leftrightarrow b, \epsilon \leftrightarrow -\epsilon) \]

\[ = \frac{s_0}{16\pi} \theta(\alpha_c(0) - J) \left[ B_{Pa}(t_c) \nu(t_c) \right] \]

\[ \times \frac{s_0}{16\pi} \int dt \frac{B_{Pb}(v)(t)}{J - 2\alpha_p(t) + 1} + (a \leftrightarrow b) \]

The sign of this expression depends on the effective value of \( J \); at low values of \( J \), which are significant for \( s \) not large, the integral will be negative and give an increasing contribution to the cross section while high values of \( J \), which are more important for larger \( s \), give a positive contribution and a decreasing cross section.

This is, of course, the same conclusion we arrived at earlier.

The complete cut discontinuity at this point is
In this section we examine the 'complete solution': the absorptive part as given by the sum of the multi-fireball series. This will be studied by summing the series as a function of $J$, and then retaining the leading singularities (to construct the familiar Regge asymptotic expansion) in transforming back to $s$. The complete solution has a number of virtues: it is computationally convenient at energies such that many terms in the series are relevant, it clearly exhibits many physically interesting "renormalization" effects, and in addition allows us to partially study the self-consistency of the initial parametrization.

One technical point requires clarification: it is perhaps not obvious that the complete solution, which involves terms with arbitrarily large energy thresholds, is relevant at a finite value of $s$ at which most (an infinite number) of these terms do not appear. In fact this is automatically taken account of by the $e^{-\Delta J}$ factors. To see this explicitly, note that from the rules of section 3 it follows that at a given value of $s$ the kinematically possible terms are those proportional to $e^{-k J}$ where $k < \log (s/s_0)$. The remainder, the difference between these and the complete solution, is proportional to $e^{-k' J}$ where $k' > \log (s/s_0)$. In the Mellin inversion (4.2) of the remainder, the integrand, which is proportional to $(s/s_0)^J e^{-k' J}$, decreases exponentially as $\text{Re} J \to +\infty$ and the contour may be closed to the right to give zero.

In summing the series, it is illuminating to begin with two simple cases. First, suppose we consider only the sum of high-mass fireballs, fig. 5.1. This gives
\[ \beta_{Pa}(0) \left[ \rho + \rho(g^2) + \rho(g^2)^2 + \cdots \right] \beta_{Pb}(0) \]

\[ = \frac{\beta_{Pa}(0) \beta_{Pb}(0)}{1 - \rho(g^2)} \]

\[ = \frac{\beta_{Pa}(0) e^{\frac{\Delta(\alpha_{P}(0)-J)}{g^2}}}{J - \alpha_{P}(0) - e^{\frac{\Delta(\alpha_{P}(0)-J)}{g^2}}} \]  \hspace{1cm} (5.1)

using the definitions in (3.9). Suppose \( g(t) \) is parametrized by, for example, a simple exponential in \( t \) so that \( g(0) \neq 0 \). Then from (3.9) \( g^2 \) has an infinite logarithmic branch point at \( J = 2\alpha_{P}(0) - 1 = \alpha_{0}(0) \), and the denominator is as shown in fig. 5.2. If \( \alpha_{P}(0) > \alpha_{0}(0) \) the denominator is negative at \( \alpha_{P}(0) \) while in the alternative it is complex there. In either case the denominator can only vanish on the real axis above \( \alpha_{P}(0) \) and \( \alpha_{0}(0) \). Since this result only involved a subset of a series of positive terms, the remaining processes cannot alter this conclusion. Thus the intercept of the leading bare pole must be renormalized upwards \([20]\), and if \( \alpha_{P}(0) = 1 \) this pole is above 1 and the Froissart bound is violated \([21]\). To avoid such an embarrassment it suffices for \( g(t) \) to vanish at \( t = 0 \), e.g., \( g(t) = -v_0 \cdot t^e^{bt} \). In this case \( g^2 \) is finite at \( J = \alpha_{P}(0) \), and the location of the zero depends on the numbers involved but need not be above one.\(^*\)

\(^*\) This device does not remove all the inconsistencies associated with \( \alpha_{P}(0) = 1 \), but suffices for our purposes in this paper.

\[ C_{ab}(J) = \langle B_{Pa} B_{Pb} \rangle + \langle B_{Pa} v \rangle \langle B_{Pb} v \rangle + \langle B_{Pa} \rangle \langle v^2 \rangle \langle B_{Pb} v \rangle + \cdots \]

\[ = \langle B_{Pa} B_{Pb} \rangle + \langle B_{Pa} v \rangle \left( \frac{1}{1 - \langle v^2 \rangle} \right) \langle B_{Pb} v \rangle . \]  \hspace{1cm} (5.2)

Again if \( v(0) \neq 0 \) and \( \alpha_{P}(0) = 1 \), the form of the denominator guarantees a pole above \( J = 1 \), which may be avoided by parametrizing \( v(t) = -v_0 \cdot t e^{bt} \). When we refer to the case \( \alpha_{P}(0) = 1 \) later, a parametrization such as this for \( v \) and \( g \) will be understood.

To find \( A_{ab}(J) \) in the general case we proceed in stages.\(^*\)

First we consider the quantity (see fig. 5.3b)

\[ C_{a}(J) = \langle B_{a} v \rangle + \langle B_{a} v \rangle \langle v^2 \rangle + \cdots \]

\[ = \langle B_{a} v \rangle \left( \frac{1}{1 - \langle v^2 \rangle} \right) . \]  \hspace{1cm} (5.3)

defined as the sum of pure low-mass diffraction coupled to particle \( a \).

\( C_{b}(J) \) is defined analogously, and (fig. 5.3c)

\(^*\) The solution can also be found directly from the integral equations (3.6), say if these are written in matrix form for high- and low-mass channels. This method would essentially be based on factorization of the physical pomeron link, while the method we present exploits factorization of the bare pomeron. The latter seems more reasonable to us.
\[ C(J) = 1 + \langle v^2 \rangle + \langle v^2 \rangle \langle v^2 \rangle + \cdots \]
\[ = \frac{1}{1 - \langle v^2 \rangle} \quad (5.4) \]
is one plus the sum of purely low-mass internal diffraction. Next we define \( G_0(J) \) as the sum of all diagrams with \( \rho(t) \) on each end, fig. 5.4a, which can be summarized in fig. 5.4b. Algebraically the latter is
\[ G_0(J) = (\langle g^2 \rangle + \langle gv \rangle \cdot C \cdot \langle gv \rangle )(1 + \rho \cdot G_0) \quad (5.5a) \]
so solving for \( G_0 \) and introducing (5.4) for \( C(J) \) gives
\[ G_0(J) = \frac{\langle g^2 \rangle + \langle (gv)^2 - \langle g^2 \rangle \langle v^2 \rangle \rangle}{1 - \rho \langle g^2 \rangle - \langle v^2 \rangle - \rho \langle (gv)^2 - \langle g^2 \rangle \langle v^2 \rangle \rangle} \]
\[ \quad (5.5b) \]
The complete absorptive part is now shown in fig. 5.5; explicitly
\[ A_{ab}(J) = C_{ab}(J) + R_{a}(J) G(J) R_{b}(J) \]
\[ \quad (5.6) \]
\( C_{ab} \) is the sum over purely low-mass fireballs defined in (5.2), \( R_{a} \) is the sum of all processes standing between incident particle \( a \) and the first high mass fireball
\[ R_{a}(J) = \beta_{Pa}(0) + \langle B_{Pa} \rangle g + C_{a}(gv) \]
\[ \quad (5.7) \]
and \( G \), the sum over high mass fireballs, is
\[ G(J) = \rho(J) + \rho(J) G_0(J) \rho(J) \]
\[ = \frac{\rho(1 - \langle v^2 \rangle)}{(1 - \rho \langle g^2 \rangle)(1 - \langle v^2 \rangle) - \rho \langle gv \rangle^2} \quad (5.8) \]
Written out in full,
\[ A_{ab}(J) = \langle B_{Pa} B_{Pb} \rangle + \frac{\langle B_{Pa} \rangle \langle B_{Pb} \rangle \langle v \rangle \langle v \rangle}{1 - \langle v^2 \rangle} \]
\[ + \left[ \beta_{Pa}(0) + \langle B_{Pa} \rangle g + \frac{\langle B_{Pa} \rangle \langle v \rangle \langle v \rangle}{1 - \langle v^2 \rangle} \right] \]
\[ \times \left[ \frac{\rho(1 - \langle v^2 \rangle)}{(1 - \rho \langle g^2 \rangle)(1 - \langle v^2 \rangle) - \rho \langle gv \rangle^2} \right] \]
\[ \times \left[ \beta_{Pb}(0) + \langle B_{Pb} \rangle g + \frac{\langle B_{Pb} \rangle \langle v \rangle \langle v \rangle}{1 - \langle v^2 \rangle} \right] \quad (5.6') \]
The solution (5.6) appears to have poles at \( J = \alpha_p(0) \) coming from \( \rho \), at \( J = \alpha_g(0) \) coming from factors like \( \langle g^2 \rangle \) (in the case \( \alpha_p(0) < 1, \ g(0) \neq 0 \)), and at the value of \( J \) for which \( \langle v^2 \rangle = 1 \), coming from the \( C \)'s. In fact these are superficial and the amplitude is regular at these points. For example, the \( \rho(J) \) in the numerator of \( C \) is canceled by the same factor in the denominator, and similar cancellations occur in the other cases.
The actual singularities of the solution will be a real pole near $J = 1$, a branch point at $J = \alpha_c(0)$, plus an assortment of lower lying poles. To see this clearly, for the moment we simplify the denominator by dropping the $\langle gv \rangle^2$ and $\langle g^2 \rangle \langle v^2 \rangle$ terms. If $g$ and $v$ were proportional (i.e., had the same $t$ dependence) these two would exactly cancel each other, and their neglect is in any case a good approximation to the extent that $g$ and/or $v$ are small.

Multiplying through by $[J - \alpha_p(0)]$, we examine the denominator of $A_{ab}(J)$ in the form

$$D(J) = J - \alpha_p(0) - e^{\frac{2}{3}(\alpha_p(0) - J)} \langle g^2 \rangle \langle v^2 \rangle = [J - \alpha_p(0)] \langle v^2 \rangle.$$

If $\alpha_p(0)$ is near one and $g$ and $v$ are small we are guaranteed a single real zero of $D$ near $J = 1$ (but above $\alpha_p(0)$) which is identified as the pomeron pole. This follows since near $J = 1$ $D(J)$ is roughly a straight line through $\alpha_p(0)$, slightly perturbed downwards by $\langle g^2 \rangle$. There cannot be any other real poles near or above $J = 1$ since $D'(J) > 1$ there. The imaginary part of the denominator is proportional to

$$e^{\frac{2}{3}(\alpha_p(0) - J)} \langle g^2 \rangle \langle v^2 \rangle [J - \alpha_p(0)] t_p(J)$$

on the real axis below $\alpha_c(0)$, which does not vanish if $g \geq v$, and so any other zeros of $D$ must be complex. Since $D(J)$ is a real analytic function, these occur in complex conjugate pairs. The possibility of these complex poles is a consequence of the thresholds and the resulting $e^{-\Delta J}$ factors [21]. We can see this immediately if we neglect the $\langle v^2 \rangle$ term, for if $\Delta = \tilde{\Delta} = 0$ Im $D(J)$ is non-vanishing in the entire $J$-plane excepting the real axis above $\alpha_c(0)$.

We will study these matters in general in much more detail in the following paper.

In addition to the shift in position of the intercept of the leading pole from $\alpha_p(0)$ to $\alpha_p(0)$ there are related "renormalization" effects in the couplings. The residue of the pomeron pole in $A_{ab}(J)$ has the factorized form $F_a(0) F_b(0)$ where from (5.6)

$$F_a(0) = \left( F_a(0) + B_a g + C_a \langle gv \rangle \right) \frac{1}{J - \alpha_p(0)}$$

where $\alpha_p(0)$ is the residue of $\alpha_p$ at the pole. Since the second factor reduces to one plus quadratic terms in $g$ as $v$ and $g, v \to 0$, we have expressed $F_a(0)$ as $\beta_a(0)$ plus first order corrections. To find the full triple pomeron coupling, $g_p(t) = g_{pp}(t, t, 0)$, we require the residue of the pomeron pole in the pomeron-particle absorptive part. We consider all terms in the complete solution of the form

$$\frac{1}{16\pi} \int dt \frac{\Delta(2\alpha_p(t) - 1)}{J - \alpha_p(t) + 1} \left| t_p(t) \right|^2 B_a(t) F_b(J_1; t)$$

and identify

$$\lim_{J \to \alpha_p(0)} \left( J - \alpha_p(0) \right) f_a(J; t) = g_p(t) F_b(0).$$

This result is shown in fig. 5.6a and gives
\[ s_p(t) = \left[ g(t) + v(t) \frac{(gv)}{1 - (v^2)} \right] (\delta(t))^2_{J=\alpha_p(0)} \]
\[ = g(t) + O(g^2, v^2). \]  

(5.11)

Note that the \( C_{ab} \) term in the complete solution does not contain the pomeron pole and does not contribute to \( s_p \). If \( g \) and \( v \) vanish at \( t = 0 \), then (5.11) insures that \( s_p \) also vanishes.

Using the methods of section 4 we can calculate the discontinuity across the two-pomeron cut; this is carried out in Appendix B and the result is

\[ \text{disc } A_{ab}(J) = \pi \frac{1}{16\pi s_0} \int_{-\infty}^{\infty} dt (J - 2\alpha_p(t) + 1) |\xi_p(t)|^2 N_{Pa}(J;t)N_{Pb}(J;t) \]

(5.12)

where the \( N's \) are the full pomeron-particle absorptive parts, given by the sum of figs. 5.6a and b:

\[ N_{Pa}(J;t) = B_{Pa}(t) + C_a v(t) + \left[ \beta_{Pa}(0) + \langle B_{Pa} g \rangle + C_a \langle gv \rangle \right] \]

\[ \times \left[ g(t) + C \langle gv \rangle v(t) \right]. \]  

(5.13)

Formula (5.12-13) is the same result as Abarbanel's [14], as expected.

In the case where \( g(0) \) and \( v(0) \) are nonzero, the loop integrals diverge logarithmically as \( J - \alpha_p(0) \),

\[ \langle g^2 \rangle = \frac{g^2(0)}{2\alpha_p} \log(J - \alpha_p(0)). \]

Thus the cut discontinuity vanishes at the tip. In the case of \( g(0) = v(0) = 0 \), the loop integrals are finite as \( J \to \alpha_p(0) \) and

\[ N_{Pa}(J;t) \to B_{Pa}(t). \]  

(5.14b)

This result can be understood from the fact that the terms in the series are constructed so that zero-mass pomerons decouple from all but elastic and quasielastic vertices. The discontinuity is nevertheless reduced in strength from the AFS value away from the tip (see also the following paper). This is an interesting illustration of the Bronzan-Jones theorem [23] on the softness of Regge cuts. If \( \alpha_p(0) < 1 \) the various singularities of \( A_{ab}(J) \) are at isolated points at \( t = 0 \), and we have a soft cut. If \( \alpha_p(0) = 1 \), however, the pole and cut collide at \( t = 0 \) and the theorem need not apply [24]. In this case, having been forced into a new parametrization of the couplings by the Froissart bound, we have a hard cut.

Since the cut discontinuity (5.12) is positive definite if particles a and b are the same, it is at first somewhat mysterious that the truncated series can have an effectively negative cut in some ranges of energy. The resolution of this paradox lies in the complex poles; these appear only in the complete solution and are responsible for the latter exhibiting the threshold structure in the cross

\[ N_{Pa}(J;t) \to B_{Pa}(t) - \frac{B_{Pa}(0)}{v(0)} v(t) + (\text{const.}) g(t) \frac{g(0)}{v(0)} v(t) \]

\[ + \frac{1}{\log(J - \alpha_p(0))}. \]  

(5.14a)
section [22]. A pair of complex poles at $J = \alpha, \alpha^*$ with residues $\beta, \beta^*$ contribute

\[ \frac{\alpha}{J - \alpha} + \frac{\beta^*}{J - \alpha^*} \]

to $A_{ab}(J)$ and

\[ 2|\beta| \frac{\text{Re}(\alpha)}{s} \cos \left[ \text{Im}(\alpha) \cdot \log \frac{s}{s_0} + \text{arg}(\beta) \right] \]

to $A_{ab}(s)$. It is the oscillation due to this term which allows the complete solution to exhibit the threshold effects of the individual terms in the series.

In Appendix C, we give numerical estimates for the various renormalization effects and the energy dependence of the total cross section.

6. DISCUSSION

We have given a simple formalism for particle production, based on embedding the two-component model in a decomposition of multiparticle production processes according to large final state rapidity gaps. The key assumptions in this work are that a fireball with no large rapidity gaps is Regge-like behaved and that it is sensible to think of "external"-pomeron scattering amplitudes which are qualitatively similar to ordinary scattering amplitudes. The resulting perturbation series for the pomeron in powers of (small) pomeron absorptive parts provides a framework in which one may take simple $J$-plane structures as a reasonable starting point and exploit the weakness of pomeron couplings to improve the description. This paper has emphasized the physical origin of the terms in the series and the interplay between the separate terms and their sum.

In the course of our arguments we have also made a variety of technical assumptions on factorizability, $t$-dependence, and so on, to simplify the labor involved. In most cases these assumptions involved quantities whose detailed behavior is not well-known experimentally or theoretically, and our choice of parametrization is the simplest reasonable one. If significant nonfactorizable contributions are required, the present algebraic formalism is not immediately applicable, but the integral equations (3.6) may be expected to give physically similar results.

In the following paper we examine the present model in more detail using some algebraic simplifications, but there are several conclusions and remarks which can be given at this stage.
1. The SRC component of total cross sections cannot be exactly constant in energy but must fall as a small power of $s$, where the power is controlled by integrals over pomeron couplings.

2. The pole and cut properties of the two-component model are modified by the presence of high mass diffraction. Factorization should hold to a better accuracy than one would suppose from the relative amounts of diffraction and SRC.

3. The total cross section may exhibit oscillations associated with the thresholds for fireball production, which can be understood in terms of complex Regge poles. $c^{\text{tot}}(\infty)$ should be either a constant or zero. From the numerical results in Appendix C, we are skeptical about the relevance of these oscillations to rising cross sections.

4. The presence of complex Regge poles confuses the question of the sign of the two-pomeron cut (see refs. [10], [14], [25]). Phenomenologically a positive cut plus a set of complex poles may be indistinguishable from a negative cut plus a different set of poles. We may have here a possible explanation for the widely-publicized disagreement about sign.

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APPENDIX A: Phase Space

To facilitate the $J$-plane analysis of section 3 we require a convenient form for the final state phase space element. Referring to fig. 3.1, we let $K_i$ be the total four-momentum of the $i$th fireball and $P_i$ the total four-momentum of the collection of particles with $(\text{mass})^2 s_i$. Then

\[ d\theta(N\text{-fireball}) = \frac{1}{2} (2\pi)^{n-4} \delta^4(p_a + p_b - \sum_{i=1}^{N} \sum_{j=1}^{n_i} p_{ij}) \]

\[ \times \left[ \frac{1}{2} (2\pi)^{-2} \delta^4(p_a + p_b - K - K_1) d^4 K_1 \delta^4(K_1^2 - u_1) \right. \]

\[ \times \left. d^4 p_2 \delta^4(p_2^2 - s_2) \right] \frac{1}{2} d u_1 d s_2 \]

\[ \times \left[ \frac{1}{2} (2\pi)^{-2} \delta^4(p_a - K_1 - P_j) d^4 K_1 \delta^4(K_1^2 - u_1) \right. \]

\[ \times \left. d^4 P_j \delta^4(P_j^2 - m_{2j}^2) \right] \frac{1}{2} d u_2 d m_{2j} \]

\[ \times \left[ \frac{1}{2} (2\pi)^{-2} \delta^4(p_a - K_2 - P_j) d^4 K_2 \delta^4(K_2^2 - u_2) \right. \]

Continued
Each factor in square brackets is the phase space element $d\phi(n_i)$ for the $n_i$ particles in the $i$th fireball, while each factor in braces is a quasi-two body phase space element whose high energy form is

$$\frac{1}{2} (2\pi)^{-2} \delta^4 \left( P_1 - K - P_{i+1} \right) d^4 K_1 \delta^+(K_1^2 - c^2_{1}) d^4 P_{i+1} \delta^+(P_{i+1}^2 - s_{i+1}^2)$$

all with the correct factors of 2 and $\pi$. (The $\omega_i$ are the azimuthal angles in the quasi-two body center-of-mass system, and if the orientation of the latter is chosen in accord with the conventions of ref. [26], the $\omega_i$ are Toller angles.) Thus

$$d\phi(N\text{-fireball}) = \frac{dt_1 \, dw_1}{32\pi^2 s_1} \cdot \frac{1}{\pi} \, du_1 \, ds_1 \cdots \cdot \frac{1}{\pi} \, du_N \, ds_N.$$

In order that the couplings in the text agree with popular convention, a small change is required. Note that the phase space element for a single particle plus a cluster of $n$ may be written

$$d\phi(1+n) = \frac{1}{8} \, (2\pi)^{4-3(n+1)} \delta^h \left( P_a + P_b - q - \sum_{l=1}^{n} P_l \right) d^h q$$

$$\times \delta^+(q^2 - m^2) \int_{1}^{n} d^4 P_1 \, \delta^+(P_1^2 - m_1^2)$$

This gives rise, as in section 3, to a triple Regge expansion

$$s^2 \frac{d\sigma(ab - cx)}{dt \, ds'} \sim \left[ \frac{1}{\pi} \right] \frac{1}{16\pi} \beta_{ac} \beta_b \frac{1}{2} g_R(t) \, A_R \left( s' \right)$$

$$= \left[ \frac{1}{\pi} \right] \frac{1}{16\pi} \beta_{ac} \beta_b \frac{2}{2} g(t) \, A_R \left( s' \right)$$

The first factor of $1/\pi$ is usually not written and so is in effect absorbed into the $R-b$ absorptive part and thence into $g(t)$.

Similarly, in a two fireball process (as in fig. 1.1f) the conventional di-triple Regge expansion associates the same factor with each reggeon-particle absorptive part. The effect of this is that triple Regge couplings defined in the popular expansion are $1/\pi$ times those which
would arise from an absorptive part with the "correct" factors of \( \pi \).

We wish to use the popular couplings, so we associate an extra factor
\( 1/\pi (1/\pi^2) \) with each pomeron-particle (pomeron-pomeron) phase space.
This corresponds to the replacement

\[
\frac{1}{\pi^2} \, du_1 \, ds_{1+1} \rightarrow du_1 \, ds_{1+1}
\]

in (A.2) and leads to (3.2).

APPENDIX B: The Cut Discontinuity

We wish to show that the discontinuity of (5.6) across the two
pomeron branch cut is given by (5.12-13). It is convenient to do this
by working backwards; consider

\[
X = \int \frac{dt}{16\pi s_0} \, \frac{\pi^*}{\pi} \, (J; t) \, \frac{B_{PB}(J; t) \delta(J - 2\alpha_p(t) + 1)}{1 - (\nu^2)}
\]

\[
= \int \frac{dt}{16\pi s_0} \, \delta(J - 2\alpha_p(t) + 1) \left\{ \begin{array}{c}
P_a(t) \, B_{PB}(t) \\
+ \frac{(B_{PB}, v)^*}{1 - (\nu^2)} \cdot v(t) \, B_{PB}(t) + B_{PB}(t) \, v(t) \, \frac{(B_{PB}, v)}{1 - (\nu^2)} \\
+ \frac{(B_{PB}, v)^*}{1 - (\nu^2)} \cdot v^2(t) \, \frac{(B_{PB}, v)}{1 - (\nu^2)} \\
+ \left[ (B_{PB}(t) + \frac{(B_{PB}, v)^*}{1 - (\nu^2)} \cdot v(t)) \cdot (g(t) + \frac{(g, v)}{1 - (\nu^2)} \cdot v(t)) \\
\cdot g \cdot R_b \right] + [a \leftrightarrow b]^* + \left[ R_a^* \, g^* (g^2(t) \\
+ \frac{(g, v)}{1 - (\nu^2)} \cdot v^2(t) + \frac{(g, v)^*}{1 - (\nu^2)} \cdot g(t) \cdot v(t) \\
+ g(t) \cdot v(t) \, \frac{(g, v)}{1 - (\nu^2)} \cdot g \cdot R_b \right]
\end{array} \right.
\]

Now using

\[
\text{disc}(xy) = \int \frac{dt}{16\pi s_0} \, x(t) \, y(t) \, \delta(J - 2\alpha_p(t) + 1)
\]
which follows from the definition (3.9b) and (4.6), and (4.8) together with the fact that \( \langle \cdots \rangle \) defines a real analytic function of \( J \), we may identify the first term in square brackets above as

\[
\text{disc} \left[ \frac{(B_{Ra} R_{Pb})}{1 - (v^2)} \right] = \text{disc} C_{ab}.
\]

Similarly the second term in brackets is

\[ (\text{disc } R_a) \cdot G \cdot R_b, \]

the third is

\[ R_a^* \cdot G^* \cdot (\text{disc } R_b), \]

and the fourth is

\[ R_a^* \cdot \text{disc } G \cdot R_b, \]

and thus

\[
X = \text{disc}(C_{ab} + R_a G R_b)
\]

\[ = \text{disc} A_{ab}. \]

APPENDIX C: Numerical Estimates

We wish to give some rough numerical estimates for some of the quantities described above. These should be regarded with caution for two reasons: (1) many of the parameters entering into our formulae have not been accurately determined from experiment yet, and (2) a number of algebraic simplifications have been made above, especially with respect to factorization. Despite these reservations, we feel the estimates are adequate for illustrating qualitative aspects of the formalism.

First we require the values of the external residue functions, for which we use data at pre-ISAR energies. For p-p scattering we have [6] \( \sigma_{tot} \approx 38 \text{ mb} \), with the two components \( \sigma_{SRC} \approx 26 \text{ mb} \), and \( \sigma_D \approx 12 \text{ mb} \). This gives \( \beta_{ppp}(0) \approx 9.8 \) and \( \beta_{ppp}(0) \approx 8.1 \). From factorization and \( \sigma_{et} \approx 6 \text{ mb} \), the single diffraction dissociation cross section is \( \sigma_{SDD} \approx 0.41 \sigma_{et} \). For the coupling of the pomeron to a low mass fireball we use

\[
\beta_{ppp}(t) = \frac{1}{s_0} \sum_p \beta_{ppp,p}^2(t) \approx \frac{1}{s_0} \beta_{ppp}^2(0) e^{bt} \left( 1 + \frac{\sigma_{SDD}}{\sigma_{et}} \right)
\]

with a uniform slope parameter \( b \approx 4 \text{ GeV}^{-2} \). We use thresholds of \( \Delta = \tilde{\Delta} = 3.0 \).

The pomeron trajectory is written as \( \alpha_{p}(t) = 1 + 0.25t \), where the intercept has been chosen at one largely for definiteness. The internal couplings are taken as \( v \approx 0 \) and \( g(t) \approx g_p(t) = -s_0 t e^{bt} \), with \( s_0 = 3 \text{ GeV}^{-4} \) and \( b = 1 \text{ GeV}^{-2} \). (Departures from \( g \approx g_p \) are of higher order.) Since we are for the most part concerned with integrals over \( g \) and \( \alpha_p \), the results should not depend critically
on the specific parametrization used. From (5.9) the shift in pomeron intercept is
\( \alpha'_p(0) - \alpha_p(0) \approx 0.027 \), corresponding to a nearly constant SRC cross section. From (5.11) the renormalized triple-pomeron coupling is
\( g_p(t) \approx -2.5 t e^t \) and the renormalized pomeron-proton residue is \( \beta_{pp}(0) = 7.7 \), corresponding to an asymptotic total cross section of \( \approx 24 \text{ mb} \). At presently accessible energies, the more relevant corrected residue is that of (the real part of) eq. (4.5), which gives \( \beta_{pp}(p) \approx 10.6 \). Thus the factorizable component of \( \sigma_{\text{tot}} \) has been enhanced by high mass diffraction above \( \sigma_{\text{SRC}} \). The leading pair of complex poles occurs at \( J = 0.41 \pm 0.27 \).

The behavior of the total cross section as a function of energy is shown as the solid line in fig. C.1. The effect of high mass diffraction has been to make \( \sigma_{\text{tot}} \) level off, as in ref. [10]. One might think that increasing \( g \) would produce a rise, but this is not the case. The dotted line in fig. C.1 results from using the same parameters as above with \( g(t) \) doubled. The problem is that increasing \( g \) also increases \( \alpha'_p(0) - \alpha_p(0) \); since \( \alpha'_p(0) \leq 1 \) an increase in \( g \) merely lowers \( \alpha'_p(0) \), so the SRC component falls faster.

These matters are discussed at length in the following paper. If we use the "realistic" pomeron-proton cross section of fig. 2.1, the result is similar, except that \( \sigma_{\text{tot}} \) tends to flatten at lower energies (\( \log s \approx \Delta \)), with a smoother transition.

This curve bears little resemblance to the observed p-p total cross section at ISR energies [27], and we are led to conclude that, at least within the present approach and approximations, high mass diffraction is not responsible for rising cross sections. This point is discussed further in the following paper.

REFERENCES

[15a] D. Leith, review talk to appear in ref. [8].


TABLE CAPTION

Table I. Processes occurring in the fireball series.

(a) Elastic and quasi-elastic scattering.
(b) Particle a coupled to a high-mass fireball.
(c) Low-mass diffractive excitation of particle a, adjacent (in rapidity) to a low-mass fireball.
(d) Low-mass diffractive excitation of particle a, adjacent to a high-mass fireball.
(e) High-mass fireball.
(f) Pomeron exchange separating two high-mass fireballs.
(g) Pomeron exchange separating two low-mass fireballs.
(h) Pomeron exchange separating one high- and one low-mass fireballs.
FIGURE CAPTIONS

Fig. 1.1 Fireball decomposition of production amplitudes.

Fig. 1.2 Fireball decomposition of the elastic absorptive part.

Fig. 2.1a Energy dependence of a single-fireball pomeron-particle absorptive part.

Fig. 2.1b An idealized form.

Fig. 3.1 Kinematics of an N-fireball process.

Fig. 3.2 Integral equations for various absorptive parts.

Fig. 3.3a The (single-fireball) pomeron-pomeron absorptive part.

Fig. 3.3b The (single-fireball) pomeron-particle absorptive part.

Fig. 5.1 Sum of high-mass fireballs.

Fig. 5.2 Denominator of eq. (5.1).

Fig. 5.3 Sums of low-mass fireballs.

Fig. 5.4a Diagramatic sum for $G_0(J)$.

Fig. 5.4b Equation for $G_0(J)$.

Fig. 5.5 The complete two-particle absorptive part.

Fig. 5.6 The complete pomeron-particle absorptive part.

Fig. C.1 p-p total cross section as computed from the fireball series.
Fig. 3.1
Fig. 5.2
Fig. 5.6a
Fig. 5.6b
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